Normal forms for Context free Grammer :-

A given CFG can be normalized by:
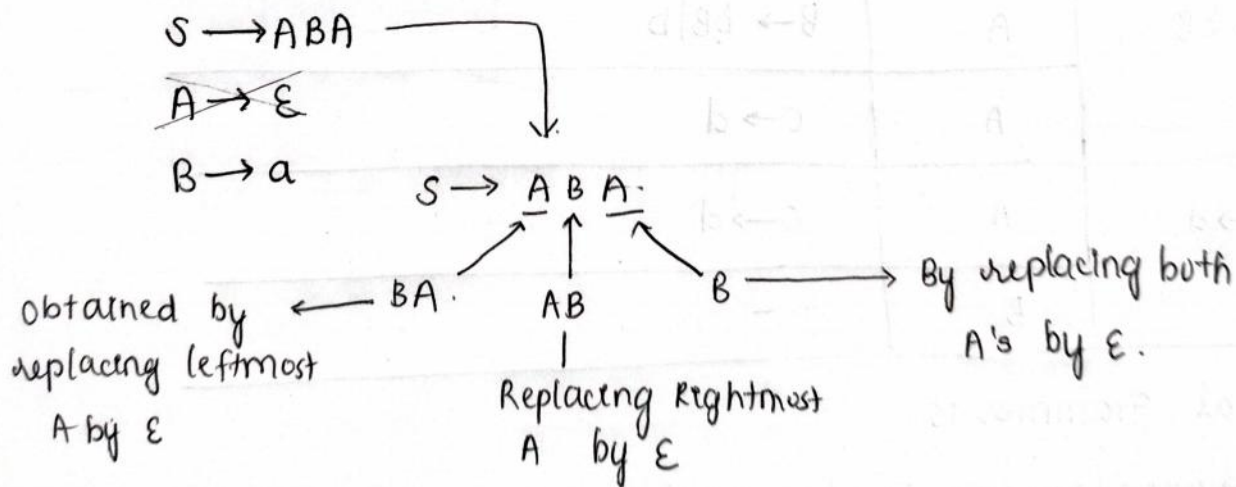
a) Eliminating ε-productions

b) Eliminating unit productions

c) Eliminating useless symbols.

Elimation of ε-productions:- $[A \rightarrow \varepsilon]$ → variable/Non-terminal

Step I:- Identify nullable variable

    a) $A \rightarrow \varepsilon$ (A) <directly>

    b) $A \rightarrow aA|B$ (A, B)
         $B \rightarrow \varepsilon$ → directly
         indirectly

Step II:- Elimination procedure

    $S \rightarrow ABA$
    $\cancel{A \rightarrow \varepsilon}$
    $B \rightarrow a$

    $S \rightarrow \underline{A\ B\ A}$

obtained by ← BA   AB   B → By replacing both
replacing leftmost   A's by ε.
A by ε

        Replacing Rightmost
        A by ε

∴ $\boxed{\begin{array}{l} S \rightarrow ABA|BA|AB|B \\ B \rightarrow a \end{array}}$ → obtained by replacing every
         occurance of A with ε.

Eliminate ε-production from the following:-

1) Given,

$S \longrightarrow ABAC$

$A \longrightarrow aA | \varepsilon$

$B \longrightarrow bB | \varepsilon$

$C \longrightarrow d.$

Sol:- **Step1:-** Identify nullable variable

$$NV = \{A, B\}$$

Step2:- Eliminating ε-productions.

Production	Nullable Variable	Production after ε-elimination
$S \rightarrow ABAC$	A	$S \rightarrow ABAC | BAC | ABC | BC$
	B	$S \rightarrow ABAC | AAC | BAC | AC | ABC | BC | C$
$A \rightarrow aA$	A	$A \rightarrow aA | a$
	B	-
$B \rightarrow bB$	A	$B \rightarrow bB | b$
	A	$C \rightarrow d$
$C \rightarrow d$	A	$C \rightarrow d$
	B	-

