Today's Content :

→ Submatrix Sum Queries → ✓

→ Sum of all Submatrices → ✓

→ Max Submatrix Sum

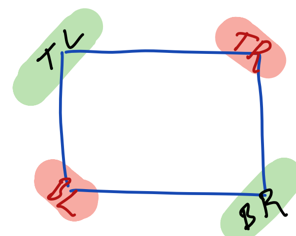
→ (Revise) : 2D matrix
Sum of all Subarrays
Kadane's

// Similar → for every query get subarray sum
↳ pf[i] = Sum from [0] - [i]

Q) Given a matrix of size N*M (rows, columns), for each query q find sum of given submatrix.

└> part of a matrix

└> Full Matrix/Single Element



Q: TL (2, 1) BR (4, 2) = $\begin{Bmatrix} 9+8+-1 \\ 2+2+6 \end{Bmatrix}$ = 26

Ex:

A :

	0	1	2	3
0	2	-1	3	2
1	3	2	6	2
2	10	9	8	2
3	4	-1	2	3
4	3	2	6	9

If (TL & BR) are fixed

or (TR & BL) are fixed

For our discussion: TL & BR

Idea: For every query iterate in submatrix & get sum.

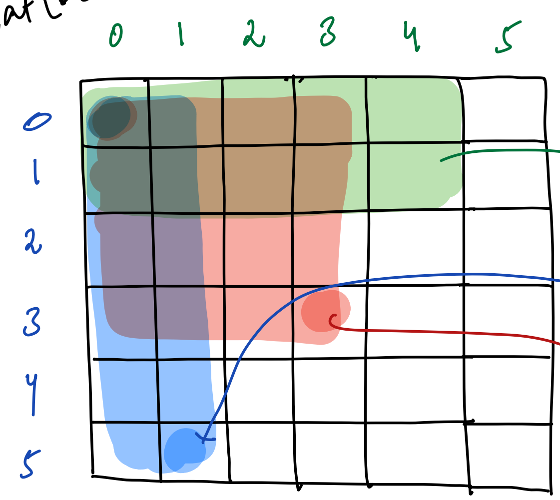
TC: Q * {N*M}

SC: O(1)

Idea2: Pf[ ][ ] : prefix matrix

Pf[i][j] : Sum of all Elements {[0, 0] - [i j]}

mat[6][6]

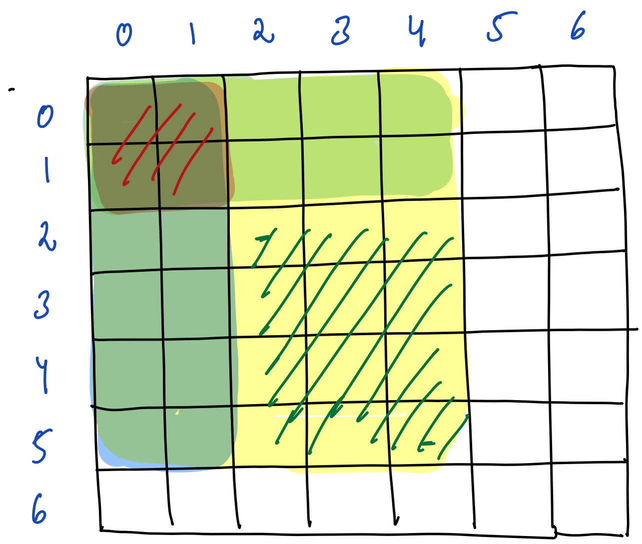


Pf[1][4] = Sum {[0, 0] - [1, 4]}

Pf[5][1] = Sum {[0, 0] - {5, 1}}

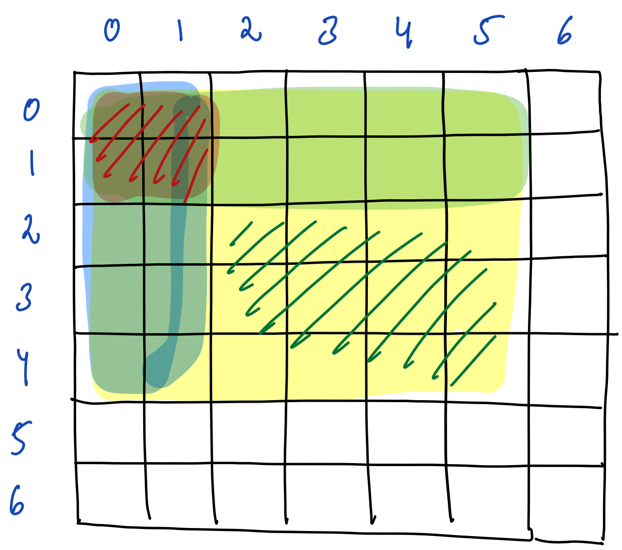
Pf[3][3] = Sum {[0, 0] - [3, 3]}

// Assume pf[][], calculate Query ?



