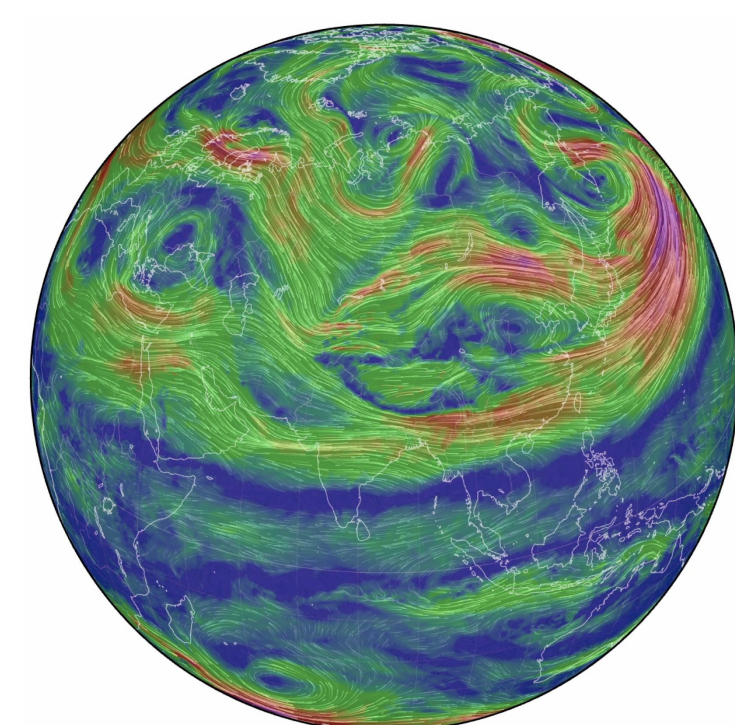


Objective

Model and Forecast:

$$\mathbf{u}(\mathbf{x}, t) = \begin{pmatrix} u_1(\mathbf{x}, t) \\ \vdots \\ u_K(\mathbf{x}, t) \end{pmatrix} \begin{matrix} \text{Temperature} \\ \vdots \\ \text{Precipitation} \end{matrix}$$



Issues with black box modeling

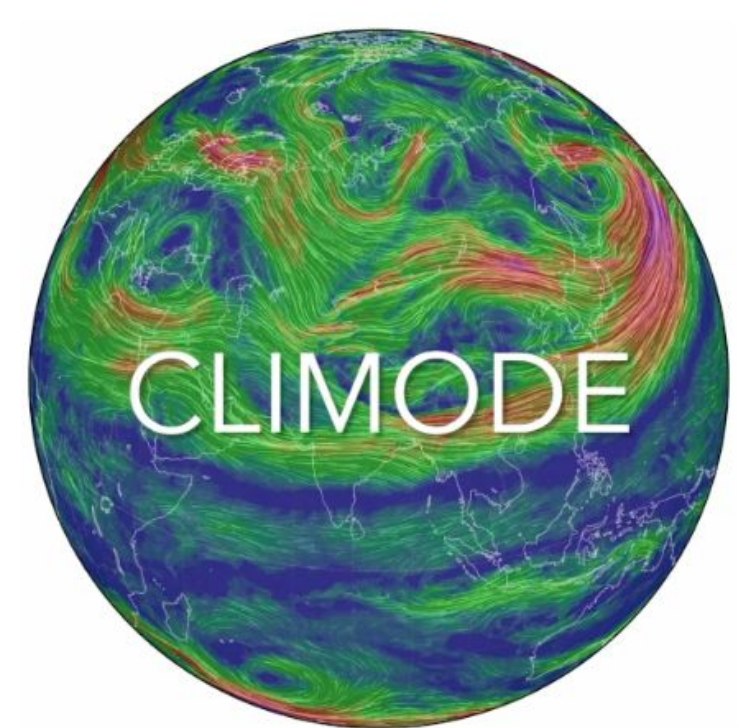
- Black box methods based on Transformers, UNets, etc overlook the **physical dynamics (P)** and **continuous time (CT) nature** and are not **compact (C)**.

