

Institute of
Data



Data Science and AI

Module 4

Introduction to Machine Learning/Regression



Agenda: Module 4

- Introduction to **Machine Learning**
- The **Predictive** Modelling Process
- **Feature** Selection
- **Measuring** the Accuracy of Regression Models
- **Overfitting**



Introduction to Machine Learning

- **Supervised** Learning
- **Unsupervised** Learning
- **Regression**
- **Classification**



“All models are **wrong**;
some are **useful**!”

George Edward Pelham Box (1919-2013)





Supervised Learning

- The main characteristic of **Supervised Learning** is to have previously known results
- Consider data collected from cases with known **input** and **output**
 - Cases can also be called **observations**, experiments, results, etc
 - Input can also be called variables; input, explanatory or independent variables; **features**; predictors
 - Output can also be called output or dependent variable; label; result; **outcome**; response
- Supervised learning relies on **induction**:
 - the inference of a general law from particular instances.



Supervised Learning

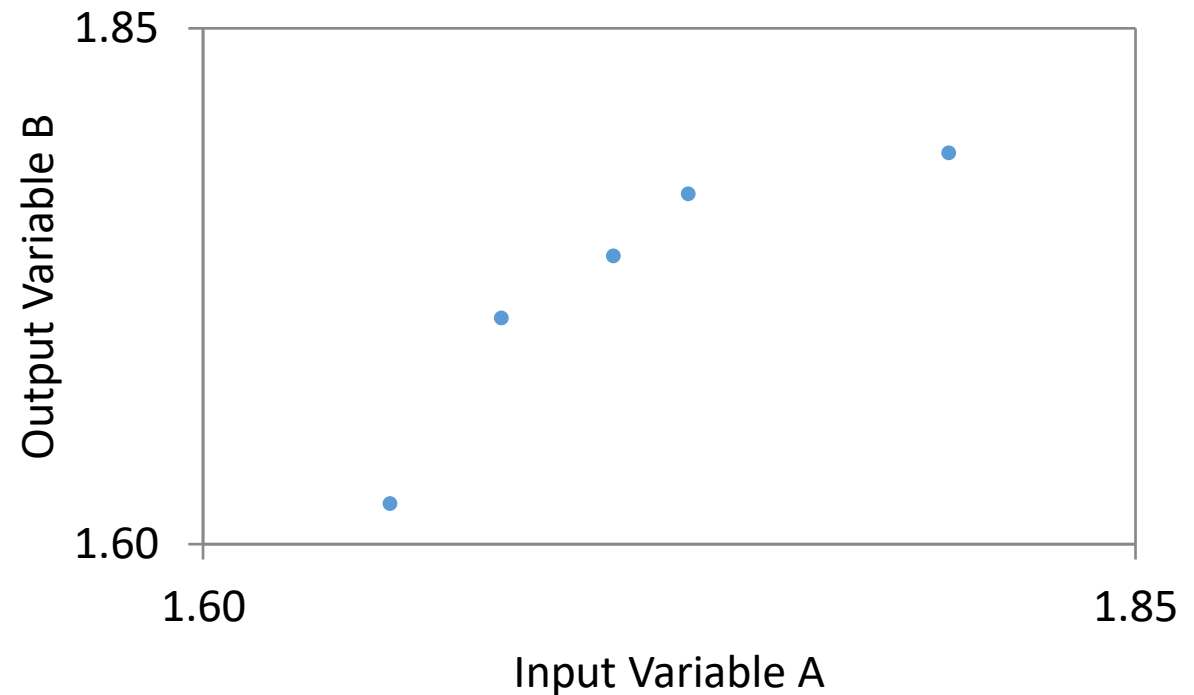
- Most **practical** Machine Learning uses Supervised Learning.
- Knowing the values of an **input** variable **A** and an output **variable B** can be the source of a model to predict **future** outcomes of **B** given a **new** input of **A**
- Definition:
 - Supervised learning is where you have input variables (x) and an output variable (Y) and you use **an algorithm** to **learn** the **mapping function** from the input to the output **$Y = f(X)$**



Supervised Learning

- The computer will find a function by the analysis of input-output pairs, so it can be used to predict future results in similar conditions

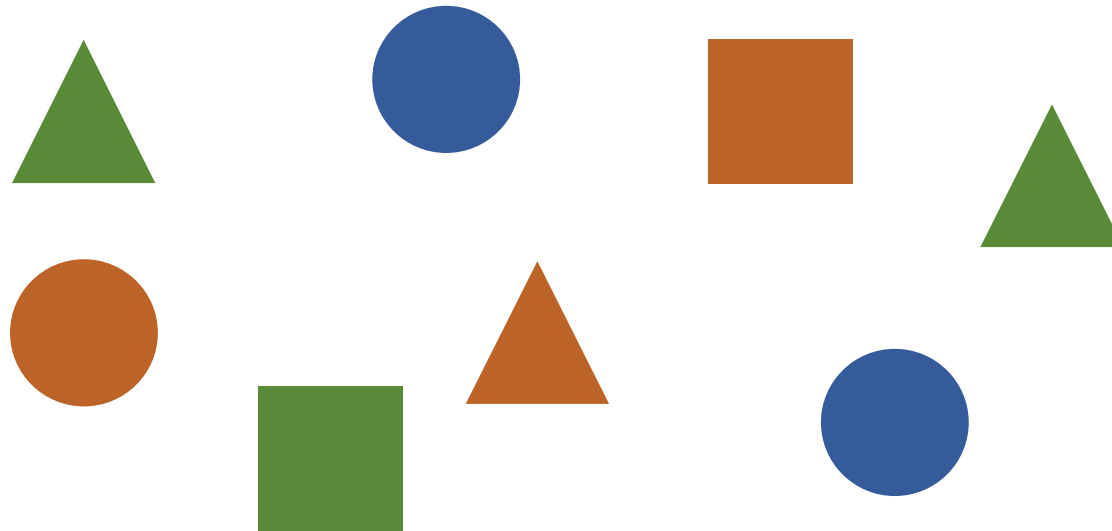
A	B
1.71	1.74
1.65	1.62
1.80	1.79
1.68	1.71
1.73	1.77





Unsupervised Learning

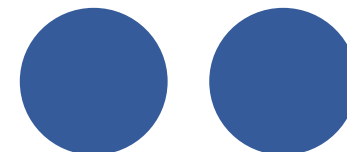
- The main characteristic of **Unsupervised Learning** is to have **unknown prior outcomes**
- Consider data collected from **observations** of which **only** the **input** is **known**. For example the collection of customers or transactions





Unsupervised Learning

- The computer will infer a function by the analysis of **similar characteristics** or **proximity** of the input examples, so it can be used to predict future results in the same conditions





