

Todays Content:

- 2D matrix questions
- Number of ways to go from $(0,0)$ → BR cell
 - ↳ // Ways, with blocked cell
- Min Cost to reach BR → TODO
- Dungeons & Dragons

Q) Number of ways to go from $(0, 0) \rightarrow (\text{BR cell})$

$$\begin{matrix} \cancel{0} & 0 \\ \cancel{1} & \end{matrix} \Rightarrow \begin{matrix} 0 \\ \cancel{1} \\ \cancel{2} \end{matrix}$$

Paths:

0	1	2
0	TL	
1		
2		BR

Cell \rightarrow right or bottom

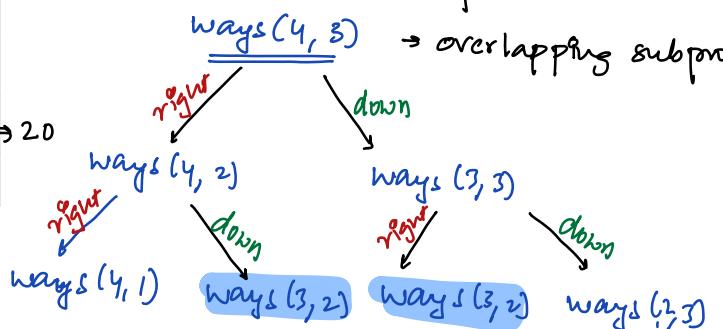
- $(0, 0) (0, 1) (0, 2) (1, 2) (2, 2)$
- $(0, 0) (0, 1) (1, 1) (1, 2) (2, 2)$
- $(0, 0) (0, 1) (1, 1) (2, 1) (2, 2)$
- $(0, 0) (1, 0) (1, 1) (1, 2) (2, 2)$
- $(0, 0) (1, 0) (1, 1) (2, 1) (2, 2)$
- $(0, 0) (1, 0) (2, 0) (2, 1) (2, 2)$

0	1	2	3
0			
1			
2			
3			
4			

//ways to reach from $(0, 0) \rightarrow (4, 3)$

\Rightarrow optimal substructure

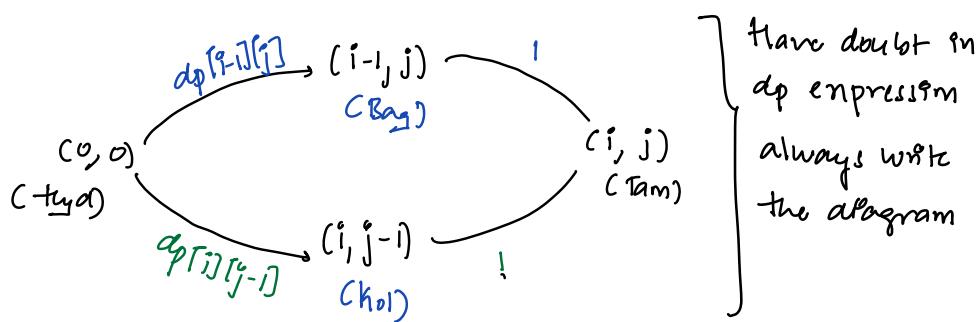
\Rightarrow overlapping subproblem



Steps:

$dp[i][j] = \# \text{ Number of ways to reach from } (0, 0) \rightarrow (i, j)$

$$dp[i][j] = dp[i-1][j] + dp[i][j-1]$$



$$dp[i][j] = \underbrace{dp[i-1][j]}_{i>0} + \underbrace{dp[i][j-1]}_{j>0}$$

Base Conditions: $\{i=0 \text{ or } j=0\}$ case fails

→ If ($i=0 \text{ or } j=0$) $dp[i][j] = 1$

Table: $dp[N][M]$

Code:

```
int ways(int N, int M) {
```

```
    int dp[N][M];
```

```
    i=0; j=0; i<N; i++ {
```

```
        j=0; j<M; j++ {
```

