Solution For The School Geometry Problems

Yogesh Choudhary

***Abstract*—This document includes different problems and solution on geometry from trigonometry and algebra.It also provides the imformation about the python and latex codes of figures.**

Download all python codes from

```
svn co svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/figures
```

1 Problem

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR.Show that:

(a) Δ ABM ≅ Δ PQN
(b) Δ ABC ≅ Δ PQR

2 Construction

2.1. We have the values of all three sides of the triangle ABC and PQR so to construct a triangle we need all three coordinates of A,B and C.After getting the all three coordinates the coordinates of the median from point A to side BC and from point P to line QR, are achieved.

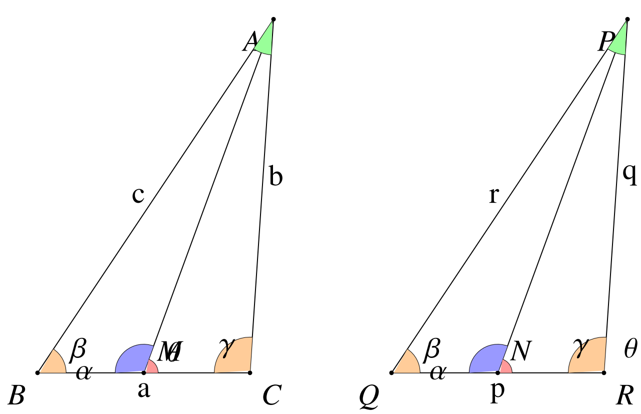

Fig. 2.1: Triangles by Latex-Tikz

Parameter	Value
a,p	3
p,q	5
c,r	6

TABLE 2.1: To construct $\triangle ACB$ and $\triangle PQR$

2.2. Finding out the coordinates of the various points in Fig. 2.1

$$x_1 = \frac{\left(a^2 + c^2 - b^2\right)}{2 * a} \tag{2.0.1}$$
$$y_1 = \sqrt{c - 1^2 - x^2} \tag{2.0.2}$$
$$x_2 = \frac{\left(p^2 + r^2 - q^2\right)}{2 * p} \tag{2.0.3}$$
$$y_2 = \sqrt{r^2 - x_2^2} \tag{2.0.4}$$
$$\begin{pmatrix} \mathbf{A} & \mathbf{P} \end{pmatrix} = \begin{pmatrix} x_1 & x_2 + 5 \\ y_1 & y_2 \end{pmatrix} \tag{2.0.5}$$
$$\begin{pmatrix} \mathbf{B} & \mathbf{Q} \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix} \tag{2.0.6}$$
$$\tag{2.0.7}$$
$$\begin{pmatrix} \mathbf{C} & \mathbf{R} \end{pmatrix} = \begin{pmatrix} a & p + 5 \\ 0 & 0 \end{pmatrix} \tag{2.0.8}$$
$$\tag{2.0.9}$$

$\because$ **M** is the midpoint of BC and **N** of QR,

$$\mathbf{M} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{2.0.10}$$
$$\mathbf{N} = \frac{\mathbf{P} + \mathbf{Q}}{2} \tag{2.0.11}$$
$$\begin{pmatrix} \mathbf{M} & \mathbf{N} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a & p + 10 \\ 0 & 0 \end{pmatrix} \tag{2.0.12}$$

The values are listed in Table. 2.2

2.3. Draw Fig. 2.1.
Solution: The following Python code generates Fig. 2.3

```
codes/Triangle.py
```

Derived Values.	
M	$\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$
N	$\begin{pmatrix} 6.5 \\ 0 \end{pmatrix}$

