BAHIR DAR UNIVERSITY

DEPARTMENT OF MATHEMATICS

BASIC MATHEMATICS FOR NATURAL SCIENCES (Math 1011)

WORKSHEET I ON CHAPTER 1 AND CHAPTER 2

1) Determine whether each of the following sentences is a proposition or not

   a) Every integer is a real number.
   b) Bahir Dar is a city in Ethiopia.
   c) $x^2 \geq 0$ for all real numbers x.
   d) Thank you for your cooperation!
   e) The sum of two odd numbers is even.
   f) The first human being lived in Africa.
   g) 9 is a prime number.
   h) I am determined to score A in Math 1011.

2) Let p ≡ It is cold, q ≡ It rains. Write each of the following in symbolic form

   a) It is neither cold nor raining.
   b) It rains whenever it is cold.
   c) It rains only if it is cold.
   d) It never rains when it is cold.

3) Let p, q and r have the truth value F, F and T respectively. Determine the truth values of each of the following:

   a) $(p \wedge \neg r) \Rightarrow \neg q$
   b) $(\neg p \wedge \neg q) \Leftrightarrow \neg(p \vee q)$
   c) $(p \wedge q) \Rightarrow \neg(q \vee r)$
   d) $[p \Rightarrow (q \vee r)] \Leftrightarrow [(p \wedge \neg q) \Rightarrow r]$

4) Determine the truth value of p in each of the following cases.

   a) $(p \Rightarrow \neg q) \vee r$ *is false*
   b) $(q \Leftrightarrow p) \Rightarrow (r \Rightarrow q)$ is false
   c) $[(p \vee q)] \Rightarrow [(p \wedge q) \vee q]$ *is false*
   d) $(p \Rightarrow q) \Rightarrow (r \Rightarrow q)$ *is false*

5) Show that

   a) $[(p \wedge q) \Rightarrow r] \equiv [p \Rightarrow (q \Rightarrow r)]$
   b) $\neg(p \Leftrightarrow q) \equiv \neg p \Leftrightarrow q \equiv p \Leftrightarrow \neg q$

6) Determine whether the following are tautologies, contradictions or neither.

   a) $(p \Rightarrow q) \Leftrightarrow (q \Rightarrow p)$
   b) $(p \Rightarrow q) \Rightarrow \neg(p \wedge \neg q)$
   c) $(\neg q \wedge (p \Rightarrow q)) \Rightarrow \neg p$
   d) $[p \Rightarrow (q \vee r)] \Leftrightarrow [(p \wedge q) \Rightarrow r]$

7) Let $U = N$, $q(x)$: x is prime, and $p(x)$: $x^2 + 1$ is composite. Determine the truth value of the following.

   a) $(\forall x)(p(x) \Rightarrow q(x))$
   b) $(\exists x)(q(x) \Rightarrow p(x))$
   c) $(\forall x)(p(x) \vee q(x))$
   d) $(\exists x)(\neg p(x) \wedge q(x))$

8) Let $U = \mathbb{R}$. Determine the truth value of the following.

   a) $(\forall x)(\forall y)(x < y \vee x \geq y)$
   b) $(\forall x, y)(\exists z)(x + y + z = 0)$
   c) $(\exists x)(\exists y)(x \leq y)$
   d) $(\forall x)(\exists y)(x + y = 2 \vee 2x - y = 1)$
   e) $(\exists x)(\forall y)(yx = 1)$

9) Let { 1, 2, 3 } be the universe. Determine the truth value of the following:

a) $(\forall x)(\forall y)[x^2 + 2y < 10]$ c) $(\exists x)(\forall y)[x^2 + 2y < 10]$

b) $(\forall x)(\exists y)[x^2 + 2y < 10]$ d) $(\exists x)(\exists y)[x^2 + 2y < 10]$

10) Write the negation of the following sentences where U = R

a) $(\exists y)(\forall x)[xy = x]$ b) $(\forall x)(\forall y)[x < y \vee x \geq y]$ c) $(\forall x)(\exists y)[x + y = 0]$

11) Write the following sentences in symbolic form.

a) Somebody is a child of somebody.

b) Somebody is a child of everybody.

c) Everybody has a parent.

d) Everybody is a child of everybody.

e) If all students at BDU are Ethiopians and no Ethiopian is a racist then no student at BDU is a racist.

