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EE22BTECH11059

Let (X, Y) have joint probability mass function

1) Marginal probability mass function of X

$$p(x, y) = \begin{cases} \frac{c}{2^{x+y+2}} & \text{if } x = 0, 1, 2, \dots, y = 0, 1, 2, \dots; x \neq y \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Then which of the following is true?
(GATE ST 2023)

- 1) $c = \frac{1}{2}$
- 2) $c = \frac{1}{4}$
- 3) $c > 1$
- 4) X and Y are independent

Solution: For $p(x, y)$ to be joint probability mass function

$$\sum_{y=-\infty}^{\infty} \sum_{x=-\infty}^{\infty} p(x, y) = 1 \quad |x \neq y \quad (2)$$

$$\sum_{y=0}^{\infty} \sum_{x=0}^{\infty} \frac{c}{2^{x+y+2}} - \sum_{x=y}^{\infty} \frac{c}{2^{x+y+2}} = 1$$

$$\sum_{y=0}^{\infty} \frac{c}{2^{y+2}} \sum_{x=0}^{\infty} 2^{-x} - \frac{c}{4} \sum_{x=0}^{\infty} \frac{1}{4^x} = 1$$

$$\sum_{y=0}^{\infty} \frac{2c}{2^{y+2}} - \frac{c}{3} = 1$$

$$\frac{2c}{4} \sum_{y=0}^{\infty} 2^{-y} - \frac{c}{3} = 1$$

$$c - \frac{c}{3} = 1$$

$$c = \frac{3}{2}$$

$$p_X(x) = \sum_{y=0}^{\infty} p(x, y) \quad |x \neq y \quad (9)$$

$$= \sum_{y=0}^{\infty} \frac{3}{2^{x+y+3}} - p_{XY}(x, x) \quad (10)$$

$$= \frac{3}{2^{x+3}} \sum_{y=0}^{\infty} 2^{-y} - \frac{3}{2^{2x+3}} \quad (11)$$

$$= \frac{3}{2^{x+2}} - \frac{3}{2^{2x+3}} \quad (12)$$

2) Marginal probability mass function of Y

$$p_Y(y) = \sum_{x=0}^{\infty} p(x, y) \quad |x \neq y \quad (13)$$

$$= \sum_{x=0}^{\infty} \frac{3}{2^{x+y+3}} - p_{XY}(y, y) \quad (14)$$

$$= \frac{3}{2^{y+3}} \sum_{x=0}^{\infty} 2^{-x} - \frac{3}{2^{2y+3}} \quad (15)$$

$$= \frac{3}{2^{y+2}} - \frac{3}{2^{2y+3}} \quad (16)$$

(3) For X and Y to be independent,

$$p(x, y) = p_X(x) p_Y(y) \quad (17)$$

$$p_X(x) p_Y(y) = \left(\frac{3}{2^{x+2}} - \frac{3}{2^{2x+3}} \right) \left(\frac{3}{2^{y+2}} - \frac{3}{2^{2y+3}} \right) \quad (18)$$

$$p(x, y) \neq p_X(x) p_Y(y) \quad (19)$$

Option (4) is incorrect.

 \therefore Only option (3) is correct.

Simulations:

(8) 1) Marginal pmf of X

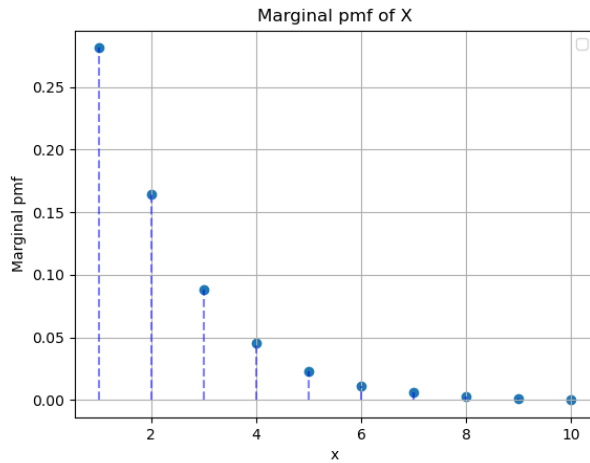


Fig. 1. Marginal pmf

Steps of simulation:

- 1) Define function p as $P(x, y) = \frac{3}{2^{x+y+3}}$ in C and run a loop to sum the function p for all values of y except $x \neq y$, for each value of x .
- 2) Store values of marginal pmf for each value of x in a dat file
- 3) Using `plt.scatter` function of python, plot the graph of Marginal pmf of X vs X. The same can be used to plot marginal pmf of Y