- Free Form Neural PDEs** does not assume any physical dynamics but gives solution for continuous time

	P	CT	C
UNet	✗	✗	✗
Neural PDE	✗	✓	✗
ClimODE	✓	✓	✓

Contributions

- Develop Neural ODEs/PDEs that:
 - Follow **P**, **CT** and **C**
 - Provides uncertainty estimates**



Method	Value-preserving	Explicit Periodicity/Seasonality	Uncertainty	Continuous-time	Parameters (M)	
FourCastNet	✗	✗	✗	✗	N/A	Pathak et al. (2022)
GraphCast	✗	✗	✗	✗	37	Lam et al. (2022)
Pangu-Weather	✗	✗	✗	✗	256	Bi et al. (2023)
ClimaX	✗	✗	✗	✗	107	Nguyen et al. (2023)
NowcastNet	✓	✗	✗	✗	N/A	Zhang et al. (2023)
ClimODE	✓	✓	✓	✓	2.8	this work

Advection PDE

Basic idea:

- Movement of a substance u (scalar field) via motion \mathbf{v} (vector field) as bulk motion.
- Value conserving dynamics and require currents.

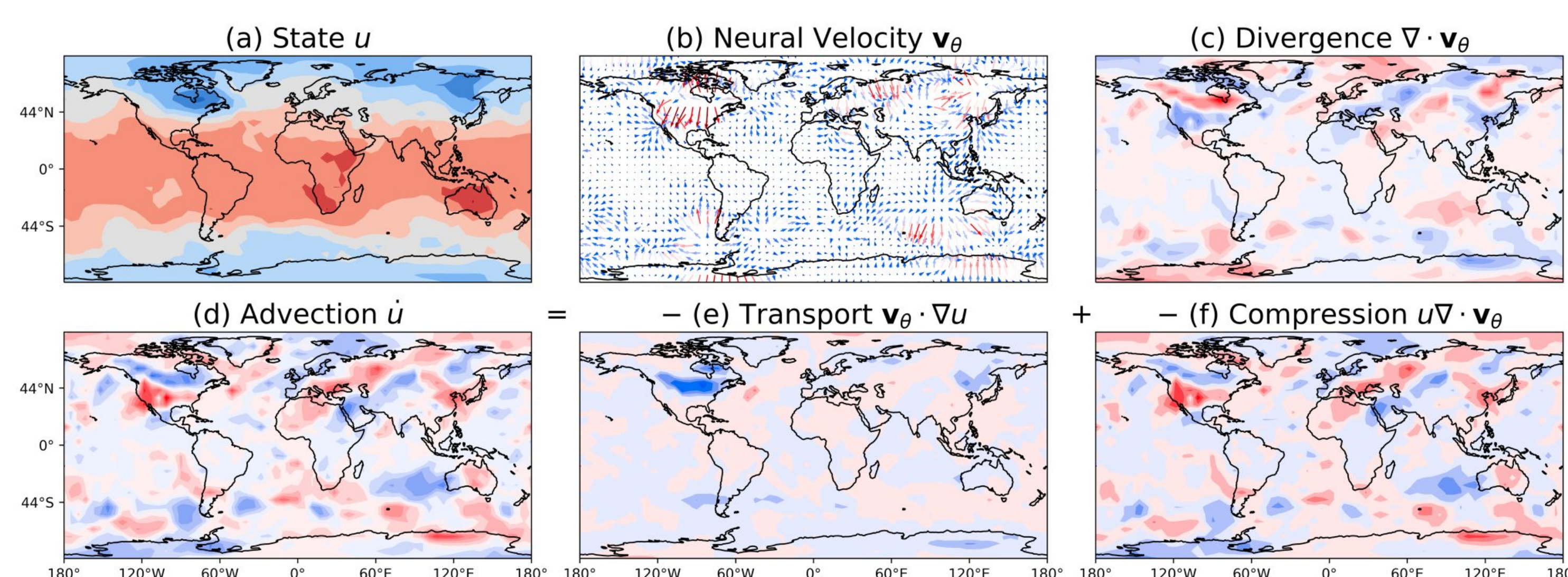
$$\frac{\partial u}{\partial t} + \underbrace{\mathbf{v} \cdot \nabla u}_{\text{Transport}} + \underbrace{u \nabla \cdot \mathbf{v}}_{\text{Compression}} = 0$$

