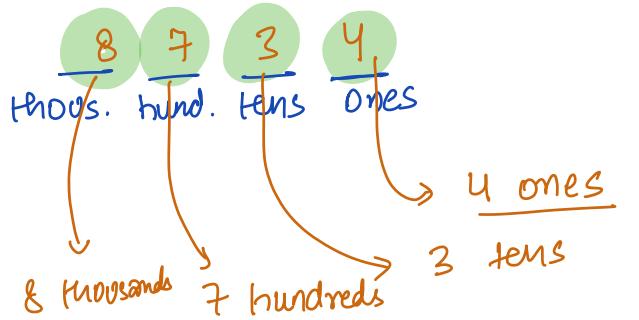


1 | 1 | 1 | 1 | 1 |

1 2 3 4 . . .

Decimal No. System.



Base 8 [Octal No. System]

$$\begin{array}{c} 2 \quad 1 \quad 0 \quad 3 \quad 6 \\ \dots \quad 8^4 \quad 8^3 \quad 8^2 \quad 8^1 \quad 8^0 \end{array}$$

$$2 \times 8^4 + 1 \times 8^3 + 0 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 \rightarrow (8734)_{10}$$

$$\text{Decimal} \rightarrow [0 \ 9]$$

$$\text{Octal} \rightarrow [0 \ 7]$$

$$\text{Ternary} \rightarrow [0 \ 2]$$

$$(1 \ 1 \ 2 \ 0)_3 \rightarrow \begin{array}{r} 1 \quad 1 \quad 2 \quad 0 \\ \dots \quad 3^3 \quad 3^2 \quad 3^1 \quad 3^0 \end{array}$$

$$1 \times 3^3 + 1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0 \rightarrow (42)_{10}$$

- QUIZ ① -

$$(125)_8 \rightarrow (\quad)_{10}$$

$$\begin{array}{ccc} 1 & 2 & 5 \\ 8^2 & 8^1 & 8^0 \end{array}$$

$$1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0 = 85$$

QUIZ 2

$$(02101)_3 \rightarrow (\quad)_{10}$$

$$\begin{array}{ccccc} 0 & 2 & 1 & 0 & 1 \\ \frac{0}{3^4} & \frac{2}{3^3} & \frac{1}{3^2} & \frac{0}{3^1} & \frac{1}{3^0} \end{array}$$

$$0 \times 3^4 + 2 \times 3^3 + 1 \times 3^2 + 0 \times 3^1 + 1 \times 3^0 = 64$$

Octal $\rightarrow [0 \ 7]$

QUIZ 3

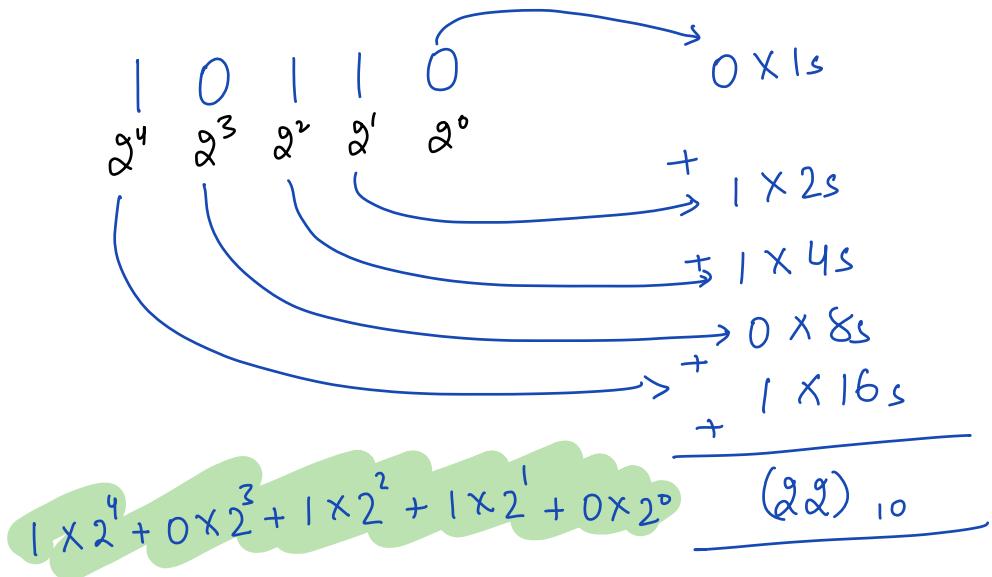
a) 1000001

b) $6854 X$

c) 2576

d) 7460

Binary No. System \Rightarrow 2 unique nos.
 [0 & 1]