∴ Final Grammer is

$S \rightarrow ABAC | AAC | BAC | AC | ABC | BC | C$

$A \rightarrow aA | a$

$B \rightarrow bB | b$

$C \rightarrow d$

2) S → AB

A → Bd|a

B → bAC|ε

C → dAB|ε

sol:- Nullable variables = {B, C}

Eliminating ε-productions:-

Production	N.V	Production after ε-elimination
S → AB	B	S → AB|A
	C	—
A → Bd	B	A → Bd|d
	C	—
A → a	B	A → a
	C	—
B → bAC	B	—
	C	B → bAC|bA
C → dAB	B	C → dAB|dA
	C	—

Final grammar is

S → AB|A

A → Bd|d|a

B → bAC|bA

C → dAB|dA

3). Given,

$$S \rightarrow XY0$$

$$X \rightarrow 0X | \varepsilon$$

$$Y \rightarrow 1Y | \varepsilon$$

Sol⁻

$$N.V = \{X, Y\}$$

Production	N.V	Production after ε-elimination
S → XY0	X	S → XY0 | Y0
	Y	S → XY0 | X0 | Y0 | 0
X → 0X	X	S → 0X | 0
	Y	–
Y → 1Y	X	–
	Y	Y → 1Y | 1

∴ Final grammer is

$$S \rightarrow XY0 | X0 | Y0 | 0$$

$$X \rightarrow 0X | 0$$

$$Y \rightarrow 1Y | 1$$

4.) Given,

$A \rightarrow 0B1 \mid 1B1$

$B \rightarrow 0B \mid 1B \mid \varepsilon$

sol^t N.V = {B}.

Eliminating ε-productions:-

Production	N.V	Production after ε-elimination
$A \rightarrow 0B1$	B	$A \rightarrow 0B1 \mid 01$
$A \rightarrow 1B1$	B	$A \rightarrow 1B1 \mid 11$
$B \rightarrow 0B$	B	$B \rightarrow 0B \mid 0$
$B \rightarrow 1B$	B	$B \rightarrow 1B \mid 1$

∴ Final grammer is

$A \rightarrow 0B1 \mid 01 \mid 1B1 \mid 11$

$B \rightarrow 0B \mid 0 \mid 1B \mid 1$

Elimination of unit production:-

Non-terminal derives a single non-terminal

$$\alpha \longrightarrow \beta$$

where $\alpha, \beta \in V$ (non-terminal)

Ex:-
$$\alpha \longrightarrow \beta$$
$$\beta \longrightarrow \beta_1 | \beta_2 | \cdots | \beta_n$$

where $\beta_1, \beta_2, \cdots, \beta_n \in (V \cup T)^*$

Sol:-
$$\alpha \longrightarrow \beta_1 | \beta_2 | \cdots | \beta_n$$
$$\beta \longrightarrow \beta_1 | \beta_2 | \cdots | \beta_n.$$

---

1). Given,
$$A \longrightarrow B$$
$$B \longrightarrow a | b.$$

Sol:-   Note:- First check to remove ε-production.

next remove unit production.

here in the given no ε-production.

S1:- No-ε productions

S2:- Identifying unit productions

$$A \longrightarrow B$$

S3:- Removal of unit productions

Unit production	Replacement symbol	Product after elimination
$A \longrightarrow B$	B	$A \longrightarrow a | b$

Final grammer:

A → a|b

B → a|b

2) S → AB

A → a

B → ε|b

C → D

D → E

E → a

Sol:- S1:- No - ε productions

S2:- unit productions

B → C
C → D
D → E

B → C → D → E { If there is dependency, go from backword }

Unit productions	Replacement symbol	Production after elimination
D → E	E	D → a
C → D	D	C → a
B → C	C	B → a|b

∴ Final grammer is

S → AB   C → a

A → a   D → a

B → a|b   E → a

3). $S \rightarrow Aa | B | d$.

$B \rightarrow A | bb$

$A \rightarrow a | bd$.

Sol:- *) No. ε-productions.

*) <u>Removal of unit productions:</u>

S1:- Identifying unit productions

$$S \rightarrow B, B \rightarrow A \quad \Rightarrow \quad S \rightarrow B \rightarrow A.$$

S2:- Removal of unit production.

