raSAT: An SMT Solver for Polynomial Constraints

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Download: http://www.jaist.ac.jp/~s1310007/raSAT/, or google “raSAT SMT”

Solving Polynomial Constraints

- Satisfiability of : $\varphi ::= g(x_1, \ldots, x_n) \diamond 0 \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi$

  where $\diamond \in \{>, \geq, <, \leq, =, \neq\}$ and $g(x_1, \ldots, x_n)$ is a polynomial.

- Example: $(x^3y - y^4 + 1 > 0) \wedge (y^3 - xy > 0)$ is satisfiable with

  $x = 2.652, y = 2.346$

- Notion: UNSAT for Unsatisfiability, SAT for Satisfiability

Applications: Invariants Generation, Round-off and Over-flow Error Analysis, Automatic Termination Prover for Term Rewriting Systems.

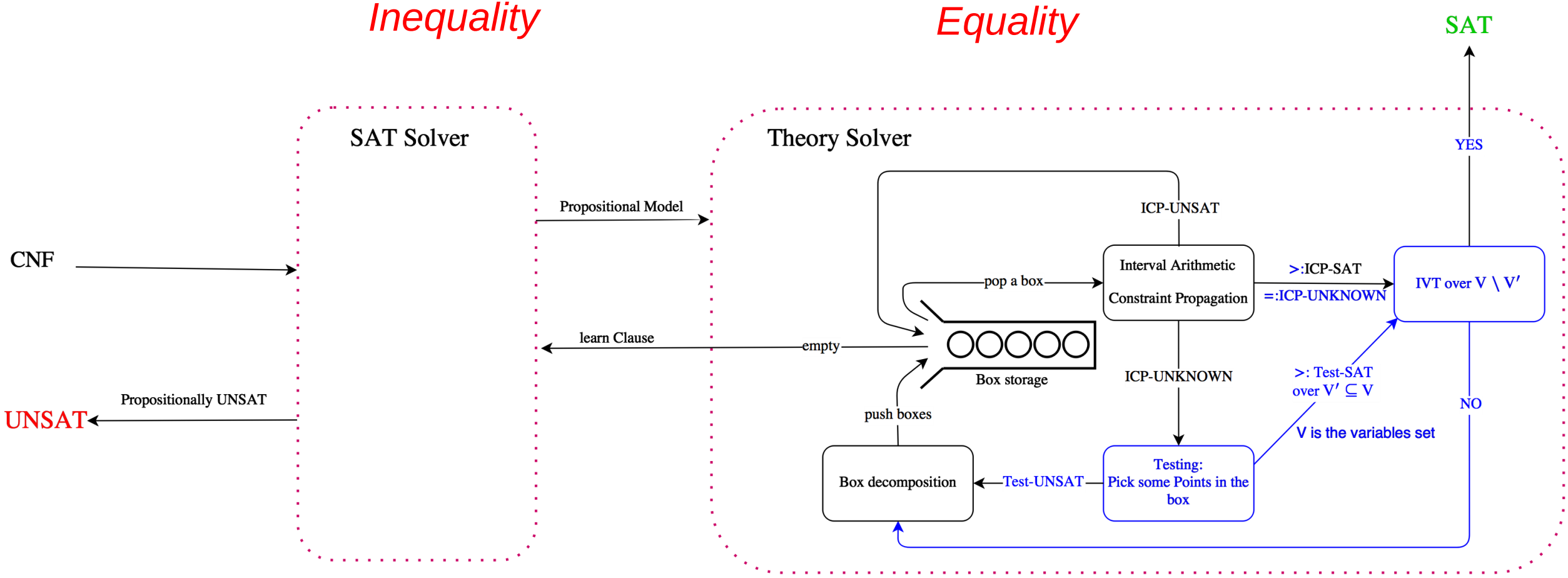
Methods for Solving Polynomial Constraints

