Agenda

1) Basics of division
2) Modular Properties
3) Divisibility Rules
4) Challenging problem.

% → Modulo / Remainder

$$\boxed{\begin{array}{l} 10 \% 4 = 2 \\ 13 \% 5 = 3 \end{array}}$$

Divisor → 5) 13 (2 → Quotient ; Dividend → 13

- 10

3 → Remainder

Divident = Divisor X Quotient + Remainder

$$\text{Remainder} = \text{Divident} - \underbrace{(\text{Divisor} \times \text{Quotient})}_{\text{greatest multiple of divisor} <= \text{dividend}}$$

QUIZ 1

$$150 \% 11 \rightarrow 150 - \underbrace{\overset{(11 \times 13)}{143}} = 7$$

QUIZ 2

$$100 \% 7 \rightarrow 100 - \underbrace{98} = 2$$

$-100 > -200$

-43

-42

-76

-37

$-76 < -43 < -42 < -37$

KBC

(kids version)

$-40 \% 7 = -40 - \{$ greatest multi of 7 $<= -40 \}$

$-42 <= -40$

$-40 - (-42)$

$-40 + 42 = 2$

$-60 \% 9 = -60 - \{ \underbrace{9 <= -60}_{-63} \}$

$= -60 - (-63)$

$= -60 + 63 = 3$

. . .

Remainder

$a \% m \rightarrow [0 \quad m-1]$

C++/Java . Python

$-40 \% 7 = -5 + 7 \longrightarrow 2$

$-60 \% 9 = -6 + 9 \longrightarrow 3$

$-30 \% 4 = -2 + 4 \longrightarrow 2$

0
1
2
3
4
5
6
7

Remainder
$\% 5$

0
1
2
3
4

$a \% 10 \rightarrow [0 \quad 9]$

Application :

1. Hashing
2. Consisent Hashing
3. Encryption

Modular Arithmetic

+ , − , * , /

$(a + b) \% M = (a \% M + b \% M) \% M$

$a = 16 \quad b = 8 \quad m = 10$

$(16 + 8) \% 10$

$(24) \% 10$

$= 4$

$(16 \% 10 + 8 \% 10) \% 10$

$(6 + 8) \% 10$

$14 \% 10 = 4$

$(a * b) \% M = (a \% M * b \% M) \% M.$

Que. Implement a power function

$\text{power}(a, n, p) \rightarrow a^n \% p$

$a = 2 \quad n = 5 \quad p = 7 \Rightarrow (2^5) \% 7$ → 32

$\boxed{32} \% 7 = 4$

$[0 \quad p-1]$

$a = 3 \quad n = 4 \quad p = 6 \Rightarrow (3^4) \% 6$

$81 \% 6 = 3$

```
power(a, n, p) {
    long ~~int~~ ans = 1
    for( i = 1 ; i <= N ; i++) {
            ans = (ans % p * a % p) % p
    }
    return ans
}
```

TC → O(N)
SC → O(1)

$a^n \rightarrow a * a * a * \ldots$ n times

$p <= 10^9$.

$(p-1)^2 = p^2$

$(10^9)^2 = 10^{18}$

$a = 10 \quad n = 40$

ans → 10^{40}

int ? $<= 10^9$

long ? $<= 10^{18}$

$(a * b) \% p$

$(a \% p * b \% p) \% p$

$(a * b) \% M = (a \% M * b \% M) \% M$.

Divisibility Rule of 3 ?

sum of all the digits should be divisible by 3.

$$4372 = 4 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 2$$

$1 \% 3 = 1$

$10 \% 3 = 1$

$10^2 \% 3 = 1$

$10^3 \% 3 = 1$

$\vdots$

$10^n \% 3 = 1$

$$(4372) \% 3 = (4 * 10^3 + 3 \times 10^2 + 7 * 10 + 2) \% 3$$

$$= \Big((4 * 10^3) \% 3 + (3 \times 10^2) \% 3 + (7 * 10) \% 3 + 2 \% 3 \Big) \% 3$$

$$= \Big((4 \% 3 * 10^3 \% 3) \% 3 + (3 \% 3 * 10^2 \% 3) \% 3 + (7 \% 3 * 10 \% 3) \% 3 + 2 \% 3 \Big) \% 3$$

$$\Big((4 \% 3) \% 3 + (3 \% 3) \% 3 + (7 \% 3) \% 3 + (2 \% 3) \Big) \% 3$$

$$(4 \% 3 + 3 \% 3 + 7 \% 3 + 2 \% 3) \% 3$$

$$(a \% m + b \% m) \% m = (a+b) \% m$$

$$(4 + 3 + 7 + 2) \% 3$$

Divisibility Rule of 4

Last two digits should be divisible by 4.