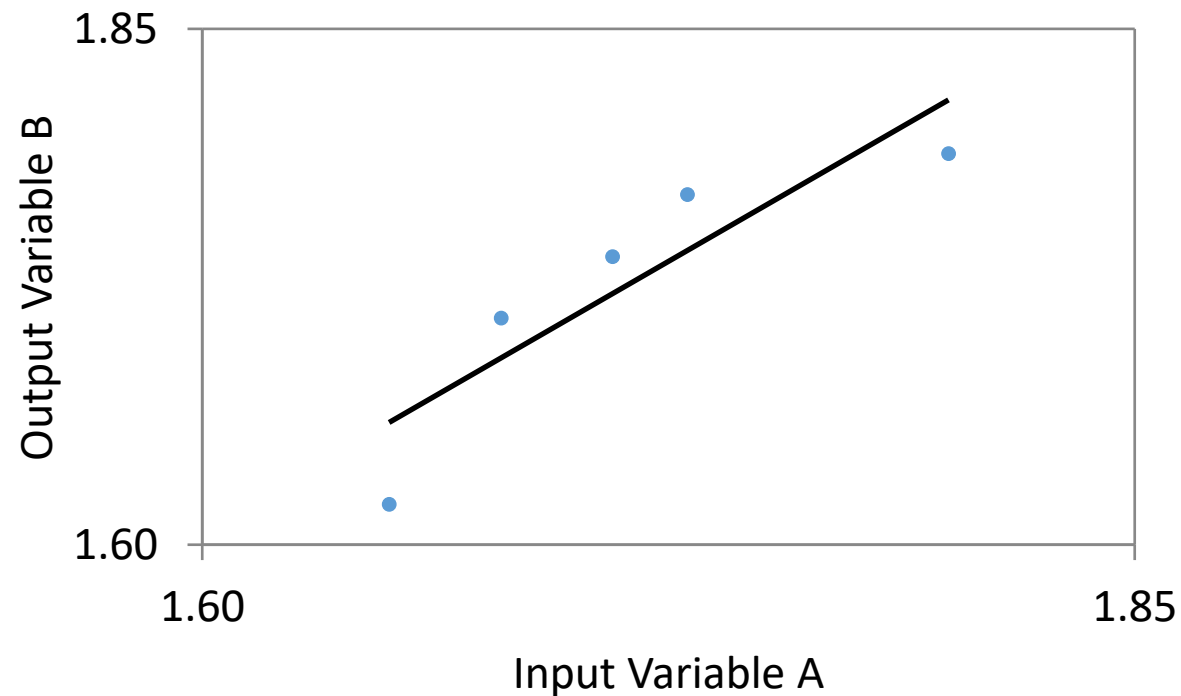
Regression

- What characterises **Regression** is to obtain a **scalar result**, a number
 - Regression results can have a value range from $-\infty$ to ∞
- Tries to answer questions like: **How Much / How Many?**
 - What will the temperature be tomorrow?
 - How many new followers will be in the next week?
- Set of statistical processes for estimating the relationships among variables
 - One independent variable: **univariate** analysis
 - More than one independent variable: **multivariate** analysis



(Univariate) Regression

- A function of changes in any one **independent** variable (holding others fixed)





Classification

- What characterises **Classification** is to identify a **class** or **group**
 - Classification results can be a value within the domain set
- Set of statistical processes to determine membership or **similarity**

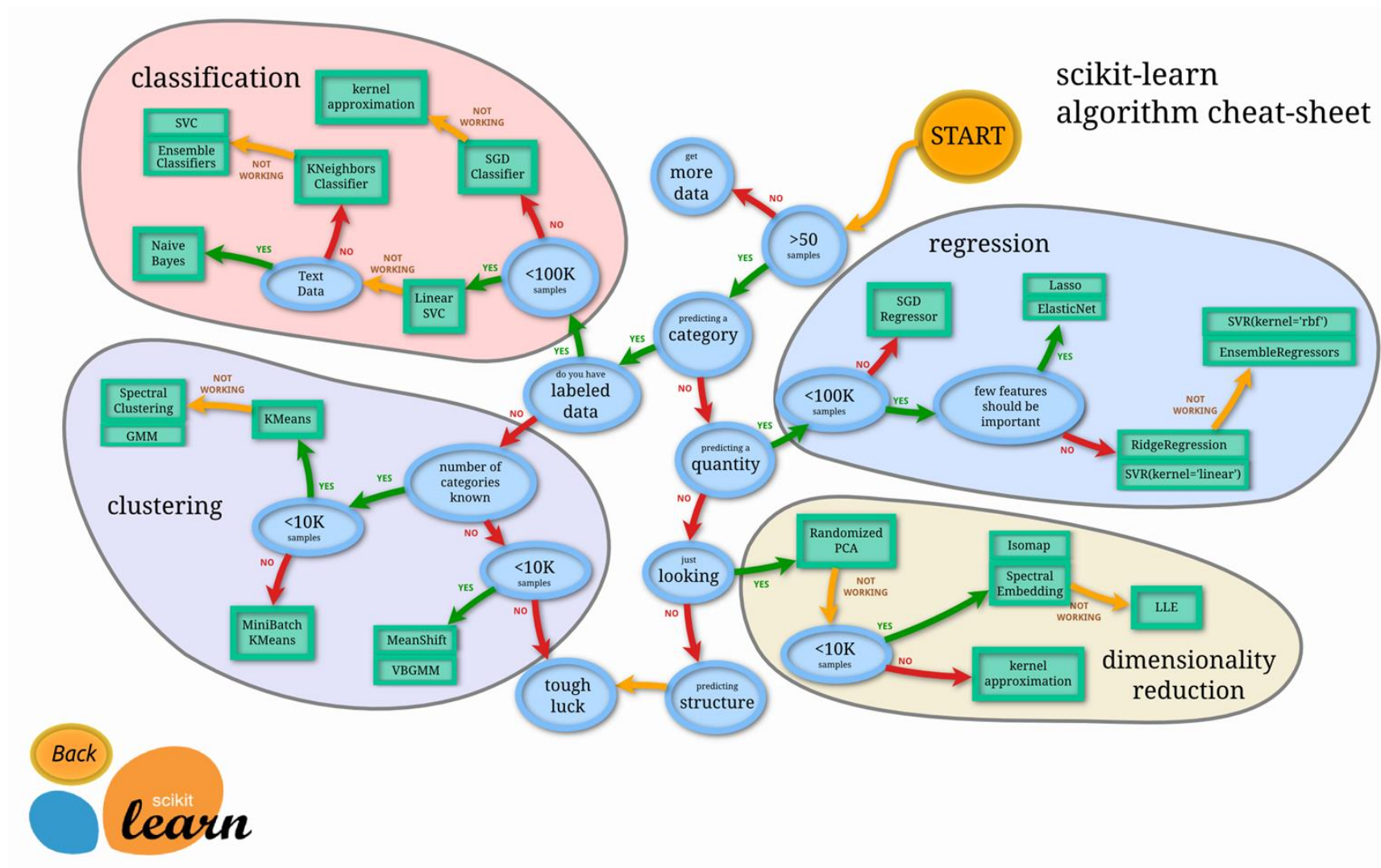


Classification

- Two classes ([Binary classification](#)): (Yes, No), (Positive, Negative)
 - Answer questions like:
 - Will the customer renew their subscription?
 - Will the website visitor click the [submit](#) button?
- More than two classes: A set of possible categories, classes or groups
 - Answer questions like:
 - Which animal is in this image?
 - What is the genre of this movie?



