Solution For School Geometry Problems

Yogesh Choudhary

***Abstract*—This document includes different problems and solution on geometry from trigonometry and linear algebra.It also provides the imformation about the python and latex codes of figures.**

Download all python codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/
    line_alg/codes
```

and latex-tikz codes from

```
svn co https://github.com/yogi13995/
    yogesh_training/tree/master/Geometry/
    line_alg/figures
```

1 Triangle

1.0.1 Problem:

1. The vertices of ΔPQR are $\mathbf{P} = \begin{pmatrix}2\\1\end{pmatrix}$,$\mathbf{Q} = \begin{pmatrix}-2\\3\end{pmatrix}$,$\mathbf{R} = \begin{pmatrix}4\\5\end{pmatrix}$. Find the equation of the median through the vertex $\mathbf{R}$.

1.1 Solution

1. We have a triangle as given bellow .First of all we will find out the midpoint of the $\mathbf{A}B$ because each median devide the side in two equal part. Finding out the point $\mathbf{S}$ as given in fig (1.1)... $\mathbf{S}$ is the midpoint of the $\mathbf{P}$ and $\mathbf{Q}$

$$\mathbf{S} = \frac{\mathbf{P}+\mathbf{Q}}{2} \tag{1.1.1.1}$$

Direction vector in the direction of RS

$$\mathbf{RS} = \mathbf{S} - \mathbf{R} \tag{1.1.1.2}$$

equation of the line going through points $\mathbf{S}$ and $\mathbf{R}$ can be given as

$$\mathbf{x} = \mathbf{R} + \lambda(\mathbf{S} - \mathbf{R}) \tag{1.1.1.3}$$

$$\mathbf{x} = \begin{pmatrix}4\\5\end{pmatrix} + \lambda\left(\begin{pmatrix}0\\2\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix}\right) \tag{1.1.1.4}$$

$$\mathbf{x} = \begin{pmatrix}4\\5\end{pmatrix} + \lambda\left(\begin{pmatrix}-4\\-3\end{pmatrix}\right) \tag{1.1.1.5}$$

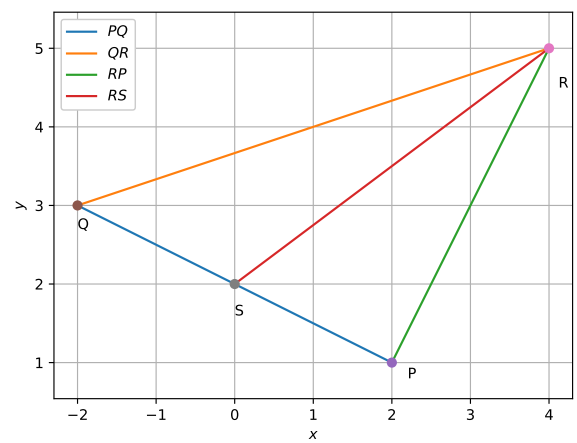

Fig. 1.1.1: triange

Path to pythone code for the above figure

```
codes/triangle/triangle.py
```

2 Quadrilateral

2.1 Problem

1. Find the area of a rhombus if its vertices are $\begin{pmatrix}3\\0\end{pmatrix}$,$\begin{pmatrix}4\\5\end{pmatrix}$,$\begin{pmatrix}-1\\4\end{pmatrix}$ and $\begin{pmatrix}-2\\-1\end{pmatrix}$ taken in order.

2.2 Solution

1. let assume that the vertices of the rhombus are$\mathbf{P}$, $\mathbf{Q}$,$\mathbf{R}$ and $\mathbf{S}$ respectively as shown in

fig(2.2.1). finding out the **SP** and **QP**...

$$\mathbf{P}-\mathbf{S}=\begin{pmatrix}3+2\\0+1\end{pmatrix} \tag{2.2.1.1}$$
$$=\begin{pmatrix}5\\1\end{pmatrix} \tag{2.2.1.2}$$
$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}4-3\\5-0\end{pmatrix} \tag{2.2.1.3}$$
$$=\begin{pmatrix}1\\5\end{pmatrix} \tag{2.2.1.4}$$

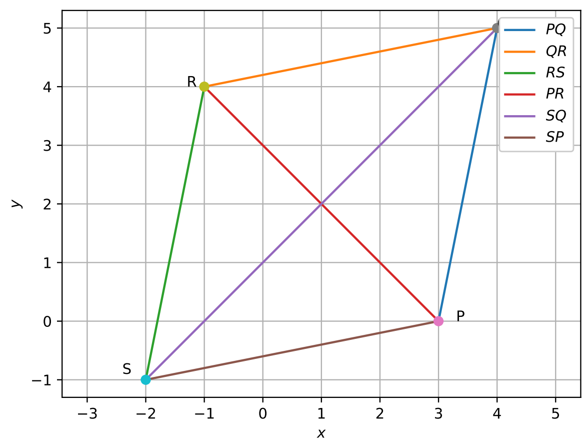

Fig. 2.2.1: quadrilateral
Path to the python code for the above figure

codes/quadrilateral/quad.py

S Area of the rhombus can be calculated as follows

$$\|\Delta\|=\|(\mathbf{P}-\mathbf{S})\times(\mathbf{Q}-\mathbf{P})\| \tag{2.2.1.5}$$
$$\|\Delta\|=\left\|\begin{pmatrix}5\\1\end{pmatrix}\times\begin{pmatrix}1\\5\end{pmatrix}\right\| \tag{2.2.1.6}$$
$$\|\Delta\|=5\times5-1\times1 \tag{2.2.1.7}$$
$$\|\Delta\|=24 \tag{2.2.1.8}$$

3 Line

3.1 Point and Vector

3.1.1 Problem:

