

Ganiti (Mathematicians) by Achyut Godbole

[1] Ancient Mathematics

- **Number Systems:**
 - Base 5 (Quinary): Easy to use with just one hand
 - Base 60 (Sexagesimal): Sumerian/Babylonian
- **Egyptians:** Trigonometry, Ratios, π (Pyramid)
- **Arabic:** Al-gebra (without negatives), Astronomy (planet positions to find *namaaz* times)
- **Omar Khayyám:** Roots of Cubic Equations, Non-Euclidean Geometry

[2] Indian Mathematics: **Salvsutra** to **Aryabhat**

- Prospered due to 2 *vedanga(s)*, *gyotish* and *kalp*
 - Trigonometry (*shalva-sutra*: construction of various shapes of *yadnya* platform)
 - Astronomy (ominous planetary positions)
 - Measurement units (fingers, human height)
 - Pythagoras theorem, π (Circle), *bakshali* (Roots)
 - $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{(3 \times 4)} + \frac{1}{(3 \times 4 \times 34)} = \frac{577}{408} = 1.4142$
- **Jains:** π (Circle), Permutations-Combinations
- **Vedic:** 16 *sutra(s)* by *swami bharati krishna teerth* for arithmetic, algebra, geometry, calculus, astronomy etc
- **Aryabhat:** 33 verses from '*aryabhatiya*' on roots, trigonometry, series, terse numbering vocab etc. His $\pi = \frac{(104 \times 8) + 62000}{2000} = \frac{62832}{2000} = 3.1416$

[3] Indian Mathematics:

Brahmagupt and *shunya*

- Buddha numbers: *talakshana* = 10^{21} , 0.11 nm
- *sankheda* copper plate: first *shunya* in place of nothing
- **Brahmagupt:** Universe started with *shunya*. Cycles after 9 with 1 and *shunya*. Laws of *shunya* in *brahmsphut-siddhant*.
 - $y + 0 = y$ (Additive Identity)
 - $y - 0 = y$ (Subtractive Identity)
 - $y \times 0 = 0$ (Multiplicative Identity)
 - $y \div 0 = 0$ (Divisive Identity) **WRONG!!**
 - Gave rise to negative numbers
 - Cyclic Quadrilateral: perimeter of circle is $\frac{1}{2} \sum \text{sides}$
- **Varahmihir:** Mathematical Astronomer, Gravitational force, Trigonometry
- Fibonacci spread Indian Numbers (book "Liber Abaci")

[4] Indian Mathematics: **Bhaskaracharya** and others

- **Mahaviracharya:** Pure Mathematics, *ganit sar sangrah*, Geometric Progression, Geometry, LCM
- **Bhaskaracharya:** *goladhyay*, *siddhant shiromani*, *leclavati*
 - Laws of Positive-Negative numbers, 6 laws of '0'. Divide by '0' is termed indefinite **CORRECT!!!**
 - Trigonometry, Limits in Calculus, π
 - Indeterminate equations of first degree (e.g. $2x = y$)
- **Madhvacharya:** Gregory-Leibniz series for π , Newton power series, Taylor series for sine-cosine
- **Neelkanth:** $\pi = \frac{2827433388233}{900000000000}$, Eclipse timings, Differentiation

[5] Greek Mathematics: **Thales** and **Pythagoras**

- **Thales:** Geo (land)-Metry(Measurement), Similar figures
- **Pythagoras:** 350+ ways to prove Pythagoras theorem
 - Childhood: Hexagons fit well, Earth is round
 - Figurate numbers, Discrete and Continuous Mathematics, $\sqrt{2}$ is not-a-number/irrational number
 - Own cult, philosophy
 - Dodecahedron (12 sided), Icosahedron (20 sided)
 - Proof needed for any theorem

[6] Greek Mathematics: **Zeno** and others

- **Zeno:** 4 paradoxes (Achilles and Tortoise), Determinants.
- **Hippocrates:** *Stoichia* (The Elements) on Geometry, If $\frac{a}{b} = \frac{b}{c}$ then $b^2 = ac$
- **Plato:** disciple of Socrates, bifurcation of the World into Things and Ideas, body and soul (mind) are separate, body dies but not the soul, agreed to irrational numbers like $\sqrt{2}$ going against Pythagoras
- **Eudoxus:** Surds (irrational numbers) acceptable, Astronomy, Validation/Proof by observations is necessary
- **Aristotle:** Disciple of Plato and teacher of Alexander, started Logic, Deductive reasoning, Philosophy, Ethics, Politics, Aesthetics, Physics, Astronomy, Botany, Zoology, Meta-Physics.