$$\text{sum}\left\{ \underset{TL}{(2,2)} - \underset{BR}{(5,4)} \right\} = ?$$

$$pf[5][4] - pf[5][1] - pf[1][4] + pf[1][1]$$

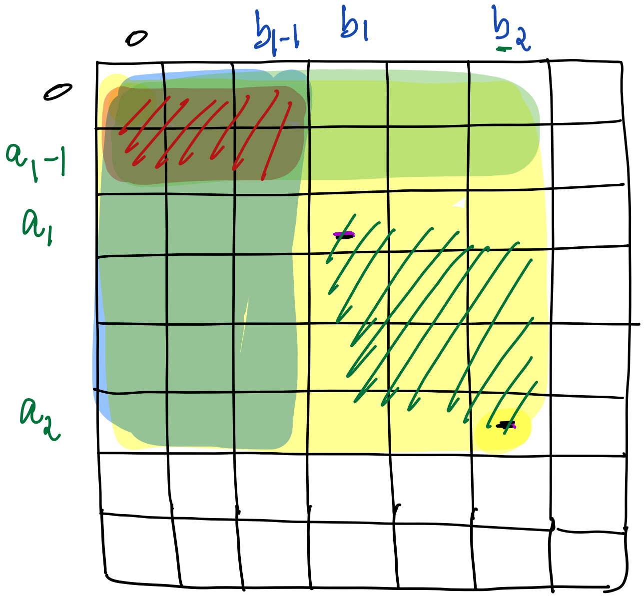


$$\text{sum}\{[0,0] - \{4,5\}\}$$

$$\underset{TL}{(2,2)} - \underset{BR}{(4,5)}$$

$$pf[4][5] - pf[4][1] - pf[1][5] + pf[1][1]$$

// Generalize $\underset{TL}{(a_1, b_1)}$ $\underset{BR}{(a_2, b_2)}$



Q : TL (a_1, b_1) BR $(a_2\ b_2)$

$Pf[a_2\ b_2] - Pf[a_2, b_1-1]$

$- Pf[a_1-1\ b_2] + Pf[a_1-1\ b_1-1]$

// Given in Question

$\left.\begin{array}{l} a_2 >= a_1 \\ b_2 >= b_1 \end{array}\right\}$

// Given TL & BR

(a_1, b_1) (a_2, b_2) } TL (a_1, b_1) BR $(a_2\ b_2)$

ans $= Pf[a_2\ b_2] - \underbrace{Pf[a_2, b_1-1]}_{\text{if } (b_1 > 0)} - \underbrace{Pf[a_1-1\ b_2]}_{\text{if } (a_1 > 0)} + \underbrace{Pf[a_1-1\ b_1-1]}_{\text{if } (a_1 > 0\ \&\&\ b_1 > 0)}$

overall TC to answer

Q Query $\Rightarrow$ $Q * (O(1)) + (N * M)$

$\Rightarrow$ TC $= (Q + N * M)$

SC =
- O(1) : If we directly update original
- Take a copy & construct on That : $O(N * M)$

Calculating pf[][] → TC: (Pf in every row + Pf in every col

$[N * (M) + M * (N)] \rightarrow \boxed{O(N*M)}$

mat[][]

a_0	b_0	c_0
a_1	b_1	c_1
a_2	b_2	c_2

$pf[1][2] = \begin{matrix} a_0 + b_0 + c_0 \\ a_1 + b_1 + c_1 \end{matrix}$

$pf[2][1] = \begin{matrix} a_0 + b_0 \\ a_1 + b_1 \\ a_2 + b_2 \end{matrix}$

→ on every row apply pf sum

a_0	a_0+b_0	$a_0+b_0+c_0$
a_1	a_1+b_1	$a_1+b_1+c_1$
a_2	a_2+b_2	$a_2+b_2+c_2$

on every col apply pf sum →

	0	1	2
0	a_0	a_0+b_0	$a_0+b_0+c_0$
1	a_0+a_1	a_0+b_0 a_1+b_1	$a_0+b_0+c_0$ $a_1+b_1+c_1$
2	$a_0+a_1+a_2$	a_0+b_0 a_1+b_1 a_2+b_2	$a_0+b_0+c_0$ $a_1+b_1+c_1$ $a_2+b_2+c_2$

