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EE22BTECH11059

47.2023 Let (X, Y) have joint probability mass function

$$p(x, y) = \begin{cases} \frac{c}{2^{x+y+2}} & \text{if } x = 0, 1, 2, \dots; x \neq y \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Then which of the following is true?

- 1) $c = \frac{1}{2}$
- 2) $c = \frac{1}{4}$
- 3) $c > 1$
- 4) X and Y are independent

Solution: For $p(x, y)$ to be joint probability mass function

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} p(x, y) = 1 \quad |x \neq y \quad (2)$$

$$\sum_{y=0}^{\infty} \sum_{x=0}^{\infty} \frac{c}{2^{x+y+2}} - \sum_{x=y} \frac{c}{2^{x+y+2}} = 1 \quad (3)$$

$$\sum_{y=-\infty}^{\infty} \frac{c}{2^{y+2}} \sum_{x=-\infty}^{\infty} 2^{-x} - \frac{c}{4} \sum_{x=0}^{\infty} \frac{1}{4^x} = 1 \quad (4)$$

$$\sum_{y=-\infty}^{\infty} \frac{2c}{2^{y+2}} - \frac{c}{3} = 1 \quad (5)$$

$$\frac{2c}{4} \sum_{y=-\infty}^{\infty} 2^{-y} - \frac{c}{3} = 1 \quad (6)$$

$$\frac{4c}{4} \frac{c}{3} = 1 \quad (7)$$

$$c - \frac{c}{3} = 1 \quad (8)$$

$$c = \frac{3}{2} \quad (9)$$

\therefore Option (3) is correct.