Unit–V

(Compulsory Question)

9. (a) State Rolle's theorem.

(b) Find nth derivative of $\frac{1}{ax+b}$

(c) Find greatest common divisor of 258, 60 by using Euclidean algorithm.

(d) If a, b, c and d are integers such that $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $a + c \equiv b + d \pmod{m}$.

(e) Prove that every cyclic group is an abelian group.

(f) Prove that the inverse of each element is unique.

(g) Show that the set Z of all integers ......, –4, –3, –2, –1, 0, 1, 2, 3, 4, ... is a group w.r.t. the operation of addition of integers.

(h) Define a ring and give two examples.

(i) If F is a field and $f(x)$ is a polynomial of degree $n \geq 1$ in $F(x)$ then the equation $f(x) = 0$ has at most n roots in F.

(j) Prove that the set I of all integers is not a group under addition and multiplication as ring composition.

Communicative English

Roll No. 6160130030

Total No. of Questions : 9] [Total No. of Printed Pages : 4

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B.C.A. (CBCS) RUSA IInd Semester Examination

4027

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] [Maximum Marks : 70

Note :– Attempt *five* questions in all, selecting *one* question from each Unit–I to IV. Question No. 9 (Unit–V) is compulsory. All questions are of equal marks.

Unit–I

1. (a) Verify Lagrange's mean value theorem for the function f defined by $f(x) = x^3 - x^2 - 6x$ in the interval $[-1, 4]$.

(b) Find nth derivative of $e^{ax} \sin(bx + C)$.

2. (a) Discuss the applicability of Cauchy's mean value theorem to $f(x)$ and $g(x)$ in $[a, b]$ where :

$$f(x) = \begin{cases} 2, & a \le x < b \\ 4, & x = b \end{cases}$$

and $g(x) = x, x \in [a, b]$.

(b) Find the nth derivative of co-efficient of $x^3 e^x \cos x$.

Unit–II

3. (a) If a and b are any *two* integers, with $b > 0$, $a \neq 0$, then there exists unique integers q and r such that $a = bq + r$, $0 \le r < |b|$.

(b) Solve the linear congruence equation :

$$11x = 2(\text{mod } 317)$$

4. (a) Find the least member and greatest member of the following set X, if they exist

$$X = \{n \in N : n^2 + 2n \le 60\}$$

(b) For any $n \in I_+$, prove that the integers $8n + 3$ and $5n + 2$ are relatively prime.

Unit–III

5. (a) Prove that the set $G = \{0, 1, 2, 3, 4\}$ is a finite abelian group of order 5 w.r.t. addition modulo 5.

(b) Show that the set Z of integers, with the ordinary addition operation, is an infinite cyclic group.

6. (a) Prove that the set G_1 of all n-rowed non-singular matrices over a field F is a non-Abelian group w.r.t. the operation of matrix multiplication.

(b) Prove that $(ab)^{-1} = b^{-1}a^{-1} \ \forall \ a, b \in G$, the inverse of the product of two elements of a group G is the product of inverse taken in reverse order.

Unit–IV

7. (a) Prove that a finite ring without zero divisors is a field.

(b) Prove that in general the set of all numbers of the form $a + (\sqrt{p})b$ where a, b are rational numbers, and p is a prime is a field w.r.t. ordinary addition and multiplication.

8. (a) Let F be a field and suppose $f(x)$ is a polynomial in $F(x)$, then prove that $x - a$ is a divisor of $f(x)$ in $F(x)$ iff $f(a) = 0$ in F.

(b) If R is a ring such that $a^2 = a \ \forall \ a \in R$, prove that :

(i) $a + a = 0 \quad \forall \ a \in R$

(ii) R is commutative ring

(d) Find the remainder when 3^{287} is divided by 23.

(e) L.C.M. (8, 12, 15, 20, 25) = ...

(f) Give an example of an infinite abelian group.

(g) The set Z of all integers is an abelian group under the binary composition $a*b = a + b + 2$. Find the identity element.

(h) Prove that non-abelian groups cannot be cyclic.

(i) Define ring and ring with unity.

(j) Prove that Z_6 = {0, 1, 2, 3, 4, 5}, the ring of integer modulo 6, is with zero divisors. 3×10=30

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Total No. of Questions : 9] [Total No. of Printed Pages : 4

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BCA (CBCS) RUSA IInd Semester Examination

4385

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Unit. Q. No. 9 is compulsory.

