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# Towards Learned Clauses Database Reduction Strategies Based on Dominance Relationship

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## Abstract

Clause Learning is one of the more important components of conflict driven clause learning (CDCL) SAT solver that is effective on industrial instances. Since the number of learned clauses is proved to be exponential in the worse case, it is necessarily to identify the most relevant clauses to maintain and delete the irrelevant ones. As reported in the literature, several learned clauses deletion strategies have been proposed. However the diversity in both the number of clauses to be removed at each step of reduction and the results obtained with each strategy creates confusion to determine which criterion is better. Thus, the problem to select which learned clauses are to be removed during the search step remains very challenging. In this paper, we propose a novel approach to identify the most relevant learned clauses without favoring or excluding any of the proposed measures, but by adopting the notion of dominance relationship among those measures. Our approach bypasses the problem of the diversity of results and reaches to a compromise between the assessments of these measures. Furthermore, the proposed approach also avoids another non-trivial problem which is the amount of clauses to be deleted at each reduction of the learned clause database.

## 1 Introduction

The SAT problem, i.e., the problem of checking whether a Boolean formula in conjunctive normal form (CNF) is satisfiable or not, is central to many domains in computer science and artificial intelligence including constraint satisfaction problems (CSP), automated planning, non-monotonic reasoning, VLSI correctness checking, etc. Today, SAT has gained a considerable audience with the advent of a new generation of solvers able to solve large instances encoding real-world problems. These solvers, often called *modern SAT solvers* [Moskewicz *et al.*, 2001; Eén and Sörensson, 2003] or CDCL (Conflict Driven Clause Learning) SAT solvers have been shown to be very efficient at solving real-world SAT instances. They are built by integrating four major components to the classical(DPLL) procedure

[Davis *et al.*, 1962]: lazy data structures [Moskewicz *et al.*, 2001], activity-based variable selection heuristics (VSIDS-like) [Moskewicz *et al.*, 2001], restart policies [Gomes *et al.*, 1998], and clause learning [Silva and Sakallah, 1999; Moskewicz *et al.*, 2001]. Although a nice combination of these components contributes to improve the efficiency of modern SAT solvers [Katebi *et al.*, 2011], clause learning remains known as the most important component [Pipatsrisawat and Darwiche, 2009]. The global idea of clause learning is that during the unit propagation process, when a current branch of the search tree leads to a conflict, moderns SAT solvers learn a conflict clause that helps unit propagation to discover one of the implications missed at an earlier level. This conflict clause expresses the causes of the conflict and is used to prune the search space. Clause learning, also known in the literature as Conflict Driven Clause Learning (CDCL), refers now to the most known and used First (UIP) learning scheme, first integrated in the SAT solver Grasp [Silva and Sakallah, 1996] and efficiently implemented in zChaff [Moskewicz *et al.*, 2001]. Most of the SAT solvers integrate this strong learning scheme. Since at each conflict, CDCL solvers learn a new clause that is added to the learned clauses database, and the number of learned clauses is proved to be exponential in the worse case, it is necessary to remove some learned clauses to maintain a database of polynomial size. Therefore, remove too many clauses can make learning inefficient, and keeping too many clauses also can alter the efficiency of unit propagation.

Managing the learned clauses database was the subject of several studies [Moskewicz *et al.*, 2001; Silva and Sakallah, 1996; Eén and Sörensson, 2003; Audemard and Simon, 2009; Audemard *et al.*, 2011; Guo *et al.*, 2014]. These strategies were proposed with the objective to maintain a learned clause database of reasonable size by eliminating clauses deemed irrelevant to the subsequent search. The general principle of these strategies is that, at each conflict, an activity is associated to the learned clauses (static strategy). Such heuristic-based activity aims to weight each clause according to its relevance to the search process. In the case of dynamic strategies, such clauses activities are dynamically updated. The reduction of the learned clauses database consists in eliminating inactive or irrelevant clauses. Although all the learned clause deletion strategies proposed in the literature are shown to be empirically efficient, identifying the most relevant clause

to maintain during the search process remains a challenging task. Our motivation in this work comes from the observation that the use of different relevant-based deletion strategies gives different performances. Our goal is to take advantage of several relevant learned clauses deletion strategies by seeking a compromise between them through a dominance relationship.

