

## Today's Content:

- Inversion Count
- Wave Array
- Smaller Subarray to Sort
- Content: { Saturday 12 AM - Monday }
- Content discussion Monday

Q8) Given 2 arrays  $A[N]$  &  $B[M]$

(Duplicates)

Count no of pairs  $(i, j)$  such that  $\underline{A[i] > B[j]}$

Ex1:  $A: \begin{matrix} 0 & 1 & 2 \\ 7 & 3 & 5 \end{matrix}$        $B: \begin{matrix} 0 & 1 & 2 \\ 2 & 0 & 6 \end{matrix} :$

$\begin{matrix} (7, 2) \\ (7, 0) \\ (7, 6) \end{matrix} \left| \begin{matrix} (3, 2) \\ (3, 0) \end{matrix} \right| \begin{matrix} (5, 2) \\ (5, 0) \end{matrix} \} \text{ans} = 7 \text{ pairs}$

Ex2:  $A: \begin{matrix} 0 & 1 & 2 \\ 3 & 1 & 6 \end{matrix}$        $B: \begin{matrix} 0 & 1 & 2 \\ 2 & 4 & 9 \end{matrix}$

$\begin{matrix} (3, 2) \\ (3, 0) \\ (6, 2) \\ (6, 0) \end{matrix} \left| \begin{matrix} (6, 2) \\ (6, 0) \end{matrix} \right| \} \text{ans} = 3 \text{ pairs}$

Brute Force Idea :

Check all pairs & get count

$c = 0$

$i = 0; i < N; i = i + 1 \{$

$j = 0; j < M; j = j + 1 \{$

$\text{if } (A[i] > B[j]) \{$

$c = c + 1$

$T_C: O(N^M)$

$S_C: O(1)$

return  $c;$

Ques 2

Eg 3:  $A[4] = \{7, 8, 2, 4\}$        $B[4] = \{3, 5, 1, 10\}$

Sorting

$A[4] = \{ \begin{matrix} & 2 \\ P_1 & 4 & 7 & 8 \end{matrix} \} \quad B[4] = \{ \begin{matrix} & 1 \\ P_3 & 3 & 5 & 10 \end{matrix} \}$

$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline 1 & 2 & 3 & 4 & 5 & 7 & 8 & 10 \\ \hline \end{array}$$

$c = \frac{1}{4} + 0 + 13 + 0 + 12 + 10 + 10 + 10 = 9$

Eg 4:  $A[4] = \{8, 5, 3, 10\}$        $B[4] = \{5, 4, 9, 2\}$

Soln

No. of Elmts in 1st

$A[4] = \{ \begin{matrix} & 0 & 1 & 2 & 3 \\ P_1 & 3 & 5 & 8 & 10 \end{matrix} \} \quad B[4] = \{ \begin{matrix} & 0 & 1 & 2 \\ P_2 & 2 & 4 & 5 & 9 \end{matrix} \}$

$\left[ P_1 \quad \frac{N-1}{2} \right]$

$= \frac{N-P_1}{2}$

$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline 2 & 3 & 4 & 5 & 5 & 8 & 9 & 10 \\ \hline \end{array}$$

$\frac{N-P_1}{2} + 0 + \frac{N-P_1}{2} + \frac{N-P_1}{2} + \frac{N-P_1}{2} + 1$

$\frac{4}{2} + 0 + \frac{3}{2} + \frac{2}{2} + \frac{2}{2} + 1$

$2 + 0 + 1.5 + 1 + 1 + 1 = 10$

## Pseudocode :

Inversions Count( int A[], int N, int B[], int M ) {

$\text{Sort}(A) \rightarrow N \log N$	$\text{Sort}(B) \rightarrow M \log M$	$SC:$ $O(N) + \log N$	$O(M) + \log M$	}
$C = 0, P_1 = 0, P_2 = 0$				