- CAD: **complete** for **general** quantified formulas, DEXP
  - ✓ Z3, SMT-RAT, QEPCAD, Redlog
- Virtual substitution: degree < 4, EXP
  - ✓ SMT-RAT, Redlog
- Gröbner basis: Equalities, DEXP
  - ✓ SMT-RAT, Mathematica, Maple, Reduce
- Interval Constraint Propagation (ICP): Inequalities, incomplete
  - ✓ iSAT3, dReal, raSAT
- Bit-blasting: Bounded variables and precision
  - ✓ miniSmt
- Linearization using CORDIC: Bounded variables and precision
  - ✓ CORD

raSAT – an SMT Solver for Polynomial Constraints

- raSAT: ICP + Testing + Intermediate Value Theorem (IVT).

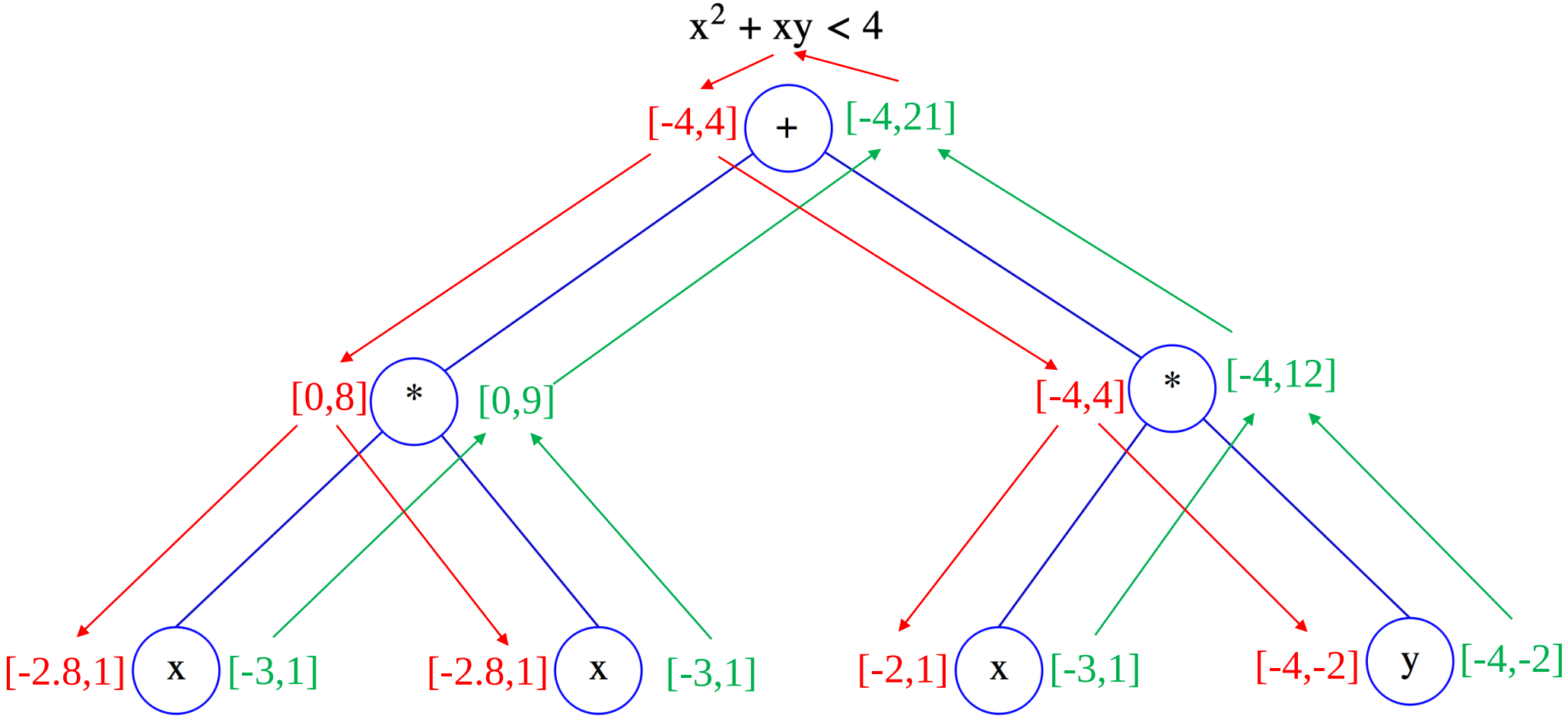
Inequality *Equality*



ICP = Interval arithmetic + Constraint propagation + Box decomposition

Interval Arithmetic and Constraint Propagation

- E.g., $x^2 + xy < 4$ $x \in [-3,1]$, $y \in [-4,-2]$

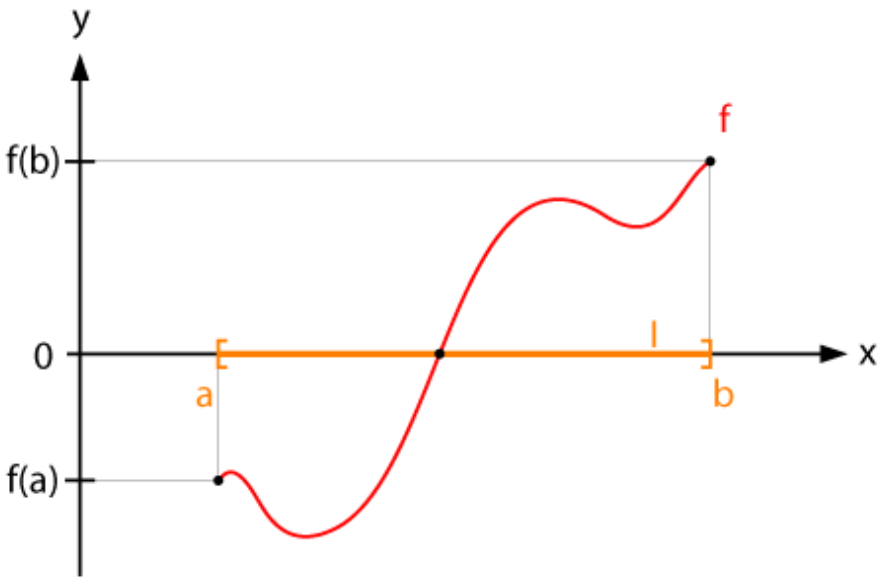


Input: x ∈[-3,1], y ∈[-4,-2]

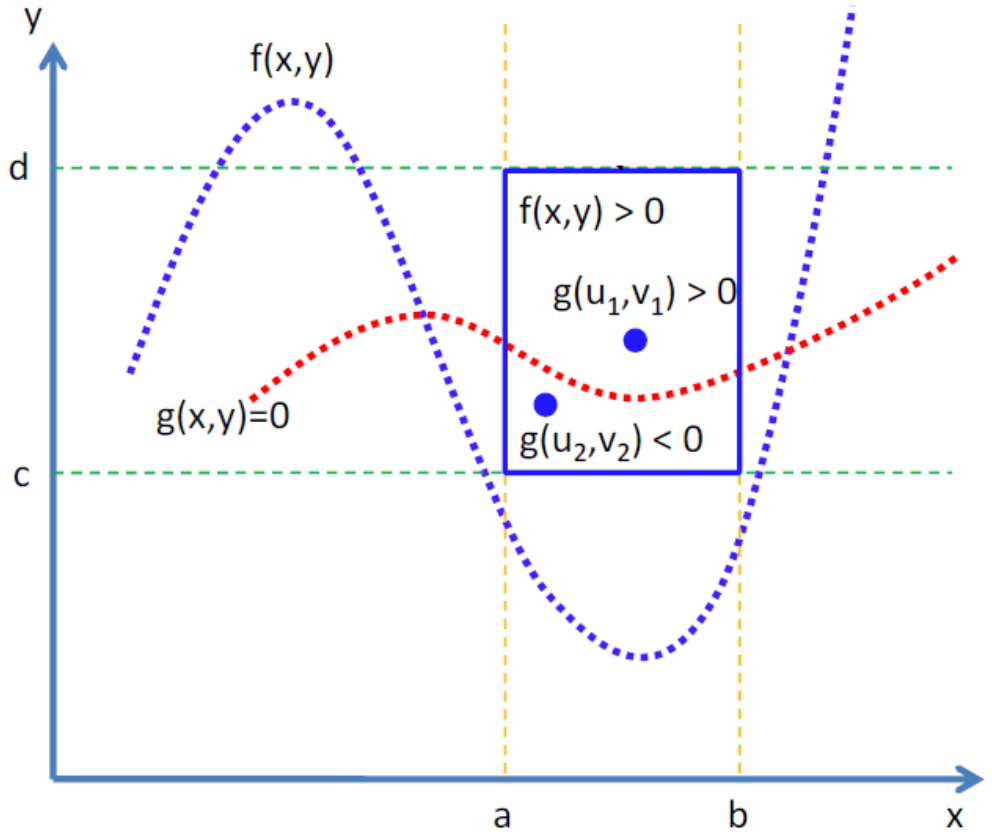
Output: ICP-UNKNOWN, x ∈[-2,1], y ∈[-4,-2]

