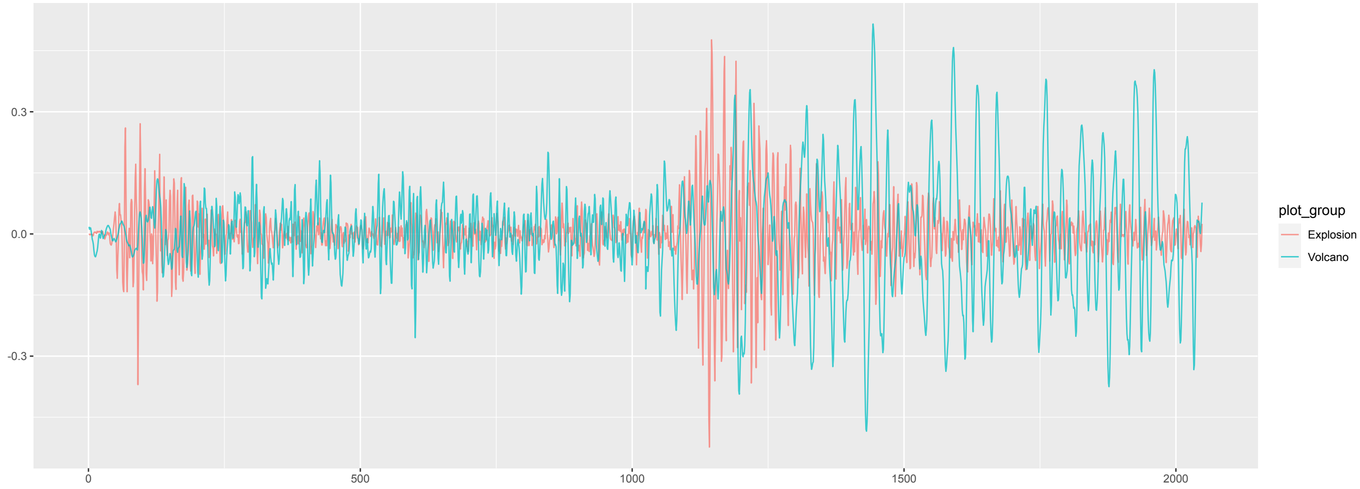
Problem 1.1

```
library(ggplot2)

hw1<-cbind(EXP6,EQ5)
colnames(hw1)<-c("Explosion","Volcano")
autoplot(hw1, facets = FALSE, alpha = 0.75)
```

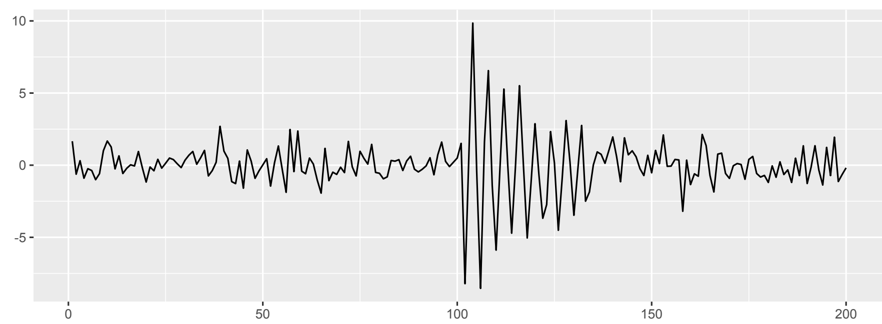


Problem 1.2

(a)

