

Solution For School Geometry Problems

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Abstract—This document includes different problems and solution on geometry from trigonometry and linear algebra. It also provides the information about the python and latex codes of figures.

Download all python codes from

```
svn co https://github.com/yogi13995/
yogesh_training/tree/master/Geometry/
line_alg/codes
```

and latex-tikz codes from

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svn co https://github.com/yogi13995/
yogesh_training/tree/master/Geometry/
line_alg/figures
```

equation of the line going through points **S** and **R** can be given as

$$\mathbf{x} = \mathbf{R} + \lambda (\mathbf{S} - \mathbf{R}) \quad (1.1.1.3)$$

$$\mathbf{x} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \lambda \left(\begin{pmatrix} 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix} \right) \quad (1.1.1.4)$$

$$\mathbf{x} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad (1.1.1.5)$$

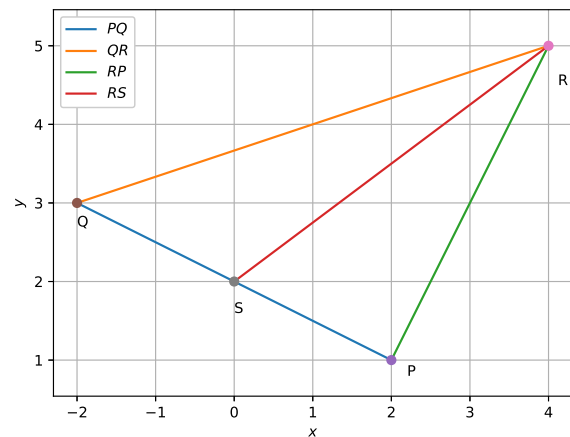


Fig. 1.1.1: triangle

Path to python code for the above figure

codes/triangle/triangle.py

1 TRIANGLE

1.0.1 Problem:

- The vertices of ΔPQR are $\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. Find the equation of the median through the vertex **R**.

1.1 Solution

- We have a triangle as given below. First of all we will find out the midpoint of the **AB** because each median divide the side in two equal part. Finding out the point **S** as given in fig (1.1)... **S** is the midpoint of the **P** and **Q**

$$\mathbf{S} = \frac{\mathbf{P} + \mathbf{Q}}{2} \quad (1.1.1.1)$$

Direction vector in the direction of RS

$$\mathbf{RS} = \mathbf{S} - \mathbf{R} \quad (1.1.1.2)$$

2 QUADRILATERAL

2.1 Problem

- Find the area of a rhombus if its vertices are $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ taken in order.

2.2 Solution

- let assume that the vertices of the rhombus are **P**, **Q**, **R** and **S** respectively as shown in

fig(2.2.1). finding out the **SP** and **QP**...

$$\mathbf{P} - \mathbf{S} = \begin{pmatrix} 3+2 \\ 0+1 \end{pmatrix} \quad (2.2.1.1)$$

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (2.2.1.2)$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 4-3 \\ 5-0 \end{pmatrix} \quad (2.2.1.3)$$

$$= \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (2.2.1.4)$$

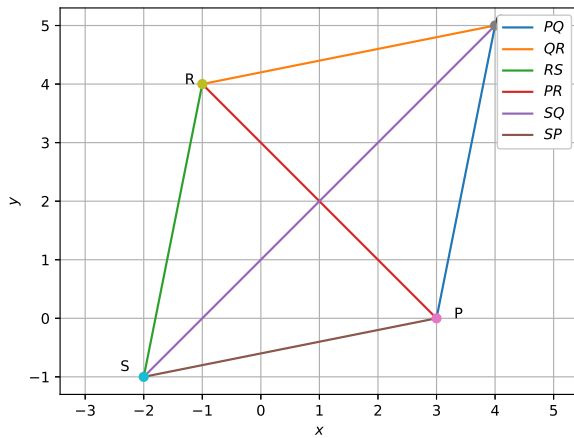


Fig. 2.2.1: quadrilateral
Path to the python code for the above figure

codes/quadrilateral/quad.py

S Area of the rhombus can be calculated as follows

$$\|\Delta\| = \|(\mathbf{P} - \mathbf{S}) \times (\mathbf{Q} - \mathbf{P})\| \quad (2.2.1.5)$$

$$\|\Delta\| = \left\| \begin{pmatrix} 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| \quad (2.2.1.6)$$

$$\|\Delta\| = 5 \times 5 - 1 \times 1 \quad (2.2.1.7)$$

$$\|\Delta\| = 24 \quad (2.2.1.8)$$

3 LINE

3.1 Point and Vector

3.1.1 Problem:

1. Name the type of Quadrilateral formed, if any, by the following points, and give reasons for your answer.

(a) $\begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

(b) $\begin{pmatrix} -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

(c) $\begin{pmatrix} 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3.1.2 Solution:

1. here

$$\mathbf{P} - \mathbf{S} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (3.1.1.1)$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (3.1.1.2)$$

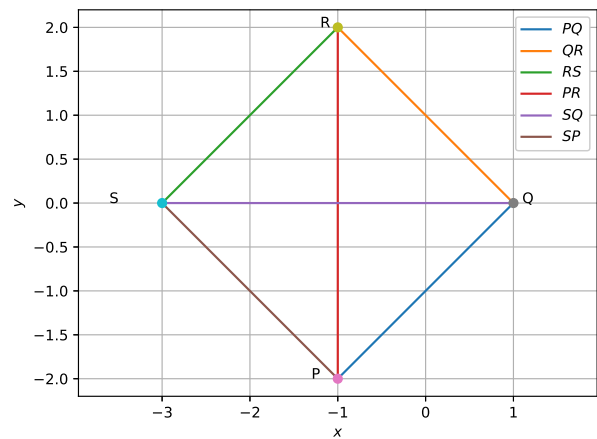


Fig. 3.1.1: quadrilateral
Path to the python code for the above figure

codes/line/quad/quad1.py

$$\mathbf{d1} = \mathbf{R} - \mathbf{P} \quad (3.1.1.3)$$

$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (3.1.1.4)$$

$$\mathbf{d2} = \mathbf{S} - \mathbf{Q} \quad (3.1.1.5)$$

$$= \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (3.1.1.6)$$

$$\|\mathbf{d1} \times \mathbf{d2}\| = \left\| \begin{pmatrix} 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| = 16 \quad (3.1.1.7)$$

$$\|(\mathbf{P} - \mathbf{S}) \times (\mathbf{Q} - \mathbf{P})\| = \left\| \begin{pmatrix} 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\| = 8 \quad (3.1.1.8)$$

$$\frac{1}{2} \|\mathbf{d1} \times \mathbf{d2}\| = \|(\mathbf{P} - \mathbf{S}) \times (\mathbf{Q} - \mathbf{P})\| = 8 \quad (3.1.1.9)$$

from above we can say that the area of the quadrilateral is equal to the half of the multiplication of its diagonals. thus this is a rhombus.

2.

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \quad (3.1.2.1)$$

$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (3.1.2.2)$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (3.1.2.3)$$

$$(\mathbf{Q} - \mathbf{P}) = (\mathbf{R} - \mathbf{P}) + (\mathbf{Q} - \mathbf{R}) \quad (3.1.2.4)$$

$$= \begin{pmatrix} 6 \\ -4 \end{pmatrix} \quad (3.1.2.5)$$

so from above we can say that \mathbf{P} , \mathbf{Q} and \mathbf{R} are linear so it can not be a quadrilateral.

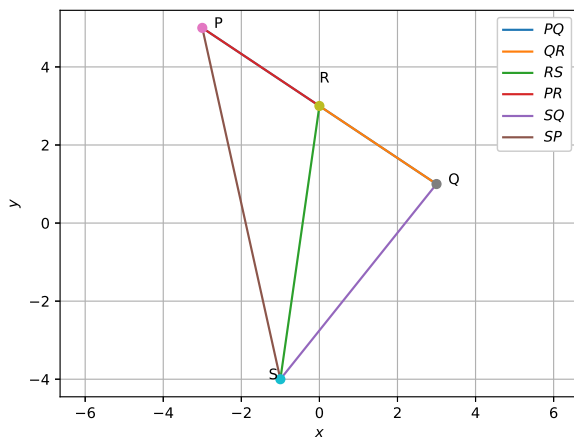


Fig. 3.1.2: quadrilateral2
Path to the python code for the above figure

[codes/line/quad/quad2.py](#)

3.