In this paper, we integrate a user-preference point of view in the SAT process. To this end, we integrate into the SAT process the idea of skyline queries [Börzsönyi *et al.*, 2001], dominant patterns [Soulet *et al.*, 2011], undominated association rules [Bouker *et al.*, 2014] in order to learn clauses in a threshold-free manner. Such queries have attracted considerable attention due to their importance in multi-criteria decision making. Given a set of clauses, the skyline set contains the clauses that are not dominated by any other clause.

Skyline processing does not require any threshold selection function, and the formal property of domination satisfied by the skyline clauses gives to the clauses a global interest with semantics easily understood by the user. This skyline notion has been developed for database and data mining applications, however it was unused for SAT purposes. In this paper, we adapt this notion to the learned clauses management process.

The paper is organized as follows. We first present some effective relevant-based learned clauses deletion strategies used in the literature. Then, our learned clauses deletion strategy based on the dominance relationship between different strategies is presented in section 3. Finally, before the conclusion, experimental results demonstrating the efficiency of our approach are presented.

## 2 On the learned clauses database management strategies

In this section, we present some efficient learned clauses relevance measures exploited in the most SAT solvers of the literature.

The most popular CDCL SAT solver *Minisat* [Eén and Sörensson, 2003] considers as relevant the clauses the most involved in recent conflict analysis and removes the learned clauses whose the involvement in recent conflict analysis is marginal. An other strategy called *LBD* for *Literal Block Distance* was proposed in [Audemard and Simon, 2009]. *LBD* based measure is also exploited by most of the best state-of-the-art SAT solver (*Glucose*, *Lingeling* [Biere, 2012]) and whose the efficiency has been proved empirically. *LBD* based measure uses the number of different levels involved in a given learned clause to quantify the quality of the learned clauses. Hence, the clauses with smaller *LBD* are considered as more relevant. In [Audemard *et al.*, 2011], a new dynamic management policy of the learned clauses database is proposed. It is based on a dynamic freezing and activation principle of the learned clauses. At a given search state, using a relevant selection function based on progress saving (PSM), it activates the most promising learned clauses while freezing irrelevant ones. In [Guo *et al.*, 2014], a new criterion to quantify the relevance of a clause using its backtrack level called *BTL* for *BackTrack Level* based clause was

proposed. From experiments, the authors observed that the learned clauses with small *BTL* values are used more often in the unit propagation process than those with higher *BTL* values. More precisely, the authors observed that the learned clauses with *BTL* value less than 3 are always used much more than the remaining clauses. Starting from this observation, and motivated by the fact that a learned clause with smaller *BTL* contains more literals from the top of the search tree, the authors deduce that relevant clauses are those allowing a higher backtracking in the search tree (having small *BTL* value). More recently, several other learned clauses database strategies were proposed in [Jabbour *et al.*, 2014; Ansótegui *et al.*, 2015]. In [Jabbour *et al.*, 2014], the authors explore a number of variations of learned clause database reduction strategies, and the performance of the different extensions of *Minisat* solver integrating their strategies is evaluated on the instances of the SAT competitions 2013/2014 and compared against other state-of-the-art SAT solvers (*Glucose*, *Lingeling*) as well as against default *Minisat*. From the performances obtained in [Jabbour *et al.*, 2014], the authors have shown that size-bounded learning strategies proposed more than fifteen years ago [Silva and Sakallah, 1996; Bayardo and Miranker, 1996; Bayardo, Jr. and Schrag, 1997] is not over and remains a good measure to predict the quality of learned clauses. They show that adding randomization to size bounded learning is a nice way to achieve controlled diversification, allows to favor the short clauses, while maintaining a small fraction of large clauses necessary for deriving resolution proofs on some SAT instances. This study opens many discussions about the learned clauses database strategies and raises questions about the effectiveness proclaimed by other strategies of the state-of-the-art [Eén and Sörensson, 2003; Audemard and Simon, 2009]. In [Ansótegui *et al.*, 2015], the authors use the community structure of industrial SAT instances to identify a set of highly useful learned clauses. They show that augmenting a SAT instance with the clauses learned by the solver during its execution does not always mean to make the instance easy. However, the authors show that augmenting the formula with a set of clauses based on the community structure of the formula improves the performance of the solver in many cases. The different performances obtained by each strategy suggests that the question on how to predict efficiently the "best" learned clauses is still open and deserves further investigation.

