Agenda

1) Negative Nos.
2) Ranges
3) Overflows
4) Questions

$$\begin{array}{cccccccc} \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

MSB
Most significant Bit

LSB
least significant Bit

$$\underbrace{1 \;\; 0 \;\; 0 \;\; 0}_{8} \;>\; \underbrace{0 \;\; 1 \;\; 1 \;\; 1}_{7}$$

$$\begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array} \;>\; \begin{array}{cccccccc} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

2^7

$$\underbrace{2^0 + 2^1 + 2^2 + \dots 2^6}$$

$$\frac{2^0(2^7-1)}{2-1} \qquad r=2,\; a=2^0$$

$$\frac{1(2^7-1)}{1} = 2^7-1$$

$$\underbrace{\frac{1}{2^{N-1}}}_{2^{N-1}} \; \frac{0}{2^{N-2}} \; \frac{0}{2^{N-3}} \; \frac{0}{2^{N-4}} \cdots \frac{0}{2^2} \; \frac{0}{2^1} \; \frac{0}{2^0} \;>\; \frac{0}{2^{N-1}} \; \underbrace{\frac{1}{2^{N-2}} \; \frac{1}{2^{N-3}} \; \frac{1}{2^{N-4}} \cdots \frac{1}{2^2} \; \frac{1}{2^1} \; \frac{1}{2^0}}_{2^0+2^1+2^2+\cdots 2^{N-2}}$$

$$2^{N-1} \qquad\qquad 2^{N-1}-1$$

MSB overpowers

Negative No. Representation in Binary

10 : 0 0 0 0 1 0 1 0

assume MSB will be 1 for ve- nos.
& 0 " ve+ nos.

-10 : 1 0 0 0 1 0 1 0

8 Bit

2/2

-3 :	1	1	0	0	0	0	0	1	1
-4 :		1	0	0	0	0	1	0	0
-7		0	0	0	0	0	1	1	1

X

					1	1		
6 :	0	0	0	0	0	1	1	0
-2 :	1	0	0	0	0	0	1	0
4	1	0	0	0	1	0	0	0

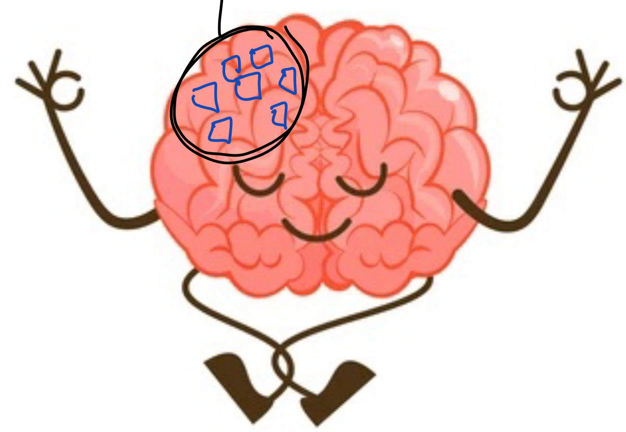
0 :	0	0	0	0	0	0	0	0
-0 :	1	0	0	0	0	0	0	0

N

↓

-N

Shift + Delete.



How to get Negative Representation of N in Binary.

$N \xrightarrow[\text{2's complement}]{} -N$

Steps

1. Toggle all the Bits of N → 1's complement
2. Add 1

Why 2's complement works?

N : 10	0	0	0	0	1	0	1	0
1's complement	1	1	1	1	0	1	0	1
							+	1
Add +1	1	1	1	1	0	1	1	0
	-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$-2^7 \times 1 + 2^6 \times 1 + 2^5 \times 1 + 2^4 \times 1 + 0 + 2^2 \times 1 + 2^1 \times 1 + 0$$

$$\underbrace{-2^7 + 2^6 + 2^5 + 2^4 + 2^2 + 2^1}_{-10}$$

```
int N;
unsigned int N;
```

4 Bits

max → 7

$$\underset{-2^3}{0} \quad \underset{2^2}{1} \quad \underset{2^1}{1} \quad \underset{2^0}{1}$$

min → -8

$$\underset{-2^3}{1} \quad \underset{2^2}{0} \quad \underset{2^1}{0} \quad \underset{2^0}{0}$$