Unit production	Replacement symbol	Production after Elimination
$B \rightarrow A$.	A	$B \rightarrow a | bd | bb$
$S \rightarrow B$	B	$S \rightarrow a | bd | bb | Aa | d$

<u>final grammer is:</u>

$S \rightarrow a | bd | bb | Aa | d$

$B \rightarrow a | bd | bb$

$A \rightarrow a | bd$

4). Given, $S \rightarrow ACA \,|\, Aa$
$\qquad A \rightarrow BB \,|\, \varepsilon$
$\qquad B \rightarrow A \,|\, bc$
$\qquad C \rightarrow b$.

Sol: There is an ε-production.

$\qquad N.V = \{A, B\}$

Production	N.V	Production after ε-Elimination
$S \rightarrow ACA$	A, B	$S \rightarrow ACA \,/\, CA \,/\, AC \,/\, C$
$S \rightarrow Aa$	A, B	$S \rightarrow Aa \,/\, a$
$A \rightarrow BB$	A, B	$A \rightarrow BB \,/\, B$
$B \rightarrow A$	A, B	$B \rightarrow A$
$B \rightarrow bC$	A, B	$B \rightarrow bC$
$C \rightarrow b$	A, B	$C \rightarrow b$

modified grammer is :-

$S \rightarrow ACA \,|\, CA \,|\, AC \,|\, C \,|\, Aa \,|\, a$

$A \rightarrow BB \,|\, B$

$B \rightarrow A \,|\, bC$

$C \rightarrow b$

Unit productions :-

$\qquad S \rightarrow C$

$\qquad A \rightarrow B \qquad \Rightarrow \qquad S \rightarrow C$

$\qquad B \rightarrow A \qquad\qquad\qquad A \rightarrow B \rightarrow A$

Unit production	Replacement symbol	Production after elimination
S → C	C	S → b
B → A	A	B → BB | bC
A → B	B	A → BB | bc

final grammer is

S → ACA \| CA \| AC \| b \| Aa \| a

A → BB \| bC

B → BB \| bc

C → b

5) Given, S → AB \| a

  A → a \| b

  B → ε \| d

Soln: *) There is a ε-production

    N.V = {B}

modified grammer after Eliminating ε-production

S → AB \| A \| a

A → a \| b

B → d

Unit productions: S → A

S → A	A	S → a | b | AB | a

∴ final grammer is

  S → a \| b \| AB

  A → a \| b

  B → d

Removal of useless symbol:-

Step1:- Remove non-generating symbols/variables.

step2:- Remove unreachable symbols

Remove non-generating symbols:

NGV $\longrightarrow$ those variables which do not produce terminals

NGV = V - GV

$(\alpha \longrightarrow a)$

$\rightarrow$ Remove all production that contain NGVs.

Remove unreachable symbols:

Unreachable symbols $\rightarrow$ those symbols that cannot be reached

from 's'

$\quad \rightarrow$ start symbol.

---

1) Given,

$S \rightarrow ABla$

$A \rightarrow B\varepsilon lb$

$B \rightarrow aB l C$

$C \rightarrow aCl B.$

Sol:- *) No ε- production.

Removal of unit production:

Step1: Identify the unit production:

$$B \rightarrow C$$
$$C \rightarrow B$$

$$\Rightarrow B \rightarrow C \rightarrow B$$

Step 2 :- Removal of unit production :-

Unit production	Replacement symbol	Production after elimination
$C \rightarrow B$	B	$C \rightarrow aC \mid aB$
$B \rightarrow C$	C	$B \rightarrow aB \mid aC$

modified grammer:

$$S \rightarrow AB \mid a$$
$$A \rightarrow BC \mid b$$
$$C \rightarrow AC \mid aB$$

Removal of useless symbols :-

Step 1 :- Removing non-generating variables :-

Variables	Generating variables	$NGV = V - GV$
$\{S, A, B, C\}$	$\{S, A\}$	$\{B, C\}$

modified grammar:

$S \rightarrow a$

$A \rightarrow b.$

Step2: Removing unreachable symbols.

