Efficient Local Search for Nonlinear Real Arithmetic

Zhonghan Wang, Bohua Zhan, Bohan Li, Shaowei Cai

Institute of Software, Chinese Academy of Sciences

VMCAI 2024 15 January 2024

Outline

Outline

Syntax of SMT(NRA)

polynomial: $p ::= x \mid c \mid p + p \mid p - p \mid p \times p$
atoms: $a ::= b \mid p = 0 \mid p > 0 \mid p < 0$
formula: $f ::= a \mid \neg f \mid f \wedge f \mid f \vee f$

SMT: Determine whether the formula is satisfied by some assignment (local search focuses), or prove unsat

Example:
$x^2 + y^2 \leq 1 \wedge x + y < 1 \wedge x + z > 0$
assignment with $\{x \to 0, y \to 0, z \to 1\}$ satisfies all clauses.

Fragment of Local Search

Input : A set of clauses F
Output: An assignment of variables that satisfy F, or failure
Initialize assignment to variables;
while $\top$ **do**
  if *all clauses satisfied* **then**
    return *success with assignment;*
  end
  if *time or step limit reached* **then**
    return *failure;*
  end
  Critical move procedure.
end

Algorithm 1: Basic Fragment of Local Search

Fragment of Local Search

$var, new\_value, score \leftarrow$ best move according to make-break score;
if *score* $>$ *0* **then**
    Perform move, assigning var to new_value;
end
else
    Update clause weight according to PAWS scheme;
    repeat
        $cls \leftarrow$ random unsatisfied clause;
        $var, new\_value, score \leftarrow$ critical move making cls satisfied;
        if *score* $\neq -\infty$ **then**
            Perform move, assigning var to new_value;
        end

Local Search for SAT and SMT

LS \ Problem	SAT	SMT
Operation (Move)	Flip	Critical Move
Score Definition	Weighted unsat clauses	
Score Computation	Cached score	No Caching, time costly

What LS for SAT brings us:
Maintain scoring information after each iteration.
Difficulty:
Predetermine critical move shift value.
Our Solution:
Introduce Scoring Boundaries.

Outline

Critical Move Value ? Interval!

Reference introduces multiple values choices for critical move, resulting in different scores.
Actually, **interval** brings the difference.

Boundary

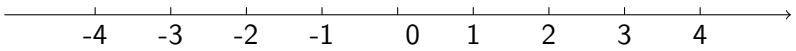
Brief. Given an arithmetic variable, maintain critical move scores for specific moving intervals.
Definition. 123
Example.
Assume boundary for variable v is

Remark

- the environment above is **block**
- the environment here is **alertblock**

Boundary Example

Assume boundary for variable v as below:



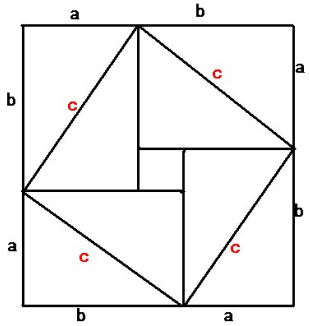
Computation of Boundary (Incremental)

```
Data: this text
Result: how to write algorithm with LaTeX2e
initialization;
while not at end of this document do
    read current;
    if understand then
        go to next section;
        current section becomes this one;
    else
        go back to the beginning of current section;
    end
end
```

Algorithm 3: How to write algorithms (copied from here)

Outline

Equality Constraints in Local Search



1. item
2. another
3. more
   - first
   - second
   - third

Relaxation

This is a text in first column.

$$E = mc^2$$

- First item
- Second item

first block

columns achieves splitting the screen

second block

stack block in columns

Relaxation (CAD view)

Restore

Outline

Outline

End

The last page.