# 47

## EE22BTECH11059

**47.2023** Let $(X, Y)$ have joint probability mass function

$$p(x,y) = \begin{cases} \frac{c}{2^{x+y+2}} & if x = 0,1,2,...; x \neq y \\ 0 & otherwise \end{cases} \tag{1}$$

Then which of the following is true?

1) $c = \frac{1}{2}$
2) $c = \frac{1}{4}$
3) $c > 1$
4) $X$ and $Y$ are independent

**Solution:** For p$(x, y)$ to be joint probability mass function

$$\sum_{y=-\infty}^{\infty}\sum_{x=-\infty}^{\infty} p(x,y) = 1 \,|x \neq y \tag{2}$$

$$\sum_{y=0}^{\infty}\sum_{x=0}^{\infty} \frac{c}{2^{x+y+2}} - \sum_{x=y} \frac{c}{2^{x+y+2}} = 1 \tag{3}$$

$$\sum_{y=0}^{\infty} \frac{c}{2^{y+2}} \sum_{x=0}^{\infty} 2^{-x} - \frac{c}{4}\sum_{x=0}^{\infty}\frac{1}{4^x} = 1 \tag{4}$$

$$\sum_{y=0}^{\infty} \frac{2c}{2^{y+2}} - \frac{c}{3} = 1 \tag{5}$$

$$\frac{2c}{4}\sum_{y=0}^{\infty} 2^{-y} - \frac{c}{3} = 1 \tag{6}$$

$$c - \frac{c}{3} = 1 \tag{7}$$

$$c = \frac{3}{2} \tag{8}$$

1) Marginal probability mass function of X

$$p_X(x) = \sum_{y=0}^{\infty} p(x,y) \tag{9}$$

$$= \sum_{y=0}^{\infty} \frac{3}{2^{x+y+3}} \tag{10}$$

$$= \frac{3}{2^{x+3}} \sum_{y=0}^{\infty} 2^{-y} \tag{11}$$

$$= \frac{3}{2^{x+2}} \tag{12}$$

2) Marginal probability mass function of Y

$$p_Y(y) = \sum_{x=0}^{\infty} p(x,y) \tag{13}$$

$$= \sum_{x=0}^{\infty} \frac{3}{2^{x+y+3}} \tag{14}$$

$$= \frac{3}{2^{y+3}} \sum_{x=0}^{\infty} 2^{-x} \tag{15}$$

$$= \frac{3}{2^{y+2}} \tag{16}$$

For X and Y to be independent,

$$p(x,y) = p_X(x)\, p_Y(y) \tag{17}$$

$$p_X(x)\, p_Y(y) = \frac{3}{2^{x+2}}\frac{3}{2^{y+2}} \tag{18}$$

$$= \frac{9}{2^{x+y+4}} \tag{19}$$

$$p(x,y) \neq p_X(x)\, p_Y(y) \tag{20}$$

Option (4) is incorrect.
$\therefore$ Only option (3) is correct.