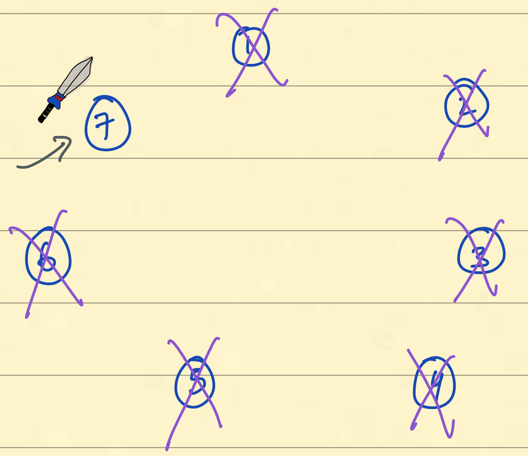
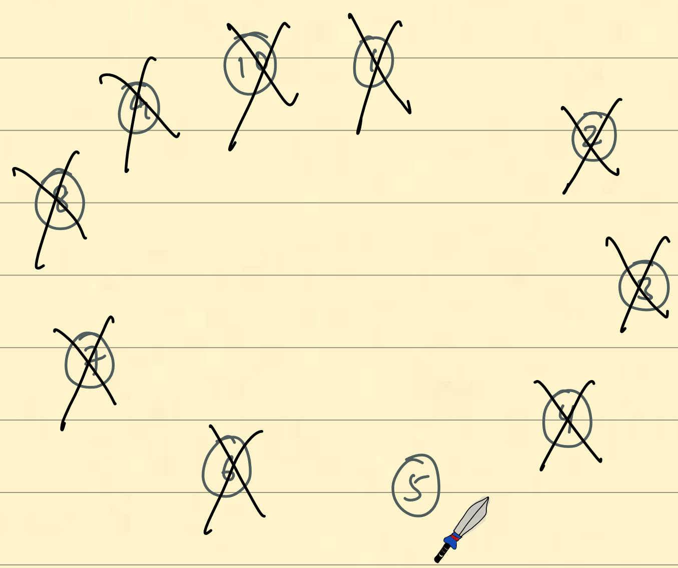
Josephus Problem

Adobe
Google

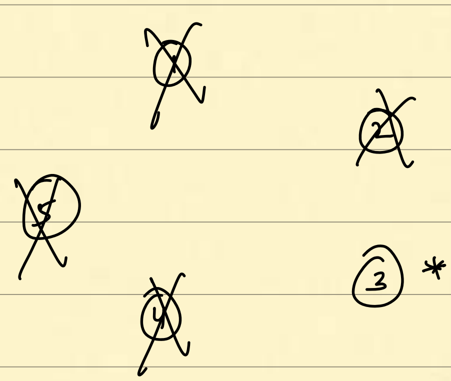
N = 7



N = 10

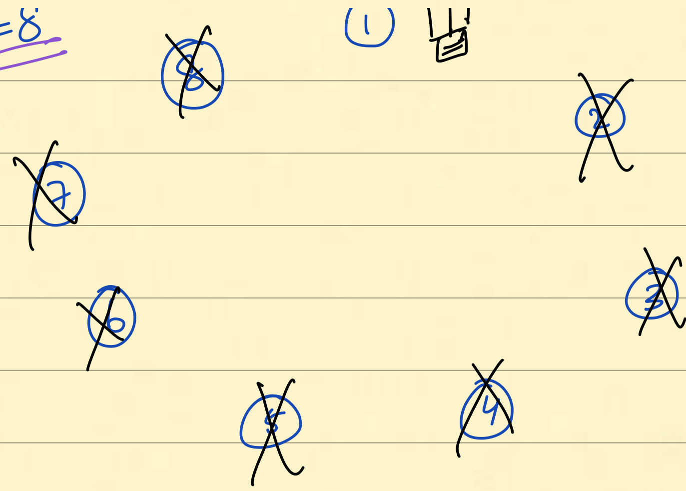


N = 5



9 : 05.