Variables	Reachable symbols	unreachable symbols
{S, A}	{S}	{A}

Final grammar:

$S \rightarrow a.$

---

2) Given,

$S \longrightarrow AB | AC$

$A \longrightarrow aAb | bAa | a$

$B \longrightarrow bbA | aaB | AB$

$C \longrightarrow abCA | aDb$

$D \longrightarrow bD | ac.$

Sol: *) There is no ε-productions and unit productions.

Removing of useless symbols:

SI: Removing non-generating variable:

variables	Generating variables	NGV = N-GV
{S, A, B, C, D}	{A, B, S}	{C, D}

modified grammer:

   $S \rightarrow AB$

   $A \rightarrow aAb \mid bAa \mid a$

   $B \rightarrow bbA \mid aaB \mid AB$.

S2:- Removing unreachable symbols:-

variables	Reachable symbols	unreachable symbols
$\{S, A, B\}$	$\{S, A, B\}$	$\emptyset$

final grammer:

   $S \rightarrow AB$

   $A \rightarrow aAb \mid bAa \mid a$

   $B \rightarrow bbA \mid aaB \mid AB$.

---

3). $T \rightarrow aaB \mid abA \mid aaT$

   $A \rightarrow aA$

   $B \rightarrow ab \mid b$

   $C \rightarrow ad$

Sol:- *) No- ε-production.

   *) ~~Remove~~ No-unit production.

Removal of useless symbol:-

S1:- Removing non-generating symbols:

variables	Generating variables	NGV = V - GV
{T, A, B, C}	{B, C, T}	{A}

modified grammar:

$T \rightarrow aaBla aT$

$B \rightarrow abl b$

$C \rightarrow ad$.

s2:- Removing unreachable symbols:

variables	Reachable symbols	Unreachable symbols
{T, B, C}	{T, B}	{C}

final grammer:

$T \rightarrow aaBla aT$

$B \rightarrow abl b$

Imp.

4) Simplify the following CFG.

$S \rightarrow ASB | \varepsilon$

$A \rightarrow aASla$

$B \rightarrow SbS | A | bb$

Sol:- Removal of ε-production:-

*) Here in the ques, start state derives ε and no other
variables are deriving ε

∴ no need to remove the ε-production.

Removal of unit production :-

S1 :- Identify the unit production.

$$B \longrightarrow A.$$

S2 :- Removal of unit production.

Unit production	Replacement symbol	Production after Elimination
B ⟶ A	A	B ⟶ aASla.

modified grammer :

S ⟶ ASB|ε

A ⟶ aASla

B ⟶ Sbsla ASla | bb

Removal of useless grammer :

S1 :- Removing non-generating symbols.

variables	generating variables	NGV = V - GV
{A,B,S}	{A,B,S}	∅

modified grammer :

S ⟶ ASB|ε

A ⟶ aAS| a

B ⟶ Sbsla ASla|bb

S2: Removal of unreachable symbols.

variables	unreachable symbols	unreachable symbols
{S, A, B}	{S, A, B}	∅

final grammar:

S → ASB|ε

A → aA S|a

B → Sbs|aAS|a|bb

Imp. ***

5) Given,

S → ABC|BaB

A → aA|BaC|aaa

B → bBb|a|D

C → CA|AC

D → ε

Sol: Removal of ε-production:

S1: Identify the nullable variable.

$$NV = \{D, B\}$$

S2: Eliminating ε-productions.

Production	Nullable variables	Production after ε-elimination
S → ABC⊀BaB	D	—
	B	S → ABC|AC.
S → BaB	D	—
	B	S → Ba|aB|a|BaB
A → aA	D,B	A → aA
A → BaC	D,B	A → Bac|ac
A → aaa	D,B	A → aaa
B → bBb	D,B	B → bBb|bb
B → a⊀	D,B	B → a
C → ⊀CA	D,B	C → CA
C → AC	D,B	C → AC

modified grammer:

S → ABC\| AC\|Ba\|aB \|a\|BaB

A → aA\| Bac\|aC\|aaa

B → bBb\|bb\|a\|D

C → CA\|AC

Removal of unit production:

S1:- Identify the unit production.

