

Today's Content:

- 1) N Sorted arrays ↗ print completed data in sorted order
↘ store complete sorted data in single array
- 2) given 2 sorted array find k smallest pair sum

1) Merge N Sorted Lists into single sorted list

Ex: $\text{mat}[\underline{N}][\underline{M}] \rightarrow \underline{\text{col index: } [0, M-1]}$

Solutions:

	0	1	2	3	4	5
0	2	7	10	17	25	34
1	-6	0	1	8	11	14
2	3	4	6	14	21	26
3	7	10	14	19	23	27

1) Store all N^*M elements in a 1D array & directly sort it

$$\underline{\text{TC: } (NM + NM \log(NM))}$$

$$\underline{\text{SC: } N^*M} \quad \begin{array}{l} \text{Merge sort: } O(N^*M) \\ \text{Quick sort: } O(1) \end{array}$$

2) Store all N^*M elements in a min-heap, & delete 1 by 1 element insert in new 1D array

$$\underline{\text{TC: } N^*M \log(N^*M) + N^*M \log(N^*M)}$$

$$\underline{\text{SC: } N^*M} \rightarrow \text{max heap size}$$

3) Apply merge on arrays itself,

In N rows $\left\{ \begin{array}{l} \text{merge first } N/2 \text{ rows} \rightarrow \text{1D array} \\ \text{merge last } N/2 \text{ rows} \rightarrow \text{1D array} \end{array} \right\} \xrightarrow{\text{merge them}}$

Total assume n elements

$$T(n) = T(n/2) + T(n/2) + n$$

$$\boxed{T(n) = 2T(n/2) + n}$$

$$T(n) = n \log n$$

$$\hookrightarrow n = N^*M$$

$$T(N^*M) = N^*M \log(N^*M)$$

idea: Pointer approach

0	1	2	3	4	5
P_1 2	P_1 7	P_1 10	P_1 17	25	34
P_2 -6	P_2 0	P_2 1	P_2 8	P_2 11	P_2 16
P_3 3	P_3 4	P_3 6	P_3 14	P_3 21	26
P_4 7	P_4 10	P_4 14	P_4 19	23	27

→ Idea: N pointers, at every step pick pointer with min value store it in 1D array & increment the pointer.
Repeat the above process till all elements are inserted in 1D-array

output

-6 0 1 2 3 4 6 7 7 8 10 10 11 14 14 16 ...

How to implement?

0	1	2	3	4	5
P_1 2	7	10	17	25	34
P_2 -6	0	1	8	11	16
P_3 3	4	6	14	21	26
P_4 7	10	14	19	23	27

→ Idea, min-heap

obs1: We need to know which row an element belongs to

obs2: We need to know current index in P_k row, col number

Data we need to insert:

val, row number, col number

For tracing
say we are storing
using 1st way

1) pair<int, pair<int, int>>, as this a single element in heap

2) Implement your custom class

```
class data {
    int val;
    int row;
    int col;
};
```

minheap<data> mh;

3) pair<int, int>

val ← $[row * M + col]$

$/M \Rightarrow$ row number

$\%M \Rightarrow$ col number

Trac:

	0	1	2	3	4	5
$\rightarrow P_1$	2	7	10	17	25	34
$\rightarrow P_2$	-6	0	1	8	11	16
P_3	3	4	6	14	21	26
P_4	7	10	14	19	23	27

Step:

	<u>getMin()</u>			<u>insert()</u>		
	val	r	c	val	r	c
1	-6	1	0	0	1	1
2	0	1	1	1	1	2
3	1	1	2	8	1	3
4	2	0	0	7	0	1
5	3	2	0	4	2	1

minheap < pair<int, pair<int, int>>
 \downarrow \downarrow \downarrow
 val r c

~~<2, 0, 0>~~ ~~<0, 1, 1>~~

~~<-6, 1, 0>~~ <1, 1, 2>

~~<3, 2, 0>~~ <8, 1, 3>

<7, 3, 0> <7, 0, 1>

<4, 2, 1>

idea: do the above approach $N \cdot M$

TC: $\{ \boxed{NM \log N} + \boxed{NM \log N} \}$
 \downarrow \downarrow
 insert in heap determine in heap

SC: $N \rightarrow$ max size of heap

```
int[] Merge(int mat[N][M]) {
```

```
    int n = mat.length; int m = mat[0].length;  
    int in[N * M]; int ind = 0 { at which index in in[] we are  
                                going to insert }
```

```
    minheap < pair < int, pair < int, int > > mh;
```

```
// Step-1 Insert entire 0th col
```

```
    for (i = 0; i < N; i++) {
```

```
        ele = mat[i][0]
```

```
        mh.Insert(pair(ele, pair(i, 0)));
```

```
    while (mh.size() > 0) {
```

```
        pair < int, pair < int, int > > d = mh.getMin();
```

```
        mh.deleteMin();
```

```
        int val = d.first, r = d.second.first, c = d.second.second
```

```
        in[ind] = val; ind++
```

```
// We need to insert next element in that row
```

```
        if (c + 1 < M) {
```

```
            val = mat[r][c + 1]
```

```
            mh.Insert(pair(val, pair(r, c + 1)))
```

```
    return in;
```

10:30 -> 10:40

TC: $N \cdot M \log N$ SC: N

28) k Smallest Pair sums:

Given 2 sorted arrays, find k smallest pair sums, a single pair only 1 time

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$
 $a[s]: \quad 2 \quad 5 \quad 8 \quad 11 \quad 13$
 $b[t]: \quad 3 \quad 7 \quad 9 \quad 12 \quad 15 \quad 16 \quad 20$

$N = 10^5$
 $M = 10^5$

k=5 output

Pairs:	Sum	i	j
1	5	0	0
2	8	1	0
3	9	0	1
4	11	0	2
5	11	2	0

? both are diff pairs

ideas:

1) Store all pairs sum in 1D array
sort & get 1st k elements

given $a[N], b[M]$
 int $c[N \times M], ind = 0$
 for $i = 0; i < N; i++$

for $j = 0; j < M; j++$

$c[ind] = a[i] + b[j]$

$ind++$

sort $c[]$ & get $[0, k-1]$

Tc: $N \times M \log(N \times M)$

Sc: $N \times M$

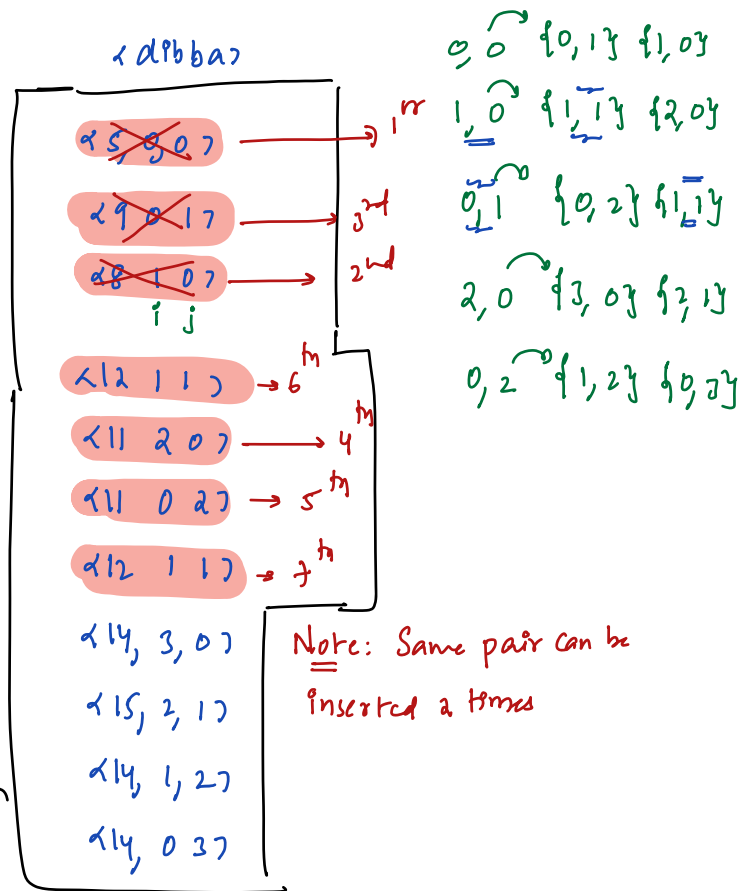
2) Generate all pairs but insert
in min heap of size k

Tc: $N \times M \log k + N \times M \log k$
 insert delete

Sc: k

0 1 2 3 4 5 6
 a[5]: 2 5 8 11 13
 b[7]: 3 7 9 12 15 16 20

Pairs	val	i	j
1 st	5	0	0
2 nd	8	1	0
3 rd	9	0	1
4 th	11	2	0
5 th	11	0	2
6 th	12	1	1
7 th	12	1	1



0, 0 → {0, 1} {1, 0}
 1, 0 → {1, 1} {2, 0}
 0, 1 → {0, 2} {1, 1}
 2, 0 → {3, 0} {2, 1}
 0, 2 → {1, 2} {0, 2}

Data Structure

- 1) getMin()
- 2) insert()
- 3) deleteMin()

Heap

- 4) Search if a pair is already present or not

i j

to check if pair is there or not
 { i + 4, i + j }

```
void kpairs(int a[], int b[]) {
```

```
    int n = a.length, m = b.length;
```

```
    minheap < pair<int, pair<int, int>>> mh;
```

```
    HashSet<String> hs
```

```
    // Insert mfo pair:
```

```
    mh.insert(pair(a[0]+b[0], pair(0, 0)));
```

```
    hs.insert("0 0");
```

```
    for (int i = 1; i <= k; i++) {
```

```
        pair<int, pair<int, int>> d = mh.getmin();
```

```
        mh.delete(d);
```

```
        int val = d.first, r = d.second.first, c = d.second.second
```

```
        // print(val)
```

```
        // for r, c possibilities → {r+1, c}, {r, c+1}
```

```
        if (c+1 < m && hs.contains(to_string(r) + " " + to_string(c+1)) == false)
```

```
            val = a[r] + b[c+1]
```

```
            mh.insert(pair(val, pair(r, c+1)))
```

```
            hs.insert(to_string(r) + " " + to_string(c+1))
```

```
        if (r+1 < n && hs.contains(to_string(r+1) + " " + to_string(c)) == false)
```

```
            val = a[r+1] + b[c]
```

```
            mh.insert(pair(val, pair(r+1, c)))
```

```
            hs.insert(to_string(r+1) + " " + to_string(c))
```

```
    }
```

insert

delete

max heap size

hashset size

TC: $k \times \log k + k \times \log k$

SC: $O(k + 2k)$