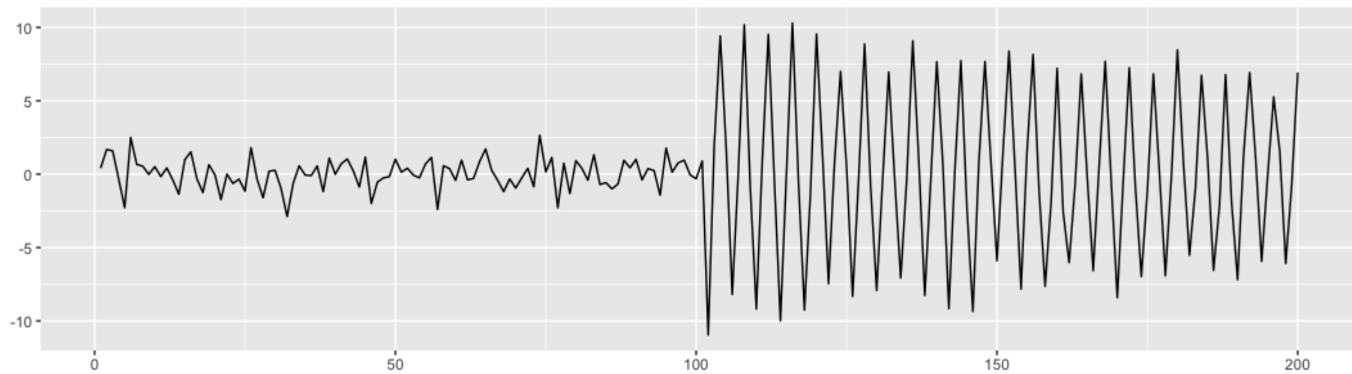
```
library(ggplot2)
library(ggfortify)
set.seed(1)

s=c(integer(100), 10*exp(-(1:100)/20)*cos(2*pi*1:100/4))
x_a=s+rnorm(200)
autoplot(as.ts(x_a))
```



(b)

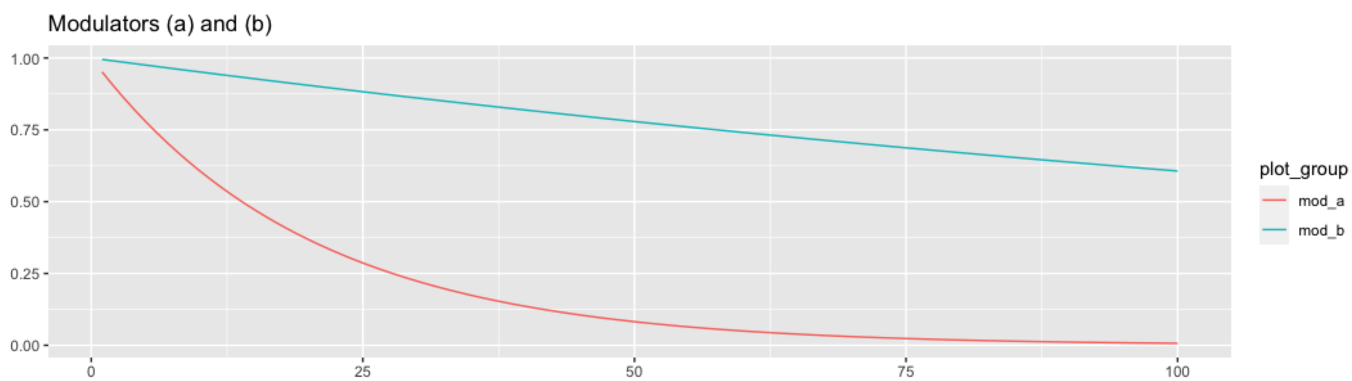
```
s=c(integer(100), 10*exp(-(1:100)/200)*cos(2*pi*1:100/4))
x_b=s+rnorm(200)
autoplot(as.ts(x_b))
```



(c)

The explosion series is similar to series **a**, and the earthquake series is similar to series **b** in their mid-to-end behavior. We see that both series **a** and the explosion series have big amplitudes in the middle that slowly become smaller and tend towards 0 but don't get there. We also see that both series **b** and the earthquake have big amplitudes throughout their mid-to-end behavior.

```
mod_a=exp(-(1:100)/20)
mod_b=exp(-(1:100)/200)
modulators<-as.ts(cbind(mod_a,mod_b))
autoplot(modulators,facets=F,main="Modulators (a) and (b)")
```