Scikit-Learn Cheat-Sheet





All Together

	Supervised	Unsupervised
Discrete	Classification	Clustering
Continuous	Regression	Dimensionality Reduction



The Predictive Modelling Process

- Outline
- Data Science **Process**
- Defining the **Response** Variable
- Defining the **Cost Function**
- A Familiar Example: **Univariate** Linear Regression
- **Multivariate** Linear Regression
- **Partitioning** Data
- Model **Training**
- Model **Testing**



Outline

- **data** | 'deɪtə |, *noun*
 - **facts** and **statistics** collected together for reference or analysis
 - (Philosophy) things **known** or **assumed** as facts, making the basis of reasoning or calculation
 - ORIGIN: mid 17th century; from Latin, the plural of *datum*
- **science** | 'saɪəns |, *noun*
 - the intellectual and practical activity encompassing the **systematic study** of the structure and behaviour of the physical and natural world through observation and experiment
 - ORIGIN: Middle English (denoting knowledge): from Old French, from Latin *scientia*, from *scire* 'know'

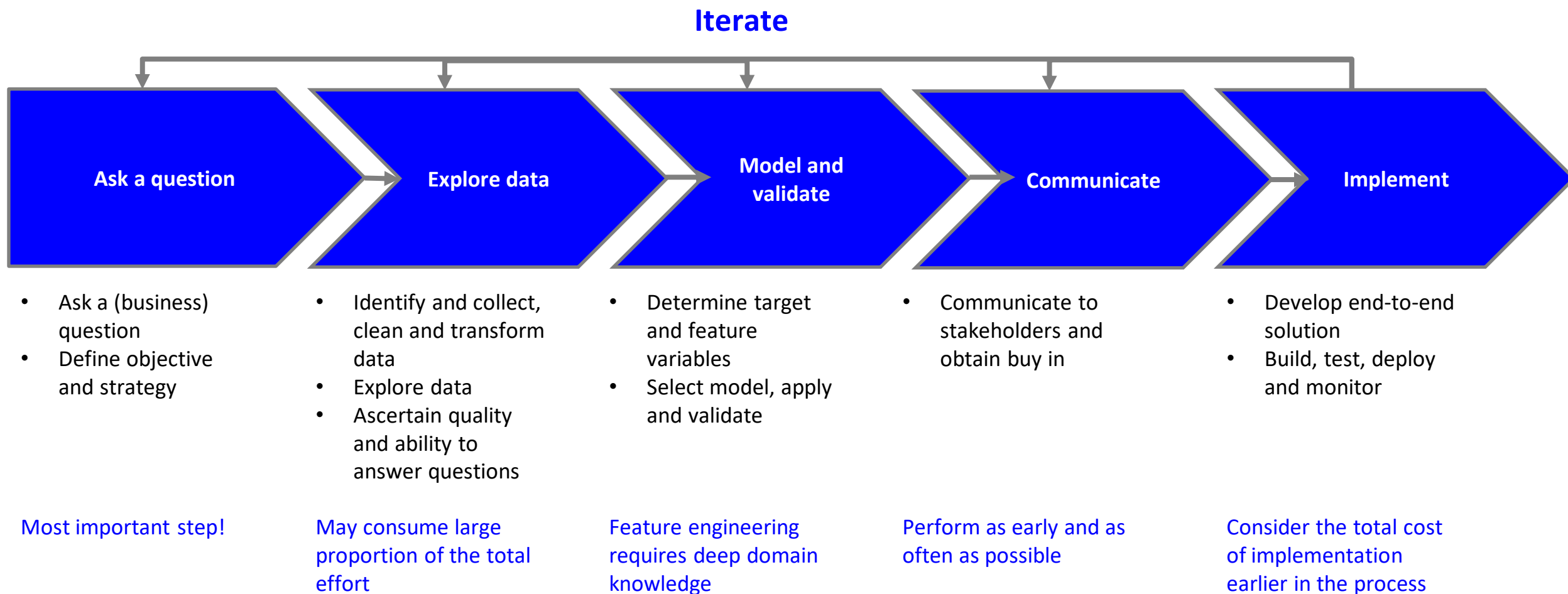


Outline

- Data Science: **Systematic** use of data to identify insights and inform decisions
- Data science is an **interdisciplinary** field that uses **scientific** methods, processes, algorithms and systems to extract *information, knowledge and insights* from both structured and unstructured data.
- Data science is a "concept to **unify** statistics, data analysis, **machine learning** and their related methods" to "understand and analyse actual phenomena" **with data**. It employs techniques and theories drawn from many fields within the context of mathematics, statistics, information science and **computer science**.

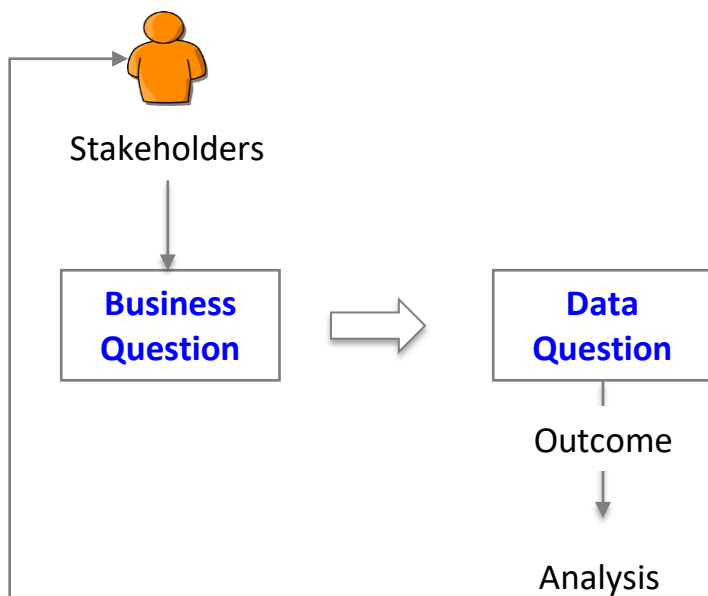


Data Science Process





Defining the Outcome/ Response Variable



- Your **question** leads to the selection of the outcome/ response variable:
 - To answer the question
 - You could frame it as a hypothesis that can be statistically tested
- The initial step to answer the question is to define which is the Response Variable and its characteristics, such as **type** and **range**
- Some **modelling techniques** that are only relevant to a particular type of variable, so knowing the nature of the response variable can reduce the number of approaches to consider



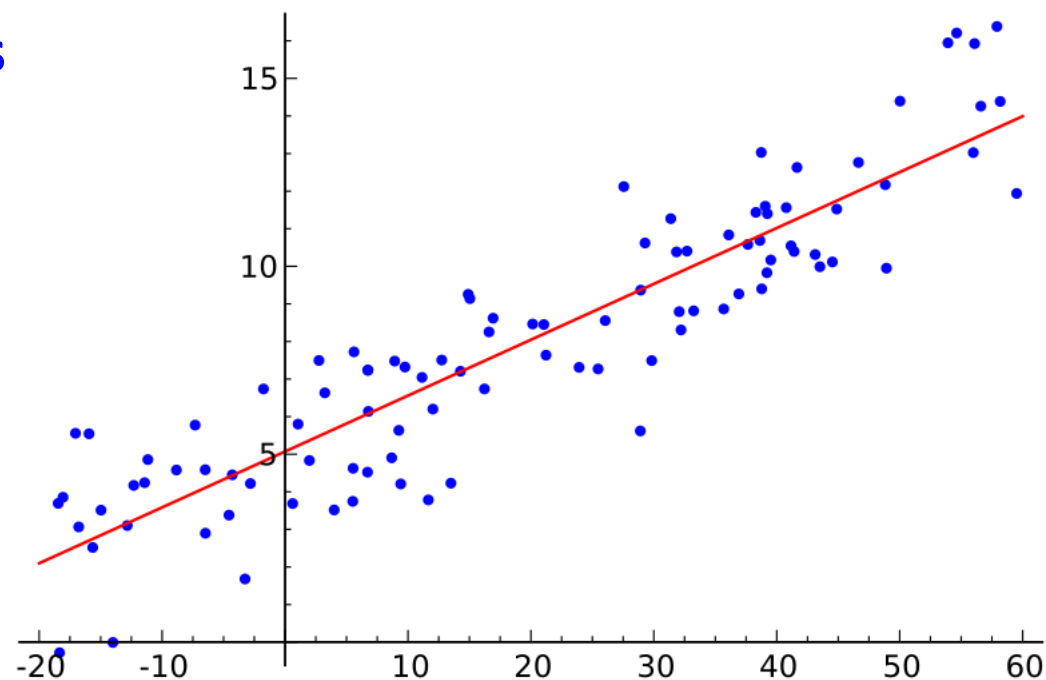
Defining the Cost Function

- A cost function calculates **how close the model can estimate the relationship** between the input variables (**X**) and output variable (**y**)
- In other words, it describes **how wrong the model is** by how far off its predictions are in comparison to reality
- Some traditional statistical methods such Linear Regression have error metrics associated with it, like MSE (Mean Squared Error)



