Journal of Physics: Conference Series

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# A Review of the Development and Applications of Number Theory

To cite this article: Keyu Yan 2019 *J. Phys.: Conf. Ser.* **1325** 012128

Journal of Physics: Conference Series **1325** (2019) 012128 doi:10.1088/1742-6596/1325/1/012128

# A Review of the Development and Applications of Number Theory

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**Abstract.** This paper mainly studies the development and applications of number theory, aiming to review the history of this discipline, and explore its influence on production and our life and its applications. Number theory is devoted originally to the study of the integers. With the contributions made by mathematicians in different ages to advancing the study of the integers, the basic system of number theory has been gradually improved, and thereby a complete and unified discipline has been formed. As a basic discipline, number theory spreads profound impacts on other disciplines, and is the foundation of many disciplines.

## 1. An Overview of Number Theory

### *1.1 The Concept of Number Theory*
Just as the name implies, number theory is a theory focusing on numbers. More than 3,000 years ago, the concept of number and arithmetic has occurred. The term for number theory was arithmetic in the early period, and was superseded by "number theory" in the early twentieth century. Number theory is a branch of mathematics. Mathematics is the foundation of many science and engineering disciplines, while number theory is the foundation of mathematics.

Number theory primarily deals with the nature of integers. Questions in number theory are concise, and the key to solve these questions is unique factor decomposition. In addition, in the process of reconstructing the unique factorization, some new concepts, such as complex integers, ideal numbers and ideals, are introduced, which also provide new research methods for number theory.

### *1.2 The Subdivisions of Number Theory*
In terms of the field, number theory can be divided into many different subdivisions, of which elementary number theory, algebraic number theory, geometric number theory, and analytic number theory are more important ones. It also includes some popular number theory subdivisions, such as transcendental number theory and combinatorial number theory. The focuses of these subdivisions and their differences are shown in the following table:

Table 1 The focuses of these subdivisions and their differences

| Subdivision | Explanation |
|---|---|
| Elementary number theory | Elementary number theory is a branch of number theory based on elementary method. In essence, it applies divisible property to mainly study divisible theory and congruence theory. The typical conclusions in this theory include the familiar congruence theorem, Euler's theorem, Chinese residual theorem and so on. |

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| | |
|---|---|
| Analytic number theory | Analytic number theory studies the integers with calculus and complex analysis. Some analytic functions, such as the Riemann function ζ which studies the properties of integers and primes, can also be employed to understand number theory. |
| Algebraic number theory | Algebraic number theory is more inclined to study the nature of various rings of integers from the perspective of algebraic structure. |
| Geometric number theory | Geometric number theory studies the distribution of the integers from the perspective of geometry. |
| Computational number theory | Computational number theory studies questions in number theory with computer algorithms. |

### *1.3 The Significance of Number Theory*

[1]For a long time, number theory only demonstrated the basic properties of mathematics, so it was classified as a discipline without direct application value. With the great and profound scientific and technological transformation brought by the emergence and development of computer, number theory has been widely used, and is no longer just a pure mathematics, but a mathematical discipline with practical application value. At present, number theory is widely and fully applied in many fields, such as computing, cryptography, physics, chemistry, biology, acoustics, electronics, communication, graphics and even musicology.

This also proves the significance of number theory that it can be widely and fully applied to many other fields involving mathematics, and has developed into a new applied mathematics discipline--applied number theory. Therefore, number theory is no longer just a pure discipline, but a veritable applied discipline. Judged from the current development trend and applications of number theory, this ancient discipline is bound to be vigorous.

