

Today's Content:

- Gcd Intro
- Properties of GCD
- GCD function
- gcd problems?
 - Check Subsequence with $\text{gcd} = 1$
 - Delete 1 element such that gcd of remaining elements in your array is max
 - Given N distinct elements, pick 2 elements (i, j) & $i \neq j$, add their difference find min in array element
 - If time permits mmz problems

GCD: Greatest Common divisor or HCF & Higher common factor

$\gcd(a, b)$: greater factor which divides both a & b

↪ $n \Rightarrow a \% n = 0$ $\&$ $b \% n = 0$, n highest factor

$$\begin{array}{r} \text{gcd}(15, 25) = 5 \\ \downarrow \qquad \downarrow \\ 1 \qquad 1 \\ \hline 3 \qquad 5 \\ 5 \qquad 25 \\ \hline 15 \end{array}$$

$$\begin{array}{r}
 \text{gcd}(12 \quad 20) = 8 \\
 \downarrow \quad \downarrow \\
 1 \quad 1 \\
 2 \quad 2 \\
 3 \quad 3 \\
 4 \quad 5 \\
 6 \quad 6 \\
 12 \quad 10 \\
 18 \quad 15 \\
 7
 \end{array}$$

$$\begin{array}{r} \text{gcd}(10, -25) = 8 \\ \downarrow \quad \downarrow \\ 1 \quad -25' \quad \xrightarrow{-1e} \\ 2 \quad -5 \quad \left[\begin{array}{l} \text{factors} \\ \hline \end{array} \right] \\ 5 \quad -1 \\ 10 \quad 1 \\ 5 \quad 25 \end{array}$$

$$\begin{array}{rcl} \text{gcd}(0, 8) & = & 8 \\ & | & \downarrow \\ & 1 & 1 \\ & | & | \\ 2 & 2 \\ \cdot & \cdot \\ & | & 4 \\ 8 & 8 \\ \vdots & \vdots & \vdots \\ 0 & 0 & n \end{array}$$

$$\begin{array}{r}
 \text{gcd}(0, -10) \\
 \downarrow y \quad \downarrow \\
 1 \quad 1 \\
 2 \quad 2 \\
 3 \quad 5 \\
 : \\
 10 \quad 10 \\
 \vdots \\
 \infty
 \end{array}$$

$$\begin{array}{ccc} \text{gcd}(-16, -24) = 8 \\ \downarrow & \downarrow \\ 1 & 1 \\ 2 & 2 \\ 4 & 3 \\ 8 & 4 \\ 16 & 6 \\ & 8 \\ & 12 \end{array}$$

$$\text{gcd}(-2, -3) = 1$$

↓ ↓
1 1
2 3

$\text{gcd}(0, 0) = \text{not defined}$

If given in Q
They will
what to

Properties of $\gcd(a, b)$

| | → abs.

$$\rightarrow \text{gcd}(a, b) = \text{gcd}(b, a) \rightarrow \text{commutative}$$

$$\rightarrow \quad \gcd(a, b) = \gcd(|a|, |b|)$$

$$\begin{array}{l} \text{gcd}(0, n) = \boxed{|n|} \quad \text{gcd}(0, 5) = 5 \\ \hookrightarrow |n| = 0 \quad \text{gcd}(0, 10) = 10 \end{array}$$

$$\gcd(a, b, c) = \gcd(\gcd(a, b), c) = \gcd(a, \gcd(b, c))$$

↳ associative

Special property:

$\int A, B > 0 \text{ eq } A > B$

$$\text{Say } \boxed{\gcd(A, B) = n} \rightarrow (A \% n = 0) \text{ & } (B \% n = 0)$$

$$\boxed{\gcd(A-B, B) = n}$$

$$\boxed{\gcd(A, B) = \gcd(A - B, B)}$$

$$(A - B) \% n = 0 \quad \checkmark$$

$$\text{By } n = 0 \checkmark$$

$$\left(\underbrace{A \% x}_0 - \underbrace{B \% x}_0 + n \right) \% n$$

$$(0 - 0 + n) \% n = 0$$

B>A:

$$\gcd(a, b) = \gcd(b-a, a)$$

Ex: $a \ b$

$$\text{gcd}(23, 5) \rightarrow \text{gcd}(18, 5) \rightarrow \text{gcd}(13, 5) \rightarrow \text{gcd}(8, 5) \rightarrow \text{gcd}(3, 5)$$

