

## Today's Content:

- Merge 2 Sorted Arrays
- Merge 2 Sorted Subarrays
- Merge Sort
- Comparator

Friday → Sorting Completed

Saturday

Monday 6pm

Monday 9pm - Content discussion  
(E, At, M, AM) ↗ optional

Wednesday - {Binary Search}

{  
Adv:  
Arrays - 3  
BPTs - 2  
Mats - 4  
Recr - 2  
Sorting - 3  
}

Q8) Given 2 sorted arrays  $A[N]$ ,  $B[m]$ , merge, create a new sorted array.

$a[3] = \{-1, 4, 8\}$   
 $b[2] = \{2, 9\}$   
 $c[5] = \{-1, 2, 4, 8, 9\} \xrightarrow{\text{red}}$

Input:  $A[N]$ ,  $B[m]$ ,  $C[N+m]$   
 1) copy  $A[] \rightarrow C[] : O(N)$   
 2) copy  $B[] \rightarrow C[] : O(m)$   
 3) Sort  $C[] : O(n+m) \log(n+m)$   
 TC:  $N + M + (N+M)\log(N+M)$

$a[7] = \{ -5, -1, 3, 7, 10, 12, 15 \}$   
 $b[5] = \{ -4, 0, 2, 8, 9 \}$

copy remaining data onto  $C[]$   
 out of bounds: no comparison

$$c[12] = \{ -5, -4, -1, 0, 2, 3, 7, 8, 9, 10, 12, 15 \}$$

//Ex1:

$A[5] = \{ 3, 8, 9, 11, 14, 20 \}$   
 $B[3] = \{ 3, 6, 25, 30 \}$

out of bounds  
 copy remaining elements to  $C[]$

$c[10] = \{ 3, 3, 6, 8, 9, 11, 14, 20, 25, 30 \}$  formed

## Pseudo Code

```
int[] merge (ptr A[], int N, ptr B[], ptr M) {
```

ptr P<sub>1</sub> = 0, P<sub>2</sub> = 0, P<sub>3</sub> = 0;

ptr C[N+M];

while (P<sub>1</sub> < N & P<sub>2</sub> < M) // out of bounds.

if (A[P<sub>1</sub>] <= B[P<sub>2</sub>]) { // A[P<sub>1</sub>] should come first

C[P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub> = P<sub>1</sub> + 1; P<sub>3</sub> = P<sub>3</sub> + 1

else { // B[P<sub>2</sub>] should come first

C[P<sub>3</sub>] = B[P<sub>2</sub>]; P<sub>2</sub> = P<sub>2</sub> + 1; P<sub>3</sub> = P<sub>3</sub> + 1

// P<sub>1</sub> is left over

while (P<sub>1</sub> < N) { C[P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub> = P<sub>1</sub> + 1; P<sub>3</sub> = P<sub>3</sub> + 1 }

// P<sub>2</sub> is left over

while (P<sub>2</sub> < M) { C[P<sub>3</sub>] = B[P<sub>2</sub>]; P<sub>2</sub> = P<sub>2</sub> + 1; P<sub>3</sub> = P<sub>3</sub> + 1 }

return C;

Iterations  $\Rightarrow (N+M)$  SC:  $\underline{\underline{O(1)}}$

Tc:  $O(N+M)$

If it is given in question to create a new array to solve

Q8) Given N array elements & 3 input indices  $s, m, e$

Also given } // Subarray  $[s \ m]$  is sorted } // Subarray  $[m+1 \ e]$  is sorted } // Sort  $\rightarrow$  Increasing }  $s \leq m < e$

Q8) Sort Subarray from  $[s \ e]$

Eg:  $ar[12] = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11$

$\boxed{s} \ m \ e$

$\boxed{2} \ 6 \ 9$

$\downarrow$

$tmp[8] = \underbrace{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7}_{e-s+1}$