N=8



ans: 1

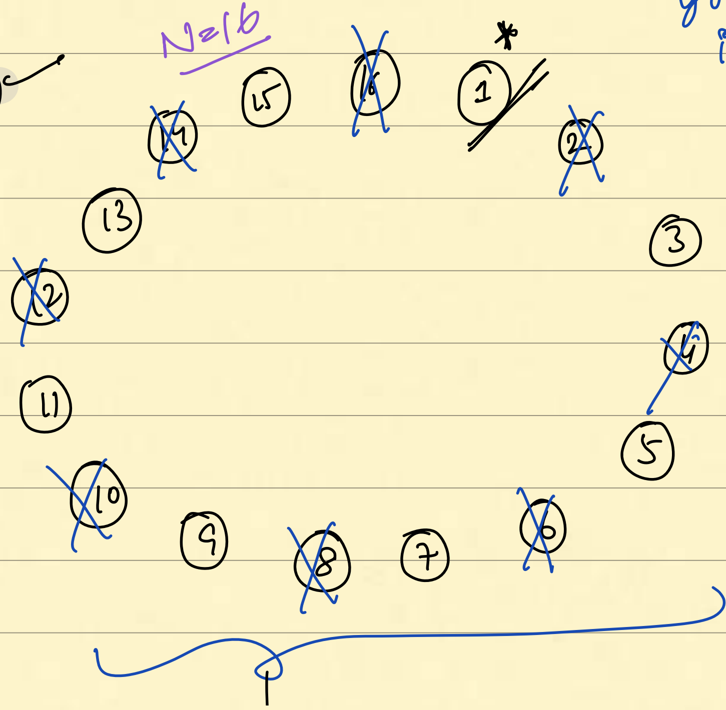
N	ans
1	1
2	1
3	3
4	1
5	3
6	5
7	7
8	1
9	3
10	5

Observations

1. Ans will always be an odd position

2. If $N = 2^n$, the ans = pos from where you started initially.

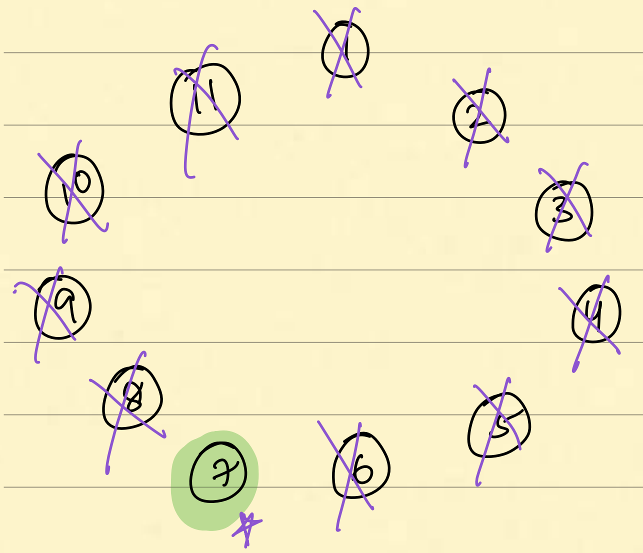
N=16



8 people

1

N = 11



remaining people

11
10
9
8

start pos

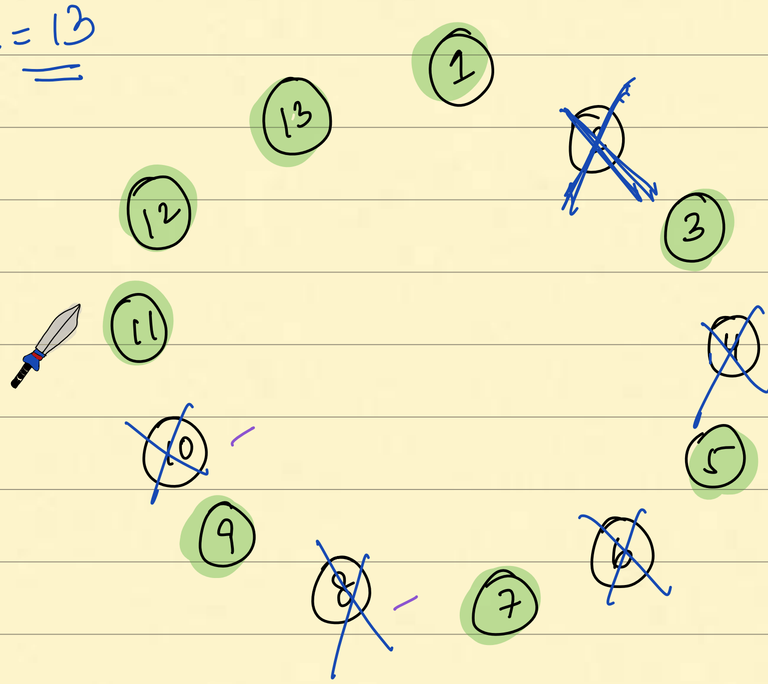
$\boxed{7}$

N = 11

$11 - 2^3 = 11 - 8 = \boxed{3}$

$2 \times 3 + 1 = 7$

N = 13



remaining

13
12
11
10
9
8 → 11

$13 - 2^3 = 5$

$2 \times 5 + 1 = 11$

$$11 - 2^3 = 11 - 8 = 5$$

Steps

① Find the power of 2 $<= N$ [say x]

TODO

② $N - 2^x = y$

③ $2 \times y + 1$ ans.