A Familiar Example: Univariate Linear Regression

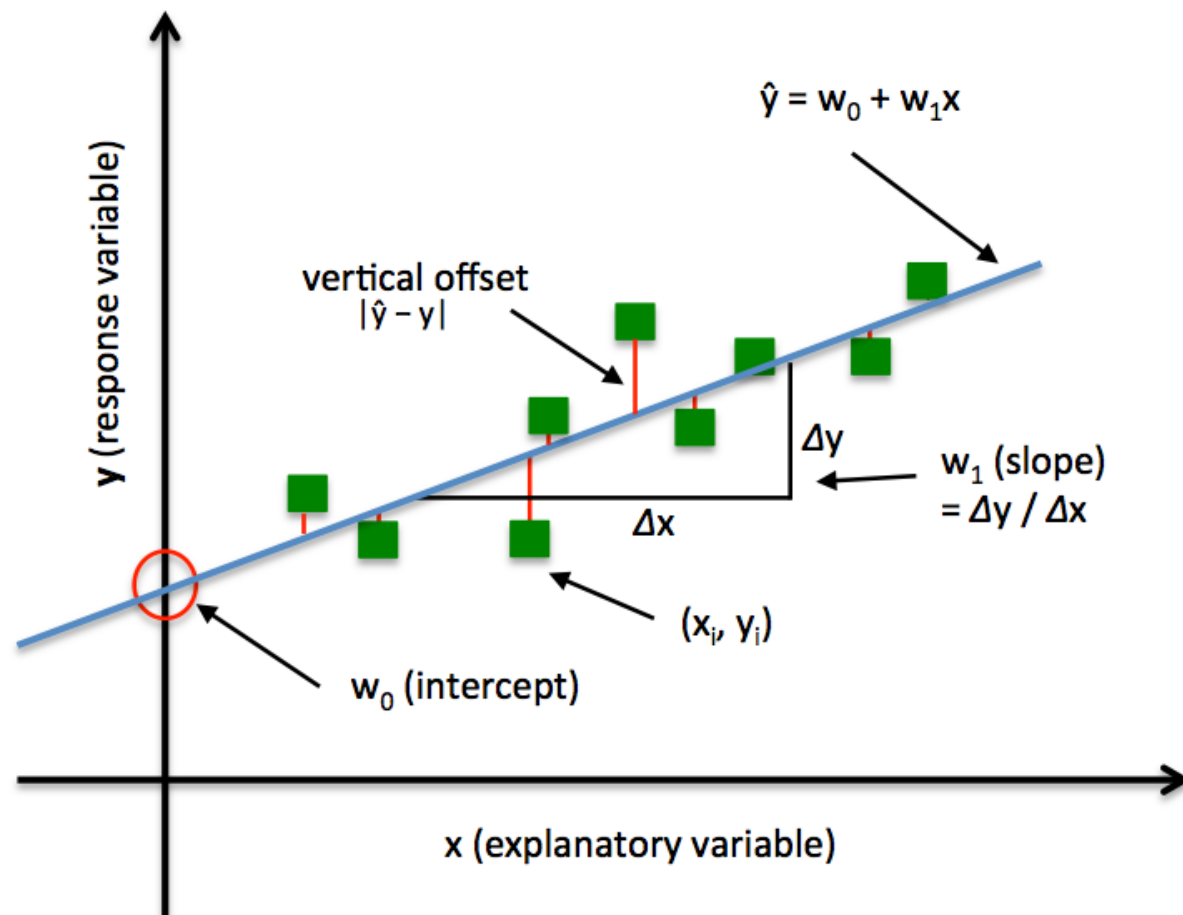
- The relationship between a **continuous variable** and other independent variables
- Explain the relationship between **x** and **y** using the starting point **a** and the power in explanation **b**
- The simplest version is just a **line** of best fit



Source: [Wikipedia](https://en.wikipedia.org/wiki/Linear_regression)



Example: Univariate Linear Regression



Source: [MLxtend](https://mlxtend.github.io/)



Multivariate Linear Regression

- However, linear regression uses linear algebra to explain the relationship between y and multiple x
- Explains the relationship
 - Between a Matrix X and a Dependent Vector y
 - By using a y-intercept α and the relative coefficients β



Multivariate Linear Regression

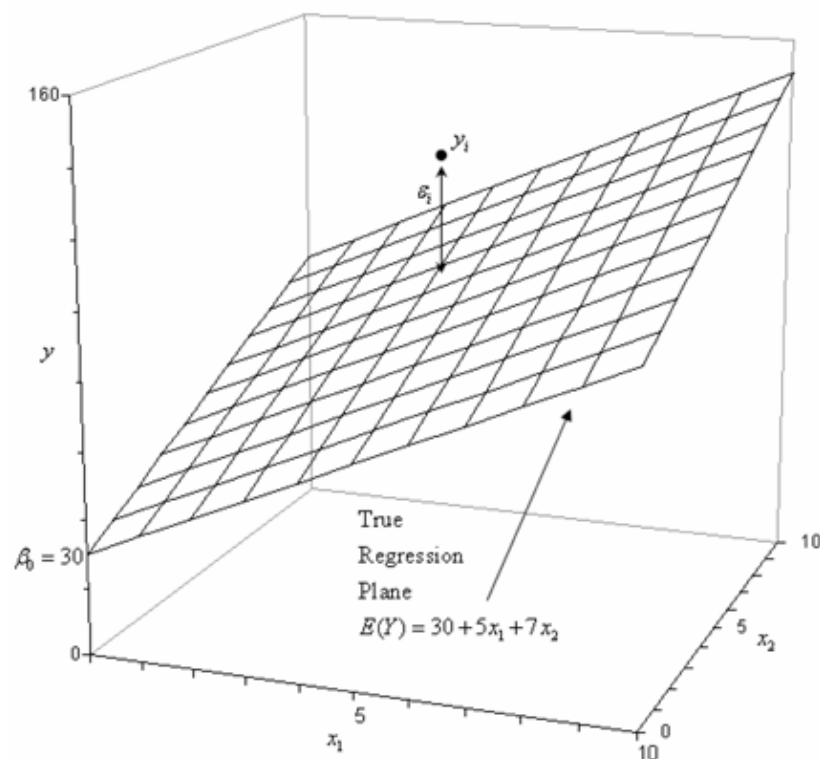
$$\mathbf{y} = \alpha + \boldsymbol{\beta} \cdot \mathbf{X} + \boldsymbol{\varepsilon}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \dots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & x_{1,3} & \dots & x_{1,m} \\ 1 & x_{2,1} & x_{2,2} & x_{2,3} & \dots & x_{2,m} \\ 1 & x_{3,1} & x_{3,2} & x_{3,3} & \dots & x_{3,m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{n,1} & x_{n,2} & x_{n,3} & \dots & x_{n,m} \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \dots \\ \beta_m \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \dots \\ \epsilon_n \end{bmatrix}$$



Multivariate Linear Regression

$$y = \alpha + \beta . X + \varepsilon$$



Source: [Reliawiki](#)

Source: [MathWorks](#)



Linear Regression

- Linear Regression works best when
 - The data is **normally** distributed (but does not have to be)
 - X s significantly **explain** y (have low p-values)
 - X s are **independent** from one another (low multicollinearity/ correlation)



Model Training, evaluation and testing

- Application of one or more **modelling** techniques on the training data aiming for a model with either or both **better performance** and **lower cost**
- Collect **evaluation** results for each technique, each set of variables
- **Refine** and **select** the best model for testing
- Optionally explore a **combination** of techniques
- **Test** and compare metrics with previous results



Discussion

What do you think about the terms **“Data Science”** and **“Machine Learning”**?