12) Identify whether the following arguments are valid or not.

a) $p \Rightarrow \sim q,\ \sim r \vee q,\ r \mid \sim p$

b) $p \Rightarrow q,\ \sim p \Rightarrow r,\ r \Rightarrow s \mid \sim q \Rightarrow s$

c) $p \vee \sim q,\ (t \vee s) \Rightarrow (p \vee r),\ \sim r \vee (t \wedge s),\ p \Leftrightarrow (t \vee s) \mid (p \wedge r) \Rightarrow (q \wedge r)$

d) If I do not pay my income taxes, then I file for an extension or I am a felon. I am not a felon and I did not file for an extension. Therefore, I paid my income taxes.

e) If I want to be a lawyer, then I need to study logic. If I do not want to be a lawyer, then I do not like to argue. Therefore, if I like to argue, then I need to study logic.

f) If I like mathematics, then I will study. Either I do not study or I pass mathematics. Passing mathematics is sufficient condition for promoting. Therefore, if I like mathematics, then I promote.

13) List the elements of the following sets:

a) $\{x / x \in N \wedge x \text{ is a factor of } 12\}$ b) $\{x / x \in Z \wedge 2 \leq |x| < 10\}$

14) If U={0,1,2,3,4,5,6,7,8,9,10} and $A \subset B \subseteq U$. Find the maximum and minimum possible number of elements in B-A.

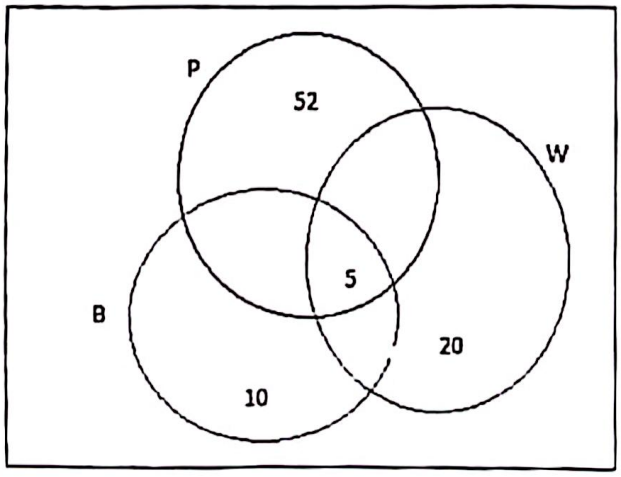
15) Prove each of the following by using properties of set operations.

a) A-(A-B) =A∩B b) (A-C)-(B-C) = (A-B)-C

16) Determine the power set of $A = \{-1, 0, 1, 2\}$.

17) There is a group of 80 persons who can drive Ford Focus or X8 BMW car or both. Out of these, 35 can drive Ford Focus and 60 can drive X8 BMW car. Find how many can drive both Ford Focus and X8 BMW cars? How many can drive Ford Focus only? How many can drive X8 BMW car only?

18) A survey of fast food was conducted on 145 teenagers. Suppose $n(P)$ represents the number of teenagers who liked Pizza, $n(B)$ represents the number of teenagers who liked Burger and $n(W)$ represents the number of teenagers who liked Whitefish sandwich as shown in the Venn diagram below.



If $n(P) = 75$, $n(B) = 60$, and $n(W) = 68$, then

a) Complete the Venn diagram

b) How many teenagers liked

i) Pizza, Burger and Whitefish?

ii) Burgers not Whitefish?

iii) Only Burgers and Pizza?

iv) Pizza or Whitefish?

v) None of these types of foods?

19) Determine GCF and LCM of

a) 60 and 72

b) 216 and 300

c) 45, 180 and 225

d) 308 and 1176

20) Use Principle of Mathematical induction to prove the following:

a) $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for any $n \in N$.

b) $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for any $n \in N$.

c) $2^{2n} - 1$ is divisible by 3 for any $n \in N$.

d) $n^3 - n$ is divisible by 3 for any natural number $n \geq 2$.

e) $2n + 1 < 2^n$ for any natural number $n \geq 3$.

21) Determine the lower bound, the upper bound, glb and lub (if any) for the following sets:

a) $A = \left\{\frac{1}{n}\right\}$ for $n \in \mathbb{N}$. b) $B = \left\{1 - \frac{1}{n}\right\}$ for $n \in \mathbb{N}$. c) $C = \left\{\frac{2^n - 5}{3^n + 3}\right\}$ for $n \in \mathbb{N}$