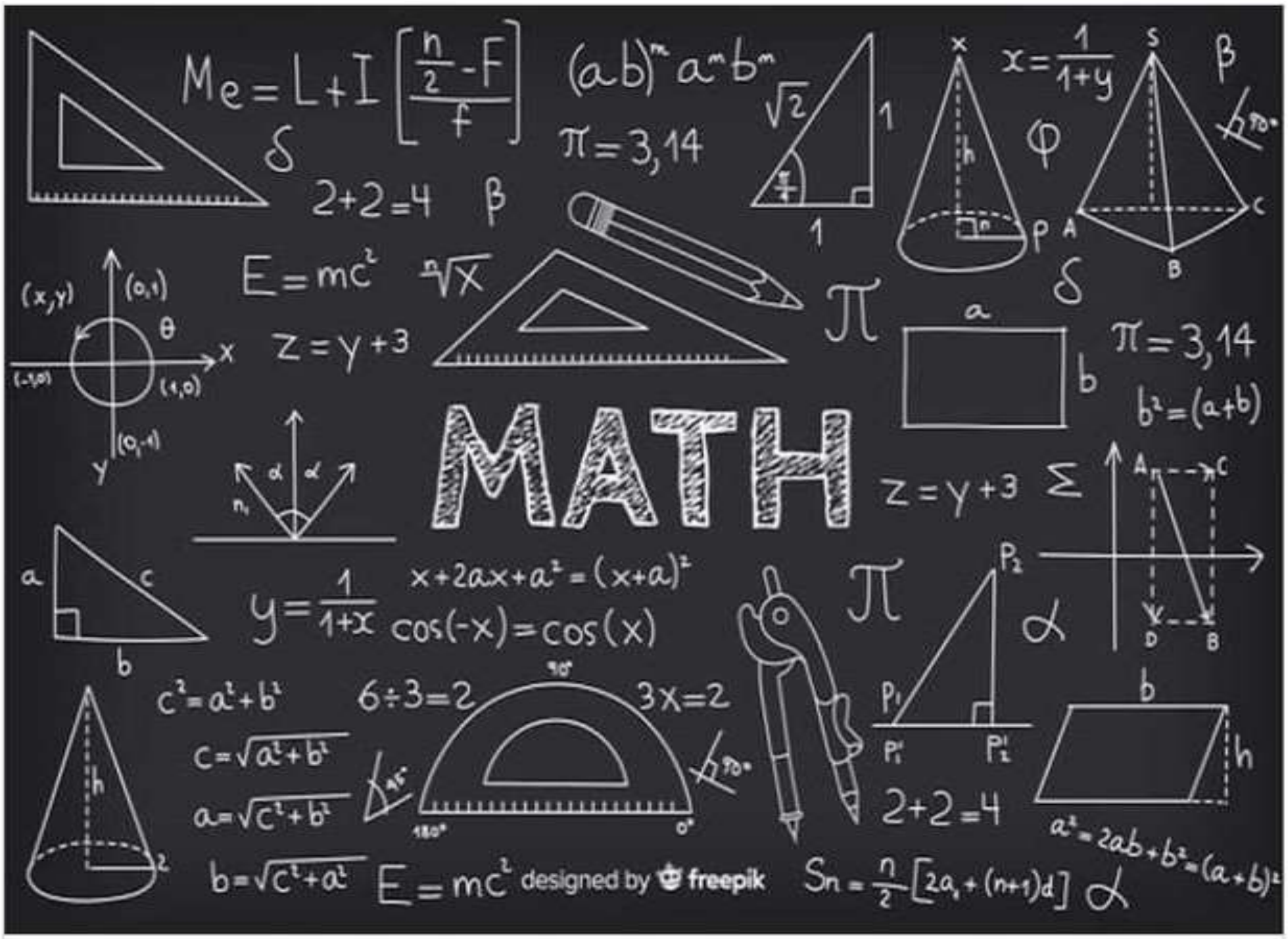


Mathematics and SAT (Quantitative reasoning) EUEE questions with detail solutions (2009E.C- 2013E.C)

Prepared by: - Haftu Ebuy
September, 2022

Prepared by: - Haftu Ebuy

Email: - haftuebuy3134@gmail.com

You Tube: - calculustube

Table of Contents

Most frequent topics in EUEE examination

Grade 11 – unit one – further on relations and functions

- Types of functions

A function f: A→B is said to be

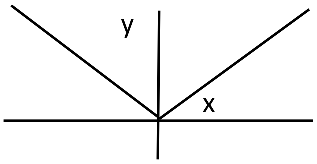
i. Odd if and only if for any x∈A, f (-x) =- f(x). Example f(x) =5x, f (2) =10, f (-2) =-10=-f (2).

ii. Even if and only if for any x∈A, f (-x) =f(x). Example f(x) =2x^2+5, f (3) =23=f (-3).

- Absolute value function

$$|x| = \begin{cases} x \; if \; x > 0 \\ 0 \; if \; x = 0 \\ -x \; if \; x < 0 \end{cases} \qquad \text{Examples,} \begin{pmatrix} |5| = 5 \\ |0| = 0 \\ |-5| = -(-5) = 5 \end{pmatrix}$$

f(x)= $|x|$



- Greatest integer function

$\lfloor x \rfloor$ = Is the greatest integer less than or equal to x.

$$\text{Examples} \begin{pmatrix} \lfloor 2.56 \rfloor = 2 & \lfloor 2 \rfloor = 2 \\ \lfloor -4.3 \rfloor = -5 & \lfloor -3 \rfloor = -3 \\ \lfloor 0 \rfloor = 0 & \lfloor 25 \rfloor = 25 \end{pmatrix}$$

- Signum function

$$\text{Sgn (x)} = \begin{cases} 1 \; if \; x > 0 \\ 0 \; if \; x = 0 \\ -1 \; if \; x < 0 \end{cases} = \begin{cases} \frac{|x|}{x} if \; x \neq 0 \\ 0 \; if \; x = 0 \end{cases} \qquad \text{Examples} \begin{pmatrix} sgn\,(3) = 1 \\ sgn\,(-5) = -1 \\ sgn\,(0) = 0 \end{pmatrix}$$

- Classification of functions

i. One to one function

A function is one to one function if x_1=x_2⇒f(x_1) = f(x_2) and x_1≠x_2⇒f(x_1) ≠f(x_2)

Examples a. f(x) =x^2

2≠-2, f(2)=4=f(-2)

f is not one to one function

b. f(x) =2x + 3

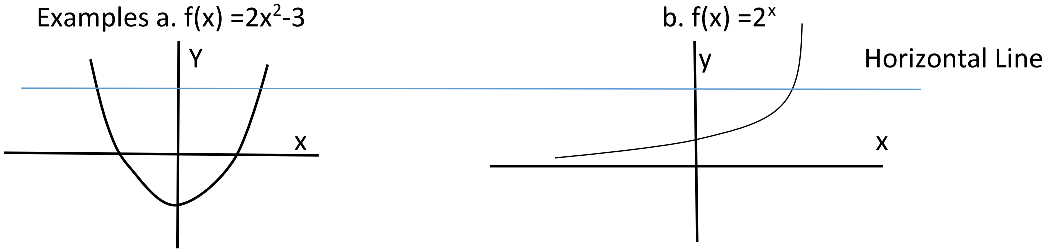
if x_1=x_2 then, f(x_1) = f(x_2)

if x_1≠x_2 then, f(x_1) ≠f(x_2)

f is one to one function.

Horizontal line test – if a function is one to one function then any horizontal line crosses the graph at most once.

Examples a. f(x) =2x^2-3 b. f(x) =2^x

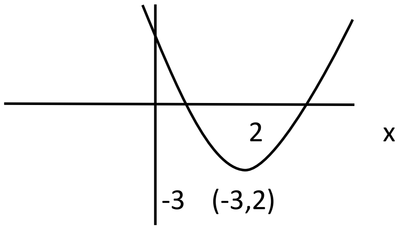


Not one to one function one to one function

ii. On to function

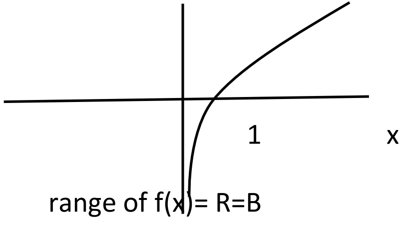
A function f: A→B is said to be on to if range of f=B

Examples a. f: R→R, f(x) = (x-2)^2-3



Range of f(x)=$\{y: y \geq -3\} \neq B$, not on to

b. f: R→R, f(x) = $log\,\frac{x}{3}$



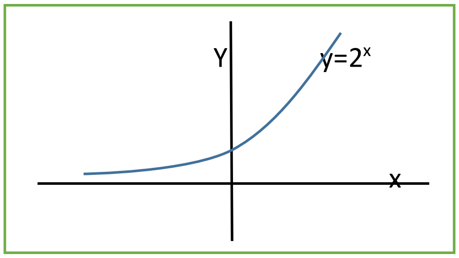
range of f(x)= R=B

on to function

iii. One to one correspondence

A function is said to be one to one correspondence if it is both one to one and on to function.

Examples f: R→(0,∞) , f(x) = 2^x



- if $x_1=x_2$ then, $f(x_1) = f(x_2)$

if $x_1 \neq x_2$ then, $f(x_1) \neq f(x_2)$

f is one to one function

- Range of f is (0,∞)=B
- F is one to one correspondence.

- Inverse of a function

- Domain of f = Range of f^{-1}
- Range of f = Domain of f^{-1}
- To find inverse, interchange x and y, then find y in terms of x.

Examples, find the inverse of

a. f(x) =- 5x+13

y= -5x +13

x= -5y+13

5y= -x+13

y=$\frac{-x+13}{5}$

f^{-1}(x)=$\frac{-x+13}{5}$

b. f(x) =log_3^{2x}

y=log_3^{2x}

x=log_3^{2y}

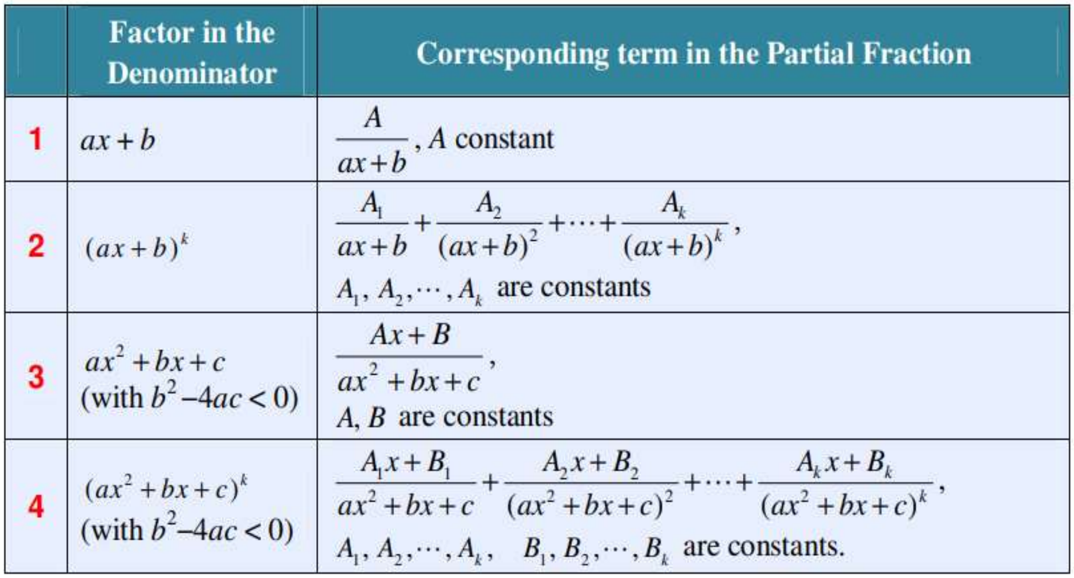
2y=3^x

y=$\frac{3^x}{2}$

f^{-1}(x)=$\frac{3^x}{2}$

Grade 11 –unit two – rational expressions and rational functions

- **decomposition of rational expressions in to partial fraction**



	Factor in the Denominator	Corresponding term in the Partial Fraction
1	$ax+b$	$\frac{A}{ax+b}$, A constant
2	$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$, $A_1, A_2, \cdots, A_k$ are constants
3	ax^2+bx+c (with $b^2-4ac<0$)	$\frac{Ax+B}{ax^2+bx+c}$, A, B are constants
4	$(ax^2+bx+c)^k$ (with $b^2-4ac<0$)	$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$, $A_1, A_2, \cdots, A_k$, $B_1, B_2, \cdots, B_k$ are constants.

Examples a. $\frac{3x+1}{x^2-1}=\frac{3x+1}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}$, 3x+1=A(x+1)+B(x-1)

$\frac{3x+1}{x^2-1}=\frac{3x+1}{(x-1)(x+1)}=\frac{1}{x-1}+\frac{2}{x+1}$ 3x+1=Ax+A+Bx=B

A+B=3, A-B=1⇒A=1 and B=2

b. $\frac{2x^2+4}{x^3-2x^2+3x}=\frac{2x^2+4}{x(x^2-2x+3)}=\frac{A}{x}+\frac{Bx+C}{x^2-2x+3}=\frac{\frac{4}{3}}{x}+\frac{\frac{-2}{3}x+\frac{8}{3}}{x^2-2x+3}=\frac{4}{3x}+\frac{-2x+8}{3(x^2-2x+3)}$

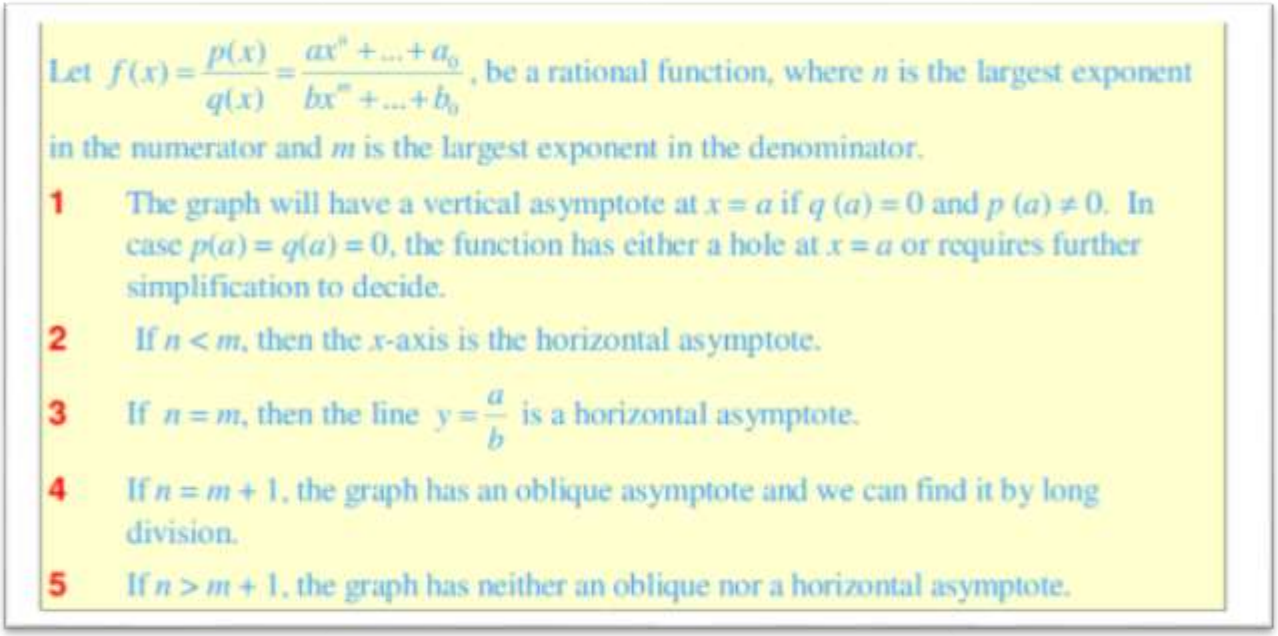
$2x^2+4=A(x^2-2x+3)+(Bx+C)x$

$2x^2+4=Ax^2-2Ax+3A+Bx^2+Cx$

2=A+B, 0= -2A+C, 4=3A, A=4/3, B=-2/3, C=8/3

- **Graphs of rational functions**

Asymptote rule



Let $f(x)=\frac{p(x)}{q(x)}=\frac{ax^n+...+a_0}{bx^m+...+b_0}$, be a rational function, where n is the largest exponent in the numerator and m is the largest exponent in the denominator.

1 The graph will have a vertical asymptote at $x=a$ if $q(a)=0$ and $p(a)\neq 0$. In case $p(a)=q(a)=0$, the function has either a hole at $x=a$ or requires further simplification to decide.

2 If $n<m$, then the x-axis is the horizontal asymptote.

3 If $n=m$, then the line $y=\frac{a}{b}$ is a horizontal asymptote.

4 If $n=m+1$, the graph has an oblique asymptote and we can find it by long division.

5 If $n>m+1$, the graph has neither an oblique nor a horizontal asymptote.

Examples

1. Find the asymptotes of f(x)=$\frac{2x^3+2x^2+3x}{x^2+x}$

$$f(x)=\frac{2x^3+2x^2+3x}{x^2+x}= f(x)=\frac{x(2x^2+2x+3)}{x(x+1)}$$

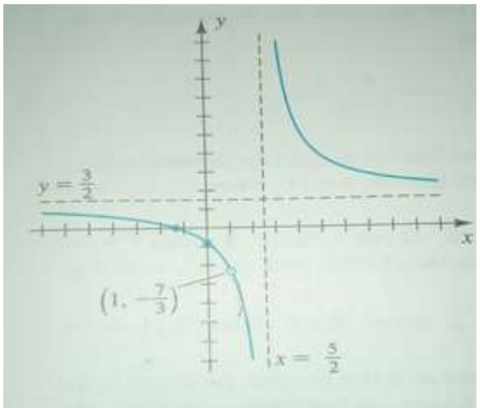
- f(0)=$\frac{0}{0}$, the graph has hole at x=0
- f(-1)=$\frac{-3}{0}$, x=-1 is vertical asymptote.
- Degree of numerator>degree of denominator, the graph has no horizontal asymptote.
-

$$\begin{array}{r|l} & 2x \\ \hline (x^2+x) & 2x^3+2x^2+3x \\ & \underline{2x^3+2x^2} \\ & \qquad\qquad 3x \end{array}$$

Y=2x is oblique asymptote

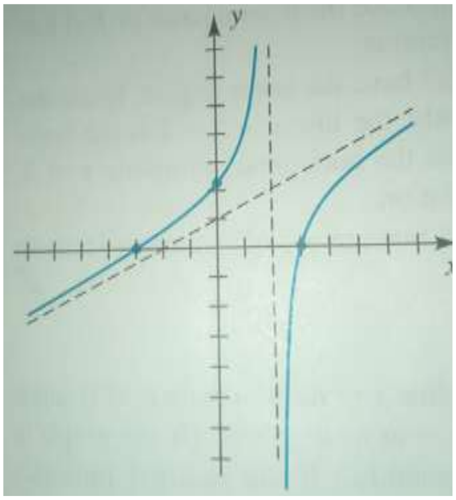
2. Sketch the graphs of

a. f(x)=$\frac{(3x+4)(x-1)}{(2x-5)(x-1)}$



- f(1)=$\frac{0}{0}$, the graph has hole at x=1
- f(5/2)=$\frac{69/2}{0}$, x=5/2 is vertical asymptote
- take coefficients of x^2,
  y=3/2 is horizontal asymptote
- y=0, x=-4/3, (-4/3,0) is x intercept
- x=0, y=-4/5, (0,-4/5) is y intercept

b. f(x)=$\frac{x^2-9}{2x-4}$



- f(2)=$\frac{-5}{0}$, x=2 is vertical
- Degree of numerator > degree of denominator
  The graph has no horizontal asymptote.

$$\begin{array}{r|l} & \frac{1}{2}x+1 \\ \hline (2x-4) & x^2-9 \\ & \underline{x^2-2x} \\ & 2x-9 \\ & \underline{2x-4} \\ & -5 \end{array}$$

- y=$\frac{1}{2}$x+1 is oblique asymptote
- x=0, y=9/4, (0, 9/4) is x intercept
- y=0, x=±3, (±3,0) are y intercepts

Grade 11- unit three- coordinate geometry

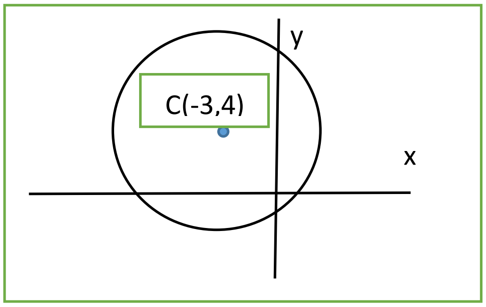
- **conic section**

i. circles

Equations $x^2+y^2=1$ if C (0,0)

$(x-h)^2+(y-k)^2=1$ if C(h,k)

Example, find center and radius of the circle with equation $x^2+y^2+6x-8y-11=0$



$x^2+y^2+6x-8y-11=0$

$x^2+6x+ y^2-8y=11$

$x^2+6x+(b/2)^2-(b/2)^2+y^2-8y+(b/2)^2-(b/2)^2=11$

$x^2+6x+(6/2)^2-(6/2)^2+y^2-8y+(-8/2)^2-(-8/2)^2=11$

$x^2+6x+(3)^2-(3)^2+y^2-8y+(-4)^2-(-4)^2=11$

$x^2+6x+9-9+y^2-8y+16-16=11$

$x^2+6x+9 +y^2-8y+16=11+9+16$

$(x+3)^2+(y-4)^2=36$ center C(-3,4), radius r=6

ii. Parabola

Equations

a. $\left.\begin{cases} x^2 = 4py & \text{if the vertex is } (0,0) \\ (x-h)^2 = 4p(y-k) & \text{if the vertex is } (h,k) \end{cases}\right\}$, the parabola opens up.

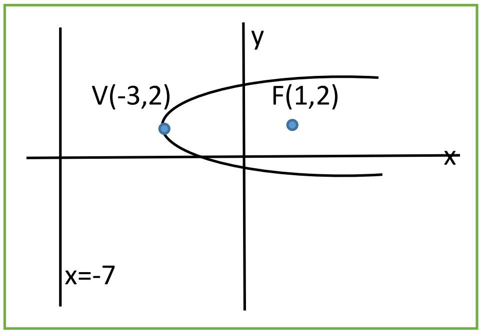
b. $\left.\begin{cases} x^2 = -4py & \text{if the vertex is } (0,0) \\ (x-h)^2 = -4p(y-k) & \text{if the vertex is } (h,k) \end{cases}\right\}$, the parabola opens down.

c. $\left.\begin{cases} y^2 = 4px & \text{if the vertex is } (0,0) \\ (y-k)^2 = 4p(x-h) & \text{if the vertex is } (h,k) \end{cases}\right\}$, the parabola opens to the right.

d. $\left.\begin{cases} y^2 = -4px & \text{if the vertex is } (0,0) \\ (y-k)^2 = -4p(x-h) & \text{if the vertex is } (h,k) \end{cases}\right\}$, the parabola opens to the left.

Examples, given equation of a parabola

$(y-2)^2=16(x+3)$, the parabola opens to the right.



- Vertex, V(h,k)=(- 3,2)
- Latus rectum, 4p=16, p=4
- Focus, F(h+p,k)

F((-3+4,2)

F(1,2)

- Directrix, x=h-p=-3-4=-7

iii. Ellipse

Equations

a. $\left.\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 & \text{if the center is } (0,0) \\ \frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 & \text{if the vertex is } (h,k) \end{cases}\right\}$, horizontal ellipse

b. $\left.\begin{cases} \frac{y^2}{a^2}+\frac{x^2}{b^2}=1 & \text{if the center is } (0,0) \\ \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1 & \text{if the vertex is } (h,k) \end{cases}\right\}$, vertical ellipse.

a>b

$$a^2 = b^2 + c^2$$

$$e = \frac{c}{a}$$

Example, given equation of an ellipse $\frac{(x-2)^2}{25}+\frac{(y+3)^2}{9}=1$, the ellipse is horizontal.

