Solution for Problemes on Optimization Techniques

Yogesh Choudhary

***Abstract*—Many problems and their solution related to the different type of optimization techniques are included in this ducument.All the solutions are provided using the linear alzebra and linear programming.Python codes are also available for the solution and diagrams.**

Download all python codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/
    optimization/codes
```

and latex-tikz codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/
    optimization/figures
```

1 Problem

1.0.1 Question:

1. Solve

$$\min_{\mathbf{x}} Z = \begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} \tag{1.0.1.1}$$

$$s.t. \quad \begin{pmatrix} -1 & -1 \\ 3 & 5 \end{pmatrix} \mathbf{x} \preceq \begin{pmatrix} -8 \\ 15 \end{pmatrix} \tag{1.0.1.2}$$

$$\mathbf{x} \succeq \mathbf{0} \tag{1.0.1.3}$$

1.0.2 Solution:

1. From the gernelised form of the eqautions

$$minZ = c^t\mathbf{x} \tag{1.0.1.1}$$

$$\text{subjected to} \tag{1.0.1.2}$$

$$A\mathbf{x} \preceq b \tag{1.0.1.3}$$

$$\text{we can find} \rightarrow \tag{1.0.1.4}$$

$$c = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{1.0.1.5}$$

$$A = \begin{pmatrix} -1 & -1 \\ 3 & 5 \end{pmatrix} \tag{1.0.1.6}$$

$$b = \begin{pmatrix} -8 \\ 15 \end{pmatrix} \tag{1.0.1.7}$$

$$\tag{1.0.1.8}$$

Solution for the above equations can be find from the python code of Lenear programing.

```
./optimization/codes
```

From the codes we get that there is no such a point which is common to the both constraints.

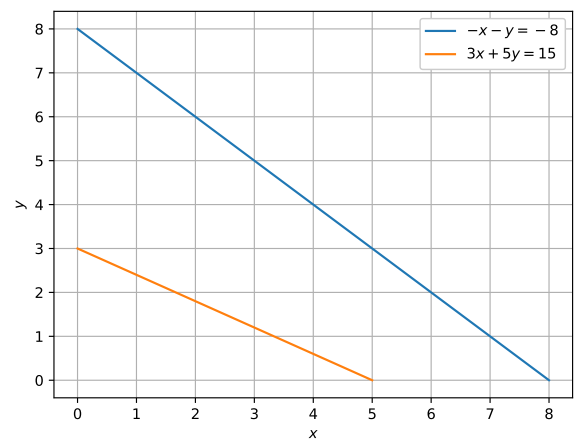

Fig. 1.0.1: lp1

Pythone codes for the above figure can be get from

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./optimization/figures/lp1.eps
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2 Problem

2.0.1 Question:

1. Solve

$$\min_{\mathbf{x}} Z = \begin{pmatrix} 200 & 500 \end{pmatrix} \mathbf{x} \tag{2.0.1.1}$$

$$s.t. \quad \begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} \mathbf{x} \preceq \begin{pmatrix} -10 \\ 24 \end{pmatrix} \tag{2.0.1.2}$$

$$\mathbf{x} \succeq \mathbf{0} \tag{2.0.1.3}$$

2.0.2 Solution:

1. From the gernelised form of the eqautions

$$minZ = c^t\mathbf{x} \tag{2.0.1.1}$$
$$\text{subjected to} \tag{2.0.1.2}$$
$$A\mathbf{x} \preceq b \tag{2.0.1.3}$$
$$\text{we can find} \rightarrow \tag{2.0.1.4}$$
$$c = \begin{pmatrix} 200 \\ 500 \end{pmatrix} \tag{2.0.1.5}$$
$$A = \begin{pmatrix} -1 & -2 \\ 3 & 4 \end{pmatrix} \tag{2.0.1.6}$$
$$b = \begin{pmatrix} -10 \\ 24 \end{pmatrix} \tag{2.0.1.7}$$
$$\tag{2.0.1.8}$$

Solution for the above equations can be find from the python code of Lenear programing

./optimization/codes

From the codes we get that the minimum value of the equation will be 2299.99 at the point $\begin{pmatrix} 4 & 3 \end{pmatrix}$.

