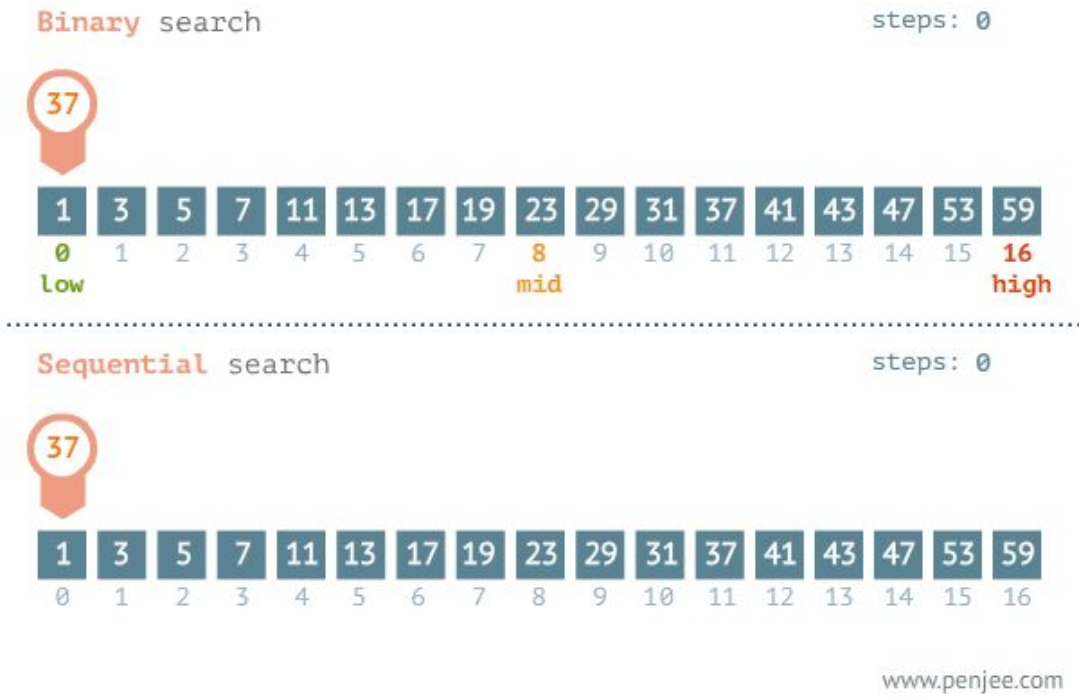
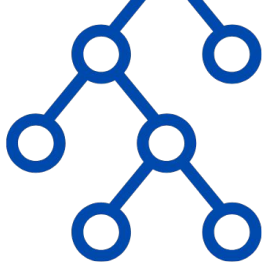
Binary Search



Lecture Flow

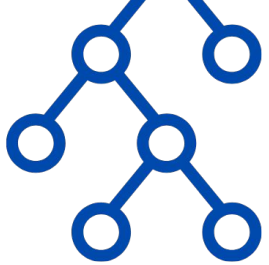
1. Prerequisites
2. Introduction
3. Naive Approach
4. Binary Search
5. Variants
6. Common Pitfalls
7. Quote of the day

Pre-requisites



1. Array
2. Sorting
3. Loops and conditional statements

Introduction

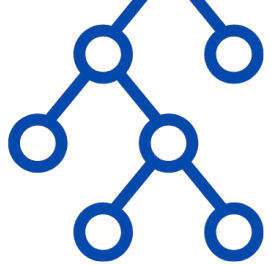


Let’s play a game.

Introduction

- Binary Search is one of the most fundamental and useful algorithms in Computer Science.
- It describes the process of searching for a specific value in an ordered collection.

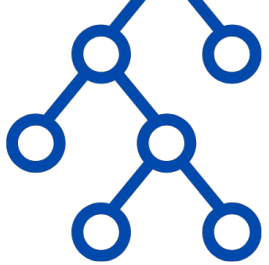
Naive Approach



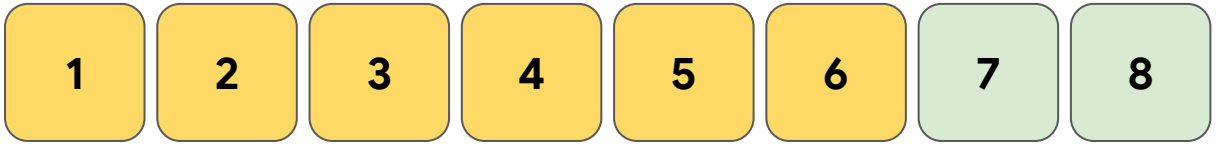
Given a sorted list of numbers search for 6

1	2	3	4	5	6	7	8

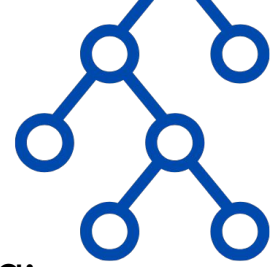
Naive Approach



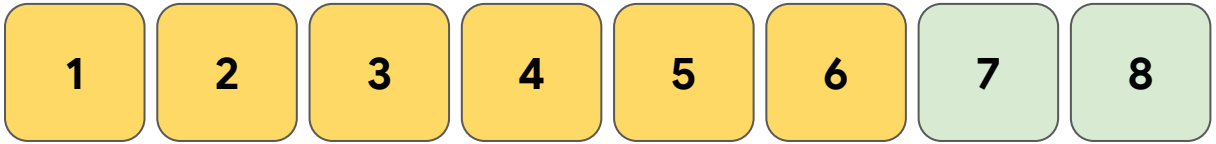
You iterate until we find the number. It takes 6 steps.