2Q Given a matrix of size N*M calculate sum of all Submatrix Sums

Ex1: $\begin{array}{c|c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 3 & 1 \\ -1 & -2 \\ 2 & 4 \end{bmatrix} \end{array}$ =

$\left\{ [3] \quad [3\ 1] \quad \begin{bmatrix}3\\-1\end{bmatrix} \quad \begin{bmatrix}3&1\\-1&-2\end{bmatrix} \quad \begin{bmatrix}3\\-1\\2\end{bmatrix} \quad \begin{bmatrix}3&1\\-1&-2\\2&4\end{bmatrix} \quad [1] \quad \begin{bmatrix}1\\-2\end{bmatrix} \quad \begin{bmatrix}1\\-2\\4\end{bmatrix} \quad [-1] \quad [-1\ -2] \quad \begin{bmatrix}-1\\2\end{bmatrix} \quad \begin{bmatrix}-1&-2\\2&4\end{bmatrix} \quad [-2] \quad \begin{bmatrix}-2\\4\end{bmatrix} \quad [2] \quad [2\ 4] \quad [4] \right\}$

ele contrib

3 * 6

1 * 6

-2 * 8

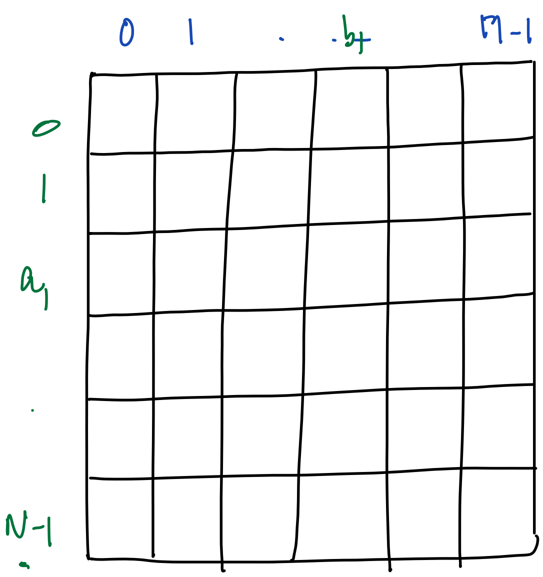
-1 * 8

2 * 6

4 * 6

36

BF Idea :



Q: ?

// Idea : $\left\{ \begin{array}{l} \text{For every matrix} \\ \text{mat[i][j]} \end{array} \right\} * \left\{ \underbrace{\begin{array}{l} \text{// Calculate in how many} \\ \text{submatrices ele mat[i][j]} \\ \text{is present = count} \end{array}} \right\}$

// How to fin Submatr → (TL & BR)

optiming Technique

// How many submatrix cell [2,3] is = ?

	0	1	2	3	4	5
0	T	T	T	T		
1	T	T	T	T		
2	T	T	T	TB	B	B
3				B	B	B
4				B	B	B

Note: To form Submatrix

TL & BR

(a_1, b_1) (a_2, b_2)

$a_2 >= a_1$
$b_2 >= b_1$

In how places we form TL = 12

and

In how many place BR = 9

cells => 12 * 9 = 108 :

// How many submatrix cell [1,2] is = ?

	0	1	2	3	4
0	T	T	T		
1	T	T	TB	B	B
2			B	B	B
3			B	B	B

Note: To form Submatrix

TL & BR

(a_1, b_1) (a_2, b_2)

In how places we form TL = 6

and

In how many place BR = 9 =>

=> 54

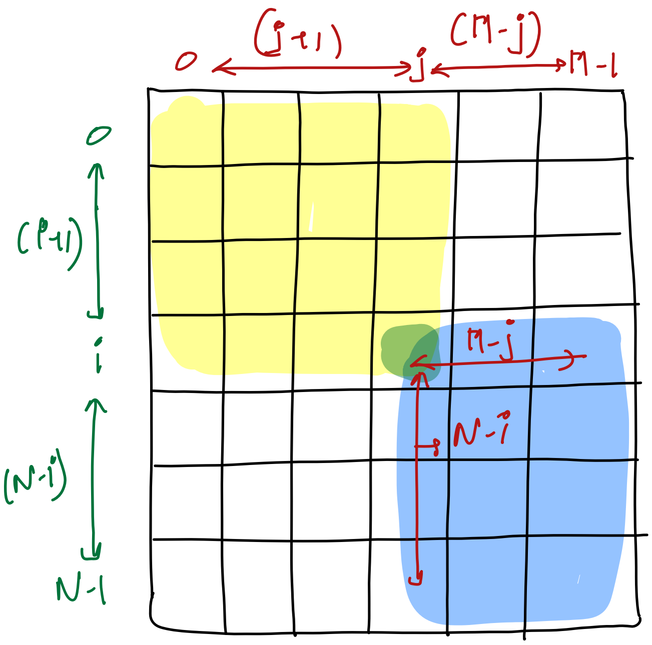
TL	BR
0 0	1, 2
0 1	1, 3
0 2	1, 4
1 0	2 2
1 1	2 3
1 2	2 4
	3 2
	3 3
	3 4