Binary to Decimal

deconstruct 22 as sum of power of 2

$$\begin{aligned}
 (22)_{10} & \rightarrow 16 + 6 \\
 & \downarrow \\
 & 4 + 2
 \end{aligned}$$

$$22 \rightarrow 16 + 4 + 2$$

$$\begin{aligned}
 2^0 &= 1 \\
 2^1 &= 2 \\
 2^2 &= 4 \\
 2^3 &= 8 \\
 2^4 &= 16 \\
 2^5 &= 32
 \end{aligned}$$

$$\begin{array}{r}
 1 \quad 0 \quad 1 \quad 1 \quad 0 \\
 \hline
 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0
 \end{array}$$

Dec \rightarrow Binary

Long Division

$$\begin{array}{r}
 2 \overline{)22} - 0 \\
 2 \overline{)11} - 1 \\
 2 \overline{)5} - 1 \\
 2 \overline{)2} - 0 \\
 2 \overline{)1} - 1 \\
 \hline
 0
 \end{array}$$

$$(1\ 0\ 1\ 1\ 0)_2.$$

You may read about other conversions.

QV12

$$= (25)_{10} \longrightarrow (?)_2$$

$$\begin{array}{r}
 2 \overline{)25} - 1 \\
 2 \overline{)12} - 0 \\
 2 \overline{)6} - 0 \\
 2 \overline{)3} - 1 \\
 2 \overline{)1} - 1 \\
 \hline
 0
 \end{array}$$

$(1\ 1\ 0\ 0\ 1)_2$

Q17

$$(37)_{10} \longrightarrow (?)_2$$

$$\begin{array}{r} 2 | 37 - 1 \\ 2 | 18 - 0 \\ 2 | 9 - 1 \\ 2 | 4 - 0 \\ 2 | 2 - 0 \\ 2 | 1 - 1 \\ \hline 0 \end{array}$$

$$(100101)_2$$

Decimal Addition

$$\begin{array}{r} 6/10 \quad 13/10 \quad 10/10 \quad 16/10 \\ \textcircled{0} \quad \textcircled{3} \quad \textcircled{4} \quad \textcircled{5} \quad \textcircled{9} \\ + \quad \textcircled{2} \quad \textcircled{8} \quad \textcircled{4} \quad \textcircled{7} \\ \hline 6 \quad 3 \quad 0 \quad 6 \\ 6 \cdot .10 \quad 13 \cdot .10 \quad 10 \cdot .10 \quad 16 \cdot .10 \end{array}$$

$\underbrace{n \cdot M}_{\downarrow} [0 \quad M-1]$

$$\begin{array}{r} 1/10 \quad 12/10 \quad 11/10 \quad 14/10 \quad 12/10 \quad 14 \\ \textcircled{0} \quad \textcircled{1} \quad \textcircled{1} \quad \textcircled{1} \quad \textcircled{1} \quad \textcircled{1} \\ + \quad \textcircled{3} \quad \textcircled{7} \quad \textcircled{7} \quad \textcircled{6} \quad \textcircled{4} \\ \hline 1 \quad 2 \quad 1 \quad 4 \quad 2 \\ 1 \cdot .10 \quad 12 \cdot .10 \quad 11 \cdot .10 \quad 14 \cdot .10 \quad 12 \cdot .10 \end{array}$$