If ($i=0 \text{ or } j=0$) { // handle Basic Conditions

$dp[i][j] = 1$

else {

$dp[i][j] = dp[i-1][j] + dp[i][j-1]$

```
    return dp[N-1][M-1];
```

depends

0	1	2
0		
1		
2		

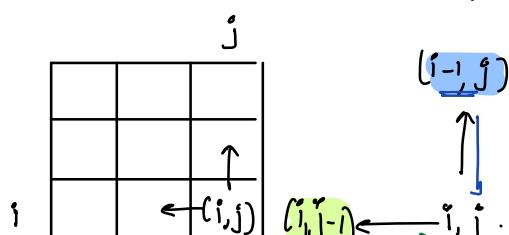
0	1	2
0		
1		
2		✓

→ 1) \leftarrow right → top \rightarrow down

2) top \rightarrow down
left \rightarrow right

TC: # $O(N \times M) + 1$

SC: # $O(N \times M)$



Note: In iteration dp while filling table it's also important on how we are iterating in table.

1) Resolve top \rightarrow down

2) Resolve left \rightarrow right

Q8) Number of ways to go from $(0, 0) \rightarrow (4, 3)$ (BB cells)

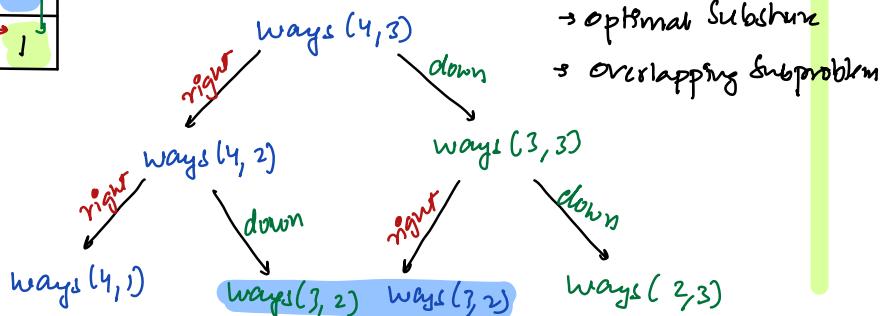
	0	1	2	3
0	1,1	1,1	1,1	1,1
1	1,1	0	1	0
2	0,0	1	1	1
3	1,0	0	1	1
4	1,0	1	1	1

// cell \rightarrow right, cell \rightarrow bottom

// $\text{mat}[i][j] = 0$, it means it blocked

// We cannot traverse a blocked cell

// ways to reach from $(0, 0) \rightarrow (4, 3)$



+ Steps :

$dp[i][j] = \# \text{ Number of ways to reach from } (0, 0) \rightarrow (i, j)$

$$dp[i][j] = \begin{cases} \text{if } (\text{mat}[i][j] == 0) \quad \left\{ \begin{array}{l} \text{if cell blocked} \\ \{ dp[i][j] = 0 \} \end{array} \right. \\ \text{else } \left\{ \begin{array}{l} \{ dp[i-1][j] + dp[i][j-1] \} \end{array} \right. \end{cases}$$

Base Condition :

if ($i == 0 \text{ || } j == 0$) { code fails }

dp_table : int $dp[N][M] = \{0\}$ & default initializing to zero

// Case-I:

Iterate in 0ⁿ jth col

$i = 0; i < n; i++ \{$

$\left. \begin{array}{l} \text{if } (\text{mat}[i][0] == 1) \{ \text{if } dp[i][0] == 1 \\ \text{else } \{ \text{break} \} \end{array} \right\}$

Case-II

Iterate in 0^m row

$j = 0; j < m; j++ \{$

$\left. \begin{array}{l} \text{if } (\text{mat}[0][j] == 1) \{ \text{if } dp[0][j] == 1 \\ \text{else } \{ \text{break} \} \end{array} \right\}$

Code:

```
int ways (int mat[][], int N, int M) {  
    int dp[N][M] = {0} // initialize to 0  
  
    Iterate m om jth col  
    i=0; i < n; i++ {  
        if (mat[i][0] == 1) { dp[i][0] = 1 }  
        else { break }  
    }  
  
    Iterate m om row  
    j=0; j < m; j++ {  
        if (mat[0][j] == 1) { dp[0][j] = 1 }  
        else { break }  
    }  
  
    i=1; i < N; i++ {  
        j=1; j < M; j++ {  
            if (mat[i][j] == 0) { dp[i][j] = 0 }  
            else {  
                dp[i][j] = dp[i-1][j] + dp[i][j-1]  
            }  
        }  
    }  
    return dp[N-1][M-1]
```

```

// int ways( int mat[][], int N, int M) {
    int dp[N][M] = -1; // ways (0, 0) → (N-1, M-1)
    if (mat[0][0] == 0) { return 0; }
    dp[0][0] = 1 // Base condition // VIMP
}