[7] Greek Mathematics: **Euclid** and geometry

- **Euclid:** Compiled "The Elements" - still a bestseller, 23 definitions of point-line-surface-plane etc, geometric postulates, 465 theorems, practical usage of geometry, Arithmetic, Perfect numbers (sum of divisors is same as the number itself), GCD (Greatest Common Divisor) algorithm, Encyclopedia of Mathematics

- **Eratosthenes:** Geography, Longitude-latitude, detection of Prime numbers, Earth's circumference,

[8] Greek Mathematics: **The Great Archimedes**

- **Archimedes:**
 - Eureka!!! (weight of the object is weight of the volume of the displaced water), Levers ("Give me a place in the Universe and I will lift the whole Earth"), Archimedian Screw to lift water
 - Started Applied Mathematics, notion of infinity (∞), "The Method" book, idea of thought experiments
 - "The Method of Exhaustion" (faceting circle for finding area), Limits (Tangent is Limit of Secant-Chord), Integration over curve (area of embedded stripes)
 - Ratio of volume of Cylinder and its maximal Sphere is $\frac{3}{2}$
 - Warfare: Catapult, Sun-rays reflectors, cranes
- **Apollonius:** Sections of a Cone (Conics) - Circle, Ellipse, Parabola, Hyperbola
- **Hipparchus:** Father of Trigonometry, Sin-Cos-Tan charts, Eclipse charts

[9] Arithmetics

- Number system with *base* = 10, popular due to fingers.
- Binary (*base* = 2) is popular in Computers.
- **Types of Numbers**
 - *Natural Numbers* like 1, 2, 3...
 - *Negative Numbers* : bigger Natural number - smaller one.
 - *Integers* or *Whole Numbers* : +/- Natural Numbers and 0
 - *Fractional Numbers* came due to Division.
 - *Rational Numbers* = *Whole* + *Fractional Numbers*.
 - *Irrational Numbers*: Cannot be expressed as ratio.
 - * *Algebraic* (roots of polynomial like $\sqrt{5}$)
 - * *Transcendental* (like π, e)
 - *Real Numbers* = *Rational* and *Irrational Numbers*
 - *sqrtNegative* gave rise to *Imaginary Numbers* using by $i = \sqrt{-1}$. *Complex Numbers* has *Real* and *Imaginary* components.

[10] Algebra

- “Arithmetica” by Diaphanous, 189 questions (infinite solutions of indeterminate equations)
- Number of variables = number of simultaneous equations
- Linear Equation, max power of 1, 2 for “Quadratic”, 3 for “Cubic”, 4 for “Quartic” and 5 for “Quintic”.
- “Quadratic” has two roots (solutions), “Cubic” has 3 and
- Brahmagupt, Bhaskaracharya had solutions to Quadratics.
- “Algebra” comes from Arabic *al-jabr* meaning “reunion of broken parts”
- Scipione del Ferro discovered method to solve cubic equations
- Mathematical evolution: fractions were discovered while working on Linear equations, negative numbers while on quadratics, complex numbers while on cubics/quartics and group theory for quintics

[11] Napier and Logarithms

- Mechanical calculators mentioned in his book “Rabdology”
- Got idea of Logarithms from Dr Craig while he was explaining Prosthaphaeresis (multiplications and divisions in terms of additions and subtractions respectively). “Logos” means ratios and “Arithmos” means numbers. He used e as base.
- $10^2 = 100 \therefore \log 100 = \log 10^2 = 2$
- $\log(a \times b) = \log a + \log b$
- Jost Bürgi, the watchmaker, had independently put forth idea of Logarithms

