12.13.3.12

12.13.3.12 If X is the number of tails in three tosses of coin, determine the standard deviation of X.

Solution: Let number of tails obtained be defined by random variable

$$
\begin{aligned}
X &= 0, 1, 2, 3 && (1)\\
n &= \text{Number of trails} && (2)\\
p &= \text{Probability of getting tails in a toss} && (3)\\
&= \frac{1}{2} && (4)\\
q &= \text{Probability of not getting tails} && (5)\\
&= (1-p) && (6)\\
&= \frac{1}{2} && (7)\\
p_X(k) &= \binom{n}{k} p^k q^{n-k} && (8)\\
E(X) &= \sum_{k=0}^{n} k p_X(k) && (9)\\
&= np && (10)\\
\sigma_X^2 &= E(X - E(X))^2 && (11)\\
&= E\left(X^2 - 2XE(X) + E(X)^2\right) && (12)\\
&= E\left(X^2\right) - 2E(X).E(X) + E(X)^2 && (13)\\
&= E\left(X^2\right) - E(X)^2 && (14)\\
E\left(X^2\right) &= \sum_{k=0}^{n} k^2 p_X(k) && (15)\\
&= \sum_{k=0}^{n} k^2 \binom{n}{k} p^k q^{n-k} && (16)\\
&= \sum_{k=0}^{n} (k + k(k-1)) \binom{n}{k} p^k q^{n-k} && (17)\\
&= \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} + k(k-1) \binom{n}{k} p^k q^{n-k} && (18)\\
&= np + \sum_{k=0}^{n} (k-1) k \frac{n!}{(n-k)!k!} p^k q^{n-k} && (19)\\
&= np + n(n-1) p^2 \sum \frac{(n-2)!}{[((n-2)-(k-2))!(k-2)!]} p && (20)\\
&= np + n(n-1) p^2 (p+q)^{n-2} && (21)\\
&= np + \left(n^2 p^2 - np^2\right)(1)^{n-2} && (22)\\
&= np + n^2 p^2 - np^2 && (23)\\
\sigma_X^2 &= np + n^2 p^2 - np^2 - (np)^2 && (24)\\
&= np(1-p) && (25)\\
&= npq && (26)
\end{aligned}
$$

On substituting values we get:

$$\sigma_X^2 = \frac{3}{4} \tag{27}$$

$$\sigma_X = \sqrt{\frac{3}{4}} \tag{28}$$