LINEAR-TIME JOINT PROBABILISTIC DATA ASSOCIATION FOR MULTIPLE EXTENDED OBJECT TRACKING

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ABSTRACT

The Joint Probabilistic Data Association (JPDA) filter for multiple object tracking is based on the assumption that at most one measurement originates from a target object. However, with the development of high-resolution sensors, it is often the case that multiple spatially distributed detections are obtained from a single object. To tackle this emerging data association challenge, this paper presents a JPDA method based on the Poisson spatial measurement model for extended objects. As the constraint that one target gets at most one measurement is relaxed, the marginal association probabilities can be obtained with linear complexity in the number of measurements and targets. The proposed method is compared to a partition-based multiple extended object tracking algorithm.

I. INTRODUCTION

With the development of short range and high resolution sensors such as Light Detection and Ranging (LIDAR), a single target object often occupies multiple sensor resolution cells and the spatial information of an object is of interest. This problem is called Extended Object Tracking (EOT), which considers the simultaneous estimation of the dynamics and the spatial information of the objects [1].

The widely-used model proposed by Gilholm *et al.* in [2], [3] employs a Poisson Points Process (PPP) in order to model the spatial extend and the number of measurements. Most EOT algorithms approximate objects as rigid shapes such as rectangular, elliptical, and star-convex shapes. In [4] an oriented bounding box model in combination with a particle filter for EOT during the DARPA Urban Grand Challenge is introduced. The Random Matrix approach [5], [6] treats the measurements as noise-corrupted detections from the object's centroid that are spread over the extent. The Random Hypersurface Model [7] parameterizes the object extent as a star-convex shape and uses nonlinear Kalman filters to estimate the shape variables. In order to parameterize the shape, [8] and [9] implement Gaussian Processes. In [10], the extended object is modelled by a set of reflector points and the Expectation Maximization (EM) algorithm is used to deal with unknown associations. In our previous works [11], [12], we explicitely parameterize an ellipse with the orientation and semi-axes lengths and employ multiplicative noise to form a nonlinear measurement equation.

As multiple measurements may stem from a single object, clustering and partitioning is one of the most intuitive solution to adopt traditional data association methods, e.g., within Probability Hypothesis Density (PHD) filter [13], Probabilistic Multihypothesis Tracking (PMHT) [14] or Joint Probability Data Association (JPDA) [15] framework. The other category of extended object tracking data association methods directly solves multiple measurements to target association by permuting all possible combinations for all measurements in the gating area. Each combination is assumed to be uniformly distributed and the innovation gain for each combination is required [16] [17]. Even through the computational load can be improved iteratively [18], the complexity of calculating candidate measurements combinations is still very high. Streit derived a JPDA intensity filter (JiFi) [19], which estimates an intensity function for each extended object. The JiFi approach is also based on the model from [2], [3] and avoids extensive calculations of association probabilities. Granström *et al.* proposed a sampling-based stochastic optimization methods to obtain the most probable association [20].

In this work, we propose a linear-time Joint Probabilistic Data Association (JPDA) filter for multiple extended object tracking (ET-JPDA). The number of measurements from an object is assumed to be Poisson distributed. In line with [3], the Poisson mean has a similar role as the detection probability in traditional tracking approach. No explicit probability of detection is employed. The Poisson model allows for an exact solution of the marginal association probabilities. For shape estimation, we employ our recently proposed method [12] that explicitly estimates the orientation and semi-axes lengths of an elliptic extended object by means of a recursive incorporation of individual measurements. The time-complexity of the overall algorithm is linear in the number of measurements and targets, due to compact expressions for the marginal association probabilities and the

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recursive incorporation of multiple measurements from one target.

The remainder of this paper is structured as follows. In Section II, the association likelihood for measurements to extended targets is derived, followed by a description of the Probabilistic Data Association (PDA) filter's update step in Section III. The algorithm is then tested in simulations in Section IV. Section V concludes this paper.