[12] René Descartes : Dawn of a new Era

- Credited with discovery of Coordinate system
- Unification of Algebra and Geometry by having equations for figures
- Called study of Optics, Astronomy, Mechanics, Hydrostatics as “Physio Mathematica”
- Out of 10 laws of the Universe he proposed, two were similar to Newton’s laws of motion.
- “Father of modern Philosophy”. Gave birth to a branch called “Epistemology” in his book “Meditations on first Philosophy”. *cogito ergo sum: I think therefore I am.*
- In solitude he wrote “Traité du monde et de la lumière”(“The World”), based on all his work.

[13] René Descartes : Coordinate Geometry

- Cartesian Coordinate system: locate a point using distances from two orthonormal axes
- Curves defined by equations of coordinate variables. Putting $x = 1, 2, 3, 6$ equation $y = 6/x$ gives $y = 6, 3, 2, 1$. Plotting gives the *curve*

- Intersection point can be found by solving simultaneous equations of the curves.
- Although most of his opinions were against the church, he did not get harassed due to support by Prince of Orange.
- His disciple, princess Elizabeth solved problem of “Kissing Circles” using Analytical Geometry.

[14] Pascal: Conics and Calculators

- Founded Projective Geometry’, Probability.
- Studied Conics and made calculator.
- Independently proved many of Euclid’s theorems at the tender age of 12.
- At 16, wrote about 400 theorems on Conics.
- Mystic Hexagon: Have 6 points on circumference of a circle (or any conics!!), extend opposite sides, all three intersections are collinear.
- Invented Hydraulic Press, Sewing needle, Calculator.
- Experimentally proved presence of vacuum. Unit of pressure is thus called by his name.
- For favorable odds at betting, Probability got discovered.
- “Pascal’s Triangle”. Coefficients of Binomial Equation.
- Cycloid: Path/Loci traced by a point on a rolling circle.
- Reverse Cycloid can be used as arcs as stronger structure

[15] The one who puzzled the world, Fermat

- Contemporary to Descartes and Pascal, he had independently discovered Coordinate Geometry.
- One of the founders of the “Calculus”. Found a simple method the tangent of a curve, as limiting value.
- Found method for maxima and minima

[16] Fermat’s last theorem

- His partial solutions written in the margins of revered ‘Arithmetica’ (by Diaphanous) are invaluable today.
- Postulated that prime numbers can be generated by $f_n = 2^{2^n} + 1$ where $n = 1, 2, 3 \dots$
- Euler proved that it fails at $n = 5$
- “Little Theorem”: $a^p - a$ is divisible by p , where a is natural and p is prime number. Modular mathematics.
- Found pair of “Amicable Numbers”, 220, 284. Sum of divisors is the other number. Symbol of love.
- $a^n + b^n = c^n$. Easy to get triplets of positive integers for $n = 1, 2$ but not for higher values. He had (for $n = 4$) but did not disclose the proof. Later Euler proved it for $n = 3$. Later some did it up to 4 million.
- In 1994 Andrew Wiles submitted the proof which was checked for 2 years. On 27 June 1997 he was awarded.

[17] Newton: Binomial theorem

- Newton dethroned Aristotle’s ideas by his “Force”.
- Proposed 3 laws of motion for prediction of movements
- It was now possible to chart trajectory of falling stone or leaf by summation of forces.
- Gravitational force does not let the Moon escape its orbit around the Earth.
- Invented “Calculus” to represent the laws of motion.
- “Binomial Theorem”: Coefficients as in Pascal’s triangle. $(a + b)^n = C_0^n a^n + C_1^n a^{n-1}b + C_2^n a^{n-2}b^2 + \dots + C_n^n b^n$

[18] Newton: The beginning of Calculus

- Debate between Leibniz (France) and Newton (English) on Calculus. Blown out of proportion to an extent, where England went much better Leibniz way for 100 years.
- Newton solved Bernoulli’s famous “Brachistochrone”(Find Curve on which rolling ball come in minimum time) problem in no time, sent anonymously.
- At 75, solved Leibniz’s touch question in just few hours.