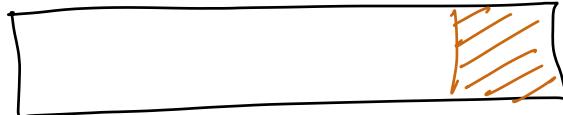
Decimal Addition

Generalize

$C \rightarrow$ quotient
(sum/B)

$$d = \text{sum} \% B$$

$$C = \text{sum} / B$$



Binary Addition

$$\begin{array}{ccccccccc}
 & 1/2 & 1/2 & 3/2 & 2/2 & 1/2 & & \\
 & 0 & 0 & 1 & 1 & 0 & & \\
 \textcircled{1} & 1 & 0 & 0 & 1 & 1 & & \\
 & 0 & 0 & 1 & 1 & 1 & & \\
 \hline
 & 1 & 1 & 1 & 0 & 1 & & \\
 & 1/2 & 1/2 & 3/2 & 2/2 & 1/2 & & \\
 \hline
 \end{array}$$

$$\underbrace{x-1 \cdot M}_{[0 \quad m-1]}$$

$$\underbrace{n-1 \cdot 2}_{[0 \quad 1]}$$

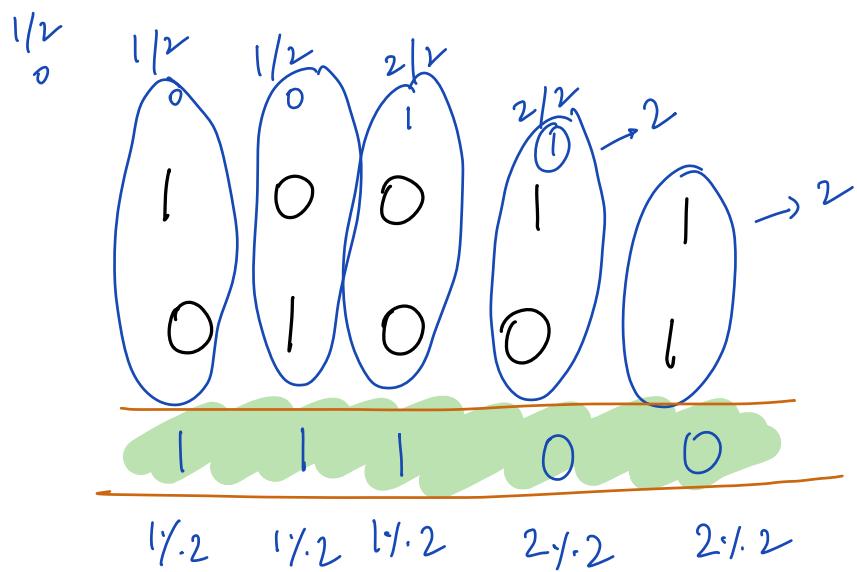
$$\begin{array}{ccccccccc}
 & 1/2 & 2/2 & 4/2 & 3/2 & 3/2 & & \\
 & 0 & 1 & 2 & 1 & 0 & 1 & \\
 \textcircled{4} & 1 & 0 & 1 & 1 & 0 & 0 & \\
 & 1 & 0 & 0 & 1 & 1 & 1 & \\
 & 0 & 1 & 1 & 1 & 1 & 1 & \\
 \hline
 & 1 & 0 & 0 & 1 & 1 & & \\
 & 1/2 & 2/2 & 4/2 & 3/2 & 3/2 & & \\
 \hline
 \end{array}$$

→ 5
 → 4
 → 3
 → 7
 → 19

$$\begin{array}{ccccc}
 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\
 | & 0 & 0 & 1 & 1
 \end{array}$$

$$1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$16 + 0 + 0 + 2 + 1 \rightarrow 19$$



Bitwise Operations

($\&$ | ^ ~ \gg \ll)

Next Session

a	b	$a \& b$	$a b$	$a ^ b$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

$$a = 4$$

$$b = 3$$

$$\begin{array}{rr} 0 & 0 \\ 1 & 1 \end{array} \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right.$$

$$a \& b$$

$$\begin{array}{r} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \hline 0 & 0 & 0 \end{array} \Rightarrow 0$$

$$\begin{array}{r} 1 & 0 \\ 0 & 1 \end{array} \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right.$$

$$a | b$$

$$\begin{array}{r} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \hline 1 & 1 & 1 \end{array} \Rightarrow 7$$

$$a ^ b$$

$$\begin{array}{r} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \hline 1 & 1 & 1 \end{array} \Rightarrow 7$$

