

ASSIGNMENT 1

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1.5.11 Obtain m, n, p in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution:

Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1)$$

Length of sides:

$$a = \|(\mathbf{C} - \mathbf{B})\| \quad (2)$$

$$= \sqrt{(1 \quad -11) \begin{pmatrix} 1 \\ -11 \end{pmatrix}} \quad (3)$$

$$= \sqrt{122} \quad (4)$$

$$b = \|(\mathbf{A} - \mathbf{C})\| \quad (5)$$

$$= \sqrt{(4 \quad 4) \begin{pmatrix} 4 \\ 4 \end{pmatrix}} \quad (6)$$

$$= \sqrt{32} \quad (7)$$

$$c = \|(\mathbf{B} - \mathbf{A})\| \quad (8)$$

$$= \sqrt{(-5 \quad 7) \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \quad (9)$$

$$= \sqrt{74} \quad (10)$$

$\mathbf{D}_3, \mathbf{E}_3, \mathbf{F}_3$ are the points of the incircle with the sides BC, CA, AB respectively.

$$AE_3 = AF_3 = m, BD_3 = BF_3 = n, CD_3 = CE_3 = p.$$

We know that,

$$AB = AF_3 + F_3B \quad (11)$$

$$BC = BD_3 + D_3C \quad (12)$$

$$CA = CE_3 + E_3A \quad (13)$$

$$(14)$$

$$\therefore c = m + n \quad (15)$$

$$a = n + p \quad (16)$$

$$b = m + p \quad (17)$$

Equations (19), (20), (21) can be written in the following manner:

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (18)$$

On swapping R_1 and R_3 , we get:

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} c \\ b \\ a \end{pmatrix} \quad (19)$$

Solving using Gaussian elimination method:

Augmented matrix:

$$\begin{pmatrix} 1 & 1 & 0 & c \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \quad (20)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2 + R_3} \begin{pmatrix} 2 & 2 & 2 & a+b+c \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \quad (21)$$

$$\xleftrightarrow{R_1 \leftarrow R_1/2} \begin{pmatrix} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \quad (22)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{-a+b+c}{2} \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \quad (23)$$

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$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & 0 & \frac{-a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{pmatrix} \quad (24)$$

From the above augmented matrix we get,

$$m = \frac{-a + b + c}{2} \quad (25)$$

$$n = \frac{a - b + c}{2} \quad (26)$$

$$p = \frac{a + b - c}{2} \quad (27)$$

Substituting $a = \sqrt{122}, b = \sqrt{32}, c = \sqrt{74}$, we get:

$$m = \frac{-\sqrt{122} + \sqrt{32} + \sqrt{74}}{2} \quad (28)$$

$$n = \frac{\sqrt{122} - \sqrt{32} + \sqrt{74}}{2} \quad (29)$$

$$p = \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \quad (30)$$