Total iterations  
 $\rightarrow N \log N + M \log M$   
 $\rightarrow N + M$

$SC: O(N+M)$

while (  $P_1 < N$  &  $P_2 < M$  ) {  $\rightarrow N+M$

If ( $A[P_1] \leq B[P_2]$ ) { //  $A[P_1]$  should come first  
|  
|  $P_1 = P_1 + 1$   
| }  
else { //  $B[P_2]$  should come first //  $A[P_1] > B[P_2]$

|  
 $C = C + \underline{N - P_1}$   
|  $P_2 = P_2 + 1$   
| }  
| }

// If one of them, they are no pairs

return C;

Q3) Google / Microsoft / De Shaw / Inversion Count

Given  $A[N]$ , find no: of pairs  $i, j$  such that  $\underline{i} < \underline{j}$  &  $A[i] > A[j]$

$$\text{Ex1: } ar[5] = \{ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \} \quad \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \\ \text{Index: } \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \quad \boxed{\text{Total = 5}}$$

$$\text{Ex2: } ar[10] = \{ \begin{matrix} 10 \\ 3 \\ 8 \\ 15 \\ 6 \\ 12 \\ 2 \\ 18 \\ 7 \\ 1 \end{matrix} \} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix} \\ \dots \quad \begin{matrix} 6 \\ 2 \\ 4 \\ 5 \\ 2 \\ 3 \\ 1 \\ 2 \\ 1 \\ 0 \end{matrix} = \boxed{26}$$

BF: Check all pairs

$$c = 0;$$

$$i = 0; i < N; i++ \}$$

$$j = i + 1; j < N; j = j + 1 \}$$

$$\text{if } (A[i] > A[j]) \}$$

$$| \\ j \quad c = c + 1$$

$$\boxed{\text{TC: } O(N^2)}$$

$$\boxed{\text{SC: } O(1)}$$

$$arr[10] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$$

$$A[] = \boxed{\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 10 & 3 & 8 & 15 & 6 \end{array}}$$

↓

Sum

$$= 3 \quad 6 \quad 8 \quad 10 \quad 15$$

$\uparrow$   
 $P_1$

$$B[] = \boxed{\begin{array}{cccccc} 5 & 6 & 7 & 8 & 9 \\ 12 & 2 & 18 & 7 & 1 \end{array}}$$

total pairs, sum

$$1 \quad 2 \quad 7 \quad 12 \quad 18$$

$\uparrow$   
 $P_2$

$$\begin{array}{c} 1 | 2 | 3 | 6 | 7 | 8 | 10 | 12 | 15 | 18 \\ + 5 \quad + 5 \quad \quad + 3 \quad \quad + 1 \end{array} = 14$$