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (3.1.3.1)$$

$$\mathbf{R} - \mathbf{S} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (3.1.3.2)$$

$$\mathbf{P} - \mathbf{S} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (3.1.3.3)$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (3.1.3.4)$$

$$(\mathbf{Q} - \mathbf{P}) = (\mathbf{R} - \mathbf{S}) \quad (3.1.3.5)$$

$$(\mathbf{P} - \mathbf{S}) = (\mathbf{Q} - \mathbf{R}) \quad (3.1.3.6)$$

Thus above equations shows that the opposites sides are parallel and equal in length

$$\mathbf{d1} = \mathbf{R} - \mathbf{P} \quad (3.1.3.7)$$

$$= \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (3.1.3.8)$$

$$\mathbf{d2} = \mathbf{S} - \mathbf{Q} \quad (3.1.3.9)$$

$$= \begin{pmatrix} -6 \\ -4 \end{pmatrix} \quad (3.1.3.10)$$

$$\|\mathbf{d1} \times \mathbf{d2}\| = \left\| \begin{pmatrix} 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 5 \end{pmatrix} \right\| \quad (3.1.3.11)$$

$$= 8 \quad (3.1.3.12)$$

$$\|(\mathbf{P} - \mathbf{S}) \times (\mathbf{Q} - \mathbf{P})\| = \left\| \begin{pmatrix} 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\| \quad (3.1.3.13)$$

$$= 10 \quad (3.1.3.14)$$

$$\|(\mathbf{P} - \mathbf{S}) \times (\mathbf{Q} - \mathbf{P})\| \neq \|(\mathbf{P} - \mathbf{S}) \times (\mathbf{Q} - \mathbf{P})\| \quad (3.1.3.15)$$

From equation (3.1.3.5), (3.1.3.6) and (3.1.3.15) we can say that it is a parallelogram.

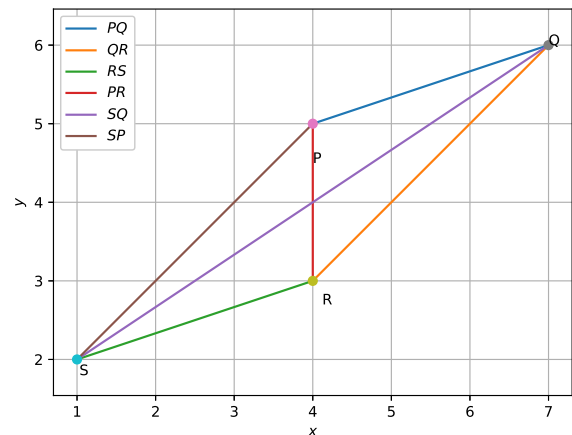


Fig. 3: quadrilateral3
Path to the python code for the above figure

[codes/line/quad/quad3.py](#)

3.2 Point on a line

3.2.1 Problem:

1. Find the ratio in which the line segment joining $\begin{pmatrix} 1 \\ -5 \end{pmatrix}, \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ is divided by the x-axis. Also find the coordinates of the point of division.

3.2.2 Solution:

1. Let assume that we have a point $C \begin{pmatrix} x \\ 0 \end{pmatrix}$ which divide the line segment AB in $k:1$ ratio.

$$(k+1) \begin{pmatrix} x \\ 0 \end{pmatrix} = k \begin{pmatrix} 1 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix} \quad (3.2.1.1)$$

$$0 = -5k + 5 \quad (3.2.1.2)$$

$$k = 1 \quad (3.2.1.3)$$

$$C = \frac{\begin{pmatrix} -3 \\ 0 \end{pmatrix}}{2} \quad (3.2.1.4)$$

$$= \begin{pmatrix} -1.5 \\ 0 \end{pmatrix} \quad (3.2.1.5)$$

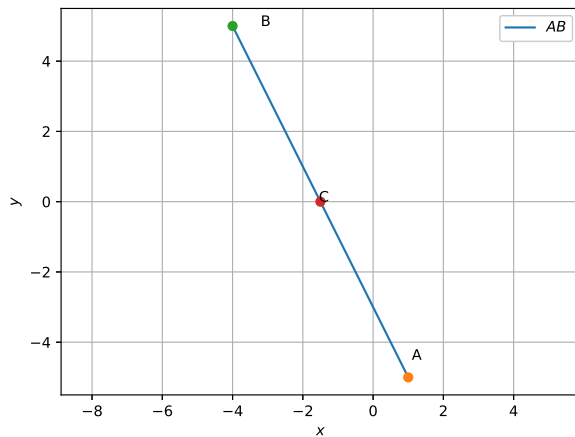


Fig. 3.2.1: line
path to the python code for the above figure

codes/line/point_on_line/points_on_line
.py

3.3 Lines and Planes

3.3.1 Problem:

1. Sketch the lines
- $\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} = 9.35$,
 - $\begin{pmatrix} 1 & -\frac{1}{5} \end{pmatrix} \mathbf{x} = 10$
 - $\begin{pmatrix} -2 & 3 \end{pmatrix} \mathbf{x} = 6$,
 - $\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 0$
 - $\begin{pmatrix} 2 & 5 \end{pmatrix} \mathbf{x} = 0$,
 - $\begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} = -2$

g) $\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 2$,

h) $\begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = 5$

3.3.2 Solution:

1. All the lines can be drawn as follow