Unit–I

1. (a) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

   (b) Verify Lagrange's mean value theorem for the function :

$$f(x) = x(x-1)(x-2) \text{ in } \left[0, \frac{1}{2}\right]$$ 5,5

2. (a) Show that :

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$$

Where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

(b) State and prove Leibnitz theorem. 5,5

Unit–II

3. (a) Show that for any *two* integers a and $b > 0$, there exists q_1 and r_1 such that

$$a = bq_1 + cr_1,\ 0 \le r_1 < \frac{b}{2},\ c = 1 \text{ or } -1.$$

(b) Find the remainder when 2^{340} is divided by 341. 5,5

4. (a) Solve $12x + 15 \equiv 0 \bmod 45$.

(b) If p is prime, show that $2(p-3)! + 1$ is a multiple of p. 5,5

Unit–III

5. (a) Show that $S = \{3^n : n \in \mathbf{Z}\}$ is a commutative group with respect to multiplication.

(b) Prove that the set of matrices :

$$G = \left\{\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} : \alpha \in \mathbf{R}\right\}$$

forms a group under matrix multiplication. 5,5

6. (a) Show that a finite semi-group G in which cancellation laws hold is a group.

(b) Prove that the group G of n n^{th} roots of unity is a cyclic group. 5,5

Unit–IV

7. (a) Prove that a ring R is commutative if and only if

$$(a+b)^2 = a^2 + 2ab + b^2 \ \forall\ a, b \in \mathrm{R}$$

(b) Show that the ring $\mathbf{Z}_p$ of integers modulo p is a field if and only if p is prime. 5,5

8. (a) Let $f(x)$ and $g(x)$ be two non-zero polynomials in R(x), R being a ring. If $f(x) + g(x) \neq 0$, then prove that :

$$\deg(f(x) + g(x)) \le \max(\deg f(x), \deg g(x))$$

(b) Find the sum and product of the polynomials :

$f(x) = 4x - 5,\ g(x) = 2x^2 - 4x + 2$ in $\mathbf{Z}_8[x]$ 5,5

Unit–V

(Compulsory Question)

9. (a) State Lagrange's mean value theorem.

(b) $\lim\limits_{x\to\infty} \dfrac{\sin x}{x} =$ ....

(c) Find 100th derivative of x^{101}.

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2212

Total No. of Questions : 9] [Total No. of Printed Pages : 3

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BCA (CBCS) IInd Semseter Examination

7072

MATHEMATICS–II

BCA-201

Time : 3 Hours] **[Maximum Marks : 70**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note:- Attempt *five* questions in all, including Question No. 9, which is compulsory and selecting *one* from each Unit I to IV. Justify each step in your solutions. All questions carry equal (14) marks.

Unit–I

1. State and prove the Rolle's theorem. Also discuss its geometric significance.

2. State Lagrange's mean value theorem and prove that if $f'(x) = 0$ for all $a < x\ b$, then $f(x)$ must be a constant function on the interval $a < x < b$.

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Unit–II

3. If n is any integer, prove that the numbers $5n + 2$ and $7n + 3$ are coprime to each other. Further obtain their least common multiple.

4. If n is an odd integer, show that $n^4 + 4n^2 + 11$ is divisible by 16.

Unit–III

5. Let G be the set of residues modulo 5. Is G a group with respect to :
   (i) addition modulo 5
   (ii) multiplication modulo 5 ?

   Justify your answers.

6. If G is a group, prove that the order of any element $a \in G$ is equal to the order of the cyclic subgroup generated by a.

Unit–IV

7. Prove that the set of square matrices of order 3, under standard matrix addition and matrix multiplication is a ring. Is it commutative ? Justify.

8. Define a field and prove that both of the cancelation laws hold in a field. Is the same true in arbitrary rings ? Justify your answer.