IVT for a single equation

IVT
when $f(x)$ is continuous



SAT detection by IVT
$f(x,y) > 0 \wedge g(x,y) = 0$



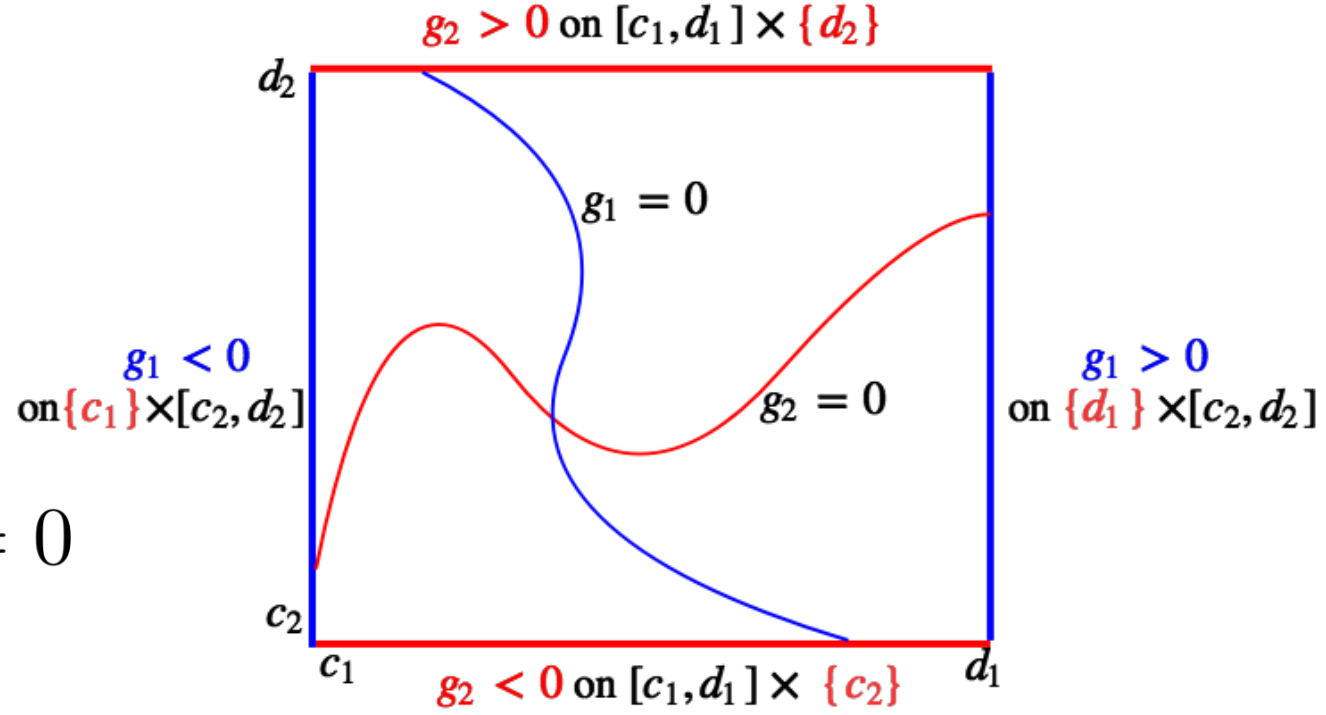
Generalized IVT for Multiple Equations

- The Generalized IVT†
  - ✓ Multiple equations
  - ✓ Requires
    |Variables| ≧ |Equations|

- Example:

$$g_1(x, y) = 0 \wedge g_2(x, y) = 0$$

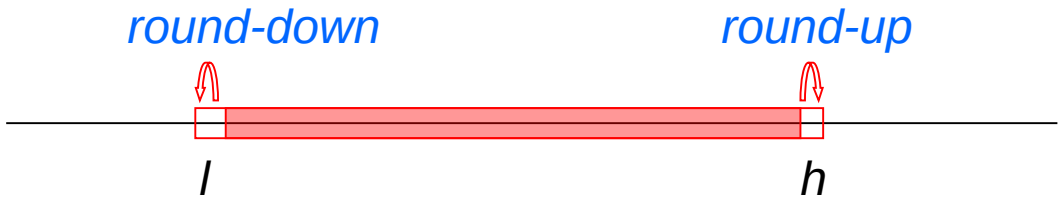
$$x \in [c_1, d_1], y \in [c_2, d_2]$$



†Neumaier, A.: Interval Methods for Systems of Equations. Cambridge Middle East Library, Cambridge University Press (1990)

raSAT is Sound