## 2. The Development of Number Theory

### *2.1 The Development of Number Theory and Algebra*

[2]Many questions in number theory have been proposed and then solved, which attracts more and more people to focus on number theory. In the long history, techniques and methods to solve problems have emerged, and some theories have been formed. Algebraic number theory has been advanced with the expansion of number field and practical applications. Bacon, the famous philosopher, said that history makes people smart, so it is necessary to explore the development of early algebraic number theory. Domestic researches on algebraic number theory are mainly comprehensive discussions on the progress of algebraic number theory. Based on the collection and collation of the relevant data, this paper is devoted to investigating the birth of algebraic number theory by analyzing the key problems in the development of two higher reciprocity laws and Fermat's theorem. With a new perspective to observe the history, this paper strives to make a more comprehensive analysis and profound thinking.

1) The stage of Arithmetic: during the period from about 3,800 to the third century, arithmetic symbols were not uniform, and algebra was separated from geometry. The ancient Greeks made the greatest contribution to number theory, including some renowned achievements, such as Euclid’s Euclidean algorithm in geometry which proposed that the number of prime numbers is infinite, and the fundamental theorem of arithmetic which was involved in elementary number theory.

2) The complete stage of number and equation theory: during the period from the 7th century to the 16th century, irrational and imaginary numbers were discovered.

a) The discovery of irrational numbers: Hipparsos of the Pythagorean school discovered the first irrational number, shocking the leaders of the school at that time. He proposed that all numbers could be expressed as the ratio of integers, which led to the first mathematical crisis.

b) Creation of arithmetic operators and solution to irrational equations: In India, the mathematician Brahmagupta introduced a group of symbols used to express concepts and describe operations in the 7th century, and Posgallo later put forward the concept of negative square root, the solution to irrational equations and the algorithm of irrational numbers in the 12th century, which fostered the study of algebra to a new stage.

c) Establishment of imaginary number theory: in the book The Great Art published in 1545 by the Milanese scholar Cardano (1501-1556), the general solution to the cubic equation was unveiled, which was known as Cardano's formula later. Cardano was the first mathematician to formulate the square root of a negative number.

3) The stage of linear algebra: during the period from the 17th century to the 19th century, the tools for solving linear problems, matrices, determinants, and vectors emerged, which provided services to the industrial society.

4) The stage of abstract algebra: during the period from the 19th century to the present, the importance of form and technique to the algebra structure was highlighted, which offered services to the information society.

## 3. The Classical Questions and Conjectures in Number Theory

### *3.1 Mersenne Prime*

Mersenne primes are derived from Mersenne numbers which refer to the positive integers of the form 2p-1, where if the exponent p is prime, p is usually defined as Mp. If a Mersenne number is prime, it is called a Mersenne prime; otherwise it is called a Mersenne number.

Prime numbers, also known as primes, refer to the numbers which are divisible only by 1 and themselves, such as 2, 3, 5 and so on. Euclid has proved that the number of primes is infinite with proofs by contradiction.

In the infinite sequence 2n-1, Mersenne numbers and Mersenne primes only account for a small proportion, but Mersenne primes are infinite.

If the exponent n is prime, Mn is a prime number. However, when n is prime, Mp may not be prime (for example, M2=4-1=3 and M3=8-1=7 are prime, while M11=2047=23*89 is not a prime number). For the time being, 51 prime numbers have been discovered, of which the largest one is M82589933 with 24862048 digits. Nowadays, distributed network computing technology has become the latest method to discover primes.

### *3.2 Goldbach Conjecture*

Goldbach conjecture is one of the oldest unsolved problems in number theory. It stated that every even integer greater than two can be written as the sum of two primes. Goldbach conjecture is associated with integer partition which is proposed by European number theorists at that time and focused on the question--“can you analyze integers as the sum of certain numbers with certain properties?” To be specific, the question is whether you can divide all integers into the sum of several complete squares, or the sum of several complete cubes. Such a partition of a given even number into the sum of two prime numbers is called Goldbach analysis. Goldbach conjecture took a long time to develop. China’s mathematician Chen Jingrun proved that every sufficiently large even number can be written as the sum of some prime number and another number which is the product of two primes.