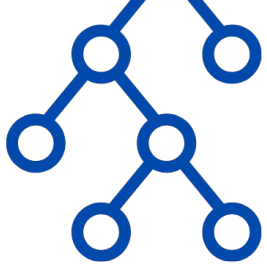
Naive Approach - Time Complexity



Worst case scenario: when the element to be searched is at the end. Eg: when target is 8. $O(n)$



Binary Search

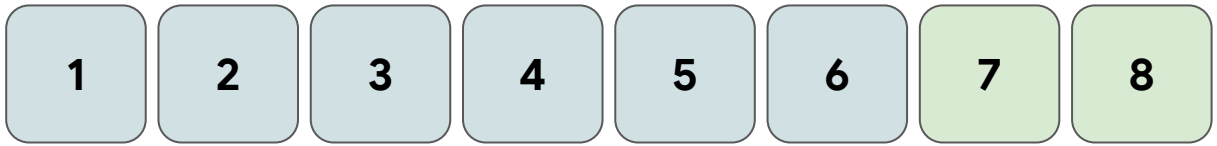


We know the numbers are sorted. Is there a room for optimization?

1	2	3	4	5	6	7	8

Binary Search

Let's call the region of the list we are looking for the number the search space.
We will start with the whole list as the search space.



Binary Search

Let's pick a number around the middle. If the number is smaller this number, we make the search space up to that number.

Otherwise, we make the search space above this number

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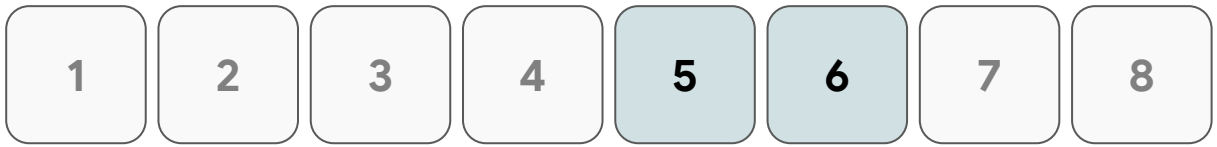


Binary Search

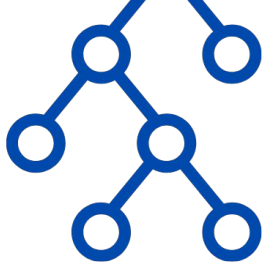
Let's pick a number around the middle. If the number is smaller this number, we make the search space up to that number.

Otherwise, we make the search space above this number: 3 steps

Pair Programming

Binary Search

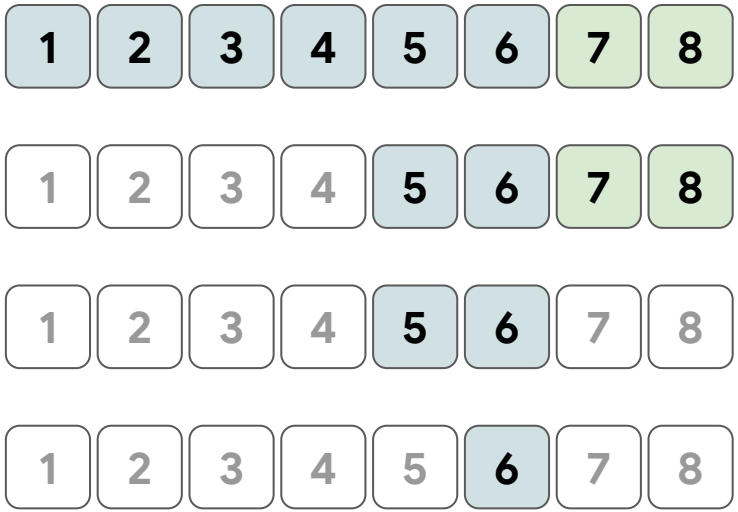
Binary Search - Implementation



```
def binary_search(arr, x):
    low, high = 0, len(arr) - 1
    while low <= high:
        mid = (high + low) // 2
        if arr[mid] < x:
            low = mid + 1
        elif arr[mid] > x:
            high = mid - 1
        else:
            return True
    return False
```

Binary Search - Time Complexity

What is the number of steps needed to make the search space size exactly 1?
On each iteration we are halving the it.