- **Soundness** under floating point arithmetic
  - ✓ Outward rounding in Interval Arithmetic (library of Alliot et al.†)



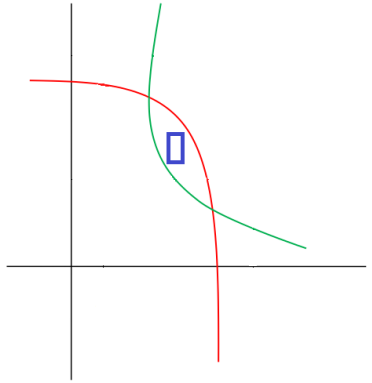
  - ✓ Confirming SAT instance by iRRAM, an guaranteed round-off error bound package: http://irram.uni-trier.de/

- Easily extended to constraints over Integers (NIA) by picking only integers as test data.

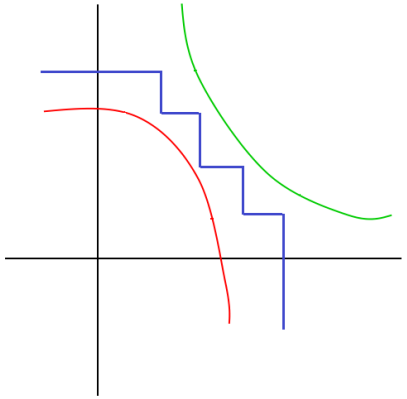
† Alliot, J.M., Gotteland, J.B., Vanaret, C., Durand, N., Gianazza, D.: Implementing an interval computation library for OCaml on x86/amd64 architectures. In: ICFP. ACM (2012)