1. Name the type of Quadriletral formed ,if any,by the following points, and give reaons for your answer.
   (a) $\begin{pmatrix}-1\\-2\end{pmatrix},\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}-1\\2\end{pmatrix},\begin{pmatrix}-3\\0\end{pmatrix}$
   (b) $\begin{pmatrix}-3\\5\end{pmatrix},\begin{pmatrix}3\\1\end{pmatrix},\begin{pmatrix}0\\3\end{pmatrix},\begin{pmatrix}-1\\-4\end{pmatrix}$
   (c) $\begin{pmatrix}4\\5\end{pmatrix},\begin{pmatrix}7\\6\end{pmatrix},\begin{pmatrix}4\\3\end{pmatrix},\begin{pmatrix}1\\2\end{pmatrix}$

3.1.2 Solution:

1. here

$$\mathbf{P}-\mathbf{S}=\begin{pmatrix}2\\2\end{pmatrix} \tag{3.1.1.1}$$
$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}2\\2\end{pmatrix} \tag{3.1.1.2}$$

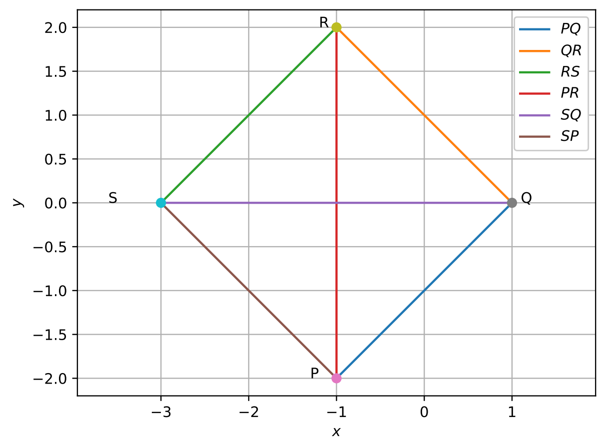

Fig. 3.1.1: quadrilateral1
Path to the python code for the above figure

codes/line/quad/quad1.py

$$\mathbf{d1}=\mathbf{R}-\mathbf{P} \tag{3.1.1.3}$$
$$=\begin{pmatrix}0\\4\end{pmatrix} \tag{3.1.1.4}$$
$$\mathbf{d2}=\mathbf{S}-\mathbf{Q} \tag{3.1.1.5}$$
$$=\begin{pmatrix}-4\\0\end{pmatrix} \tag{3.1.1.6}$$
$$\|\mathbf{d1}\times\mathbf{d2}\|=\left\|\begin{pmatrix}5\\1\end{pmatrix}\times\begin{pmatrix}1\\5\end{pmatrix}\right\| \quad =16 \tag{3.1.1.7}$$
$$\|(\mathbf{P}-\mathbf{S})\times(\mathbf{Q}-\mathbf{P})\|=\left\|\begin{pmatrix}2\\2\end{pmatrix}\times\begin{pmatrix}2\\2\end{pmatrix}\right\| \quad =8 \tag{3.1.1.8}$$
$$\frac{1}{2}\|\mathbf{d1}\times d2\|=\|(\mathbf{P}-\mathbf{S})\times(\mathbf{Q}-\mathbf{P})\| \quad =8 \tag{3.1.1.9}$$

from above we can say that tha area of the quadrileteral is equal to the half of the multiplication of its diogonals.thus this is a rhombus.

2.

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}6\\-4\end{pmatrix} \tag{3.1.2.1}$$
$$\mathbf{R}-\mathbf{P}=\begin{pmatrix}3\\-2\end{pmatrix} \tag{3.1.2.2}$$
$$\mathbf{Q}-\mathbf{R}=\begin{pmatrix}3\\-2\end{pmatrix} \tag{3.1.2.3}$$
$$(\mathbf{Q}-\mathbf{P})=(\mathbf{R}-\mathbf{P})+(\mathbf{Q}-\mathbf{R}) \tag{3.1.2.4}$$
$$=\begin{pmatrix}6\\-4\end{pmatrix} \tag{3.1.2.5}$$

so from above we can say that $\mathbf{P},\mathbf{Q}$ and $\mathbf{R}$ are linear so it can not be a quadrilateral.

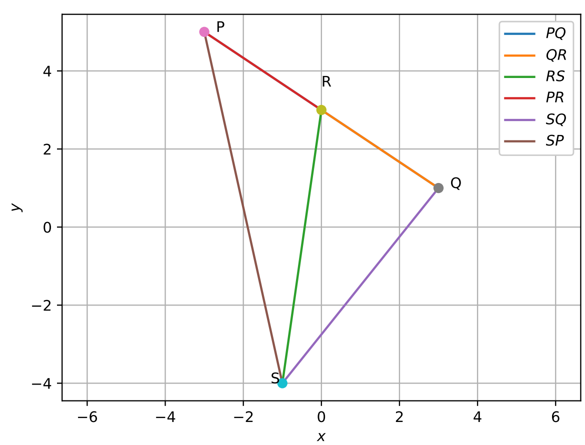

Fig. 3.1.2: quadrilateral2

Path to the python code for the above figure

codes/line/quad/quad2.py

3.

