

12.13.3.12

12.13.3.12 If X is the number of tails in three tosses of coin, determine the standard deviation of X .

Solution: Let number of tails obtained be defined by random variable

$$X = 0, 1, 2, 3 \quad (1)$$

$$n = \text{Number of trails} \quad (2)$$

$$p = p_X = \text{Probability of getting tails in a toss} \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

$$q = \text{Probability of not getting tails} \quad (5)$$

$$= (1 - p) \quad (6)$$

$$= \frac{1}{2} \quad (7)$$

$$p_X(k) = \binom{n}{k} p^k q^{n-k} \quad (8)$$

$$E(X) = \sum_{k=0}^n k p_X(k) \quad (9)$$

$$= \sum_{k=0}^3 k \binom{3}{k} p^k q^{3-k} \quad (10)$$

$$= \frac{3}{2} \quad (11)$$

$$\sigma_X^2 = E(X - E(X))^2 \quad (12)$$

$$= E(X^2 - 2XE(X) + E(X)^2) \quad (13)$$

$$= E(X^2) - 2E(X) \cdot E(X) + E(X)^2 \quad (14)$$

$$= E(X^2) - E(X)^2 \quad (15)$$

$$E(X^2) = \sum_{k=0}^n k^2 p_X(k) \quad (16)$$

$$= \sum_{k=0}^3 k^2 p_X(k) \quad (17)$$

$$= 3 \quad (18)$$

$$\sigma_X = \sqrt{3 - \frac{3^2}{2}} \quad (19)$$

$$= \sqrt{\frac{3}{4}} \quad (20)$$