$P_1 = s, P_2 = m+1$

$P_3 = 0$

$copy \ tmp[] \rightarrow ar[]$

$ar[12] = 4 \ 8 \ \underbrace{-1 \ 2 \ 3 \ 4 \ 6 \ 7 \ 9 \ 11}_{2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9} \ 13 \ 0$

Pseudocode

word merge( int A[], int s, int m, int e) {

int tmp[e-s+1]; // Indcn: [0, e-s]

int P<sub>1</sub>=s, P<sub>2</sub>=m+1, P<sub>3</sub>=0

while( P<sub>1</sub> <= m && P<sub>2</sub> <= e ) {

if( A[P<sub>1</sub>] <= A[P<sub>2</sub>]) {

    tmp[P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub>++; P<sub>3</sub>++

else

    tmp[P<sub>3</sub>] = A[P<sub>2</sub>]; P<sub>2</sub>++; P<sub>3</sub>++

while( P<sub>1</sub> <= m ) { tmp[P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub>++; P<sub>3</sub>++ }

while( P<sub>2</sub> <= e ) { tmp[P<sub>3</sub>] = A[P<sub>2</sub>]; P<sub>2</sub>++; P<sub>3</sub>++ }

// tmp[]  $\xrightarrow{\text{copy}}$  ar[]

// Indcn: [0, e-s] [s, s+1, e] // Can write in your own way

i = 0; i <= e-s; i++) {

A[i+s] = tmp[i]

TC: O(N)

SC: O(N)

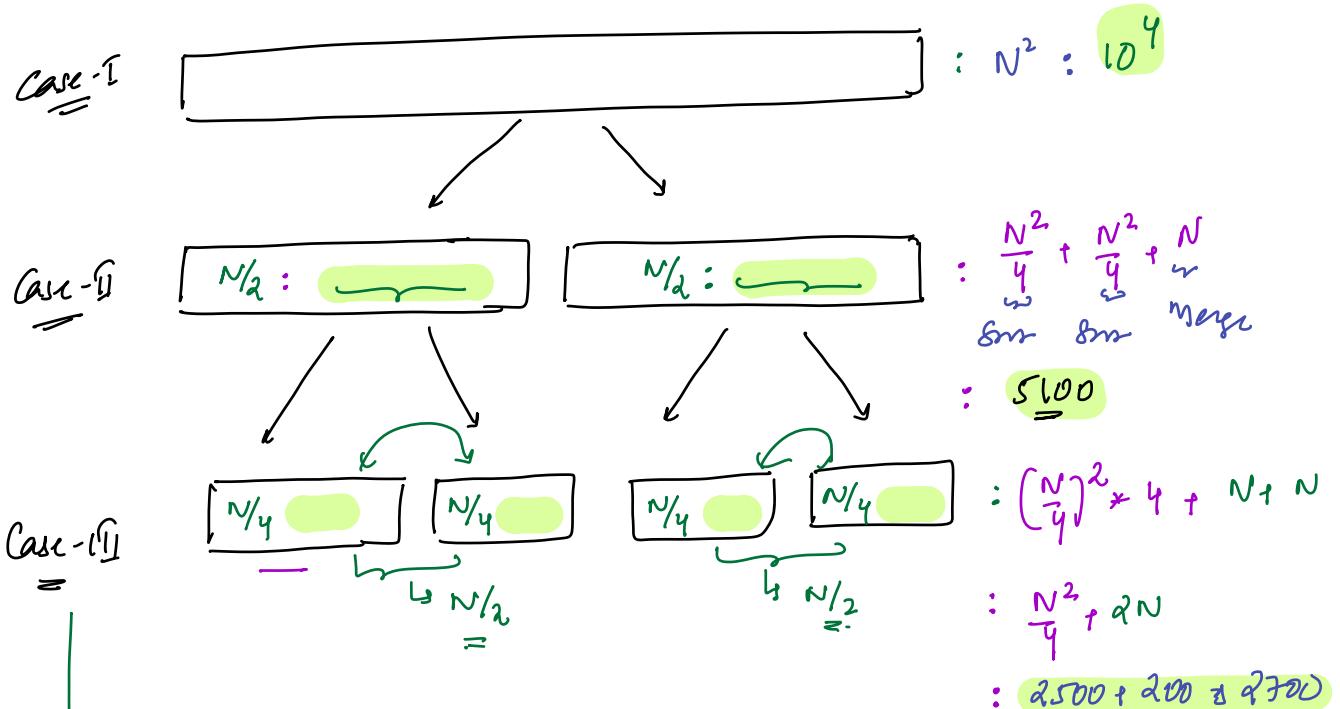
TODO: Merge 2 sorted  
subarrays TC: O(N) ?