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}3\\1\end{pmatrix} \tag{3.1.3.1}$$
$$\mathbf{R}-\mathbf{S}=\begin{pmatrix}3\\1\end{pmatrix} \tag{3.1.3.2}$$
$$\mathbf{P}-\mathbf{S}=\begin{pmatrix}3\\3\end{pmatrix} \tag{3.1.3.3}$$
$$\mathbf{Q}-\mathbf{R}=\begin{pmatrix}3\\3\end{pmatrix} \tag{3.1.3.4}$$
$$(\mathbf{Q}-\mathbf{P})=(\mathbf{R}-\mathbf{S}) \tag{3.1.3.5}$$
$$(\mathbf{P}-\mathbf{S})=(\mathbf{Q}-\mathbf{R}) \tag{3.1.3.6}$$

Thus above equations shows that the opposites sides are parallel and equal in length

$$\mathbf{d1}=\mathbf{R}-\mathbf{P} \tag{3.1.3.7}$$
$$=\begin{pmatrix}0\\-2\end{pmatrix} \tag{3.1.3.8}$$
$$\mathbf{d2}=\mathbf{S}-\mathbf{Q} \tag{3.1.3.9}$$
$$=\begin{pmatrix}-6\\-4\end{pmatrix} \tag{3.1.3.10}$$
$$\|\mathbf{d1}\times\mathbf{d2}\|=\left\|\begin{pmatrix}5\\1\end{pmatrix}\times\begin{pmatrix}1\\5\end{pmatrix}\right\| \tag{3.1.3.11}$$
$$=8 \tag{3.1.3.12}$$
$$\|(\mathbf{P}-\mathbf{S})\times(\mathbf{Q}-\mathbf{P})\|=\left\|\begin{pmatrix}2\\2\end{pmatrix}\times\begin{pmatrix}2\\2\end{pmatrix}\right\| \tag{3.1.3.13}$$
$$=10 \tag{3.1.3.14}$$
$$\|(\mathbf{P}-\mathbf{S})\times(\mathbf{Q}-\mathbf{P})\|\neq\|(\mathbf{P}-\mathbf{S})\times(\mathbf{Q}-\mathbf{P})\| \tag{3.1.3.15}$$

From equation (3.1.3.5),(3.1.3.6)and (3.1.3.15) we can say that it is a parallelogram.

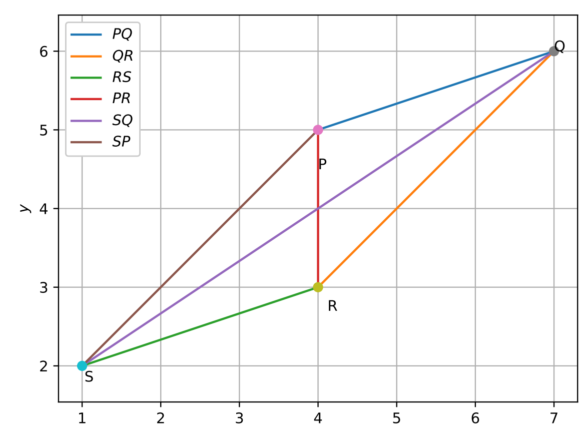

Fig. 3: quadrilateral3

Path to the python code for the above figure

codes/line/quad/quad3.py

3.2 Point on a line

3.2.1 Problem:

1. Find the ratio in wich the line segment joining $\begin{pmatrix}1\\-5\end{pmatrix},\begin{pmatrix}-4\\5\end{pmatrix}$ is devided by the x-axis. Also find the coardinates of the point of division.

3.2.2 Solution:

1. Let assume that we have a point $\mathbf{C}\begin{pmatrix} x \\ 0 \end{pmatrix}$ which devide the linesegment **AB** in k:1 ratio.