What do they mean?

Are they meaningful/ practical term?

Any Questions?



Partitioning Data

- Models are **simplified representations of reality** created from data that can be used to estimate future outcomes better
- To validate the models, it is necessary **to simulate existing data and future data**, hence the need to use **existing** data to play those roles
- The **Quality** of data is more critical than the **Quantity** of data
 - Data must be **representative**
 - Large quantities may incur in higher storage and computation costs without improving the model's performance



Partitioning Data

- Existing data can be **split** into (percentages shown are common, not a rule):
 - Training (70%) and Testing (30%), or
 - Training (65%), Validation (20%) and Testing (15%)
- **Training** (Learning) Data is used during the development of the model
- **Validation** Data can be used to improve the performance of the model
- **Test** Data is used to check the performance of the model with **unseen** data
- Data split must be **random**



Lab 4.1.1: Linear Regression from scratch

- Purpose
 - To develop familiarity with the Data Science Process
- Resources
 - Sample data
- Materials
 - Jupyter Notebook (Lab-4.1.1)



Lab 4.1.2: Splitting Data

- Purpose
 - To develop familiarity with the Data Science Process
 - Split data for model development and evaluation
- Resources
 - Sample data from SciKit-Learn
- Materials
 - Jupyter Notebook (Lab-4.1.2)



Feature Selection

- Forward and Backward Stepwise Feature Selection
- Feature Engineering



Forward and Backward Stepwise Feature Selection

- **Occam's razor** (also **Ockham's razor** or "law of parsimony") is the problem-solving principle that the simplest solution tends to be the right one.
 - **When presented with competing hypotheses to solve a problem, one should select the solution with the fewest assumptions**
- The idea is attributed to William of Ockham (c. 1287–1347), who was an English Franciscan friar, scholastic philosopher, and theologian

([Wikipedia](#))



Forward and Backward Stepwise Feature Selection

- Forward Feature Selection
 - Take **each** of the predictors **individually** and select the variable from the model that explains the output variable the best
 - Repeat the process by **adding** the remaining predictors **individually** until there is **no** significant improvement
- Backward Feature Selection
 - Take **all** of the predictors and create a model
 - **Remove each** of the predictors **individually** and keep the model that explains the output variable the best
 - Repeat the process by removing the remaining predictors individually until there is a significant reduction in performance



Feature Engineering

- **Feature Engineering** can be described as the process of **transforming** the variables into forms or domains **suitable** both to the **underlying problem** and to computation
- Modelling techniques manipulate **numbers** to optimise parameters and coefficients used in functions
- Not all techniques can manipulate nominal and ordinal variables (classes, categories)



Feature Engineering

- There are **no specific rules** in cases when the data manipulation relates to **particular problems**, industries or businesses
- Some data manipulations are mode specific to the chosen modelling approaches, such:
 - Variables transformation from nominal/ordinal to numeric
 - A common practice is to use one hot encoding
 - **Change of scale** or range for numeric variables
 - Variables with **different orders of magnitude** (A:0-10, B:300-9000) can be **normalised** to be closer to each other (A:0-1, B:0-1)



Lab 4.2: Feature Selection

- Purpose
 - To understand Forward Feature Selection
- Resources
 - Sample data from SciKit-Learn
- Materials
 - Jupyter Notebook (Lab-4_2_1, Lab-4_2_2)



Measuring the Accuracy of Regression Models

- Introduction
- **R-Squared**
- **Mean Squared Error** (MSE)



Introduction

- It is possible to compare the known results with the ones predicted by the model when using **Supervised modelling**
- When comparing known and predict results it is possible to make calculations and create metrics on how close or similar there outcomes are
- Such metrics can both tell how **“good”** or how **“efficient”** a model is and how it compares to other models as most metrics have standard results



R-Squared

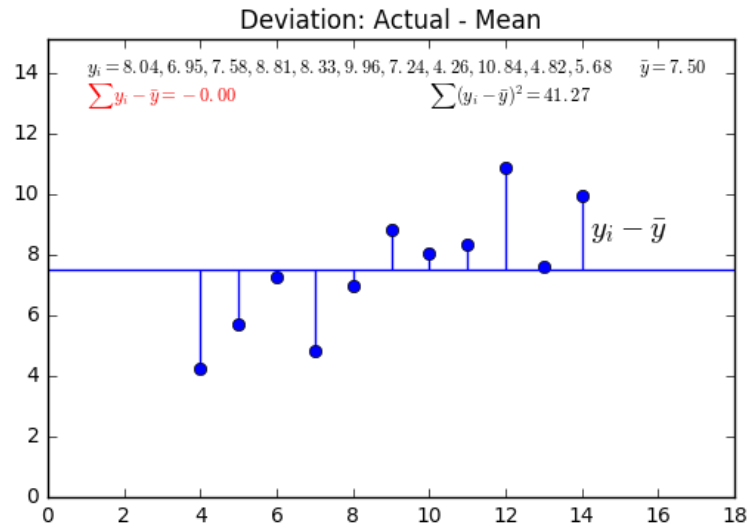
- Is the central metric introduced for Linear Regression
- It determines how much of the variation in **y** is **explained** by the change in **X**
 - But does it tell the magnitude or scale of error?
- The value range is between:
 - 0: the model does not model explain any variability in **y**
 - 1: the model **explains full variability** in the target variable
- Exploring cost or loss functions can be used to **refine** the models



R-Squared and Sum of Squares (SS)