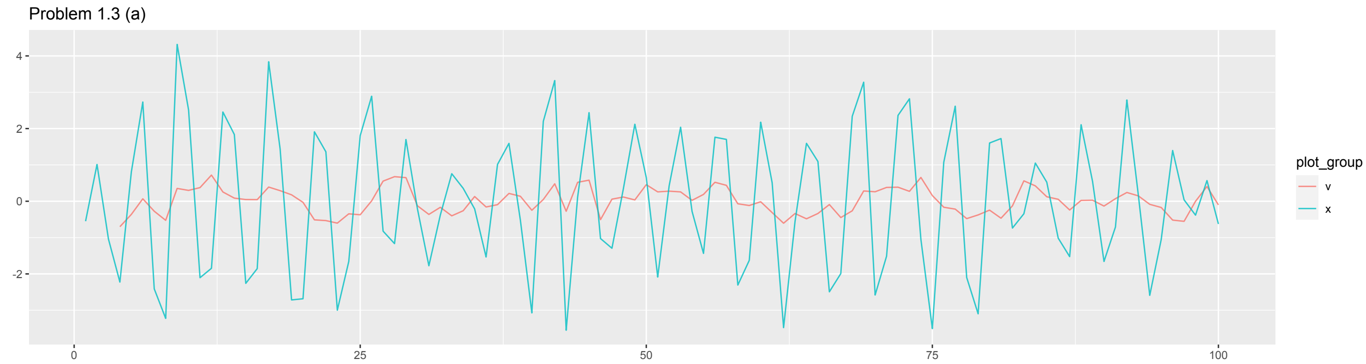


Problem 1.3

(a)

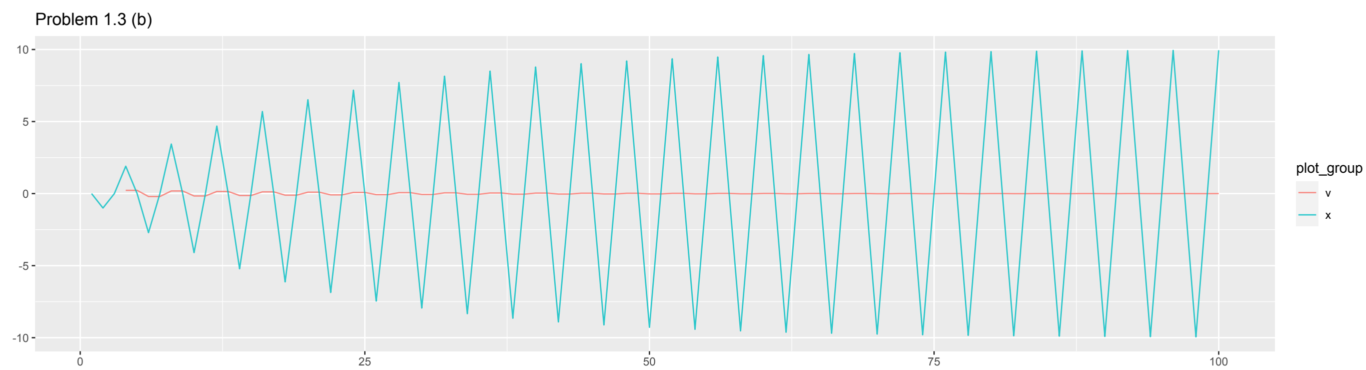
```
library(ggfortify)
library(astsa)

w <- rnorm(150)
x <- filter(w, filter=c(0, -.9), method="recursive")[-(1:50)]
v <- filter(x, rep(1/4, 4), sides = 1)
autoplot(as.ts(cbind(x, v)), main = "Problem 1.3 (a)", facets = F, alpha=0.8)
```



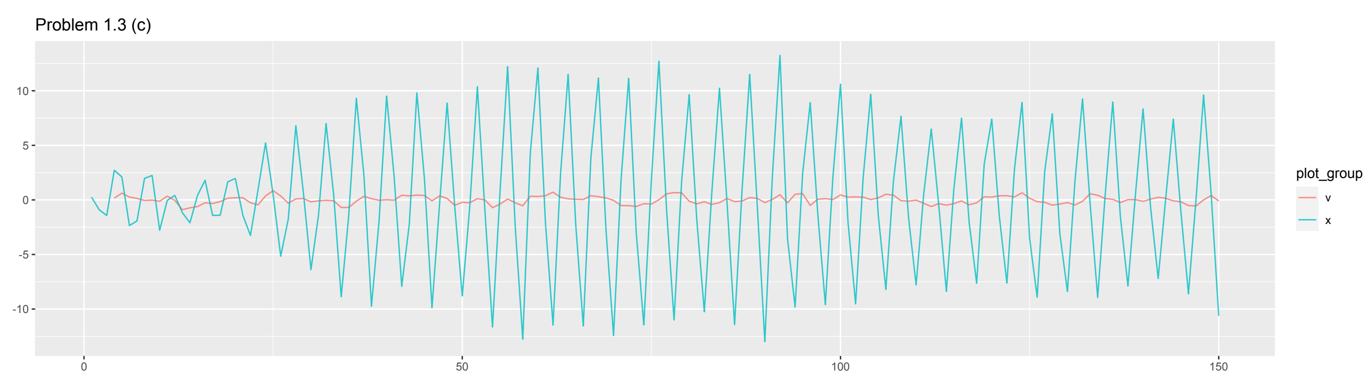
(b)

```r
f <- cos(2*pi*(1:100)/4)
x <- filter(f, filter=c(0, -.9), method="recursive")
v <- filter(x, rep(1/4, 4), sides = 1)
autoplot(as.ts(cbind(x, v)), main = "Problem 1.3 (b)", facets = F, alpha=0.8)
```