$N = 98$

① 64

② $98 - 64 = 34$

③ $34 \times 2 + 1 = 69$

$N = 6$

① 4

② $6 - 4 = 2$

③ $2 \times 2 + 1 = 5$

$N = 8$

① 8

(2) $8 - 8 = 0$

(3) $0 \times 2 + 1 = 1$

Google Fb MS Apple
Adobe Rubrik.

Que. Majority Element

Given an arr[], return if there exists a no. with freq > N/2

Eg: arr[6] : {1, 6, 1, 1, 2, 1} $\frac{6}{2} = 3$

freq(1) > 3 freq > 3

Ans 1

Eg: {3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3} N = 11

$\frac{11}{2} = 5$

ans = 3

freq > 5

Eg: {4, 6, 5, 3, 4, 5, 6, 4, 4, 4}

freq(maj) > $\frac{10}{2}$ N=10

> 5

Ans: -INF [Doesn't exist].

Brute Force

① {3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3}

TC → $O(N^2)$
SC → $O(1)$

② Sorting

{1 2 3 3 3 3 3 3 4 5 6}

TC → $O(N \log N)$

③ Hashmap

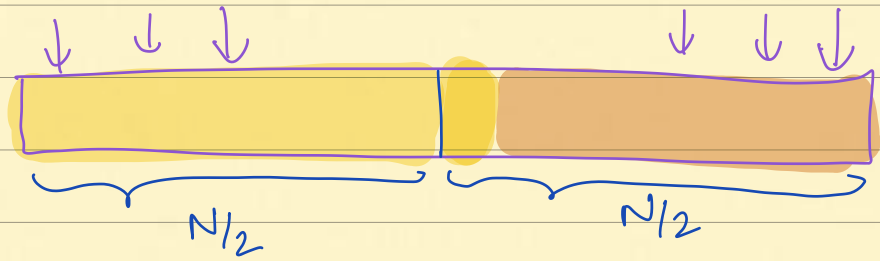
{3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3}

3 → 0
4 → 0
1 → 0
5 → 0
6 → 0

TC → $O(N)$
SC → $O(N)$

④ TC → $O(N)$
SC → $O(1)$

QUIZ: How many majority ele can be there at max in array of size N.



Conclusion

1. You can have only 1 majority element.

2. freq(major) > freq(rest)

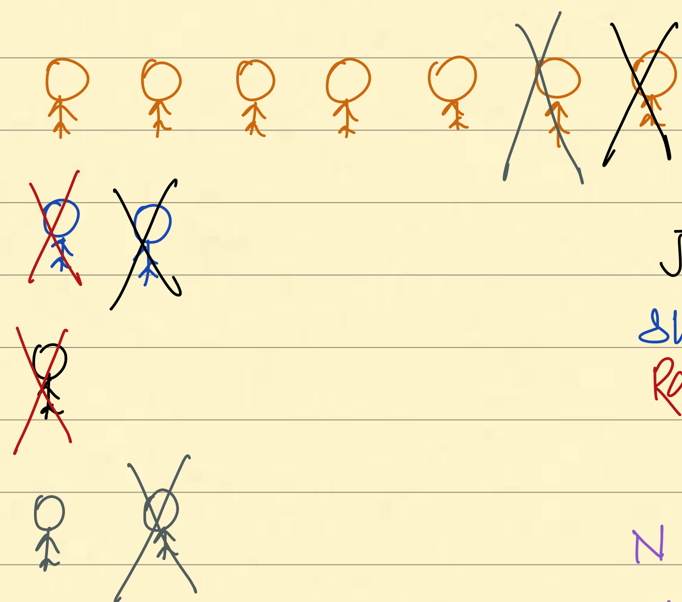
Our own Election

Paridhi

Yuvaraj

Abhishek

Seth



Jay
Sharik
Ravi

$N = 12$

$\frac{N}{2} = 6$

major > 6

N	N/2	Major
12	6	7
10	5	6
8	4	5

Conclusion : If you remove two distinct seats, still majority will remain majority.