16 8 4 2 1

QUIZ
=

$$a = 13$$
$$b = 10$$

$$\begin{array}{r} 2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \\ 1 \quad 1 \quad 0 \quad 1 \\ 1 \quad 0 \quad 1 \quad 0 \\ \hline & & & & 10 \\ & & & & -8 \\ & & & & \hline & & & & 2 \\ & & & & -2 \\ & & & & \hline & & & & 0 \end{array}$$

$$a \& b$$

$$1 \ 0 \ 0 \ 0 = 8$$

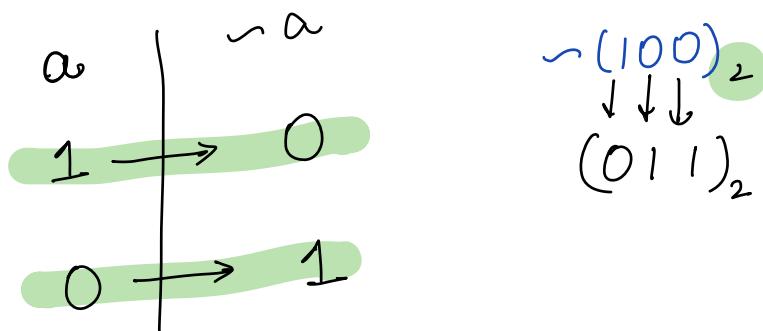
$$a \mid b$$

$$1 \ 1 \ 1 \ 1 = 15$$

$$a \sim b$$

$$0 \ 1 \ 1 \ 1 = 7$$

NOT operators



Simple Observation

	2^3	2^2	2^1	2^0
1	0	0	0	1
5	0	1	0	1
3	0	0	1	1
11	1	0	1	1
9	1	0	0	1

	2^3	2^2	2^1	2^0
4	0	1	0	0
6	0	1	1	0
10	1	0	1	0
8	1	0	0	0
2	0	0	1	0

10:34

→ Break:

Quiz
=

$$\begin{array}{r} a = 11 \\ \hline a | 1 \end{array}$$

$$\begin{array}{r} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 1 & 1 \end{array}$$

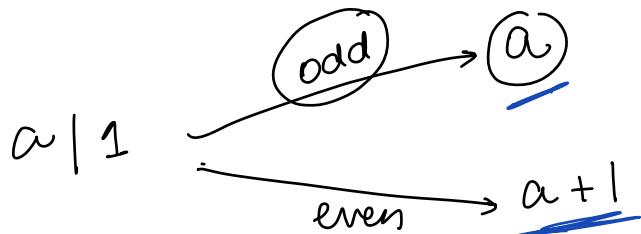
$$a = 10$$

$$\underline{a | 1}$$

$$\begin{array}{r} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 1 & \underline{1} \end{array}$$

For even no.,

$$\begin{array}{l} 0 | 1 \rightarrow 1 \\ 0 | 0 \rightarrow 0 \\ 1 | 1 \rightarrow 1 \\ 1 | 0 \rightarrow 1 \end{array}$$



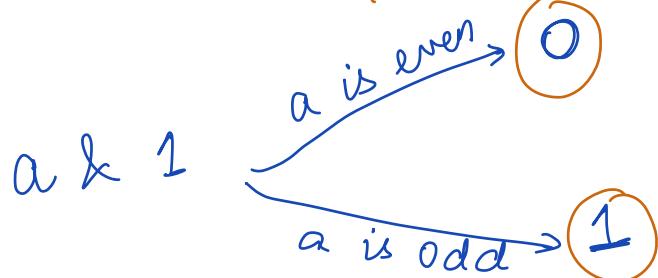
$a = n$

$a \& 1$

$$\begin{array}{r} 0 \& 1 \\ 1 \& 1 \end{array} \rightarrow \begin{array}{r} 0 \\ 1 \end{array}$$

$$\begin{array}{r} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{r} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}$$