(c)

```r
fw <- f+w
x <- filter(fw, filter=c(0, -.9), method="recursive")
v <- filter(x, rep(1/4, 4), sides = 1)
autoplot(as.ts(cbind(x, v)), main = "Problem 1.3 (c)", facets = F, alpha=0.8)
```



(d)

Problem 1.4

Show the following:

$$\mathbf{E}[(x_s - \mu_s)(x_t - \mu_t)] = \mathbf{E}[x_s x_t] - \mu_s \mu_t$$

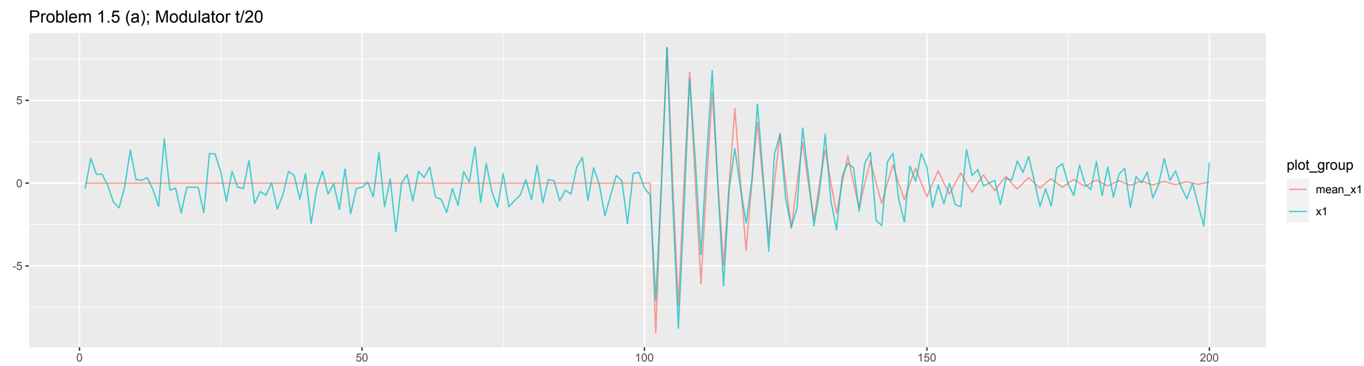
Proof:

$$\mathbf{E}[(x_s-\mu_s)(x_t-\mu_t)] = \mathbf{E}[x_s x_t - x_s\mu_t - x_t\mu_s + \mu_s\mu_t] = \mathbf{E}[x_s x_t] - \mu_t\mathbf{E}[x_s] - \mu_s\mathbf{E}[x_t] + \mu_s\mu_t$$
$$\mathbf{E}[x_s x_t] - \mu_t\mu_s - \mu_s\mu_t + \mu_s\mu_t = \mathbf{E}[x_s x_t] - \mu_s\mu_t$$
$$\implies \mathbf{E}[(x_s-\mu_s)(x_t-\mu_t)] = \mathbf{E}[x_s x_t] - \mu_s\mu_t$$