& SC: O(1) Not possible ?

? Is O: 3pm break?  $\Rightarrow$  tmp merge

508) Given  $\frac{N}{2}$  Array elements, sort them.  $\{ \text{BS/SS} \rightarrow$

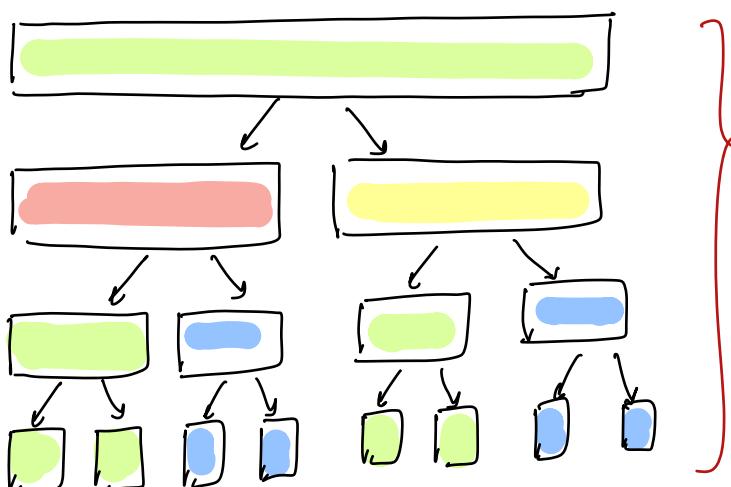
$\underline{\underline{N=100}}$



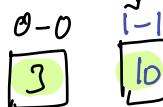
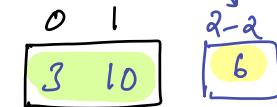
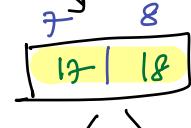
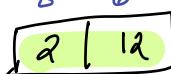
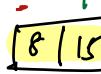
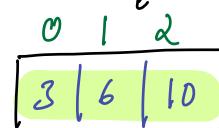
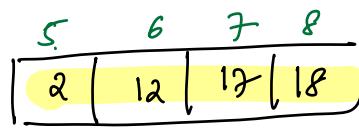
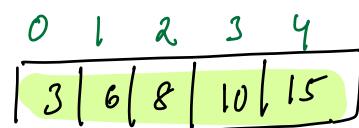
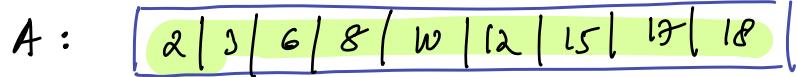
The moment subarray size = 1

WC cannot decide.