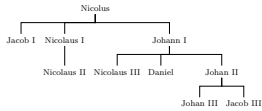
[19] Sidelined Leibniz

- Leibniz had much better-simpler Calculus, good notations, but Newton was made immortal by British.
- Wrote “De Arte Combinatoria” for Combinatorial Analysis. Permutations and Combinations.
- First steps in “Artificial Intelligence”. Complex thought can be put forth using small-simple equations.
- Calculator (“Leibniz’s Wheel”) could do multiplications, divisions and square-root also.
- Worked in many fields such as Optics, Mechanics, Statistics, Logic, Probability, History, Law and Politics.
- Follower of “Rationalism” with Descartes and Spinoza.

[20] Calculus

- Derivative: Interval closes to get the tangent (instantaneous rate of change) at a point. $\lim_{x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$
- Integration: Area under curve summation of strips with width tending to 0. $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int f(x) dx$
- Helped Physics, Electromagnetism, Conduction of heat (Boundary value problems), Economics (inflation) etc.

[21] Bernoulli : A Mathematics Family



- 8 mathematicians in the family. 120 disciples including Euler.
- Jacob I solved problem of Isochrone curves. On this curve, ball from any height comes down in the same time. Cycloid takes min time than plane. Parabola takes most.
- He also found “Separation of Variables” in Integration, Curve of a Sagging chain (Catenary), “Logarithmic Spiral” ($r = e^{a\theta}$),
- Johann I was paid for a rule known as L’Hospital’s rule.
- Daniel was father of “Mathematical Physics” and worked on Hydraulics (Conversation of Energy), Theory of Vibrating Strings, Kinetic Theory of Gases etc.

[22] Euler and Graph Theory

- Breathed Mathematics. Wrote beautifully and poetically.
- Number Theory, Mathematical Analysis, Astronomy, Light-Sound, Cartography, Graph Theory, Topology ...
- Disciple of Johann I Bernoulli. Later overtook him.
- Proved that: In a network, with ≥ 2 odd vertices, its impossible to return to the original point without crossing any edge twice. Possible for < 2 . Königsberg bridges have 4 odd vertices, so cant come back.
- Euler Path: Can trace edge only ones but need not come back. Possible for ≤ 2 odd points, but not for > 2 .
- Graph Theory: “Network formula” Invariant $v + f = e + 1$
- For Polyhedrons: $v - e + f = 2$. Birth of Topology.

[23] Euler: an Architect of Mathematics

- Wrote “The Elements of Algebra”, had Number theory.
- Euler’s constant e . Depicts Exponential Growth $y = e^x$.

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = 2.71 \dots$$
- Discovered $e^{i\pi} + 1 = 0$ and $e^{ix} = \cos x + i \sin x$
- Literally lived mathematics. Won Paris Academy price 12 times. But in turn lost eyesight of one eye.
- Wrote textbooks, on tides, chemistry, geography and also Mathematics in Music. About 75 volumes of 300-600 pages each. Plus 3000+ pages notes. $\frac{1}{3}$ of all the Mathematics, Mechanics in last 75 years of 18th century.

[24] Lagrange: a Humble, Savant Mathematician

- “Calculus of Variations” (Geodesic: distance on a surface, Min-Max of functions etc), Number Theory, Differential Equations, Numerical Analysis (Bisection method).
- Found definitive solution to $x^2 - ny^2 = 1$ by method of “Continued Fraction”. One equation with two variables - not possible. For certain n you can get pairs of x and y .

[25] Lagrange and Laplace

- Stepping ahead of Newton and Leibniz (who used calculus to find rate of change), Laplace found “Differential Equations” (rate of change depends on variables e.g. $\frac{dy}{dx} = x$). $\frac{\partial x}{\partial y}$ for Partial Differential.
- “Treaties on Celestial Mechanics” is considered next part of the “Principia Mathematica”. Motion of planets.
- Laplace wrote “Meccanica Analitica” (Analytical Mechanics).
 $E_{total} = E_{potential} + E_{kinetic}$. Had 4th dimension as ‘time’. Hated geometry so mostly equations.
- Lagrange started decimal (Metric) system for measures.