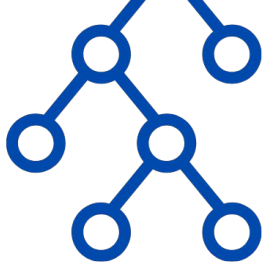



```
1*2*2* ... *2    =  problem size
Number of steps
     Times
```

```
2^(number of steps) = problem size
```

```
Number of steps = log2(problem size)
```

Note



```
mid = (high + low) / 2
```

Unlike in python, this could result in an overflow, in lower-level languages. So use this instead.

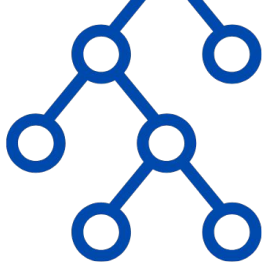
```
mid = low + (high - low) / 2
```

A more general way of thinking

Not all binary search problems are about finding the right position in a sorted list.

Problem Link

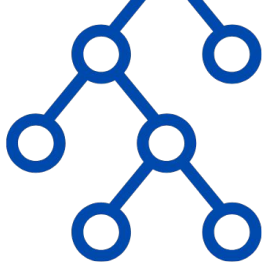
A more general way of thinking



In this problem we are looking for the first time we get a defect.

Not Defect	Not Defect	Not Defect	Not Defect	Not Defect	Defect	Defect	Defect

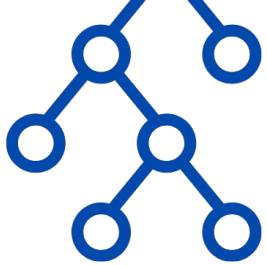
A more general way of thinking



Cut the search space by half every time

Not Defect	Not Defect	Not Defect	Not Defect	Not Defect	Defect	Defect	Defect

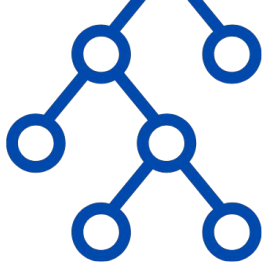
A more general way of thinking



Cut the search space by half every time

Not Defect	Not Defect	Not Defect	Not Defect	Not Defect	Defect	Defect	Defect

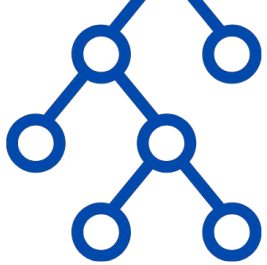
A more general way of thinking



Cut the search space by half every time

Not Defect	Not Defect	Not Defect	Not Defect	Not Defect	Defect	Defect	Defect

A more general template



```
def binarySearch(low=1, high=1, key = lambda x: True):

    while low <= high:
        mid = low + (high - low)//2
        if key(mid):
            high = mid-1
        else:
            low = mid+1
    return low
```

Variants

Variant 1 - Over an input space

This is the variant we have seen, where we search for a particular value in the given input

In other words, the search space is given explicitly by the problem.

Variant 1 - Over an input space

Pair Programming

Problem Link

Variant 2 - Over an output space

The search is applied over the a possible output range. For every choice there is usually a linear check to validate the choice

This is trickier than the first variant.

Variant 2 - Over an output space

Pair Programming

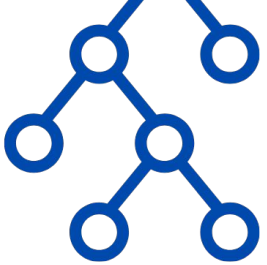
Problem Link

Pair Programming

1. Find Minimum in Rotated Sorted Array
2. Koko Eating Bananas

Python Bisect Library

Bisect Lib – `bisect_left`

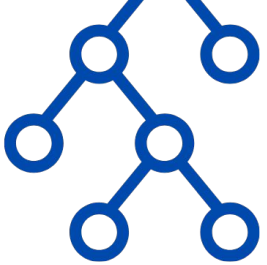


- returns first position of a number which is >= target

For example, `array = [1, 3, 3, 5, 6, 8, 17]`

```
>>> print(bisect_left(array,  3))  → 1
>>> print(bisect_left(array,  0))  → 0
>>> print(bisect_left(array,  7))  → 5
```

Bisect Lib – `bisect_right`



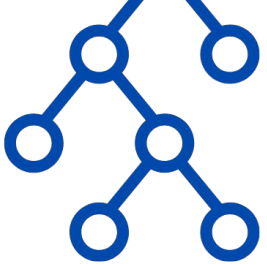
- returns first position of a number which is > target

For example, `array = [1, 3, 3, 5, 6, 8, 17]`

```
>>> print(bisect_right(array,  3))  → 3
>>> print(bisect_right(array,  0))  → 0
>>> print(bisect_right(array,  7))  → 5
```

Common Pitfalls

Common Pitfalls



- Checking the right condition for loop termination;
  `low < high vs low <= high vs low + 1 < high`
- Off-by-one on the search space
- Un reachable loop-termination condition

Practice Problems

1. H-Index II - LeetCode
2. Search a 2D matrix
3. Heaters
4. The Meeting Place Cannot Be Changed

Quote of the Day

“Believe you can and you're halfway there.”

~ ***Theodore Roosevelt***