

1. (100 points) Consider the following polynomial with a real variable, x ;

$$p(x) = 3x^3 - 5x^2 - 16x + 12$$

The polynomial and its root are shown in the figure below.

- (15 points) Write a Python function for $p(x)$ that takes x as a formal parameter and returns $p(x)$.
- (15 points) Write a Python function for the derivative of $p(x)$ that takes x as a formal parameter and returns the derivative of $p(x)$. Take the derivative of $p(x)$ analytically with respect to x before writing the function.
- (45 points) Write a Python function for root finding using the Newton-Raphson method that takes an initial guess, x_0 , as a formal parameter, and returns its root, x_r , of $p(x)$ such that $p(x_r) \sim 0$. At each iteration, the function is expected to print the **iteration number**, the **new guess** and the **absolute value of $p(x)$ at the guess**. Iterations should stop when the absolute value of $p(x)$ at a new guess is less than 10^{-6} . The function will return that guess as x_r .
- (25 points) Write a main program that will get the initial guess, x_0 , from a user using the **input** statement, and call the functions developed earlier as

needed to find the root of the polynomial. It should call the function developed for question 1.b to find the root. The program is supposed to print the root right after locating it.