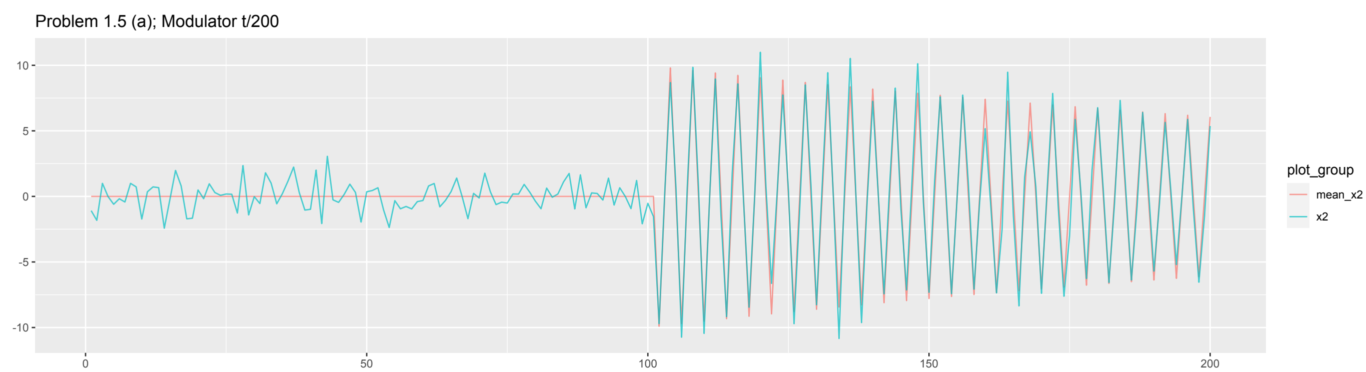
Problem 1.5

(a)

```
mean_x1 <- c(integer(100), 10*exp(-(1:100)/20)*cos(2*pi*1:100/4))
x1 <- mean_x1 + rnorm(200)
autoplot(as.ts(cbind(x1,mean_x1)), main = "Problem 1.5 (a); Modulator t/20", facets = F, alpha=0.7)
```




```
mean_x2 <- c(integer(100), 10*exp(-(1:100)/200)*cos(2*pi*1:100/4))
x2 <- mean_x2 + rnorm(200)
autoplot(as.ts(cbind(x2,mean_x2)), main = "Problem 1.5 (a); Modulator t/200", facets = F, alpha=0.7)
```



(b)

Problem 1.6

(a)

No, x_t is not stationary because it fails the first requirment, that is

$$E[x_t] = E[\beta_1 + \beta_2 t + w_t] = E[\beta_1] + E[\beta_2 t] + E[w_t] = \beta_1 + \beta_2 t$$

which is dependent on time and that is not allowed in stationary time series.

(b)

In order to be stationary we need to satisfy two requirements which we show below.
1.) The mean function is not dependent on time.

$$E[y_t] = E[x_t - x_{t-1}] = E[x_t] - E[x_{t-1}] = E[\beta_1 + \beta_2 t + w_t - \beta_1 - \beta_2(t-1) - w_{t-1}] = \beta_2$$

2.)

(a)

(b)

$)$

$$
\begin{aligned}
cov(y_t, y_{t-1}) &= E[(y_t - \mu_t)(y_{t-1} - \mu_{t-1})] \\
&= E[(\beta_2 + w_t - w_{(t-1)} - \beta_2)(\beta_2 + w_{(t-1)} + w_{(t-2)} - \beta_2)] \\
&= E[(w_t - w_{(t-1)})(w_{(t-1)} + w_{(t-2)})] \\
&= E(w_t w_{t-1} - w_{t-1}^2 - w_{t-1} w_{t-2} + w_t w_{t-2}) \\
&= 0
\end{aligned}
$$

(c)

$$Cov(V_t, V_{t+h}) = E((V_t - \mu_t)(V_{t+h} - \mu_{t+h}))$$

$$E((V_t - (\beta_1 + \beta_2 t))(V_{t+h} - (\beta_1 + \beta_2(t+h))))$$

$$V_t \cdot V_{t+h} - V_t \mu_{t+h} - V_{t+h}\mu_t + \mu_t \mu_{t+h}$$

$$E((V_t - \beta_1 - \beta_2 t)(V_{t+h} - \beta_1 - \beta_2(t+h)))$$

$$E((V_t - \beta_1 - \beta_2 t)(V_{t+h} - \beta_1 - \beta_2 t - \beta_2 h))$$

$$V_t V_{t+h} - V_t(\beta_1 + \beta_2(t+h)) - V_{t+h}(\beta_1 + \beta_2 t) + (\underbrace{\beta_1 + \beta_2(t+h)}_{\beta_1 + \beta_2 t + \beta_2 h})(\beta_1 + \beta_2 t)$$

$$V_t V_{t+h} - V_t \beta_1 - V_t t \beta_2 - V_t h \beta_2 - V_{t+h}\beta_1 - V_{t+h} t \beta_2 + \beta_1^2 + \beta_1\beta_2 t + \beta_1\beta_2 t + \beta_2^2 t^2 + \beta_1\beta_2 h + \beta_2^2 t h$$