On the other hand, it is important to note that the efficiency of most of these state-of-the-art learned clauses management strategies heavily depends on the cleaning frequency and on the amount of clauses to be deleted each time. Generally, all the CDCL SAT solvers using these strategies exactly delete half of the learned clauses at each learned clauses database reduction step. For example, the CDCL SAT solver *Minisat* [Eén and Sörensson, 2003] and *Glucose* [Audemard and Simon, 2009] delete half of the learned clauses at each cleaning. Therefore, the efficiency of this amount of learned clauses to delete (e.g the half) at each cleaning step of the learned clauses database has not been demonstrated theoretically, but instead experimentally. For our knowledge, there are not many studies in the literature on how to determine the amount of clauses to be deleted each time. This paper

proposes an approach to identify the relevant learned clauses during the resolution process without favor any of the best reported relevant measures and which frees itself of the amount of clauses to be removed at each time: the amount of learned clauses to delete corresponds at each time to the number of learned clauses dominated by one particular learned clause of the set of the current learned clauses which is called in the following sections, the reference learned clause.

### 3 Detecting undominated learned Clauses

We present now our learned clauses relevant measure based on dominance relationship. We first motivate this approach with a simple example, and then propose an algorithm allowing to identify the relevant clauses with some technical details.

#### 3.1 Motivating example

Let us consider the following relevant strategies: *LBD* [Audemard and Simon, 2009], *SIZE* (which consider as relevant the clause of the short size) and the relevant measure use by *minisat* [Eén and Sörensson, 2003] that we denote here *CVSIDS*. Suppose that we have in the learned clauses database, the clauses  $c_1$ ,  $c_2$  and  $c_3$  with:

- $SIZE(c_1) = 8$ ,  $LBD(c_1) = 3$ ,  $CVSIDS(c_1) = 1e^{100}$ ,
- $SIZE(c_2) = 6$ ,  $LBD(c_2) = 5$ ,  $CVSIDS(c_2) = 1e^{200}$ ,
- $SIZE(c_3) = 5$ ,  $LBD(c_3) = 4$ ,  $CVSIDS(c_3) = 1e^{300}$ .

The question we ask is the following: which one is relevant? In [Audemard and Simon, 2009], the authors consider the clause  $c_1$  which has the most small LBD measure as the most relevant. In contrast, the authors of [Jabbour et al., 2014] and [Goldberg and Novikov, 2007] prefer the clause  $c_3$  while the preference of the authors of *Minisat* [Eén and Sörensson, 2003] leads to the clause  $c_3$ . Our approach copes with the particular preference at one measure by finding a compromise between the different relevant measures through the dominance relationship. Hence, for the situation described above, only the clause  $c_2$  is irrelevant because it is dominated by the clause  $c_3$  on the three given measures.

#### 3.2 Formalization

During the search process, the CDCL SAT solvers learn a set of clauses which are stored in the learned clauses database  $\Delta$ ,  $\Delta = \{c_1, c_2, \dots, c_n\}$ . At each cleaning step, we evaluate these clauses with respect to a set  $\mathcal{M} = \{m_1, m_2, \dots, m_k\}$  of relevant measures. We denote  $m(c)$  the value of the measure  $m$  for the clause  $c$ ,  $c \in \Delta$ ,  $m \in \mathcal{M}$ . Since the evaluation of learned clauses varies from a measure to another one, using several measures could lead to different outputs (relevant clauses with respect to a measure). For example, if we consider the motivating example,  $c_1$  is the best clause with respect to the *LBD* measure whereas it is not the case according to the evaluation of *SIZE* measure which favors  $c_3$ . This difference of evaluations is confusing for any process of learned clauses selection. Hence, we can utilize the notion of dominance between learned clauses to address the selection of relevant ones. Before, formulating the dominance relationship between learned clauses, we need to define it at the level

of measure values. To do that, we define dominance value as follows:

**Definition 1 (dominance value)** *Given a learned clauses relevant measure  $m$  and two learned clauses  $c$  and  $c'$ , we say that  $m(c)$  domine  $m(c')$ , denoted by  $m(c) \succeq m(c')$ , iff  $m(c)$  is preferred to  $m(c')$ . If  $m(c) \succeq m(c')$  and  $m(c) \neq m(c')$  then we say that  $m(c)$  strictly dominates  $m(c')$ , denoted  $m(c) \succ m(c')$ .*

**Definition 2 (dominance clause)** *Given two learned clauses  $c, c'$ , the dominance relationship according to the set of learned clauses relevant measures  $\mathcal{M}$  is defined as follows:*

- $c$  dominates  $c'$ , denoted  $c \succeq c'$ , iff  $m(c) \succeq m(c')$ ,  $\forall m \in \mathcal{M}$ .
- If  $c$  dominates  $c'$  and  $\exists m \in \mathcal{M}$  such that  $m(c) \succ m(c')$ , then  $c$  strictly dominates  $c'$  and we note  $c \succ c'$ .

To discover the relevant learned clauses a naive approach consists in comparing each clause with all other ones. However, the number of learned clauses is proved to be exponential which makes pairwise comparisons costly. In the following, we show how to overcome this problem by defining at each cleaning step of learned clauses database, a particular learned clause denoted by  $\tau$  that we call here *current reference learned clause* which is an undominated clause of  $\Delta$  according to the set of learned clauses relevant measures  $\mathcal{M}$ . At each cleaning step, all the learned clauses dominated by  $\tau$  will be considered as the irrelevant learned clauses and thus deleted from the learned clauses database.

To define *current reference learned clause*, we need a new relevant measure based on all the learned clauses relevant measures of  $\mathcal{M}$ . We call this new measure *Degree of compromise*, in short *DegComp* defines as follows:

**Definition 3 (Degree of compromise)** *Given a learned clause  $c$ , the degree of compromise of  $c$  with respect to the set of learned clauses relevant measures  $\mathcal{M}$  is defined by  $DegComp(c) = \frac{\sum_{i=1}^n \widehat{m_i}(c)}{|\mathcal{M}|}$ , where  $\widehat{m_i}(c)$  corresponds to the normalized value of the clause  $c$  on the measure  $m_i$ .*

In fact, in practice, measures are heterogeneous and defined within different scales. For example the values of the learned clauses relevant measures in [Eén and Sörensson, 2003] are very high, in exponential order while the values of the relevant measures in [Audemard and Simon, 2009] are smallest ones. Hence, in order to avoid that the measures with the higher values make marginal the measures with smallest values in the computation of the comprise degree of a given learned clauses, it is recommend to normalize the measures values. In our case here, we choose to normalize all the measures in the interval  $[0, 1]$ . More precisely, each value of measure  $m(c)$  of any learned clause  $c$  must be normalized into  $\widehat{m(c)}$  within  $[0, 1]$ . The normalization of a given measure  $m$  is performed depending on its domain and the statistical distribution of its active domain. We recall that the active domain of a measure  $m$  is the set of its possible values. It is worth mentioning, **the normalization of a measure does not modify the dominance relationship** between two given values. If we consider the learned clause  $c_1$  given



in the motivating example in the section 3.1, with its three values :  $DegComp(c_1) = \frac{CVSIDS(c_1) + LBD(c_1) + SIZE(c_1)}{3}$ , then, we have,  $DegComp(c_1) = \frac{\frac{1}{1e100} + \frac{3}{nVars()} + \frac{8}{nVars()}}{3}$ , with  $nVars()$  the number of variables of the Boolean formula.

After giving the necessary definitions (current reference learned clause and Degree of compromise), the following lemma offers a swifter solution rather than pairwise comparisons, to find relevant clauses based on dominance relationship.