- Center C(h,k)=(2,-3)
- $a^2=25$, a=5
- $b^2=9$, b=3 $a^2=b^2+c^2$

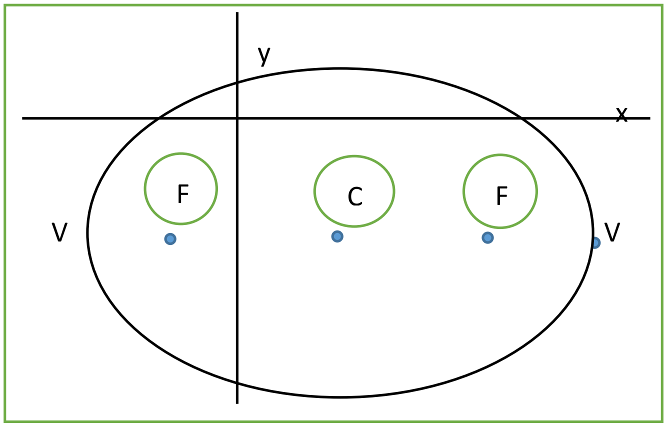
  $c=\sqrt{a^2-b^2}=\sqrt{5^2-3^2}=\sqrt{25-9}=\sqrt{16}=4$
- Vertex V(h-a,k), (h+a,k)

  V(2-5,-3), (2+5,-3)

  V(-3,-3), (7,-3)
- Focus F(h-c,k), (h+c,k)

  F(2-4,-3), (2+4,-3)

  F(-2,-3), (6,-3)



iv. Hyperbola

Equations

a. $\begin{cases}\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 & if\ the\ center\ is\ (0,0)\\ \frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 & if\ the\ vertex\ is\ (h,k)\end{cases}$, horizontal transverse axis.

b. $\begin{cases}\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 & if\ the\ center\ is\ (0,0)\\ \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1 & if\ the\ vertex\ is\ (h,k)\end{cases}$, vertical transverse axis.

$c^2=a^2+b^2$

$e=\frac{c}{a}$

Example, given an equation of a hyperbola, $\frac{(y+3)^2}{16}-\frac{(x+2)^2}{9}=1$, vertical transverse axis

- Center C(h,k)=(-2,-3)
- $b^2=16$, a=4
- $b^2=9$, b=3 $c^2=a^2+b^2$

  $c=\sqrt{a^2+b^2}=\sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}=5$
- Vertex V(h,k-a), (h,k+a)

  V(-2,-3-4), (-2,-3+4)

  V(-2,-7), (-2,1)
- Focus F(h,k-c), (h,k+c)

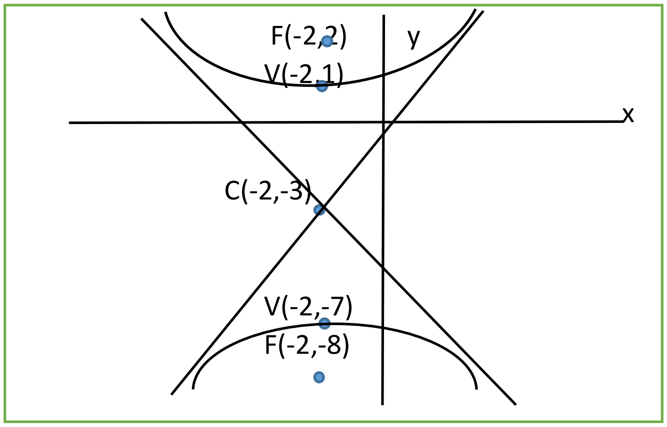
  F(-2,-3-5), (-2,-3+5)

  F(-2,-8), (-2,2)
- Asymptote,

  $y-k=\pm\frac{a}{b}(x-h)$

  $y+3=\pm\frac{4}{3}(x+2)$

  $y=\frac{4}{3}x-\frac{1}{3}$ or $y=\frac{-4}{3}x-\frac{17}{3}$



- **Parallel and perpendicular lines**

Two lines are said to be parallel if they have the same slope and perpendicular if the product of their slopes is -1.

Grade 11- unit four – mathematical reasoning

- Statement (proposition) – a statement which has a truth value (T or F).

- **Logical connectives**

Statement		Negation		Conjunction	Disjunction	Implication	Bi-implication
p	q	$\neg p$	$\neg q$	$p\wedge q$	$p\vee q$	$p\Longrightarrow q$	$p\Longleftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T
Rules		$\neg p$ is true if p is false		T iff Both are T	F iff Both are F	F iff P=T , q=F	T iff They have same truth .v

- **Contradiction and tautology**

A compound statement is

i. Tautology – if it always have a truth value of T.
ii. Contradiction – if it always have a truth value of F.

Examples

a. $(p\Longrightarrow q)\Longleftrightarrow(\neg p v q)$

$(p\Longrightarrow q)\Longleftrightarrow(\neg p v q)$					
p	q	$\neg p$	$p\Longrightarrow q$	$\neg p v q$	→
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T
Tautology					

b. $(p\Longrightarrow q)\Longleftrightarrow(p\wedge\neg q)$

$(p\Longrightarrow q)\Longleftrightarrow(p\wedge\neg q)$					
p	q	$\neg q$	$p\Longrightarrow q$	$p\wedge\neg q$	→
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F
Contradiction					

- **Converse and contrapositive**

Given a compound statement $p\Longrightarrow q$, then

i. $q\Longrightarrow p$ is converse of $p\Longrightarrow q$, and
ii. $\neg q\Longrightarrow\neg p$ is contrapositive of $p\Longrightarrow q$

Example, let p= 2 is a prime number
q= 6 is divisible by 3

i. $q\Longrightarrow p$= if 2 is a prime number, then 6 is divisible by 3.
ii. Converse, $q\Longrightarrow p$= if 6 is divisible by 3, then 2 is a prime number.
iii. Contrapositive, $\neg q\Longrightarrow\neg p$= if 6 is not divisible by 3, then 2 is not a prime number.

- **Quantifiers**

Two quantifiers, $(\exists x)$, fore some x, there exist

$(\forall x)$, for all x

Examples – given the universe is a set of natural numbers, find the truth value of

a. $(\exists x)(4x-3=2x+1)$ T, example x=2

b. $(\exists x)(x^2<0)$ F, no natural number < 0

C. $(\forall x)(\ x^2>0)$ T, every square of natural number is > 0

D. $x^2>6$ F, example x=2

- Relationship between quantifiers

i. $\neg(\forall x)$P(x) = $(\exists x)(\ \neg p(x))$

ii. $\neg(\exists x)$P(x) = $(\forall x)(\ \neg p(x))$

Examples, let the universe is a set of real numbers, find the truth values of the negation of

a. $(\exists x)(x^2<0)$

$\neg(\exists x)(x^2<0)$ =$(\forall x)$ $\neg(x^2<0)$ =$(\forall x)$ $(x^2\geq 0)$ =T

b. $(\forall x)$(2x-1=0)

$\neg(\forall x)$(2x-1=0) = $(\exists x)$ $\neg(2\text{x}-1=0)$= $(\exists x)$ $(2\text{x}-1\neq 0)$=T

- Quantifiers occurring in combination

Consider set of real numbers as a universe

i. $(\exists x)(\exists y)$p(x,y) is T if we get one x and one y that satisfy p

Example, $(\exists x)(\exists y)$(2x-y=6) T, eg. x=4, y=2

ii. $(\exists x)(\forall y)$p(x,y) one x for every y that satisfy p

Example, $(\exists x$ $(\forall y)$($x+y^2>0$) T, eg. x=1

iii. $(\forall x)(\exists y)$p(x,y) every x has its own y satisfy p

Example, $(\forall x)(\exists y)$(x-1=y) T, eg. x=1 has y=0, x=3 has y=2

iv. $(\forall x)(\forall y)$p(x,y) every x and every y satisfy p

Example, $(\forall x)(\forall y)$(x+y=0) F, eg. x=1, y=2

- Validity

An argument $P_1,P_2,P_3,\ldots\ldots,P_n \vdash Q$ is valid, if Q is true whenever all the premise $P_1,P_2,P_3,\ldots\ldots,P_n$ are true.

Example, check the validity of p, q⟹p ⊢ q

p	q	q⟹p
T	T	T
T	F	F
F	T	T
F	F	T

The premises p and q⟹p are true in the first row, the conclusion q is also true in the first row. Thus, the argument p, q⟹p ⊢ q is valid.

Grade 11 – unit five – statistics and probability

- **Statistics**
- **Measures of location and measures of dispersion**

i. Mean ($\bar{x}$) = sum of the variables divided by the number of variables.

$$\begin{cases} \bar{x} = \dfrac{\sum x_i}{n} \ or \ \bar{x} = \dfrac{\sum f_i x_i}{\sum f} - - - - - - - - \ if \ the \ data \ is \ un \ grouped \\ \bar{x} = \dfrac{\sum f_i x_c}{\sum f} - - - - - - - - - - - - - - - - \ if \ the \ data \ is \ grouped \end{cases}$$

ii. Median (md) = the middle term of the data.

$$\begin{cases} md = \left(\dfrac{n+1}{2}\right)^{th} \ item \ - - - - - - - - - - - if \ the \ data \ is \ un \ grouped \ and \ n = odd \\ md = \dfrac{\left(\frac{n}{2}\right)^{th} \ item + \left(\frac{n}{2}+1\right)^{th} \ item}{2} - - - - if \ the \ data \ is \ un \ grouped \ and \ n = even \\ md = B_L + \left(\dfrac{\frac{n}{2} - cf_b}{f}\right) i - \ if \ the \ data \ is \ grouped \end{cases}$$

iii. Quartiles (Q_k) = values that divide a set of data in to four equal parts.

There are three quartiles Q_1, Q_2, and Q_3.

Quartiles class $(\frac{kn}{4})^{th}$ item

$$\begin{cases} Q_k = k\left(\dfrac{n+1}{4}\right)^{th} \ item \ - - - - - - - - - - - if \ the \ data \ is \ un \ grouped \ and \ n = odd \\ Q_k = \dfrac{\left(\frac{kn}{4}\right)^{th} \ item + \left(\frac{kn}{4}+1\right)^{th} \ item}{2} - - - - if \ the \ data \ is \ un \ grouped \ and \ n = even \\ Q_k = B_L + \left(\dfrac{\frac{kn}{4} - cf_b}{f}\right) i - \ if \ the \ data \ is \ grouped \end{cases}$$

iv. Deciles (D_k) = values that divide a set of data in to ten equal parts.

There are nine deciles D_1, D_2, ……., and D_9.

Decile class $(\frac{kn}{10})^{th}$ item

$$\begin{cases} D_k = k\left(\dfrac{n+1}{10}\right)^{th} \ item \ - - - - - - - - - - - if \ the \ data \ is \ un \ grouped \ and \ n = odd \\ D_k = \dfrac{\left(\frac{kn}{10}\right)^{th} \ item + \left(\frac{kn}{10}+1\right)^{th} \ item}{2} - - - - if \ the \ data \ is \ un \ grouped \ and \ n = even \\ D_k = B_L + \left(\dfrac{\frac{kn}{10} - cf_b}{f}\right) i - \ if \ the \ data \ is \ grouped \end{cases}$$

v. Percentiles (P_k) = values that divide a set of data in to hundred equal parts.

There are ninety nine percentiles P_1, P_2, …………., and P_{99}.

Percentile class $(\frac{kn}{100})^{th}$ item

$$\begin{cases} P_k = k\left(\frac{n+1}{100}\right)^{th}\ item\ ----------if\ the\ data\ is\ un\ grouped\ and\ n = odd \\ P_k = \dfrac{\left(\frac{kn}{100}\right)^{th}\ item + \left(\frac{kn}{100}+1\right)^{th}\ item}{2} ----if\ the\ data\ is\ un\ grouped\ and\ n = even \\ P_k = B_L + \left(\dfrac{\frac{kn}{100} - cf_b}{f}\right)i ------------------\ if\ the\ data\ is\ grouped \end{cases}$$

Where, B_L=lower boundary of the median/quartile/decile/percentile class.

Cf_b= cumulative frequency which comes before the median/quartile/decile/percentile class.

f=frequency of the median/quartile/decile/percentile class.

i= class width.

n= total number of data (sum of the frequencies)

vi. Mode (m_0)=most frequent item

$$\begin{cases} m_0 = an\ item\ with\ high\ frequency --------if\ the\ data\ is\ un\ grouped \\ m_0 = B_L + \left(\frac{d_1}{d_1+d_2}\right)i -----------------\ if\ the\ data\ is\ grouped \end{cases}$$

Where d_1= difference between the frequencies of the modal class and the previous class.

d_2= difference between the frequencies of the modal class and the next class.

vii. Range (R) = the difference between maximum and minimum values of the data.

$$\begin{cases} R = x_{max} - x_{min} ---------------------if\ the\ data\ is\ un\ grouped \\ R = \underset{of\ the\ last\ class}{upper\ class\ bouodary} - \underset{of\ the\ first\ class}{lower\ class\ boundary} -----\ if\ the\ data\ is\ grouped \end{cases}$$

viii. Variance (δ^2)= is an average of squared deviations of each item from the mean.

$$\begin{cases} \delta^2 = \frac{\sum(x_i-\bar{x})^2}{n}\ or\ \delta^2 = \frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i} -------------if\ the\ data\ is\ un\ grouped \\ \delta^2 = \frac{\sum f_i(x_c-\bar{x})^2}{\sum f_i} -----------------------if\ the\ data\ is\ grouped \end{cases}$$

ix. Standard deviation (s.d)(δ) = positive square root of variance.

Examples

1. Given the following un grouped data. Find mean, variance, standard deviation, range, median, mode, 3rd quartile, 6th decile, and 86th percentile.

X	2	6	7	8	10
f	3	5	9	2	6

Solution

Total number of data = 3+5+9+2+6 = 25 = odd

a. Mean=$\bar{x} = \frac{\sum f_i x_i}{\sum f} = \bar{x} = \frac{3*2+5*6+9*7+2*8+6*10}{3+5+9+2+6} = \frac{6+30+63+16+60}{25} = \frac{175}{25} = 7$

b. Variance=$\delta^2 = \frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i} = \frac{3(2-7)^2+5(6-7)^2+9(7-7)^2+2(8-7)^2+6(10-7)^2}{3+5+9+2+6} = \frac{75+5+0+2+54}{25} = \frac{136}{25}$

c. Standard deviation= $\delta = \sqrt{\frac{136}{25}}$

d. Range =R= max- min= 10 -2 =8

e. Median =md= $\left(\frac{n+1}{2}\right)^{th}\ item = \left(\frac{25+1}{2}\right)^{th}\ item = (13)^{th}\ item = 7$

f. Mode= m_0=7, most frequent

g. 3rd quartile=Q$_3$= $3\left(\frac{25+1}{4}\right)^{th}\ item = \left(\frac{78}{4}\right)^{th}\ item$

$$= (19.5)^{th}\ item = \frac{19^{th}\ it + 20^{th}\ it}{2} = \frac{8+10}{2} = 9$$

h. 6th decile=D$_6$= $6\left(\frac{25+1}{10}\right)^{th}\ item = (15.6)^{th}\ item$

$$= (15)^{th}\ item + 0.6\left(16^{th} item - 15^{th} item\right) = 7 + 0.6(7-7) = 7$$

i. 86th percentile=P$_{86}$= $86\left(\frac{25+1}{100}\right)^{th}\ item = (22.36)^{th}\ item$

$$= (22)^{nd}\ item + 0.36\left(23^{rd} item - 22^{nd} item\right) = 10 + 0.36(10-10) = 10$$

2. Given the following grouped data. Find mean, variance, standard deviation, range, median, mode, 3rd quartile, 6th decile, and 86th percentile.

X	3-7	8-12	13-17	18-22
f	2	2	10	6

Solution

x	f	cf	x$_c$	fx$_c$	Class boundary	x$_c$-$\bar{x}$	(x$_c$-$\bar{x}$)^2	f(x$_c$-$\bar{x}$)^2
3-7	2	2	5	10	2.5-7.5	-10	100	200
8-12	2	4	10	20	7.5-12.5	-5	25	50
13-17	10	14	15	150	12.5-17.5	0	0	0
18-22	6	20	20	120	17.5-22.5	5	25	150
$\sum f$=20		$\sum f x_c$=300			$\sum f(x_c-\bar{x})^2$=400			

a. Mean=$\bar{x} = \frac{\sum f_i x_c}{\sum f} = \frac{300}{20}$=15

b. Variance=$\delta^2 = \frac{\sum f_i (x_c-\bar{x})^2}{\sum f_i} = \frac{400}{20} = 20$

c. Standard deviation=s.d= $\delta = \sqrt{20} = 2\sqrt{5}$

d. Range=R=upper bou. Of last class – lower bou. Of first class=22.5-2.5=20

e. Median class = 10th +11th items are in the 3rd class

$$\text{Md}= B_L + \left(\frac{\frac{n}{2}-cf_b}{f}\right) i\text{=12.5+}\left(\frac{\frac{20}{2}-4}{10}\right)5\text{=12.5+}\frac{30}{10}\text{=12.5+3=15.5}$$

f. 3rd quartile class=$(\frac{kn}{4})^{th}$ item=$(\frac{3*20}{4})^{th}$ item=15th item=4th class

$$Q_3=B_L + \left(\frac{\frac{kn}{4}-cf_b}{f}\right) i = 17.5+\left(\frac{\frac{3*20}{4}-14}{6}\right)5 = 17.5+\left(\frac{1}{6}\right)5 = 17.5+0.83=18.33$$

g. 6th decile class=$(\frac{kn}{10})^{th}$ item=$(\frac{6*20}{10})^{th}$ item=12th item=3th class

$$D_6 = B_L + \left(\frac{\frac{kn}{10}-cf_b}{f}\right) i\text{=12.5} + \left(\frac{\frac{6*20}{10}-4}{10}\right)5 = 12.5 + \left(\frac{8}{10}\right)5 = 12.5 + 4 = 16.5$$

h. 86th percentile class=$(\frac{kn}{100})^{th}$ item=$(\frac{86*20}{100})^{th}$ item=17.2th item=4th class

$$P_{86} = B_L + \left(\frac{\frac{kn}{100}-cf_b}{f}\right) i = 17.5 + \left(\frac{\frac{86*20}{100}-14}{6}\right)5 = 17.5 + \left(\frac{3.2}{6}\right)5 = 17.5 + 2.66 = 20.16$$

i. Modal class= 3rd class

$$\text{Mode}=m_0 = B_L + \left(\frac{d_1}{d_1+d_2}\right) i\text{=12.5} + \left(\frac{8}{8+4}\right)5 = 12.5 + \left(\frac{40}{12}\right) = 12.5 + 3.33 = 15.83$$

- **Probability**
- P(E)=$\frac{n(E)}{n(S)}$, n(E)= number of elements of an event, n(S)=number of elements of sample space
- Note, $0 \leq P(E) \leq 1$, if P(E)=0, E is impossible event.
  If P(E)=1, E is sure event.
- Complement of an event, $P(E)+P(E^{I})=1$
- P(A∩B) = P(A)*P(B)
- P(AUB)=P(A)+P(B)-P(A∩B)
- P(A/B)= P(A) - P(A∩B)
- Examples

1. A box contains 3 white, 4 red, and 5 blue same size balls. If one ball is drawn at random, then what is the probability that the ball drawn is

a. Red?	e. not red?
b. White?	f. blue or white?
c. Blue?	g. not green?
d. Not blue?	h. yellow?

Solution

n(S)=3+4+5=12, n(W)=3, n(R)=4, n(B)=5

a. Red? P(R)=$\frac{n(R)}{n(S)} = \frac{4}{12} = \frac{1}{3}$

b. White? P(W)=$\frac{n(W)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

c. Blue? P(B)=$\frac{n(B)}{n(S)} = \frac{5}{12}$

d. Not blue? P(not B)= 1- P(B) = $1 - \frac{n(B)}{n(S)} = 1 - \frac{5}{12} = \frac{7}{12}$

e. Not red? P(not R)=1 – P(R) = $1 - \frac{n(R)}{n(S)} = 1 - \frac{4}{12} = 1 - \frac{1}{3} = \frac{2}{3}$

f. Blue or white? P(BUW)= P(B) + P(W) =$\frac{5}{12} + \frac{3}{12} = \frac{8}{12} = \frac{2}{3}$

g. Not green? P(not G)= 1- P(BG) = $1 - \frac{n(G)}{n(S)} = 1 - \frac{0}{12} = 1 = 0 = 1$, sure event

h. Yellow? P(Y)=$\frac{n(Y)}{n(S)} = \frac{0}{12} = 0$

2. Three coins tossed together. What is the probability of getting
   a. At least two heads?
   b. Exactly two tails?

Solution

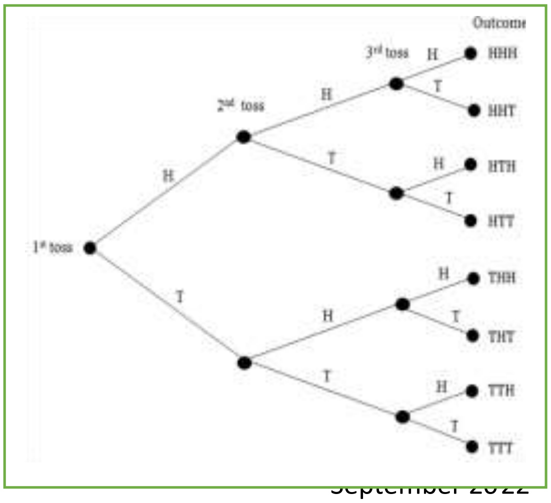
S={HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}, n(S) = 8

a. E={HHH,HHT,HTH,THH}, n(E) = 4

P(E)=$\frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

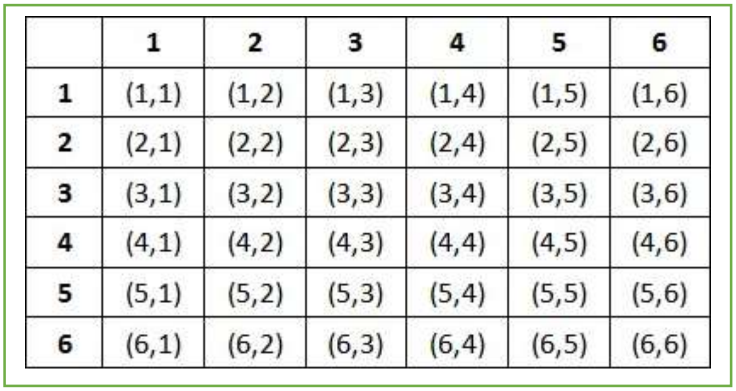
b. E={ HTT,THT,TTH,}, n(E) = 3

P(E)=$\frac{n(E)}{n(S)} = \frac{3}{8}$



3. Two dice tossed together. What is the probability that the sum of the numbers shown is
   a. 8 b. 12 c. 14

Solution



	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

n(S)=36

a. E={(2,6), (3,5), (4,4), (5,3), (6,2)}, n(E) = 5, P(E)=$\frac{n(E)}{n(S)} = \frac{5}{36}$

b. E={(6,6) }, n(E) = 1, P(E)=$\frac{n(E)}{n(S)} = 36$

c. E={ }, n(E) = 0, P(E)=$\frac{n(E)}{n(S)} = \frac{0}{8} = 0$

- Multiplication rule

Examples

1. A ball contains 3 red, 5 blue, and 6 black balls. If three balls are drawn at random one after the other with replacement. Find the probability that
   a. The first ball is red, the second blue, and the third black.
   $$P=\frac{3}{14} * \frac{4}{14} * \frac{6}{14} = \frac{72}{2744}$$
   b. All the drawn balls are red.
   $$P=\frac{3}{14} * \frac{3}{14} * \frac{3}{14} = \frac{27}{2744}$$
   c. The first two are black and the third is blue.
   $$P=\frac{6}{14} * \frac{6}{14} * \frac{5}{14} = \frac{180}{2744}$$
2. A ball contains 3 red, 5 blue, and 6 black balls. If three balls are drawn at random without replacement. Find the probability that
   a. Red followed by blue and then followed by black.
   $$P=\frac{3}{14} * \frac{5}{13} * \frac{6}{12} = \frac{90}{2184}$$
   b. All the drawn balls are black.
   $$P=\frac{6}{14} * \frac{5}{13} * \frac{4}{12} = \frac{9}{2184}$$
   c. Blue followed by blue, followed by red.
   $$P=\frac{5}{14} * \frac{4}{13} * \frac{3}{12} = \frac{60}{2184}$$
3. A ball contains 3 red, 5 blue, and 6 black balls. If three balls are drawn at random without replacement. Find the probability that same colors are drawn no matter which color comes first.
   a. 3 red balls followed by 5 blue balls, and then followed by 6 black balls.
   $$P=\frac{3}{14} * \frac{2}{14} * \frac{1}{14} + \frac{5}{14} * \frac{4}{14} * \frac{3}{14} + \frac{6}{14} * \frac{5}{14} * \frac{4}{14} = \frac{6}{2744} + \frac{60}{2744} + \frac{120}{2744} = \frac{186}{2744}$$

- Factorials, permutation, and combinations
- Factorials , for any whole number n, n!=n*(n-1)*(n-2)*(n-2)……..3*2*1

Note, 0!=1

Examples

1. Find
   a. 8!=8*7*6*5*4*3*2*1=40320
   b. 12 !=479001600
2. In how many ways can 7 students be seated in a row?

7!=7*6*5*4*3*2*1=5040

3. In how many ways can 6 students be seated around a table (circular)?

(n-1)!= $(6-1)!$= 5!=5*4*3*2*1=120

- Permutation (ordered selection)

P(n,r)=nPr=$\frac{n!}{(n-r)!}$

Examples

1. Find
   a. P(8,4)= $\frac{8!}{(8-4)!}$=$\frac{8!}{4!}$=$\frac{8*7*6*5*4!}{4!}=8*7*6*5=1680$
   b. P(15,11)= $\frac{15!}{(15-11)!}$=$\frac{8!}{4!}$=$\frac{15*14*13*12*11*10*9*8*7*6*5*4!}{4!}$

$$=15*14*13*12*11*10*9*8*7*6*5=54486432000$$

2. In how many ways can 10 different books be arranged on a shelf?

P(10,10)= $\frac{10!}{(10-10)!}$=$\frac{8!}{4!}$=$\frac{10!}{0!}$=$10!=3628800$

- Combinations (un ordered selection)

C(n,r)=$\frac{n!}{(n-r)!r!}$

Examples

1. In how many ways can a committee of 5 people be selected from 8 people?

C(8,5)=$\frac{8!}{(8-5)!5!}$ = $\frac{8!}{3!5!}$ $=$ $\frac{8*7*6*5!}{3!5!}$ = $\frac{8*7*6}{6}$ = 8*7=56

2. A group of 6 students be formed from 10 boys and 12 girls. In how many ways could this be done if the group is consists of
   a. 3 girls and 3 boys?