Time evolution \dot{u}

Neural Transport Model

We model weather/climate as a spatiotemporal process $\mathbf{u}(\mathbf{x}, t) = (u_1(\mathbf{x}, t), \dots, u_K(\mathbf{x}, t))$ of K quantities as an advection PDE,

$$\dot{u}_k(\mathbf{x}, t) = -\mathbf{v}_k(\mathbf{x}, t) \cdot \nabla u_k(\mathbf{x}, t) - u_k(\mathbf{x}, t) \nabla \cdot \mathbf{v}_k(\mathbf{x}, t)$$



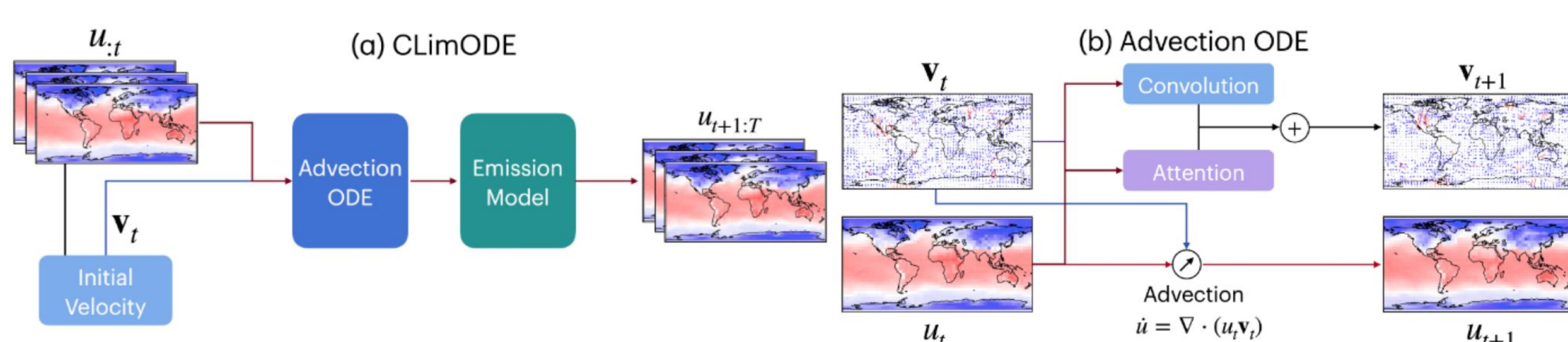
Flow velocity \mathbf{v}

We model the flow velocity via a NN, as a function of observed values $\mathbf{u}(t)$, $\nabla \mathbf{u}$, $\mathbf{v}(t)$ and spatio-temporal embeddings ψ

$$\dot{\mathbf{v}}_k(\mathbf{x}, t) = f_\theta(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \psi)$$

To capture **local** and **global** effects, we propose a hybrid network

$$f_\theta(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \psi) = f_{\text{conv}}(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \psi) + f_{\text{att}}(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \psi)$$



Training Objective

Optimise the log likelihood over the observations,

$$\log p(\mathbf{y} | \theta, \phi) \propto \sum_{i=1}^N \log \mathcal{N}(\mathbf{y}_i | \mathbf{u}_\theta(t_i) + \mu_\phi(t_i), \text{diag}(\sigma_\phi(t_i)))$$

- Solve $\mathbf{u}(t)$ forward with neural velocity
- Evaluate likelihood
- Backpropagate wrt θ, ϕ