(Compulsory Question)

9. Attempt any *seven* out of the following : 7×2
   (i) Prove that the function $f(x) = \cos x - e^x - x$ is strictly decreasing.
   (ii) State Cauchy's mean value theorem.
   (iii) Prove that the n^{th} derivative of $\sin x$ is $\sin\left(x+\frac{1}{2}n\pi\right)$.
   (iv) Write down the domain on which the function $f(x) = e^x - x$ is increasing.
   (v) True or False : $g.c.d.\ (m, n) = g.c.d.\ (m + 1, n + 1)$, for any $m, n \in \mathbb{N}$.
   (vi) Find out the maximum value of $n \in \mathbb{N}$ such that 10^n divides 24 !. How many consecutive zeros are there ?
   (vii) Is the set of cube roots of unity form a group under addition ?
   (viii) Let G be a cyclic group of order 6, generated by an element a. Write down the orders of all the elements of G.
   (ix) Prove that a group of order 5 is always cyclic.
   (x) Write down a non-Abelian group of order 6.

Rajwinder kaur

Roll No. 62111[redacted]

Total No. of Questions : 9] [Total No. of Printed Pages : 4

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BCA (CBCS) RUSA IInd Semester Examination

3743

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Unit. Q. No. **9** is compulsory.

Unit–I

1\. (a) Verify Lagrange's mean value theorem for the function $f(x) = (x - 2)(x - 3)(x - 4)$ in the interval [0, 4]. Also find the value of 'C'.

(b) If $y = (ax + b)^m$ and $y_n = \frac{d^n y}{dx^n}$, show that :

$$y_n = m(m - 1)(m - 2) \ldots (m - n + 1)(ax + b)^{m-n} a^n$$ 5,5

2. (a) By using mean value theorem find the approximate value of $\sqrt{66}$.

(b) Apply Leibnitz theorem to find the third order derivative of :

$$f(x) = (x^2 + 1)e^{2x} \text{ at } x = 0$$ 5,5

Unit–II

3. (a) Find the greatest common divisor of 275 and 200 and express it in the from $m.275 + n.200$.

(b) Show that the relation of divisibility in the set of integer is reflexive, transitive but not symmetric. 5,5

4. (a) Prove that if a and b be two integers, then $a \equiv b(\text{mod } m)$ if and only if a and b has the same remainder when divided by m.

(b) For positive integer a and b show that :

GCD $(a, b) \times$ LCM $(a, b) = (a \times b) = ab$ 5,5

Unit–III

5. (a) Show that the set of all 2×2 non-singular matrices from an infinite non-abelian group under the composition of matrix multiplication.

(b) Show that the set $G = \{1, \omega, \omega^2\}$ of cube roots of unity from a finite abelian group of order 3 under multiplication of complex number. 5,5

6. (a) Let $M_g(I) = \left\{\begin{bmatrix} a, b \\ c, d \end{bmatrix}\right\}$ where $a, b, c, d \in I$, show that $M_g(I)$ from a monoid under the operation of matrix multiplication.

(b) Define a cyclic group and prove that every cyclic group is abelian but converse is not true. 5,5

Unit–IV

7. (a) Prove that the set of integer is a ring with respect to usual addition and multiplication.

(b) Prove that <R, +, .> where R is set of all reals, is a commutative ring with unity. 5,5

8. (a) Let R be a ring. Then show that the following conditions are equivalent :

(i) R has no zero divisor

(ii) R satisfies left cancellation law

(iii) R satisfies right cancellation law

(b) Show that the set of rationals 'Q' is a field under composition of addition $\oplus$ and multiplication $\odot$, given as :

$$a \oplus b = a + b - 1 \text{ and } a \odot b = a + b - ab$$ 5,5

Unit–V

(Compulsory Question)

9. (i) Discuss applicability of Rolle's theorem for the function $f(x) = |x|$ in $[-3, 3]$.

(ii) If $f(x) = x^2 \sin 2x$, find the value of $f'''(0)$.

(iii) If a/b and a/c, then show that $a/(bx + cy) \ \forall \ x, y \in z$.

(iv) Solve the following congruence $3x \equiv 4 \pmod 5$.

(v) Find the LCM of (272, 1479).

(vi) Define a group.

(vii) Show that the set $G = \{1, 2, 3, \ldots\ldots\ldots, n - 1\}$ does not form a group under multiplication modulo n, where 'n' is a composite number.

(viii) Let R bearing such that $x^2 = x \ \forall \ x \in R$, show that R is a commutative ring.

(ix) Define a ring and a ring with unity.

(x) Prove that every field is an integral domain.

3×10=30