II. DATA ASSOCIATION

Let $Y_k = \{y_k^j\}_{j=1}^{M_k}$ be the set of available measurements at time k and $X_k = \{x_k^i\}_{i=1}^{N_k}$ the set of unknown extended targets. Each measurement arises from an extended target or clutter, where the number of clutter is Poisson distributed with rate parameter λ_0. Clutter is assumed to be uniformly distributed in the surveillance region with density of ρ. In this work, extended object tracking differentiates itself from traditional point target tracking mainly in the following two aspects: (i) the state vector x_k^i contains not only kinematic parameters but also shape (size and orientation) information, (ii) an extended target produces fluctuating number of measurements. The objective of multiple extended object tracking is to tackle the measurement-to-target assignment and estimate X_k.

II-A. Poisson Point Process

In this work, we employ the Poisson Point Process (PPP) for extended object measurement modeling as described in [2], [3]. Each extended target x_k^i produces n_k^i measurements, where n_k^i is Poisson distributed with rate parameter of λ_i, i.e.,

$$p(n_k^i|x_k^i) = \underbrace{e^{(-\lambda_i)}\frac{(\lambda_i)^{n_k^i}}{n_k^i!}}_{:=\mathcal{P}(\lambda_i;n_k^i)} \ . \tag{1}$$

As it is noted in [2], the rate parameter λ_i plays a similar role as detection probability in traditional tracking. Target x_k^i is missed-detected when $n_k^i = 0$. A measurement y from an extended object x_k^i is an independent random draw from the spatial distribution $p(y|x_i)$. Assume an association event θ (see Section II-C) assigns measurement set Y^i to target x^i, i.e., we have

$$p(Y_k^i|\theta, x_k^i) = \prod_{y^j\in Y_k^i} p(y^j|x_k^i) \ , \tag{2}$$

where $p(y^j|x_k^i)$ is the spatial distribution of measurement y^j given object x_k^i, which is described in the next section. For multiple targets X_k, we get

$$p(Y_k|\theta, X_k) = \prod_{i=1}^{N_k}\prod_{y^j\in Y_k^i} p(y^j|x_k^i) \ . \tag{3}$$

II-B. Spatial Distribution

In order to model the spatial distribution, we employ the multiplicative error measurement model [12] for ellipses, which is based on an explicit measurement function for relating the shape parameters with a measurement. The state vector x_k^i consists of the kinematic state r_k^i and the shape parameters p_k^i. Both r_k^i and p_k^i are Gaussian distributed with mean of $\hat{r}_k^i$ and $\hat{p}_k^i$ with respective covariance $C_k^{r_i}$ and $C_k^{p_i}$. For the data association in the JPDA filter, we define

$$G_{ij} = \mathcal{N}(y^j; \bar{y}_{k|k-1}^i, C_{k|k-1}^{y_i}) \ , \tag{4}$$

where $\bar{y}_{k|k-1}^i$ is the predicted measurement of object i at time k and $C_{k|k-1}^{y_i}$ is its covariance, see [12] for the detailed formulas. Note that (4) is in the same form as equation (2) in [2], where the spatial dependent rate function of inhomogeneous Poisson process is $\lambda_i G_{ij}$.

II-C. Joint Probabilistic Data Association

At time step k, M_k measurements need to be assigned to N_k targets or background clutter. The mapping between measurements and an extended target is $many$-1, with measurements being unknown and fluctuating over time, i.e., multiple measurements can be assigned to one target. We denote an association hypotheses as a $1\times M_k$ random vector $\theta = [\theta_j]_{j=1}^{M_k}$ with

$$\theta_j = \begin{cases} 0, & \text{if } y_j \text{ is a clutter} \ , \\ i, & \text{if } y_j \text{ is assigned to } x_i,\ i\in\{1,\cdots,N_k\} \ . \end{cases} \tag{5}$$

Example realizations of θ are illustrated in Fig. 1.

An association event implicitly specifies

- Y_k^i, the set of measurements that is assigned to object i,
- Y_k^0, the set of clutter measurements,
- n_k^i, the cardinality of Y_k^i, and
- n_k^0, the number of clutters, i.e., $n_k^0 = M_k - \sum_{i=1}^{N_k} n_k^i$.