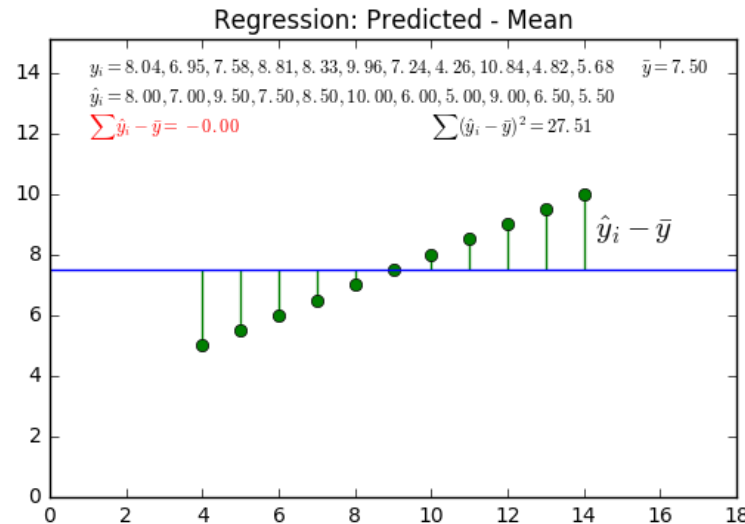
Total SS

$$SS_{tot} = \sum_{i=1}^n (y_i - \bar{y})^2$$



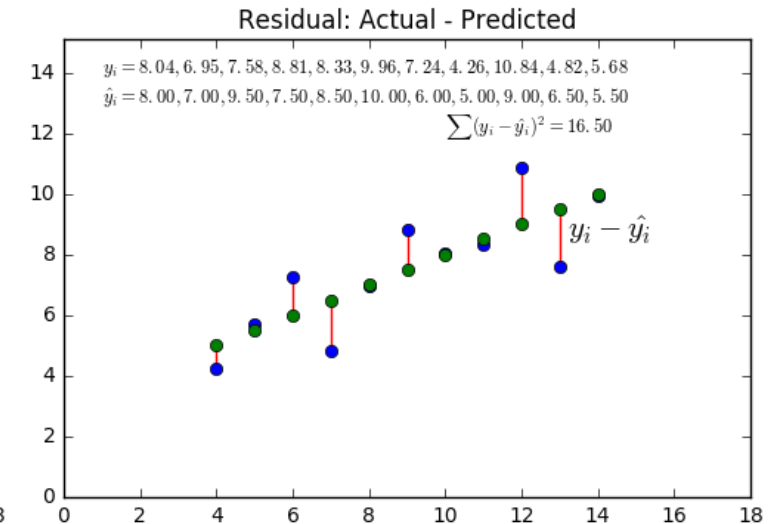
Regression SS

$$SS_{reg} = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$$



Residual SS

$$SS_{res} = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$





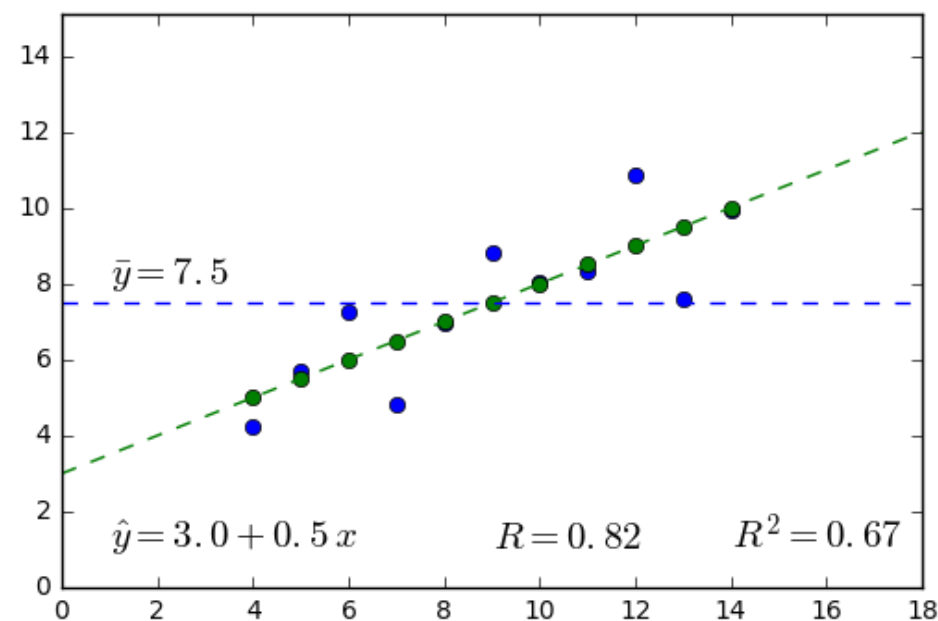
R-Squared and Residuals

- In a data set with **n** cases where the outputs are: $y_i = y_1, \dots, y_n$
- And the **predicted** values are (note the *y-hat*): $\hat{y}_i = \hat{y}_1, \dots, \hat{y}_n$
- SS: Sum of Squares
- The **mean of y** is (note the *y-bar*):

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$SS_{tot} = SS_{reg} + SS_{res}$$

$$R^2 = \frac{SS_{reg}}{SS_{tot}}$$





Adjusted R-Squared

- The Adjusted R-Squared is the R-Squared adjusted for the number of predictors in the model
- It incorporates a model's degrees of freedom
- Adjusted R-Squared only increases if the new term improves the model accuracy

$$\text{Adjusted } R^2 = 1 - (1 - R^2) \left[\frac{N - 1}{N - (p + 1)} \right]$$

- where
 - R² = Sample R-Squared
 - N = Total sample size
 - p = Number of predictors



Residual Error

- In linear models, the **residual error** must be normal with a median close to zero
- Individual residuals are useful to see the error of specific points, but it does not provide an overall picture for optimisation
- It is necessary a metric to summarise the error in the model into one value
 - **Mean Squared Error:** the mean residual error in our model



Mean Squared Error (MSE)

- Calculate the difference between each target y and the model's predicted value \hat{y} (i.e. the residual)
- Take the mean of the squared residual errors

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

- The MSE is always greater or equal to zero, with smaller errors being better
- An MSE of 0 means that actual and predicted results are identical



Minimising the Error, MSE as Cost Function

- The linear regression method with MSE is also known as:
 - Ordinary Least Squares
- Meaning that given a matrix X
 - Solve for the least amount of square error for y
- Assuming that X is unbiased
 - Saying it is representative of the population



Lab 4.3: Measurements

- Purpose
 - To compare model complexity
- Resources
 - Sample data from SciKit-Learn
- Materials
 - Jupyter Notebook (Lab-4_3)



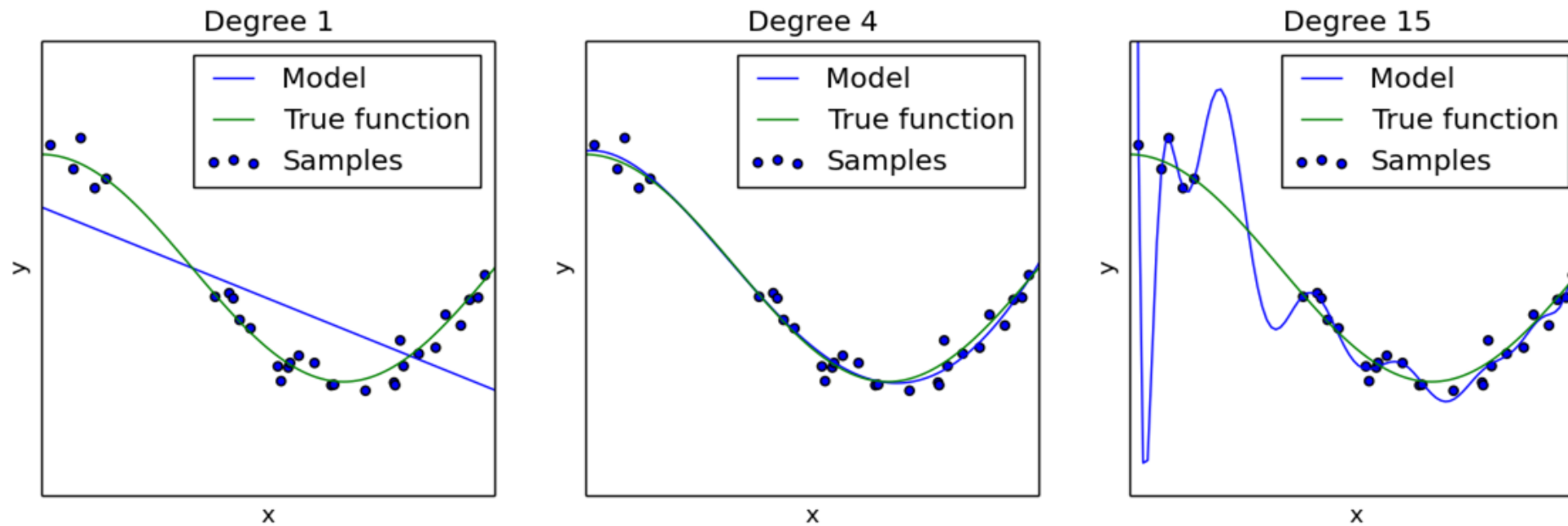
Overfitting

- **Detecting** Overfitting
- **Avoiding** Overfitting
- **Cross-Validation**
- **Regularisation**