$$(k+1)\begin{pmatrix} x \\ 0 \end{pmatrix} = k\begin{pmatrix} 1 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{3.2.1.1}$$
$$0 = -5k + 5 \tag{3.2.1.2}$$
$$k = 1 \tag{3.2.1.3}$$
$$\mathbf{C} = \frac{\begin{pmatrix} -3 \\ 0 \end{pmatrix}}{2} \tag{3.2.1.4}$$
$$= \begin{pmatrix} -1.5 \\ 0 \end{pmatrix} \tag{3.2.1.5}$$

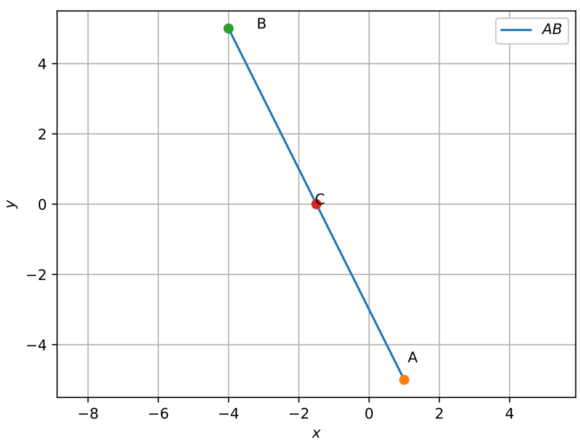

Fig. 3.2.1: line

path to the python code for the above figure

```
codes/line/point_on_line/points_on_line
.py
```

3.3 Lines and Planes

3.3.1 Problem:

1. Sketch the lines
   a) $\begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{x} = 9.35,$
   b) $\begin{pmatrix} 1 & -\frac{1}{5} \end{pmatrix}\mathbf{x} = 10$
   c) $\begin{pmatrix} -2 & 3 \end{pmatrix}\mathbf{x} = 6,$
   d) $\begin{pmatrix} 1 & -3 \end{pmatrix}\mathbf{x} = 0$
   e) $\begin{pmatrix} 2 & 5 \end{pmatrix}\mathbf{x} = 0,$
   f) $\begin{pmatrix} 3 & 0 \end{pmatrix}\mathbf{x} = -2$
   g) $\begin{pmatrix} 0 & 1 \end{pmatrix}\mathbf{x} = 2,$
   h) $\begin{pmatrix} 2 & 0 \end{pmatrix}\mathbf{x} = 5$

3.3.2 Solution:

1. All the lines can be drawn as follow
   a) put $\mathbf{x}\begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ 0 \end{pmatrix} = \frac{187}{20} \tag{3.3.1.1}$$
$$x = \frac{187}{40} \tag{3.3.1.2}$$

put $\mathbf{x}\begin{pmatrix} 0 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} = \frac{187}{20} \tag{3.3.1.3}$$
$$y = \frac{187}{60} \tag{3.3.1.4}$$
$$\mathbf{P1} = \begin{pmatrix} \frac{187}{40} \\ 0 \end{pmatrix} \tag{3.3.1.5}$$
$$\mathbf{Q1} = \begin{pmatrix} 0 \\ \frac{187}{60} \end{pmatrix} \tag{3.3.1.6}$$

   b) put $\mathbf{x}\begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} 1 & -\frac{1}{5} \end{pmatrix}\begin{pmatrix} x \\ 0 \end{pmatrix} = 10 \tag{3.3.1.7}$$
$$x = 10 \tag{3.3.1.8}$$

put $\mathbf{x}\begin{pmatrix} 0 \\ y \end{pmatrix}$ in equation

$$\begin{pmatrix} 1 & -\frac{1}{5} \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} = 10 \tag{3.3.1.9}$$
$$y = -50 \tag{3.3.1.10}$$
$$\mathbf{P2} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{3.3.1.11}$$
$$\mathbf{Q2} = \begin{pmatrix} 0 \\ -50 \end{pmatrix} \tag{3.3.1.12}$$

   c) put $\mathbf{x}\begin{pmatrix} x \\ 0 \end{pmatrix}$ in equation

$$\begin{pmatrix} -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ 0 \end{pmatrix} = 6 \tag{3.3.1.13}$$
$$x = -3 \tag{3.3.1.14}$$

put $\mathbf{x}\begin{pmatrix}0\\y\end{pmatrix}$ in equation

$$\begin{pmatrix}-2 & 3\end{pmatrix}\begin{pmatrix}0\\y\end{pmatrix} = 6 \tag{3.3.1.15}$$

$$y = 2 \tag{3.3.1.16}$$

$$\mathbf{P3} = \begin{pmatrix}-3\\0\end{pmatrix} \tag{3.3.1.17}$$

$$\mathbf{Q3} = \begin{pmatrix}0\\2\end{pmatrix} \tag{3.3.1.18}$$

d) there is no constant in the line equation thus it passes through the origin.
put $\mathbf{x}\begin{pmatrix}3\\y\end{pmatrix}$ in equation

$$\begin{pmatrix}1 & -3\end{pmatrix}\begin{pmatrix}3\\y\end{pmatrix} = 0 \tag{3.3.1.19}$$

$$y = 1 \tag{3.3.1.20}$$

$$\mathbf{P4} = \begin{pmatrix}0\\0\end{pmatrix} \tag{3.3.1.21}$$

$$\mathbf{Q4} = \begin{pmatrix}3\\1\end{pmatrix} \tag{3.3.1.22}$$

e) there is no constant in the line equation thus it passes through the origin
put $\mathbf{x}\begin{pmatrix}1\\y\end{pmatrix}$ in equation

$$\begin{pmatrix}2 & -1\end{pmatrix}\begin{pmatrix}1\\y\end{pmatrix} = 0 \tag{3.3.1.23}$$

$$y = 1 \tag{3.3.1.24}$$

$$\mathbf{P5} = \begin{pmatrix}0\\0\end{pmatrix} \tag{3.3.1.25}$$

$$\mathbf{Q5} = \begin{pmatrix}1\\2\end{pmatrix} \tag{3.3.1.26}$$

f) put $\mathbf{x}\begin{pmatrix}x\\0\end{pmatrix}$ in equation

$$\begin{pmatrix}3 & 0\end{pmatrix}\begin{pmatrix}x\\0\end{pmatrix} = -2 \tag{3.3.1.27}$$

$$x = -\frac{2}{3} \tag{3.3.1.28}$$

we can see in this equation the value of x coordinate does not depend on the y coordinate so we can say that it is parallel to the y-axis.

g) put $\mathbf{x}\begin{pmatrix}x\\0\end{pmatrix}$ in equation

$$\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}0\\y\end{pmatrix} = 2 \tag{3.3.1.29}$$

$$y = 2 \tag{3.3.1.30}$$

we can see in this equation the value of y coordinate does not depend on the x coordinate so we can say that it is parallel to the x-axis.