Based on Goldbach conjecture of even numbers, the conjecture that every odd integers greater than 7 can be written as the sum of three primes has been proposed, which is called the weak Goldbach conjecture. It has been proved in 2013.

### *3.3 Fibonacci Sequence*

Fibonacci sequence, defined by Italian mathematician Leonardo Fibonacci, refers to a series of numbers in which beginning from the third number in the sequence, each number is the sum of the two preceding ones. The nth number in the sequence can be denoted by f(n), and its recursive sequence can be expressed as the following formula.

$$\mathrm{f(n)} = \mathrm{f(n-1)} + \mathrm{f(n-2)}$$

Applications of Fibonacci Sequence:

1) Golden ratio: as the number of items in the sequence increases, the ratio of the former to the latter increases closer to the golden ratio.

2) Pascal triangle: the numbers on diagonals of Pascal triangle add to the Fibonacci sequence.

3) Area of a rectangle:the squares of the first few numbers in the Fibonacci sequence are treated as different small quadrilateral areas, and they can be combined into large quadrilateral areas.

### *3.4 The Significance of Mathematical Conjectures*

Apart from the above conjectures, there are many other conjectures. Most of the mathematical conjectures are based on observation, verification, induction and generalization of a large number of facts. Such a method of abstracting the general and common properties from the special properties is an important driver of mathematical research. The expression and research of mathematical conjectures vividly reflect the application of dialectics in mathematics. Moreover, mathematical conjectures promote the study of mathematical methodology.

Furthermore, mathematical conjectures often play the role as the important indicator of mathematical development. Fermat conjecture gave birth to algebraic number theory, while Goldbach conjecture promoted the development of screening methods. Riemann conjecture proved the prime number theorem, while the Four-color conjecture was solved by computer, and thereby a new era of machine verification has been opened. Therefore, mathematical conjectures are not only the precious gemstones, but also the key driver in the development of mathematics.

## 4. Applications of Number Theory

### *4.1 Cryptography*

With the development of network encryption technology, number theory has found its own place--cryptography. Professor Wang Xiaoyun who cracked the MD5 code a few years ago is from the number theory school of Shandong University. Because of the irregular appearance of prime factors in composite numbers, it is very difficult to decompose composite numbers into the product of prime numbers. At the same time, it is this difficulty that enlightens people to use it to design difficult codes.

When studying number theory, especially cryptography, we pursue deterministic algorithm rather than probabilistic algorithm, and we will only lower our requirements and apply probabilistic algorithm if there is no deterministic algorithm.

### *4.2 Computer Animation*

Linear transformation is usually used to make images, and computer graphics are to build graphics on display devices through algorithms and programs, so linear transformation technology can be used to make computer animation. Computer graphics mainly consist of image representation, storage and computation. With the improvement of software capabilities, linear transformation technology is commonly used in computer animation.

### *4.3 Machine Translation*

The main algorithm of machine translation is based on the statistical method, with the accuracy of 90%. In addition, this algorithm is also used in image search technology. The core of this method is that the language units of source language and target language can be represented by vectors, and the lexical vectors of different languages can be projected onto a two-dimensional plan for analysis. Experimental results show that the lexical vectors of different languages do have some relations similar to linear relations, so it is of significance to classify machine translation as a linear transformation.

### *4.4 Other Basic Fields*

Number theory also plays a surprising role in other theories. In quantum theory, Hermite operator is one of the most basic concepts. Apart from that, number theory is also widely used in non-mathematical disciplines, such as information science, theoretical physics, quantum chemistry, and so on.

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## 5. Conclusion and Prospects

This paper mainly discusses the basic concept, theory, development process and applications of number theory. As a foundation of science and engineering disciplines, the development trend and level of mathematics has a profound influence on other disciplines. By reviewing the development of number theory and its applications, this paper aims to help readers acquire the origin and development of number theory, and its future trend in the combination of computer science. In today's society, with the rapid development of computer field, number theory or even mathematical discipline will make greater strides in the future.

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