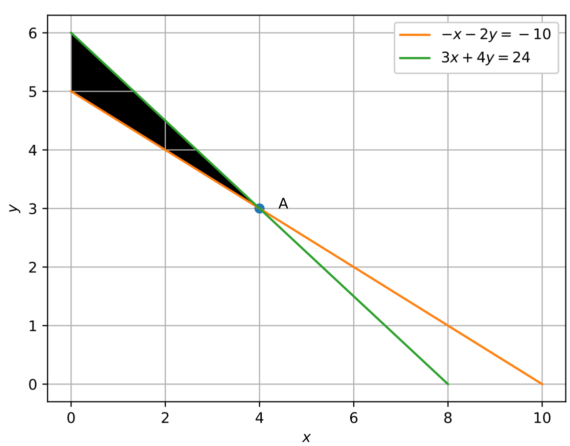

Fig. 2.0.1: lp2

Pythone codes for the above figure can be get from

./optimization/figures/lp2.eps

3 Problem

3.0.1 Question:

1. Maximise Z=3x+4y
subject to the constraints : x+y≤4, x≥0, y≥ 0.

3.0.2 Solution:

1. given that →

$$Z = 3x + 4y \tag{3.0.1.1}$$
$$\text{subjected to} \tag{3.0.1.2}$$
$$x + y \leq 4 \tag{3.0.1.3}$$
$$x \geq 0 \tag{3.0.1.4}$$
$$y \geq 0 \tag{3.0.1.5}$$

Comparing above equation to the gernelised form→

$$maxZ = c^t\mathbf{x} \tag{3.0.1.6}$$
$$\text{subjected to} \tag{3.0.1.7}$$
$$A\mathbf{x} \preceq b \tag{3.0.1.8}$$
$$\text{we can find} \rightarrow \tag{3.0.1.9}$$
$$c = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{3.0.1.10}$$
$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.0.1.11}$$
$$b = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \tag{3.0.1.12}$$

Solution for the above equations can be find from the python code of Lenear programing

./optimization/codes

From the codes we get that the maximum value of the equation will be 16 at the point $\begin{pmatrix} 0 & 4 \end{pmatrix}$.

4 Problem

4.0.1 Question:

1. Z=-3x+4y
subject to x+2y≤8, 3x+2y≤12, x≥0, y≥0.

4.0.2 Solution:

1. given that →

$$Z = -3x + 4y \tag{4.0.1.1}$$
$$\text{subjected to} \tag{4.0.1.2}$$
$$x + 2y \leq 8 \tag{4.0.1.3}$$
$$3x + 2y \leq 12 \tag{4.0.1.4}$$
$$x \geq 0 \tag{4.0.1.5}$$
$$y \geq 0 \tag{4.0.1.6}$$

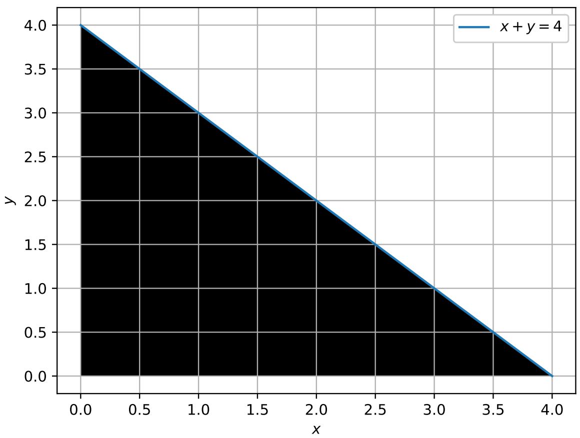

Fig. 3.0.1: lp3

Pythone codes for the above figure can be get from

./optimization/figures/lp3.eps

Comparing above equation to the gernelised form→

$$minZ = c^t\mathbf{x} \tag{4.0.1.7}$$

$$\text{subjected to} \tag{4.0.1.8}$$

$$A\mathbf{x} \le b \tag{4.0.1.9}$$

$$\text{we can find} \rightarrow \tag{4.0.1.10}$$

$$c = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{4.0.1.11}$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4.0.1.12}$$

$$b = \begin{pmatrix} 8 \\ 12 \\ 0 \\ 0 \end{pmatrix} \tag{4.0.1.13}$$

Solution for the above equations can be find from the python code of Lenear programing

./optimization/codes

From the codes we get that the minimum value of the equation will be 12 at the point $\begin{pmatrix} 4 & 0 \end{pmatrix}$.