4 Bit [-8 7]

# bits	Min	Max
2	$\underset{-2^1}{1}\ \underset{2^0}{0} \rightarrow -2$	$0\ 1 \rightarrow 1$
5	$\underset{-2^4}{1}\ \underset{2^3}{0}\ \underset{2^2}{0}\ \underset{2^1}{0}\ \underset{2^0}{0} \rightarrow -16$	$0\ 1\ 1\ 1\ 1 \rightarrow 15$
⋮		
N	$\underset{-2^{N-1}}{1}\ \underset{2^{N-2}}{0} \cdots \underset{2^2}{0}\ \underset{2^1}{0}\ \underset{2^0}{0} \rightarrow -2^{N-1}$	$\underbrace{\underset{-2^{N-1}}{0}\ \underset{2^{N-2}}{1} \cdots \underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{1}}_{2^{N-1}-1}$

Range $\{-2^{N-1},\ 2^{N-1}-1\}$

1 Byte $\Rightarrow$ 8 Bits $\{-2^7 \quad 2^7-1\}$

Short int
2 Bytes $\Rightarrow$ 16 Bits $\{-2^{15} \quad 2^{15}-1\} \rightarrow \{-32768,\ 32767\}$

int
4 Bytes $\Rightarrow$ 32 bits $\{-2^{31} \quad 2^{31}-1\}$

$\{-2\times10^9 \quad 2\times10^9-1\}$

long 8 Bytes $\Rightarrow$ 64 bits, $\{-2^{63} \quad 2^{63}-1\}$

$\{-8 \times 10^{18} \quad 8 \times 10^{18}\}$

Approximation

$$2^{10} = 1024 \simeq 10^3$$

$$(2^{10})^3 \simeq (10^3)^3$$

$$2^{30} \simeq 10^9$$

$$2^{10} \simeq 10^3$$

$$(2^{10})^6 \simeq (10^3)^6$$

$$2^{60} \simeq 10^{18}$$

$$\underbrace{8 \times 2^{60}}_{} \simeq 8 \times 10^{18}$$

$$2^{63} \simeq \underline{8 \times 10^{18}}$$

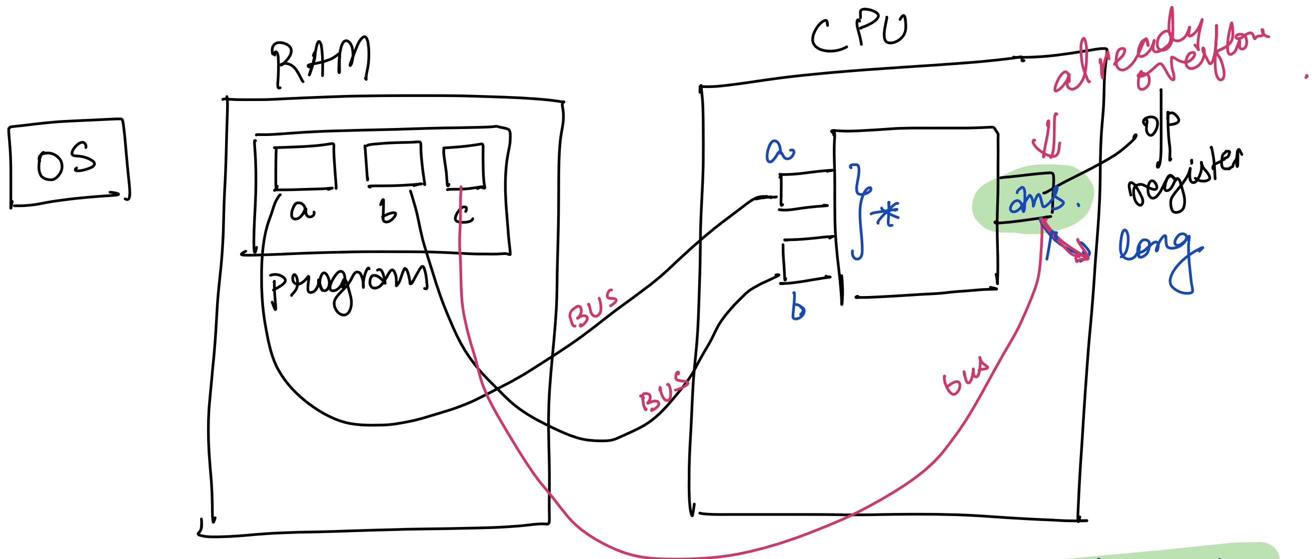
Overflow

$int\ a = 10^5,\ b = 10^6$

$int\ c = \underbrace{a \times b}_{10^5 \times 10^6 = 10^{11}}$ ✗ —— 1

long c = a × b ✗ —— 2

long c = long(a × b) ✗ —— 3



long c = (long)(a) * b

long c = a * long(b)