h) put $\mathbf{x}\begin{pmatrix}x\\0\end{pmatrix}$ in equation

$$\begin{pmatrix}2 & 0\end{pmatrix}\begin{pmatrix}x\\0\end{pmatrix} = 5 \tag{3.3.1.31}$$

$$x = -\frac{5}{2} \tag{3.3.1.32}$$

we can see in this equation the value of x coordinate does not depend on the y coordinate so we can say that it is parallel to the y-axis.

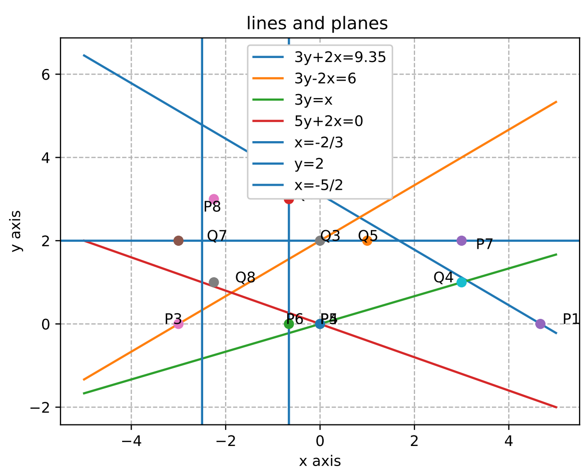

Fig. 3.3.1: lines
Path to the python code for the above figure

codes/line/lines_and_planes/
plane_and_line.py

3.4 Motion

3.4.1 Problem:

1. A hicker stands on the edge of a cliff 490m above the ground and throws a stone horizan-

tally with an initial speed of $15ms^-1$. Neglecting air rsistance , find the time taken by the stone to reach the ground ,and the speed with which it hits the ground .

3.4.2 Solution:

1. given $\implies$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 490 \end{pmatrix} \tag{3.4.1.1}$$

$$\mathbf{a} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \tag{3.4.1.2}$$

$$\mathbf{v_A} = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} \tag{3.4.1.3}$$

$$\mathbf{v_B} = \mathbf{v_A} + \mathbf{at} \tag{3.4.1.4}$$

$$\mathbf{d} = \mathbf{v_A}t + \frac{1}{2}\mathbf{a}t^2 \tag{3.4.1.5}$$

$$\mathbf{B} = \mathbf{A} + \mathbf{d} \tag{3.4.1.6}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 490 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \tag{3.4.1.7}$$

$$\begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 490 \end{pmatrix} + \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} t^2 \tag{3.4.1.8}$$

$$490 = \frac{1}{2}9.8t^2 \tag{3.4.1.9}$$

$$t = 10 \tag{3.4.1.10}$$

$$\mathbf{v_B} = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 9.8 \end{pmatrix} 10 \tag{3.4.1.11}$$

$$= \begin{pmatrix} 1.5 \\ 98 \end{pmatrix} \tag{3.4.1.12}$$

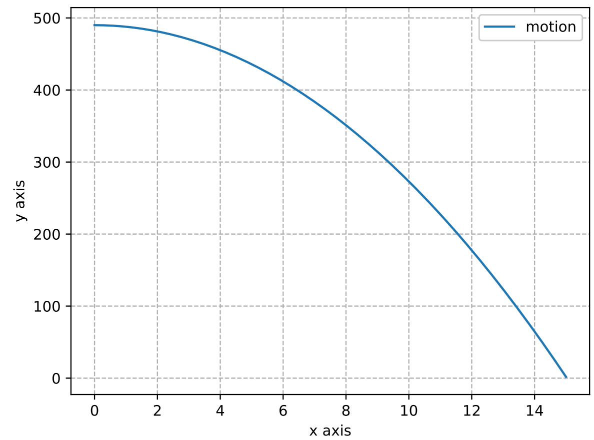

Fig. 3.4.1: motion

path to the python code for the above figure

codes/line/motion/motion.py

3.5 Matrix

3.5.1 Problem:

1. Construct a 2 x 2 matrix .$\mathbf{A} = [a_i j]$,whose elements are given by:
   a) $a_{ij} = \frac{(i+j)^2}{2}$
   b) $a_{ij} = \frac{i}{j}$
   c) $a_{ij} = \frac{(i+2j)^2}{2}$

3.5.2 Solution:

1. Formation of matrix can be done as follow

a)

$$\tag{3.5.1.1}$$

$$a_{ij} = \frac{(i+j)^2}{2} \tag{3.5.1.2}$$

$$a_{11} = 2 \tag{3.5.1.3}$$

$$a_{12} = 4.5 \tag{3.5.1.4}$$

$$a_{21} = 4.5 \tag{3.5.1.5}$$

$$a_{22} = 8 \tag{3.5.1.6}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{3.5.1.7}$$

$$\mathbf{A} = \begin{pmatrix} 2 & 4.5 \\ 4.5 & 8 \end{pmatrix} \tag{3.5.1.8}$$

b)

$$a_{ij} = \frac{i}{j} \tag{3.5.1.9}$$

$$a_{11} = 1 \tag{3.5.1.10}$$

$$a_{12} = 0.5 \tag{3.5.1.11}$$

$$a_{21} = 2 \tag{3.5.1.12}$$

$$a_{22} = 1 \tag{3.5.1.13}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{3.5.1.14}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 0.5 \\ 2 & 1 \end{pmatrix} \tag{3.5.1.15}$$

c)

$$a_{ij} = \frac{(i+2j)^2}{2} \quad (3.5.1.16)$$
$$a_{11} = 4.5 \quad (3.5.1.17)$$
$$a_{12} = 12.5 \quad (3.5.1.18)$$
$$a_{21} = 8 \quad (3.5.1.19)$$
$$a_{22} = 18 \quad (3.5.1.20)$$
$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (3.5.1.21)$$
$$\mathbf{A} = \begin{pmatrix} 4.5 & 12.5 \\ 2 & 18 \end{pmatrix} \quad (3.5.1.22)$$

Path to pythone codes to get above matrices is..