5 Problem

5.0.1 Question:

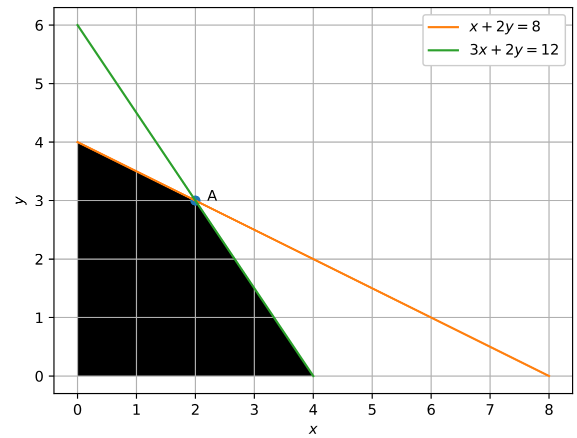

Fig. 4.0.1: lp4

pythone codes for the above figure can be get from

./optimization/figures/lp4.eps

1. Maximise Z=5x+3y subject to 3x+5y≤15, 5x+2y≤10, x≥0, y≥0.

5.0.2 Solution:

1. given that →

$$Z = 5x + 3y \tag{5.0.1.1}$$

$$\text{subjected to} \tag{5.0.1.2}$$

$$3x + 5y \le 15 \tag{5.0.1.3}$$

$$5x + 2y \le 10 \tag{5.0.1.4}$$

$$x \ge 0 \tag{5.0.1.5}$$

$$y \ge 0 \tag{5.0.1.6}$$

Comparing above equation to the gernelised form→

$$maxZ = c^t\mathbf{x} \tag{5.0.1.7}$$
$$\text{subjected to} \tag{5.0.1.8}$$
$$A\mathbf{x} \leq b \tag{5.0.1.9}$$
$$\text{we can find } \rightarrow \tag{5.0.1.10}$$
$$c = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{5.0.1.11}$$
$$A = \begin{pmatrix} 3 & 5 \\ 5 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5.0.1.12}$$
$$b = \begin{pmatrix} 15 \\ 10 \\ 0 \\ 0 \end{pmatrix} \tag{5.0.1.13}$$

Solution for the above equations can be find from the python code of Lenear programing

./optimization/codes/lp5.eps

From the codes we get that the maximum value of the equation will be 12.37 at the point $\begin{pmatrix} 1.05 & 2.37 \end{pmatrix}$.

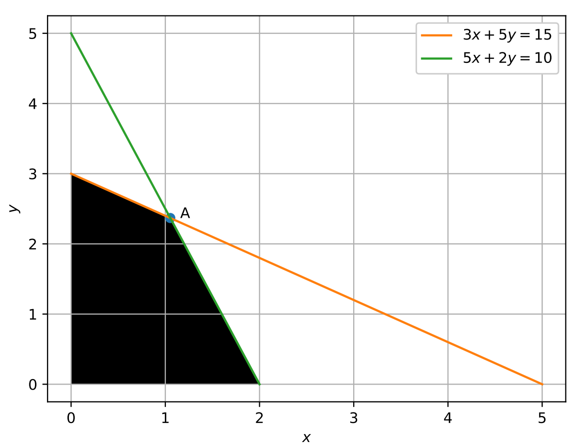

Fig. 5.0.1: lp5

Pythone codes for the above figure can be get from

./optimization/figures/lp5.eps

6 Problem

6.0.1 Question:

1. Minimise Z=3x+5y such that x+3y≥3, x+y≥2, x,y≥0.

6.0.2 Solution:

1. given that →

$$Z = 3x + 5y \tag{6.0.1.1}$$
$$\text{subjected to} \tag{6.0.1.2}$$
$$x + 3y \geq 3 \tag{6.0.1.3}$$
$$x + y \geq 2 \tag{6.0.1.4}$$
$$x \geq 0 \tag{6.0.1.5}$$
$$y \geq 0 \tag{6.0.1.6}$$

