12.13.3.12

12.13.3.12 If X is the number of tails in three tosses of coin, determine the standard deviation of X.

Solution: Let number of tails obtained be defined by random variable

$$X = 0, 1, 2, 3 \tag{1}$$

$$n = \text{Number of trails} \tag{2}$$

$$p = p_X = \text{Probability of getting tails in a toss} \tag{3}$$

$$= \frac{1}{2} \tag{4}$$

$$q = \text{Probability of not getting tails} \tag{5}$$

$$= (1 - p) \tag{6}$$

$$= \frac{1}{2} \tag{7}$$

$$p_X(k) = \binom{n}{k} p^k q^{n-k} \tag{8}$$

$$E(X) = \sum_{k=0}^{n} k p_X(k) \tag{9}$$

$$\sigma_X^2 = E(X - E(X))^2 \tag{10}$$

$$= E\left(X^2 - 2XE(X) + E(X)^2\right) \tag{11}$$

$$= E\left(X^2\right) - 2E(X).E(X) + E(X)^2 \tag{12}$$

$$= E\left(X^2\right) - E(X)^2 \tag{13}$$

$$E\left(X^2\right) = \sum_{k=0}^{n} k^2 p_X(k) \tag{14}$$

On substituting values we get:

$$E(X) = \sum_{k=0}^{3} k \binom{3}{k} p^k q^{3-k} \tag{15}$$

$$= \frac{3}{2} \tag{16}$$

$$E\left(X^2\right) = \sum_{k=0}^{3} k^2 p_X(k) \tag{17}$$

$$= 3 \tag{18}$$

$$\sigma_X = \sqrt{3 - \frac{3}{2}^2} \tag{19}$$

$$= \sqrt{\frac{3}{4}} \tag{20}$$