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B.C.A. UG (CBCS) RUSA IIIrd Semester Examination

4210

MATHEMATICS-III

BCA–0301

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Part of B, C, D and E. Part-A is compulsory. Marks for Part-B, C, D and E are 10 each and Part-A is of 30 marks.

Part–A

1. (i) Write order and degree of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$$

(ii) What is particular solution of differential equation ?

(iii) Find differential equation that will represent family of straight line $y = mx + c$.

(iv) Find modulus and argument of complex number $1 + \sqrt{3}i$.

(v) Find x and y if $2x + (3x + y)i = 4 + 10i$.

(vi) Show that :

$$1 + i^{10} + i^{100} + i^{1000} = 2$$

(vii) The finite field of order p^n is usually denoted by $GF(p^n)$. In this case p is prime number and n is real number. (True/False)

(viii) Only even prime number is 2. (True/False)

(ix) If $a, b, c, d \in I$ and a/b, c/d then ac/bd.

(True/False)

(x) If a and b are relatively prime then any common divisor of ac and b is a divisor of c.

(True/False)

1×10=10

(xi) Verify that $y = e^{-3x}$ is a solution of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0.$$

(xii) Solve :

$$\frac{dy}{dx} = \frac{-x}{y}$$

(xiii) Find square root of $-15 - 8i$.

(xiv) Find g.c.d. of 35 and 56 and express it as linear combination of these numbers.

(xv) Show that $5^{48} - 1$ is divisible by 24.

Part–B

2. (a) Form the differential equation representing the family of curves $y = A \cos 2t + B \sin 2t$ where A and B are arbitary constant.

(b) Solve :

$$\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$$

3. (a) Solve :

$$x^3\frac{d^3y}{dx^3} + 6x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 4y = 0$$

(b) Show that $y = (a + bx)e^{2x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$. 10

Part–C

4. (a) If $(x + iy)^3 = u + iv$ then show that

$$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2).$$

(b) If 1, w, w^2 are cube roots of unity, then show that $(1 + w - w^2)^6 = 64$. 10

5. (a) If α, β are roots of $x^2 - 2x + 4 = 0$, then prove that $\alpha^6 + \beta^6 = 128$.

(b) Find the five fifth roots of unity and show that their sum vanishes. 10

Part–D

6. (a) Solve $3x^2 + 9x + 7 \equiv 0 \pmod{13}$

(b) Use Chinese Remainder Theorem solve

$17x \equiv 9 \pmod{276}$ 10

7. Prove that the congruence $x^2 \equiv a \pmod{p}$ where p is an odd prime and g.c.d. $(a, p) = 1$ has exactly two solutions or no solutions. 10

Part–E

8. (a) Prove that $(Q, +, \cdot)$ is a field.

(b) Show that $x^4 + 8 \in Q[x]$ is irreducible over Q. 10

9. (a) Let F be the field of rational numbers. Determine the degree of spliting field of the Polynomial $x^3 - 2$ over F.

(b) Show that any two finite fields having the same number of elements are isomorphic. 10