Roll No. [redacted]

Total No. of Questions : 9] [Total No. of Printed Pages : 4

(2021)

BCA (CBCS) RUSA IIIrd Semester Examination

4042

MATHEMATICS–III

BCA–0301

Time : 3 Hours] [Maximum Marks : 70

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Parts–B, C, D and E. Marks are indicated with questions for Parts–B, C, D and E.

Part–A

1. (A) (i) Write order and degree of the differential equation :

$$\left(\frac{d^3y}{dx^3}\right)+\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+3y=0$$

(ii) The set of solutions of homogeneous differential equation are finite. (True/False)

(iii) Find modulus and argument of complex number $-\sqrt{3}+i$.

(iv) Show that :

$$1+i^{10}+i^{100}-i^{1000}=0$$

(v) Write $\frac{3-i}{2+7i}$ in standard form.

(vi) Zero is an odd number. (True/False)

(vii) Any natural number n can be expressed as a product of prime numbers. (True/False)

(viii) If a, b, d, r and s are integers and d divides a, d divides b. Then d divides $(ra + sb)$.

(True/False)

(ix) $(\mathbb{Z}_4, +_4, \times_4)$ is a finite field. (True/False)

(x) There exist a finite field of order 21.

(True/False)

1×10=10

(B) (i) Find the square root of $8 - 7i$.

(ii) Solve :

$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0$$

(iii) Solve :

$$\frac{dy}{dx} = -\frac{x^3}{y^2}$$

(iv) Find g.c.d. of 45 and 66 and express it as a linear combination of these numbers.

(v) Show that $6^{68} - 1$ is divisible by 35. 4×5=20

Part–B 10 each

2. (a) Solve :

$$\frac{d^2y}{dx^2} + 6y = e^{2x} + \sin 2x$$

(b) Solve :

$$\frac{dy}{dx} + \frac{1}{x}y = \sin x$$

3. (a) Solve :

$$\frac{d^2y}{dx^2} - y = x^2$$

(b) Form the differential equation representing the family of curves $y = e^{-x}(A \cos 3t + B \sin 3t)$ where A and B are arbitrary constants.

Part–C

10 each

4. (a) Compute $(5 + 5i)^3$.

   (b) Find the four fourth root of unity and show that their sum vanishes.

5. (a) If 1, w, w^2 are cube roots of unity, then show that $(1 + w - 2w^2)^3 = -27$.

   (b) Find the cube root of $z = -1 + 3i$.

Part–D

10 each

6. (a) Solve :

$$x^2 + 3x + 11 \equiv 0 (\text{mod } 13).$$

   (b) Find an integer that has a remainder 3 when divided by 7 and 13.

7. State and prove Chinese Remainder Theorem.

Part–E

10 each

8. (a) Prove that $(\mathbb{Z}_5, +_5, \times_5)$ is a field.

   (b) Show that $x^3 + x + 1 \in F_2[x]$ is irreducible over F_2.

9. (a) Prove that characteristic of a finite field is always prime.

   (b) Find the result of $(x^5 + x^4 + x^2 + x) + (x^4 + x^3 + x^2 + x + 1)$ in $GF(2^8)$.