

Week 3.1 Classification and Representation

Classification

Classification: $y = 0 \text{ or } 1$

$h_{\theta}(x)$ can be > 1 or < 0

Logistic Regression: $0 \leq h_{\theta}(x) \leq 1$

Classification

- Linear Regression is not a good idea

Hypothesis Representation

Sigmoid Function

- we want $0 \leq h_{\theta}(x) \leq 1$
- $g(\theta^T x)$
- where $g(z) = \frac{1}{1+e^{-z}}$ (**Sigmoid function/Logistic function**)

- Our goal: fit parameters θ of our data

Logistic Regression Model

Want $0 \leq h_{\theta}(x) \leq 1$

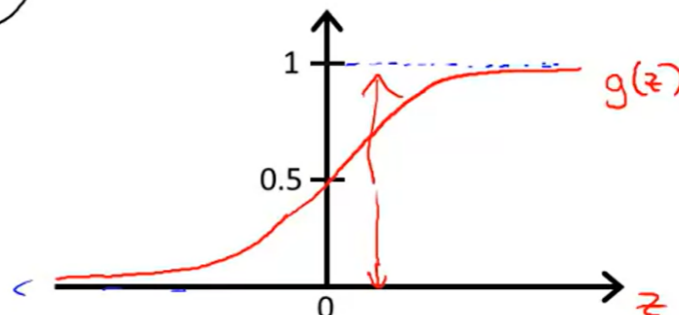
$$h_{\theta}(x) = g(\theta^T x)$$

$$\rightarrow g(z) = \frac{1}{1 + e^{-z}}$$

$\theta^T x$

Sigmoid function
Logistic function

$$h_{\theta}(x) = \frac{1}{1 + e^{-\theta^T x}}$$



Parameters θ

Interpretation

- $h_{\theta}(x)$: estimated probability that $y = 1$ on input x

Interpretation of Hypothesis Output

$h_{\theta}(x)$ = estimated probability that $y = 1$ on input x

Example: If $x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{tumorSize} \end{bmatrix}$

$$h_{\theta}(x) = 0.7$$

$y = 1$

Tell patient that 70% chance of tumor being malignant

$$h_{\theta}(x) = P(y=1|x;\theta)$$

$y = 0 \text{ or } 1$

"probability that $y = 1$, given x , parameterized by θ "

$$\rightarrow P(y=0|x;\theta) + P(y=1|x;\theta) = 1$$

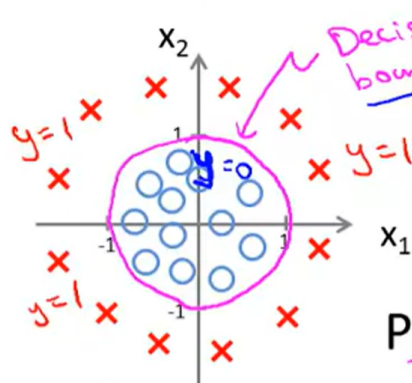
$$P(y=0|x;\theta) = 1 - P(y=1|x;\theta)$$

Decision Boundary

- predict $y = 1$ when $z > 0$
- predict $y = 0$ when $z < 0$
- Decision Boundary: the separate line

Nonlinear Decision Boundary

Non-linear decision boundaries



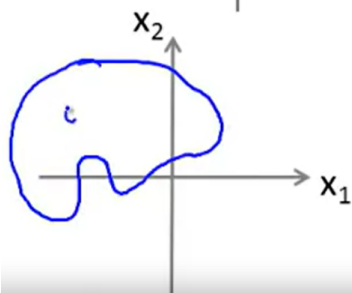
$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

$\begin{matrix} \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{matrix}$

$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Predict "y = 1" if $-1 + x_1^2 + x_2^2 \geq 0$

$\boxed{x_1^2 + x_2^2 = 1}$
 $\underbrace{-1 + x_1^2 + x_2^2}_{x_1^2 + x_2^2 \geq 1} \geq 0$



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_1^2 x_2 + \theta_5 x_1^2 x_2^2 + \theta_6 x_1^3 x_2 + \dots)$$