   C(12,3)*C(10,3)=26,400

   b. All are girls?

   C(12,6)*C(10,0)=924

   c. At least 2 boys?

   C(10,6)*C(12,0)+ C(10,5)*C(12,1)+ C(10,4)*C(12,2)+ C(10,3)*C(12,3)+ C(10,2)*C(12,4)

   =20*1+252*12+210*66+120*220+45*495=65769

   d. At most 4 girls?

   C(22,6) – [C(12,5)*C(10,1)+C(12,6)* C(10,0)]

   =74613 – (7920+924) = 65769

3. A box contains 3 red and 2 blue pens. If two pens are selected at random. Find the probability that
   a. All the pens are red. b. One red and one blue c. Both have the same color

$P=\frac{C(3,2)*C(2,0)}{C(5,2)}=\frac{3}{10}$ $P=\frac{C(3,1)*C(2,1)}{C(5,2)}=\frac{3}{5}$ $P=\frac{C(3,2)}{C(5,2)}+\frac{C(2,2)}{C(5,2)}=\frac{3}{10}+\frac{1}{10}=\frac{4}{10}=\frac{2}{5}$

Grade 11 – unit six – matrices and determinants

- **Matrices**
- Equality of matrices,

Example, find the values of x, y, and z, so that A=B. A=$\begin{pmatrix} 2 & 4 & 0 \\ 3 & 8 & 0 \\ -5 & 6 & 3 \end{pmatrix}$, B=$\begin{pmatrix} x & y+3 & 0 \\ 3 & 2z & 0 \\ -5 & 6 & 3 \end{pmatrix}$

Solution

A=B, $\begin{pmatrix} 2 & 4 & 0 \\ 3 & 8 & 0 \\ -5 & 6 & 3 \end{pmatrix}$=$\begin{pmatrix} x & y+3 & 0 \\ 3 & 2z & 0 \\ -5 & 6 & 3 \end{pmatrix}$, 2=x, 4=y+3, y= 1, 8=2z, 4=z

- Addition and subtraction- we can find sum or difference of two or more matrices if they have the same order.

Examples, find a. 2A+3B, b. 3A-2B if A=$\begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & -3 \\ 1 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix}$, and B=$\begin{pmatrix} 8 & 3 & 1 \\ 0 & 2 & 1 \\ 3 & 4 & 1 \\ -1 & -2 & -1 \end{pmatrix}$

Solution

Note, A and B have the same order, which is 4x3

a. 2A+3B

$$=2\begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & -3 \\ 1 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix}+3\begin{pmatrix} 8 & 3 & 1 \\ 0 & 2 & 1 \\ 3 & 4 & 1 \\ -1 & -2 & -1 \end{pmatrix}$$

$$=\begin{pmatrix} 4 & 8 & 0 \\ 0 & 4 & -6 \\ 2 & 4 & 6 \\ 0 & 8 & 0 \end{pmatrix}+\begin{pmatrix} 24 & 9 & 3 \\ 0 & 6 & 3 \\ 9 & 12 & 3 \\ -3 & -6 & -3 \end{pmatrix}$$

$$=\begin{pmatrix} 4+24 & 8+9 & 0+3 \\ 0+0 & 4+6 & -6+3 \\ 2+9 & 4+12 & 6+3 \\ 0-3 & 8-6 & 0-3 \end{pmatrix}$$

$$=\begin{pmatrix} 28 & 17 & 3 \\ 0 & 10 & -3 \\ 11 & 16 & 9 \\ -3 & 2 & -3 \end{pmatrix}$$

b. 3A – 2B

$$=3\begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & -3 \\ 1 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix}-2\begin{pmatrix} 8 & 3 & 1 \\ 0 & 2 & 1 \\ 3 & 4 & 1 \\ -1 & -2 & -1 \end{pmatrix}$$

$$=\begin{pmatrix} 6 & 12 & 0 \\ 0 & 6 & -9 \\ 3 & 6 & 9 \\ 0 & 12 & 0 \end{pmatrix}-\begin{pmatrix} 16 & 6 & 2 \\ 0 & 4 & 2 \\ 6 & 8 & 2 \\ -2 & -4 & -2 \end{pmatrix}$$

$$=\begin{pmatrix} 6-16 & 12-6 & 0-2 \\ 0-0 & 6-4 & -9-2 \\ 3-6 & 6-8 & 9-2 \\ 0+2 & 12+4 & 0+2 \end{pmatrix}$$

$$=\begin{pmatrix} -10 & 6 & -2 \\ 0 & 2 & -11 \\ -3 & -2 & 7 \\ 2 & 16 & 2 \end{pmatrix}$$

- Multiplication- we can find the product of matrices AB if number of columns of A is equal to number of rows of B.

Examples, find AB if A=$\begin{pmatrix} 3 & 2 & -1 \\ -2 & 4 & 2 \\ 3 & 2 & -2 \end{pmatrix}$, and B=$\begin{pmatrix} 0 & 2 \\ 2 & -1 \\ -3 & -2 \end{pmatrix}$

Solution

$$AB=\begin{pmatrix} 3 & 2 & -1 \\ -2 & 4 & 2 \\ 3 & 2 & -2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 2 & -1 \\ -3 & -2 \end{pmatrix}$$

$$=\begin{pmatrix} 3(0)+2(2)-1(-3) & 3(2)+2(-1)-1(-2) \\ -2(0)+4(2)+2(-3) & -2(2)+4(-1)+2(-2) \\ 3(0)+2(2)-2(-3) & 3(2)+2(-1)-2(-2) \end{pmatrix}=\begin{pmatrix} 7 & 6 \\ 2 & -12 \\ 10 & 8 \end{pmatrix}$$

- Transpose of a matrix- interchange columns with rows

Examples, find transpose of A, if A=$\begin{pmatrix}28 & 17 & 3\\ 0 & 10 & -3\\ 11 & 16 & 9\\ -3 & 2 & -3\end{pmatrix}$, A^T=$\begin{pmatrix}28 & 0 & 11 & -3\\ 17 & 10 & 16 & 2\\ 3 & -3 & 9 & -3\end{pmatrix}$

- Systematic and skew systematic

  A matrix A is systematic if A=A^T, and skew systematic if A+A^T=0

  Examples

1. If A=$\begin{pmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{pmatrix}$, then find AA^T

$$\text{AA}^T=\begin{pmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{pmatrix}\begin{pmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{pmatrix}$$

$$=\begin{pmatrix}sin^2x+cos^2x & \cos x\sin x - sinx\cos x\\ \sin x\cos x-\cos x\sin x & sin^2x+cos^2x\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\text{I}$$

2. Find the values of x and y , so that A=$\begin{pmatrix}0 & -3 & 4\\ y+2 & 0 & 3\\ -4 & x-5 & 0\end{pmatrix}$,

A+A^T=0

$$\begin{pmatrix}0 & -3 & 4\\ y+2 & 0 & 3\\ -4 & x-5 & 0\end{pmatrix}+\begin{pmatrix}0 & y+2 & -4\\ -3 & 0 & x-5\\ 4 & 3 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & -3+y+2 & 0\\ y+2-3 & 0 & 3+x-5\\ 0 & x-5+3 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

$y+2$–3=0, 3+x-5=0

$y=1,$ $x=2$

- **Determinants**

a. $|A|=|A^T|$ d. use the signs $\begin{pmatrix}+ & - & +\\ - & + & -\\ + & - & -\end{pmatrix}$

b. $|A^{-1}|=\frac{1}{|A|}$

c. If A is nxn, $|kA|=k^n|A|$

Examples,

1. Find the determinants of a. A=$\begin{pmatrix}1 & 5\\ 7 & 3\end{pmatrix}$ b. B=$\begin{pmatrix}1 & 3 & 3\\ 0 & 2 & -1\\ 2 & 1 & 2\end{pmatrix}$

<u>Solution</u>

a. $|A|=\begin{vmatrix}1 & 5\\ 7 & 3\end{vmatrix}=3-35=-32$

b. $|B|=\begin{vmatrix}1 & 3 & 3\\ 0 & 2 & -1\\ 2 & 1 & 2\end{vmatrix}=1\begin{vmatrix}2 & -1\\ 1 & 2\end{vmatrix}-0\begin{vmatrix}3 & 3\\ 1 & 2\end{vmatrix}+2\begin{vmatrix}3 & 3\\ 2 & -1\end{vmatrix}$

= 1(4+1) – 0(6-3) + 2(- 3 - 6)= 5 – 0 – 18= -13

2. Find x if $\begin{vmatrix}x+1 & 2 & 1\\ 1 & 1 & 2\\ x-1 & 1 & x\end{vmatrix}=0$

$$(x+1)\begin{vmatrix}1 & 2\\ 1 & x\end{vmatrix}-1\begin{vmatrix}2 & 1\\ 1 & x\end{vmatrix}+(x-1)\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}=0$$

(x+1)(x - 2) – 1(2x -1)+(x-1)(4 -1) =0

X^2-2x+x-2-2x+1+4x-x-4+1=0

X^2-4=0

X=$\pm$2

- Inverse of square matrices
  a. Only square matrices have an inverse
  b. $A^{-1}=\frac{1}{|A|}adjA$, where adjoit of A, adjA is transpose of the matrix of cofactors.
  c. $(AB)^{-1}=B^{-1}A^{-1}$
  d. $AA^{-1}=I$, where I is an identity matrix.
  e. A has an inverse if and only if $|A| \neq 0$.

Examples

Find the inverse of a. $A=\begin{pmatrix}4 & 5\\ 2 & 3\end{pmatrix}$ b. $A=\begin{pmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{pmatrix}$

Solution

a. Let $A^{-1}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, then $AA^{-1}=I$

$$\begin{pmatrix}4 & 5\\ 2 & 3\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}4a+5c & 4a+5d\\ 2a+3c & 2b+3d\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\begin{cases}4a+5c=1\\ 2a+3c=0\end{cases} \qquad \begin{cases}4a+5d=0\\ 2b+3d=1\end{cases}$$

C= -1, a=2/3 d=2, b=-5/2

$$A^{-1}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}\frac{2}{3} & -5/2\\ -1 & 2\end{pmatrix}$$

b. $A=\begin{pmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{pmatrix}$

$$|A|=\begin{vmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{vmatrix}=2\begin{vmatrix}3 & 1\\ 2 & 4\end{vmatrix}-3\begin{vmatrix}4 & 1\\ 1 & 4\end{vmatrix}+4\begin{vmatrix}4 & 3\\ 1 & 2\end{vmatrix}$$

= 2(12-2) – 3(16-1) + 4(8 - 3)= 20 – 45 + 20= -5

Matrix of co factors

$$C=\begin{pmatrix}\begin{vmatrix}3 & 1\\ 2 & 4\end{vmatrix} & -\begin{vmatrix}4 & 1\\ 1 & 4\end{vmatrix} & \begin{vmatrix}4 & 3\\ 1 & 2\end{vmatrix}\\ -\begin{vmatrix}3 & 4\\ 2 & 4\end{vmatrix} & \begin{vmatrix}2 & 4\\ 1 & 4\end{vmatrix} & -\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}\\ \begin{vmatrix}3 & 4\\ 3 & 1\end{vmatrix} & -\begin{vmatrix}2 & 4\\ 4 & 1\end{vmatrix} & \begin{vmatrix}2 & 3\\ 4 & 3\end{vmatrix}\end{pmatrix}$$

$$=\begin{pmatrix}(12-2) & -(16-1) & (8-3)\\ -(12-8) & (8-4) & -(4-3)\\ (3-12) & -(2-16) & (6-12)\end{pmatrix}=\begin{pmatrix}10 & -15 & 5\\ -4 & 4 & -1\\ -9 & 14 & -6\end{pmatrix}$$

$$\text{adj } A=C^T=\begin{pmatrix}10 & -4 & -9\\ -15 & 4 & 14\\ 5 & -1 & -6\end{pmatrix}$$

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{-5}\begin{pmatrix}10 & -4 & -9\\ -15 & 4 & 14\\ 5 & -1 & -6\end{pmatrix}=\begin{pmatrix}-2 & 4/5 & 9/5\\ 3 & -\frac{4}{5} & -14/5\\ -1 & 1/5 & 6/5\end{pmatrix}$$

- Systems of linear equations
- Cramer's rule

a. $\begin{cases}a_1x+b_1y=c_1\\ a_2x+b_2y=c_2\end{cases}$, has a solution if the determinant of the coefficient matrix is different from zero.

D=$\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}$, Dx=$\begin{vmatrix}c_1 & b_1\\ c_2 & b_2\end{vmatrix}$, Dy=$\begin{vmatrix}a_1 & c_1\\ a_2 & c_2\end{vmatrix}$, $x=\frac{Dx}{D}$ $y=\frac{Dy}{D}$

b. $\begin{cases} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{cases}$, has a solution if the determinant of the coefficient matrix is different from zero.

D= $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, Dx= $\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, Dy= $\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, Dz= $\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

$$x=\frac{Dx}{D} \qquad y=\frac{Dy}{D} \qquad z=\frac{Dz}{D}$$

Example

Find the solution set of a. $\begin{cases} 2x+y=7 \\ 3x-2y=0 \end{cases}$ b. $\begin{cases} -x+4y-z=1 \\ 2x-y+z=0 \\ x+y+z=1 \end{cases}$

<u>Solution</u>

a. D= $\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix}=-7,$ Dx= $\begin{vmatrix} 7 & 1 \\ 0 & -2 \end{vmatrix}=-14,$ Dy= $\begin{vmatrix} 2 & 7 \\ 3 & 0 \end{vmatrix}-21,$ $x=\frac{-14}{-7}=2\,,\quad y=\frac{-21}{-7}=3$

Solution set $=\{(x,y)\}=\{(2,3)\}$

b. D= $\begin{vmatrix} -1 & 4 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}=-1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}-4\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

= -1(-1-1) – 4(2-1) -1(2+1)= 2 – 4 - 3= -5

Dx= $\begin{vmatrix} 1 & 4 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}=1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}-0\begin{vmatrix} 4 & -1 \\ 1 & 1 \end{vmatrix}+1\begin{vmatrix} 4 & -1 \\ -1 & 1 \end{vmatrix}$

= 1(-1-1) – 0 + 1(4 - 1)= - 2 + 3 = 1

Dy= $\begin{vmatrix} -1 & 1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}=-1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

= -1(0-1) – 1(2-1) - 1(2 - 0)= 1 – 1 - 2= -2

Dz= $\begin{vmatrix} -1 & 4 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}=-1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}-4\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}+1\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

= -1(-1-0) – 4(2-0) + 1(2 + 1)= 1 – 8 + 3= -4

$$x=\frac{Dx}{D}=\frac{-1}{5} \qquad y=\frac{Dy}{D}=\frac{2}{5} \qquad z=\frac{Dz}{D}=\frac{4}{5}$$

Solution set $=\{(x,y,z)\}=\left\{\left(\frac{-1}{5},\frac{2}{5},\frac{4}{5}\right)\right\}$

- Echelon form of a matrix

Given a system of linear equations $\begin{cases} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{cases}$,

$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$, the system has infinitely many solutions

$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, the system has no solution

$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ 0 & 0 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, the system has one solution

Example, find the values of a and b if the system $\begin{cases} 3x-2y+z=b \\ 5x-8y+9z=3 \\ 2x+y+az=-1 \end{cases}$, has

a. One solution
b. Infinitely many solutions
c. No solution

<u>Solution</u>

$$\begin{pmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{pmatrix}\begin{pmatrix} b \\ 3 \\ -1 \end{pmatrix}, R_2 \longrightarrow \frac{-5}{3}R_1 + R_2 \quad and\ R_3 \longrightarrow \frac{-2}{3}R_1 + R_3$$

$$\begin{pmatrix} 3 & -2 & 1 \\ 0 & -\frac{14}{3} & \frac{22}{3} \\ 0 & \frac{7}{3} & \frac{-2a}{3} \end{pmatrix}\begin{pmatrix} b \\ \frac{9-5b}{3} \\ \frac{-2b-3}{3} \end{pmatrix}, R_2 \longrightarrow \frac{-5}{3}R_1 + R_2 \quad and\ R_3 \longrightarrow \frac{-2}{3}R_1 + R_3$$

$$\begin{pmatrix} 3 & -2 & 1 \\ 0 & 1 & \frac{-11}{7} \\ 0 & \frac{7}{3} & \frac{-2a}{3} \end{pmatrix}\begin{pmatrix} b \\ \frac{5b-9}{14} \\ \frac{-2b-3}{3} \end{pmatrix}, R_3 \longrightarrow \frac{-7}{3}R_2 + R_3$$

$$\begin{pmatrix} 3 & -2 & 1 \\ 0 & 1 & \frac{-11}{7} \\ 0 & 0 & a+3 \end{pmatrix}\begin{pmatrix} b \\ \frac{5b-9}{14} \\ \frac{1-3b}{2} \end{pmatrix},$$

a. a+3$\neq 0$,	b. a+3=0 and $\frac{1-3b}{2} = 0$	c. a+3=0 and $\frac{1-3b}{2} \neq 0$
a$\neq -3$	a= - 3, b=1/3	a= - 3, b$\neq$1/3

Grade 11- unit seven – the set of complex numbers

- note, $\sqrt{-1} = i$, or $i^2 = -1$, example, $\sqrt{-4} = \sqrt{4(-1)} = \sqrt{4}\sqrt{-1} = 2i$
- z=x+yi is a complex number with real part=x and imaginary part=y

  Example

  1. a. z=3+2i, b. z=$\sqrt{16} - \sqrt{-25}$ =4 – 5i
  2. simplify (note, i^n is 1 if n is divisible by 4, example i^{100}=$(i^4)^{25} = 1^{25} = 1$
     a. i^2= - 1 d. i^{24}=1
     b. i^3=i*i^2= - i e. i^{25}=i*i^{24}=i
     c. i^4=i^2*i^2=- 1* - 1=1 f. i^{26}=i^2i^{24}=i^2= - 1
- operations on complex numbers

  if z=x+zi, and w=a+bi, then

  a. z=w, if and only if x=a and y=b d. 2z+5w=(2x+5a)+(2y+5b)i
  b. z+w=(x+a)+(y+b)I e. 3z-w=(3x-a)+(3y-b)i
  c. z-w=(x-a) + (y-b)I f. zw=(x+yi)(a+bi)=(xa-yb)+(xb+ay)
- conjugate of a complex number

  The conjugate of z=x+yi is given by $\bar{z}$ =x – yi

  $z\bar{z} = x^2 + y^2$ = real number
- modulus of a complex number

  The modulus of z=x+yi is given by $|z| = \sqrt{x^2 + y^2}$

  Examples, find the conjugate and modulus of

  a. z=$\frac{3-4i}{1+2i} = \frac{3-4i}{1+2i} * \frac{1-2i}{1-2i} = \frac{3-6i-4i-8}{1^2+2^2} = \frac{-5-10i}{5} = -1 - 2i$

  Conjugate, $\bar{z}$ =-1 + 2i

  Modulus, $|z| = |\ \bar{z}| = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$
- polar form of a complex number

  If z=x+yi, then the polar form of z is given by z=r(cosθ + i sinθ)

  Where, θ= $\tan^{-1}(\frac{y}{x})$ is the argument of z, and $r = \sqrt{x^2 + y^2}$

  Z^n=r^n(cos$(n\theta)$ + i sin$(n\theta)$), argument of z^n is nθ

  Examples

  1. given a complex z=4+4$\sqrt{3}i$
     a. express z in polar form
     b. find z^{10}
  2. covert the polar form z=$\sqrt{2}(\cos\frac{3}{4}\pi - i\sin\frac{3}{4}\pi)$ in to Cartesian form

  Solution

  1. z=4+4$\sqrt{3}i$
     a. θ= $\tan^{-1}(\frac{y}{x})$= $\tan^{-1}(\frac{4\sqrt{3}}{4})$= $\tan^{-1}(\sqrt{3})$=60^0=$\frac{\pi}{3}$

     $$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (4\sqrt{3})^2} = \sqrt{16+48} = \sqrt{64} = 8$$

     Z= r(cosθ + i sinθ)= z=8(cos $\frac{\pi}{3}$ + i sin $\frac{\pi}{3}$)

     Z^{10}=r^{10}(cos 10θ + i sin 10θ)= 8^{10}(cos $\frac{10\pi}{3}$ + i sin $\frac{10\pi}{3}$))

     b. z=$\sqrt{2}(\cos\frac{3}{4}\pi - i\sin\frac{3}{4}\pi)$, $\frac{3\pi}{4} = \frac{3\pi}{4} * \frac{180}{\pi}$=$270^0$

     z=$\sqrt{2}(\cos 270^0 - i\sin 270^0) = \sqrt{2}(0 - i(-1))$ =0+i=i

- identities
  a. z+(-z)=0, -z is additive inverse and 0 is additive identity
  b. z($\frac{1}{z}$) =1, $\frac{1}{z}$ is multiplicative inverse and 1 is multiplicative identity

Examples

1. find the additive inverse of z=3 – 4i

Solution

-z= -3 +4i

2. find the multiplicative inverse of z=$\frac{1-3i}{4+5i}$

Solution

$$\frac{1}{z}=\frac{1}{\frac{1-3i}{4+5i}}=\frac{4+5i}{1-3i}=\frac{4+5i}{1-3i}*\frac{1+3i}{1+3i}=\frac{4+12i+5i-15}{1^2+3^2}=\frac{-11+17i}{10}$$

Grade 11- unit eight – vectors and transformation of the plane

- vectors

Vector quantities are quantities which have both magnitude and direction.

Examples, given vectors v=(3,-2) and u=(2,6), then find

a. u+v=(3+2,-2+6)=(5,4)

b. 3v-2u=3(3,-2)-2(2,6)=(9,-6) - (4,12)= (9-4,-6-12) = (5,-18)

- Magnitude (norm) of a vector

Magnitude of a vector u=(x,y) is given by $|u| = \sqrt{x^2 + y^2}$

- Unit vector

A unit vector is a vector with magnitude 1.