$\boxed{\text{gcd}(23 \% 5, 5) = \text{gcd}(3, 5)}$

Given $A, B \geq 0 \quad A > B$

$$\begin{aligned}
 \text{gcd}(A, B) &= \text{gcd}(B, A - B) \\
 &= \text{gcd}(B, A - 2B) \\
 &= \text{gcd}(B, A - 3B) \\
 &\vdots \\
 &= \text{gcd}(B, \boxed{A - nB}) \\
 &\quad \downarrow D_i \quad \& \quad \downarrow \text{Divs} \\
 &\quad \text{Subtract} \quad \text{man nos of } B \text{ from } A
 \end{aligned}$$

$A > B$

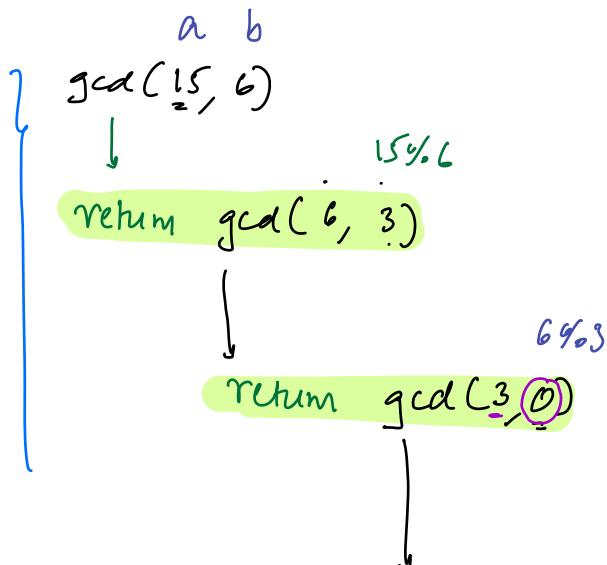
$$\text{gcd}(A, B) = \text{gcd}(B, A \% B)$$

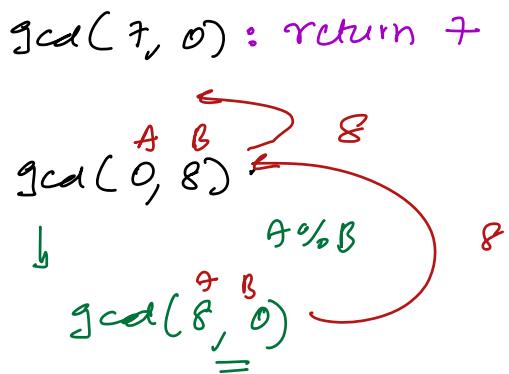
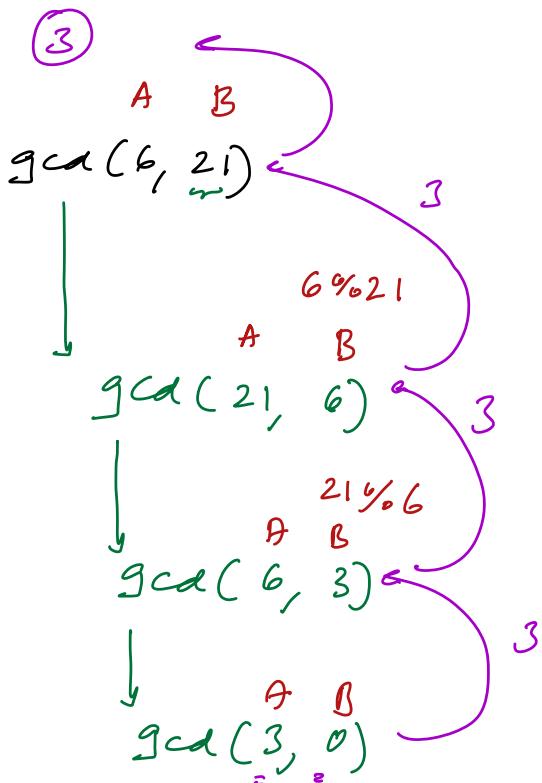
Euclidean gcd
 left side > right side

Note: If both 0, handle separately

```

//int gcd(int a, int b) {
  if(b == 0) {return a;}
  if(a == 0) {return b;}
  // no need
  return gcd(b, a % b)
}
  
