Efficient Local Search for Nonlinear Real Arithmetic

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Outline

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Syntax of SMT(NRA)

polynomial: $p ::= x \mid c \mid p + p \mid p - p \mid p \times p$
atoms: $a ::= b \mid p = 0 \mid p > 0 \mid p < 0$
formula: $f ::= a \mid \neg f \mid f \wedge f \mid f \vee f$

SMT: Determine whether the formula is satisfied by some assignment (local search focuses), or prove unsat

Example:
$x^2 + y^2 \leq 1 \wedge x + y < 1 \wedge x + z > 0$
assignment with $\{x \to 0, y \to 0, z \to 1\}$ satisfies all clauses.

Fragment of Local Search

Input : A set of clauses F
Output: An assignment of variables that satisfy F, or failure
Initialize assignment to variables;
while $\top$ **do**
  if *all clauses satisfied* **then**
    return *success with assignment;*
  end
  if *time or step limit reached* **then**
    return *failure;*
  end
  Critical move procedure.
end

Algorithm 1: Basic Fragment of Local Search

Fragment of Local Search

```
var, new_value, score ← best move according to
  make-break score;
if score > 0 then
    Perform move, assigning var to new_value;
end
else
    Update clause weight according to PAWS
      scheme;
    repeat
        cls ← random unsatisfied clause;
        var, new_value, score ← critical move
          making cls satisfied;
        if score ≠ −∞ then
            Perform move, assigning var to
              new_value;
        end
```

Local Search for SAT and SMT

LS \ Problem	SAT	SMT
Operation (Move)	Flip	Critical Move
Score Definition	Weighted unsat clauses	
Score Computation	Cached score	No Caching, time costly

What LS for SAT brings us:
Maintain scoring information after each iteration.
Difficulty:
Predetermine critical move shift value.
Our Solution:
Introduce Scoring Boundaries.

Outline

Make-break Intervals

make-break intervals[1]

Combination of (in)feasible intervals of arithmetic varaible x with respect to all clauses.

Example

Current assignment: $\{x \mapsto 1, y \mapsto 1, z \mapsto 1\}$

Calculate infeasible set for each clause.

- $x^2 + y^2 \leq 1$ (unsat): $(-\infty, 0) \cup (0, \infty)$.
- $x + y < 1$ (unsat): $[0, \infty)$.
- $x + z > 0$ (sat): $(-\infty, -1]$.

Combined information: x: $(-\infty, -1] \mapsto 0$, $(-1, 0) \mapsto 1$, $[0, 0] \mapsto 1$, $(0, \infty) \mapsto 0$.

[1] Bohan Li and Shaowei Cai. "Local Search For SMT On Linear and Multilinear Real Arithmetic". In: *CoRR* abs/2303.06676 (2023). accepted for FMCAD.. DOI: 10.48550/arXiv.2303.06676. arXiv: 2303.06676. URL: https://doi.org/10.48550/arXiv.2303.06676.

Traditional Computation

Input : unsat clauses F
Output: Best critical move (variable, value)
foreach *variable* v *in unsat clauses* **do**
  foreach *unsat clause* c *with* v **do**
    Compute interval-score info of v in c.
  end
  Combine interval-score information.
  Update best var-value move.
end
return *best critical move*

Repeated computation:

- variable's (in)feasible set
- clause's sat staus

Boundary

Definition. A quadruple $\langle val, is\_open, is\_make, cid \rangle$, where val is a real number, is_open and is_make are boolean values, and cid is a clause identifier.

Meaning

- val : make-break value.
- is_open : active or not at val point.
- is_make : make or break, increase or decrease score.
- cid : causing clause.

Sorting First ordered by val, then by is_open ($\bot < \top$).