```

$\text{func}(\text{mat}, \text{dp}, \underbrace{\begin{matrix} i \\ N-1, M-1 \end{matrix}}_{\substack{\hookrightarrow \text{It always reducing} \\ \rightarrow \text{It can never touch } N, M}}$

```

int func( int mat[][], int dp[][], int i, int j) {

```

$\text{if}(i < 0 \text{ || } j < 0) \{ \text{return } 0; \}$ } In recursive codes
 $\text{if}(\text{mat}[i][j] == 0) \{ \text{return } 0; \}$ advantage, writing
 $\text{if}(\underline{\underline{\text{dp}[i][j] == -1}} \{ \text{base conditions become easy}$

$\hookrightarrow \text{calculating } \text{dp}[i][j] \text{ for first time}$

$$\text{dp}[i][j] = \left\{ \begin{array}{c} \text{func}(\text{mat}, \text{dp}, i-1, j) \\ + \\ \text{func}(\text{mat}, \text{dp}, i, j-1) \end{array} \right\}$$

$\text{return } \text{dp}[i][j]$

$11:00 \text{ pr} \rightarrow 11:05 \text{ pm}$

TC: $O(\underline{\underline{N^m}}) * O(1)$ SC: $O(\underline{\underline{N^m}})$

3Q) Min cost to reach from $(0,0) \rightarrow \underline{\text{BR cell}}$

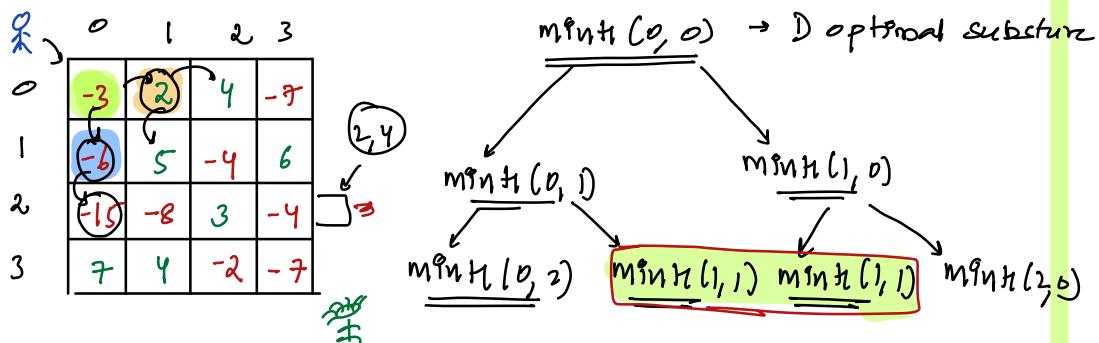
	0	1	2	3	4
0	2	1	3	7	2
1	4	2	1	3	1
2	3	2	3	1	7
3	6	1	3	2	4
4	1	5	2	7	3

→ TODO

// Given $mat[N][M]$, where each cell $mat[i][j]$ indicates health gained, minimum health required at ≥ 0 so that we can reach $[N-1][M-1]$.

- 1) Cell \rightarrow right Cell \rightarrow bottom
- 2) If health reaches ≤ 0 at any place you are dead
- 3) We are starting at $(0, 0) \rightarrow (N-1, M-1)$

$$\begin{array}{|c|c|} \hline -3 & -5 \\ \hline -2 & 1 \\ \hline 2 & 4 \\ \hline 5 & -4 \\ \hline \end{array} \Rightarrow \begin{array}{l} H=4 \\ H=5 \\ H=6 \end{array}$$



$dp[i][j] = \min \text{ health required to start } (i, j) \text{ to reach } N-1, M-1$

Expression :