```
codes/line/matrix/matrix.py
```

3.6 Determinents

3.6.1 Problem:

1. If $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then show that $\left|3A\right| = 27\left|A\right|$

3.6.2 Solution:

1.

$$\left|3A\right| = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \quad (3.6.1.1)$$
$$= \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix} \quad (3.6.1.2)$$
$$\left|3A\right| = 108 \quad (3.6.1.3)$$
$$\left|A\right| = 4 \quad (3.6.1.4)$$
$$\left|3A\right| = 27\left|A\right| \quad (3.6.1.5)$$

hence proved

3.7 Linear inequalities

3.7.1 Problem:

1. the marks obtained by the student of class XI in first and second terminal examination are 62 and 48,respectively.Find the minimum marks in annual examination to have an average of at least 60 marks.

3.7.2 Solution:

1. let assume that the student get $\mathbf{x}$ marks in the annual examination so now...

$$\frac{62+48+x}{3} \geq 60 \quad (3.7.1.1)$$
$$x + 110 \geq 180 \quad (3.7.1.2)$$
$$x \geq 70 \quad (3.7.1.3)$$

4 Circle

4.1 Problem

1. find the area enclosed by the circle $\left(x\right) = a$ area of circle

4.2 Solution

1.

$$||\Delta|| = 2\pi r^2 \quad (4.2.1.1)$$
$$||\Delta|| = 2\pi a^2 \quad (4.2.1.2)$$

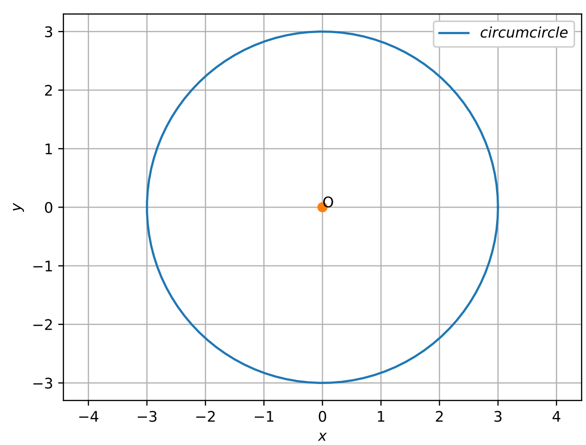

Fig. 4.2.1: circle

path to the code for the above figure

```
codes/circle/circle.py
```

4.3 Problem

1. Does the point lie inside, outside or on the circle $\mathbf{x}^T\mathbf{x} = 25$?

4.4 solution

1. general equation for the circle can be given as
$$\mathbf{x}^T\mathbf{x} + 2\mathbf{O}^T\mathbf{x} + \|O\|^2 - \mathbf{r}^2 = 0 \tag{4.4.1.1}$$
given equation of circle
$$\mathbf{x}^T\mathbf{x} - 25 = 0 \tag{4.4.1.2}$$
comparing bothe of equation we can find the value of r and value of **O**
$$\mathbf{r} = 4 \tag{4.4.1.3}$$
$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4.4.1.4}$$
$$\mathbf{A} - \mathbf{O} = \begin{pmatrix} -2.5 \\ 3.5 \end{pmatrix} \tag{4.4.1.5}$$
$$\|\mathbf{A} - \mathbf{O}\| = 18.5 \tag{4.4.1.6}$$
$$18.5 < \mathbf{r} \tag{4.4.1.7}$$
Thus it is clear that the length of OA is shorter than that of r so point**A**exist inside the circle.

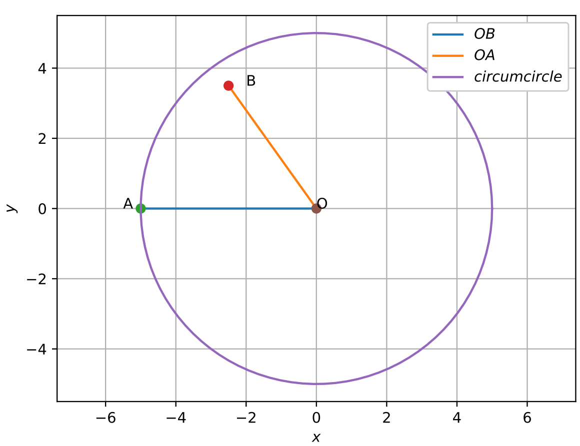

Fig. 4.4.1: circle
path to the code for the above figure

codes/circle/circle2.py

5 Conics

5.1 Problem

1. Find the zeroes of the following Quadratic polynomials and verify the relationship between the zeroes and the coefficients.
   a) $x^2 - 2x - 8$
   b) $4u^2 + 8u$
   c) $4s^2 - 4s + 1$
   d) $t^2 - 15$
   e) $6x^2 - 3 - 7x$
   f) $3x^2 - 2x - 8$

5.2 Solution

1.