[26] Monge: New Pathways in Mathematics

- Started “Projective Geometry”. For everyday use.
- Founded “Engineering Drawing”, “Differential Geometry”. Building machines, structures made possible.
- “Descriptive Geometry”: 2D projections/views of 3D.
- Wrote “Application de l’analyse á la géométrie”. Line on Curved Surface. Was later called as “Non-Euclidean Geometry” or “Riemann Geometry”.
- Worked on Matrices, which are used in “Finite Element Analysis”.
- Wrote “Art of Manufacturing Canon”!!

[27] Monge-Fourier: Victims of King-friendship

- By 14, consumed all 6 volumes by Bézou. Won prizes.
- Changed Education system which was based on cramming, reciting notes. Started Q&A dialogs in class.
- Wrote first paper “On Propagation of Heat in Solid Bodies”, reviewed by Monge, Laplace, Lagrange.
- In the book “The Analytical Theory of Heat” he mentioned that “any periodic function can be presented by sum of sines and cosines”, called “Fourier Series”.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$
 Used in Signal Processing, or any phenomenon showing periodicity.
- Representation of Discontinuous Curves by equations.

[28] Carl Gauss: The Emperor of Mathematics

- Considered equivalent to Archimedes and Newton.
- Topped 2000 years old Euclidean Geometry.
- Major mistake was that he kept his work a secret.
- Laid foundation of “Modular Mathematics” (Clock Mathematics, as, e.g. 16 hrs, is actually 4pm, with periodicity of 12 hours. $16 = 4 \pmod{12}$). Security apps.
- “Method of Least Squares” to fit a curve between points.
- Instead of predicting Primes, tried their occurrences in given ranges. From $1 \rightarrow n = \frac{n}{\log_e n}$

[29] Carl Gauss: A Scientist

- Predicted position of planet “Ceres Ferdinandea” using Newton’s “Inverse Square Law”. Became a star at 24.
- Napoleon spared Göttingen as Gauss was director there.
- Strong memory, had Logarithm Tables by heart.
- 4 Phases: Arithmetic till 1800, Astronomy up-to 1820, Geodesic and imaginary numbers till 1830 and Electromagnetism and light up-to 1840.

[30] Abel and Quintic Equations

- Died at tender age of just 26. Fall of a glorious star.
- Rewrote proofs by Newton, Euler, Gauss, Lagrange.
- Proved which Quintic will have “radical” solution (factorization possible). Galois group over rationals.
- Published 22 papers in first 3 editions of “Crelle” journal.
- Found integrations for Transcendental functions.
- Fourier opposed Able and Jacobi as their mathematics was not applied. But their Quintic is now used in Bose-Einstein Condensed state, Cosmology etc.

[31] Rebellious Galois and Group Theory

- Died at 21. “Remember my name!! My fate did not give me long enough life to do something extra-ordinary”
- At 15, mastered Calculus, Analytical functions like a pro
- Sensing his death he wrote all his invaluable mathematics. About 60 pages. Still not deciphered fully.
- A “Group” (denoted by set S and Operation \oplus) follows:
 - If $x, y \in S$ then $x \oplus y \in S$. E.g S has integers. $x = 2, y = 3$ and \oplus is Addition, then $2 + 3 = 5 \in S$
 - If $x, y, z \in S$ then $(x \oplus y) \oplus z = x \oplus (y \oplus z)$
 - S needs to have some Identity element e , so that $x \oplus e = x$. 0 for Addition, 1 for Multiplication \otimes .

- for each x there exists an inverse element x' so that $x \oplus x' = e$. $-x$ for Addition, $\frac{1}{x}$ for Multiplication.

- “All integers with Addition” is a group but “Integers from 1 to 10 and Multiplication” is not (Rule 1).