Given an association event θ, $T = \{n_k^i\}_{i=0}^{N_k}$ is determined. We obtain

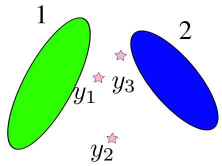
$$\begin{aligned} p(\theta|X_k) &= p(\theta, T|X_k) \\ &= p(\theta|T, X_k)p(T|X_k) \ . \end{aligned} \tag{6}$$

Together with (1), we obtain

$$p(T|X_k) = \prod_{i=0}^{N_k}\mathcal{P}(\lambda_i; n_k^i) \ . \tag{7}$$

Different association events with equal T are assumed to be uniformly distributed, i.e.,

$$\begin{aligned} p(\theta|T, X_k) &= \frac{1}{\binom{M_k}{n_k^1}\binom{M_k-n_k^1}{n_k^2}\cdots\binom{M_k-n_k^1-\cdots-n_k^{N_k-1}}{n_k^{N_k}}} \\ &= \frac{1}{M_k!}\prod_{i=0}^{N_k} n_k^i! \ . \end{aligned} \tag{8}$$



θ	Likelihood of θ
$[0,0,0]$	$3!\mathcal{P}(\lambda_0;3)(\rho)^3\mathcal{P}(\lambda_1;0)\mathcal{P}(\lambda_2;0)$
$[1,0,0]$	$2!\mathcal{P}(\lambda_0;2)(\rho)^2\mathcal{P}(\lambda_1;1)G_{11}\mathcal{P}(\lambda_2;0)$
$[1,1,0]$	$\mathcal{P}(\lambda_0;1)\rho 2!\mathcal{P}(\lambda_1;2)G_{11}G_{1,2}\mathcal{P}(\lambda_2;0)$
$[2,1,2]$	$\mathcal{P}(\lambda_1;1)G_{12}2!\mathcal{P}(\lambda_2;2)G_{21}G_{23}$
$\vdots$	$\vdots$

Fig. 1: In this example, there are two elliptical objects and three measurements. If all measurements are clutter, the corresponding association event is denoted as $[0,0,0]$. A further association hypotheses $\theta = [2,1,2]$ states that the first and third measurements are assigned to target 2 and the second measurement is associated with target 1. Note that the joint likelihood can be simplified in the form of (11). For example, the likelihood of $\theta = [2,1,2]$ is $\lambda_1 G_{12}\lambda_2 G_{21}\lambda_2 G_{23}$.

Substitution of (7) and (8) into (6) gives, after cancellation of $n_k^i!$,

$$p(\theta|X_k) = \frac{1}{M_k!}\prod_{i=0}^{N_k} e^{\lambda_i}(\lambda_i)^{n_k^i} \quad . \tag{9}$$

Applying Bayes rule, together with (3) and (9), the posterior of θ is

$$\begin{aligned} p(\theta|Y_k,X_k) &\propto p(Y_k|\theta,X_k)p(\theta|X_k) \ , & (10)\\ &\propto (\lambda_0)^{n_k^0}\rho^{n_k^0}\prod_{i=1}^{N_k}\prod_{y_j\in Y_k^i}\lambda_i G_{ij} \ . & (11)\end{aligned}$$

The marginal association probability that measurement j associated with target i is

$$\begin{aligned} \beta_{ij} &= p(\theta_j = i|Y_k,X_k) & (12)\\ &= \sum_{\theta \text{ with } \theta_i=j} p(\theta|Y_k,X_k). & (13)\end{aligned}$$

The key insight for deriving a compact closed-form expression for (12) under the Poisson model is as follows: According to [2], [3], the likelihood for the Poisson model corresponds to the likelihood used in the Probabilistic Multi-Hypotheses Tracker (PMHT) [21], i.e., multiple measurements are independently assigned to different targets, where the rate parameters λ_i represent the association probabilities. For this reason, the marginal association probabilities can significantly be simplified in the similar way as the *Many-2-1* approach in [22] and *LMIPDA* [23], i.e., we obtain

$$\begin{aligned} \beta_{ij} &\propto \lambda_i G_{ij}\prod_{m=1,m\neq j}^{M_K}\left(\sum_{n=1}^{N_k}\lambda_n G_{nm}+\lambda_0\rho\right) & (14)\\ &\propto \frac{\lambda_i G_{ij}}{S_j} \ , & (15)\end{aligned}$$

with

$$S_j = \sum_{n=1}^{N_k}\lambda_n G_{nj} + \lambda_0\rho.$$

The calculation of marginal association probabilities is linear in the number of objects and linear in the number of measurements, i.e., $O(N_k M_k)$.