**Lemma 1** *Let  $c$  be a learned clause having the minimal degree of compromise with respect to the set of learned clauses relevant measures  $\mathcal{M}$ , then  $c$  is an undominated clause.*

**Proof 1** *Let  $c$  be a learned clause having the minimal degree of compromise with respect to the set of learned clauses relevant measures  $\mathcal{M}$ , we suppose that there exists a learned clause  $c'$  that strictly dominates  $c$ , which means that  $\forall m \in \mathcal{M}$ ,  $m(c') \succeq m(c)$  and  $\exists m' \in \mathcal{M}$ ,  $m'(c') \succ m'(c)$ . Hence, we have  $DegComp(c') < DegComp(c)$ . The latter inequality contradicts our hypothesis, since  $c$  has the minimal degree of compromise with respect to  $\mathcal{M}$ .*

**Property 1** *Let  $\mathcal{M}$  the set of learned clauses relevant measures,  $\forall c, c', c''$  three learned clauses, if  $c \succ c'$  and  $c' \succ c''$  then  $c \succ c''$ .*

During the search process, at each cleaning step of the learned clauses database, we first find the learned clause  $cMin$  having the minimal degree of compromise with respect to  $\mathcal{M}$ . Then, we delete from the learned clauses database all the clauses dominated by  $cMin$ .

Searching for all undominated clauses during each cleaning step can be time consuming, such that we only compute the undominated clauses with respect to the reference learned clause during each reduction step.

### 3.3 Algorithm

In this section, after presenting the general scheme of a deletion strategy of learned clauses ( $reduceDB(\Delta)$ ) adopted by most of the reported solvers, we propose an algorithm allowing to discover relevant learned clauses by using dominance relationship.

Algorithm 1 depicts the general scheme of a learned clause deletion strategy ( $reduceDB(\Delta)$ ). This algorithm first sorts the set of learned clauses according to the defined criterion and then deletes half of the learned clauses. In fact, this algorithm take a learned clauses database of size  $n$  and outputs a learned clauses database of size  $n/2$ . This is different from our approach which first searches the learned clause having the smallest degree of compromise (called reference learned clause) and then removes all the learned clauses that it dominates. The algorithm 2 depicts our learned clause deletion strategy. It is important to note that the clauses whose size (number of literals) and LBD are less than or equal to 2 are not concerned by the dominance relationship. These learned clauses are considered as more relevant and are maintained in the learned clauses database. Hence, the  $minDegComp$

function of our algorithm 2 looks the learned clause of minimal degree of compromise among the learned clauses of size and LBD greater than 3.

---

#### Algorithm 1: Deletion Strategy: $reduceDB$ function

---

**Input:**  $\Delta$ : The learned clauses database of size  $n$   
**Output:**  $\Delta$  The new learned clauses database of size  $n/2$

```

1 sortLearntClauses(); /* by the defined criterion */
2 limit = n/2;
3 ind = 0;
4 while ind < limit do
5   clause =  $\Delta[ind]$ ;
6   if clause.size() > 2 and clause.lbd() > 2 then
7     removeClause();
8   else
9     saveClause();
10  ind++;
11 return  $\Delta$ ;

```

---



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#### Algorithm 2: $reduceDB$ -Dominance-Relationship

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**Input:**  $\Delta$ : The learned clauses database;  $\mathcal{M}$ : a set of relevant measures  
**Output:**  $\Delta$  The new learned clauses database

```

1 cMin = minDegComp( $\mathcal{M}$ ); /* cMin the clause having minimal degree of compromise according to  $\mathcal{M}$  */
2 ind = 0;
3 while ind <  $|\Delta|$  do
4    $c = \Delta[ind]$ ; /* a learned clause */
5   if c.size() > 2 and c.lbd() > 2 and dominates(cMin, c,  $\mathcal{M}$ ) then
6     removeClause();
7   else
8     saveClause();
9   ind++;
10 return  $\Delta$ ;

11 Function dominates (cMin: a clause, c: a clause,  $\mathcal{M}$ )
12 i = 0;
13 while i <  $|\mathcal{M}|$  do
14    $m = \mathcal{M}[i]$ ; /* a relevant measure */
15   if  $m(c) \succeq m(cMin)$  then
16     return FALSE;
17   i++;
18 return TRUE;