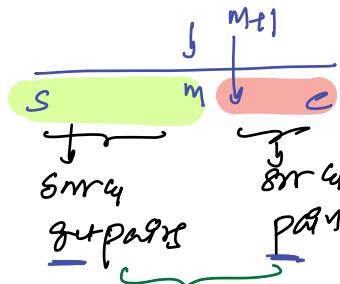
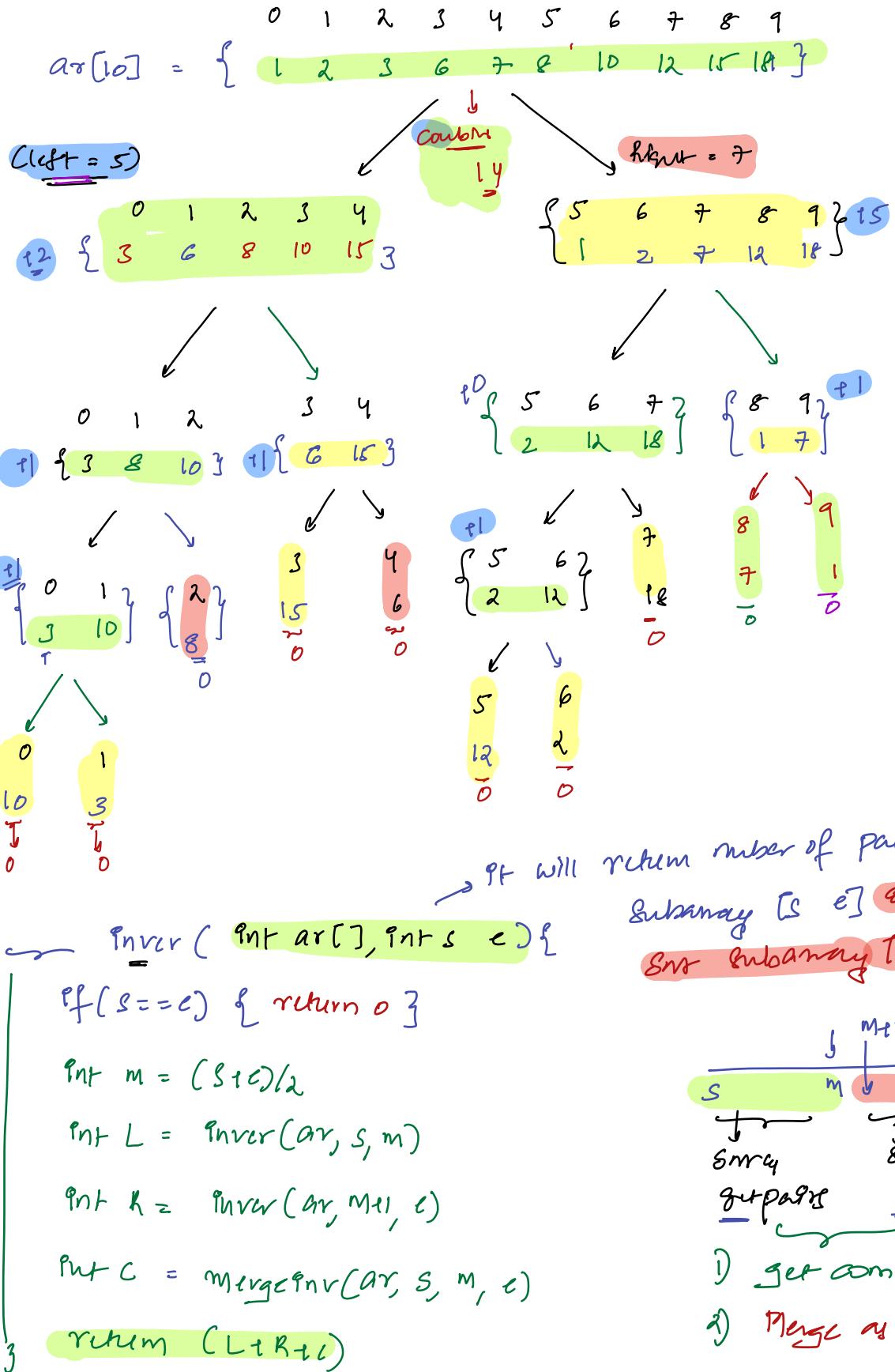
Total Pairs =

$$\{ \text{Total pair } A \} + \{ \text{Total pair } B \} + \{ \text{Total pair Between } \underline{A \text{ & } B} \}$$

We need to sort  $A, B$

so that we need can

get pairs between them



- 1) get combine pair
- 2) Merge as well

ptr mergeInv ( int A[], int s, int m, int c ) {

ptr tmp [ e-s+1 ]; int C = 0

ptr P<sub>1</sub> = s, P<sub>2</sub> = m+1, P<sub>3</sub> = 0

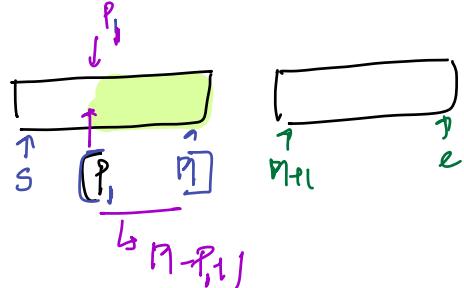
while ( P<sub>1</sub> <= m && P<sub>2</sub> <= e ) {