Idea: Meng Sort



idea:  $0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \rightarrow [0 \underline{8}]$



//code: main()

Ass: given an subarray from  $[s \underline{e}]$ , array have to be sorted  $[s \underline{c}]$

void MergeSort (int A[], int s, int e) {

if (s == e) { return; }

$$m = (s + e) / 2$$

MergeSort(A, s, m)

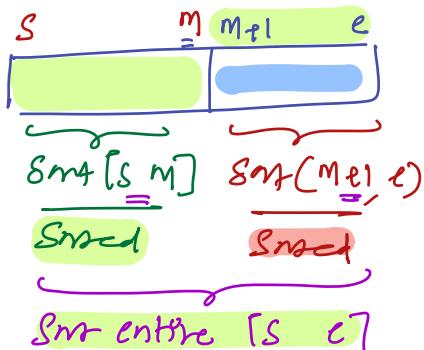
MergeSort(A, m+1, e)

merge(A, s, m, e)

MergeLogic

$$T(N) = 2T(N/2) + N$$

$$T(1) = 1$$



void merge(int A[], int s, int m, int e) {

int tmp[e-s+1]; // endcn: [0, e-s]

int P<sub>1</sub>=s, P<sub>2</sub>=m+1, P<sub>3</sub>=0

while(P<sub>1</sub><=m && P<sub>2</sub><=e) {

if(A[P<sub>1</sub>] <= A[P<sub>2</sub>]) {

tmp[P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub>++; P<sub>3</sub>++

else {

tmp[P<sub>3</sub>] = A[P<sub>2</sub>]; P<sub>2</sub>++; P<sub>3</sub>++

}

while(P<sub>1</sub><=m) { tmp[P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub>++; P<sub>3</sub>++ }

while(P<sub>2</sub><=e) { tmp[P<sub>3</sub>] = A[P<sub>2</sub>]; P<sub>2</sub>++; P<sub>3</sub>++ }

// tmp[] copy → ar[]

// endcn: [0, | e-s] [s, s+1, e] // Can write in your own way

i=0; i<=e-s; i++) {

A[i+s] = tmp[i]

TC: O(N)

SC: O(N)

TODO: Merge 2 sorted

subarrays TC: O(N) ?

or SC: O(1) Not possible

Slo: 3pm break? } → 7min max

// code:

main() {

    // given arr[N] we need to sort

    MergeSort (arr, 0, N-1)

}

### Recursion Relation

$$\left\{ \begin{array}{l} T(N) = 2T(N/2) + N \\ T(1) = 1 \end{array} \right.$$

TODD    k

T<sub>C</sub>:

S<sub>C</sub>:

$$T(N) = 2T(N/2) + N$$

$$T(N/2) = 2T(N/4) + N/2$$

$$= 2[2T(N/4) + N/2] + N$$

$$= 4T(N/4) + 3N \rightarrow 2^2 T(N/2) + 3N$$

$$T(N/4) = 2T(N/8) + N/4$$

$$= 4[2T(N/8) + N/4] + 3N$$

$$= 8T(N/8) + 7N \rightarrow 2^3 T(N/2) + 7N$$

$$T(N/8) = 2T(N/16) + N/8$$

$$= 8[2T(N/16) + N/16] + 7N$$

$$= 16T(N/16) + 15N \rightarrow 2^4 T(N/2) + 15N$$

// generalized expression :

$$\hookrightarrow 2^k T\left(\frac{N}{2^k}\right) + kN \quad ] \quad T(1) = 1$$

$\Rightarrow \frac{N}{2^k} = 1$

$N = 2^k$

$k = \log_2 N$

$\left\{ \begin{array}{l} 1 \\ 2 \\ \log_2 N = N \end{array} \right\}$

from

$\boxed{a^{\log_2 N} = N}$

$$\Rightarrow 2^{\log_2 N} T\left(\frac{N}{2^{\log_2 N}}\right) + (\log_2 N)(N)$$

$$= (N) T(1) + N \log N$$

$$= \boxed{\Theta(N \log N)}$$

SC:  $O(\log N + N)$

$\downarrow$

$\left\{ \begin{array}{l} \text{recursion} \\ \text{and store} \end{array} \right\}$

$\left\{ \begin{array}{l} \text{space complexity for} \\ \text{merging} \end{array} \right\}$

Sorting : 1  $\rightarrow$  { Merge Sort }

Sorting : 2 - { All problems }

Sorting : 3  $\rightarrow$  { Algorithms }