Overfitting

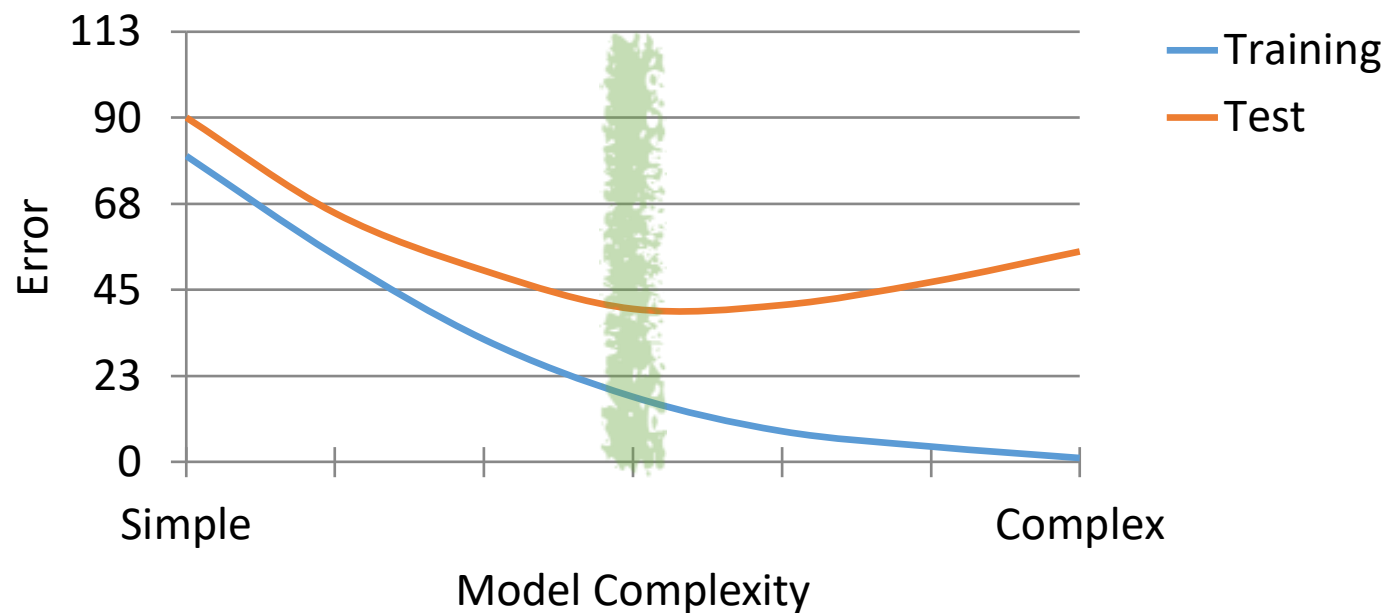
- The first model **poorly explains** the data (underfitting)
- The second model describes **the general curve** of the data
- The third model drastically **overfits** the model, bending to every point





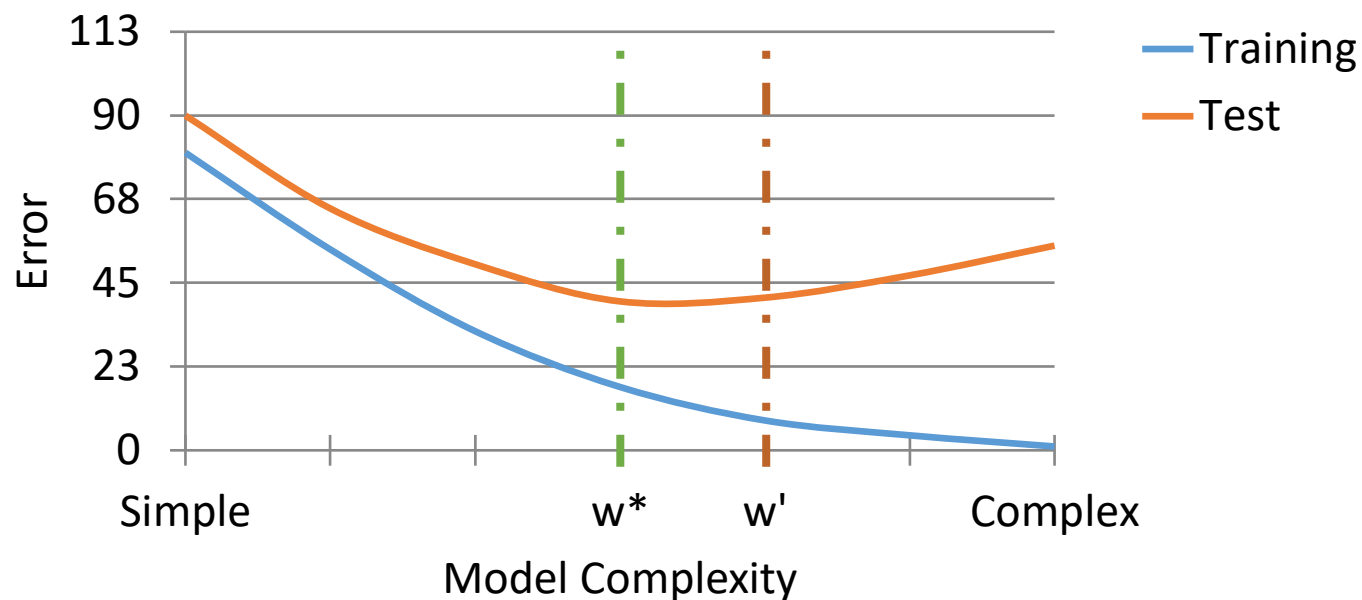
Detecting Overfitting

- Mathematically, modelling is to **fit** a curve to a set of known data points, so overfitting is to find a curve that fits the known data points very closely but does **not** generalise well to **unknown** data points





Detecting Overfitting



- Overfitting:
 - $\text{Training Error}(w^*) > \text{Training Error}(w')$
 - $\text{Test Error}(w^*) < \text{Test Error}(w')$



Avoiding Overfitting

- Have the data split into **a training** set and a **test** set
 - Works well only with the availability of large datasets
- To find out the best model (training error) the **residual sum of squares** is used
- Varies model complexity
 - Includes varying the set of **variables** and data manipulation
- Continuously updated the parameters/coefficients/weights
- Use techniques such as **Cross-Validation** and **Regularisation**



Cross-Validation

- The goal of cross-validation is to test a model's ability to predict new data that was not used in estimating it
 - Used to flag problems like overfitting and to give an insight on how the model will **generalise** to an independent dataset
- General Process
 - Generate several models on different **cross-sections** of the data
 - Partition the data in separate data sets **alternating their roles** as Training and Test
 - Measure the **performance** of each iteration
 - Take the **overall performance**
 - Usually the mean of the collected results