```

---

## 4 Experiments

For our experiments, we use three relevant measures for the dominance relationship in order to assess the efficiency of our approach. Notice that the user can choose to combine different other measures. We use SIZE [Goldberg and Novikov, 2007], LBD [Audemard and Simon, 2009] and CVSIDS [Eén and Sörensson, 2003] measures. All these measures have been proved effective in the literature [Eén and Sörensson,

son, 2003; Audemard and Simon, 2009; Jabbour *et al.*, 2014]. It is possible to use more relevant measures, but it should be noted that by adding a measure to  $\mathcal{M}$ , the number of relevant learned clauses maintained may decrease or increase. The decrease can be explained by the fact that a learned clause can be dominated with respect to a set of measures  $\mathcal{M}$  and undominated with respect to  $\mathcal{M}'$ , such that  $\mathcal{M} \subset \mathcal{M}'$ . For example, if two learned clauses  $c$  and  $c'$  are undominated with respect to  $\mathcal{M}$ , there is a possibility that one of them dominates the other by removing one measure. The increase can be explained by the fact that a learned clause can be dominated with respect to  $\mathcal{M}$  and undominated with respect to  $\mathcal{M}'$ . For example, consider a learned clause  $c$  which dominates another learned clause  $c'$  with respect to  $\mathcal{M}$ , by adding a measure  $m$  to  $\mathcal{M}$ , such that  $m(c') \succ m(c)$ , then  $c'$  is no longer dominated by  $c$ .

We run the SAT solvers on the 300 instances taken from the last SAT-RACE 2015 and on the 300 instances taken from the last SAT competition 2016. All the instances are pre-processed by SatElite [Eén and Biere, 2005] before running the SAT solver. The experiments are made using Intel Xeon quad-core machines with 32GB of RAM running at 2.66 Ghz. For each instance, we used a timeout of 1 hour of CPU time for the SAT-RACE, and 10000s for the SAT Competition. We integrate our approach in *Glucose* and made a comparison between the original solver and the one enhanced with the new deletion learned clause strategy using dominance relationship called *DegComp-Glucose*.

#### 4.1 Number of solved instances and CPU time

Table 1 presents results on SAT-RACE. We use the source code of *Glucose* 3.0 with the measure *LBD* (written *LBD-Glucose* or *Glucose* in what follows). We then replace *LBD* by each of the other measures : *SIZE-Glucose* that considers the shortest clauses as the most relevant, *CVSIDS-Glucose* that maintains the learned clauses most involved in recent conflict analysis, and finally our proposal *DegComp-Glucose*. Table 1 shows the comparative experimental evaluation of the four measures as well as *Minisat* 2.2. In the second column of Table 1, we give the total number of solved instances (#Solved). We also mention, the number of instances proven satisfiable (#SAT) and unsatisfiable (#UNSAT) in parenthesis. The third column shows the average CPU time in seconds (total time on solved instances divided by the number of solved instances). On the SAT-RACE 2015, our approach *DegComp-Glucose* is more efficient than the others in terms of the number of solved instances (see also Figure 1). In fact the original solver *Glucose* solves 236 instances while it is enhanced with our dominance approach as 12 more instances are solved. In fact, solving such additional instances is clearly significant in practical SAT solving. The *CVSIDS-Glucose* solver solves 4 more instances than *Glucose* 3.0. *Minisat* 2.2 is the worst solver among the five solvers.

Table 2 shows 5 instances of the SAT-RACE 2015 solved by our approach but not solved by *LBD-Glucose*, *SIZE-Glucose*, nor *CVSIDS-Glucose*. The time used to solved those instances may also explained the increase of the average running time of *DegComp-Glucose*. In addition we also

Solvers	#Solved (#SAT - #UNSAT)	Average Time
<i>Minisat</i> 2.2	209 (134 - 75)	585.19 s
<i>SIZE-Glucose</i>	230 (131 - 99)	533.86 s
<i>CVSIDS-Glucose</i>	240 (140 - 100)	622.23 s
<i>LBD-Glucose</i>	236(136 - 100)	<b>481.66 s</b>
<i>DegComp-Glucose</i>	<b>248 (146 - 102)</b>	571.31 s

Table 1: Comparative evaluation on SAT-RACE-2015.

find that there is none instance solved by all the other solvers and not solved by our approach (as detailed later). This shows on the one hand that the application of dominance between different relevant measures does not degrade the performance of all the solvers but instead takes advantage of the performance of each relevant measure, considering the SAT-RACE dataset.

