Roll No. ....

Total No. of Questions : 9] [Total No. of Printed Pages : 4

(2043)

BCA (CBCS) RUSA IInd Semester Examination

4205

MATHEMATICS–II

Paper : BCA-0201

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Unit. Q. No. **9** is compulsory.

Unit–I

1. (a) By using Lagrange's mean value theorem, prove that :

$$|\sin x - \sin y| \le |x - y| \ \forall \ x, y \in R$$

(b) State and prove Rolle's theorem. 5,5

2. (a) Find the nth derivative of $e^{3x} \sin^2 x \cos^3 x$.

(b) State and prove Leibnitz's theorem. 5,5

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Unit–II

3. (a) Prove that $9^n - 8^n - 1$ is divisible by 8.

   (b) Let a and b be two positive integers. Prove that :

   $$[g.c.d.\ (a, b)]\cdot[l.c.m.\ (a, b)] = ab$$ 5,5

4. (a) Prove that the congruence is an equivalence relation.

   (b) Find the remainder when 4444^{4444} is divisible by 9. 5,5

Unit–III

5. (a) Show that the set of all positive rational numbers under the composition defined by $a * b = \frac{ab}{3}$ forms an infinite abelian group.

   (b) Show that the set G = {0, 1, 2, 3} forms a group under addition modulo 4. 5,5

6. (a) Show that the set of n-nth roots of unity forms a cyclic group under multiplication.

   (b) Show that the set :

   $$G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ where } a, b, c\ d \in R \text{ s.t. } ad - bc = 1 \right\}$$

   forms a non-abelian group. 5,5

Unit–IV

7. (a) Show that a ring R is commutative iff :

   $$(a + b)^2 = a^2 + b^2 + 2ab\ \forall\ a, b \in R$$

   (b) Show that the set F = {0, 1, 2, 3, 4, 5, 6} forms a field w.r.t. addition and multiplication modulo 7. 5,5

8. (a) If in a ring R, $x^3 = x$ for all $x \in R$, then show that R is commutative.

   (b) Show that for every prime p, the ring Z/pz with usual modulo operations, is a field. 5,5

Unit–V

9. (i) Let $f(x) = x^3$ in $[-1, 1]$ and $g(x) = x^4$ in $[-1, 1]$. Is Cauchy's mean value theorem applicable to $f(x)$ and $g(x)$ in $[-1, 1]$.

   (ii) Find the nth derivative of $e^x \sin x$.

   (iii) Prove that $2^{4n} - 1$ is divisible by 15.

   (iv) For any two integers a and b not both zero, prove that a and b are relatively prime integers if and only if $\exists$ integers x and y such that $ax + by = 1$.

(v) Solve :

$$5x \equiv 2 \pmod{26}$$

(vi) Find the remainder when 16^{53} is divided by 7.

(vii) Prove that in a group G, $a \in G \Rightarrow (a^{-1})^{-1} = a$.

(viii) Prove that a group G in which every element is its own inverse is an abelian group.

(ix) If R is a ring such that $x^2 = x \ \forall \ x \in R$, then show that $2x = 0$.

(x) Prove that $(Z, +, \cdot)$ is not a field. $3 \times 10 = 30$