TABLE 2.2: To construct madian AM and PN

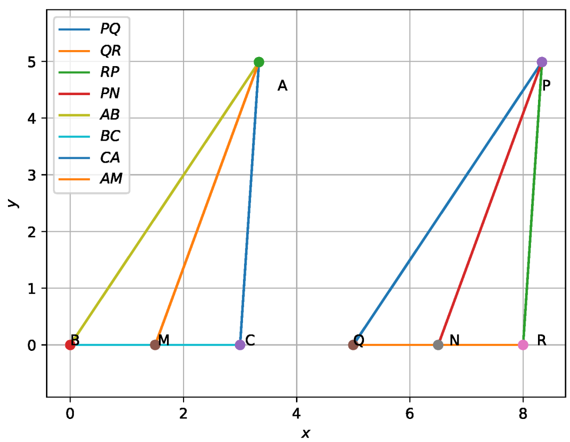

Fig. 2.3: Triangles generated using python

and the equivalent latex-tikz code generating Fig.2.1 is

```
figs/triangle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/triangle_fig.tex
```

3 Solution

3.1. given that →

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{P}-\mathbf{Q}\| = c = r \tag{3.0.1}$$
$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{P}-\mathbf{N}\| = m = n \tag{3.0.2}$$
$$\|\mathbf{B}-\mathbf{C}\| = \|\mathbf{Q}-\mathbf{R}\| = a = p \tag{3.0.3}$$

3.1 Solution.a)

3.1. From equation (3.0.3)...

$$\frac{1}{2}\|\mathbf{B}-\mathbf{C}\| = \frac{1}{2}\|\mathbf{Q}-\mathbf{R}\| \tag{3.1.1}$$
$$\|\mathbf{B}-\mathbf{M}\| = \|\mathbf{Q}-\mathbf{N}\| \tag{3.1.2}$$

3.2. From fig [2.1] ...

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{P}-\mathbf{Q}\| \tag{3.1.3}$$
$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{P}-\mathbf{N}\| \tag{3.1.4}$$
$$\|\mathbf{B}-\mathbf{M}\| = \|\mathbf{Q}-\mathbf{N}\| \tag{3.1.5}$$

$\implies$ from SSS congurance rule $\Delta ABM \cong \Delta PQN$

3.2 Solution.b)

3.1. given that →

$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{P}-\mathbf{N}\| = m = n \tag{3.2.1}$$

from equation (3.0.3)...

$$\frac{1}{2}\|\mathbf{B}-\mathbf{C}\| = \frac{1}{2}\|\mathbf{Q}-\mathbf{R}\| \tag{3.2.2}$$
$$\|\mathbf{M}-\mathbf{C}\| = \|\mathbf{N}-\mathbf{R}\| \tag{3.2.3}$$

$$\because \Delta ABM \cong \Delta PQN \tag{3.2.4}$$
$$\implies \angle AMB = \angle PNQ \tag{3.2.5}$$
$$180 - \angle AMB = 180 - \angle PNQ \tag{3.2.6}$$
$$\angle AMC = \angle PNR \tag{3.2.7}$$

from equation (3.0.2),(3.2.3) and (3.2.10)...

$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{P}-\mathbf{N}\| \tag{3.2.8}$$
$$\|\mathbf{M}-\mathbf{C}\| = \|\mathbf{N}-\mathbf{R}\| \tag{3.2.9}$$
$$\angle AMC = \angle PNR \tag{3.2.10}$$

$\implies$ from SAS congurancy $\Delta AMC \cong \Delta PNR$
$\implies \|\mathbf{A}-\mathbf{C}\| = \|\mathbf{P}-\mathbf{R}\|$
from equation (3.0.1),(3.0.3) and...

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{P}-\mathbf{Q}\| \tag{3.2.11}$$
$$\|\mathbf{B}-\mathbf{C}\| = \|\mathbf{Q}-\mathbf{R}\| \tag{3.2.12}$$
$$\|\mathbf{A}-\mathbf{C}\| = \|\mathbf{P}-\mathbf{R}\| \tag{3.2.13}$$

$\implies$ from SSS congurancy $\Delta ABC \cong \Delta PQR$