$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} D & E \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} + k = 0 \tag{5.2.1.1}$$

2.
$$\begin{bmatrix} \mathbf{x} \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \end{bmatrix} + \begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \end{bmatrix} - 8 \tag{5.2.2.1}$$
$$x^2 - 2x - 8 = 0 \tag{5.2.2.2}$$
$$(x - 4)(x + 2) = 0 \tag{5.2.2.3}$$
$$\alpha = 4, \beta = -2 \tag{5.2.2.4}$$
quadratic equation can be represented as
$$ax^2 + bx + c = 0 \tag{5.2.2.5}$$
$$\alpha + \beta = -\frac{b}{a} = 2 \tag{5.2.2.6}$$
$$\alpha \times \beta = \frac{c}{a} = -8 \tag{5.2.2.7}$$

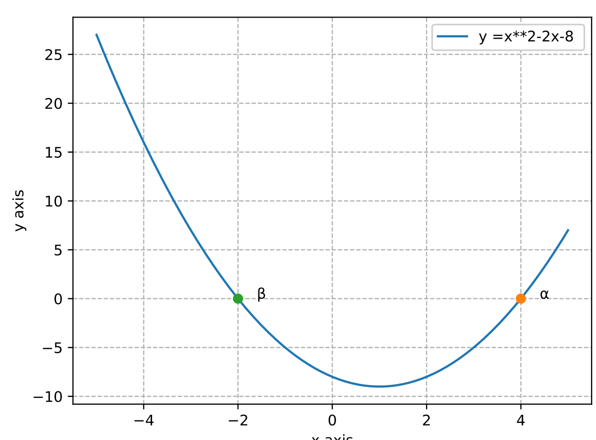

Fig. 5.2.2: equation 1
path to the python code for the above figure is

codes/conics/perabola2.py

3.

$$\begin{bmatrix}\mathbf{x}\end{bmatrix}^T \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} + \begin{bmatrix}8 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} \quad (5.2.3.1)$$

$$4u^2 + 8u = 0 \quad (5.2.3.2)$$

$$(4u)(u+2) = 0 \quad (5.2.3.3)$$

$$\alpha = 0, \beta = -2 \quad (5.2.3.4)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.3.5)$$

$$\alpha + \beta = -\frac{b}{a} = -2 \quad (5.2.3.6)$$

$$\alpha \times \beta = \frac{c}{a} = 0 \quad (5.2.3.7)$$

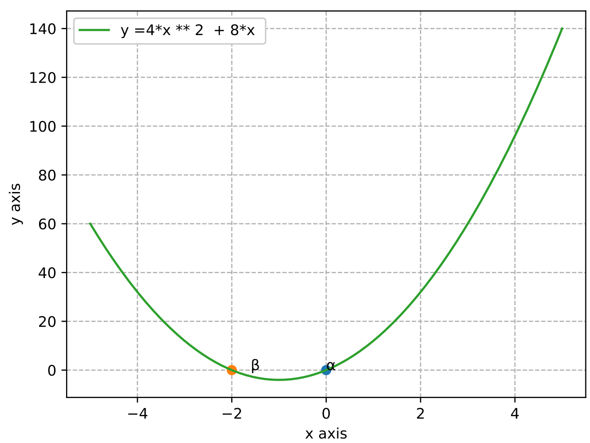

Fig. 5.2.3: equation 2
path to the python code for the above figure is

codes/conics/perabola2.py

4.

$$\begin{bmatrix}\mathbf{x}\end{bmatrix}^T \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} + \begin{bmatrix}-4 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} + 1 \quad (5.2.4.1)$$

$$4s^2 - 4s + 1 = 0 \quad (5.2.4.2)$$

$$(2s-1)(2s-1) = 0 \quad (5.2.4.3)$$

$$\alpha = \frac{1}{2}, \beta = \frac{1}{2} \quad (5.2.4.4)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.4.5)$$

$$\alpha + \beta = -\frac{b}{a} = 1 \quad (5.2.4.6)$$

$$\alpha \times \beta = \frac{c}{a} = \frac{1}{4} \quad (5.2.4.7)$$

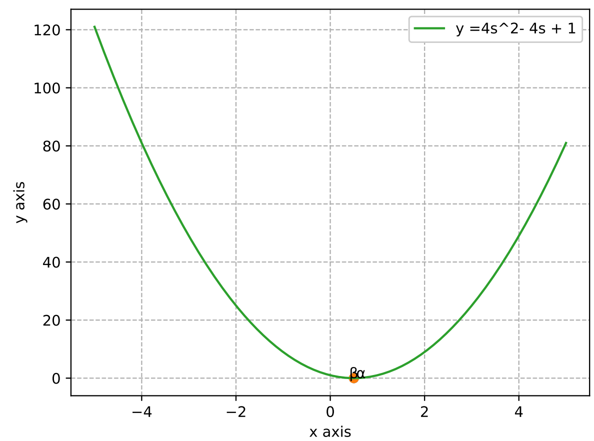

Fig. 5.2.4: equation 3
path to the python code for the above figure is

codes/conics/perabola3.py

5.