- a) put $\mathbf{x} \begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = \frac{187}{20} \quad (3.3.1.1)$$

$$x = \frac{187}{40} \quad (3.3.1.2)$$

- put $\mathbf{x} \begin{pmatrix} 0 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = \frac{187}{20} \quad (3.3.1.3)$$

$$y = \frac{187}{60} \quad (3.3.1.4)$$

$$\mathbf{P1} = \begin{pmatrix} \frac{187}{40} \\ 0 \end{pmatrix} \quad (3.3.1.5)$$

$$\mathbf{Q1} = \begin{pmatrix} 0 \\ \frac{187}{60} \end{pmatrix} \quad (3.3.1.6)$$

- b) put $\mathbf{x} \begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 1 & -\frac{1}{5} \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 10 \quad (3.3.1.7)$$

$$x = 10 \quad (3.3.1.8)$$

- put $\mathbf{x} \begin{pmatrix} 0 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} 1 & -\frac{1}{5} \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = 10 \quad (3.3.1.9)$$

$$y = -50 \quad (3.3.1.10)$$

$$\mathbf{P2} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \quad (3.3.1.11)$$

$$\mathbf{Q2} = \begin{pmatrix} 0 \\ -50 \end{pmatrix} \quad (3.3.1.12)$$

- c) put $\mathbf{x} \begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 6 \quad (3.3.1.13)$$

$$x = -3 \quad (3.3.1.14)$$

put $\mathbf{x} \begin{pmatrix} 0 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} -2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = 6 \quad (3.3.1.15)$$

$$y = 2 \quad (3.3.1.16)$$

$$\mathbf{P3} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (3.3.1.17)$$

$$\mathbf{Q3} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (3.3.1.18)$$

d) there is no constant in the line equation thus it passes through the origin.

put $\mathbf{x} \begin{pmatrix} 3 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 3 \\ y \end{pmatrix} = 0 \quad (3.3.1.19)$$

$$y = 1 \quad (3.3.1.20)$$

$$\mathbf{P4} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3.3.1.21)$$

$$\mathbf{Q4} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (3.3.1.22)$$

e) there is no constant in the line equation thus it passes through the origin

put $\mathbf{x} \begin{pmatrix} 1 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ y \end{pmatrix} = 0 \quad (3.3.1.23)$$

$$y = 1 \quad (3.3.1.24)$$

$$\mathbf{P5} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3.3.1.25)$$

$$\mathbf{Q5} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (3.3.1.26)$$

f) put $\mathbf{x} \begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = -2 \quad (3.3.1.27)$$

$$x = -\frac{2}{3} \quad (3.3.1.28)$$

we can see in this equation the value of x coordinate does not depend on the y coordinate so we can say that it is parallel to the y-axis.

g) put $\mathbf{x} \begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = 2 \quad (3.3.1.29)$$

$$y = 2 \quad (3.3.1.30)$$

we can see in this equation the value of y coordinate does not depend on the x coordinate so we can say that it is parallel to the x-axis.

h) put $\mathbf{x} \begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 5 \quad (3.3.1.31)$$

$$x = -\frac{5}{2} \quad (3.3.1.32)$$

we can see in this equation the value of x coordinate does not depend on the y coordinate so we can say that it is parallel to the y-axis.