III. SEQUENTIAL PDA MEASUREMENT UPDATE

After the marginal association probabilities have been determined, each target's state vector is updated in a PDA fashion by incorporating the association probabilities. In this work, the state vector update is based on the single extended object tracker in [12]. If the origin of the measurements is known, the tracker in [12] updates kinematic and shape parameters independently and sequentially. Given a measurement y^j which is assigned to the i-th target with probability β_{ij}, the traditional PDA updates state vector x^i and covariance C_x^i as [24]

$$\begin{aligned} x_j^i &= x_{j-1}^i + \beta_{ij}C_{xy}^{ij}(C_y^j)^{-1}(y^j - \bar{y}^j) \ , & (16)\\ C_{x_j}^i &= (1-\beta_{ij})C_{x_{j-1}}^i + \beta_{ij}\left(C_{x_{j-1}}^i - C_{xy}^{ij}(C_y^j)^{-1}(C_{xy}^{ij})^{\mathrm{T}}\right) & \\ &\quad +\beta_{ij}(1-\beta_{ij})\underbrace{C_{xy}^{ij}(C_y^j)^{-1}\beta_{ij}(y^j-\bar{y}^j)}_{\bullet}(\bullet)^{\mathrm{T}} & \\ &= C_{x_{j-1}}^i - \beta_{ij}\left(C_{xy}^{ij}(C_y^j)^{-1}(C_{xy}^{ij})^{\mathrm{T}}\right) & \\ &\quad +\beta_{ij}^3(1-\beta_{ij})\underbrace{C_{xy}^{ij}(C_y^j)^{-1}(y^j-\bar{y}^j)}_{\star}(\star)^{\mathrm{T}} \ , & (17)\end{aligned}$$

where $\bar{y}^j$ is the expected value of measurement y^j, C_{xy}^{ij} and C_y^j are the cross-covariance between x_{j-1}^i and y^j and covariance of y^j, respectively, and x_j^i and $C_{x_j}^i$ are the state and covariance of target i updated with measurements $1,\ldots,j$. For extended objects, we can directly apply (16) and (17) for the kinematic and shape parameters. The resulting formulas are similar to (16) and (17), except

- state x^i need to be replaced by kinematic variable r^i and shape variable p^i,
- measurement y^j refers to the pseudo-measurement for the shape update,
- covariance C_y^j and cross-covariance C_{xy}^{ij} are given in [12].