Case-I :

$$\begin{array}{|c|} \hline -5 \\ \hline \end{array}$$

$$n = 5 = 1$$

Case-II

$$\begin{array}{|c|} \hline -2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline -5 & 6 \\ \hline \end{array}$$

$$n = 6$$

Case-III

$$\begin{array}{|c|c|} \hline -4 & -5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 \\ \hline \end{array}$$

$$n = 10$$

Case-IV

$$\begin{array}{|c|c|} \hline -3 & -2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline -4 & -5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 10 \\ \hline \end{array}$$

$$n = 11$$

Case-V

$$\begin{array}{|c|c|} \hline 6 & -2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline -4 & -5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 8 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 10 \\ \hline \end{array}$$

$$n + 6 = \min(8, 10)$$

$$n + 6 = 8, n = 2$$

Case-VI

Case-VII

$$\begin{array}{|c|c|} \hline -3 & -2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline -4 & -5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 10 \\ \hline \end{array}$$

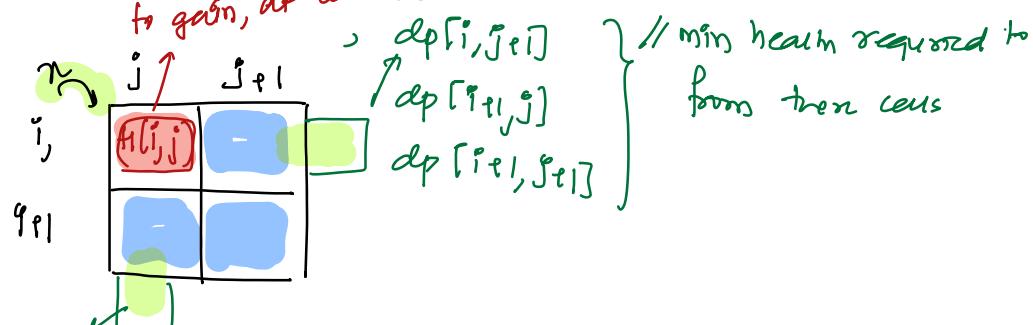
$$n = 10$$

$$n + -3 = \min(8, 10)$$

$$n = \min(8, 10) + 3$$

$$n = 11$$

to gain at cell (i, j)



$$n + H(i, j) = \min(dp[i, j+1], dp[i+1, j])$$

$$n = \min(dp[i, j+1], dp[i+1, j]) - H(i, j)$$

$$dp[i, j] = \min(dp[i, j+1], dp[i+1, j]) - H(i, j)$$

Edge Case, all case $dp[i, j]$ should be 1

Case-VIII

$$n \rightarrow \text{at least } n=1$$

$$\begin{array}{|c|} \hline 10 \\ \hline \end{array}$$

Case-IX

$$\begin{array}{|c|c|} \hline 1 & -2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 9 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline -4 & -5 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 8 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 10 \\ \hline \end{array}$$

$$n + 9 = \min(8, 10)$$

$$n + 9 = 8$$

$$n = -1$$

dp State: $dp[i][j] = \min \text{ health required } [i, j] \rightarrow [N-1, M-1]$

dp Expression: $dp[i][j] = \min(1, \underbrace{\min(dp[i, j+1], dp[i+1, j])}_{\text{Edge Case}} - h[i, j])$

Base Case Cond: $\{ i == N-1 \text{ or } j == M-1 \} // \text{won't work}$

dp tab: $dp[N][M]$

$\min(1 + h[1, 1], N, M) \{$

$\text{int } dp[N][M] = -1$

$dp[N-1][M-1] = \min(1, 1 - h[N-1][M-1])$

$\quad \quad \quad // \min \text{ cost to start at } (0, 0)$

$\text{func}(h, N, M, \underline{dp}, \underline{g})$

}

$\text{int func}(h[T][T], N, M, \underline{dp}, i, j) \{$

$\text{if}(i == N \text{ or } j == M) \{ \text{return INT_MAX} \}$

$\text{if}(dp[i][j] == -1) \{ \text{Edge Case}$

$\text{int } a = \text{fun}(h, N, M, \underline{dp}, i, j+1)$

$\text{int } b = \text{fun}(h, N, M, \underline{dp}, i+1, j)$

$dp[i][j] = \min(1, \underline{\min(a, b)} - h[i][j])$

$\text{return } dp[i][j]$

}

	0	1	2
0	3/4 → 6; 10 -3		
1	-5	9/4 -2	10
2	2	-4 -5/4	10

0	1	2	
0	1	1	11
1	6	1	8
2	8	10	6