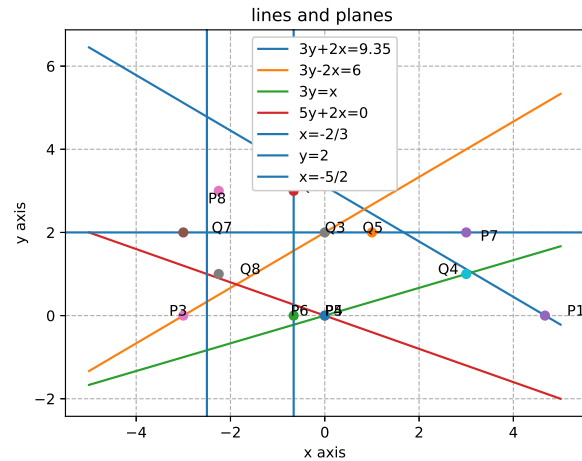


Fig. 3.3.1: lines

Path to the python code for the above figure

codes/line/lines_and_planes/
plane_and_line.py

3.4 Motion

3.4.1 Problem:

1. A hicker stands on the edge of a cliff 490m above the ground and throws a stone horizon-

tally with an initial speed of 15ms^{-1} . Neglecting air rsistance , find the time taken by the stone to reach the ground ,and the speed with which it hits the ground .

3.4.2 Solution:

1. given \Rightarrow

$$\mathbf{A} = \begin{pmatrix} 0 \\ 490 \end{pmatrix} \quad (3.4.1.1)$$

$$\mathbf{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \quad (3.4.1.2)$$

$$\mathbf{v}_A = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} \quad (3.4.1.3)$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{a}t \quad (3.4.1.4)$$

$$\mathbf{d} = \mathbf{v}_A t + \frac{1}{2} \mathbf{a} t^2 \quad (3.4.1.5)$$

$$\mathbf{B} = \mathbf{A} + \mathbf{d} \quad (3.4.1.6)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 490 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \quad (3.4.1.7)$$

$$\begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 490 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \quad (3.4.1.8)$$

$$490 = \frac{1}{2} 9.8 t^2 \quad (3.4.1.9)$$

$$t = 10 \quad (3.4.1.10)$$

$$\mathbf{v}_B = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} 10 \quad (3.4.1.11)$$

$$= \begin{pmatrix} 1.5 \\ 98 \end{pmatrix} \quad (3.4.1.12)$$

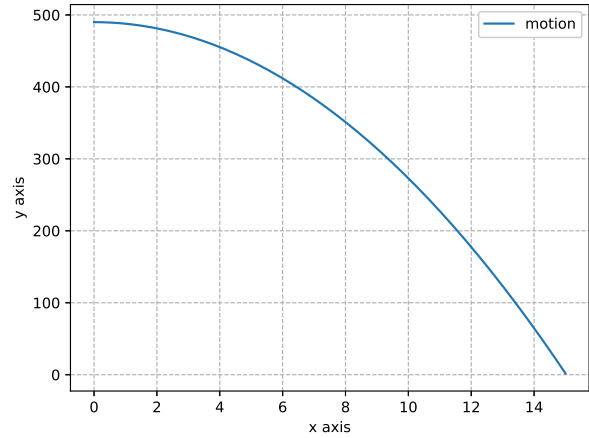


Fig. 3.4.1: motion path to the python code for the above figure

codes/line/motion/motion.py

a)

$$(3.5.1.1)$$

$$a_{ij} = \frac{(i+j)^2}{2} \quad (3.5.1.2)$$

$$a_{11} = 2 \quad (3.5.1.3)$$

$$a_{12} = 4.5 \quad (3.5.1.4)$$

$$a_{21} = 4.5 \quad (3.5.1.5)$$

$$a_{22} = 8 \quad (3.5.1.6)$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (3.5.1.7)$$

$$\mathbf{A} = \begin{pmatrix} 2 & 4.5 \\ 4.5 & 8 \end{pmatrix} \quad (3.5.1.8)$$

b)

$$a_{ij} = \frac{i}{j} \quad (3.5.1.9)$$

$$a_{11} = 1 \quad (3.5.1.10)$$

$$a_{12} = 0.5 \quad (3.5.1.11)$$

$$a_{21} = 2 \quad (3.5.1.12)$$

$$a_{22} = 1 \quad (3.5.1.13)$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (3.5.1.14)$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0.5 \\ 2 & 1 \end{pmatrix} \quad (3.5.1.15)$$

3.5 Matrix

3.5.1 Problem:

1. Construct a 2 x 2 matrix $\mathbf{A} = [a_{ij}]$, whose elements are given by:

a) $a_{ij} = \frac{(i+j)^2}{2}$

b) $a_{ij} = \frac{i}{j}$

c) $a_{ij} = \frac{(i+2j)^2}{2}$

3.5.2 Solution:

1. Formation of matrix can be done as follow

c)

$$a_{ij} = \frac{(i+2j)^2}{2} \quad (3.5.1.16)$$

$$a_{11} = 4.5 \quad (3.5.1.17)$$

$$a_{12} = 12.5 \quad (3.5.1.18)$$

$$a_{21} = 8 \quad (3.5.1.19)$$

$$a_{22} = 18 \quad (3.5.1.20)$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (3.5.1.21)$$

$$\mathbf{A} = \begin{pmatrix} 4.5 & 12.5 \\ 2 & 18 \end{pmatrix} \quad (3.5.1.22)$$

Path to python codes to get above matrices is..

