Solution For Problem 8.1.26

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Question

Exercise 8.1(Q no.36)

Line l is the bisector of $\angle A$ and **B** is any point on l. **BP** and **BQ** are perpendiculars from **B** to the arms of $\angle A$ show that :

a)$\Delta APB \cong \Delta AQB$

b)$\mathbf{BP} = \mathbf{BQ}$

Codes and Figures

The python code for the figure is

```
./code/angle.py
```

The latex- tikz code is

```
./figs/angle.tex
```

The above latex code can be compiled as standalone document

```
./figs/angle_fig.tex
```

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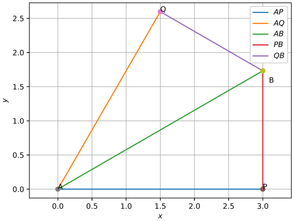
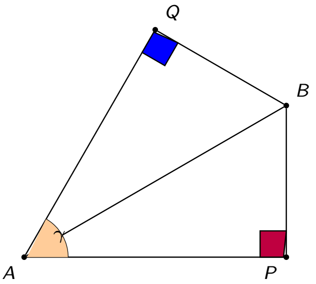


(a) By Python



(b) By Latex-tikz

Construction method

The tables below are the values used for constructing the triangles in both Python and Latex-Tikz.

Input Values.	
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
P	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
$\angle PAQ$	60

Table: To construct $\angle ACB$ and

The steps for constructing $\triangle ACB$ are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} r * \cos 0 \\ r * \sin 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\left(\mathbf{B} - \mathbf{P}\right)^T \left(\mathbf{A} - \mathbf{P}\right) = 0$$

$$\angle\gamma = \frac{\angle\theta}{2} = 30$$

$$\mathbf{Q} = \begin{pmatrix} b * \cos\gamma - 3 \\ b * \sin\gamma - 0 \end{pmatrix} = \begin{pmatrix} 0 - 3 \\ 0 - 0 \end{pmatrix}$$

$$3 * (b * \cos\gamma - 3) = 0$$

$$b * \cos\gamma = 3$$

$$b = 3.4641$$

$$\mathbf{B} = \begin{pmatrix} 3.4641 * \cos 30 \\ 3.4641 * \sin 30 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.732 \end{pmatrix}$$

Derived Values for $\angle PAQ$.	
Q	$\begin{pmatrix} 1.5 \\ 2.59 \end{pmatrix}$
P	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
B	$\begin{pmatrix} 3 \\ 1.7 \end{pmatrix}$

Table: To construct madians AN and PN

Solution a)

from the ΔAPB and ΔAQB...

$$\|\mathbf{A} - \mathbf{P}\| = \|\mathbf{A} - \mathbf{P}\|$$

$$\angle AQB = \angle APB$$

AB is bisector of $\angle QAP$

$$\implies \angle AQB = \angle APB$$

thus from ASA conguransy

$$\Delta APB \cong \Delta AQB$$

Solution b)

$$\Delta APB \cong \Delta AQB$$

$$\implies \|\mathbf{BQ}\| = \|\mathbf{BP}\|$$

Hence proved