Random variables

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Question 10.13.3.42
At a fete, cards bearing numbers 1 to 1000, one number on a card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that
(i) the first player wins a prize
(ii) the second player wins a prize, if the first has won?
Solution: Let

Variable	Value	Description
X_i	0	i^{th} player doesn't win a prize
	1	i^{th} player wins a prize

If n^2 is the value of the chosen number that is greater than 500 and also a perfect square, then

$$n^2 \in (500, 1000] \tag{1}$$
$$\implies n \in (22.36, 31.62] \tag{2}$$

n can take 9 integer values in the above interval.

$$Pr(X_1 = 1) = \frac{9}{1000} \tag{3}$$
$$= 0.009 \tag{4}$$

If first player gets a number greater than 500 which is a perfect square then the second player can get a number from the remaining 8 numbers in the above interval to win a prize.

1) Probability that first player wins a prize

$$= Pr(X_1 = 1) \tag{5}$$
$$= 0.009 \tag{6}$$

2) Probability that second player wins given that the first player has won prize

$$= \Pr((X_2 = 1)|(X_1 = 1)) \tag{7}$$
$$= 0.008 \tag{8}$$