Sources and Uncertainty

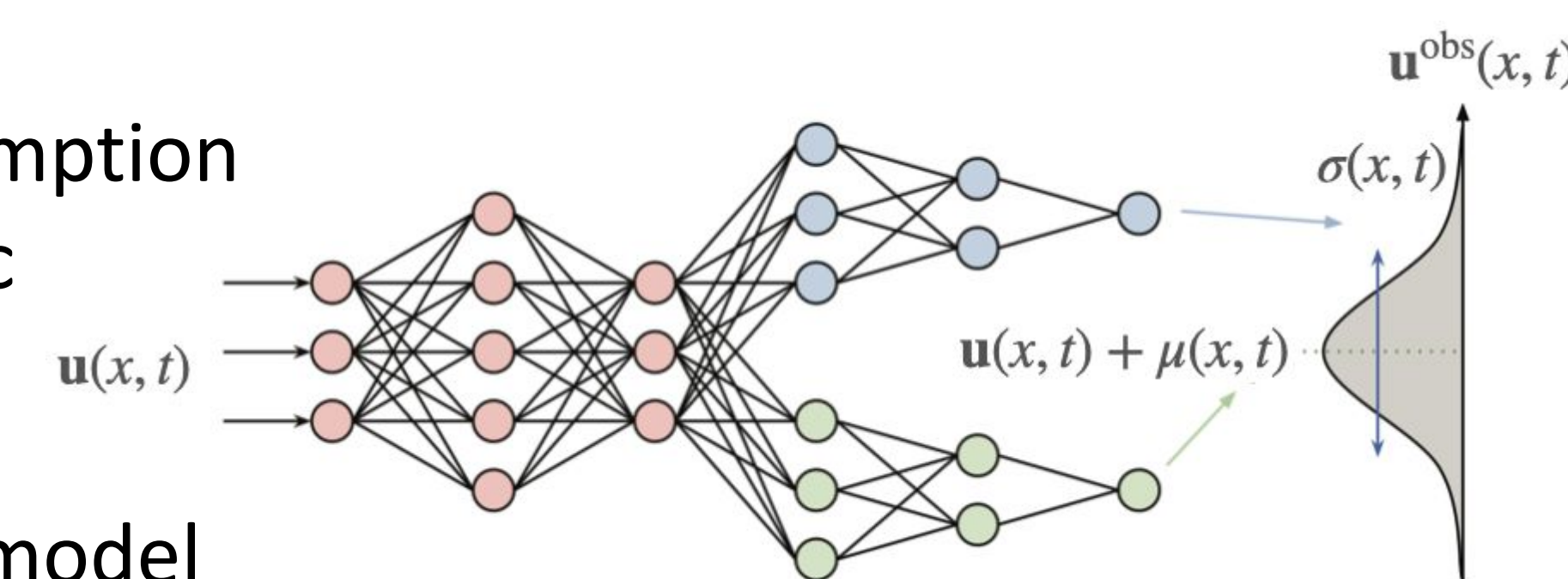
Limitation:

- Closed system assumption
- ODE is deterministic

Idea:

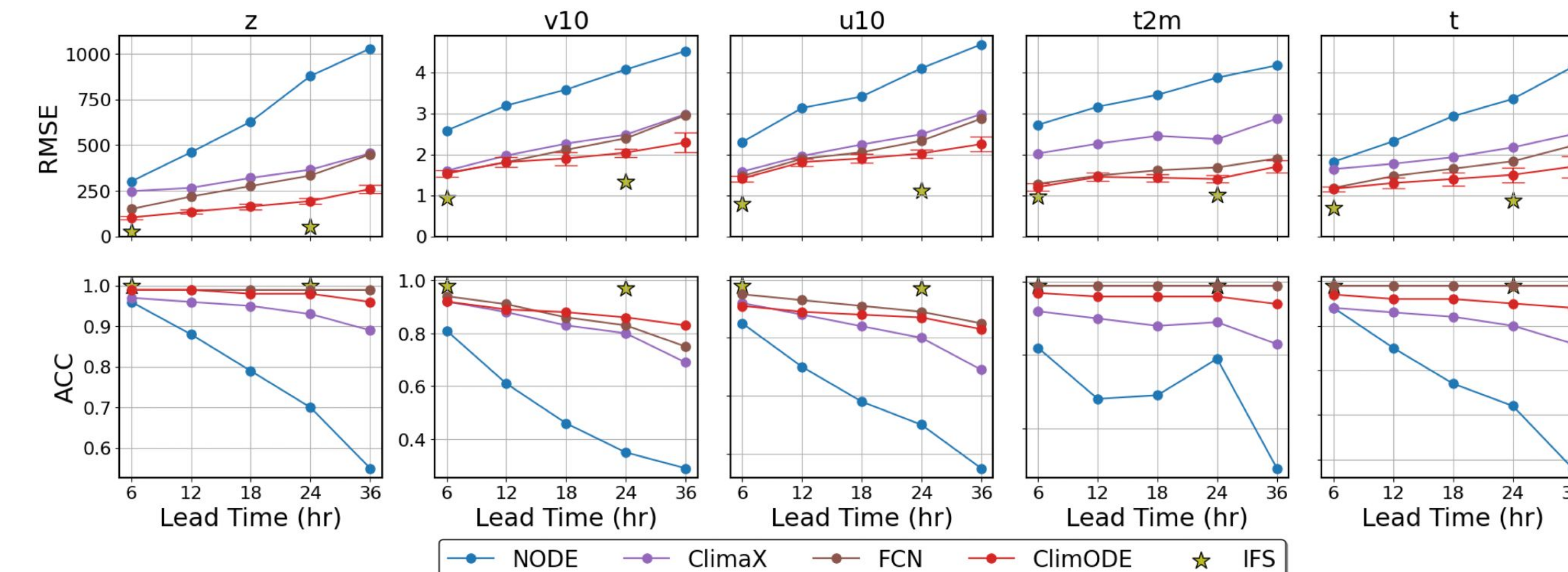
- Gaussian Emission model
- ODE becomes mean forecast