```





$A > B$

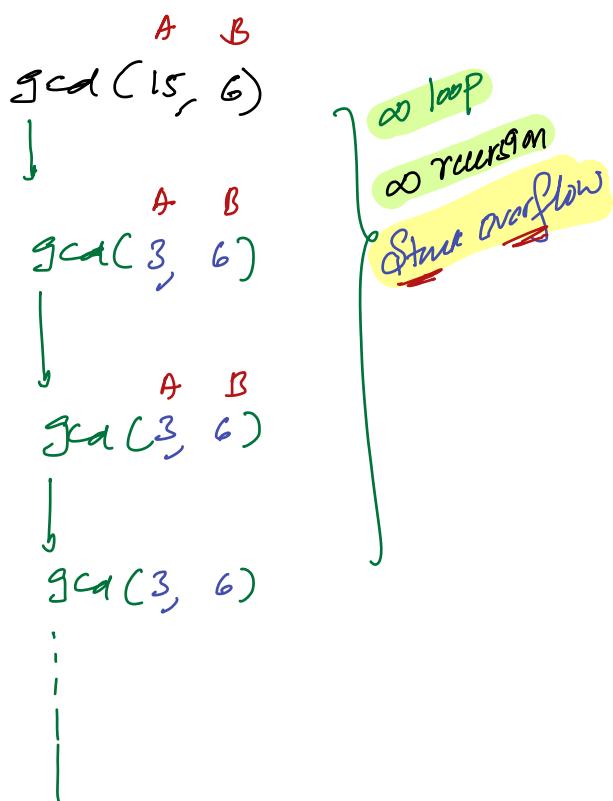
Put $\text{gcd}(\text{int } a, \text{int } b)$

If ($b == 0$) {return a ;
return $\text{gcd}(a \% b, b)$ }

// Correct Code

```
int gcd(a, b){  
    if(b == 0) {return a;  
    return gcd(b, a % b)  
}
```

// $\text{gcd}(|a|, |b|) \rightarrow$ To avoid negative cases



// TC $\rightarrow O(\log_2(\max(A, B)))$

Ex1: $N \rightarrow N-1 \rightarrow N-2 \rightarrow \dots \rightarrow 1$: N iterations

Ex2: $N \rightarrow N/2 \rightarrow N/4 \rightarrow N/8 \dots L$: \log_2^N iterations

$a \geq b$

not clear // $\text{gcd}(a, b) = \text{gcd}(\underbrace{a \% b}_{\{a\}}, b) \Rightarrow O(\log_2 \frac{\max(A, B)}{2})$

// $b < a$

$b < \frac{a}{2}$

$\{b = \frac{a}{2}\}$

$a \% b < b < a/2$

$a \% b < [b = a/2]$

$a \% b < \frac{a}{2}$

$a \% b < a/2$

$b > \frac{a}{2}$ $\left\{ \begin{array}{l} a \% b = \\ a - b \\ 2b > a \\ 2b - a > 0 \\ a - 2b < 0 \end{array} \right.$

// (\rightarrow) in both sides

$a - 2b < 0$

// and a both sides

$2a - 2b < a$

$2(a - b) < a$

$(a - b) < a/2$

Case 3: How many times we can do b

$a \% b = a - b$: (We can do n times)
 $a - 2b \neq$

$\left\{ \begin{array}{l} \text{Break: 10:40 break} \\ \quad \end{array} \right\}$

$(a \% b) < a/2$

// given N array elements calculate gcd of entire array

↳ gcd of entire array:

$$ar[3] \rightarrow \{ 6 \underset{6}{\cancel{12}} \underset{3}{\cancel{15}} \} \rightarrow \underline{\underline{15}} \rightarrow \text{gcd}(\text{man}, \text{men}) \neq \text{gcd}(com)$$

Ex: $ar[3] \rightarrow \{ 5 \underset{5}{\cancel{6}} \underset{10}{\cancel{3}} \}$

$$ar[4] \rightarrow \{ 8 \underset{8}{\cancel{12}} \underset{12}{\cancel{12}} \underset{10}{\cancel{10}} \} \quad \left. \begin{array}{l} \text{gcd} = 0 \\ \text{gcd} = 1 \end{array} \right\} \quad \begin{array}{l} \text{gcd}(5, 10) = 5 \\ \text{gcd}(0, 5) = 5 \\ \text{gcd}(1, 5) = 1 \end{array}$$

```
int gcd(a, b){  
    if(b == 0) { return a; }  
    return gcd(b, a % b);  
}
```

```
int gcdarr(int ar[], int N){  
    int ans = 0; // ans = ar[0]  
    for(int i = 0; i < N; i++) {  
        ans = gcd(ans, ar[i]);  
    }  
    return ans;  
}
```