if ( A[P<sub>1</sub>] <= A[P<sub>2</sub>] ) {

tmp [P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub>++; P<sub>3</sub>++

else

tmp [P<sub>3</sub>] = A[P<sub>2</sub>]; P<sub>2</sub>++; P<sub>3</sub>++



only extra part  
C = C + (m - P<sub>1</sub>)

while ( P<sub>1</sub> <= m ) { tmp [P<sub>3</sub>] = A[P<sub>1</sub>]; P<sub>1</sub>++; P<sub>3</sub>++ }

while ( P<sub>2</sub> <= e ) { tmp [P<sub>3</sub>] = A[P<sub>2</sub>]; P<sub>2</sub>++; P<sub>3</sub>++ }

i = 0; i <= e-s; i++ }

A[i+s] = tmp[i]

return C;

}

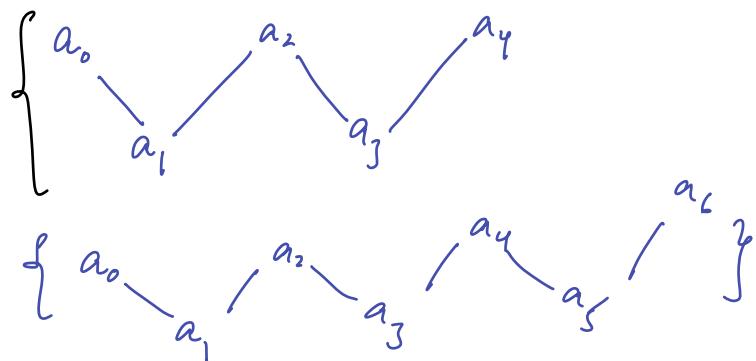
TC: O(N log N)

SC: O(N)

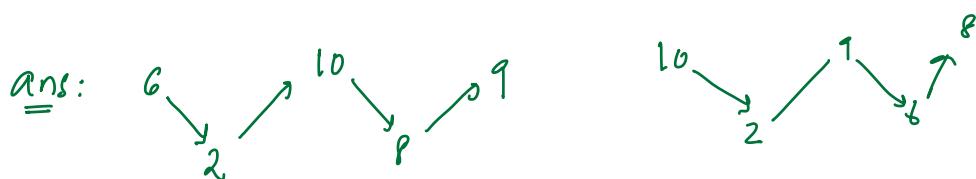
{ 10:45 break

longest

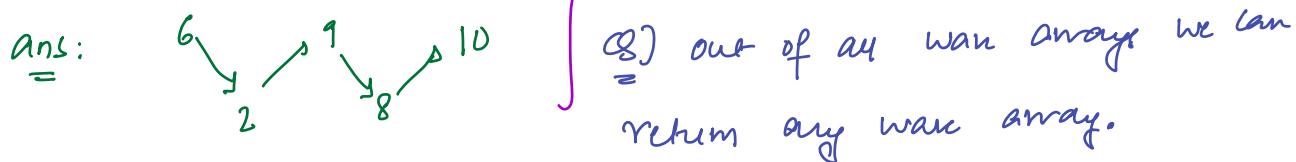
Q Given  $N$  distinct array elements re-arrange array in  
a wave form