Comparing above equation to the gernelised form→

$$minZ = c^t\mathbf{x} \tag{6.0.1.7}$$
$$\text{subjected to} \tag{6.0.1.8}$$
$$A\mathbf{x} \leq b \tag{6.0.1.9}$$
$$\text{we can find } \rightarrow \tag{6.0.1.10}$$
$$c = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{6.0.1.11}$$
$$A = \begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix} \tag{6.0.1.12}$$
$$b = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{6.0.1.13}$$

Solution for the above equations can be find from the python code of Lenear programing.

./optimization/codes/lp6.eps

From the codes we get that the miniimum value of the equation will be 7 at the point $\begin{pmatrix} 1.5 & 0.5 \end{pmatrix}$.

7 Problem

7.0.1 Question:

1. Maximise Z=3x+2y subject to x+2y≤10, 3x+y≤15, x,y≥0.

7.0.2 Solution.7:

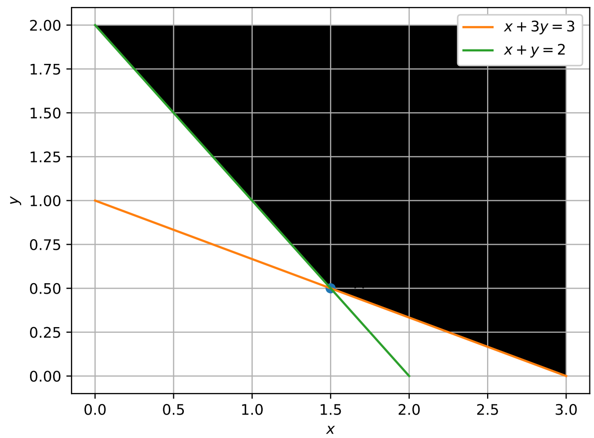

Fig. 6.0.1: lp6
pythone codes for the above figure can be get from

./optimization/figures/lp6.eps

1. given that →

$$Z = 3x + 2y \tag{7.0.1.1}$$
$$\text{subjected to} \tag{7.0.1.2}$$
$$x + 2y \leq 10 \tag{7.0.1.3}$$
$$3x + y \leq 15 \tag{7.0.1.4}$$
$$x \geq 0 \tag{7.0.1.5}$$
$$y \geq 0 \tag{7.0.1.6}$$

comparing above equation to the gernelised form→

$$maxZ = c^t\mathbf{x} \tag{7.0.1.7}$$
$$\text{subjected to} \tag{7.0.1.8}$$
$$A\mathbf{x} \leq b \tag{7.0.1.9}$$
$$\text{we can find } \rightarrow \tag{7.0.1.10}$$
$$c = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{7.0.1.11}$$
$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \tag{7.0.1.12}$$
$$b = \begin{pmatrix} 10 \\ 15 \end{pmatrix} \tag{7.0.1.13}$$

Solution for the above equations can be find from the python code of Lenear programing.

./optimization/codes/lp7.eps

From the codes we get that the maximum value of the equation will be 18 at the point $\begin{pmatrix} 4 & 3 \end{pmatrix}$.