$$
\begin{aligned}
&E[V_t V_{t+h}] - \beta_1 E[V_t] - \beta_2 t E[V_t] - \beta_2 h E[V_t] - \beta_1 E[V_{t+h}] - t\beta_2 E[V_{t+h}] + \beta_1^2 + \beta_1\beta_2 t + \beta_1\beta_2 t + \beta_2^2 t^2 + \beta_1\beta_2 h + \beta_2^2 t h \\
&- \beta_1(\beta_1 + \beta_2 t) - \beta_2 t(\beta_1 + \beta_2 t) - \beta_2 h(\beta_1 + \beta_2 t) - \beta_1(\beta_1 + \beta_2(t+h)) - t\beta_2(\beta_1 + \beta_2(t+h)) + \beta_1^2 + \beta_1\beta_2 t + \beta_1\beta_2 t + \beta_2^2 t^2 + \beta_1\beta_2 h + \beta_2^2 t h \\
&\cancel{-\beta_1^2} \cancel{- \beta_1\beta_2 t} \cancel{- \beta_1\beta_2 t} \cancel{- \beta_2^2 t^2} \cancel{- \beta_1\beta_2 h} - \beta_1\beta_2 t h - \beta_1^2 - \beta_1\beta_2 t - \beta_1\beta_2 h - \beta_1\beta_2 t - \beta_2^2 t - \beta_2^2 h \cancel{+ \beta_1^2} \cancel{+ \beta_1\beta_2 t} \cancel{+ \beta_1\beta_2 t} \cancel{+ \beta_2^2 t^2} \cancel{+ \beta_1\beta_2 h} + \beta_2^2 t h
\end{aligned}
$$

The mean of the moving average is:

$$E[v_t] = E[\frac{1}{2q+1}\sum_{j=-q}^{q} x_{t-j}] = \frac{1}{2q+1}(2q+1)E[x_t] = E[x_t] = \beta_1 + \beta_2 t$$

The simplified expression for the covariance is:

Problem 1.8

(a)

$x_0 = 0$
$x_1 = \delta + w_1,$
$x_2 = \delta + (\delta + w_1) + w_2 = 2\delta + \sum_{k=1}^{2} w_k$
$x_3 = \delta + (2\delta + \sum_{k=1}^{2} w_k) + w_2 = 3\delta + \sum_{k=1}^{3} w_k$

... (By Induction)
$x_t = \delta + ((t-1)\delta + \sum_{k=1}^{t-1} w_k) + w_t = t\delta + \sum_{k=1}^{t} w_k$

(b)

$$E[x_t] = E[t\delta + \sum_{k=1}^{t} w_k] = E[t\delta] + E[\sum_{k=1}^{t} w_k] = t\delta$$

$$cov(x_s, x_t) = E[(s\delta + \sum_{k=1}^{s} w_k - s\delta)(t\delta + \sum_{k=1}^{t} w_k - t\delta)] = E[(\sum_{k=1}^{s} w_k)(\sum_{k=1}^{t} w_k)] = 0$$

(the last equality is from the fact that expectation of two IID random variables multiplied is the product of each of their expectations and when we simplify that we should get a lot of sums of products of w_k terms which all have mean 0)

Need to check this -- it doesn't feel correct.

(c)

x_t is not stationary because the mean function is not time homogeneous so it doesn't satisfy the 1st condition of stationarity.

$y_t = x_t - x_{t-1}$

$= \delta t + \sum_{i=1}^{t} w_k - \delta(t-1) - \sum_{i=1}^{t-1} w_k$

$= \delta t - \delta t + \delta + w_t$

$= \delta + w_t$

$E[y_t] = \delta + E[w_t] = \delta$

$cov(y_t, y_{t+h}) = E[(y_t - \delta)(y_{t+h} - \delta)]$

$= E[y_t y_{t+h} - \delta y_{t+h} - \delta y_t + \delta^2]$

$= E[y_t y_{t+h}] - 2\delta^2 + \delta^2$

$= E[y_t y_{t+h}] - \delta^2$

10 11 0

$s = K_n$

$x \cdot \mathbb{1}^T = (1, \dots, 1, -1, \dots, -1)$

$= \underbrace{1 + \dots + 1}_{} + \underbrace{-1, \dots, -1}_{}$ $\Theta(s$

0 1 2 $n-1$ $-n$

0

$\Theta(s) \geq$

$= E[(\delta + w_t)(\delta + w_{t-1})] - \delta^2$

$= E[\delta^2 + \delta w_t + \delta w_{t-1} + w_t w_{t-1}]$

$= \delta^2 + 2\delta + 0$

$= \delta^2 + 2\delta$

(d)

(e)