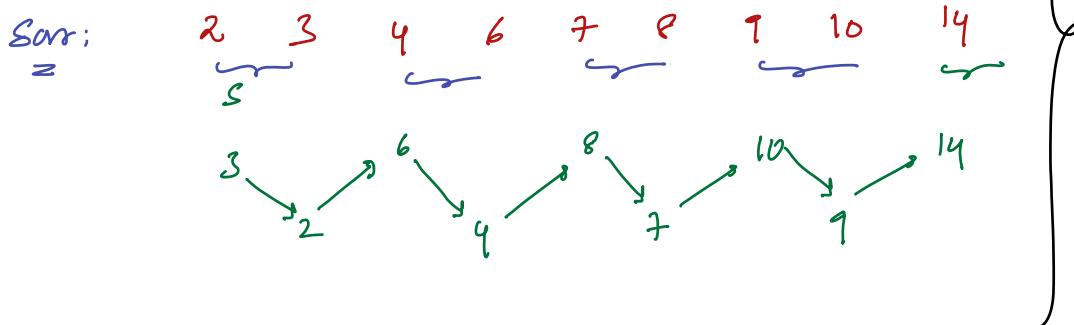
In: 6 8 2 9 10



} More than 1 wave is possible



In: 8 2 4 10 9 3 14 6 7



Idea: Sort array

Swap adjacent pairs

$$i=0; j \leq N; i = i+2$$

// take care of Edge Case

Swap  $a[i], a[i+1]$

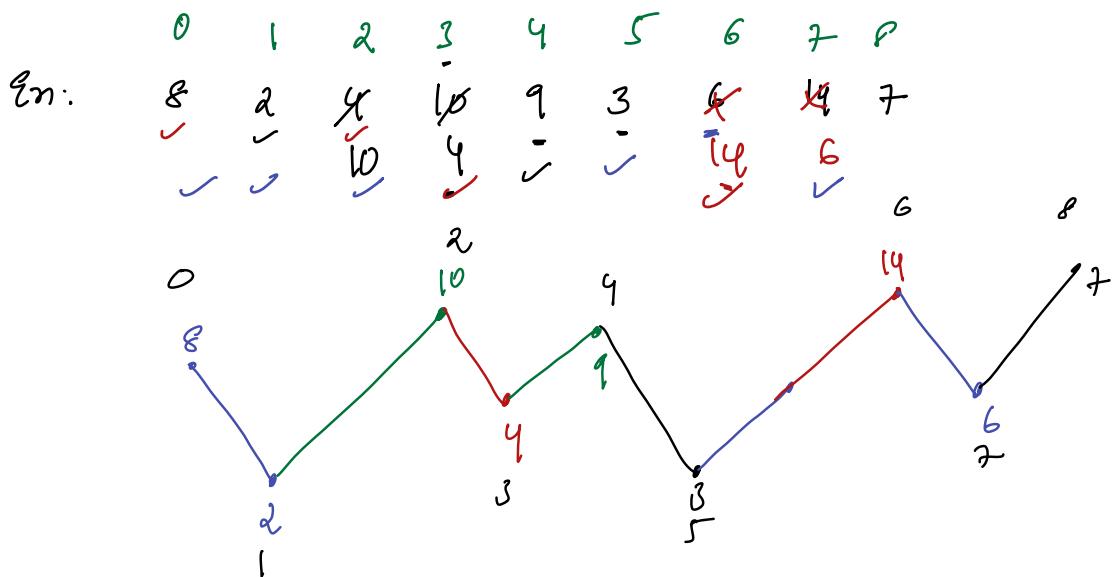
$$Tl: O(N \log N + N)$$

$$TC: O(N \log N)$$

$$SC: O(N)$$

Histogrammally Bef Warr

If this is question we need to  
Sort



Idea 2 TC: O(N) SC: O(1)