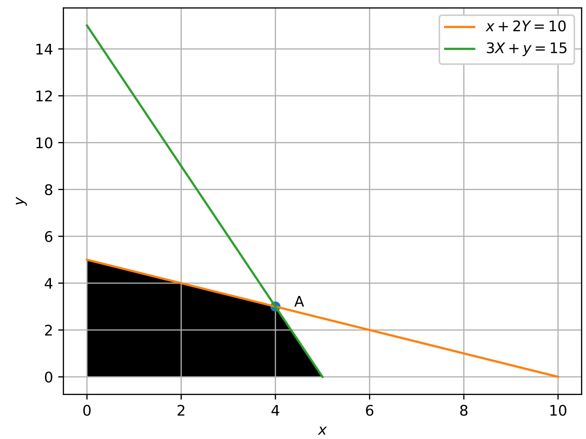

Fig. 7.0.1: lp7
pythone codes for the above figure can be get from

./optimization/figures/lp7.eps

8 Problem

8.0.1 Question:

1. Minimise Z=x+2y subject to 2x+y≥3, x+2y≥6, x,y≥0.
Show that the minimum of Z occurs at more than two points.

8.0.2 Solution:

1. given that →

$$Z = x + 2y \tag{8.0.1.1}$$
$$\text{subjected to} \tag{8.0.1.2}$$
$$2x + y \geq 3 \tag{8.0.1.3}$$
$$x + 2y \geq 6 \tag{8.0.1.4}$$
$$x \geq 0 \tag{8.0.1.5}$$
$$y \geq 0 \tag{8.0.1.6}$$

Comparing above equation to the gernelised form→

$$miniZ = c^t\mathbf{x} \tag{8.0.1.7}$$
$$\text{subjected to} \tag{8.0.1.8}$$
$$A\mathbf{x} \leq b \tag{8.0.1.9}$$
$$\text{we can find} \rightarrow \tag{8.0.1.10}$$
$$c = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{8.0.1.11}$$
$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \tag{8.0.1.12}$$
$$b = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \tag{8.0.1.13}$$

Solution for the above equations can be find from the python code of Lenear programing.

./optimization/codes/lp8.eps

From the codes we get that the miniimum value of the equation will be 6 but from the line eqaution 2 we can see that it gives 6 on each point thus for minimum value of 6 there are more than 2 points .

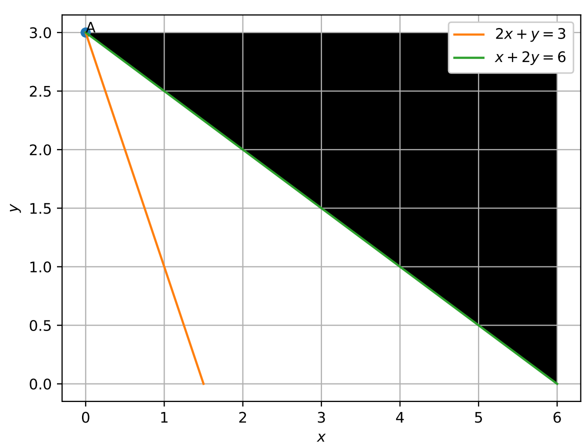

Fig. 8.0.1: lp8

Pythone codes for the above figure can be get from

./optimization/figures/lp8.eps

9 Problem

9.0.1 Question:

1. Minimise and Maximise Z=5x+10y subject to x+2y≤120, x+y≥60, x-2y≥0, x,y≥0.

9.0.2 Solution:

1. given that →

$$Z = 5x + 10y \tag{9.0.1.1}$$
$$\text{subjected to} \tag{9.0.1.2}$$
$$x + 2y \leq 120 \tag{9.0.1.3}$$
$$x + y \geq 60 \tag{9.0.1.4}$$
$$x - 2y \geq 0 \tag{9.0.1.5}$$
$$x \geq 0 \tag{9.0.1.6}$$
$$y \geq 0 \tag{9.0.1.7}$$

Comparing above equation to the gernelised form→

$$maxZ = c^t\mathbf{x} \tag{9.0.1.8}$$
$$\text{subjected to} \tag{9.0.1.9}$$
$$A\mathbf{x} \leq b \tag{9.0.1.10}$$
$$\text{we can find} \rightarrow \tag{9.0.1.11}$$
$$c = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \tag{9.0.1.12}$$
$$A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & -2 \end{pmatrix} \tag{9.0.1.13}$$
$$b = \begin{pmatrix} 120 \\ 60 \\ 0 \end{pmatrix} \tag{9.0.1.14}$$

Solution for the above equations can be find from the python code of Lenear programing.

