Unit–V

(Compulsory Question)

9. (a) State Rolle's theorem.

   (b) Find nth derivative of $\frac{1}{ax+b}$.

   (c) Find greatest common divisor of 258, 60 by using Euclidean algorithm.

   (d) If a, b, c and d are integers such that $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $a + c \equiv b + d \pmod{m}$.

   (e) Prove that every cyclic group is an abelian group.

   (f) Prove that the inverse of each element is unique.

   (g) Show that the set Z of all integers ......, –4, –3, –2, –1, 0, 1, 2, 3, 4, ... is a group w.r.t. the operation of addition of integers.

   (h) Define a ring and give two examples.

   (i) If F is a field and $f(x)$ is a polynomial of degree $n \geq 1$ in F(x) then the equation $f(x) = 0$ has at most n roots in F.

   (j) Prove that the set I of all integers is not a group under addition and multiplication as ring composition.

Communicative English

Roll No. 61600130030

Total No. of Questions : 9] [Total No. of Printed Pages : 4

(1048)

B.C.A. (CBCS) RUSA IInd Semester Examination

4027

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] [Maximum Marks : 70

Note :– Attempt *five* questions in all, selecting *one* question from each Unit–I to IV. Question No. 9 (Unit–V) is compulsory. All questions are of equal marks.

Unit–I

1. (a) Verify Lagrange's mean value theorem for the function f defined by $f(x) = x^3 - x^2 - 6x$ in the interval [–1, 4].

   (b) Find nth derivative of $e^{ax} \sin(bx + C)$.

2. (a) Discuss the applicability of Cauchy's mean value theorem to $f(x)$ and $g(x)$ in $[a, b]$ where :

$$f(x) = \begin{cases} 2, & a \le x < b \\ 4, & x = b \end{cases}$$

and $g(x) = x,\ x \in [a, b]$.

(b) Find the nth derivative of co-efficient of $x^3 e^x \cos x$.

Unit–II

3. (a) If a and b are any *two* integers, with $b > 0$, $a \neq 0$, then there exists unique integers q and r such that $a = bq + r$, $0 \le r < |b|$.

(b) Solve the linear congruence equation :

$$11x = 2 (\text{mod } 317)$$

4. (a) Find the least member and greatest member of the following set X, if they exist

$$X = \{n \in N : n^2 + 2n \le 60\}$$

(b) For any $n \in I_+$, prove that the integers $8n + 3$ and $5n + 2$ are relatively prime.

Unit–III

5. (a) Prove that the set $G = \{0, 1, 2, 3, 4\}$ is a finite abelian group of order 5 w.r.t. addition modulo 5.

(b) Show that the set Z of integers, with the ordinary addition operation, is an infinite cyclic group.

6. (a) Prove that the set G_1 of all n-rowed non-singular matrices over a field F is a non-Abelian group w.r.t. the operation of matrix multiplication.

(b) Prove that $(ab)^{-1} = b^{-1}a^{-1}\ \forall\ a, b \in G$, the inverse of the product of two elements of a group G is the product of inverse taken in reverse order.

Unit–IV

7. (a) Prove that a finite ring without zero divisors is a field.

(b) Prove that in general the set of all numbers of the form $a + (\sqrt{p})b$ where a, b are rational numbers, and p is a prime is a field w.r.t. ordinary addition and multiplication.

8. (a) Let F be a field and suppose $f(x)$ is a polynomial in $F(x)$, then prove that $x - a$ is a divisor of $f(x)$ in $F(x)$ iff $f(a) = 0$ in F.

(b) If R is a ring such that $a^2 = a\ \forall\ a \in R$, prove that :

(i) $a + a = 0 \quad \forall\ a \in R$

(ii) R is commutative ring