(d) Find the remainder when $3^{287}$ is divided by 23.

(e) L.C.M. (8, 12, 15, 20, 25) = ............ .

(f) Give an example of an infinite abelian group.

(g) The set Z of all integers is an abelian group under the binary composition $a*b = a + b + 2$. Find the identity element.

(h) Prove that non-abelian groups cannot be cyclic.

(i) Define ring and ring with unity.

(j) Prove that $Z_6 = \{0, 1, 2, 3, 4, 5\}$, the ring of integer modulo 6, is with zero divisors. 3×10=30



Roll No. ................................

Total No. of Questions : 9] [Total No. of Printed Pages : 4

(1049)

**BCA (CBCS) RUSA IInd Semester Examination**

# 4385

**MATHEMATICS-II**

**Paper : BCA-0201**

Time : 3 Hours] [Maximum Marks : 70

*Note* :– Attempt *one* question from each Unit. Q. No. 9 is compulsory.

## Unit–I

1. (a) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

   (b) Verify Lagrange's mean value theorem for the function :

   $$f(x) = x(x-1)(x-2) \text{ in } \left[0, \frac{1}{2}\right]$$ 5,5

2. (a) Show that :

   $$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$$

   Where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

(b) State and prove Leibnitz theorem. 5,5

## Unit–II

3. (a) Show that for any *two* integers $a$ and $b > 0$, there exists $q_1$ and $r_1$ such that

$$a = bq_1 + cr_1,\ 0 \le r_1 < \frac{b}{2},\ c = 1 \text{ or } -1.$$

(b) Find the remainder when $2^{340}$ is divided by 341. 5,5

4. (a) Solve $12x + 15 \equiv 0 \mod 45$.

(b) If $p$ is prime, show that $2(p - 3)! + 1$ is a multiple of $p$. 5,5

## Unit–III

5. (a) Show that $S = \{3^n : n \in \mathbf{Z}\}$ is a commutative group with respect to multiplication.

(b) Prove that the set of matrices :

$$G = \left\{ \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} : \alpha \in \mathbf{R} \right\}$$

forms a group under matrix multiplication. 5,5

6. (a) Show that a finite semi-group G in which cancellation laws hold is a group.



(b) Prove that the group G of $n$ $n^{th}$ roots of unity is a cyclic group. 5,5

## Unit–IV

7. (a) Prove that a ring R is commutative if and only if

$$(a + b)^2 = a^2 + 2ab + b^2 \ \forall\ a, b \in R$$

(b) Show that the ring $\mathbf{Z}_p$ of integers modulo $p$ is a field if and only if $p$ is prime. 5,5

8. (a) Let $f(x)$ and $g(x)$ be two non-zero polynomials in R($x$), R being a ring. If $f(x) + g(x) \neq 0$, then prove that :

$$\deg(f(x) + g(x)) \le \max(\deg f(x), \deg g(x))$$

(b) Find the sum and product of the polynomials :

$f(x) = 4x - 5,\ g(x) = 2x^2 - 4x + 2$ in $\mathbf{Z}_8[x]$ 5,5

## Unit–V

### (Compulsory Question)

9. (a) State Lagrange's mean value theorem.

(b) $\lim\limits_{x \to \infty} \dfrac{\sin x}{x} =$ ................. .

(c) Find 100th derivative of $x^{101}$.