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Total No. of Questions : 9] [Total No. of Printed Pages : 4

(1109)

BCA UG (CBCS) RUSA IIIrd Semester Examination

3600

MATHEMATICS–III

BCA–0301

Time : 3 Hours] [Maximum Marks : 70

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Part–B, C, D and E. Marks are indicated with questions for Part–B, C, D and E.

Part–A

1. (i) Write order and degree of the differential equation :

$$\frac{d^2y}{dx^2}+\sin\left(\frac{dy}{dx}\right)+y=0$$

(ii) Every differential equation with degree one is linear differential equation. (True/False)

(iii) If $f(x)$ and $g(x)$ are two solutions of second order differential equation then their linear combination is also a solution. (True/False)

(iv) Roots of $z^2 + 2z + 1 = 0$ are complex numbers. (True/False)

(v) Find modulus and argument of complex number $1 + i$.

(vi) $\sin^2 z + \cos^2 z = 1$ holds for all complex numbers. (True/False)

(vii) Prime numbers are finite. (True/False)

(viii) The congruence $3^{100} \equiv 1 \pmod{10}$ is not true. (True/False)

(ix) Algebraic structure $(\mathbb{Z}_n, +_n, \times_n)$ is field for every positive integer n. (True/False)

(x) Characteristic of finite field is not always prime number. (True/False)

1×10=10

(xi) Find the differential equation that will represent family of circle having centre at $(a, 0)$ and radius r.

(xii) Solve :

$$\frac{d^4y}{dx^4} - y = 0$$

(xiii) Find the real part of the complex number i^i.

(xiv) Find GCD of 39 and 45 and express it as linear combination of these numbers.

(xv) Find the unit digit of 7^{45}. 4×5=20

Part–B

10 each

2. (a) Solve :

$$\frac{d^2y}{dx^2} + 2y = e^x \sin x$$

$a^2 - b^2 =$

(b) Find the equation of family of the circle :

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

in the xy plane.

3. (a) Find the differential equation from the relation $ax^2 + by^2 = 1$.

$y = ax^2 + by^2$
$y' = 2ax + 2by\frac{dy}{dx}$
$y'' = 2a + 2b\frac{dy}{dx}$

(b) Solve :

$$\frac{d^3y}{dx^3} + \frac{dy^2}{dx^2} - \frac{dy}{dx} - y = \sin 2x$$

Part–C

10 each

4. (a) State and Prove De-Moivre's theorem.

   (b) Compute $(3 + 3i)^5$. $(3+3i))$

5. (a) Find the cube root of $z = -1 + i$. $x = (6)^{1/3}$ $z =$

   (b) Find the square root of $8 - 6i$. $x^3 - 1 = 0$

   $x^3 = 1$

Part–D

10 each

6. (a) Solve :

$$x^2 + 2x - 1 \equiv 0 \pmod 7.$$

   (b) Solve :

$$7x \equiv 2 \pmod{31}.$$

7. Find the smallest positive integers that when divided by 3, 5, 7 we get remainder 1, 4, 6, respectively.

Part–E

10 each

8. (a) Prove that $(\mathbb{Z}_7, +_7, \times_7)$ is a field.

   (b) Show that $x^2 + x + 1$ is irreducible over GF(2).

9. (a) In $(\mathbb{Z}_5, +_5, \times_5)$ solve : $2^{m-1}+1$

$$(x^4 + 4x^3 + 3x^2 + 2) + (2x^4 + 3x^3 + 4x + 4)$$

$$\text{and } (x^2 + 2x + 4).(x^2 + 3x + 3)$$

   (b) Find idempotent and nilpotent element of $(\mathbb{Z}_8, +_8, \times_8)$.

$x^3+x+1 \overline{) x^2 + 1}$

$x^2 + 1 + x$

x

$x^2+x+1 \overline{) x^3 + 1}$ x

$x^3 + x^2 + x$