[32] Foundations of Non-Euclidean Geometry

- Ancient geometry by Babylonians, Egyptians, Indians was 2D-Planar. Measuring land, building pyramids etc.
- 13 volumes of “The Elements” by Euclid in 300 BC laid the foundation. Based 465 theorems on 5 postulates and 5 common notions.
 1. A straight line segment can be drawn between two points.
 2. Any straight line segment can be extended indefinitely.
 3. A circle can be drawn having the segment as radius and one endpoint as center.
 4. All right angles are congruent.
 5. If two lines with the sum of the inner angles less than 180, then they intersect if extended far enough.
- Euclid’s fifth postulate cannot be proven as a theorem, although this was attempted by many people
- On Curved surface, parallel lines meet. Sum of internal angles is more than 180, as each of base angle was 90.

[33] Lobachevsky and Bolyai: Pioneers of the new-age Geometry

- Lobachevsky changed Euclid’s 5th postulate to create self-consistent Non-Euclidean geometry
- Became director of Kazan Univ at a young age. Toiled to make it prosperous. Good administrator. Hard working.
- Bolyai wrote 26 page paper “Science of Absolute Space”
- According to Euclid, one line can be drawn parallel to a given line at a point outside. According to Lobachevsky and Bolyai, multiple and according to Riemann, none.
- Hermann Grassmann developed n -dimensional geometry.

[34] Riemann: Mathematical wealth of an impoverished

- Finished Legendre’s 859 pages book on numbers and logarithms in just 6 days. Could recollect after 2 years.
- Like Euclid, own definitions of point, line and plane.
- On Curved surface, line between two points, not unique.
- Distances between points depend on curvature of surface
- Metric Tensor for distances. Differential Geometry.

[35] Riemann, Hilbert and Hardy: Zeta functions

- Hilbert’s 23 “Unsolved Problems in Mathematics” had “Fermat’s Last Theorem” and Zeta functions were 8th
- $\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$, with $x = 1$ is a divergent series. Riemann found $\zeta(2) = \frac{\pi^2}{6}$. For $x = 4, 6$, in terms of π^4, π^6 . But could not still find $\zeta(3), \zeta(5), \zeta(7)$. Not for negative x .
- Hilbert’s doctoral thesis on “Theory of Invariants”. Redeveloped Euclid’s geometry to propose 21 hypothesis.
- Einstein got help from Hilbert in the “General Theory of Relativity” and never denied it also.
- At Trinity, Hardy and Littlewood wrote 100 papers.

[36] Poincaré and Topology

- Algebra, Analysis, Geometry, Astronomy, Mathematical Physics etc. 100s of volumes. Brought diagrams back.
- Doctorate under Hermite on Differential Equations.
- Descartes saw Geometry as Algebra, Poincaré, opposite.
- Topological equivalence is not on exact shape but connectivity, holes etc. Homeomorphism. Sphere-Cube, Cup-Donut are equivalent. Deformable. Rubber Sheet.
- “Ancient Utility Problem”: Connect 3 houses and 3 utilities without crossing. Not possible in 2D (provable by Jordan Curve theorem) but easier in 3D. Just lift.
- “Theory of Knots”: Started by Vandermonde as part of “Geometry of Positions”. Shapes, weights irrelevant.
- Tait’s Conjectures: Alternating knots with 7 crossings.

[37] Cantor: Set Theory and Infinity

- Infinity is an old concept. In “Yajurveda”: infinity remains infinity even after taking something out of it.
- Galileo’s Paradox: Whole numbers are infinite and seemingly higher Natural numbers are also infinite. “More” infinite? Bolzano’s “Paradoxes of Infinity”.
- Solved Trigonometry problem, left by Dirichlet, Riemann.
- Six papers on Set Theory in “Mathematische Annalen”.
- Equivalence of Sets: Cardinality, 1-to-1 Correspondence.
- Uncountable Infinite sets contain irrationals like $\sqrt{2}$

[38] Ramanujan: Cursed Angle

- Brought to light by Hardy, comparable to Newton, Euler.
- Got solutions in dreams. wrote them after getting up.
- Failed in College 4 times. Lost Scholarship. Ran away.