A unit vector in the direction of v is given by $\frac{1}{|v|} * v$

Examples,

Find a unit vector in the direction of u=(4,-3)

Solution

Unit vector=$\frac{1}{|u|} * u = \frac{1}{\sqrt{4^2+(-3)^2}} * (4,-3) = \frac{1}{5} * (4,-3) = (\frac{4}{5}, \frac{-3}{5})$

i=(1,0) and j=(0,1) are unit vectors

u=(x,y)=xi+yj, i*i=j*j=1, and i*j=0

- Dot product

If u=xi+yj and v=ai+bj, then u.v=xa+yb,

u.v=$|u||v| \cos\theta, \theta\ is\ the\ angle\ between\ u\ and\ v.$

Example, find the angle between the vectors u=(1,-1) and v=(-2,2)

Solution

u.v=$|u||v| \cos\theta$

(i-j).(-2i+2j)=$\sqrt{1^2 + (-1)^2}\sqrt{(-2)^2 + 2^2}\ \cos\theta$

-2- 2=$\sqrt{2}\sqrt{8}\cos\theta$

-4=$\sqrt{16}\cos\theta$

$-4 = 4\cos\theta$

$-1 = \cos\theta$

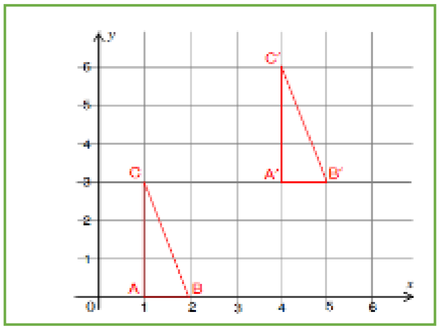
$\theta = cos^{-1}(-1) = 180^0$

- Transformation of the plane

Translation, rotation and reflection are rigid motions.

A. Translation – if every point of figure is moved along the same direction through the same direction, then the transformation is called translation (parallel movement).

If u=(h,k) is a translation vector, then T(x,y)=(x+h,y+k)



Examples,

1. if T is a translation that takes p(3,-2) in to P'(4,2), then find the image of a. p(2,-1) b. Q(-3,5) c. R(1,2)

Solution

Translation vector u=(4-3,2+2)=(1,4)=(h,k)

a. T(2,-1)=(2+h,-1+k)=(2+1,-1+4)=(3,3), P'(3,3)

b. T(-3,5)=(-3+h,5+k)=(-3+1,5+4)=(-2,9), Q'(-2,9)

c. T(1,2)=(1+h,2+k)=(1+1,2+4)=(2,6), R'(2,6)

2. If a translation T takes the origin in to (4,2), then find the image of

   a. Y=2x+9 b. $(x-3)^2+(y+2)^2=5$ c. $\frac{x^2}{9}-\frac{y^2}{4}=1$

Solution

Translation vector (4,2) – (0,0) = (4,2)

a. l y=2x+9

l' y-2=2(x-4)+9

y-2=2x-8+9

y=2x-8+9+2

y=2x+3

b. C $(x-3)^2+(y+2)^2=5$

C' $(x-4-3)^2+(y-2+2)^2=5$

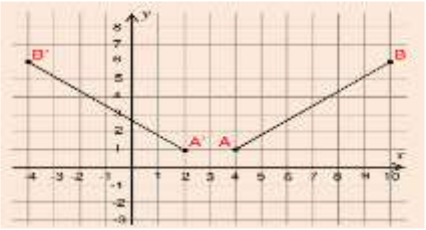
$(x-7)^2+y^2=5$

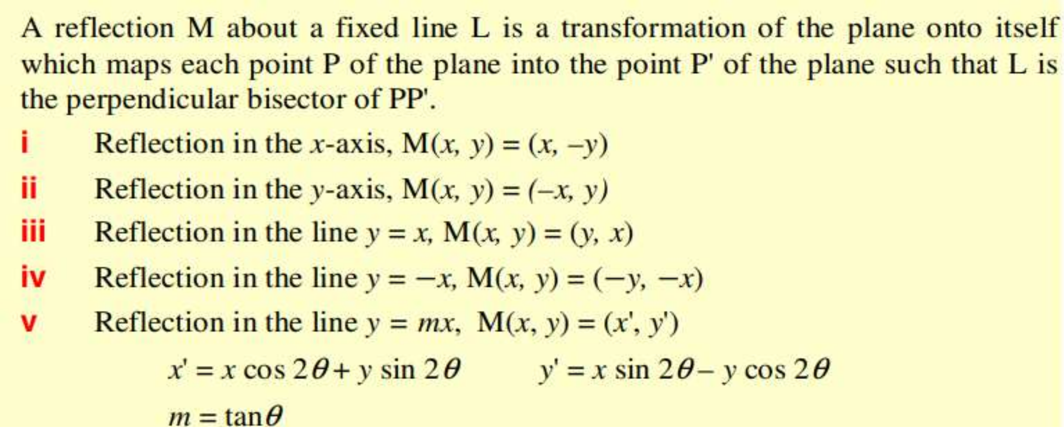
c. H $\frac{x^2}{9}-\frac{y^2}{4}=1$

H' $\frac{(x-4)^2}{9}-\frac{(y-2)^2}{4}=1$

In the circle and hyperbola only the centers are translated

B. Reflection- Let L be a fixed line in the plane. A reflection M about a line L is a transformation of the plane onto itself which carries each point P of the plane into the point P' of the plane such that L is the perpendicular bisector of PP'.





A reflection M about a fixed line L is a transformation of the plane onto itself which maps each point P of the plane into the point P' of the plane such that L is the perpendicular bisector of PP'.

i Reflection in the x-axis, $M(x, y) = (x, -y)$

ii Reflection in the y-axis, $M(x, y) = (-x, y)$

iii Reflection in the line $y = x$, $M(x, y) = (y, x)$

iv Reflection in the line $y = -x$, $M(x, y) = (-y, -x)$

v Reflection in the line $y = mx$, $M(x, y) = (x', y')$

$x' = x\cos 2\theta + y\sin 2\theta$ $y' = x\sin 2\theta - y\cos 2\theta$

$m = \tan\theta$

Examples,

1. Find the image of (3,2) when reflected about the line y=x

Solution

Y=x, m=1 or $\theta = 45^0 = \frac{\pi}{4}$, the image of p(3,2) is p'(2,3)

2. Find the image of (3,2) when reflected by the line y=$\frac{1}{\sqrt{3}}x$

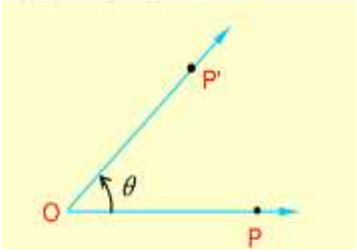
Solution

$$\theta = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^0 = \frac{\pi}{6}$$

X'=x cos 2θ + y sin 2θ=3 cos $\frac{\pi}{6}$ + 2 sin $\frac{\pi}{6}$ = 3($\frac{1}{2}$)+2($\frac{\sqrt{3}}{2}$)= $\frac{3+2\sqrt{3}}{2}$

y'=x sin 2θ - y cos 2θ=3 sin $\frac{\pi}{6}$ - 2 cos $\frac{\pi}{6}$ = 3($\frac{\sqrt{3}}{2}$)-2($\frac{1}{2}$)= $\frac{3\sqrt{3}-2}{2}$

M(3,2)=($\frac{3+2\sqrt{3}}{2}$, $\frac{3\sqrt{3}-2}{2}$)

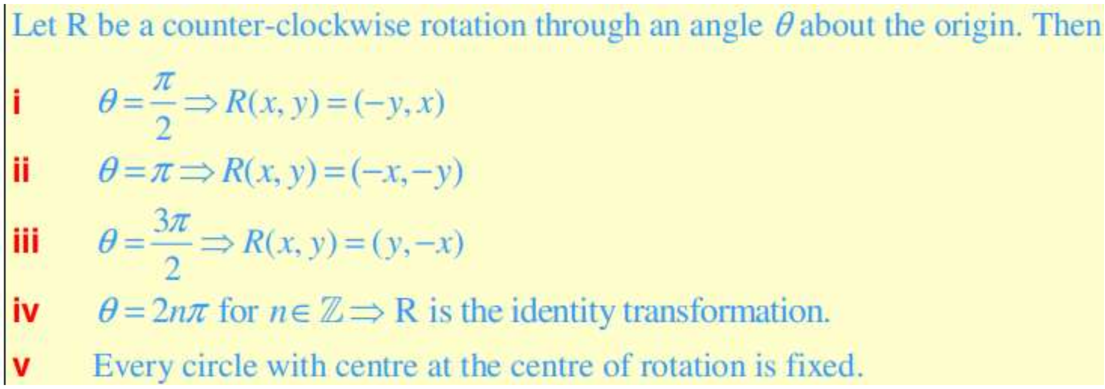


C. Rotation-A rotation R about a point O through an angle θ is a transformation of the plane onto itself which carries every point P of the plane into the point P' of the plane such that OP = OP' and m (∠POP') = θ. O is called the center of rotation and θ is called the angle of rotation.

Let R be a rotation through angle θ about the origin.

If $R_θ$ (x, y) = (x', y'), then x' = x cos θ – y sin θ

y' = x sin θ + y cos θ



Let R be a counter-clockwise rotation through an angle θ about the origin. Then

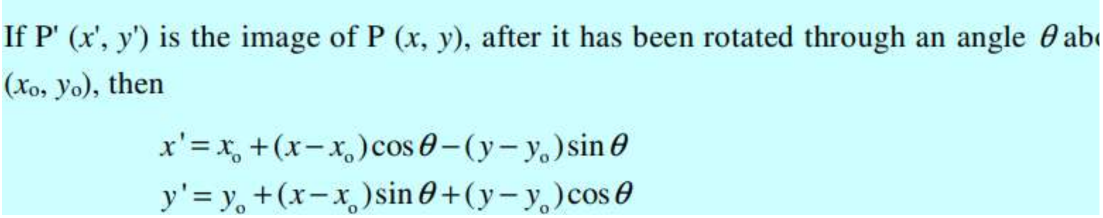
i $\theta = \frac{\pi}{2} \Rightarrow R(x, y) = (-y, x)$

ii $\theta = \pi \Rightarrow R(x, y) = (-x, -y)$

iii $\theta = \frac{3\pi}{2} \Rightarrow R(x, y) = (y, -x)$

iv $\theta = 2n\pi$ for $n \in \mathbb{Z} \Rightarrow$ R is the identity transformation.

v Every circle with centre at the centre of rotation is fixed.



If P' (x', y') is the image of P (x, y), after it has been rotated through an angle θ ab

(x_o, y_o), then

$$x' = x_o + (x - x_o)\cos\theta - (y - y_o)\sin\theta$$
$$y' = y_o + (x - x_o)\sin\theta + (y - y_o)\cos\theta$$

Example

1. Find the image of the circle $(x-3)^2+(y+5)^2=1$, when it is rotated through $\frac{5\pi}{3}$.

Solution

Rotate the center (x,y)=(3,-5)

x' = x cos θ – y sin θ= 3 cos $\frac{5\pi}{3}$ + 5 sin $\frac{5\pi}{3}$ =3 cos (45^0) + 5 sin (-45^0)=$\frac{3\sqrt{2}}{2} - \frac{5\sqrt{2}}{2} = -\sqrt{2}$

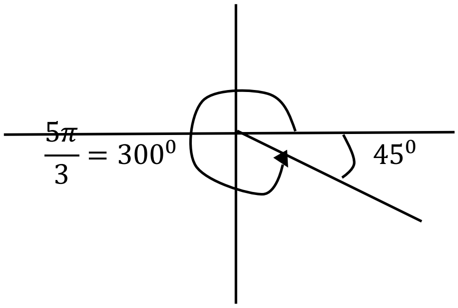
y' = x sin θ + y cos θ= 3 sin $\frac{5\pi}{3}$ + 5 cos $\frac{5\pi}{3}$ = 3 sin (-45^0) + 5 cos (45^0)= $\frac{-3\sqrt{2}}{2} + \frac{5\sqrt{2}}{2} = \sqrt{2}$

Circle, $(x-3)^2+(y+5)^2=1$

Image, $(x+\sqrt{2})^2+(y-\sqrt{2})^2=1$

Reference angle to $\frac{5\pi}{3}$ is 45^0, and $\frac{5\pi}{3}$ is fourth quadrant angle.

Sine is negative and cosine is positive in the fourth quadrant.



Grade 11 – unit nine – further on trigonometric functions

- **Trigonometric functions**

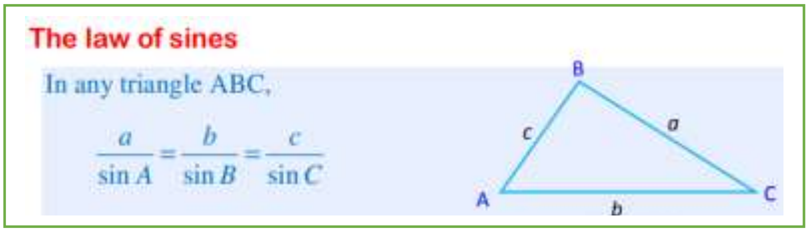
number	Function	Domain	Range	Amplitude	Period	Phase angle	Phase shift
1	Y=sin x	R	[-1,1]	1	2π	0	0
2	Y=$\frac{1}{\sin x}$= csc x	R/{kπ,k∈Z}	(-∞,-1]U[1, ∞)		2π		
3	Y=cos x	R	[-1,1]	1	2π	0	0
4	Y=$\frac{1}{\cos x}$= sec x	R/{($\frac{2k+1}{2}$)π,k∈Z}	(-∞,-1]U[1, ∞)		2π		
5	Y=tan x	R/{($\frac{k\pi}{2}$),k∈Z,k≠0}	R		π		
6	Y=$\frac{1}{\tan x}$= cot x	R/{$k\pi$,k∈Z}	R		π		
7	Y=a sin(kx+b)+c	R	[c-IaI,c+IaI]	IaI	$\frac{2\pi}{k}$	-b	$\frac{-b}{k}$
8	Y=a cos(kx+b)+c	R	[c-IaI,c+IaI]	IaI	$\frac{2\pi}{k}$	-b	$\frac{-b}{k}$

Example F(x)=$3-\sin\left(\frac{1}{2}x+\frac{\pi}{4}\right)=-\sin\left(\frac{1}{2}x+\frac{\pi}{4}\right)+3=a\sin(kx+b)+c$

a= - 1, k= ½ , b=$\frac{\pi}{4}$, c=3

- Domain=R
- Range = [c-IaI,c+IaI]= [3-I-1I,3+I-1I]= [3 - 1,3 + 1]= [2,4]
- Amplitude = IaI= I-1I= 1
- Period = $\frac{2\pi}{k}=\frac{2\pi}{\frac{1}{2}}=4\pi$
- Phase angle= - b= - $\frac{\pi}{4}$
- Phase shift = $\frac{-b}{k}$=$\frac{\frac{-\pi}{4}}{\frac{1}{2}}=\frac{-\pi}{2}$

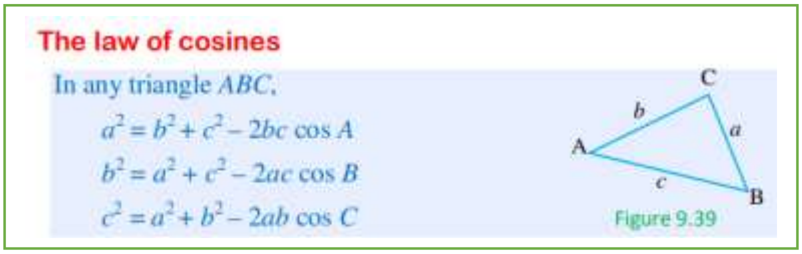
- Applications of trigonometric functions



The law of sines

In any triangle ABC,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



The law of cosines

In any triangle ABC,

$$a^2=b^2+c^2-2bc\cos A$$
$$b^2=a^2+c^2-2ac\cos B$$
$$c^2=a^2+b^2-2ab\cos C$$

Examples

1. Two drivers A and B leave the same place at the same time. If A drives 80 km/hr in the direction of N 30° E and B drives 90km/hr in the direction of E 60°N, how far apart are they after $1\frac{1}{2}$ hours?

Solution

AO=80 km/hr*$\frac{3}{2}$hr=120 km

BO=90 km/hr*$\frac{3}{2}$hr=135 km

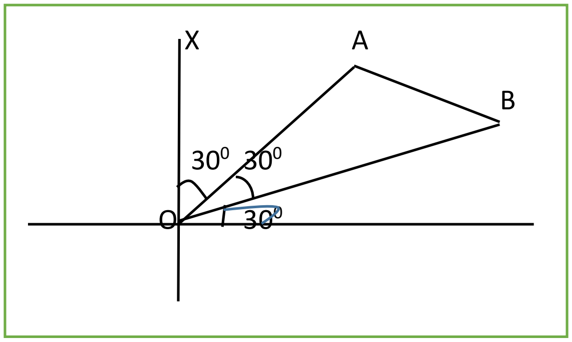
$(AB)^2=(AO)^2+(BO)^2 - 2(AO)(BO)\cos 30^0$

$(AB)^2=(120)^2+(135)^2 - 2(120)(135)0.866$

$(AB)^2=14400+18225 - 16200*0.866$

$(AB)^2=32625 - 14029=18569$

AB=136



2. The angle of elevation of the top of a building is found to be 70° as measured from a point on a level ground. If the angle of elevation of a point on the building that is 3 m below the top is 60° as measured from the same point on the ground, find the height of the building.

Solution

$\tan 70^0=\frac{3+y}{x}$

$2.7475=\frac{3+y}{x}$

2.7475=3+y

$X=\frac{3+y}{2.7475}$

$\tan 60^0=\frac{y}{x}$

$1.7321=\frac{y}{x}$

y=1.7321x

$y=1.7321(\frac{3+y}{2.7475})$

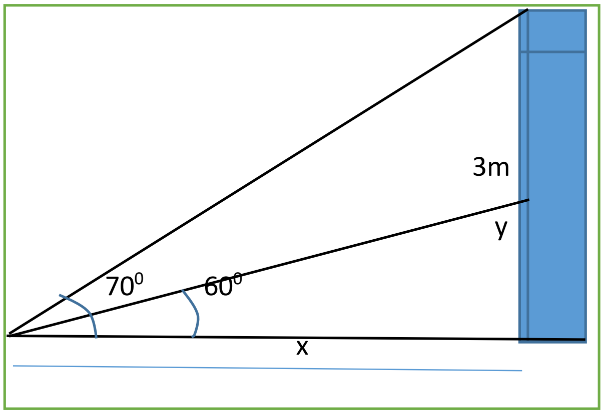
Y=0.6304(3+y)

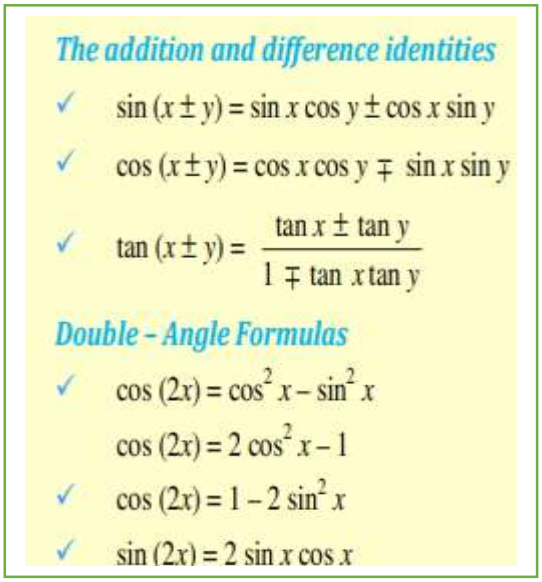
y- 0.6304y=1.8912

0.37y=1.8912

$Y=\frac{1.8912}{0.37} = 5.12$

Height of the building = 3+y=3+5.12=8.12m





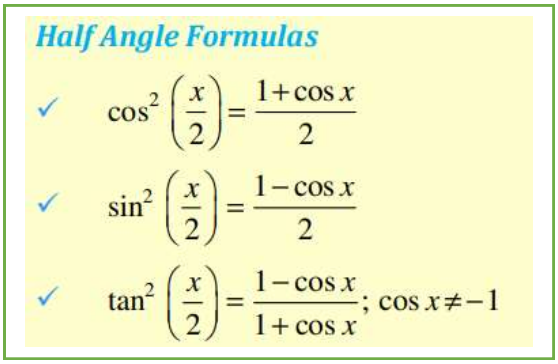
The addition and difference identities

- ✓ $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
- ✓ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
- ✓ $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

Double - Angle Formulas

- ✓ $\cos(2x) = \cos^2 x - \sin^2 x$

  $\cos(2x) = 2\cos^2 x - 1$
- ✓ $\cos(2x) = 1 - 2\sin^2 x$
- ✓ $\sin(2x) = 2\sin x \cos x$



Half Angle Formulas

- ✓ $\cos^2\left(\frac{x}{2}\right) = \dfrac{1+\cos x}{2}$
- ✓ $\sin^2\left(\frac{x}{2}\right) = \dfrac{1-\cos x}{2}$
- ✓ $\tan^2\left(\frac{x}{2}\right) = \dfrac{1-\cos x}{1+\cos x}; \cos x \neq -1$

Grade 11 – unit ten – introduction to linear programming

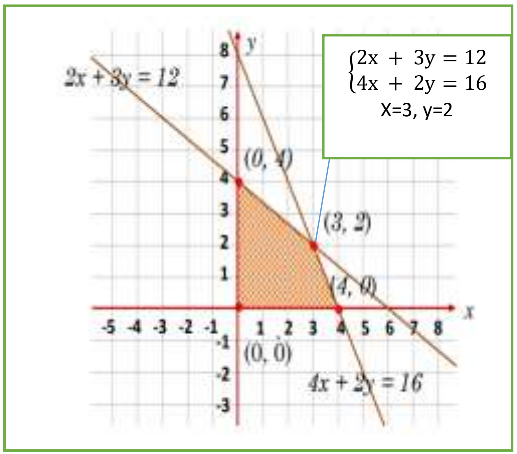
- Maximum and minimum values

Examples

1. A furniture company makes tables and chairs. To produce a table it requires 2 hrs on machine A, and 4 hrs on machine B. To produce a chair it requires 3 hrs on machine A and 2 hrs on machine B. Machine A can operate at most 12 hrs a day and machine B can operate at most 16 hrs a day. If the company makes a profit of Birr 12 on a table and Birr 10 on a chair, how many of each should be produced to maximize its profit?

Solution

Let x be the number of tables to be produced and
y be the number of chairs to be produced.
From machine A: $2x + 3y \leq 12$
From machine B: $4x + 2y \leq 16$
$x \geq 0$ and $y \geq 0$
$P = 12x + 10y$
The profit: $P = 12x + 10y$ at each vertex is
At (0, 0), $P = 12 (0) + 10 (0) = 0$
At (0, 4), $P = 12 (0) + 10 (4) = 40$
At (3, 2), $P = 12 (3) + 10 (2) = 56$
At (4, 0), $P = 12 (4) + 10 (0) = 48$
Therefore, the profit is maximum at the
Vertex (3, 2), so the company should
Produce 3 tables and 2 chairs per day to get the maximum profit of Birr 56.

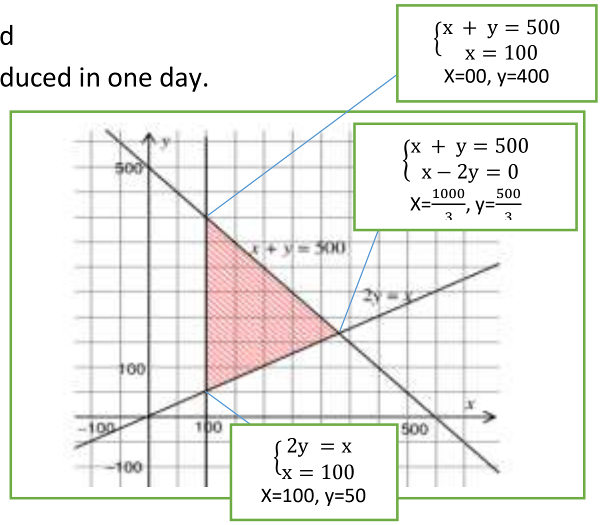


$\begin{cases} 2x + 3y = 12 \\ 4x + 2y = 16 \end{cases}$
X=3, y=2

2. Suppose a shoe factory produces both low-grade and high-grade shoes. The factory produces at least twice as many low-grade as high-grade shoes. The maximum possible production is 500 pairs of shoes. A dealer calls for delivery of at least 100 high-grade pairs of shoes per day. Suppose the operation makes a profit of Birr 2.00 per a pair of shoes on high-grade shoes and Birr 1.00 per pairs of shoes on low-grade shoes. How many pairs of shoes of each type should be produced for maximum profit?