Edge Case

$i=0; j \leq (N-1); i = i+1$  → check for every position

If  $j \% 2 = 0$ : In that from  $i, i+1$  it should decrease

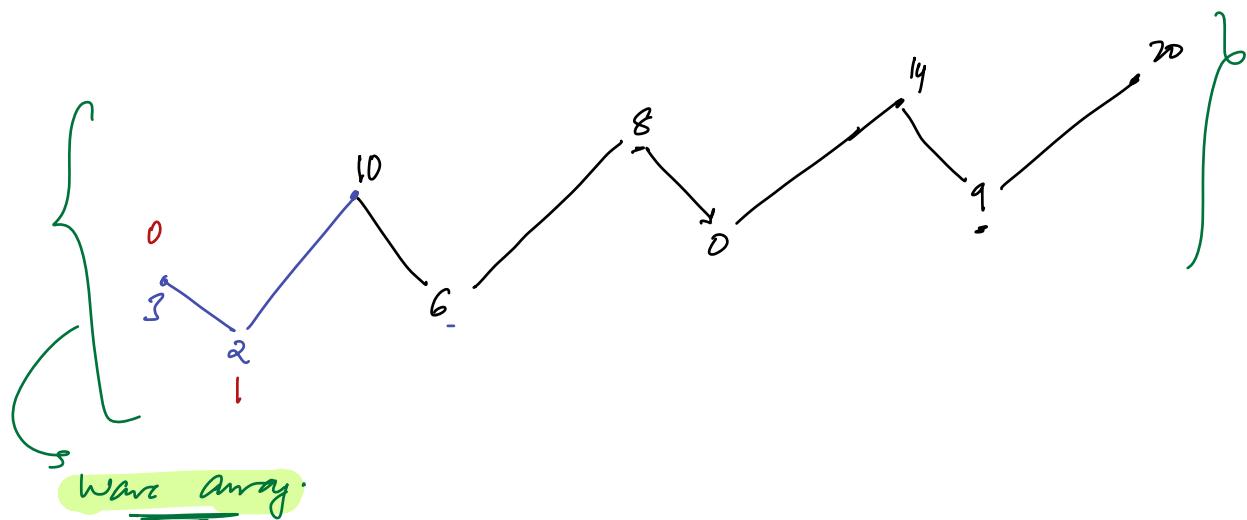
If  $(a[i] < a[i+1]) \{ \text{swap} \}$

If  $j \% 2 = 1$ : In that from  $i, i+1$  it should increase

If  $(a[i] > a[i+1]) \{ \text{swap} \}$

En:

3	2	10	10	8	0	9	14	20
✓	✓	✓	✓	✓	✓	✓	✓	✓
		6	8	8	0	14	9	



// Storing in line:

0	1	2	3	4	5	<u>6</u>	7
1	3	4	6	8	10	14	20

Inplace

[0 6] is erased, new elem at index 7

Comp:

- 6 7 :  $ar[6] > ar[7]$  : swap
- 5 6 :  $ar[5] > ar[6]$  : swap
- 4 5 :  $ar[4] > ar[5]$  : swap
- 3 4 :  $ar[3] > ar[4]$  : swap
- 2 3 :  $ar[2] > ar[3]$  : swap
- 1 2 :  $ar[1] > ar[2]$  : (no swap)

[0 7] is sorted

```
// Ex:    0   1   2   3   4   5
           1   3   7   11  14  20
```

[0 4]

<u>i</u>	$ar[i] > ar[i+1]$
<u>4</u>	$ar[4] > ar[5]$ , swap, $i = i--$
<u>3</u>	$ar[3] > ar[4]$ , swap, $i = i--$
<u>2</u>	$ar[2] > ar[3]$ , swap, $i = i--$
<u>1</u>	$ar[1] > ar[2]$ swap, $i = i--$
<u>0</u>	$ar[0] > ar[1]$ swap, $i = i--$

-1 : Recall (Topic)

$i = N-1$  /  $i+1$  out four bands

// ar[N]:

for(  $int i = 0 ; i \leq N-1 ; i = i+1$  ) {

[0 i] sorted  $\leftarrow [i+1] \rightarrow [0 i+1]$  as sorted

$j = i;$

while(  $j \geq 0$  &  $ar[j] > ar[j+1]$  ) {

Swap(ar[j], ar[j+1])

$j--$

}

insertion sort

TC:  $O(N^2)$

SC:  $O(1)$

// given array [0 0] is smart

p=0: // ar[s]:

0	1	2	3	4
3	2	-4	-1	0
=				

p=1: ar[s]:

0	1	2	3	4
2	3	-4	-1	0
=				

p=2: ar[s]:

0	1	2	3	4
-4	2	3	-1	0
=				

p=3: ar[s]:

0	1	2	3	4
-4	-1	2	3	0
=				

p=4: ar[s]:

0	1	2	3	4
-4	-1	0	2	3
=				

array smart

) no need to print array

Note:

If entire array is sorted Insertion Sort:  $\Theta(N)$

