ASSIGNMENT 1

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1.5.11 Obtain m, n, p in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution :

Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

Length of sides:

$$a = \|(\mathbf{C} - \mathbf{B})\| \tag{2}$$

$$= \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix}} \tag{3}$$

$$= \sqrt{122} \tag{4}$$

$$b = \|(\mathbf{A} - \mathbf{C})\| \tag{5}$$

$$= \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}} \tag{6}$$

$$= \sqrt{32} \tag{7}$$

$$c = \|(\mathbf{B} - \mathbf{A})\| \tag{8}$$

$$= \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \tag{9}$$

$$= \sqrt{74} \tag{10}$$

$\mathbf{D_3}, \mathbf{E_3}, \mathbf{F_3}$ are the points of the incircle with the sides BC, CA, AB respectively.

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$AE_3 = AF_3$=m , $BD_3 = BF_3$=n , $CD_3 = CE_3$=p. We know that,

$$AB = AF_3 + F_3B \tag{11}$$

$$BC = BD_3 + D_3C \tag{12}$$

$$CA = CE_3 + E_3A \tag{13}$$

$$\therefore c = m + n \tag{14}$$

$$a = n + p \tag{15}$$

$$b = m + p \tag{16}$$

Equations (19), (20), (21) can be written in the following manner:

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{17}$$

On swapping R_1 and R_3, we get:

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} c \\ b \\ a \end{pmatrix} \tag{18}$$

Solving using Gaussian elimination method:
Augmented matrix:

$$\begin{pmatrix} 1 & 1 & 0 & c \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \tag{19}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2 + R_3} \begin{pmatrix} 2 & 2 & 2 & a+b+c \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \tag{20}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/2} \begin{pmatrix} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \tag{21}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{-a+b+c}{2} \\ 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \end{pmatrix} \tag{22}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & 0 & \frac{-a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{pmatrix} \tag{23}$$

From the above augmented matrix we get,

$$m = \frac{-a + b + c}{2} \tag{24}$$

$$n = \frac{a - b + c}{2} \tag{25}$$

$$p = \frac{a + b - c}{2} \tag{26}$$

Substituting $a = \sqrt{122}, b = \sqrt{32}, c = \sqrt{74}$, we get:

$$m = \frac{-\sqrt{122} + \sqrt{32} + \sqrt{74}}{2} \tag{27}$$

$$n = \frac{\sqrt{122} - \sqrt{32} + \sqrt{74}}{2} \tag{28}$$

$$p = \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \tag{29}$$