Solution

Let x denote the number of high grade pairs of shoes, and
y denote the number of low grade pairs of shoes produced in one day.
$2y \geq x$
$x + y \leq 500$
$x \geq 100$
$x \geq 0 , y \geq 0$
Evaluating $2x + y$ at each vertex, we obtain
$2(100) + 400 = 600$
$2(100) + 50 = 250$
$2(\frac{1000}{3})+ \frac{500}{3} = \frac{2500}{3}$
The maximum profit is realized by producing
$\frac{1000}{3}$ Shoe of high-grade and $\frac{500}{3}$ shoe of low-grade.



$\begin{cases} x + y = 500 \\ x = 100 \end{cases}$
X=00, y=400

$\begin{cases} x + y = 500 \\ x - 2y = 0 \end{cases}$
X=$\frac{1000}{3}$, y=$\frac{500}{3}$

$\begin{cases} 2y = x \\ x = 100 \end{cases}$
X=100, y=50

Grade 11 – unit eleven – mathematical applications in business

- Ratio

  Example,

  1. If a profit of birr 12,000 is divided for four partners A,B,C, and D at a ratio 3:2:1:4 respectively, then find the amount of money received by each partners.

Solution

$$\frac{12000}{3+2+1+4}=\frac{12000}{10}=1200$$

A receives 3*1200=3600

B receives 2*1200=2400

C receives 1*1200=1200

D receives 4*1200=4800

Total=12000

- Mark up (m.u)= selling price(s.p) – cost price(c.p)

  Example

  1. A car costs birr 2,500,000 and sold by birr 4,200,000.find
     a. Markup.
     b. Markup percent with respect to the selling price.
     c. Markup percent with respect to the cost price.

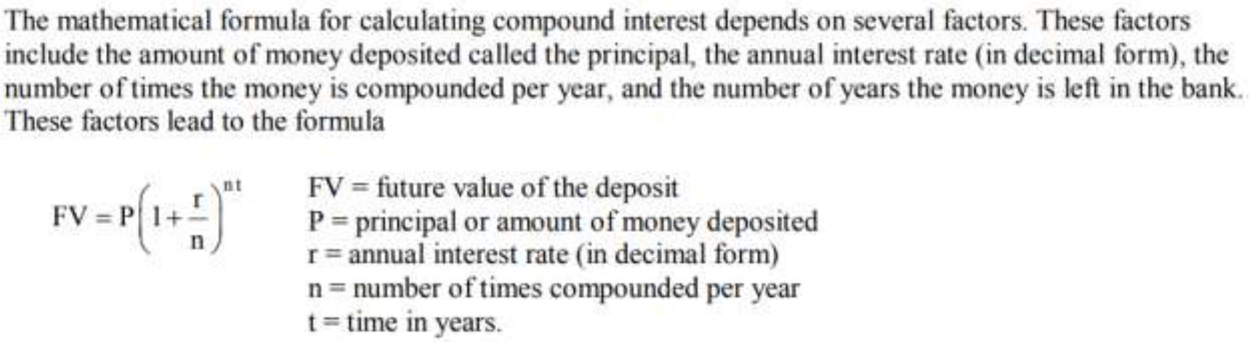
Solution

a. m.u = s.p - c.p = 4,200,000 – 2,500,000 = 1,700,000

b. m.u % w.r.t s.p = $\frac{m.u}{s.p} * 100\%$=$\frac{1700000}{4200000} * 100\%$= 40.5 %

c. m.u % w.r.t c.p = $\frac{m.u}{c.p} * 100\%$=$\frac{1700000}{2500000} * 100\%$= 37.8 %

- compound interest



The mathematical formula for calculating compound interest depends on several factors. These factors include the amount of money deposited called the principal, the annual interest rate (in decimal form), the number of times the money is compounded per year, and the number of years the money is left in the bank. These factors lead to the formula

$$FV = P\left(1+\frac{r}{n}\right)^{nt}$$

FV = future value of the deposit
P = principal or amount of money deposited
r = annual interest rate (in decimal form)
n = number of times compounded per year
t = time in years.

Examples

Example 1: If you deposit $4000 into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years?

$$FV = 4000\left(1+\frac{0.06}{4}\right)^{4(5)}$$

Plug in the giving information, P = 4000, r = 0.06, n = 4, and t = 5.

$$FV = 4000(1.015)^{20}$$

$$FV = 4000(1.346855007)$$

Use the order or operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 5387.42$$

Round your final answer to two decimals places.

After 5 years there will be $5387.42 in the account.

Example 2: If you deposit $6500 into an account paying 8% annual interest compounded monthly, how much money will be in the account after 7 years?

$$FV = 6500\left(1+\frac{0.08}{12}\right)^{12(7)}$$

Plug in the giving information, P = 6500, r = 0.08, n = 12, and t = 7.

$$FV = 6500(1.00666666)^{84}$$

$$FV = 6500(1.747422051)$$

Use the order or operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 11358.24$$

Round your final answer to two decimals places.

After 7 years there will be $11358.24 in the account.

Example 3: How much money would you need to deposit today at 9% annual interest compounded monthly to have $12000 in the account after 6 years?

$$12000 = P\left(1+\frac{0.09}{12}\right)^{12(6)}$$

Plug in the giving information, FV = 12000, r = 0.09, n = 12, and t = 6.

$$12000 = P(1.0075)^{72}$$

$$12000 = P(1.712552707)$$

Use the order or operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$P = 7007.08$$

Divide and round your final answer to two decimals places.

Grade 12 – unit one – sequences and series

- arithmetic sequence

$\{a_n\}$ is arithmetic if the terms have common difference (d)

Example: 5, 10, 15, 20, …. is an arithmetic sequence with d=20-15=15-10=10-5=5

nth term of an arithmetic sequence is given by $A_n=A_1+(n-1)d$

Sum of the first n terms an arithmetic sequence $S_n=\frac{n}{2}(2(A_1)+(n-1)d)$

Examples

1. if $\{a_n\}$ is an arithmetic sequence with first term $a_1=5$, and the fifth term $a_5=21$, then find the partial sum $\sum_{n=1}^{30}(a_n)$
2. What is the sum of all multiples of four that are between 30 and 301?

<u>Solution</u>

1. $\sum_{n=1}^{30}(a_n)=S_{30}=\frac{n}{2}(2(A_1)+(n-1)d)$

$=\frac{30}{2}(2(5)+(30-1)4)$

$=15(10+(29)4)$

$=15(10+116)$

$=15(126)$

$=1890$

$A_n=A_1+(n-1)d$

$A_5=A_1+(5-1)d$

$21=5+4d$

$21-5=4d$

$16=4d$

$4=d$

2. The terms are 32,36,40,…………,300, $A_1=32$, d= 4, $A_n=300$

$S_{68}=\frac{n}{2}(2(A_1)+(n-1)d)$

$=\frac{68}{2}(2(32)+(68-1)4)$

$=34(64+(67)4)$

$=34(64+268)$

$=34(332)$

$=11288$

$A_n=A_1+(n-1)d$

$300=32+(n-1)4$

$300=32+4n-4$

$300-32+4=4n$

$272=4n$

$68=n$

- Geometric sequence

$\{G_n\}$ is a geometric sequence if the consecutive terms have common ratio.

Example, $9,3,1,\frac{1}{3},\frac{1}{9},\frac{1}{27}$……. Is a geometric sequence with $r=\frac{1/27}{1/9}=\frac{1/9}{1/3}=\frac{1/3}{1}=\frac{1}{3}=\frac{3}{9}=\frac{1}{3}$

nth term of a geometric sequence is given by $G_n=G_1r^{n-1}$

The sum of the first n terms of geometric sequence is $S_n=\begin{cases}G_1\frac{(1-r^n)}{(1-r)}, if\ r\neq 1\\ nG_1, if\ r=1\end{cases}$

Examples

1. Find the fourth term G_4 and the sum S_6 of the sequence with terms 2,8,32,-----

<u>Solution</u>

$G_1=2$, $r=\frac{32}{8}=\frac{8}{2}=4$

$G_n=G_1r^{n-1}$

$G_4=G_1r^{4-1}$

$G_4=(2)(4)^3$

$G_4=(2)(64)=128$

$S_n=G_1\frac{(1-r^n)}{(1-r)}$

$S_6=2\frac{(1-4^6)}{(1-4)}$

$S_6=2\frac{(1-4096)}{(1-4)}=2\frac{(-4095)}{(-3)}$

$S_6=\frac{8190}{3}=2730$

- Sigma notation and partial sum

  Examples: find the partial sum

  1. $\sum_{n=1}^{5}(5n-4) = A_1 + A_2 + A_3 + A_4 + A_5 = 1 + 6 + 11 + 16 + 21 = 55$
  2. $\sum_{n=1}^{30}\left((-1)^n\left(\frac{1}{n}+\frac{1}{n+1}\right)\right) =-\left(1+\frac{1}{2}\right)+\left(\frac{1}{2}+\frac{1}{3}\right)-\left(\frac{1}{3}+\frac{1}{4}\right)+\cdots+(\frac{1}{30}+\frac{1}{31})$

     $= -1-\frac{1}{2}+\frac{1}{2}+\frac{1}{3}-\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{30}+\frac{1}{31}$

     $= -1+\frac{1}{31} = \frac{-31+1}{31} = \frac{-30}{31}$

- **Infinite series**

  If $\{G_n\}$ is a geometric infinite series with common ratio r<1, then the sum of the series is $S = \frac{G_1}{1-r}$

  Examples: find the sum of series

  1. $\sum_{n=1}^{\infty}\left((\frac{2}{3})^n\right)$ 2. $\sum_{n=1}^{\infty}\left(\frac{2^n+5^n}{10^n}\right)$

  Solution

  1. $\sum_{n=1}^{\infty}\left(\left(\frac{2}{3}\right)^n\right) = \frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+ ------, r = \frac{2}{3}, G_1 = \frac{2}{3}$

     $$S = \frac{G_1}{1-r} = S = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{2}{3} * \frac{3}{1} = 2$$

  2. $\sum_{n=1}^{\infty}\left(\frac{2^n+5^n}{10^n}\right) = \sum_{n=1}^{\infty}\left(\left(\frac{2}{10}\right)^n+\left(\frac{5}{10}\right)^n\right) = \sum_{n=1}^{\infty}\left(\left(\frac{1}{5}\right)^n+\left(\frac{1}{2}\right)^n\right)$

     $= \left(\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+ ------\right)+\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+ ------\right)$

     $r = \frac{1}{5}, G_1 = \frac{1}{5}$ $r = \frac{1}{2}, G_1 = \frac{1}{2}$

     $S_1 = \frac{G_1}{1-r} = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{5} * \frac{5}{4} = \frac{1}{4}$ $S_2 = \frac{G_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} * \frac{2}{1} = 1$

     $S = S_1 + S_2 = \frac{1}{4} + 1 = \frac{5}{4}$

- **Applications of arithmetic and geometric progressions**

  Examples

  1. A factory that produces cement had sales Birr 100,000.00 the first day and sales increased by Birr 8,000.00 every day during each successive days. Find the total sales of the factory during the first 30 days.

     Solution

     A$_1$=100,000, d=8,000, $S_{30} = \frac{n}{2}(2(A_1)+(n-1)d)$

     $= \frac{30}{2}(2(100000)+(30-1)8000) = 15(200000+(29)8000)$

     $=15(200000+232000) = 15(432000) = 6{,}480{,}000$

  2. The population of a certain town is increasing at a rate of 2.5% per year. If the population is currently 100,000, what will the population be 10 years from now?

     Solution

     r=1+0.025=1.025, G$_1$=100,000

     $G_{10} = G_1 r^n = G_1 r^{10} = 100{,}000(1.025)^{10} = 100{,}000 * 1.2800845 = 128008.45$

Grade 12 – unit two – introduction to limits and continuity

- **Limits of sequences of numbers**
- Bounded sequences - A sequence is bounded if it has both upper and lower bonds.

  Example, $(\frac{n}{n+1})_{n=1}^{\infty}$

  The terms are$\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},----,1,$ $\frac{1}{2}$ is min and 1 is max

  Lower bounds $\le\frac{1}{2}$, which are $\frac{1}{2}$, 0,-1,-2, -----, greatest lower bound, glb=$\frac{1}{2}$

  Upper bounds$\geq$ 1, which are 1, 2, 3, ------, least upper bound, lub=1
- Convergence of sequences

  A sequence is convergent if $\underset{n\to\infty}{lim}a_n = L$, L$\in$ R, divergent if $\underset{n\to\infty}{lim}a_n = \infty, or\underset{n\to\infty}{lim}a_n = -\infty,$

  Examples; note $\frac{a}{\infty} = 0$

  1. $(\frac{n}{n+1})_{n=1}^{\infty}$, $\underset{n\to\infty}{lim}(\frac{n}{n+1})$=$\underset{n\to\infty}{lim}(\frac{\frac{n}{n}}{\frac{n}{n}+\frac{1}{n}})$=$\underset{n\to\infty}{lim}(\frac{1}{1+\frac{1}{n}})$= 1, convergent
  2. $(\frac{n^3}{2n+3})_{n=1}^{\infty}$, $\underset{n\to\infty}{lim}(\frac{\frac{n^3}{n^3}}{\frac{2n}{n^3}+\frac{1}{n^3}})$=$\underset{n\to\infty}{lim}(\frac{1}{\frac{2}{n^2}+\frac{1}{n^3}})$=$\underset{n\to\infty}{lim}(\frac{1}{0})$= ∞, divergent

- **Limits of functions**

  Examples

  1. $\underset{n\to2}{lim}(3x^2+5x)$ =$3(2)^3+5(2) = 34$
  2. $\underset{n\to2.}{lim}\frac{x^2-4}{x-2} = \underset{n\to2.}{lim}\frac{(x+2)(x-2)}{x-2} = \underset{n\to2}{lim}(x+2) = 4$
- One side limit

  $\underset{n\to L}{lim}f(x)\ exists\ if\ \underset{n\to L^-}{lim}f(x) = \underset{n\to L^+}{lim}f(x)$

  Examples, find the limit of

  1. $\underset{n\to o^+}{lim}\frac{|x|}{x} = 1,\ o^+$is a positive number.
  2. $\underset{n\to o^-}{lim}\frac{|x|}{x} = -1,\ o^-$ is a negative positive number.
  3. $\underset{n\to 0}{lim}\frac{|x|}{x}, does\ not\ exist$, because $\underset{n\to o^-}{lim}\frac{|x|}{x} = -1 \neq \underset{n\to o^+}{lim}\frac{|x|}{x} = 1$
  4. $\underset{n\to1^-}{lim}\frac{x^4-1}{|x-1|} = \underset{n\to1^-}{lim}\frac{(x-1)(x^3+x^2+x+1)}{|x-1|} = \underset{n\to1^-}{lim}\frac{(x-1)}{|x-1|} * \underset{n\to1^-}{lim}(x^3+x^2+x+1) = (-1)(4) = -4$

- **Continuity of a function**

  A function is continues if - i. $\underset{n\to L}{lim}f(x)\ exists\ or\ \underset{n\to L^-}{lim}f(x) = \underset{n\to L^+}{lim}f(x)$

  ii. $\underset{n\to L}{lim}f(x) = f(x)$

  iii. L is on the domain of f(x)

  Examples, check the continuity

  1. f(x)= $\begin{cases} 5x-2\ if\ x>2 \\ 8\ if\ x=2 \\ 12x-16\ if\ x<2 \end{cases}$
     a. $\underset{n\to2^-}{lim}f(x) = \underset{n\to2}{lim}(12x-16) = 12(2)-16 = 24-16 = 8$

        $\underset{n\to2^+}{lim}f(x) = \underset{n\to2}{lim}(5x-2) = 5(2)-2 = 10-2 = 8$

        $\underset{n\to2^-}{lim}f(x) = \underset{n\to2^+}{lim}f(x)$ Thus, $\underset{n\to2}{lim}f(x)\ exists$
     b. f(2)=8, $\underset{n\to2}{lim}f(x) = f(2)$
     c. 2 is in the domain of f

        F(x) is continues at x=2

2. f(x)= $\begin{cases} 5x - 2 \; if \; x > 1 \\ 8 \; if \; x = 1 \\ 12x - 16 \; if \; x < 1 \end{cases}$

$\lim_{n\to 1^-} f(x) = \lim_{n\to 1} (12x - 16) = 12(1) - 16 = 12 - 16 = -4$

$\lim_{n\to 1^+} f(x) = \lim_{n\to 1} (5x - 2) = 5(1) - 2 = 5 - 2 = 3$

$\lim_{n\to 1^-} f(x) \neq \lim_{n\to 1^+} f(x)$ Thus, $\lim_{n\to 1} f(x) \;\; does\, notexist$

$f \; is\, not\, continues\, on\, 1$

- applications of limits

Note: $i. \lim_{n\to 0} \frac{\sin x}{x} = 1,$ $ii. \lim_{n\to 0} \frac{\tan x}{x} = 1,$ $iii. \lim_{n\to \infty} (1 + \frac{1}{x})^x = e,$

Examples, find

1. $\lim_{n\to 0} \frac{3x+\sin 5x}{4x-\sin 2x} = \lim_{n\to 0} \frac{\frac{3x+\sin 5x}{x}}{\frac{4x-\sin 2x}{x}} = \lim_{n\to 0} \frac{3+\frac{\sin 5x}{x}}{4-\frac{\sin 2x}{x}} = \frac{3+\lim_{n\to 0} \frac{\sin 5x}{x}}{4-\lim_{n\to 0} \frac{\sin 2x}{x}} = \frac{3+5\lim_{n\to 0} \frac{\sin 5x}{5x}}{4-2\lim_{n\to 0} \frac{\sin 2x}{2x}} = \frac{3+5}{4-2} = 4$

2. Find the value of k so that f(x)= $\begin{cases} \frac{\tan 2x}{x}, if \; x > 0 \\ k - e^{2x} \; if \; x \leq 0 \end{cases}$ is continues at x=0.

If f continue then the limit exists or both sides limit are equal

$$\lim_{n\to 0^-} f(x) = \lim_{n\to 0} f(x)$$

$$\lim_{n\to 0} (k - e^{2x}) = \lim_{n\to 0} \left(\frac{\tan 2x}{x}\right)$$

$$k - e^0 = \lim_{n\to 0} \left(\frac{2 \; \tan 2x}{2x}\right)$$

$$k - 1 = 2\lim_{n\to 0} \left(\frac{\tan 2x}{2x}\right)$$

$$k - 1 = 2$$

$$k = 3$$

Grade 12 – unit three – introduction to differential calculus

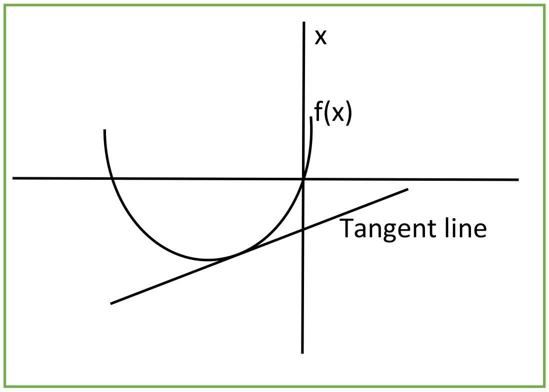
- Graphical definition of derivatives

A tangent line is a line which touches a curve at exactly one point.

Slope of a tangent line to a curve at (x,y) is given by $m = f^{l}(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

Example

1. Find the equation of the tangent line to the curve of f(x)= x^2 + 4x at x=1



<u>Solution</u>

Slope, $m = f^{l}(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{f(1+h)-f(1)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{(1+h)^2+4(1+h)-\left(1^2+4(1)\right)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{1+2h+h^2+4+4h-5}{h}\right)$

$= \lim_{h\to 0}\left(\frac{h^2+6h}{h}\right) = \lim_{h\to 0} h\left(\frac{h+6}{h}\right) = \lim_{h\to 0}(h+6) = 6$

Equation $\frac{y-y_1}{x-x_1} = m$ (x_1,y_1)=(1,f(1))=(1,5)

$\boldsymbol{\frac{y-5}{x-1} = 6}$

y-5=6x-6

y=6x -1

- Derivatives of functions

a. f(x)=x^n, $f^{l}(x) = nx^{n-1}$
b. f(x)=a^x, $f^{l}(x) = a^x \ln a$
c. f(x)=e^x, $f^{l}(x) = e^x$
d. f(x)=$log\,{}^{x}_{a}$, $f^{l}(x) = \frac{1}{x\ln a}$
e. f(x)=$\ln x$, $f^{l}(x) = \frac{1}{x}$
f. f(x)=$\sin x$, $f^{l}(x) = \cos x$
g. f(x)=$\cos x$, $f^{l}(x) = -\sin x$
h. f(x)=$\sin(ax+b)$, $f^{l}(x) = a\ \cos(ax+b)$
i. f(x)=$\cos(ax+b)$, $f^{l}(x) = -a\sin(ax+b)$
j. f(x)=$(ax+b)^n$, $f^{l}(x) = na(ax+b)^{n-1}$

Example, find the derivative of

1. f(x)= x^4+6x^2-8x+9, f^{l}(x)=4x^3+12x-8, note derivative of constant number is zero
2. f(x)=$\frac{1}{x}$ = x^{-1}, f^{l}(x)= - x^{-1-1}= - x^{-2} = $\frac{-1}{x^2}$
3. f(x)=$\sqrt{x} = x^{\frac{1}{2}}$, f^{l}(x)=$\frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$
4. f(x)=$\sin(5x-4)$, $f^{l}(x) = 5\cos(5x-4)$
5. f(x)=$\cos(-3x+9)$, $f^{l}(x) = -(-3)\sin(-3x+9) = 3\sin(-3x+9)$
6. f(x)=$(5x+5)^8$, $f^{l}(x) = 8(5)(5x+5)^{8-1} = 40(5x+5)^7$

- rules of integration

a. sum rule $[f(x)+g(x)]^{l} = f^{l}(x)+g^{l}(x)$
b. difference rule $[f(x)-g(x)]^{l} = f^{l}(x)-g^{l}(x)$
c. product rule $[f(x)g(x)]^{l} = f^{l}(x)g(x)+g^{l}(x)f(x)$
d. quotient rule $[f(x)/g(x)]^{l} = \frac{f^{l}(x)g(x)-g^{l}(x)f(x)}{g(x)^2}$
e. chain rule $[f(g(h(x)))]^{l} = f^{l}\left(g\left(h(x)\right)\right)g^{l}(f(x))h^{l}(x)$

Examples, find the derivative of

1. f(x)= $3x^2$+cos x – e^x, fl(x)=6x – cos x – e^x
2. f(x)= $\tan x = \frac{\sin x}{\cos x}, f^l(x) = \frac{(\sin x)^l(\cos x)-(\cos x)^l(\sin x)}{(\cos x)^2} = \frac{\sin x(\sin x)-(-\sin x)(\sin x)}{(\cos x)^2}$

$$= \frac{sin^2x+cos^2x}{(\cos x)^2} = \frac{1}{cos^2x} = sec^2x$$

$$similarly, \text{f(x)} = \cot x = \frac{\cos x}{\sin x}, f^l(x) = -csc^2x$$

$$\text{f(x)} = \sec x = \frac{1}{\cos x}, f^l(x) = \sec x \tan x$$

$$\text{f(x)} = \csc x = \frac{1}{\sin x}, f^l(x) = -\csc x \cot x$$

3. $f(x) = \ln\sqrt{x^2+1}, f^l(x) = \left(\ln\sqrt{x^2+1}\right)^l\left(\sqrt{x^2+1}\right)^l(x^2+1)^l$

$$= \frac{1}{\sqrt{x^2+1}} * \frac{1}{2\sqrt{x^2+1}} * 2x, \qquad \text{note}(\ln x)^l = \frac{1}{x}, \ \ (\sqrt{x})^l = \frac{1}{2\sqrt{x}}$$

$$= \frac{2x}{2(x^2+1)} = \frac{x}{x^2+1}$$

- Higher derivative

Examples, find fourth derivative of

1. f(x) = $x^5+7x^3-8x^2+2$

$f^l(x) = 5x^4+21x^2-16x$

$f^{ll}(x) = 20x^3+42x-16$

$f^{lll}(x) = 60x^2+42$

$f^4(x) = 120x$

- equation of a tangent line

1. find the equation of the tangent line to the curve of f(x), at the given point

a. $f(x) = \sqrt{9-x^2},\ at\ x = 2,$ (x_1,y_1)=(2,f(2))=(2, $\sqrt{5}$)

<u>*solution*</u>

$$f^l(x) = \frac{-2x}{2\sqrt{9-x^2}}$$

$$slope, m = f^l(2) = \frac{-2(2)}{2\sqrt{9-4}} = \frac{-2}{\sqrt{5}}$$

$$equation, \frac{y-y_1}{x-x_1} = m$$

$$\frac{y-\sqrt{5}}{x-2} = \frac{-2}{\sqrt{5}}$$

$$\sqrt{5}\,y - 5 = -2x + 4$$

$$\sqrt{5}\,y = -2x + 4 + 5$$

$$y = \frac{-2x}{\sqrt{5}} + \frac{9}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}x + \frac{9\sqrt{5}}{5}$$

- Differentiation

Examples, check the differentiability of f(x) at the given point

1. f(x) = cos x at x= 2π

f^l(x)= - sin x

$f^l(2\pi)$= - sin (2π) =0, f is differentiable at x= 2π

2. f(x)=$\sqrt{x}, at\ x = 0$

f^l(x)= $\frac{1}{2\sqrt{x}}$, f^l(0)= $\frac{1}{0} = \nexists$, f is not differentiable at x= 0

Grade 12 – unit four – applications of differential calculus

- Critical numbers

a. If $f^{l}(c)=0$, c is critical number.

b. If f is not differentiable at c (if f has no derivative at c), c is critical number.