    $B \longrightarrow D$.

S2:- Removal unit production.

Unit production	Replacement symbol	Production after Elimination
$B \longrightarrow D$.	D	ϕ

    modified grammer:

    $S \longrightarrow ABC \mid AC \mid BaJaB \mid BaB \mid a$.

    $A \longrightarrow aA \mid Ba \ Clac \mid aaa$

    $B \longrightarrow bBb \mid bb \mid a$

    $C \longrightarrow CA \mid AC$

Removal of useless production:

S1:- Removing non-generating symbols.

variables	Generating variables	NGV = V−GV
$\{S, A, B, C\}$	$\{S, A, B\}$	$\{C\}$

    modefied grammer:

    $S \longrightarrow BaB \mid BaJaB \mid a$

    $A \longrightarrow aA \mid aaa$

    $B \longrightarrow bBb \mid bb \mid a$

Removing unreachable symbols:

variables	reachable Variables	unreachable variables
{S, A, B}	{S, B}	{A}

final grammer:

$S \rightarrow BaB \mid Ba \mid aB \mid a$

$B \rightarrow bBb \mid bb \mid a$

Chomsky Normal Form (CNF):-

A context free grammer is said to be in Chomsky normal form, if all productions are of the form

Non-Terminal $\longrightarrow$ Non-Terminal . Non-Terminal

Non-Terminal $\longrightarrow$ Terminal

Ex:- $S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow b$

Note:- To convert CFG into CNF, we need a reduced CFG
ie., that CFG which is free of

   a) ε- productions
   b) unit - productions
   c) useless symbols.

Ex. Consider,

$S \longrightarrow aaaaA$

$A \longrightarrow aaaa$.

Sol:

Production	Conversion
$S \longrightarrow aaaaA$	$S \longrightarrow CA$
	$C \longrightarrow BB$
	$B \longrightarrow XX$
	$X \longrightarrow a$
$A \longrightarrow aaaa$	$A \longrightarrow BB$
	$B \longrightarrow XX$
	$X \longrightarrow a$

final grammer in CNF:

$S \longrightarrow CA$

$C \longrightarrow BB$

$B \longrightarrow XX$

$X \longrightarrow a$

$A \longrightarrow BB$

---

1) Given,

$S \longrightarrow ABa$

$A \longrightarrow aab$

$B \longrightarrow Ad$

Sol:

  * no ε-production

  * no unit-production

  * no useless symbols.

Production	Conversion
S → ABa	S → CD
	C → AB
	D → a
A → aab	A → EF
	E → XX
	X → a
	F → b
B → Ad	B → AY
	Y → d

final grammer in CNF :-

S → CD
C → AB
D → a
A → EF
E → XX
X → a
F → b
B → AY
Y → d

2) S → aBa|abba

A → ab|AA

B → aB|a

Sol: Removal of useless symbols:-
~~Production~~

S1: Remove non-generating symbols.

variable	Generating variable	NGV = V - GV
{S, A, B}	{S, A, B}	∅

modified grammer:

S → aBa|abba

A → ab|AA

B → aBa

S2: Remove unreachable symbols ~~states~~.

Variable	reachable symbols	unreachable symbols
{S, A, B}	{S, B}	{A}

modified grammer:

S → aBa|abba

B → aB|a

Production	Conversion
S → aBa	S → XY SX → YB Y → a
S → abba	S → CD C → YZ D → ZY Y → a Z → b
B → aB	B → YB Y → a
B → a	B → a

Final grammar in CNF:

S → XY | CD

X → YB

Y → a

C → YZ

D → ZY

Z → b

B → YB | a

3) Given,

S → aSb | ab | Aa

A → aab.

Sol⁻ *) nθ. ε-production, unit production, use symbol.

⇒ the grammer is in reduced form

Production	Conversion
S → aSb	S → XY X → a Y → SZ Z → b
S → ab	S → XZ
S → Aa	S → AZ ~~Y → A~~
A → aab	A → CZ C → XX

S → XY | XZ | AZ

X → a

Z → b

Y → SZ

A → CZ

C → XX