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BT-I/D-18

31017

MATHEMATICS

(Applied Mathematics–I)

Paper : AS-105 (N)

Opt. (II)

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all. Select at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Using Gauss-Jordan method, find the inverse of the

$$\text{matrix} \begin{bmatrix} -2 & -1 & -1 \\ 12 & 8 & 6 \\ 10 & 5 & 6 \end{bmatrix}. \qquad 7\frac{1}{2}$$

(b) Test for the consistency, and solve

$x + 2y + z = 3;$

$2x + 3y + 2z = 5;$

$3x - 5y + 5z = 2,$

$3x + 9y - z = 4.$ $7\frac{1}{2}$

2. (a) Find the eigen values and eigen vectors of the real matrix $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$; where a and b are non-zero constants. $7\frac{1}{2}$

(b) Reduce $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form, also find the matrix of transformation. $7\frac{1}{2}$

UNIT–II

3. (a) Find the nth derivative of the following

(i) $\cos^4 x$

(ii) $\sin x \sin 2x \sin 3x.$ (3+4½)

(b) Show that

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\right].$$

$7\frac{1}{2}$

4. (a) Show that the asymptotes of the cubic $x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$ cuts the curve in three points which lie on the straight line $x - y + 1 = 0.$ $7\frac{1}{2}$

(b) Trace the curve $r = 1 - 2 \sin \theta.$ $7\frac{1}{2}$

UNIT–III

5. (a) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u. \quad 7\frac{1}{2}$$

(b) If z is a function of x and y and $x = u \cos \alpha - v \sin \alpha$, $y = u \sin \alpha + v \cos \alpha$, then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}. \quad 7\frac{1}{2}$$

6. (a) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ then find the value of

$$\frac{\partial(u, v, w)}{\partial(x, y, z)}. \quad 7\frac{1}{2}$$

(b) Find the shortest and longest distance from the point (1, 2, –1) to the sphere $x^2 + y^2 + z^2 = 24$, using Lagrange's method. $7\frac{1}{2}$

UNIT–IV

7. (a) The area bounded by $y^2 = 4x$ and the line $x = 4$ is revolved about the line $x = 4$. Find the volume of solid of revolution. $7\frac{1}{2}$

(b) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx$ by changing the order of integration. $7\frac{1}{2}$

8. (a) Evaluate $\iint r^3 \, dr \, d\theta$; over the area bounded between the circles $r = 2 \cos \theta$ and $r = 4 \cos \theta$. $7\frac{1}{2}$

(b) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log 2} e^{x+y+z} \, dz \, dy \, dx$. $7\frac{1}{2}$