Examples, find the critical numbers

1. $f(x)= x^2+\sqrt{x}$ 2. $f(x) = x^4- 8x^2$

Solution

1. $f(x)= x^2-\sqrt{x}$

$f^{l}(x)= 2x - \frac{1}{2\sqrt{x}}$

a. $f^{l}(0)= 0 - \frac{1}{0} = \nexists$, f is not differentiable at x= 0, 0 is critical number

b. $f^{l}(x)=0$

$2x - \frac{1}{2\sqrt{x}} =0$

$4x^{\frac{3}{2}} - 1 = 0$

$X=\pm1$, - 1 and 1 are critical numbers.

$from\ a\ and\ b - 1{,}0{,}1\ are\ critical\ numbers$

2. $f(x) = x^4 - 8x^2$

$f^{l}(x)= 4x^3 – 16x$

a. f is differentiable everywhere.

b. $f^{l}(x)=0$

$4x^3 – 16x=0$

$4x(x^2 - 4)=0$

$4x(x+2)(x - 2) =0$

X=0, x=2, x= - 2 are critical numbers.

- Extreme values

Example, find maximum and minimum values.

1. $f(x)= x^4 - 8x^2$ on [-3,3]

Solution

$f(x)=x^4 - 8x^2$ -2,0,2 are critical numbers

$f(- 3)=x^4 - 8x^2 = 9$ max

$f(3)=x^4 - 8x^2 = 9$ max

$f(- 2)=x^4 - 8x^2 = - 16$ min

$f(2)=x^4 - 8x^2 = - 16$ min

$f(0)=x^4 - 8x^2 = 0$

The extreme values are at -3, 3 or at the critical numbers.

Maximum value is 9

Minimum value is - 16

- Rolle's theorem

Let f be function that satisfies the following conditions:

a. f is continuous on the closed interval [a, b]

b. f is differentiable on the open interval (a, b)

c. f (a) = f (b)

Then, there is a number c in (a, b) such that f '(c) = 0

Examples, find a number c that satisfy the three conditions of Rolle's Theorem for $f(x)= x^2 – 4x + 1$, on [0,4].

Solution

a. f is continues on [0,4]

b. f is differentiable on (0,4)

c. f(0) = f(4) = 1

f '(x) = 2x – 4, f '(c) = 0

2c – 4 =0, c=2

- **The mean-value theorem**

Let f be a function that satisfies the following conditions:

a. f is continuous on the closed interval [a, b]

b. f is differentiable on the open interval (a, b)

Then there is a number c in (a, b) such that f '(c) = $\frac{f(b)-f(a)}{b-a}$

Examples, find a number c that satisfy the two conditions of mean value theorem for

1. f(x)= $3x^2 + 2x + 5$, on [- 1,1].

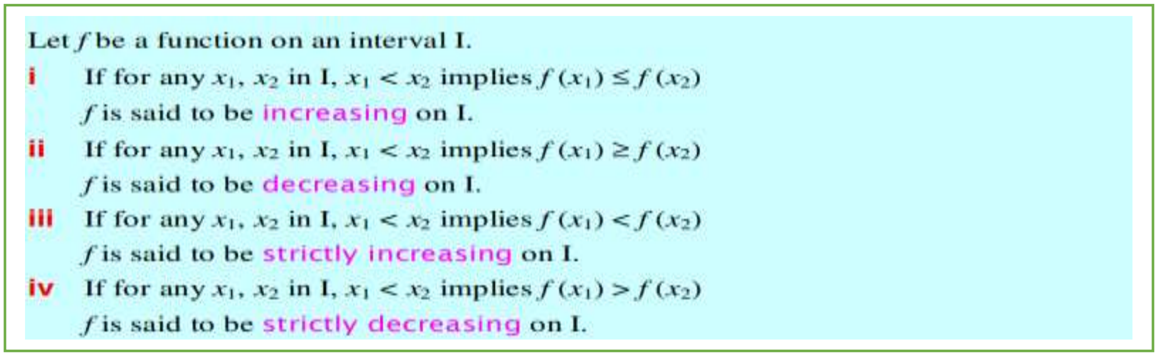
Solution

a. f is continues on [-1, 1]

b. f is differentiable on (- 1,1)

f '(x) = 6x + 2, f '(c) = $\frac{f(1)-f(-1)}{1+1}$, 6c+2 = $\frac{10-6}{2}$, 6c+2=2, c=0

- **Increasing and decreasing functions**



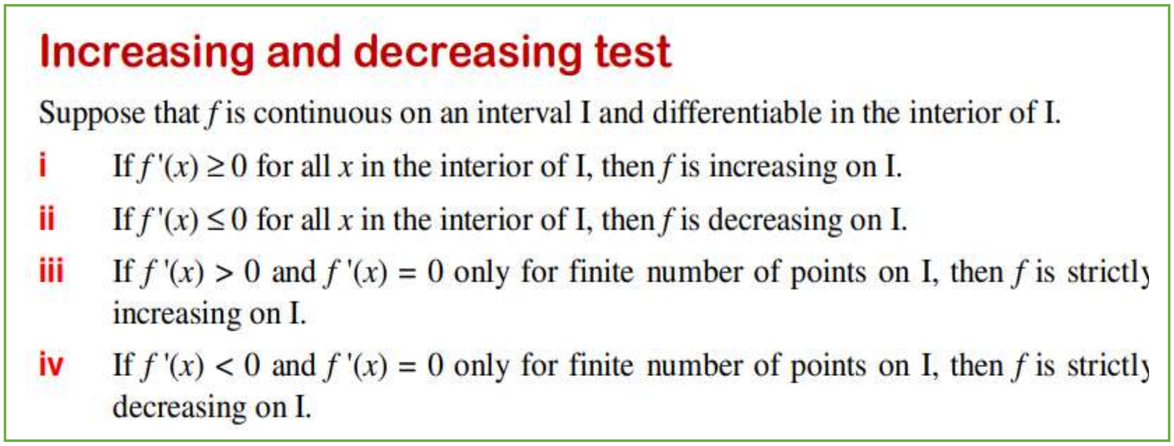
Let *f* be a function on an interval I.

i If for any x_1, x_2 in I, $x_1 < x_2$ implies $f(x_1) \leq f(x_2)$
f is said to be increasing on I.

ii If for any x_1, x_2 in I, $x_1 < x_2$ implies $f(x_1) \geq f(x_2)$
f is said to be decreasing on I.

iii If for any x_1, x_2 in I, $x_1 < x_2$ implies $f(x_1) < f(x_2)$
f is said to be strictly increasing on I.

iv If for any x_1, x_2 in I, $x_1 < x_2$ implies $f(x_1) > f(x_2)$
f is said to be strictly decreasing on I.



Increasing and decreasing test

Suppose that *f* is continuous on an interval I and differentiable in the interior of I.

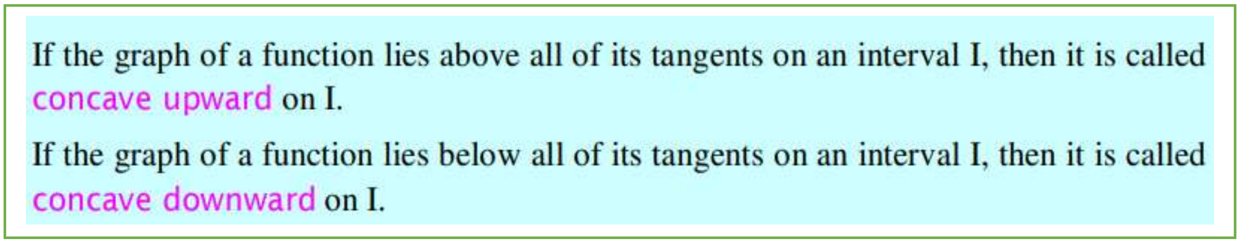
i If $f'(x) \geq 0$ for all *x* in the interior of I, then *f* is increasing on I.

ii If $f'(x) \leq 0$ for all *x* in the interior of I, then *f* is decreasing on I.

iii If $f'(x) > 0$ and $f'(x) = 0$ only for finite number of points on I, then *f* is strictl increasing on I.

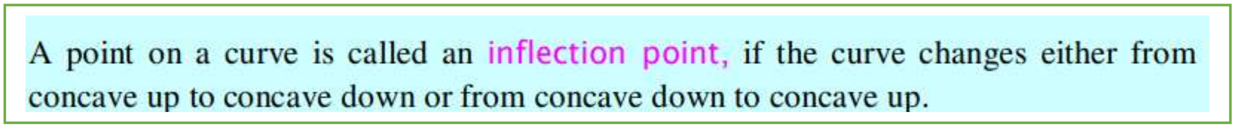
iv If $f'(x) < 0$ and $f'(x) = 0$ only for finite number of points on I, then *f* is strictl decreasing on I.

- **Concavity and inflection point**



If the graph of a function lies above all of its tangents on an interval I, then it is called concave upward on I.

If the graph of a function lies below all of its tangents on an interval I, then it is called concave downward on I.

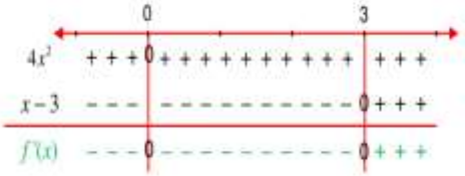


A point on a curve is called an inflection point, if the curve changes either from concave up to concave down or from concave down to concave up.

Examples

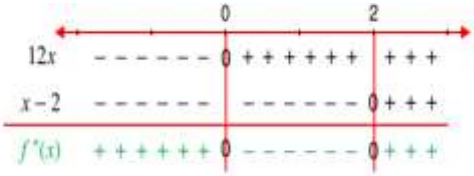
1. Find critical numbers, monotonicity intervals, concavity intervals, inflection points
   $f(x) = 4x^3 - 12x^2$

f ' (x) = 0 ⇒ $4x^3 - 12x^2 = 0$
⇒ $4x^2 (x - 3) = 0$ ⇒ $4x^2 = 0$ or $x - 3 = 0$
⇒ x = 0 or x = 3 Hence x = 0 and x = 3 are critical numbers of f.



a. f is strictly decreasing on (−∞, 3].
b. f is strictly increasing on [3, ∞)
c. The sign of f ' changes sign from negative to positive and hence by the first derivative test, f(3) = –27 is the relative minimum value of f.

f "(x) = $12x^2 - 24x$,
f " (x) = 0 ⇒ 12x (x – 2) = 0
⇒ 12x = 0 or x – 2 = 0 ⇒ x = 0 or x = 2



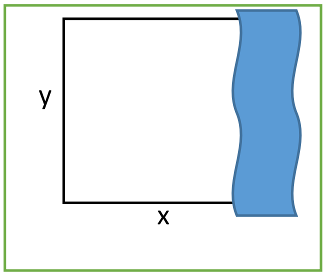
a. the graph of f is concave upward on (−∞, 0) and) (2,∞)
b. the graph of f is concave downward on (0, 2) .
c. (0, f (0)) = (0, 0) and (2, f (2)) = (2, –16) are the inflection points of the graph of f.

- **minimization and maximization problems**

1. A farmer has 2000 m of fencing to enclose a rectangular region adjacent to a river. No fencing is required along the river. Find the dimensions that maximize the area.

2x+y=2000
Y=2000 – 2x

A=xy
A=x(2000 – 2x)
A= - $2x^2$+2000x
A^{l}=0
-4x+2000=0
X=500m
y=2000 -2x
=2000 – 1000 = 1000m



- **Rate of change**

1. Find $\frac{dy}{dx}$

a. $x^2 + 2xy = 10$

$$2x + 2y + 2x\frac{dy}{dx} = 0$$
$$2x\frac{dy}{dx} = -2x - 2y$$
$$\frac{dy}{dx} = \frac{-x-y}{x}$$

b. $x^3y^3 - 5xy^2 + 2x^2y = xy$

$$3x^2y^3 - 5y^2 + 4xy + 3x^3y^2\frac{dy}{dx} - 10xy\frac{dy}{dx} + 2x^2\frac{dy}{dx} = y + x\frac{dy}{dx}$$
$$\frac{dy}{dx}(3x^3y^2 - 10xy + 2x^2 - x) = -3x^2y^3 + 5y^2 - 4xy + y$$
$$\frac{dy}{dx} = \frac{-3x^2y^3 + 5y^2 - 4xy + y}{3x^3y^2 - 10xy + 2x^2 - x}$$

Grade 12 – unit five – introduction to integral calculus

- Integration of some functions

a. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

b. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

c. $\int e^x dx = e^x + c$

d. $\int a^x dx = \frac{a^x}{\ln a} + c$

e. $\int \cos x\, dx = \sin x + c$

f. $\int \sin x\, dx = -\cos x + c$

g. $\int \frac{1}{x} dx = \ln|x| + c$

h. $\int \sin(ax+b)\, dx = \frac{-\cos(ax+b)}{a} + c$

Examples, find the integral

1. $\int x^5 dx = \frac{x^6}{6} + c$
2. $\int \cos 2x\, dx = \frac{\sin 2x}{2} + c$
3. $\int sin^2 x\, dx = \int \frac{1-\cos 2x}{2} = \frac{1}{2}x - \frac{\sin 2x}{4} + c$
4. $\int x^n dx = \int \frac{1+\cos 2x}{2} = \frac{1}{2}x + \frac{\sin 2x}{4} + c$

- **Techniques of integration**

A. Integration by substitution

1. $\int x(2x^2+5)dx$ let, $2x^2+5 = u, 4x\, dx = du,\ \ x\, dx = \frac{du}{4}$

$= \int u\ \frac{du}{4} = \frac{1}{4}\int u\, du = \frac{1}{8}u^2 + c = \frac{1}{8}(2x^2+5)^2 + c$

2. $\int 2x^2\sqrt{1-x^3}dx$ let, $1-x^3 = u,\ \ -3x^2\, dx = du,\ \ 2x^2\, dx = \frac{-2du}{3}$

$= \int \sqrt{u}\ \frac{-2du}{3} = \frac{-2}{3}\int u^{\frac{1}{2}}\, du = \frac{-2}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{-4}{9}(u)^{\frac{3}{2}} + c = \frac{-4}{9}(1-x^3)^{\frac{3}{2}} + c$

3. $\int \frac{2x+2}{x^2+2x}dx$ let, $x^2+2x = u, (2x+2)\, dx = du,$

$$= \int \frac{1}{u} du = \ln|u| + c = \ln(x^2+2x) + c$$

B. Integration by partial fraction

1. $\int \frac{4x+1}{x^2-3x+2}dx = \int \frac{4x+1}{(x-3)(x-1)}dx$ 4x+1=Ax-A+Bx-3B

$= \int (\frac{A}{x-3} + \frac{B}{x-1})dx$ 4=A+B, 1=- A -3B

$= \int (\frac{3/2}{x-3} + \frac{5/2}{x-1})dx$ A= 3/2 B=5/2

$= \frac{3}{2}\int \frac{1}{x-3}dx + \frac{5}{2}\int \frac{1}{x-1}dx = \frac{3}{2}\ln|x-3| + \frac{5}{2}\ln|x-1| + c$

2. $\int \frac{x^2+4}{x^2-1}dx = \int \frac{x^2+4}{(x+1)(x-1)}dx$ x^2 + 4=Ax-A+Bx+B

$= \int (\frac{A}{x+1} + \frac{B}{x-1})dx$ 4= - A+B, 0=A + B

$= \int (\frac{-2}{x+1} + \frac{2}{x-1})dx$ A= - 2 B=2

$= -2\int \frac{1}{x+1}dx + 2\int \frac{1}{x-1}dx = -2\ln|x+1| + 2\ln|x-1| + c = 2\ln\left|\frac{x-1}{x+1}\right| + c$

C. Integration by part

Use LATE to let u,

L= logarithms, example log x, ln x

A= algebraic expression, example x, x^2, $5x^3$

T= trigonometry, example sin x, cos x, sec x

E= exponential, example e^x, 2^x, 5^x

Derivate u and integrate dv

1. $\int x\sin x\,dx$ let x=u and sin x dx=dv
   dx = du - cos x = v

$\int x\sin x\,dx = uv - \int v\,du$

$= -x\cos x - \int -\cos x\ dx = -x\cos x + \int cosx\,dx = -x\cos x + \sin x + c$

2. $\int x\,e^x dx$ let x=u and e^x dx=dv
   dx = du e^x= v

$\int x\,e^x dx = uv - \int v\,du$

$= xe^x - \int e^x\,dx = xe^x - e^x + c = e^x(x-1) + c$

3. $\int e^x\cos x\,dx$ let cos x=u and e^xdx=dv
   -Sin x dx = du e^x = v

$\int e^x\cos x\,dx = uv - \int v\,du$

$= e^x\cos x + \int e^x\sin x\ dx$

let sin x=u and $e^x dx = dv$

Cos x dx=du $e^x = v$

$\int e^x\cos x\,dx = e^x\cos x + \int e^x\sin x\ dx$

$\int e^x\cos x\,dx = e^x\cos x + uv - \int v\,du$

$\int e^x\cos x\,dx = e^x\cos x + e^x\sin x - \int e^x\cos x\ dx$

$\int e^x\cos x\,dx + \int e^x\cos x\ dx = e^x\cos x + e^x\sin x$

$2\int e^x\cos x\,dx = e^x\cos x + e^x\sin x$

$\int e^x\cos x\,dx = \frac{1}{2}e^x(\cos x + \sin x) + c$

- **Definite integral**

$$if\ F^l(x) = f(x), then \int_a^b f(x)dx = F(b) - F(a)$$

1. $\int_1^2(2x^2 + 3x\,)dx$

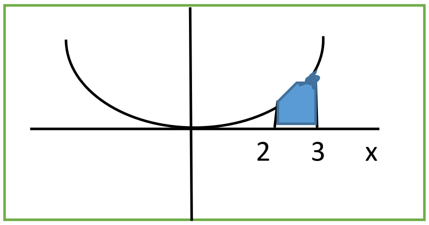
$= \left(\frac{2}{3}x^3 + \frac{3}{2}x^2\right)/_1^2 = \left(\frac{2}{3}(2)^3 + \frac{3}{2}(2)^2\right) - \left(\frac{2}{3})(1)^3 + \frac{3}{2}(1)^2\right) = \frac{16}{3} + \frac{12}{2} - \frac{2}{3} - \frac{3}{2} = \frac{55}{6}$

- **Area of regions**

Area, A=$\int_a^b f(x)\,dx$, $A = \int_a^b[f(x) - g(x)]dx$

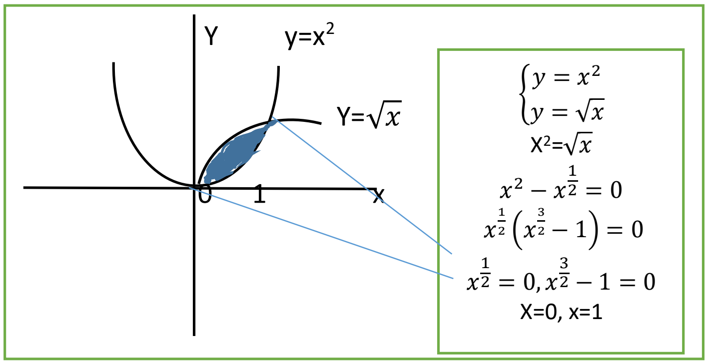
1. Find the area of the region enclosed by f(x)= x^2 and the x axis between 2 and 3.

$$A = \int_2^3 x^2\,dx = \frac{x^3}{3}/_2^3$$
$$= \frac{3^3}{3} - \frac{2^3}{3}$$
$$= \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$$



2. Find the area of the region enclosed by the curves f(x)= x^2 , g(x)=$\sqrt{x}$, and the x axis.

$$A = \int_0^1\left[x^2 - \sqrt{x}\right]dx$$
$$= \left[\frac{x^3}{3} - \frac{2}{3}x^{\frac{3}{2}}\right]/_0^1$$
$$= \frac{1}{3}$$



$\begin{cases} y = x^2 \\ y = \sqrt{x} \end{cases}$

X²=$\sqrt{x}$

$x^2 - x^{\frac{1}{2}} = 0$

$x^{\frac{1}{2}}\left(x^{\frac{3}{2}} - 1\right) = 0$

$x^{\frac{1}{2}} = 0, x^{\frac{3}{2}} - 1 = 0$

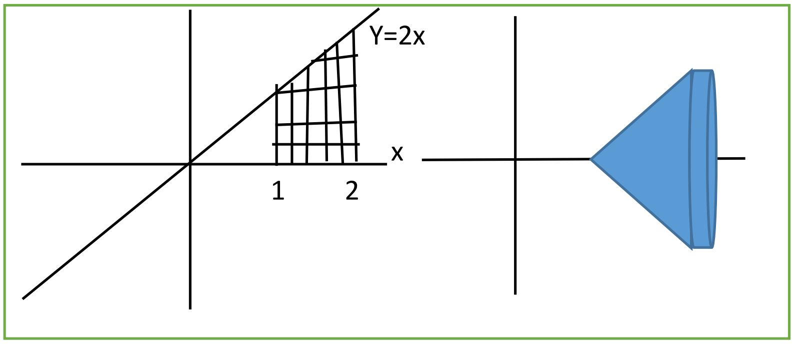
X=0, x=1

- Volume of regions

Volume, $V=\int_a^b \pi f(x)^2 dx$, $\quad V=\int_a^b[\pi f(x)^2-\pi g(x)^2]dx$

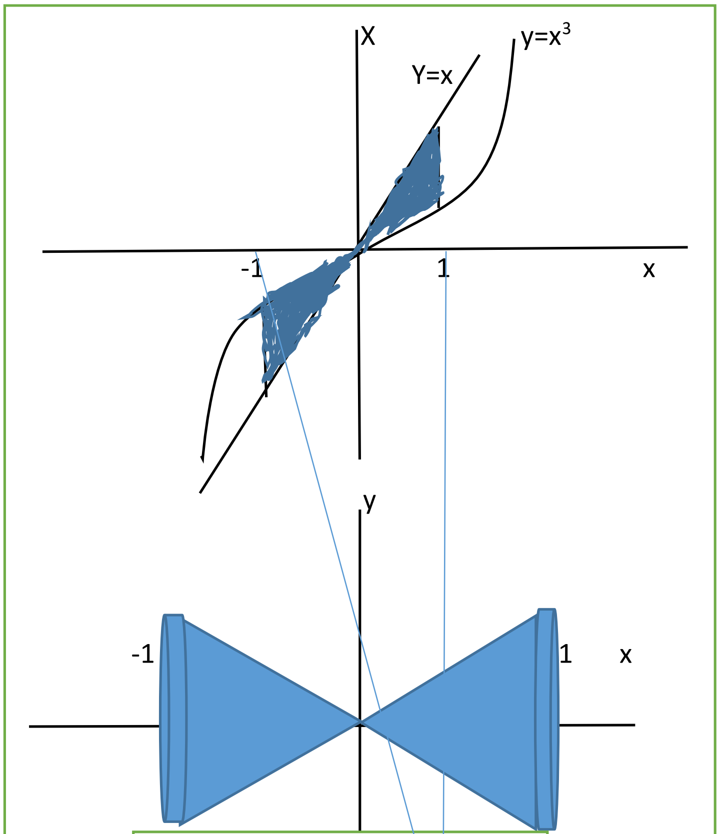
1. Find the volume of the solid of revolution about the x axis generated by revolving the region enclosed by f(x)= 2x and the vertical lines x=1 and x=2.