$x \geq 2$

$10^2 \% 4 = 0$

$10^3 \% 4 = 0$

$10^4 \% 4 = 0$

$\vdots$

$\boxed{10^x \% 4 = 0}$

$(3\ 4\ 8\ 4) \% 4$

$(3\times10^3 + 4\times10^2 + 8\times10 + 4) \% 4$

$((3\times10^3) \% 4 + (4\times10^2) \% 4 + (8\times10) \% 4 + (4 \% 4)) \% 4$

$0 \quad 0$

$((8\times10) \% 4 + 4 \% 4) \% 4$

$(80 \% 4 + 4 \% 4) \% 4$

$(a \% m + b \% m) \% m = (a+b) \% m$

$(80 + 4) \% 4$

$(84) \% 4$

Break

10:25

$10^5 \to 6$ digits $(\underbrace{100000})$

Google.

$10^9 \to 10$ digits

$10^{18} \to 19$ digits

$10^{100} \to 101$ digits

$\underline{10^{100000}} \to \underline{10^5+1}$ digits

Que. N & P, calc N % P

$1 <= \underline{N} <= \underline{10^{100000}}$

$1 <= P <= 10^9$

Int ?

long ?

N is given in form of character array.

Array of characters.

eg

$A = \{3, 8, 4, 3, 6, 8, 9\}$

unit place

$P = 3130$

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$([3,\ 8,\ 4,\ 3,\ 6,\ 8,\ 9])\ \%\ P$$

$$\big((3 \times 10^{6})\ \%\ P + (8 \times 10^{5})\ \%\ P + (4 \times 10^{4})\ \%\ P + (3 \times 10^{3})\ \%\ P + (6 \times 10^{2})\ \%\ P + (8 \times 10^{1})\ \%\ P + (9 \times 10^{0})\ \%\ P\big)\ \%\ P$$

$N = 10^{5}$

0	1	2	3	4	5	6

$N = 7$

$$(\underbrace{A[0]}_{a} \times \underbrace{10^{N-1}}_{b})\ \%\ P$$

$$(a\ \%\ P * b\ \%\ P)\ \%\ P$$

power function

Generalized

a_0	a_1	a_2	. .	. . .

N len of array

$$\left(a_0 \times 10^{N-1} + a_1 \times 10^{N-2} + a_2 \times 10^{N-3} + \dots\right) \% p$$

i		
0	$\to N-1$	$N-1-0$
1	$\to N-2$	$N-1-1$
2	$\to N-3$	$N-1-2$
⋮		⋮
		$N-1-i$

$$\left((a_0 \times 10^{N-1})\%p + (a_1 \times 10^{N-2})\%p + (a_2 * 10^{N-3})\%p \dots\right)\%p$$

$$\left(\underbrace{a_0\%p}_{\text{very easy}} * \underline{10^{N-1}\%p}\right)\%p + \left(\underbrace{a_1\%p}_{\text{very easy}} * \underline{10^{N-2}\%p}\right)\%p + \dots$$

```
ans = 0
for( i = 0 ; i < N ; i++ ) {        -> O(N)
    ans = (ans %p + (A[i] %p * power(10, N-1-i, p)) %p ) %p
}
return ans.
```

$$ans = \Big(\underbrace{ans}_{a}\%p + \underbrace{\left(A[i]\%p * power(10, N-1-i, p)\right)\%p}_{b \,\to\, O(N)}\Big)\%p$$

TC → $O(N^2)$

i

$0 \rightarrow 10^{N-1} \% p$

$1 \rightarrow 10^{N-2} \% p$

$2 \rightarrow 10^{N-3} \% p$

$3 \rightarrow 10^{N-4} \% p$

$\vdots \qquad \vdots$

i

$0 \rightarrow 10^{N-1} \% p \Rightarrow (10 \times 10^{N-2}) \% p \Rightarrow (10 \% p \times 10^{N-2} \% p) \% p$

$1 \rightarrow 10^{N-2} \% p \Rightarrow (10 \times 10^{N-3}) \% p \Rightarrow (10 \% p \times 10^{N-3} \% p) \% p$

$2 \rightarrow 10^{N-3} \% p \Rightarrow (10 \times 10^{N-4}) \% p \Rightarrow (10 \% p \times 10^{N-4} \% p) \% p$

$3 \rightarrow 10^{N-4} \% p \Rightarrow (10 \times 10^{N-5}) \% p \Rightarrow (10 \% p \times 10^{N-5} \% p) \% p$

$\vdots$

$N-4 \rightarrow 10^{3} \% p \Rightarrow (10 \times 10^{2}) \% p \Rightarrow (10 \% p \times 10^{2} \% p) \% p$

$N-3 \rightarrow 10^{2} \% p \Rightarrow (10 \times 10^{1}) \% p \Rightarrow (10 \% p * 10 \% p) \% p$

$N-2 \rightarrow 10^{1} \% p \Rightarrow (10 \times 1) \% p \rightarrow (10 \% p * 1 \% p) \% p$

$N-1 \rightarrow 10^{0} \% p \Rightarrow 1 \% p \Rightarrow 1 \% p.$

```
r = 1
for( i=N-1;  i>=0 ;  i--) {
    ans = ans + (A[i] % p * r % p) % p
    r = (r * 10) % p
}.
return ans.
```

TC → O(N)
SC → O(1)

Bye Bye .