Instances	LBD	SIZE	CVSIDS	DegComp
jgiraldezlevy.2200.9086.08.40.8	-	-	-	<b>93.71</b>
manthey_DimacsSorterHalf_37_3	-	-	-	<b>2642.88</b>
14packages-2008seed.040-NOTKNOWN	-	-	-	<b>1713.46</b>
manthey_DimacsSorter_37_3	-	-	-	<b>2673.39</b>
jgiraldezlevy.2200.9086.08.40.2	-	-	-	<b>3195.03</b>

Table 2: Instances solved by *DegComp-Glucose* and not solved by the others on SAT-RACE.

Figure 1 shows the cumulated time results i.e. the number of instances (x-axis) solved under a given amount of time in seconds (y-axis). This figure gives for each technique the number of solved instances (#instances) in less than  $t$  seconds. It confirms the efficiency of our dominance relationship approach. From this figure, we can observe that *DegComp-Glucose* is generally faster than all the other solvers, even if the average running time of *LBD-Glucose* is the lowest one (see Table 1). Although *DegComp-Glucose* needs additional time to compute the dominance relationship, the quality of the remained clauses on SAT-RACE helps to improve the time needed to solved the instances.

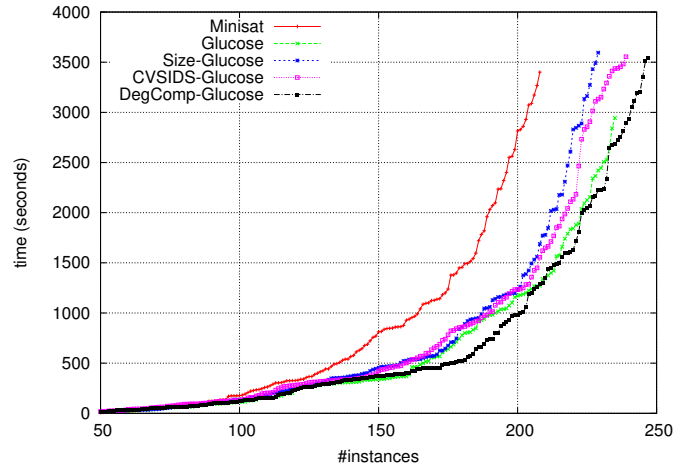


Figure 1: Evaluation on SAT-RACE-2015

Table 3 presents results on the instances of the SAT Competition 2016. Here *LBD-Glucose* and *CVSIDS-Glucose*

Solvers	#Solved (#SAT - #UNSAT)	Average Time
Minisat 2.2	138 (65 - 73)	1194.85 s
SIZE-Glucose	156 (67 - 89)	1396.73 s
CVSIDS-Glucose	165 (67 - <b>98</b> )	1368.99 s
LBD-Glucose	165 (68 - 97)	<b>1142.33 s</b>
DegComp-Glucose	164 ( <b>69</b> - 95)	1456.34 s

Table 3: Comparative evaluation on SAT-Competition-2016.

