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Total No. of Questions : 9] [Total No. of Printed Pages : 4

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BCA (CBCS) RUSA IIIrd Semester Examination

4513

MATHEMATICS–III

BCA–0301

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Parts–B, C, D and E. Marks are indicated with questions for Parts–B, C, D and E.

Part–A

1. (A) (i) Write degree and order of the differential equation :

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \sin x$$

(ii) Modulus of a zero complex number is zero, but its magnitude is not defined.

(True/False)

(iii) The principal value of the amplitude of $x + iy$ is not necessarily the principal value of $\tan^{-1}\frac{y}{x}$ which lies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. (True/False)

(iv) Roots of $x^2 + 2x + 2 = 0$ are complex numbers. (True/False)

(v) Find modulus and argument of complex number $-\sqrt{3}+i$.

(vi) $(\cos\theta + i\sin\theta)^n = \frac{\cos\theta}{n} + i\frac{\sin\theta}{n}$, where n is a +ve integer.

(vii) If y_1 and y_2 are two solutions of second order differential equation, then their linear combination is also a solution. (True/False)

(viii) Even prime numbers are infinite. (True/False)

(ix) Let a, n $(n \geq 1)$ be any integers such that g.c.d. $(a, n) = 1$. Then $a^{\phi(n)} \equiv 1 \pmod{n^2}$. (True/False)

(x) The algebraic structure $(\mathbb{Z}_p, +_p, \cdot_p)$ is not a field for p prime number. 1×10=10

(B) (xi) Solve the differential equation :

$$\operatorname{sce}^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

(xii) Show that point representing the complex numbers $1 + i$, $3 - 3i$, $-3 + 9i$ lie on a straight line.

(xiii) Find g.c.d. (36, 45) and express it as linear combination of these numbers.

(xiv) Prove that every finite integral domain is a field.

(xv) Prove that $(\mathbb{Z}_5, +_5, \cdot_5)$ is a field. 4×5=20

Part–B 10 each

2. (a) Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$.

(b) Find the differential equation from the relation :

$$\alpha x^2 + \beta y^2 = 150$$

3. (a) Solve : $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+1)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

(b) Solve : $\frac{d^2y}{dx^2} - 4y = x\sin 2x$.

Part–C 10 each

4. (a) If n is an integer, then show that :

$$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\frac{n\pi}{6}$$

(b) Use de Moivre's theorem to solve the equation :

$$x^4 - x^3 + x^2 - x + 1 = 0$$

5. (a) State and prove de Moivre's Theorem.

(b) Express $\sin^7 \theta$ in terms of cosines or sines of multiples of θ.

Part–D 10 each

6. (a) Find the remainder when 2^{23} is divided by 29.

(b) If $a \equiv b \pmod{m}$, $c \equiv d \pmod{m}$, then prove that $ax + cy \equiv (bx + dy) \pmod{m}$ for $x, y \in z$, the set of integers.

7. (a) For $n = 5, 8, 12, 20$ and 25, find all +ve integers less than n and relative prime to n.

(b) Show that if a and b are positive integers, then $ab =$ l.c.m. (a, b). g.c.d. (a, b).

Part–E 10 each

8. (a) Prove that $(\mathbb{Z}_{11}, +_{11}, \cdot_{11})$ is a field.

(b) Show that $x^3 + x + 1$ is irreducible over GF(2).

9. (a) Find all nilpotent and idempotent elements of $(\mathbb{Z}_6, +_6, \cdot_6)$.

(b) Let F be a field such that $|F| = 4$. Then find irreducible polynomials over F of degree 2, 3 and 4.