codes/line/matrix/matrix.py

3.6 Determinants

3.6.1 Problem:

1. If $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then show that $|3\mathbf{A}| = 27|\mathbf{A}|$

3.6.2 Solution:

1.

$$|3\mathbf{A}| = 3 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} \quad (3.6.1.1)$$

$$= \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix} \quad (3.6.1.2)$$

$$|3\mathbf{A}| = 108 \quad (3.6.1.3)$$

$$|\mathbf{A}| = 4 \quad (3.6.1.4)$$

$$|3\mathbf{A}| = 27|\mathbf{A}| \quad (3.6.1.5)$$

hence proved

3.7.2 Solution:

1. let assume that the student get x marks in the annual examination so now...

$$\frac{62 + 48 + x}{3} \geq 60 \quad (3.7.1.1)$$

$$x + 110 \geq 180 \quad (3.7.1.2)$$

$$x \geq 70 \quad (3.7.1.3)$$

4 CIRCLE

4.1 Problem

1. find the area enclosed by the circle $(x) = a$ area of circle

4.2 Solution

1.

$$\|\Delta\| = 2\pi r^2 \quad (4.2.1.1)$$

$$\|\Delta\| = 2\pi a^2 \quad (4.2.1.2)$$

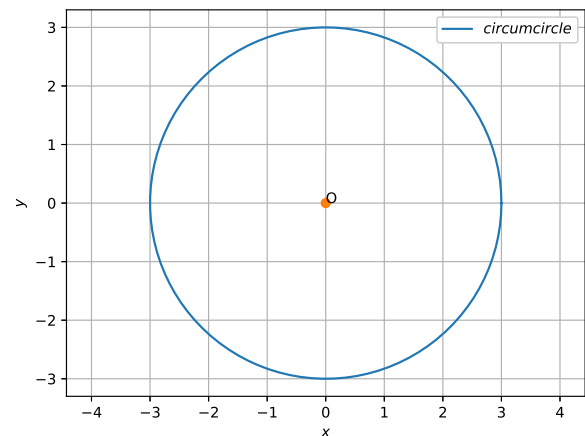


Fig. 4.2.1: circle
path to the code for the above figure

codes/circle/circle.py

3.7 Linear inequalities

3.7.1 Problem:

1. the marks obtained by the student of class XI in first and second terminal examination are 62 and 48, respectively. Find the minimum marks in annual examination to have an average of at least 60 marks.

5 CONICS

5.1 Problem

1. Find the zeroes of the following Quadratic polynomials and verify the relationship between the zeroes and the coefficients.
- a) $x^2 - 2x - 8$

- b) $4u^2 + 8u$
- c) $4s^2 - 4s + 1$
- d) $t^2 - 15$
- e) $6x^2 - 3 - 7x$
- f) $3x^2 - 2x - 8$

5.2 Solution

1.

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} D & E \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} + k = 0 \quad (5.2.1.1)$$

2.