$$u_k^{\text{obs}}(\mathbf{x}, t) \sim \mathcal{N}(u_k(\mathbf{x}, t) + \mu_k(\mathbf{x}, t), \sigma_k^2(\mathbf{x}, t)), \quad \mu_k(\mathbf{x}, t), \sigma_k(\mathbf{x}, t) = g_\phi(\mathbf{u}(\mathbf{x}, t), \psi)$$



Experiments

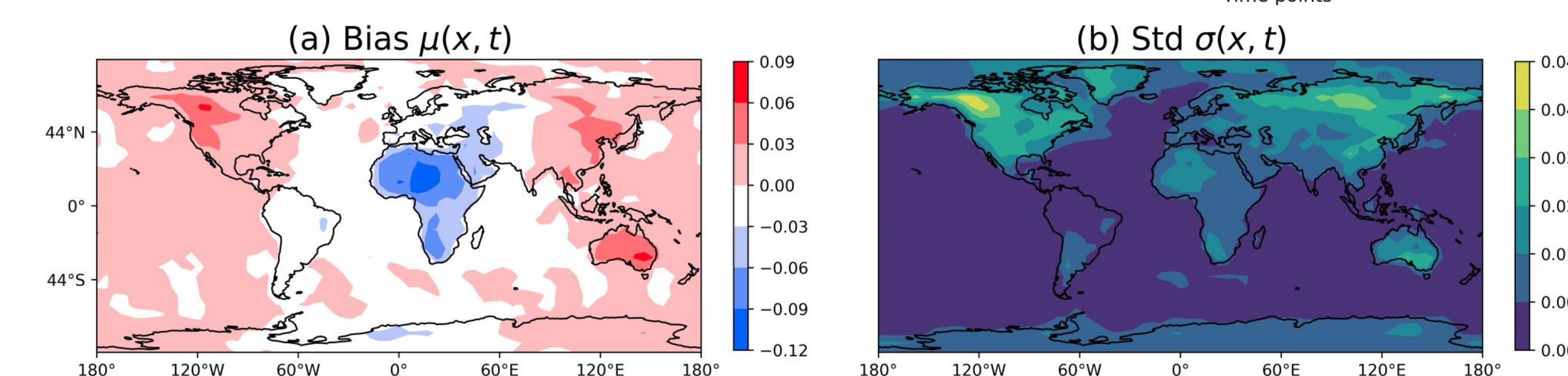
Global Forecasting



Interpretability

Bias: Explains day-night cycle

Uncertainty: Highest on land and in north according to diurnal cycle



References

- Climax: A foundational model for weather and climate, ICML 2023
- FourCastNet: A Global Data-driven High resolution weather model, PASC 2023
- Learning skillful medium-range global weather forecasting, Science, 2023
- ECMWF, IFS Documentation CY48R1
- Neural Ordinary Differential Equations, NeurIPS 2018