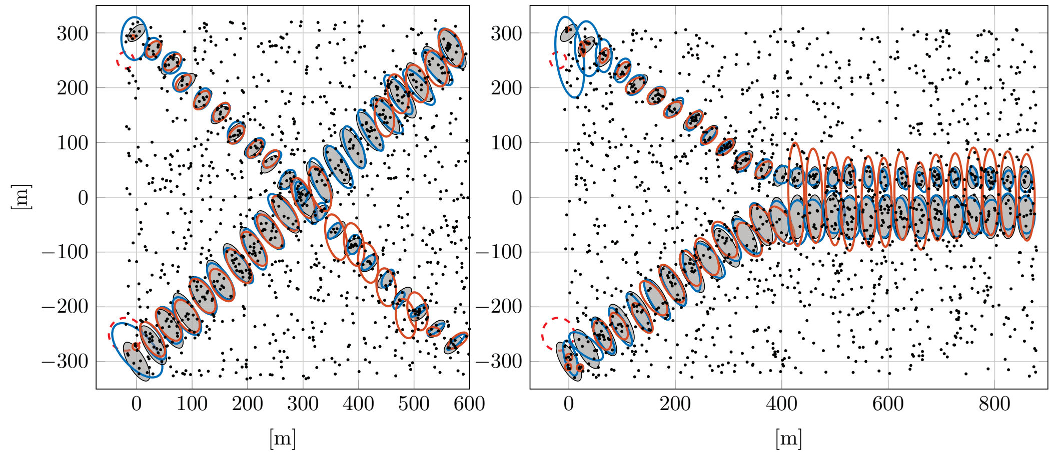


Fig. 2: One example run of the simulated scenario. Gray ellipses illustrate the ground truth. Red dashed ellipses show the initial estimates. Cyan and Orange ellipses are the estimates from our method (ET-JPDA) and the GGIW-PHD approach.

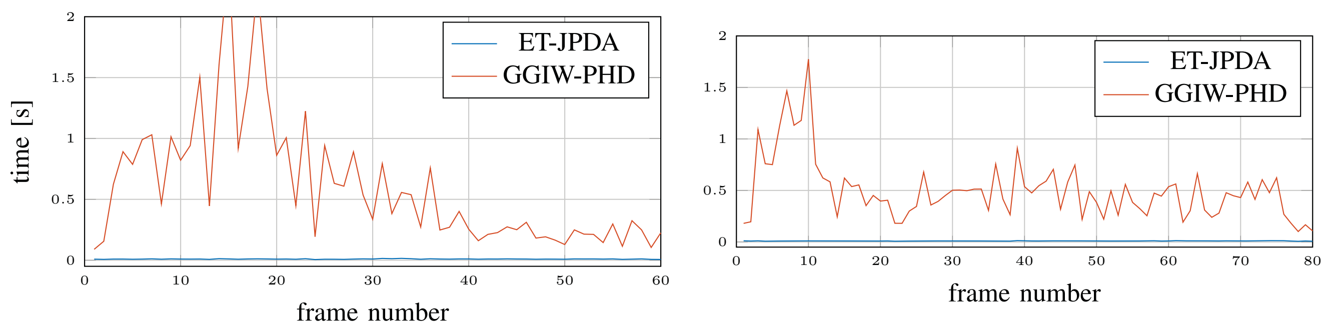


Fig. 3: Mean computation time of the ET-JPDA and GGIW-PHD method for 500 Monte Carlo runs.

IV. SIMULATION

In this section, we apply the proposed method on two simulated scenarios and compare it with the Gaussian Mixture PHD filter for extended object tracking (GGIW-PHD) [25].

In the first scenario, we simulate two targets whose trajectories cross and in the second scenario, two spatially close trajectories are considered. In both scenarios, the clutter is Poisson distributed with mean of 40. The semi-axes lengths of the ground truth ellipses are $[40, 15]$ and $[20, 10]$, respectively. The number of measurements for the targets is Poisson distributed with mean of nine and seven. Both move according to a constant velocity model. Simulated trajectories and one example run of the comparison are given in Fig. 2. We can observe that GGIW-PHD tends to lost tracks or merges close targets when they are crossing or approaching each other due to the explicit clustering. The ET-JPDA method employs the predicted measurement likelihood for association and could differentiate targets in both challenging scenarios. Note that GGIW-PHD estimates the number of targets while the ET-JPDA assumes the cardinally is known. Hence, the GGIW-PHD solves a significantly more complex problem. We performed the comparison for 500 Monte Carlo runs and depict the computation time of both method in Fig. 3. ET-JPDA requires no clustering and partitioning, it is linear in number of measurements and targets. Both methods are implemented in MATLAB and performed on a Windows 7 PC with Intel(R) i5 (3.3GHz) processor.

V. CONCLUSION

In this work, we proposed a linear exact JPDA solution for extended target tracking based on the PPP model for extended targets. It requires no permutation of possible measurement set combination. In the current version of our algorithm, the number of tracked targets is fixed. Hence, in the future, we will incorporate the existence likelihood and focus on developing a fast Joint Integrated Probabilistic Data Association (JIPDA) filter for extended target tracking.

ACKNOWLEDGMENT

This work was supported by the German Research Foundation (DFG) under grant BA 5160/1-1.

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