12.13.3.12

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If X is the number of tails in three tosses of coin, determine the standard deviation of X.
Solution: Let

$$n = \text{Number of trails} \tag{1}$$
$$p = \text{Probability of getting tails in a toss} \tag{2}$$
$$= \frac{1}{2} \tag{3}$$
$$q = \text{Probability of not getting tails} \tag{4}$$
$$= (1-p) \tag{5}$$
$$= \frac{1}{2} \tag{6}$$
$$\sigma_{X^2}^2 = E\left(X - E\left(X\right)\right)^2 \tag{7}$$
$$= E\left(X^2 - 2XE\left(X\right) + E\left(X\right)^2\right) \tag{8}$$
$$= E\left(X^2\right) - 2E\left(X\right).E\left(X\right) + E\left(X\right)^2 \tag{9}$$
$$= E\left(X^2\right) - E\left(X\right)^2 \tag{10}$$
$$E\left(X\right) = np \tag{11}$$
$$E\left(X\right)^2 = \sum_{X=0}^{n} X^2.P_r\left(X\right) \tag{12}$$
$$= \sum_{X=0}^{n} [X + X.\left(X-1\right)].P_r\left(X\right) \tag{13}$$
$$= XP_r\left(X\right) + \sum \left(X-1\right)X.P_r\left(X\right) \tag{14}$$
$$= np + \sum \left(X-1\right)X \binom{n}{X}.p^X.q^{n-X} \tag{15}$$
$$= np + \sum \left(X-1\right)X \cdot \frac{n!}{(n-X)!X!} p^X q^{n-X} \tag{16}$$
$$\tag{17}$$
$$= np + \sum \left(X-1\right)X \cdot \frac{n\left(n-1\right)(n-2)!}{[((n-2)-(X-2))!\left(X\right)\left(X-1\right)(X-2)!]} p^2 p^{X-2} \cdot q^{(n-2)-(X-2)} \tag{18}$$
$$= np + n\left(n-1\right)p^2 \sum \frac{(n-2)!}{[((n-2)-(X-2))!\left(X-2\right)!]} p^{X-2} \cdot q^{(n-2)-(X-2)} \tag{19}$$
$$= np + n\left(n-1\right) \cdot p^2 \cdot (p+q)^{n-2} \tag{20}$$
$$\tag{21}$$
$$= np + \left(n^2p^2 - np^2\right) \cdot (1)^{n-2} \tag{22}$$
$$= np + n^2p^2 - np^2 \tag{23}$$
$$\sigma_X^2 = \left(np + n^2p^2 - np^2\right) - n^2p^2 \tag{24}$$
$$= np\left(1-p\right) \tag{25}$$
$$\sigma_X = \sqrt{np(1-p)} \tag{26}$$
$$= \sqrt{\frac{3}{4}} \tag{27}$$