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - 8 \quad (5.2.2.1)$$

$$x^2 - 2x - 8 = 0 \quad (5.2.2.2)$$

$$(x - 4)(x + 2) = 0 \quad (5.2.2.3)$$

$$\alpha = 4, \beta = -2 \quad (5.2.2.4)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.2.5)$$

$$\alpha + \beta = -\frac{b}{a} = 2 \quad (5.2.2.6)$$

$$\alpha \times \beta = \frac{c}{a} = -8 \quad (5.2.2.7)$$

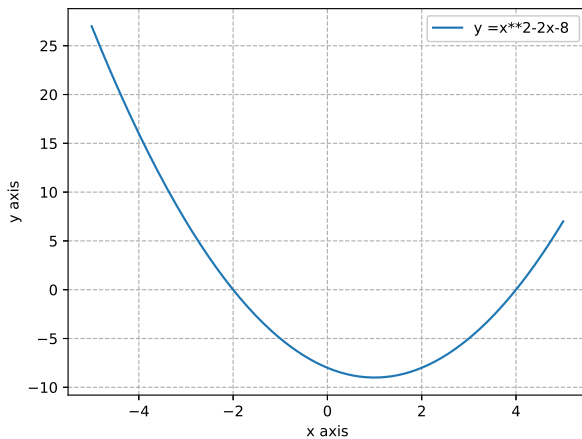


Fig. 5.2.2: equation 1

path to the python code for the above figure is

codes/conics/perabola2.py

3.

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (5.2.3.1)$$

$$4u^2 + 8u = 0 \quad (5.2.3.2)$$

$$(4u)(u + 2) = 0 \quad (5.2.3.3)$$

$$\alpha = 0, \beta = -2 \quad (5.2.3.4)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.3.5)$$

$$\alpha + \beta = -\frac{b}{a} = -2 \quad (5.2.3.6)$$

$$\alpha \times \beta = \frac{c}{a} = 0 \quad (5.2.3.7)$$

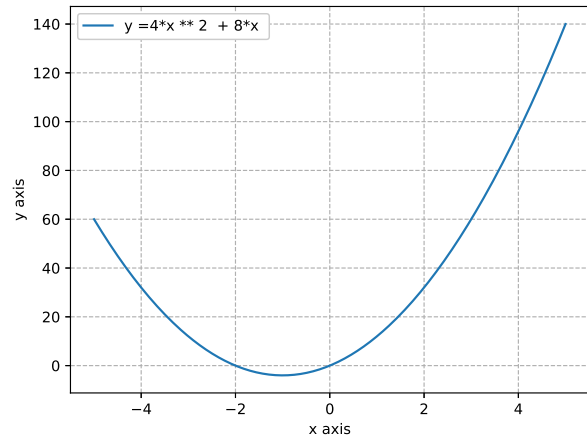


Fig. 5.2.3: equation 2

path to the python code for the above figure is

codes/conics/perabola2.py

4.

$$\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 1 \quad (5.2.4.1)$$

$$4s^2 - 4s + 1 = 0 \quad (5.2.4.2)$$

$$(2s - 1)(2s - 1) = 0 \quad (5.2.4.3)$$

$$\alpha = \frac{1}{2}, \beta = -\frac{1}{2} \quad (5.2.4.4)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.4.5)$$

$$\alpha + \beta = -\frac{b}{a} = 1 \quad (5.2.4.6)$$

$$\alpha \times \beta = \frac{c}{a} = \frac{1}{4} \quad (5.2.4.7)$$

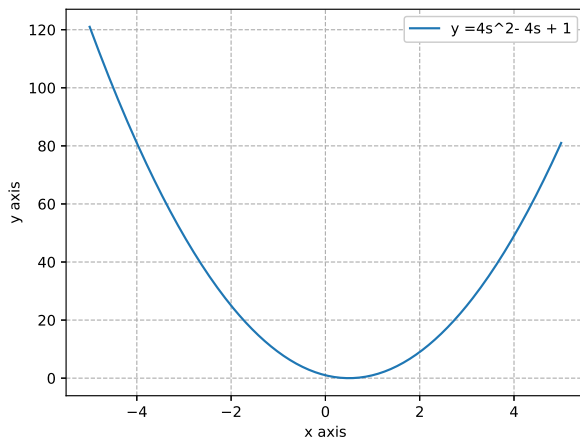


Fig. 5.2.4: equation 3

path to the python code for the above figure is

codes/conics/perabola3.py

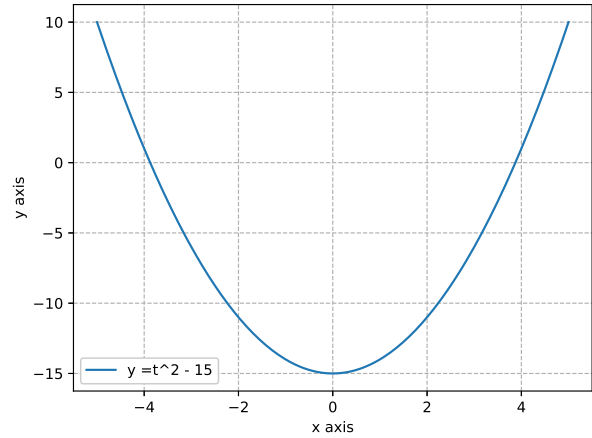


Fig. 5.2.5: equation 4