ME: 3

0	1	2	3	4	5	6	7	8	9	10
3	4	3	6	1	3	2	5	3	3	3

0	1	2	3	4	5	6	7	8	9	10
3	4	3	6	1	3	2	5	3	3	3

0	1	2	3	4	5	6	7	8	9
3	4	3	6	1	3	2	3	3	3

$\{3, 4, 3, 6, 1, 3, 2, 3, 3, 3\}$

$N = 10$

$N/2 = 5$

$freq > 5$

Moore's Voting Algorithm

0	1	2	3	4	5	6	7	8	9	10
3	4	3	6	1	3	2	5	3	3	3
↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

m = ~~1~~ ~~2~~ 3

cnt → ~~1~~ ~~0~~ ~~1~~ ~~0~~ ~~1~~ ~~0~~ ~~1~~ ~~0~~ ~~1~~ ~~2~~ 3

$\{1, 6, 1, 1, 2, 1\}$

$m = \cancel{1}\ 1$

$cnt = \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{2}\ \cancel{1}\ 2$

$\{4, 1, 1, 7, 1, 5, 1, 1, 1, 5, 7\}$

$m = \cancel{4}\ 1$

$cnt = \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{2}\ \cancel{3}\ \cancel{2}\ 1$

$\{4, 6, 5, 3, 4, 5, 6, 4, 4, 4\}$ → 5

$m = \cancel{4}\ \cancel{5}\ \cancel{4}\ \cancel{6}\ 4$

$cnt = \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{0}\ \cancel{1}\ \cancel{0}\ \cancel{1}\ 2$

$\frac{N}{2} = 5$

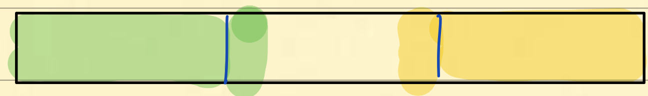
$freq(4) \le 5$

TC → $O(n)$

SC → $O(1)$

Break : 10 : 55

To find the ele with freq. $> \frac{N}{2}$



Atmost 2 ele with freq > $\frac{N}{3}$

[4 1 3 7 7 1 1 1 7 7] → $9/3 = 3$ > 3

maj1 = ~~4~~ 7   maj2 = ~~1~~ 1

Cnt1 = ~~1~~ ~~0~~ ~~1~~ ~~2~~ 3   Cnt2 = ~~1~~ ~~0~~ ~~1~~ ~~2~~ 3

Tricky Code !

Check a lot of edge cases.

Google | Adobe | Goldman Sachs | MS

Que. Doors

There are N doors. Initially all doors are closed

All doors having no. as multiple of 1 are toggled.

All doors having no. as multiple of 2 are toggled

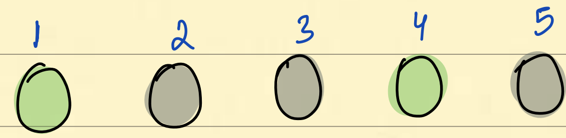
All doors having no. as multiple of 3 are toggled

⋮

All doors having no. as multiple of N are toggled

Tell no. of open doors at the end.

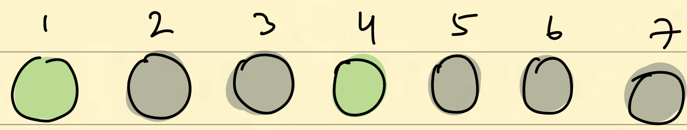
N: 5



sqrt(5) = $\boxed{2}$

Ans: 2

N: 7



$\underline{\text{sqrt}(7)}$

$= \boxed{2}$

Open → 2

N:20

Sqrt(20)

9 → 1, 3, 9

15 → 1, 3, 5, 15

18 → 1, 2, 3, 6, 9, 18

factors

A no. will get toggled only by its factor.

$C \xrightarrow{\text{1st}} O \xrightarrow{\text{2nd}} C \xrightarrow{\text{3rd}} O \xrightarrow{\text{4th}} C \xrightarrow{\text{5th}} O$

For a no.,

if factors are odd → open

even → closed

24 → 1, 2, 3, 4, 6, 8, 12, 24

factors appear in pairs.

4 → 1, 2, 4
16 → 1, 2, 4, 8, 16
25 → 1, 5, 25

Only for perfect sq., factors will be odd.

N = 100

$\boxed{i \times i <= N}$

$i * i == N$
$i^2 == N$
$i == \sqrt{N}$

i	i x i	
1	1 * 1	= 1 ≤ 100
2	2 * 2	= 4 ≤ 100
3	3 * 3	= 9 ≤ 100
4	4 * 4	= 16 ≤ 100
5	5 * 5	= 25 ≤ 100
⋮		⋮
10	10 * 10	= 100 = 100
11	11 * 11	= 121 > 100

$\boxed{i * i <= N}$

i = 10

10 X 10 == N

sqrt(N)

sqrt(100)

= 10

sqrt(20)

≈ 4

Ans → sqrt(N).

$TC \rightarrow O(\sqrt{N})$

$SC \rightarrow O(1)$

for (i = 1 ; i*i <= N; i++)

N = 17

sqrt (N)

$1 * 1 \leq 17$

$2 * 2 \leq 17$

$3 * 3 \leq 17$

$4 * 4 \leq 17$

sqrt (17) = 4