$$\begin{bmatrix}\mathbf{x}\end{bmatrix}^T \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} + \begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} - 15 \quad (5.2.5.1)$$

$$t^2 - 15 = 0 \quad (5.2.5.2)$$

$$\alpha = \sqrt{15}, \beta = -\sqrt{15} \quad (5.2.5.3)$$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \quad (5.2.5.4)$$

$$\alpha + \beta = -\frac{b}{a} = 0 \quad (5.2.5.5)$$

$$\alpha \times \beta = \frac{c}{a} = -15 \quad (5.2.5.6)$$

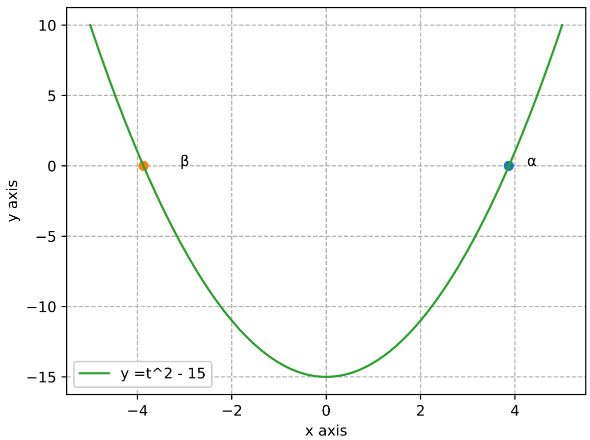

Fig. 5.2.5: equation 4
path to the python code for the above figure is

$\boxed{\text{codes/conics/perabola4.py}}$

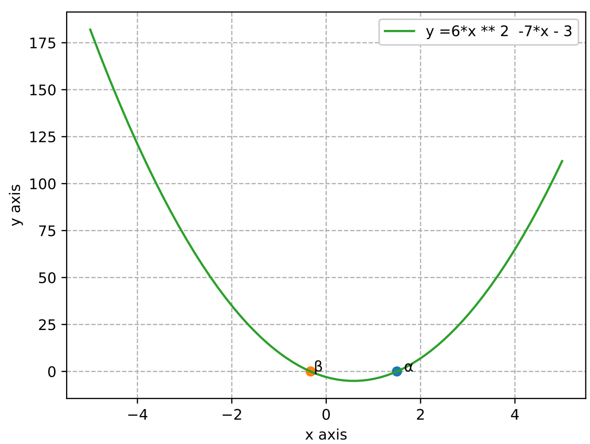

Fig. 5.2.6: equation 5
path to the python code for the above figure is

$\boxed{\text{codes/conics/perabola5.py}}$

6.

$$\begin{bmatrix}\mathbf{x}\end{bmatrix}^T \begin{bmatrix}6 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} + \begin{bmatrix}-7 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} - 3 \tag{5.2.6.1}$$

$$6x^2 - 3 - 7x = 0 \tag{5.2.6.2}$$

$$(2x - 3)(3x \ + 1) = 0 \tag{5.2.6.3}$$

$$\alpha = \frac{3}{2}, \beta = -\frac{1}{3} \tag{5.2.6.4}$$

$$\alpha + \beta = -\frac{b}{a} = \frac{7}{6} \tag{5.2.6.5}$$

$$\alpha \times \beta = \frac{c}{a} = -\frac{1}{2} \tag{5.2.6.6}$$

7.

$$\begin{bmatrix}\mathbf{x}\end{bmatrix}^T \begin{bmatrix}3 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} + \begin{bmatrix}-1 & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\end{bmatrix} - 4 \tag{5.2.7.1}$$

$$3x^2 - 2x - 8 = 0 \tag{5.2.7.2}$$

$$(3x + 4)(x - 2) = 0 \tag{5.2.7.3}$$

$$\alpha = 2, \beta = -\frac{4}{3} \tag{5.2.7.4}$$

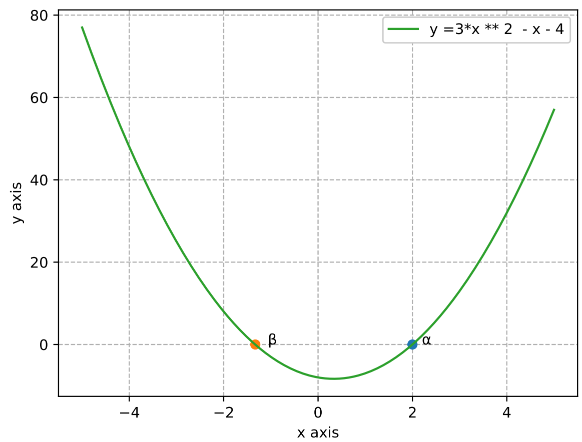

Fig. 5.2.7: equation 6
path to the python code for the above figure is

$\boxed{\text{codes/conis/perabola6.py}}$

quadratic equation can be represented as

$$ax^2 + bx + c = 0 \tag{5.2.7.5}$$

$$\alpha + \beta = -\frac{b}{a} = \frac{1}{3} \tag{5.2.7.6}$$

$$\alpha \times \beta = \frac{c}{a} = -\frac{8}{3} \tag{5.2.7.7}$$