$$V=\int_a^b \pi f(x)^2 dx$$
$$V=\int_1^2 \pi (2x)^2 dx$$
$$V=\pi\int_1^2 4x^2 dx$$
$$V=4\pi\frac{x^3}{3}/_1^2$$
$$V=4\pi\frac{2^3}{3}-4\pi\frac{1^3}{3}$$
$$V=4\pi\frac{7}{3}=\frac{28}{3}\pi$$



2. Find the volume of the solid of revolution about the x axis generated by revolving the region enclosed by f(x)= x and g(x)=x^3

$$V=\int_a^b[\pi f(x)^2-\pi g(x)^2]dx$$
$$V=\int_{-1}^1[\pi (x)^2-\pi (x^3)^2]dx$$
$$V=2\pi\int_0^1[x^2-x^6]dx$$
$$V=2\pi\left[\frac{x^3}{3}-\frac{x^7}{7}\right]/_0^1$$
$$V=2\pi\frac{4}{7}=\frac{8\pi}{7}$$



$$\begin{cases} y=x^2 \\ y=x \end{cases}$$

X³=x

$x^3-x=0$

$x(x^2-1)=0$

X(x+1)(x - 1)=0

$x=0, x+1=0, x-1=0$

X=0, x= -1, x=1

Grade 12 – unit six – three dimensional geometry and vectors in space

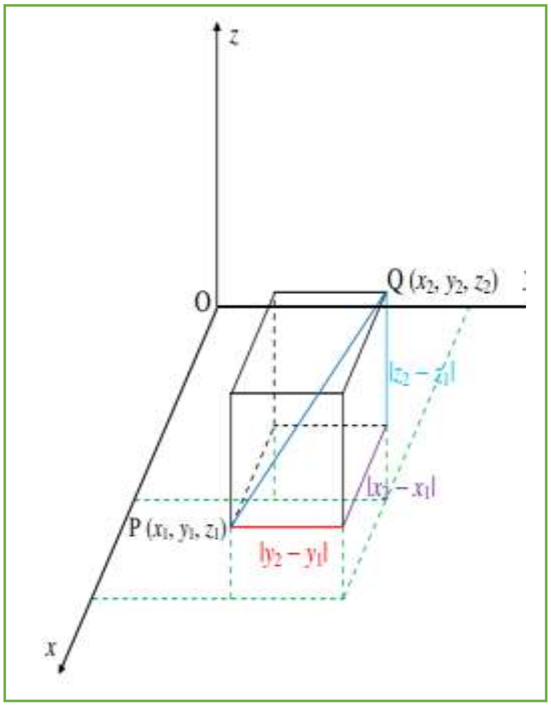
- Distance between two points in space

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Example, given two points P(2,-3,0) and Q(-5,3,$\sqrt{15}$)

a. find the distance between P and Q

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(-5-2)^2 + (3+3)^2 + (\sqrt{15}-0)^2}$$
$$= \sqrt{(-7)^2 + (6)^2 + (\sqrt{15})^2}$$
$$= \sqrt{49+36+15}$$
$$= \sqrt{100} = 10$$



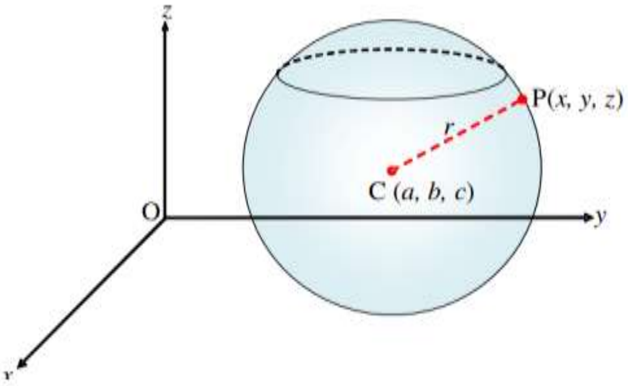
b. Find the midpoint

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right) = M\left(\frac{2-5}{2}, \frac{-3+3}{2}, \frac{0+\sqrt{15}}{2}\right) = M(\frac{-3}{2}, 0, \frac{\sqrt{15}}{2})$$

- **Equation of a sphere**

Center (a,b,c),

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$



Examples, find the center and radius of the sphere with equation

1. $x^2 + y^2 + z^2 - 4x + 6y - 8z - 7 = 0$

$x^2 - 4x + y^2 + 6y + z^2 - 8z - 7 = 0$

$x^2 - 4x + 4 - 4 + y^2 + 6y + 9 - 9 + z^2 - 8z + 16 - 16 - 7 = 0$

$(x-2)^2 - 4 + (y+3)^2 - 9 + (y-4)^2 - 16 - 7 = 0$

$(x-2)^2 + (y+3)^2 + (y-4)^2 = 7 + 4 + 9 + 16$

$(x-2)^2 + (y+3)^2 + (y-4)^2 = 36, \quad center\ C(2,-3,4), r = 6$

- **Vector in space**

Let, V (x , y , z) = xi + yj + zk and u (a , b , c) = ai + bj + ck

a. Magnitude or norm of v, $|v| = \sqrt{x^2 + y^2 + z^2}$
b. Addition, v+u= (x+a) i + (y+b) j + (z+c) k
c. Subtraction, v-u= (x-a) i + (y-b) j + (z-c) k
d. Scalar multiplication, 2v = 2xi + 2yj + 2zk
e. Dot product, i.i=j.j=k.k=1
   i.j=i.k=j.k=0
   u.v= xa + yb +zc
f. Angle between two vectors
   $u.v = |u||v|\cos\theta$
   Examples, given U(-1,1,1), V(2,2,2), then find
   a. -2u+3v= - 2(-1,1,1)+3(2,2,2)=(2,-2,-2)+(6,6,6)=(8,4,4)=8i+4j+4k
   b. $|-2u + 3v| = \sqrt{8^2 + 4^2 + 4^2} = \sqrt{96}$
   c. u.v= -1(2)+1(2)+1(2)= -2+2+2=4
   d. $cosine\ between\ u\ and\ v$
      u.v= -1(2)+1(2)+1(2)= -2+2+2=4
      $|u| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$
      $|v| = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12}$
      $u.v = |u||v|\cos\theta$
      $4 = \sqrt{3}\sqrt{12}\cos\theta$
      $4 = \sqrt{36}\cos\theta$
      $4 = 6\cos\theta$
      $\frac{2}{3} = \cos\theta$

Grade 12 – unit seven – mathematical proofs

- **Different types of proofs**

1. Direct proof.

   Implication $p \Longrightarrow q$

   Using direct proof, if P is T, then q must be T.

2. Method of cases or exhaustion.

   Every possible case is considered

   Example, if f(x)= 3x+8 and g(x)= 5-x, then proof that f(x)>g(x) for every x is an element of whole number.

   Case I: - let x=0, f(0) =3(0) +8=8
   g(0)=5-0=5, f(x)>g(x)

   Case II:- let x>0, example x=3 f(3) =3(3) +8=17
   g(3)=5-3=2, f(x)>g(x)

3. In direct proof.

   To proof $p \Longrightarrow q$, is true

   First proof the contrapositive $\neg q \Longrightarrow \neg p$ is true.

   Example, proof that 'if 3 is an even number, then 21 is divisible by 3'

   Let p=3 is an even number = F
   q= 21 is divisible by 3 = T

   First proof the contra positive $\neg q \Longrightarrow \neg p$ is true.

   $\neg q \Longrightarrow \neg p = \neg T \Longrightarrow \neg F = F \Longrightarrow T = T$

   Thus $p \Longrightarrow q$ is true.

4. Proof by contradiction.

   If P = T and q = T then $p \Longrightarrow q$=T, let q=F, or let $p \Longrightarrow q = F$

   Example, proof that '$\sqrt{2}$ is an irrational number'

   Proof, assume that $\sqrt{2}$ is a rational (not irrational) number

   And rational number is a number which can be expressed as Q= { $\frac{a}{b}, b \neq 0, a, b \in Z$ }

   And $\sqrt{2}$ cannot be expressed as rational expression.

   Thus, our assumption $\sqrt{2}$ is a rational is not true.

   Therefore $\sqrt{2}$ is an irrational number.

5. Dis proofing by counter example.

   Dis proof using different types of examples.

   Example, 'the sum of any two irrational numbers is an irrational number'

   x = $\sqrt{3} + 3$ *and* $y = 25 - \sqrt{3}$ are irrational numbers

   x + y = $\sqrt{3} + 3 + 25 - \sqrt{3} = 28$

   28 is not an irrational number, disproved using counter example.

- **Principle and application of mathematical induction.**

Principle of mathematical induction:- If for a given assertion involving a natural number n, you can show that

a. The assertion is true for n = 1.
b. If it is true for n = k, then it is also true for n = k + 1.
   Then the assertion is true for every natural number n.

Example, Use Mathematical Induction to prove that $n^3 - n$ is divisible by 3.

Proof: 1. the assertion is true when $n = 1$ because $1^3 - 1 = 0$ and 0 is divisible by 3.

2. For $n = k \geq 1$, assume that $k^3 - k$ is divisible by 3 is true for a natural number k and you must show that this is also true for $n = k + 1$.

That means you have to show that $(k + 1)^3 - (k + 1)$ is divisible by 3. Now, observe that

$(k + 1)^3 - (k + 1) = (k^3 + 3k^2 + 3k + 1) - (k + 1) = (k^3 - k) + (3k^2 + 3k) = (k^3 - k) + 3(k^2 + k)$

Since by the assumption $k^3 - k$ is divisible by 3 and $3(k^2 + k)$ is clearly divisible by 3, (as it is 3 times some integer),

You notice that the sum $(k^3 - k) + 3(k^2 + k)$ is divisible by 3.

Thus, it follows that $(k + 1)^3 - (k + 1)$ is divisible by 3.

Therefore, by the principle of mathematical induction, $k^3 - k$ is divisible by 3 for any natural number k.

Grade 12 – unit eight – further on statistics

- **Mean deviation (MD)**
- Un grouped data

Mean deviation from the mean $(MD_{\bar{x}})$

$$MD_{\bar{x}} = \frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{n} \quad or\ MD_{\bar{x}} = \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum f}$$

Mean deviation from the median (MD_{md})

$$MD_{md} = \frac{\sum_{i=1}^{n}|x_i - md|}{n} \quad or\ MD_{md} = \frac{\sum_{i=1}^{n} f_i|x_i - md|}{\sum f}$$

Mean deviation from the mode (MD_{m_o})

$$MD_{m_o} = \frac{\sum_{i=1}^{n}|x_i - m_o|}{n} \quad or\ MD_{m_o} = \frac{\sum_{i=1}^{n} f_i|x_i - m_o|}{\sum f}$$

- Grouped data

Mean deviation from the mean $(MD_{\bar{x}})$

$$MD_{\bar{x}} = \frac{\sum_{i=1}^{n} f_i|x_c - \bar{x}|}{\sum f}$$

Mean deviation from the median (MD_{md})

$$MD_{md} = \frac{\sum_{i=1}^{n} f_i|x_c - md|}{\sum f}$$

Mean deviation from the mode (MD_{m_o})

$$MD_{m_o} = \frac{\sum_{i=1}^{n} f_i|x_c - m_o|}{\sum f}$$

Examples

Find Mean deviation from the mean$(MD_{\bar{x}})$, Mean deviation from the median(MD_{md}), and Mean deviation from the mode (MD_{m_o})

1. Given ungrouped data

X	3	5	6	7
f	2	5	2	1

Solution

x	f	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$	$\lvert x_i - md\rvert$	$f_i\lvert x_i - md\rvert$	$\lvert x_i - m_0\rvert$	$f_i\lvert x_i - m_0\rvert$
3	2	6	2	4	2	4	2	4
5	5	25	0	0	0	0	0	0
6	2	12	1	2	1	2	1	2
7	1	7	2	2	2	2	2	2
	$\sum f$=10	$\sum f_i x_i = 50$		$\sum f_i\lvert x_i - \bar{x}\rvert$=8		$\sum f_i\lvert x_i - md\rvert$=8		$\sum f_i\lvert x_i - m_0\rvert$=8

$\bar{x} = \frac{\sum f_i x_i}{\sum f} = \frac{50}{10} = 5$, $MD_{\bar{x}} = \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum f} = \frac{8}{10} = 0.8$

$md = \frac{5^{th}\ item + 6^{th}\ item}{2} = \frac{5+5}{2} = 5$, $MD_{md} = \frac{\sum_{i=1}^{n} f_i|x_i - md|}{\sum f} = \frac{8}{10} = 0.8$

$m_0 = 5, most\ frequent$, $MD_{m_0} = \frac{\sum_{i=1}^{n} f_i|x_i - m_0|}{\sum f} = \frac{8}{10} = 0.8$

2. Given a groped data

X	3 - 7	8 - 12	13 - 17	18 - 22
f	2	2	10	6

Solution

x	f	x_c	f_ix_c	$\lvert x_c-\bar{x}\rvert$	$f_i\lvert x_c-\bar{x}\rvert$	$\lvert x_c-md\rvert$	$f_i\lvert x_c-md\rvert$	$\lvert x_c-m_0\rvert$	$f_i\lvert x_c-m_0\rvert$	cf	Class boundary
3-7	2	5	10	10	20	10.5	21	10.83	21.66	2	2.5 – 7.5
8-12	2	10	20	5	10	5.5	11	5.83	11.66	4	7.5 – 12.5
13-17	10	15	150	0	0	0.5	5	0.83	8.3	14	12.5 – 17.5
18-22	6	20	120	5	30	4.5	27	4.17	25.02	6	17.5 – 22.5
$\sum f$ =20		$\sum f_ix_c$ =300		$\sum f_i\lvert x_c-\bar{x}\rvert$=60		$\sum f_i\lvert x_c-md\rvert$=64		$\sum f_i\lvert x_c-m_0\rvert$=66.64			

$$\bar{x}=\frac{\sum f_ix_i}{\sum f}=\frac{300}{20}=15$$

$$MD_{\bar{x}}=\frac{\sum_{i=1}^{n}f_i\lvert x_i-\bar{x}\rvert}{\sum f}=\frac{60}{20}=3$$

$$meadian\ class=third\ class$$

$$md=B_L+\left(\frac{\frac{n}{2}-cf_b}{f}\right)i=12.5+\left(\frac{\frac{20}{2}-4}{10}\right)5=12.5+\frac{30}{10}=12.5+3=15.5$$

$$MD_{md}=\frac{\sum_{i=1}^{n}f_i\lvert x_c-md\rvert}{\sum f}=\frac{64}{20}=3.2$$

Modal class= 3rd class

$$m_0=B_L+\left(\frac{d_1}{d_1+d_2}\right)i=12.5+\left(\frac{8}{8+4}\right)5=12.5+\left(\frac{40}{12}\right)=12.5+3.33=15.83$$

$$MD_{m_0}=\frac{\sum_{i=1}^{n}f_i\lvert x_i-m_0\rvert}{\sum f}=\frac{66.64}{20}=3.332$$

- **Coefficient of variability (CV)**

$$CV=\frac{standard\ deviasion}{mean}*100\%=\frac{s.d}{\bar{x}}*100\%$$

For any two data's A and B

$if\ CV_A>CV_B, then\ A\ is\ more\ variable\ and\ B\ is\ more\ consistent$

Examples

1. The mean and standard deviation of gross incomes of two companies are given below

Company	mean	Standard deviation
A	6,000	120
B	10,000	220

Calculate CV of each company and find which company is more consistent.

Solution

$$CV_A=\frac{s.d}{\bar{x}}*100\%=\frac{120}{6000}*100\%=0.2,$$

$$CV_B=\frac{s.d}{\bar{x}}*100\%=\frac{220}{10000}*100\%=2.2$$

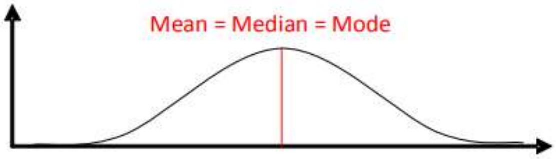
$CV_B>CV_A,$

$B\ is\ more\ variable\ and$

$A\ is\ more\ consistent.$

- **Skewness based on mean, median, and mode**

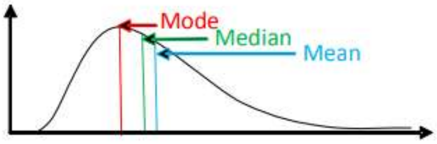
  i For a unimodal distribution in which the values of mean, median and mode coincide (i.e., Mean = Median = Mode), the distribution is said to be symmetrical.



Symmetrical distribution

a

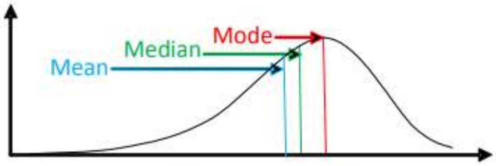
  ii If the mean is the largest in value, and the median is larger than the mode but smaller than the mean, then the distribution is positively skewed. That is, if Mean > Median > Mode, then the distribution is positively skewed (skewed to the right).



Positively skewed distribution

b

  iii If the mean is smallest in value, and the median is larger than the mean but smaller than the mode, then the distribution is negatively skewed. That is, if Mean < Median < Mode then the distribution is negatively skewed (skewed to the left).



Negatively skewed distribution

c

-

Grade 12 – unit nine – mathematical applications for business and consumers

- **Real estate expenses**

  A mortgage is an amount that is borrowed, often from a bank, to buy real estate. A person will not be able to get a loan for the full purchase price of the house, because the lender expects him/her to pay a percentage of the purchase price immediately. The money that is paid at the time of purchase is called the down payment (usually 20%) and the remaining money still to be paid is known as the mortgage or balance due. Thus,

  Mortgage = purchase cost – down payment
- Amortization is a process in which a debt is "retired" in a given length of time of equal payments. The payments include compound interest. At retirement, the borrower has paid the entire amount of the principal and the interest.
- Note that the monthly mortgage payment includes the payment of both the principal and the interest on the mortgage. The interest charged during any one month is charged against the unpaid balance of the loan.

  The amortization formula is given by:

  $P.P = P(\frac{i}{1-(1+i)^{-n}})$,

  Where p.p ≡ periodic payment

  P ≡ principal

  i ≡ interest rate per payment interval

  n ≡ number of payments made

  Example

1. Ato Toga purchased a condominium for Birr 140,000 and made a down payment of 15%. The savings-and-loan association from which he purchased his mortgage charges an annual interest rate of 9.5% on Toga's 20-year mortgage. Find the monthly mortgage payment. (Given that$(1.00792)^{-240} = 0.15057$)

<u>Solution</u>

$$i = 9.5\% = \frac{0.095}{12} = 0.00792$$

$$n = 20 * 12 = 240$$

$$motgage, p = purchase\ cost - down\ paiment$$

$$= 140000 - 0.15 * 140000$$

$$= \ 140000 - 21000$$

$$= 119000$$

$$P.P = P\left(\frac{i}{1-(1+i)^{-n}}\right) = 119000\left(\frac{0.00792}{1-(1+0.00792)^{-240}}\right)$$

$$= 119000\left(\frac{0.00792}{1-(1.00792)^{-240}}\right)$$

$$= 119000\left(\frac{0.00792}{1-0.15057}\right)$$

$$= 119000\left(\frac{0.00792}{0.84943}\right)$$

$$= 119000 * 0.0093239$$

$$= 1{,}109.54$$

QUESTIONS and DETAIL SOLUTIONS OF EUEE (2009E.C-2013E.C)

Instruction: - each question consists of four alternatives. Choose the correct answer from the given alternatives.

Grade 11 unit one – further on relations and functions

2009 E.C

1. Which of the following functions is a one to one correspondence?

A. F:[0, ∞)→ ℝ defined by f(x) =|x|

B. B. F: ℝ:→ [0, ∞) defined by f(x)= x^2

C. F:: ℝ:→ [0, ∞) defined by f(x)= 3^x

D. F:: ℝ:→ [0, ∞) defined by f(x)= $\log_2{}^x$

Solution

$A)\ f : (0,\infty] \to R\ , f(x) = |x|$

$\Rightarrow x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

f *is one funtion*

$\Rightarrow$ *Range of* $f(x) = |x| = \{x : x \geq 0\}$

f *is not one to onefunction*

$\Rightarrow f$ *is not one to one correspondence*

$B)\ f : R \to [0,\infty)\ , f(x) = x^2$

take -2 *and* 2

$-2 \neq 2\ , f(-2) = 4 = f(2)$

f *is not one to one function*

$\Rightarrow f$ *is not one to one correspondence.*

$C)\ f : R \to [0,\infty)\ , f(x) = 3^x$

$f : R \to [0,\infty)\ , f(x) = 3^x$

$\Rightarrow x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

f *is one to one function*

rangeof $f = (0,\infty)$ f *is not on to function*

$\Rightarrow f$ *is not one to one corespendence*

Answer D

$D)\ f : (0,\infty) \to R\ , f(x) = \log_{2}^{x}$

$f : (0,\infty) \to R\ , f(x) = \log_{2}^{x}$

$\Rightarrow x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

f *is one to one function*

range of $f = [0,\infty), f$ *is on to function*

$\Rightarrow f$ *is one to one correspondance*

A function is said to be one to one correspondence, if it is both one to one and on to.

