Roll No. ....

Total No. of Questions : 9] [Total No. of Printed Pages : 4

(2111)

BCA (CBCS) RUSA IIIrd Semester Examination

4513

MATHEMATICS-III

BCA-0301

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Parts–B, C, D and E. Marks are indicated with questions for Parts–B, C, D and E.

Part–A

1. (A) (i) Write degree and order of the differential equation :

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\sin x$$

(ii) Modulus of a zero complex number is zero, but its magnitude is not defined.

(True/False)

(iii) The principal value of the amplitude of $x + iy$ is not necessarily the principal value of $\tan^{-1}\frac{y}{x}$ which lies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. (True/False)

(iv) Roots of $x^2 + 2x + 2 = 0$ are complex numbers. (True/False)

(v) Find modulus and argument of complex number $-\sqrt{3}+i$.

(vi) $(\cos\theta + i\sin\theta)^n = \frac{\cos\theta}{n} + i\frac{\sin\theta}{n}$, where n is a +ve integer.

(vii) If y_1 and y_2 are two solutions of second order differential equation, then their linear combination is also a solution. (True/False)

(viii) Even prime numbers are infinite. (True/False)

(ix) Let a, n $(n \geq 1)$ be any integers such that g.c.d. $(a, n) = 1$. Then $a^{\phi(n)} \equiv 1 \pmod{n^2}$. (True/False)

(x) The algebraic structure $(\mathbb{Z}_p, +_p, \cdot_p)$ is not a field for p prime number. 1×10=10

(B) (xi) Solve the differential equation :

$$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$$

(xii) Show that point representing the complex numbers $1 + i$, $3 - 3i$, $-3 + 9i$ lie on a straight line.

(xiii) Find g.c.d. (36, 45) and express it as linear combination of these numbers.

(xiv) Prove that every finite integral domain is a field.

(xv) Prove that $(\mathbb{Z}_5, +_5, \cdot_5)$ is a field. 4×5=20

Part–B 10 each

2. (a) Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$.

(b) Find the differential equation from the relation :

$$\alpha x^2 + \beta y^2 = 150$$

3. (a) Solve : $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+1)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

(b) Solve : $\frac{d^2y}{dx^2} - 4y = x\sin 2x$.

Part–C 10 each

4. (a) If n is an integer, then show that :

$$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$$

(b) Use de Moivre's theorem to solve the equation :

$$x^4 - x^3 + x^2 - x + 1 = 0$$

5. (a) State and prove de Moivre's Theorem.

(b) Express $\sin^7 \theta$ in terms of cosines or sines of multiples of θ.

Part–D 10 each

6. (a) Find the remainder when 2^{23} is divided by 29.

(b) If $a \equiv b \pmod{m}$, $c \equiv d \pmod{m}$, then prove that $ax + cy \equiv (bx + dy) \pmod{m}$ for $x, y \in z$, the set of integers.

7. (a) For $n = 5, 8, 12, 20$ and 25, find all +ve integers less than n and relative prime to n.

(b) Show that if a and b are positive integers, then $ab = \text{l.c.m.}(a, b) . \text{g.c.d.}(a, b)$.

Part–E 10 each

8. (a) Prove that $(\mathbb{Z}_{11}, +_{11}, \cdot_{11})$ is a field.

(b) Show that $x^3 + x + 1$ is irreducible over GF(2).

9. (a) Find all nilpotent and idempotent elements of $(\mathbb{Z}_6, +_6, \cdot_6)$.

(b) Let F be a field such that $|F| = 4$. Then find irreducible polynomials over F of degree 2, 3 and 4.

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BCA UG (CBCS) RUSA IIIrd Semester Examination

3600

MATHEMATICS–III

BCA–0301

Time : 3 Hours] [Maximum Marks : 70

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Part–B, C, D and E. Marks are indicated with questions for Part–B, C, D and E.

Part–A

1. (i) Write order and degree of the differential equation :

$$\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) + y = 0$$

(ii) Every differential equation with degree one is linear differential equation. (True/False)

(iii) If $f(x)$ and $g(x)$ are two solutions of second order differential equation then their linear combination is also a solution. (True/False)

(iv) Roots of $z^2 + 2z + 1 = 0$ are complex numbers. (True/False)

(v) Find modulus and argument of complex number $1 + i$.

(vi) $\sin^2 z + \cos^2 z = 1$ holds for all complex numbers. (True/False)

(vii) Prime numbers are finite. (True/False)

(viii) The congruence $3^{100} \equiv 1 \pmod{10}$ is not true. (True/False)

(ix) Algebraic structure $(\mathbb{Z}_n, +_n, \times_n)$ is field for every positive integer n. (True/False)

(x) Characteristic of finite field is not always prime number. (True/False)

1×10=10

(xi) Find the differential equation that will represent family of circle having centre at $(a, 0)$ and radius r.

(xii) Solve :

$$\frac{d^4y}{dx^4} - y = 0$$

(xiii) Find the real part of the complex number i^i.

(xiv) Find GCD of 39 and 45 and express it as linear combination of these numbers.

(xv) Find the unit digit of 7^{45}. 4×5=20

Part–B

10 each

2. (a) Solve :

$$\frac{d^2y}{dx^2} + 2y = e^x \sin x$$

$a^2 - b^2 =$

(b) Find the equation of family of the circle :

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

in the xy plane.

3. (a) Find the differential equation from the relation $ax^2 + by^2 = 1$.

$y = ax^2 + by^2$

$y' = 2ax + 2by\frac{dy}{dx}$

(b) Solve :

$$\frac{d^3y}{dx^3} + \frac{dy^2}{dx^2} - \frac{dy}{dx} - y = \sin 2x$$

Part–C

10 each

4. (a) State and Prove De-Moivre's theorem.

   (b) Compute $(3 + 3i)^5$. $(3+3i))$

5. (a) Find the cube root of $z = -1 + i$. $x = (6)^{1/5}$

   (b) Find the square root of $8 - 6i$. $x^3 - 1 = 0$, $x^3 = 1$, $x =$

Part–D

10 each

6. (a) Solve :

$$x^2 + 2x - 1 \equiv 0 \pmod{7}.$$

   (b) Solve :

$$7x \equiv 2 \pmod{31}.$$

7. Find the smallest positive integers that when divided by 3, 5, 7 we get remainder 1, 4, 6, respectively.

Part–E

10 each

8. (a) Prove that $(\mathbb{Z}_7, +_7, \times_7)$ is a field.

   (b) Show that $x^2 + x + 1$ is irreducible over GF(2).

9. (a) In $(\mathbb{Z}_5, +_5, \times_5)$ solve : $2^{m-1}+1$

$$(x^4 + 4x^3 + 3x^2 + 2) + (2x^4 + 3x^3 + 4x + 4)$$

$$\text{and } (x^2 + 2x + 4).(x^2 + 3x + 3)$$

   (b) Find idempotent and nilpotent element of $(\mathbb{Z}_8, +_8, \times_8)$.

$(x^3+x+1)\overline{)x^2+1}(1$
$\underline{x^2+1+x}$
x

$x^2+x+1\overline{)x^3+1}(x$
$\underline{x^3+x^2+x}$