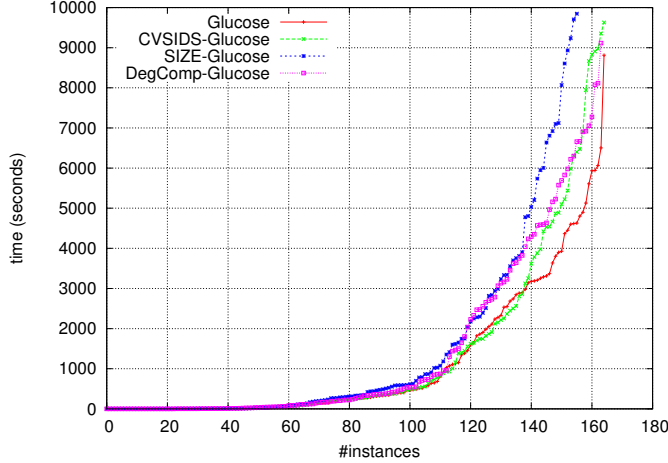


Figure 2: Evaluation on SAT competition 2016

solve one more instance than *DegComp-Glucose* which remains competitive, and solves the greatest number of satisfiable instances. Figure 2 presents the cumulated time results on the instances of the SAT competition 2016. It comes out from this second dataset that *LBD-Glucose* is more efficient than the others including our approach which remains competitive wrt the number of solved instances.

This outcome gives credit to the NO FREE Lunch theorem [Wolpert and Macready, 1997]. We also think that the aggregated function may not be unique for all the datasets, such that it is necessary to explore the efficient combination of the preferred measures.

## 4.2 Common solved instances

In table 4, the intersection between two relevant measures gives the number of common instances solved by each measure. For example, *LBD* and *SIZE* solved 219 instances in common, while 234 instances are solved by *LBD* and *DegComp*. We can see that our approach solves the largest number of instances in common with each of the aggregated measures. More precisely, the number of common instances solved with another measure is lower than the number of common instances solved with our approach.

Measures	LBD	SIZE	CVSIDS	DegComp
LBD	<b>236</b>			<b>234</b>
SIZE	219	<b>230</b>		<b>225</b>
CVSIDS	233	221	<b>240</b>	<b>238</b>
DegComp				<b>248</b>

Table 4: Common solved Instances from SAT-RACE-2015.

To get more details, Table 5 gives the number of instances commonly solved by the considered relevant measures. This table allows to see the number of common instances solved by one, two, three or four measures. For example, there are 218 common instances solved by the four deletion strategies, while 44 instances are not solved by none of them. We can observe that 1, 1, 5, and 5 are the number of instances solved alone by respectively *LBD* and *CVSIDS*, *SIZE* and *DegComp*. Moreover, there is no instance solved by the three strategies (*LBD*, *SIZE* and *CVSIDS*) and not solved by our approach *DegComp*.

Measures		DegComp		¬DegComp	
		CVSIDS	¬CVSIDS	CVSIDS	¬CVSIDS
LBD	SIZE	218	1	<b>0</b>	0
	¬SIZE	1	1	1	1
¬LBD	SIZE	3	3	0	<b>5</b>
	¬SIZE	3	<b>5</b>	1	44

Table 5: Detailed of common instances with SAT-RACE.

## 4.3 Combined measures

Table 6 gives the number of instances solved with our dominance approach wrt the measures used in the dominance relations. From this table, we can see that the number of instances solved by using two measures (instead of three) in the dominance relationship is always lower than the number of instances solved (248) by using three measures.

Measures	LBD	SIZE	CVSIDS	DegComp
LBD	<b>236</b>			
SIZE	223	<b>230</b>		
CVSIDS	239	242	<b>240</b>	
DegComp				<b>248</b>

Table 6: Combining two measures on SAT-RACE-2015.

## 5 Conclusion and Future Works

In this paper, we propose an approach that addresses the learned clauses database management problem. We have shown that the idea of dominance relationship between relevant measures is a nice way to take profit of each measure. This approach is not hindered by the abundance of relevant measures which has been the issue of several works. The proposed approach avoids another non-trivial problem which is the amount of learned clauses to be deleted at each reduction step of the learned clauses database. The experimental results show that exploiting the dominance relationship improves the performance of CDCL SAT solver, at least on the SAT-RACE 2015. For the case of SAT-Competition, we still have to find a good dominance relation. The instances categories might also be an issue which should be explored.

To the best of our knowledge, this is the first time that dominance relationship has been used in the satisfiability domain to improve the performance of a CDCL SAT solver. Our approach opens interesting perspectives. In fact, any new relevant measure of learned clauses can be integrated into the dominance relationship.

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