# Problems On Geometry Of Circle

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***Abstract*—This document prooves a circle theoram with the help of different figures and tables written in python and latex .**

Download all python codes from

```
svn co svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/circle/
    codes
```

and latex-tikz codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/circle/
    figures
```

## 1 Problem

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

## 2 Construction

2.1. We need draw the a circume circle for that we have all three sides of triangle.In order to construct the circumecircle first of all we will find all three coordinates of the triangle using the sides.

2.2. Values of all three sides of the triangle are as given in the table .

| Parameter | Value |
|---|---|
| a | 5 |
| b | 4 |
| c | 6 |

TABLE 2.2: To construct circumecircle

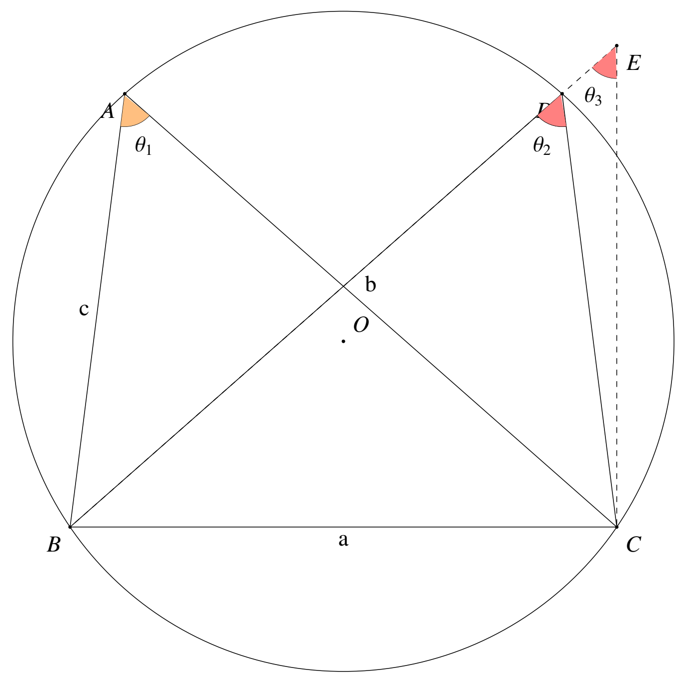


Fig. 2.2: circumecircle generated by latex

2.3. Finding out the coordinates of the various points in Fig. 2.2

$$x_1 = \frac{\left(a^2 + c^2 - b^2\right)}{2 * a} \tag{2.0.1}$$

$$y_1 = \sqrt{c^2 - x_1^2} \tag{2.0.2}$$

$$x_2 = \frac{\left(a^2 + b^2 - c^2\right)}{2 * a} \tag{2.0.3}$$

$$y_2 = \sqrt{b^2 - x_2^2} \tag{2.0.4}$$

$$x_3 = a \tag{2.0.5}$$

$$y_3 = \frac{a}{x_2} * y_2 \tag{2.0.6}$$

$$\left(\mathbf{A}\right) = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{2.0.7}$$

$$\left(\mathbf{B}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

$$\left(\mathbf{C}\right) = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2.0.9}$$

$$\left(\mathbf{D}\right) = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \tag{2.0.10}$$

$$\left(\mathbf{E}\right) = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \tag{2.0.11}$$

2.4. Finding the circumecentre $\rightarrow$
let assume that circumecentre of the triangle ABC is **O**

$$\|\mathbf{A}-\mathbf{O}\| = \|\mathbf{B}-\mathbf{O}\| = \|\mathbf{C}-\mathbf{O}\| = \|\mathbf{D}-\mathbf{O}\| \tag{2.0.12}$$

$$\|\mathbf{A}-\mathbf{O}\|^2 - \|\mathbf{B}-\mathbf{O}\|^2 = 0 \tag{2.0.13}$$

Which can be simplified as

$$\left(\mathbf{A}-\mathbf{B}\right)^T \mathbf{O} = \frac{(\|A\|^2 - \|B\|^2)}{2} \tag{2.0.14}$$

Similarly,

$$\left(\mathbf{B}-\mathbf{C}\right)^T \mathbf{O} = \frac{(\|B\|^2 - \|C\|^2)}{2} \tag{2.0.15}$$

can be combined to form the matrix equation

$$\mathbf{N}^T = \mathbf{c} \tag{2.0.16}$$

$$\mathbf{O} = \mathbf{N}^{-T}\mathbf{c} \tag{2.0.17}$$

Where

$$\mathbf{N} = \begin{pmatrix} \mathbf{A}-\mathbf{B} & \mathbf{B}-\mathbf{C} \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{c} = \frac{1}{2}\left(\|A\|^2 - \|B\|^2 \, \|B\|^2 - \|C\|^2\right) \tag{2.0.19}$$

2.5. Finding **R** of circumecircle

$$\mathbf{R} = \|\mathbf{B}-\mathbf{O}\| \tag{2.0.20}$$

The values are listed in Table. 2.5

| Derived Values. | |
|---|---|
| **O** | $\begin{pmatrix} 2.5 \\ 1.7008 \end{pmatrix}$ |

TABLE 2.5: circumecentre of the triangle

2.6. Drawing Fig. 2.6.
The following Python code generates Fig. 2.6

```
codes/c_circle.py
```

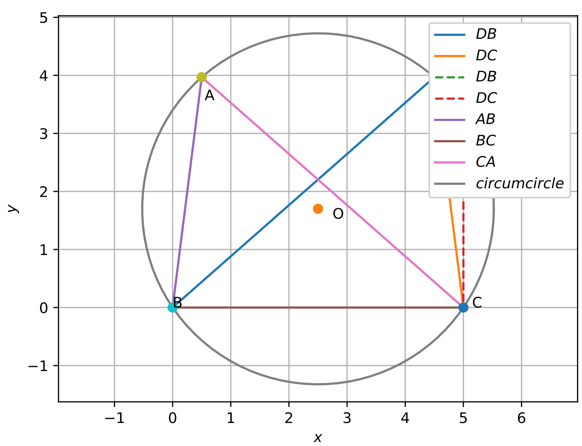


Fig. 2.6: circumecircle generated using python

and the equivalent latex-tikz code generating Fig.2.1 is

```
figs/C_circle.tex
```

The above latex code can be compiled as a standalone document as

```
figs/C_circle_slone.tex
```

## 3 Solution

3.1. Let assume that circle intersect at **E** Now,

$$\angle BAC = \angle BEC \tag{3.0.1}$$

3.2. But,given that

$$\angle BAC = \angle BDC \tag{3.0.2}$$

3.3. So from

$$\angle BEC = \angle BDC \tag{3.0.3}$$

3.4. in triangle CDE

$$\angle BDC = \angle CED + \angle DCE \tag{3.0.4}$$

$$\angle BEC = \angle BEC + \angle DCE \tag{3.0.5}$$

$$\angle BEC - \angle BEC = \angle DCE \tag{3.0.6}$$

$$\angle DCE = 0 \tag{3.0.7}$$

3.5. Thus from above we can say that point D exist on the circle.