- Value of π upto 6 decimal places with $k = 0$ in $\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$. 14 places with $k = 1$.
- “Highly Composite”: more factors than any previous. 24. Found 102 till 6746328388800 with 269 equations.
- Youngest (age 31) to get fellowship of Royal Academy.
- “Congruence in the number of partitions” with Hardy.
- Taxi number 1729 is $12^3 + 1^3$ and $10^3 + 9^3$. Lowest such.

[39] Neumann, Nash and Game Theory

- Game Theory started by Émile Borel. Later developed by Neumann for gambling. Made famous by Nash (movie “Beautiful Mind”). Nobel for “Nash Equilibrium”.
- Contributed to Eniac computer. “Standard Programs”.
- Nash’s doctorate “Non-Cooperative Games”. 27 pages.
- Game theory applicable in interaction games like Poker, Bridge but not in independent games like Gambling.
- “Prisoner’s Dilemma”: Cooperation than selfishness.
- “Nash Equilibrium”: Strategies benefiting all, work.

[40] Women Mathematicians

- Theona: 6th century BC. Disciple and wife of Pythagoras.
- Hypatia: 4th century AD. Apollonius’s conics. Astronomical charts. Unmarried. Was killed brutally.
- Agnesi: Studied curve “versiera” aka “Witch of Agnesi”.
- Sophie Germain: Studied under male name “Antoine-August Le Blanc”. Lagrange become her mentor-philosopher. Vibration of Elastic Plates.
- Sonya Kovalevskaya: Doctorate in-absentia. Differential Equations. Saturn rings. Rigid body “Kovalvesky Top”.
- Emmy Noether: Applications of “Theory of Relativity”

[41] Statistics

- “There are lies, damn lies and Statistics”- Mark Twain
- First scientific study was by John Graunt, for epidemics. Study showed patterns, convincing of reason, not luck.
- Halley (Comet fame) synthesized data as Life Tables and found interesting inferences like 346 survived till 50 out of 710 alive at 6. Out of 242 alive at 60, 41 reached 80.
- Measurements like heights, weights, behavioral traits follow “Normal Distribution” (Bell or Gaussian Curve)
- Data characterized (summary) by Mean, Mode, Median
- Standard Deviation shows how data is away from Mean
- Correlation of two variables is none if 0, strong if 1 or -1

[42] Probability

- Chance-Likelihood of an event happening is Probability.
- Cardano wrote “On Casting the Die” on gambling
- “Problem of Points”: Pascal and Fermat found probability of a player winning at an intermediate stage.
- Kolmogorov formalized it by postulates and theorems
- Independent-Separable events like tossing a coin. Dependent are like blue-red ball example. Multiplying individual probabilities at each stage gives final value.
- Used in Insurance, Genetics, Mechanics, Physics etc.

[43] Logic

- Syllogism by Aristotle: Deduction by two propositions.
- Augustus De Morgan, George Boole, Gottlob Frege used operator notation in logic. Boolean Algebra - equations.
- AND (Intersection), OR (Union), NOT (Complement)
- Frege: “Axiomatic Predicate Logic”. Predicate are properties of subject. “is green” of a ball. Russell collapsed Frege’s 10 year work, by his paradox (Barber).

[44] Epilogue

- Ancient Mathematics: Arithmetic, Algebra, Geometry

- 15-17th century: Conics, Calculus, Logarithms
- 18th century: Graph Theory, Differential Geometry
- 19th century: Non-Euclidean Geometry, N-Dimensional
- 20th century: Topology, Chaos, Fractals, Probability
- Indians: Ashutosh Mukherjee (Algebraic Curves), Vishnu Narayana (Riemann Geometry), Pralhad Vaidya (Matrix - Astronomy), D D Kosambi (Tensor Analysis), Harish Chandra (Lie Groups), Shriram Abhyankar (Singularities), Mahalanobis (Statistics-Distance), Ravindra Kulkarni (Differential Geometry).