I will discuss in
next session

$$\text{gcd}(5, 5) = 5$$

$$\text{gcd}(10, 10) = 10$$

$$\left. \begin{array}{l} \text{ans} = ar[0] \\ \text{ans} = \underset{i=0}{\text{gcd}}(ar[0], \text{ans}) \\ \text{ans} = ar[0] \end{array} \right\}$$

TC: $N \times (\log_2 \text{man array elem})$

TC: O(N)

Gcd of entire array: O(N)

TODD:

Hint 1: for every gcd we use
same ans variable

Hint 2: ans \rightarrow either same
or decrease

// Subsequence Basics

none n more

→ A sequence generated by deleting ≥ 0 elements
from array ↑

$$ar[8] = \{ 3 \ 2 \ 1 \ 6 \ 4 \ 8 \ 10 \ 9 \}$$

$$: \{ 3 \ 2 \ 6 \ 10 \ 9 \} \checkmark$$

$$: \{ \underline{2} \ \underline{6} \ \underline{10} \ \underline{9} \ \underline{3} \} \cup (\text{order matters})$$

$$ar[4] = \{ 6 \ 9 \ 0 \ 8 \}$$

$$: \{ 9 \ 8 \} \rightarrow \checkmark$$

$$: \{ 6 \ 9 \ 8 \} \rightarrow \checkmark$$

$$: \{ 0 \ 9 \ 8 \} \times \rightarrow \text{order is matter}$$

$$: \{ 6 \ 9 \ 0 \ 8 \} \rightarrow \checkmark$$

28) Given an array return if there exists a subsequence with gcd = 1

$$\underline{\underline{E_{11}}}: \quad ar[5] = \{4 \ 6 \ 3 \ 8\} \rightarrow \{4 \ 3 \ 8\}$$

$$\underline{\text{Ex2:}} \quad \text{ar}[5] = \{ \underline{16} \quad 10 \quad 6 \quad 15 \quad \underline{27} \} \rightarrow \{ 16 \quad 10 \quad 27 \}$$

$$\underline{\underline{Q_{23}}}: \quad ar[4] = \{ 6 \quad 12 \quad 3 \quad 18 \} \rightarrow \{ \text{No Subsequence} \}$$

qdal: for every subsequence get gcd of entire = 1

$\stackrel{TC}{=} (2^N * (\text{TC to get gcd of entire subsequence}))$

Egny: $\{3 \ 6\} \rightarrow (\text{No Subsequence})$

Idea: $\text{ar}[6] = A \ B \ C \ D \ F \ F \ \}$ Subsequen $\text{gcd} = 1$

// gcd of entire array

$$\begin{aligned} \text{gcd}(A, B, C, D, E, F) &= \text{gcd}(\text{gcd}(A, D, F), \underbrace{\text{gcd}(B, C, F)}_!) \\ &= \text{gcd}(\text{gcd}(A, D, E), !) \end{aligned}$$

1
≈

final obs:

⇒ If there is a Subsequence with gcd = 1

↳ gcd of entire array $\text{== } 1$: (Subsequence V)

$\neq 1$: (Subsequence X)

Delete one:

Given N array elements, we have to delete 1 Element, such that gcd of remaining array is max

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 80 & 15 \\ \cancel{*} & \underbrace{\quad}_{\text{gcd}(\quad)} & \quad & \quad & : 1 \end{array}$$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 80 & 15 & : 3 \rightarrow \underline{\underline{\text{ans}}} \\ \cancel{*} & \cancel{*} & \underbrace{\quad}_{\text{gcd}(\quad)} & \quad & \quad \end{array}$$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & \cancel{*} & \underbrace{\quad}_{\text{gcd}(\quad)} & : 1 \end{array}$$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & \cancel{*} & \underbrace{\quad}_{\text{gcd}(\quad)} & : 1 \end{array}$$