path to the python code for the above figure is

codes/conics/perabola4.py

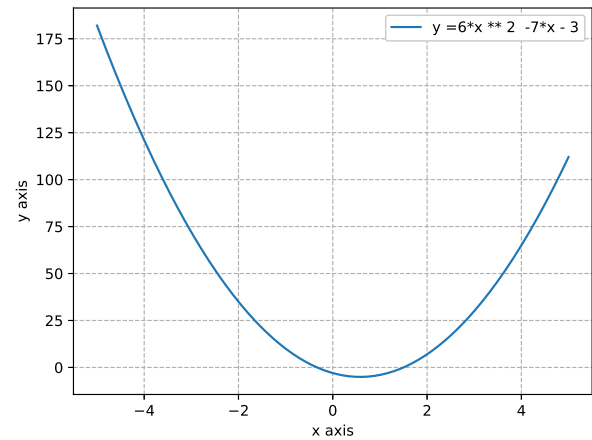


Fig. 5.2.6: equation 5

path to the python code for the above figure is

codes/conics/perabola5.py

5.

$$[\mathbf{x}]^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{x}] + [0 \ 0] [\mathbf{x}] - 15 \quad (5.2.5.1)$$

$$t^2 - 15 = 0 \quad (5.2.5.2)$$

$$\alpha = \sqrt{15}, \beta = -\sqrt{15} \quad (5.2.5.3)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.5.4)$$

$$\alpha + \beta = -\frac{b}{a} = 0 \quad (5.2.5.5)$$

$$\alpha \times \beta = \frac{c}{a} = -15 \quad (5.2.5.6)$$

6.

$$[\mathbf{x}]^T \begin{bmatrix} 6 & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{x}] + [-7 \ 0] [\mathbf{x}] - 3 \quad (5.2.6.1)$$

$$6x^2 - 3 - 7x = 0 \quad (5.2.6.2)$$

$$(2x - 3)(3x + 1) = 0 \quad (5.2.6.3)$$

$$\alpha = \frac{3}{2}, \beta = -\frac{1}{3} \quad (5.2.6.4)$$

$$\alpha + \beta = -\frac{b}{a} = \frac{7}{6} \quad (5.2.6.5)$$

$$\alpha \times \beta = \frac{c}{a} = -\frac{1}{2} \quad (5.2.6.6)$$

7.

$$[\mathbf{x}]^T \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{x}] + \begin{bmatrix} -1 & 0 \end{bmatrix} [\mathbf{x}] - 4 \quad (5.2.7.1)$$

$$3x^2 - 2x - 8 = 0 \quad (5.2.7.2)$$

$$(3x + 4)(x + 1) = 0 \quad (5.2.7.3)$$

$$\alpha = -1, \beta = -\frac{4}{3} \quad (5.2.7.4)$$

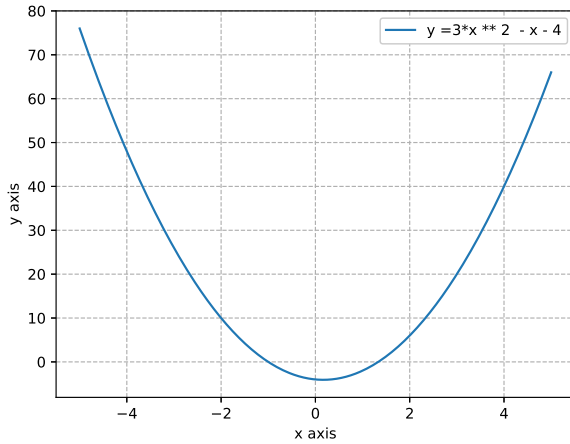


Fig. 5.2.7: equation 6

path to the python code for the above figure is

codes/conis/perabola6.py

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.7.5)$$

$$\alpha + \beta = -\frac{b}{a} = \frac{2}{3} \quad (5.2.7.6)$$

$$\alpha \times \beta = \frac{c}{a} = -\frac{8}{3} \quad (5.2.7.7)$$