Date

2212

Total No. of Questions : 9] [Total No. of Printed Pages : 3

(1056)

BCA (CBCS) IInd Semseter Examination

7072

MATHEMATICS–II

BCA-201

Time : 3 Hours] **[Maximum Marks : 70**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/ continuation sheet will be issued.

Note:- Attempt *five* questions in all, including Question No. 9, which is compulsory and selecting *one* from each Unit I to IV. Justify each step in your solutions. All questions carry equal (14) marks.

Unit–I

1. State and prove the Rolle's theorem. Also discuss its geometric significance.

2. State Lagrange's mean value theorem and prove that if $f'(x) = 0$ for all $a < x\ b$, then $f(x)$ must be a constant function on the interval $a < x < b$.

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Unit–II

3. If n is any integer, prove that the numbers $5n + 2$ and $7n + 3$ are coprime to each other. Further obtain their least common multiple.

4. If n is an odd integer, show that $n^4 + 4n^2 + 11$ is divisible by 16.

Unit–III

5. Let G be the set of residues modulo 5. Is G a group with respect to :

   (i) addition modulo 5

   (ii) multiplication modulo 5 ?

   Justify your answers.

6. If G is a group, prove that the order of any element $a \in G$ is equal to the order of the cyclic subgroup generated by a.

Unit–IV

7. Prove that the set of square matrices of order 3, under standard matrix addition and matrix multiplication is a ring. Is it commutative ? Justify.

8. Define a field and prove that both of the cancelation laws hold in a field. Is the same true in arbitrary rings ? Justify your answer.

(Compulsory Question)

9. Attempt any *seven* out of the following : 7×2

   (i) Prove that the function $f(x) = \cos x - e^x - x$ is strictly decreasing.

   (ii) State Cauchy's mean value theorem.

   (iii) Prove that the n^{th} derivative of $\sin x$ is $\sin\left(x+\frac{1}{2}n\pi\right)$.

   (iv) Write down the domain on which the function $f(x) = e^x - x$ is increasing.

   (v) True or False : $g.c.d.\ (m, n) = g.c.d.\ (m + 1, n + 1)$, for any $m, n \in \mathbb{N}$.

   (vi) Find out the maximum value of $n \in \mathbb{N}$ such that 10^n divides 24 !. How many consecutive zeros are there ?

   (vii) Is the set of cube roots of unity form a group under addition ?

   (viii) Let G be a cyclic group of order 6, generated by an element a. Write down the orders of all the elements of G.

   (ix) Prove that a group of order 5 is always cyclic.

   (x) Write down a non-Abelian group of order 6.

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