47

EE22BTECH11059

47.2023 Let (X, Y) have joint probability mass function

$$p(x,y) = \begin{cases} \frac{c}{2^{x+y+2}} & if\, x = 0,1,2,...; x \neq y \\ 0 & otherwise \end{cases} \quad (1)$$

Then which of the following is true?

1) $c = \frac{1}{2}$
2) $c = \frac{1}{4}$
3) $c > 1$
4) X and Y are independent

Solution: For p(x, y) to be joint probability mass function

$$\sum_{y=-\infty}^{\infty}\sum_{x=-\infty}^{\infty} p(x,y) = 1 \,|x \neq y \quad (2)$$

$$\sum_{y=0}^{\infty}\sum_{x=0}^{\infty} \frac{c}{2^{x+y+2}} - \sum_{x=y} \frac{c}{2^{x+y+2}} = 1 \quad (3)$$

$$\sum_{y=0}^{\infty} \frac{c}{2^{y+2}} \sum_{x=0}^{\infty} 2^{-x} - \frac{c}{4}\sum_{x=0}^{\infty}\frac{1}{4^x} = 1 \quad (4)$$

$$\sum_{y=0}^{\infty} \frac{2c}{2^{y+2}} - \frac{c}{3} = 1 \quad (5)$$

$$\frac{2c}{4}\sum_{y=0}^{\infty} 2^{-y} - \frac{c}{3} = 1 \quad (6)$$

$$c - \frac{c}{3} = 1 \quad (7)$$

$$c = \frac{3}{2} \quad (8)$$

Marginal probability mass function of X

$$p_X(x) = \sum_{y=0}^{\infty} p(x,y) \quad (9)$$

$$= \sum_{y=0}^{\infty} \frac{3}{2^{x+y+3}} \quad (10)$$

$$= \frac{3}{2^{x+3}} \sum_{y=0}^{\infty} 2^{-y} \quad (11)$$

$$= \frac{3}{2^{x+2}} \quad (12)$$

Similary, Marginal probability mass function of Y

$$p_Y(y) = \sum_{x=0}^{\infty} p(x,y) \quad (13)$$

$$= \sum_{x=0}^{\infty} \frac{3}{2^{x+y+3}} \quad (14)$$

$$= \frac{3}{2^{y+3}} \sum_{x=0}^{\infty} 2^{-x} \quad (15)$$

$$= \frac{3}{2^{y+2}} \quad (16)$$

$$x \neq y \quad (17)$$

When x takes a particular integer, y cannot take the same number and vice-versa
Option (4) is incorrect.
$\therefore$ Only option (3) is correct.