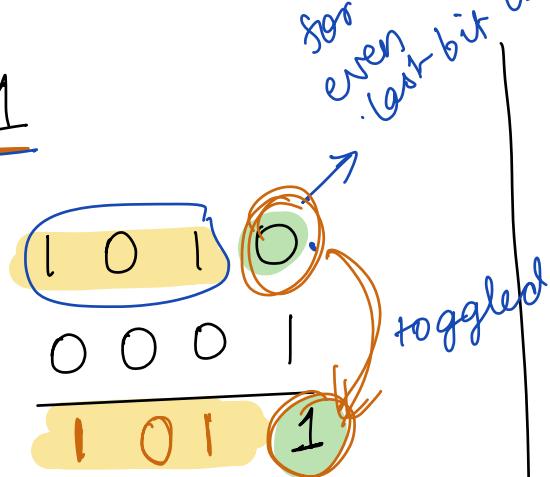


```
if ( $a \& 1 == 0$ ) {
    print("a is even")
}
else {
    print("a is odd")
}
```

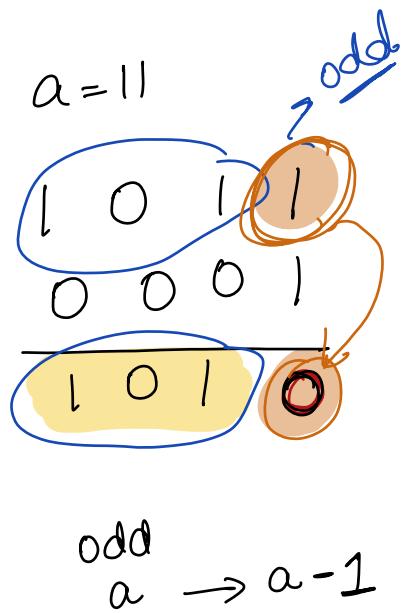
QUIZ

$a \wedge 1$

$a = 10$

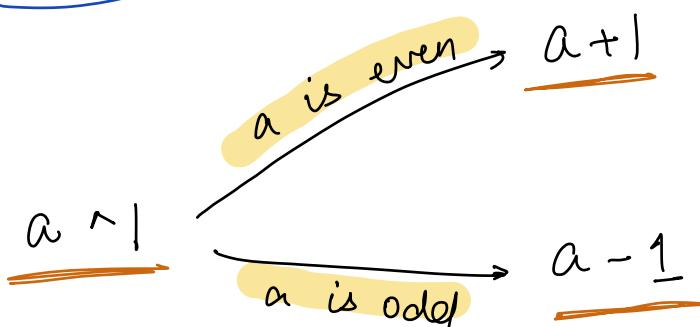


even
 $a \rightarrow a + 1$



$$\begin{aligned} 1 \wedge 0 &\rightarrow 1 \\ 0 \wedge 0 &\rightarrow 0 \end{aligned}$$

$$\begin{aligned} 1 \wedge 1 &= 0 \\ 0 \wedge 1 &= 1 \end{aligned}$$



$$a | a \rightarrow a \text{ (same)}$$

$$a \& a \rightarrow a \text{ (same)}$$

$$a \wedge a \rightarrow 0$$

$$a \wedge 0 \rightarrow a$$

$$a \wedge b = b \wedge a$$

$$a \& b = b \& a$$

$$a | b = b | a$$

commutative
property.