Que. Given N array ele, calc sum of array elements

```
long
~~int~~ sum = 0
for(i=0; i<N; i++){
    sum += A[i]
}
return sum.
```

Constraints

$1 <= N <= 10^5$

$1 <= arr[i] <= 10^6$

$\boxed{1 <= sum <=}$

$\underbrace{\{10^6, 10^6, \ldots 10^6\}}_{10^5}$

$10^6 \times 10^5 = 10^{11}$

Break

$\boxed{10:40}$

Ques. Given N array elements, each ele is either 0 or 1.

Calc no. of subarrays whose OR of all elements = 1

OR (1)

Ex: { 0, 1, 0, 0, 1 }

(indices: 0 1 2 3 4)

[0 0] → 0
[0 1] → 1
[0 2] → 1
[0 3] → 1
[0 4] → 1

[1 1] → 1
[1 2] → 1
[1 3] → 1
[1 4] → 1

[2 2] → 0
[2 3] → 0
[2 4] → 1
[3 3] → 0
[3 4] → 1
[4 4] → 1

ans : 11

Brute Force

TC → $O(N^2)$
SC → $O(1)$

} Try this out yourself.

Observation

If in a subarray even if there is a single 1, then its OR = 1 } How to check?

If we find out in how many subarray there are all 0s, then we are done?

ans : #totals − # subarrays having all 0s.

$\frac{n(n+1)}{2}$ −

{0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0}

cnt = 3 cnt = 0 1 2 cnt 0 1 2 cnt 0 0 1 2

$\frac{3(4)}{2} = 6$ $\frac{2(3)}{2} = 3$ $\frac{2(3)}{2} = 3$ $\frac{2(3)}{2} = 3$

15

ans = $\frac{13(14)}{2} - 15 = 76$

0 1 0 0 0 1 0 0 1 1 0 0

$cnt = \cancel{0}$ $\quad$ zeros = 0

$\cancel{1}$ $\quad (1 \times 2)/2 = 1$

$\cancel{0}$ $\quad$ 1

$\cancel{1}$ $\quad (3 \times 4)/2 = 6$

$\cancel{2}$

$\cancel{3}$ $\quad$ 7

$\cancel{0}$ $\quad (2 \times 3)/2 = 3$

$\cancel{1}$

$\cancel{2}$ $\quad$ 10

$\cancel{0}$ $\quad (2 \times 3)/2 = 3$

$\cancel{1}$

2 $\quad$ 13

ans: $\frac{12 \times 13}{2} - 13$

$78 - 13 = 65$

Unset x continuous bits in a no. N

$x : 3$

$N : 29$

4 3 2 1 0

1 1 1 0 1

0 0 0

1 1 0 0 0

4 3 2 1 0

1 1 1 0 1

1 1 0 0 0

0 0 1 1 1 $\rightarrow$ 1 1 0 0 0

N-1

N

0 1 0 0 0 → 0 0 1 1 1

↓

1 1 0 0 0

N: 29

1 1 1 0 1

4 3 2 1 0

0 1 0 0 0 → 0 0 1 1 1

1 1 1 0 1

4 3 2 1 0

$x : 3$

yesterday

$(1 \ll x)$

$\sim(2^x - 1) \,\&\, N$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	1	0	1

N: 93

$x : 4$

k

2^x

2^4 0 0 1 0 0 0 0

$2^4 - 1$ $\sim$(0 0 0 1 1 1 1)

⇒ 1 1 1 0 0 0 0   mask

1 0 1 1 1 0 1

(&) 1 1 1 0 0 0 0

1 0 1 0 0 0 0

TC O(1)

SC O(1)

N: 0 1 0 1 0 0 → N−1

$\boxed{0\ 1\ 0\ 0\ 1\ 1}$

1 0 1 1 1 0 1

& 1 1 1 0 0 0 0

1 0 1 0 0 0 0

1 0 1 1 1 0 1

0 0 1 0 0 0 0

~(0 0 0 1 1 1 1)

||

1 1 1 0 0 0 0

1 0 1 1 1 0 1

(k) 1 1 1 0 0 0 0

1 0 1 0 0 0 0

4 3 2 1 0

N: 1 0 1 1 0

0 0 1 0 0

1 1 0 1 1

$x = 3$

$(1<<x)$