Problems On Geometry Of Circle

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Question

Exercise 8.5(Q no.13)

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.

Codes and Figures

The python code for the figure is

./code/c_circle.py

The latex- tikz code is

./figs/C_circle.tex

The above latex code can be compiled as standalone document

./figs/C_circle_fig.tex

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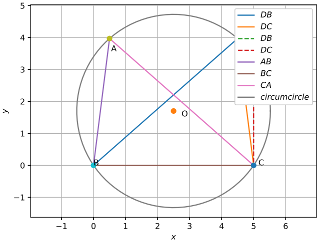
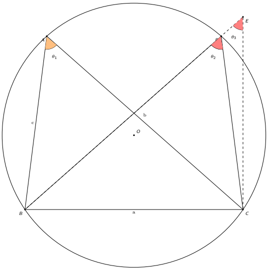


(a) By Python



(b) By Latex-tikz

Construction method

A circumecircle is to be made having two triangles with vertices on the circle and a point **e** outside of the circle,with the help of the three sides of a triangle as input as shown in the table below.

Initial Input Values.	
a	5
b	4
c	6

Table: To construct the circumecircle

Finding out the all points given in the figures

$$(i)\mathbf{A} = \begin{pmatrix} 0.5 \\ 3.968 \end{pmatrix} (ii)\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(iii)\mathbf{C} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} (iv)\mathbf{D} = \begin{pmatrix} 4.5 \\ 3.98 \end{pmatrix}$$

$$(v)\mathbf{E} = \begin{pmatrix} 5 \\ 4.40 \end{pmatrix}$$

Now,calculating circumecentre of the triangle

$$\|\mathbf{A}-\mathbf{O}\|^2 - \|\mathbf{B}-\mathbf{O}\|^2 = 0$$

Which can be simplified as

$$(\mathbf{A}-\mathbf{B})^T\mathbf{O} = \frac{(\|A\|^2 - \|B\|^2)}{2}$$

Similarly,

$$(\mathbf{B}-\mathbf{C})^T\mathbf{O} = \frac{(\|B\|^2 - \|C\|^2)}{2}$$

Above equation can be combinrd as $\rightarrow$

$$\mathbf{O} = \mathbf{N}^{-T}\mathbf{c}$$

Where

$$\mathbf{N} = \begin{pmatrix} \mathbf{A}-\mathbf{B} & \mathbf{B}-\mathbf{C} \end{pmatrix} \tag{1}$$

$$\mathbf{c} = \frac{1}{2}\begin{pmatrix} \|A\|^2 - \|B\|^2 \\ \|B\|^2 - \|C\|^2 \end{pmatrix}$$

Finding out the **R**

$$\mathbf{R} = \|\mathbf{B}-\mathbf{O}\|$$

$$\mathbf{R} = 3.023$$

Derived Values for *triangleDCB*.	
O	$\begin{pmatrix} 2.5 \\ 1.70 \end{pmatrix}$

Table: circumecentre of the triangle

Solution)

Let assume that circle intersect at **E**
Now,

$$\angle BAC = \angle BEC$$

(Angle in the same segment are equal)
But given that

$$\angle BAC = \angle BDC$$

So from above equations

$$\angle BEC = \angle BDC$$

From the triangle DEC

$$\angle BDC = \angle CED + \angle DCE$$

(Exterier angle property of triangle)

$$\angle BEC = \angle BEC + \angle DCE$$

$$\angle BEC - \angle BEC = \angle DCE$$

$$\angle DCE = 0$$

From above we can say that can not exist out of the periphery of the circle