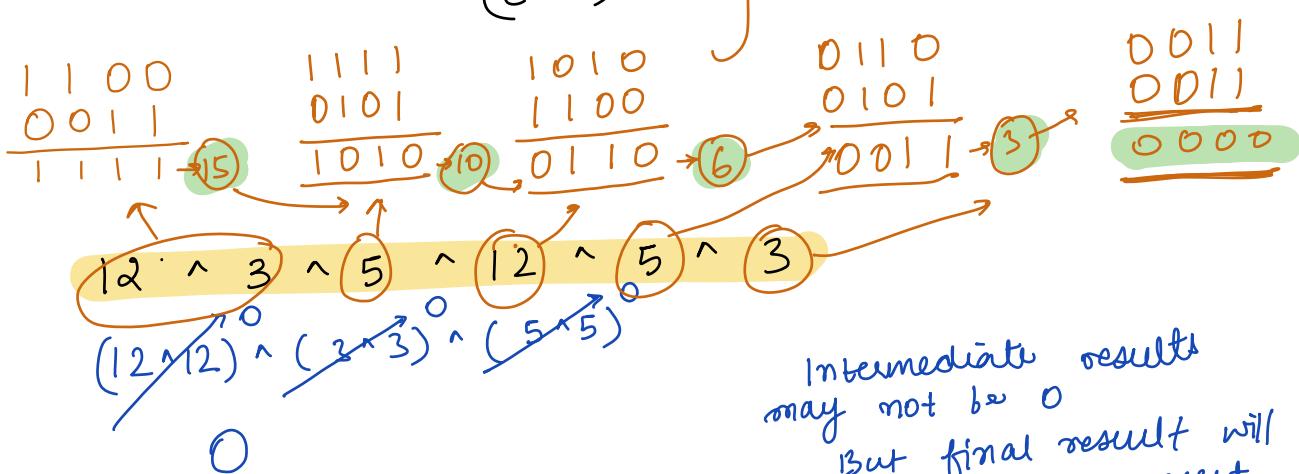
$$a \wedge b \wedge c = (a \wedge b) \wedge c$$

$$= (a \wedge c) \wedge b$$

$$= (c \wedge b) \wedge a$$

some

associative
property.



Intermediate results
may not be 0
But final result will
be 0 if ele are present
in pairs.

If $\underline{a \wedge b = K}$

$$a \wedge K = b$$

$$b \wedge K = a$$

$$a \wedge b = K$$

xor with a on both sides

$$\cancel{a \wedge a \wedge b} = K \wedge a$$

$$0 \wedge b = K \wedge a$$

$$b = K \wedge a$$

$$a \wedge b = K$$

xor with b both sides

$$\cancel{a \wedge b \wedge b} = K \wedge b$$

$$a \wedge 0 = K \wedge b$$

$$a = K \wedge b$$

Single Number

Adobe MS
Dyo Amazon
Amdocs

Ques. Given an array, all ele appear even no. of times
except 1 ele that appears odd no. of times.
Find the ele that appears odd no. of times.



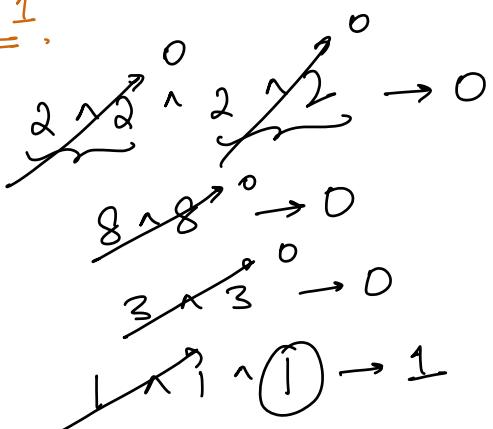
Time $\rightarrow O(N)$
Space $\rightarrow O(1)$

$$a \wedge a \rightarrow 0$$

$$a \wedge a \wedge a \wedge a \rightarrow 0$$

$$a \wedge a \wedge a \rightarrow a$$

$$\text{ans} = 1$$



.....

$x = 0$
 for($i=0$; $i < N$; $i++$) {
 | $x = x \wedge A[i]$ $TC \rightarrow O(N)$
 |
 }
 return x

 $\frac{1}{N} \rightarrow \frac{4 \text{ Bytes}}{(N \times 4 \text{ Bytes})} \rightarrow 32 \text{ bits}$

 $O(N)$

Dec to Octal

$$\begin{array}{r}
 8 | 8734 \\
 \hline
 8 | 1091 \\
 \hline
 8 | 136 \\
 \hline
 8 | 17 \\
 \hline
 8 | 2 \\
 \hline
 0
 \end{array}
 - 6 \\
 - 3 \\
 - 0 \\
 - 1 \\
 - 2$$

$(21036)_8$