raSAT is Incomplete

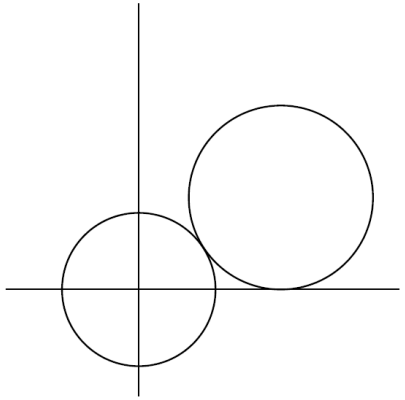
- Inequality



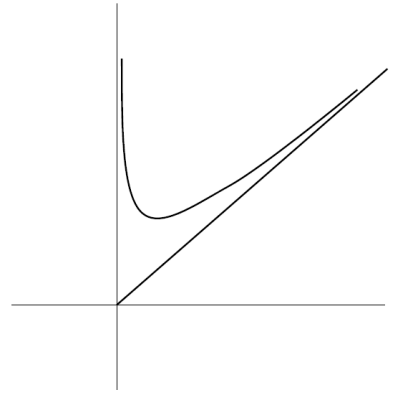
✓ SAT Detection



✓ UNSAT Detection



✗ Kissing case



✗ Convergence

- Equality
  - ✓Generalized IVT requires |Variables| ≧ |Equations|
  - ✓Potentially 0-dim ideal case when |Variables| < |Equations|

SMT-COMP results in QF_NRA and QF_NIA

- 2014: raSAT 0.1 (QF_NRA) solved **88** problems, 3rd among 3.
- 2015: raSAT 0.2 (QF_NRA) solved **7952** problems, 3rd among 6.
  (QF_NIA) solved **7917** problems, 2nd among 7.
- 2016: raSAT 0.3/0.4 preliminary results on 29th June, 2016

QF_NRA	Z3-4.4.1	Yices-2.4.2	SMT-RAT	raSAT 0.4	raSAT 0.3	CVC4
Solved No.	10056	10019	9026 (4 errors)	9024	8431	2694
Time (sec)	24785.38	61989.88	51053.15	11176.39	13576.52	150.24

QF_NIA	Z3-4.4.1	Yices-2.4.2	SMT-RAT	AProVE	CVC4	raSAT 0.4	ProB	raSAT 0.3
Solved No.	8566	8451	8443	8273	8231	8017	7557	7544
Time (sec)	27718.2	8523.4	6234.5	8527.66	161418.04	159247.55	13586.05	70228.9

https://www.starexec.org/starexec/secure/details/job.jsp?id=16079,16070

Observations from experiments

- SAT is detected on several large constraints solely by raSAT:
  (in comparison with dReal, iSAT3, SMT-RAT, Z3)
  - ✓ zankl/matrix-2-all-3.smt2: 57 variables
  - ✓ zankl/matrix-2-all-8.smt2: 17variables
  - ✓ zankl/matrix-3-all-5.smt2: 81variables
  - ✓ zankl/matrix-4-all-3.smt2: 139 variables
  - ✓ zankl/matrix-4-all-9.smt2: 193variables

  where Z3 4.4 solely solves many, among them some large ones are
  - ✓ zankl/matrix-3-all-7.smt2: 75 variables
  - ✓ zankl/matrix-4-all-12.smt2: 200variables
  - ✓ zankl/matrix-5-all-6.smt2: 258variables
- **Not** much good in UNSAT detection → needs to improve UNSAT core.
- Adding IVT to raSAT increased **1330** SAT detection in benchmarks.

Future Work: Make ICP complete by Algebraic methods

- Make the procedure complete: ICP + CAD
  - ✓ Idea from Loup et al.†
  - ✓ Call CAD when **a box** in ICP becomes **small**
  - ✓ The small box of ICP guides CAD to reduce the numbers of polynomials / cells in Projection / Lifting phases.
- Improve UNSAT detection: When ICP-UNSAT or Test-UNSAT, reduce the search space by clause learning.
  - ✓ Idea from Jovanovic et al.‡
  - ✓ Project only polynomials in ICP(Test)-UNSAT constraints
  - ✓ Compute only the cell that contains the UNSAT box or UNSAT test case
  - ✓ Learn the negation of the constraint representing such a cell

† Loup, U., et.al.: A symbiosis of interval constraint propagation and cylindrical algebraic decomposition. In: CADE. LNAI 7898, pp.193–207 (2013)

‡ Jovanovic, D., de Moura, L.M.: Solving non-linear arithmetic. In: IJCAR, pp.339–354 (2012)