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 24 & 16 & 18 & 80 & \cancel{*} & : 2 \\ \underbrace{\quad}_{\text{gcd}(\quad)} & \quad & \quad & \quad & \quad & \quad \end{array}$$

// gcd of entire array:

$$\text{if skip } i^{\text{th}} \text{ elem} = \frac{\text{gcd}(\quad)}{\text{arr}[i]}$$

Initial: Remove every element & get gcd of entire array

$$TC: N * [\underbrace{\text{get gcd of entire array}}_{\in O(N)}] \Rightarrow O(N^2)$$

// While taking gcd, skip the element we want to remove

// Assume N=7: 0 1 2 3 4 5 6

Delete gcd:

$$0 \quad \text{gcd}([1-6])$$

$$1 \quad \text{gcd}(\text{gcd}[0-0], \text{gcd}[2-6])$$

$$2 \quad \text{gcd}(\text{gcd}[0-1], \text{gcd}[3-6])$$

$$3 \quad \text{gcd}(\text{gcd}[0-2], \text{gcd}[4-6])$$

$$4 \quad \text{gcd}(\text{gcd}[0-3], \text{gcd}[5-6])$$

$$5 \quad \text{gcd}(\text{gcd}[0-4], \text{gcd}[6-6])$$

$$6 \quad \text{gcd}([0-5])$$

// Pf gcd[i] = gcd of all Elements [0 i] }

// Sf gcd[i] = gcd of all Elements [i N-1] }

Optimization:

// Construct $Pf\text{gcd}[N]$;

$$Pf\text{gcd}[0] = ar[0]$$

$i=1; i < N; i = i + 1 \{$

$$\left. \begin{array}{l} Pf\text{gcd}[i] = \underbrace{\text{gcd}}_{[0, i]} (\underbrace{Pf\text{gcd}[i-1]}_{[0, i-1]}, \underbrace{ar[i]}_{[i]}) \\ [0, i-1] \leftrightarrow [i] \end{array} \right\}$$

// Construct $Sf\text{gcd}[N]$

$$Sf\text{gcd}[N-1] = ar[N-1]$$

$i = N-2; i >= 0; i-- \{$

$$\left. \begin{array}{l} Sf\text{gcd}[i] = \underbrace{\text{gcd}}_{[i, N-1]} (\underbrace{Sf\text{gcd}[i+1]}_{[i+1, N-1]}, \underbrace{ar[i]}_{[i]}) \\ \text{gcd}[i, N-1] \leftarrow \text{gcd}[i+1, N-1] \leftrightarrow [i] \end{array} \right\}$$

// Delete Every element

ans = 0;

i = 0; i < N; i++) {

// delete i^m

if left $\rightarrow [0, \underline{i-1}] \rightarrow \text{Pf gcd}[i-1]$

right $\rightarrow [\underline{i+1}, N-1] \rightarrow \text{sf gcd}[i+1]$

if $i=0$ Edge Case

if $i=N-1$ Edge Case

TODO: handle Edge Cases

ans = max(ans, gcd(left, right))

}

Remember Subtract: $\frac{a}{m} - \frac{b}{m} = \frac{a-b}{m}$

$$A\%m = B\%m$$

$$A\%m - B\%m = 0$$

$$A\%m - B\%m + m = m$$

Apply $\%m$ on both sides

$$(A\%m - B\%m + m)\%m = 0$$

\downarrow

$$(A-B)\%m = 0$$

$$\frac{m}{B-A} : A > B$$
$$\frac{m}{B-A} : B > A$$

3Q) Pubg: $\{ \text{TODO} \} \rightarrow \text{Hint: (Repeated Subtraction)}$

N players in pubg. Each player has initial health $A[i]$

If player i attacks j \rightarrow { health not effected to person who attacks }

$\begin{cases} \text{if } (A[i] >= A[j]) \{ \\ \quad \text{player j dies} \\ \quad A[j] = 0 \\ \} \\ \text{if } (A[i] < A[j]) \{ \\ \quad A[j] = A[j] - A[i] \\ \} \end{cases}$

Ex: $\frac{A}{6} \quad \frac{B}{10}$

② find min health of last person standing

{ 6, 4 }

Observation:

Conclude:

$$\underline{E_n} = \{12, 2\epsilon\}$$

↓

\mathcal{E}_n :

{ 6 4 8 }