From the codes we get that the miniimum value of the equation will be 300 at the point (60,0).
Maximum value of the equation is 600 but from the line eqaution 2 we can see that it gives 600 on each point thus maximum value of function will on the line $x + 2y = 120$ from the point J.

10 Problem

10.0.1 Question:

1. Minimise and Maximise Z=x+2y subject to x+2y≥100, 2x-y≤0, 2x+y≤200; x,y≥0.

10.0.2 Solution:

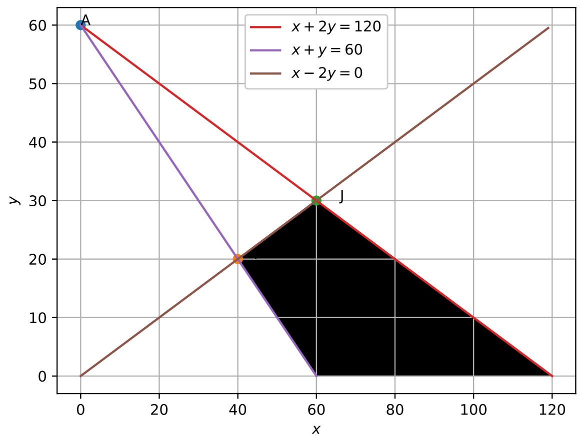

Fig. 9.0.1: lp9

pythone codes for the above figure can be get from

./optimization/figures/lp9.eps

1. given that →

$$Z = x + 2y \tag{10.0.1.1}$$
$$\text{subjected to} \tag{10.0.1.2}$$
$$x + 2y \geq 100 \tag{10.0.1.3}$$
$$2x - y \leq 0 \tag{10.0.1.4}$$
$$2x - y \leq 200 \tag{10.0.1.5}$$
$$x \geq 0 \tag{10.0.1.6}$$
$$y \geq 0 \tag{10.0.1.7}$$

Comparing above equation to the gernelised form→

$$maxZ = c^t\mathbf{x} \tag{10.0.1.8}$$
$$\text{subjected to} \tag{10.0.1.9}$$
$$A\mathbf{x} \preceq b \tag{10.0.1.10}$$
$$\text{we can find } \rightarrow \tag{10.0.1.11}$$
$$c = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{10.0.1.12}$$
$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 2 & 1 \end{pmatrix} \tag{10.0.1.13}$$
$$b = \begin{pmatrix} 100 \\ 0 \\ 200 \end{pmatrix} \tag{10.0.1.14}$$

Solution for the above equations can be find from the python code of Lenear programing.

./optimization/codes/lp10.eps

From the codes we get that the maximum value of the equation will be at 400 the point (0,200).

Maximum value of the equation is 100 but from the line eqaution 1 we can see that it gives 100 on each point thus maximum value of function will on the line $x + 2y = 120$ from the point (0,50) to point A.

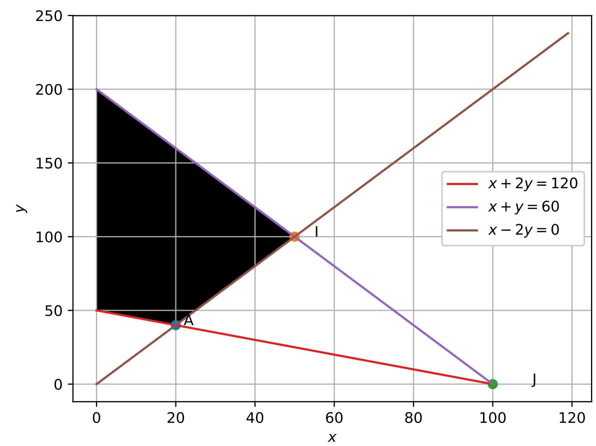

Fig. 10.0.1: lp10

pythone codes for the above figure can be get from

./optimization/figures/lp10.eps