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EE22BTECH11059

47.2023 Let (X, Y) have joint probability mass function

$$p(x, y) = \begin{cases} \frac{c}{2^{x+y+2}} & if x = 0, 1, 2, ...; x \neq y \\ 0 & otherwise \end{cases} \tag{1}$$

Then which of the following is true?

1) $c = \frac{1}{2}$
2) $c = \frac{1}{4}$
3) $c > 1$
4) X and Y are independent

Solution: For p(x, y) to be joint probability mass function

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} p(x, y) = 1 \,|x \neq y \tag{2}$$

$$\sum_{y=0}^{\infty} \sum_{x=0}^{\infty} \frac{c}{2^{x+y+2}} - \sum_{x=y} \frac{c}{2^{x+y+2}} = 1 \tag{3}$$

$$\sum_{y=0}^{\infty} \frac{c}{2^{y+2}} \sum_{x=0}^{\infty} 2^{-x} - \frac{c}{4} \sum_{x=0}^{\infty} \frac{1}{4^x} = 1 \tag{4}$$

$$\sum_{y=0}^{\infty} \frac{2c}{2^{y+2}} - \frac{c}{3} = 1 \tag{5}$$

$$\frac{2c}{4} \sum_{y=0}^{\infty} 2^{-y} - \frac{c}{3} = 1 \tag{6}$$

$$\frac{4c}{4} \frac{c}{3} = 1 \tag{7}$$

$$c - \frac{c}{3} = 1 \tag{8}$$

$$c = \frac{3}{2} \tag{9}$$

Marginal probability mass function of X

$$p_X(x) = \sum_{y=0}^{\infty} p(x, y) \tag{10}$$

$$= \sum_{y=0}^{\infty} \frac{3}{2^{x+y+3}} \tag{11}$$

$$= \frac{3}{2^{x+3}} \sum_{y=0}^{\infty} 2^{-y} \tag{12}$$

$$= \frac{3}{2^{x+2}} \tag{13}$$

Similary, Marginal probability mass function of Y

$$p_Y(y) = \sum_{x=0}^{\infty} p(x, y) \tag{14}$$

$$= \sum_{x=0}^{\infty} \frac{3}{2^{x+y+3}} \tag{15}$$

$$= \frac{3}{2^{y+3}} \sum_{x=0}^{\infty} 2^{-x} \tag{16}$$

$$= \frac{3}{2^{y+2}} \tag{17}$$

$$x \neq y \tag{18}$$

When x takes a particular integer, y cannot take the same number and vice-versa
Option (4) is incorrect.
$\therefore$ Only option (3) is correct.