Boundary

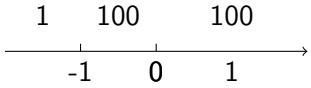
Current assignment: $\{x \mapsto 1, y \mapsto 1, z \mapsto 1\}$

- $x^2 + y^2 \leq 1$: starting score 0, boundary set $\{(0, \bot, \top, 1), (0, \top, \bot, 1)\}$, indicating no change for large negative values, *make* at boundary $[0, \cdots$, followed by *break* at boundary $(0, \cdots$.
- $x + y < 1$: starting score 3, boundary set $\{(0, \bot, \bot, 2)\}$, indicating *make* at large negative values, and *break* at boundary $[0, \ldots$.
- $x + z > 0$: starting score -2, boundary set $\{(-1, \top, \top, 3)\}$, indicating *break* at large negative values, and *make* at boundary $(-1, \ldots$.

sorted boundary set:
$\{(-1, \top, \top, 3), (0, \bot, \top, 1), (0, \bot, \bot, 2), (0, \top, \bot, 1)\}$

Boundary Example

boundary set:
$\{(-1, \top, \top, 3), (0, \bot, \top, 1), (0, \bot, \bot, 2), (0, \top, \bot, 1)\}$



starting score: Score when x moves to -∞.
Maintain and Change: We maintain the boundary info for all arithmetic varaibles, unless the neighbour does a critical move.

Algorithm for computing boundary

Input : Variable v that is modified
Output: Make-break score for all variables
$S \leftarrow \{\}$; `// set of updated variables`
for *clause* cls *that contains* v **do**
  for *variable* v' *appearing in* cls **do**
    add v' to S;
    recompute starting score and boundary of v' with respect to cls;
  end
end
for *variable* v' *in S* **do**
  recompute best critical move and score in terms of boundary information;
end

Outline

1. Problem - Nonlinear Real Arithmetic
   - Search Space of SMT(NRA)
   - Current Existing Methods
2. Incremental Computation of Variable Scores
   - Scoring Boundary for Arithmetic Variable
   - Incremental Computation
3. Temporary Relaxation of Equality Constraints
   - Difficulty in Local Search
   - Relaxation Method
4. Experiment

Previous Algorithm and Difficulty

Number complexation in Local Search

When a variable chooses a complex value, the iteration is much slower, but sometimes we have to ...

Reference[2] ignores equalities constraints due to its accurate value complexation.

We introduce Relaxation into strick equality constraints, resulting in temporary interval candidate (rather than a point).

[2] Bohan Li and Shaowei Cai. "Local Search For SMT On Linear and Multilinear Real Arithmetic". In: *CoRR* abs/2303.06676 (2023). accepted for FMCAD.. DOI: 10.48550/arXiv.2303.06676. arXiv: 2303.06676. URL: https://doi.org/10.48550/arXiv.2303.06676.

Local Search with Relaxation

Input : A set of clauses F
Output: An assignment of variables that satisfy F, or failure
Initialize assignment to variables;
while $\top$ **do**
  if *all clauses satisfied* **then**
    $success \leftarrow$ find exact solution;
    if *success* **then**
      return *success with assignment*;
    end
    else
      Restore relaxed constraints to original form;
      $success \leftarrow$ find exact solution by limited local search;
      if *success* **then**
        return *success with assignment*;
      end
    end
  end
  if *time or step limit reached* **then**
    return *failure*;
  end
  Proceed traditional local search (slack).
end

Outline

Overall Result

Category	#inst	Z3	cvc5	Yices	Ours	Unique
20161105-Sturm-MBO	120	0	0	0	**88**	88
20161105-Sturm-MGC	2	**2**	0	0	0	0
20170501-Heizmann	60	3	1	0	**8**	6
20180501-Economics-Mulligan	93	**93**	89	91	90	0
2019-ezsmt	61	**54**	51	52	19	0
20200911-Pine	237	**235**	201	**235**	224	0
20211101-Geogebra	112	**109**	91	99	101	0
20220314-Uncu	74	73	66	**74**	70	0
LassoRanker	351	155	**304**	122	272	13
UltimateAtomizer	48	**41**	34	39	27	2
hycomp	492	**311**	216	227	304	11
kissing	42	**33**	17	10	**33**	1
meti-tarski	4391	**4391**	4345	4369	4351	0
zankl	133	70	61	58	**100**	27
Total	6216	5570	5476	5376	**5687**	148

References

[LC23] Bohan Li and Shaowei Cai. "Local Search For SMT On Linear and Multilinear Real Arithmetic". In: *CoRR* abs/2303.06676 (2023). accepted for FMCAD. DOI: 10.48550/arXiv.2303.06676. arXiv: 2303.06676. URL: https://doi.org/10.48550/arXiv.2303.06676.