Common types of Cross-Validation

Exhaustive cross-validation

- All distinct ways to divide the original sample are used to learn and test
- Leave-p-out
 - Use p observations as the test set and the remaining as the training set
 - This is repeated in all ways to cut the original sample on a validation set of p observations and a training set



Common types of Cross-Validation

Non-exhaustive cross-validation

- Do not compute all ways of splitting the original sample; these methods are **approximations** of leave-p-out cross-validation
- k-fold
 - Randomly partition the original sample into **k equal sized subsamples**
 - In stratified k-fold cross-validation, select folds so that the **mean response value is approximately equal** in all the folds
- Holdout method
 - **Randomly** assign data points to two sets **d0** and **d1**, usually called the training and the test sets, respectively
 - The test set is usually smaller than the training set

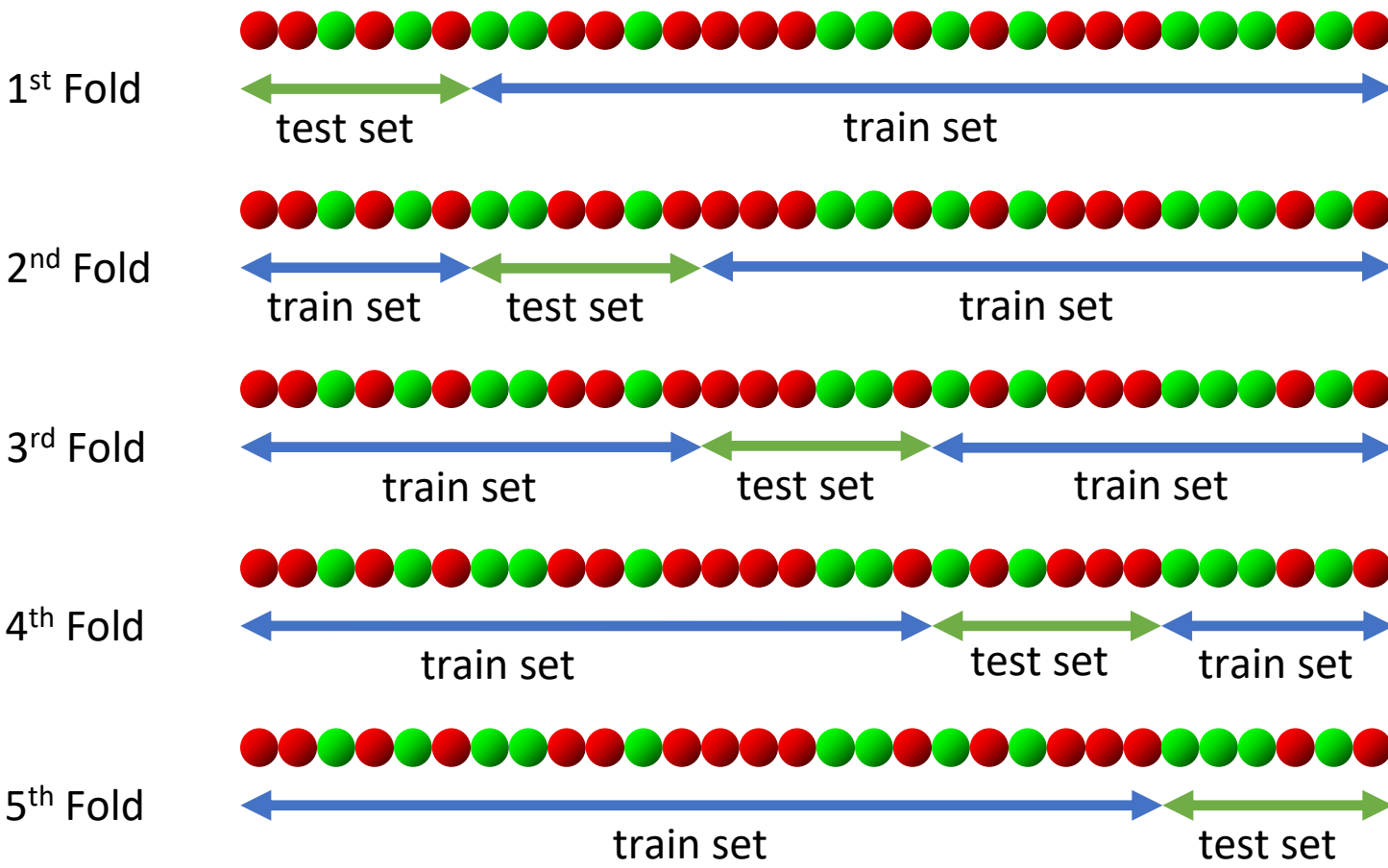


k-Fold Cross-Validation

- k-fold cross-validation
 - Split the data into **k group**
 - **Train** the model in all segments **except one**
 - **Test** model performance on the **remaining set**
- If $k = 5$, split the data into five segments and generate five models



Cross-Validation





Cross-Validation

- Computational issues
 - Straightforward to implement if the **prediction method** being studied is available
 - If the prediction method is **expensive** to train, cross-validation can be very slow since the training must be carried out repeatedly
- Limitations
 - Cross-validation only yields meaningful results if the validation set and training set are drawn from the same population and only if human biases are controlled
- Cross validation for time-series models
 - Since the order of the data is important, cross-validation might be problematic for time-series models
 - A more appropriate approach might be to use **forward chaining**



Regularisation

- Form of regression, that **constrains or shrinks** the coefficient estimates towards zero
 - This technique discourages a more complex model to avoid the risk of overfitting
- Given a set of p coefficient (β_p), the fitting procedure involves a loss function (**RSS** or **SS_{res}**)
 - The coefficients are chosen, such that they minimise the loss function by adding an “extra cost”
- The estimated coefficients do not generalise well with unseen data if there is noise in the training data



Regularisation

- There are multiple forms of regularisation, the most common being:
 - **Ridge Regression**
 - Adds cost by the inclusion of a new parameter called lambda (λ)
 - **Lasso Regression**
 - Adds cost by the addition of a new parameter called lambda (λ)
 - Can force coefficients to be zero, hence effectively removing them from the model



Regularisation - Ridge Regression

- The RSS is added a shrinkage quantity λ

$$RSS + \lambda \sum_{j=1}^p \beta_j^2$$

- The coefficients are estimated by minimising the new function
- The coefficient λ is the parameter that controls how much to penalise the model
- The Ridge Regression force coefficients to stay low



Regularisation - Ridge Regression

- When $\lambda=0$, the penalty term has no effect and the estimates produced by Ridge regression will be equal to Least Squares
- It is necessary to standardise the predictors or bring the predictors to the same scale before performing Ridge regression



Regularisation - Lasso Regression

- It penalises the high coefficients by using $|\beta_j|$
- Ridge regression will shrink the coefficients for least important predictors, very close to zero, but never make them precisely zero
 - The final model will include all predictors
- Lasso regression can force some of the coefficient estimates to be equal to zero when the tuning parameter λ is sufficiently large
 - The lasso method also performs **variable selection** and is said to yield sparse models

$$RSS + \lambda \sum_{j=1}^p |\beta_j|$$



Lab 4.4: Regularisation

- Purpose
 - To compare some regularisation approaches
- Resources
 - Sample data from SciKit-Learn
- Materials
 - Jupyter Notebook (Lab-4_4)



Questions?



End of Presentation!