for every TL & BR

You get unique submatrix =

[a b] => b-a+1 =

// Given mat[N][M], in how many submatrices cell [i, j] present?



[i, N-1] → [N-1 - i + 1] → [N-i]

Note: To form submatrix

TL & BR

(a_1, b_1) (a_2, b_2)

In how many places we form TL = $(i+1)*(j+1)$

and

In how many places BR = ? $(N-i)(M-j)$

Total no. of cells → TL * BR

$= (i+1)*(j+1)*(N-i)*(M-j)$

Total Contribution of mat[i][j]

```
i = 0; i < N; i++){
    j = 0; j < M; j++){
        ans = ans + mat[i][j] * (i+1)*(j+1) * (N-i)*(M-j)
    }
}
```

TL places: i

BR places: j

$(i+1)$ $(N-i)$

$(j+1)$ $(M-j)$

11 PM break

TC: $O(N*M)$

SC: $O(1)$

(Red color)

30Q) Given a matrix [N * M]

P1: Given a matrix find max submatrix sum

Note: Where submatrix starts row = 0 & row = N-1

0 2

Ex: mat[3][5]

	0	1	2	3	4	5
0	-3	2	3	4	-6	4
1	5	5	-5	2	2	-7
2	-4	-3	1	-1	1	4

	0	1	2	3	4	5		
ar[] =	-2	4	-1	5	-3	1	=	8 ans

48) Given a matrix find max submatrix sum ?

Note: Where submatrix starts at row = 0 & can end anywhere?

end = (0, 1, 2, 3)

Ex: mat[4][6]

2	-4	1	3	-1	2
1	3	2	-7	3	3
0	-1	1	3	4	-7
1	-1	-6	4	-4	6

mat[N][M]

Each row will have M El.

Case-I

								ans
		0	0	0	0	0	0	
st	end							
0	0	2	-4	1	3	-1	2	5
		2	-4	1	3	-1	2	
Add 1st row →		1	3	2	-7	3	3	
0	1 2	3	-1	3	-4	2	5	8
2nd row ←		0	-1	1	3	4	-7	
0	2	3	-2	4	-1	6	-2	10 ans
3rd row ←		1	-1	-6	4	-4	6	
0	3	4	-3	-2	3	2	4	9

ans = mat[0][0] // submatrix start & ending at 0th row

row can start anywhere

st = 0 ; st < N; st++){

int sum[M] = {0}

row can end anywhere

end = st ; end < N ; end++){

// Add data at end row to your sum[]

i = 0; i < M ; i++){

sum[i] = sum[i] + mat[end][i]

}

ans = max(ans, kadanes(sum[], M))

}

}

return ans;

Max Submatrix Sum

TC: $O(N^2 * (M + M))$

$\Rightarrow O(N^2 M)$

SC: $O(M)$

Tracing → max submatrix sum start = 0 & end anywhere?

2	-4	1	3	-1	2
1	3	2	-7	3	3
0	-1	1	3	4	-7
1	-1	-6	4	-4	6

st   end   ar[6] 0 0 0 0 0 0

0   0   ar[6] 2 -4 1 3 -1 2 → ans = 5

1st row : 1 3 2 -7 3 3

0   1   ar[6] : 3 -1 3 -4 2 5 → ans = 8

2nd row : 0 -1 1 3 4 -7

0   2   ar[6] : 3 -2 4 -1 6 -2 → ans = 10 → final ans

3rd row : 1 -1 -6 4 -4 6

0   3   ar[6] = 4 -3 -2 3 2 4 → ans = 9

Pseudo Code

start = 0 & end anywhere?

st = 0

```
int ar[M] = {0};
end = 0; end < N; end++) {
    // Add end row's data to ar[]
    i = 0; i < M; i++) {
        ar[i] = ar[i] + mat[end][i];
    }
    ans = max(ans, kadanes(ar[], M))
}
```