2. If f(x)= $\sqrt{x^3}$ and (fog)(x)= $\sqrt[4]{x}$, then what is the value of g(8)?

A. $\sqrt[3]{2}$ B. 2 C. $\sqrt{2}$ D. $2\sqrt{2}$

Solution

$$f(x) = \sqrt{x^3}\ ,\ (fog)(x) = \sqrt[4]{x}\ , g(x) = ?$$

$$(fog)(x) = f\big(g(x)\big) = \sqrt[4]{x}$$

$$\sqrt{(g(x))^3} = \sqrt[4]{x}$$

$$(g(x))^3 = \left(\sqrt[4]{x}\right)^2$$

$$g(x) = \left(\sqrt[4]{x}\right)^{\frac{2}{3}}$$

$$g(x) = x^{\frac{2}{3}*\frac{1}{4}} = x^{\frac{2}{12}} = x^{\frac{1}{6}}.$$

$$g(8) = 8^{\frac{1}{6}} = 8^{\frac{1}{3}*\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}$$

Answer C

3. Which one of the following is the inverse of f(x) = $8x^3+2$?

A. $f^{-1}(x)= 1/8x^3+2$ B. $f^l(x) = \frac{\sqrt[3]{x-2}}{2}$ C. $f^{-1}(x)=8x^{-3}-2$ D. $f^{-1}(x)=1/8\ \sqrt[3]{x} - 2$

Solution

$$f(x) = 8x^3 + 2$$

$$y = 8x^3 + 2$$

$$x = 8y^3 + 2$$

$$8y^3 = x - 2$$

$$y^3 = \frac{x-2}{8}, y = \sqrt[3]{\frac{x-2}{8}}, y = \frac{\sqrt[3]{x-2}}{2},\ f^{-1}(x) = \frac{\sqrt[3]{x-2}}{2}\ , Answer\ B$$

4. What is the solution set of $\frac{1}{1+\frac{1}{x}}-\frac{1}{1-\frac{1}{x}}=\frac{x+\frac{1}{x}}{x-\frac{1}{x}}$

A. {} B. $\{-1\}$ C. $\{1\}$ D. {-1, 1}

Solution

$$\frac{1}{1+\frac{1}{x}}-\frac{1}{1-\frac{1}{x}}=\frac{x+\frac{1}{x}}{x-\frac{1}{x}} \qquad U=R\,/\,\{-1\,,0\,,1\}$$

$$\frac{1}{\frac{x+1}{x}}-\frac{1}{\frac{x-1}{x}}=\frac{\frac{x^2+1}{x}}{\frac{x^2-\ 1}{x}}$$

$$\frac{x}{x\ +1}-\frac{x}{x-1}=\frac{x^2+1}{x^2-\ 1}$$

$$\frac{x^2-x-\ x^2-x}{x^2-1}=\frac{x^2+1}{x^2-1}$$

$$-2x=\ x^2+1$$

$$x^2+2x+1=0$$

$$(x+1)(x+1)=0$$

$$x+1=0$$

$$x=\ -1$$

$$-1\ is\ not\ in\ the\ \ universe\ s, s=\{\ \}$$

$$Answer\ A$$

Grade 11 unit one – further on relations and functions

2010 E.C

1. Which one of the following is true about signum, absolute value and greatest integer function?

A. $|x| = x\text{sgn}(x)$, for all $x \in \mathbb{R}$ C. $\text{sgn}(x) \le \lfloor x \rfloor$,, for all x≤ 0

B. $\text{sgn}(x) = \pm|x|$, for all all $x \in \mathbb{R}$ D. $\text{sgn}(x) \le \lfloor x \rfloor$,, for all x≥ 0

Solution

$$A. |x| = \text{x sgnx} = \text{x}\begin{cases} 1 \text{ if } x > 0 \\ \qquad 0 \text{ if } x = 0 \\ -1 \text{ if } x < 0 \\ \qquad true \end{cases}$$

$$\text{B. sgn(x)} = \pm|\text{x}| \text{ False eg. } \text{sgn}(2) = 1 \ne \pm 2$$

$$\text{C. sgn(x)} \le \lfloor x \rfloor, \ \text{, x} \le 0 \text{ False eg. sgn } (-1.5) = -1 \ne \lfloor -1.5 \rfloor = -2,$$

$$\text{D. sgn(x)} \le \lfloor x \rfloor, \text{x} \ge 0 \text{ False eg. sgn}(0.5) \ne \lfloor 0.5 \rfloor \quad \text{Answer A}$$

2. Let $f(x)=x- x^2$ and $g(x)=1/x$. then what is (f(1/x)) is equal

A. $x\text{-}x^2$ B. $\frac{x-1}{x^2}$ C. $\frac{1}{x^2-1}$ D. $\frac{x^2}{x-1}$

Solution

$$\text{f(x)} = \text{x} - \text{x}^2,\ \ \text{g(x)} = \frac{1}{x},\ \text{find} \qquad \text{g}\left(\text{f}\left(\frac{1}{x}\right)\right)$$

$$\text{f(x)} = \text{x} - \text{x}^2, \text{f}\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2},$$

$$\text{g}(\text{f}\left(\frac{1}{x}\right) = \text{g}\left(\frac{x-1}{x^2}\right) = \frac{\frac{1}{x-1}}{x^2} = \frac{x^2}{x-1}$$

Answer D

3. If $f(x) = \frac{3x+1}{x-2}$, Then what is the range of f(x)?

A. $\mathbb{R} \setminus \{2\}$ B. $\mathbb{R} \setminus \{3\}$ C. $\mathbb{R} \setminus \{1/3\}$ D. $\mathbb{R}$

Solution

$$x = \frac{3y+1}{y-2}$$
$$xy - 2x = 3y + 1$$
$$xy - 3y = 2x + 1$$
$$y(x-3) = 2x + 1$$
$$y = \frac{2x+1}{x-3}$$
$$f^{-1}(x) = \frac{2x+1}{x-3}$$

$\text{Domf}(x) = \text{range of } f^{-1}(x),$

$\text{Range of } f(x) = \text{Dom 0f } f^{-1}(x) \quad , \text{range } = \Re/\{3\}$

Answer B

Grade 11 unit one – further on relations and functions

2011 E.C

1. If f is the greatest integer function and g is the absolute value function, then what is the value of $(fog)\left(\frac{3}{2}\right) + (gof)\left(\frac{-4}{3}\right)$

A. 1 B. – 1 C. 3 D. 2

Solution

$$f(X) = \lfloor x \rfloor \quad , \quad g(X) = |x|$$
$$= (fog)\left(\frac{3}{2}\right) + (gof)\left(\frac{-4}{3}\right)$$
$$f\left(g\left(\frac{3}{2}\right)\right) + g\left(f\left(\frac{-4}{3}\right)\right)$$
$$where \quad g\left(\frac{3}{2}\right) = |\tfrac{3}{2}| = \tfrac{3}{2}{=}1.5$$
$$and\ f\left(\frac{-4}{3}\right) = \left\lfloor \frac{-4}{3} \right\rfloor = \lfloor -1.33 \rfloor = -2$$
$$= f\left(\tfrac{3}{2}\right) + g(-2) = \lfloor 1.5 \rfloor + |-2| = 1 + 2 = 3,$$

Answer C

2. Which one of the following is equal to $f(x) = \sqrt{(x+4)^2}$ for every x∈ ℝ?

A. g(x)=x+4 B. g(x)=x+2 C. g(x)=|x|+4 D. g(x)=|x+4|

Solution

$$(x) = \sqrt{(x+4)^2} = |x+4|$$

Answer D

3. What is the simplified form of $\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}}$?

A. $\frac{a^2b^2}{b^3-a^3}$ B. $\frac{a2b2}{b2-a2}$ C. $\frac{a3-b3}{ab}$ D. $\frac{a3-b3}{a-b}$

solution

$$\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}} = \frac{\frac{1}{ab}}{\frac{1}{a^3}-\frac{1}{b^3}}$$
$$= \frac{\frac{1}{ab}}{\frac{b^3-a^3}{(ab)^3}} = \frac{1}{ab} * \frac{(ab)^3}{b^3-a^3}$$
$$= \frac{a^2b^2}{b^3-a^3}$$

Answer A

4. If f(x)= ax-b and f^{-1}(x+1)= $\frac{1}{2}x + 2$, for each x∈ ℜ, then what must be the values of a and b?

   A. a=2, b=3 B. a=2, b=2 C. a=1, b=1 D. a=1/2, b=-2

Solution

$$f(x) = ax - b, \qquad f^{-1}(x - 1) = \frac{1}{2}x + 2$$

$$y = ax - b$$

$$x = ay - b$$

$$ay = x + b$$

$$y = \frac{x+b}{a}, f^{-1}(x) = \frac{x+b}{a}$$

$$f^{-1}(x+1) = \frac{x+1+b}{a} = \frac{x}{a} + \frac{1+b}{a}$$

$$\Rightarrow \frac{1}{2}x + 2 = \frac{x}{a} + \frac{1+b}{a}$$

$$\frac{1}{2} = \frac{1}{a}, \qquad 2 = \frac{1+b}{2}$$

$$2 = a \qquad 2 = \frac{1+b}{2}$$

4=1+b

4-1=b

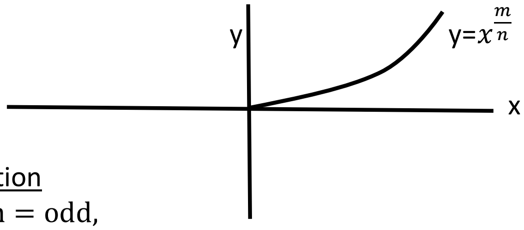
3=b

Answer A

Grade 11 unit one – further on relations and functions

2012 E.C

1. The following graph is the graph of the function y= $x^{\frac{m}{n}}$, where m and n are positive integers and n≠ 0. Which one of the following is true about m and n?
   - A. M is odd, n is even and m > n
   - B. M is even, n is odd and m < n
   - C. M is odd, n is even and m < n
   - D. M is even, n is odd and m > n



Solution

$\Rightarrow$ m = odd,

n = even,

m > n

Answer A

2. Let $f(x) = \sqrt{x+2}$, and $g(x) = x^2$-1. What are the domain and range of the composition of f with g, (f o g) respectively?
   - A. ℝ and $[1, \infty)$
   - B. ℝ and $[0, \infty)$
   - C. $[0, \infty)$and $[1, \infty)$
   - D. $[0, \infty)$and $[0, \infty)$

Solution

$$f(x) = \sqrt{x+2} ,$$

$$g(x) = x^2 - 1$$

$$(fog)(x) = f(g((x)) = \sqrt{x^2 - 1 + 2} = \sqrt{x^2 + 1}$$

$$\text{Dom} = \Re, \ \text{range} = \{y: y \geq 1\}=[1, \infty)$$

Answer A

3. Let f(x)=3-2x. What is the inverse of f?
   A. $f^{-1}(x)=2x-3$ B. $f^{-1}(x)=3+2x$ C. $f^{-1}=\frac{3}{2}-\frac{x}{2}$ D. $f^{-1}=\frac{x}{2}-\frac{3}{2}$

Solution

$$f(x) = 3 - 2x$$
$$y = 3 - 2x$$
$$x = 3 - 2y$$
$$2y = -x + 3$$
$$y = \frac{-x+3}{2}$$
$$y = \frac{-x}{2} + \frac{3}{2} = f^{-1}(x)$$

Answer C

To find inverse interchange x and y , then find y interms of x

Grade 11 unit one – further on relations and functions

2013 E.C

1. What are the domain and range of the function $f(x) = 2x^{3/4}$ respectively?
   A. [0, ∞) and (0, ∞) B. R and [0, ∞) C. [0, ∞) and {0, ∞) D. [0, ∞) and R

Solution

$$f(x) = 2x^{3/4} \quad = 2\sqrt[4]{x^3}, x \geq 0, \qquad \text{domain} = [0, \infty)$$

$$y = 2x^{3/4}$$
$$x = 2y^{3/4}$$
$$x^{4/3} = y\left(2^{4/3}\right)$$

Note i) Domain of f(x)=range of $f^{-1}(x)$

ii) Range of f(x)=Domain of $f^{-1}(x)$

$$y = f^{-1}(x) = \left(\frac{x}{2}\right)^{4/3},$$

Domaino of $f^{-1}(x) =$ R= range of f(x)

Answer D

2. Which one of the following defines a one-to-one function?
   A. f = {(x, y): y = 3x-1)}
   B. f= {(x, y}: x is a sister of y}
   C. f = {(x, y): y is a father of X}
   D. f = {(x, y): Y = X^2 +1)}

Horizontal line test
If a horizontal line touches the graph atmost once, then the function is one to one.
Y=3x-1 is one to one

Solution

A) $y = 3x - 1, \quad x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2) -$ one to one function

B) $f = \{(s, B_1), (s, B_2)\}$ eg: −s has two brothers B_1, B_2, Not one to one function

C) $f = \{(s_1, F), (s_2, F)\}$eg − Father F has two s and s_1, s_2, Not one to one function

D) $y = x^2 + 1 \Rightarrow -2 \neq 2$ but $f(2) = f(-2)$, not one to one function

Answer A

3. The inverse of the function y = 3x – 5 is equal to
   A. $y = \frac{x+3}{5}$ B. $y = \frac{x+5}{3}$ C. $y = -5x + 3$ D. $y = \frac{x-5}{3}$

Solution

$$y = 3x - 5$$
$$x = 3y - 5$$
$$3y = x + 5$$
$$y = \frac{x+5}{3}$$

Answer B

Grade 11 unit two – rational expressions and rational functions

2009 E.C

1. Which one of the following is true about the graph of f(x)=$\frac{x^3-x}{x^4-x^3}$

   A. The vertical asymptote of the graph are x=0 and x=1

   B. A horizontal asymptote of the graph is y=1

   C. The graph intersects its horizontal asymptote at a point(-1, 0)

   D. The graph intersects the vertical line x=1 at a point (1, 2)

Solution

$$\frac{x^3-x}{x^4-x^3}=\frac{x(x^2-1)}{x^3(x-1)}$$

$$A)\ vertical\ Asymptote\ \Rightarrow\ no\ vertical\ Asymptote\ but, hole\ at\ x=0\ , x=1$$

$$B)\ \lim_{x\to\infty}\left(\frac{x^3-x}{x^4-x^3}\right)=\lim_{x\to\infty}\left(\frac{\frac{x^3}{x^4}-\frac{x}{x^4}}{\frac{x^4}{x^4}-\frac{x^3}{x^4}}\right)=\lim_{x\to\infty}\left(\frac{\frac{1}{x}-\frac{1}{x^3}}{1-\frac{1}{x}}\right)$$

$$=\left(\frac{\frac{1}{\infty}-\frac{1}{\infty}}{1-\frac{1}{\infty}}\right)=\left(\frac{0-0}{1-0}\right)=\frac{0}{1}=0$$

$$y=0\ is\ horozontal\ Asymptote$$

$$C)\ f(x)=Horozontal\ Asymptote$$

$$\frac{x^3-x}{x^4-x^3}=0$$

$$x^3-x=0$$

$$x(x^2-1)=0$$

$$x=0\ \ ,\qquad x=\pm1\quad ,Hole\ at\ x=0\ ,1$$

$$\Rightarrow(-1\ ,f(-1))=\ (-1\ ,0)$$

$$The\ graph\ crosses\ the\ Horozontal\ Asymptote\ at\ (-1\ ,0\ )$$

$$D)\ The\ graph\ never\ crosses\ its\ vertical\ asymptote$$

$$Answer\ C$$

2. What is the partial fraction decomposition of $\frac{X^2+X+1}{(X+2)(X^2+1)}$?

   A. $\frac{2}{5(X+2)}+\frac{3x+1}{5(X^2+1)}$ B. $\frac{5}{3(X+2)}+\frac{2x+1}{3(X^2+1)}$ C. $\frac{3}{5(X+2)}+\frac{2x+1}{5(X^2+1)}$ D. $\frac{2}{3(X+2)}+\frac{2x+1}{3(X^2+1)}$

Solution

$$\frac{X^2+X+1}{(X+2)(X^2+1)}=\frac{A}{X+2}+\frac{Bx+C}{x^2+1}$$

$$\frac{(x^2+x+1)}{(x+2)(x^2+1)}=\frac{Ax^2+A+(Bx+c)(x+2)}{(x+2)(x^2+1)}$$

$$x^2+x+1=Ax^2+A+Bx^2+2Bx+CX+2C$$

$$1=A+B\ldots..(i)\qquad 1=2B+C\ldots\ldots(ii)\qquad 1=A+2C\ldots.(iii)$$

$$\Downarrow\qquad\qquad \text{From iii}\quad A=1-2C$$

$$1=A+B, 1=1-2C+B, \text{B-2C=0}\ldots\ldots(iv),$$

$$\text{combine ii and iv}-2\begin{cases}B-2C=0\\ \underline{2B+C=1}\end{cases}\ ,\quad \begin{array}{c}A=1-2C\\ =1-\frac{2}{5}=3/5\end{array}\ ,\quad \begin{array}{c}A+B=1\\ \frac{3}{5}+B=1, B=1-\frac{3}{5}=\frac{2}{5}\end{array}$$

$$-2B+4C=0$$

$$\underline{2B+\ C=1}$$

$$5C=1,\ C=1/5$$

$$\frac{A}{X+2}+\frac{BX+C}{X^2+1}=\frac{3}{5(X+2)}+\frac{\frac{2}{5}X+\frac{1}{5}}{X^2+1}=\frac{3}{5(X+2)}+\frac{2x+1}{5(X^2+1)}$$

Answer C

Grade 11 unit two – rational expressions and rational functions

2010 E.C

1. Which one of the following is true about the graph of $f(x)=\frac{2x^3+2x^2+3x}{x^2+x}$?
   - A. The graph has y-intercept at (0, 3)
   - B. The graph has at least one x-intercept
   - C. The vertical asymptote of the graph is only x=-1 and its oblique asymptote is y=2x
   - D. The vertical asymptote of the graph are at x=0 and x=-1 but it has no horizontal asymptote.

Solution

$$f(x)=\frac{2x^3+2x^2+3x}{x^2+x}=\frac{x(2x^2+2x+3)}{x(x+1)}$$

$\Rightarrow$No x − intercept

$\Rightarrow$hole at x = 0

$\Rightarrow$No y − intercept

$\Rightarrow$vertical asymptote, x = −1

$$\begin{array}{r|l} & 2x \\ \hline \Rightarrow x^2+x & 2x^3+2x^2+3x \\ & 2x^3+2x^2 \\ & \overline{3x} \end{array}$$, use long division

obligue asymptote $y=2x,$

$2x^2+2x+3=0$

is a quadratic equation with no root

b^2-4ac<0

2^2-4(2)(3)=-20<0

Answer C

Grade 11 unit two – rational expressions and rational functions

2011 E.C

1. Which one of the following is true about the graph of f(x)= $\frac{X^2+5x+6}{X^2-4}+3$
   - A. The graph has a hole at x=2
   - B. The vertical asymptote of the graph are x=-2 and x=2
   - C. The graph y-intercept at (0, -3/2)
   - D. The horizontal asymptote of the graph is y=4

Solution

$$\frac{X^2+5x+6}{X^2-4}+3$$

$$=\frac{X^2+5X+6+3(X^2-4)}{X^2-4}$$

$$=\frac{X^2+5X+6+3X^2-12}{X^2-4}$$

$$=\frac{4X^2+5X-6}{X^2-4}$$

$$=\frac{(4x-3)(X+2)}{(X+2)(X-2)}$$

$$Vertical\ \ Asymptote\ \ X=2$$

$$hole\ at\ X=-2$$

$$Horozontal\ Asymptote, take\ the\ coeficients\ of\ X^2, y=4$$

$$y-\ intercept\ , f(0)=\frac{-6}{4}=\frac{-3}{2}=(0\,,-\frac{3}{2})$$

Answer C

Grade 11 unit two – rational expressions and rational functions

2012 E.C

1. Let f(x)= $\frac{x^2-x}{2x^2-x-1}$ which of the following is NOT true about f and its graph?
   - A. The line x =-1/2 is a vertical asymptote.
   - B. Graph of f has hole at a point (1, 1/3)
   - C. F is an even function.
   - D. The line y= ½ is a horizontal asymptote.

Solution

$$f(x) = \frac{x^2-x}{2x^2-x-1} = \frac{x(x-1)}{(2x+1)(x-1)} \qquad 2x^2 - x - 1 \quad .$$

$$\text{A. V. asymptote : } 2x + 1 = 0 \qquad 2x^2 - 2x + x - 1$$

$$2x = -1 \qquad 2x(x-1) + (x+1)$$

$$x = \frac{-1}{2} \qquad (2x+1)(x-1)$$

B. hole at x=1

C. $f(x) \neq f(-x)$, *not even function*

D. horizontal asymptote, $y = \frac{1}{2}$

Answer C

2. Given a rational f(x)=$\frac{2x+4}{x-1}$, which of the following is NOT true about its graph
   - A. A horizontal asymptote, y=2
   - B. A vertical asymptote, x=1
   - C. y-intercept (0, -4)
   - D. x-intercept (-4, 0)

Solution

$$f(x) = \frac{2x+4}{x-1}$$

− vertical asymptote $x = 1$

−Horizontal asymptote, y=2

−Y intercept, (0,-4)

−X intercept, (-2,0)

Answer D

3. Which one of the following is zero for the function $f(x) = x^3 - 3x^2 + 3x - 1$?
   - A. 0
   - B. 1
   - C. – 2
   - D. – 1

Solution

> $f(x) = x^3 - 3x^2 + 3x - 1$ is a polynomial function. Zero of a polynomial function is a number which makes the function xero.
>
> If f(c)=0, then c is zero of the function f

$$f(x) = x^3 - 3x^2 + 3x - 1$$

A. $f(0) = 0^3 - 3(0)^2 + 3(0) - 1 = -1$

B. $f(1) = 1^3 - 3(1)^2 + 3(1) - 1 = 1 - 3 + 3 - 1 = 0$

C. $f(-2) = (-2)^3 - 3(-2)^2 + 3(-2) - 1 = -8 - 12 - 6 - 1 = -27$

D. $f(-1) = (-1)^3 - 3(-1)^2 + 3(-1) - 1 = -1 - 3 - 3 - 1 = -8$

Answer B

Grade 11 unit two – rational expressions and rational functions

2013 E.C

1. Which of the following is the universal set of the expression $\frac{1}{x^2}+\frac{x+1}{x^2+x}$?

A. R\ {0} B. R/ {0,1} C. R/{-1, 0} D. R/ {0,1, -1}

Solution

$$f(x)=\frac{1}{x^2}+\frac{x+1}{x^2+x}$$

$$=\frac{x^2+x+x^3+x^2}{x^2(x^2+x)}$$

The denominator is zero at -1 and 0

F is not real number at -1 and 0

The universe does not contain the numbers -1 and 0.

U= $R/\{-1,0\}$

Answer C

2. The graph of a rational function is given as in the figure 1 below.

Which of the following in NOT true about the graph?

A. Y = 0 is its horizontal asymptote.
B. X = 1 is its vertical asymptote.
C. It's increasing on (-∞, 1)
D. It is decreasing on (1, ∞)

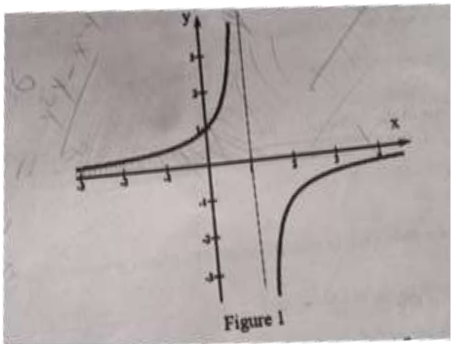


Figure 1

Solution

A) y=0 is horizontal asymptote = True
B) X=1 is vertical asymptote= True .
C) Increasing on $(-\infty,1)$ and $(1,\infty)$ = True
D) From choice C =False

Answer D

Grade 11 unit three – coordinate geometry

2009 E.C

1. What is the equation of a line that passes through point (a, a) in xy- plane if it is parallel to a line that passes through points (a, b) and (b, a) where a≠b?

A. Y=x B. Y=-x C. Y=-x+2a D. Y=2x-a

Solution

$$Passing\ through\ (a\,,a)\,,parallel\ to\ (a\,,b\,)-(\,b\,,a\,)$$

$$(x_1,y_1)-(x_2,y_2)$$

$$m_1=\frac{y_2-y_1}{x_2\ -x_1}=\frac{a-b}{b-a}=\frac{-(\,b-a)}{(b-a)}=-1$$

$$equation,\frac{y-y_2}{x-x_2}=m_2,\qquad m_1=m_2=-1$$

$$\frac{y-\ a}{X-a}=-1$$

$$y-a=-x+a$$

$$y=-x+a+a$$

$$y=-x+2a$$

Answer C

Two lines are said to be parallel to each other if they have the same slope.

Two lines are said to be perpendicular to each other if the product of their slopes is -1.

2. What are the values of the center (C) and radius (r) of a circle $x^2+y^2-4x+6y=5$?
   A. C=(-2, 3), $r=3\sqrt{2}$
   B. C=(2, -3), $r=3\sqrt{2}$
   C. C=(2,-3), $r=2\sqrt{3}$
   D. C=(-2,3), $r=2\sqrt{3}$

Solution

Completing the square method

- To make a quadratic expression perfect square
  1. Make coefficient of x^2 unit.
  2. Add $\left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

$$x^2 + y^2 - 4x + 6y = 5$$
$$x^2 - 4x + y^2 + 6y = 5$$
$$x^2 - 4x + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + y^2 + 6y + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 = 5$$
$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + y^2 + 6y + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 = 5$$
$$x^2 - 4x + 4 - 4 + y^2 + 6y + 9 - 9 = 5$$
$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 5 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 18$$
$$c(2,-3)\ ,\ r^2 = 18$$
$$r = \sqrt{18}$$
$$r = 3\sqrt{2}$$

Answer B

3. What is the radius of the largest possible circle that can be inscribed in the ellipse given by $5(x-1)^2+3y^2=15$?

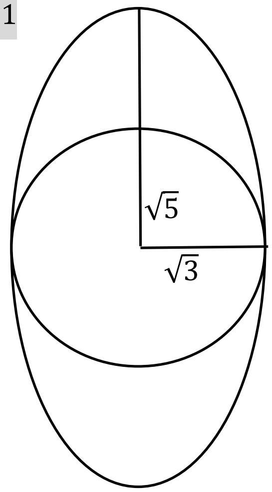
A. $\sqrt{3}$ B. $\sqrt{5}$ C. 3 D. 5

Solution

First of all write the equation as general form of ewuation of an ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \text{ or } \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$
$$5(x-1)^2 + 3y^2 = 15$$
$$\frac{5(x-1)^2}{15} + \frac{3y^2}{15} = \frac{15}{15}$$
$$\frac{(x-1)^2}{3} + \frac{y^2}{5} = 1$$
$$a^2 = 5\ ,\ a = \sqrt{5}$$
$$Major\ Axis = 2a = 2\sqrt{5}$$
$$b^2 = 3\ ,b = \sqrt{3}$$
$$Minor\ Axis = 2b = 2\sqrt{3}$$

Answer A



4. Suppose the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b} = 1$ is reciprocal to that of the eccentricity of the ellipse $x^2+4y^2=4$. If the hyperbola passes through the focus of the ellipse, then what is the equation of the hyperbola?

A. $X^2-2y^2=2$ B. $X^2-3y^2=3$ C. $X^2/3- y^2/2$ D. $X^2-y^2=1$

Solution

$Focus\ f\ the\ ellipse\ is\ vertex\ of\ the\ hyperbilo$

$$\Rightarrow ellipse:\ x^2 + 4y^2 = 4$$

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$a^2 = 4\ ,\qquad a = 2$$

$$b^2 = 1\ ,\qquad b = 1$$

$$c^2 = a^2 - b^2 = 4 - 1 = 3$$

$$c = \sqrt{3}$$

$$\Rightarrow F\,(-c\,,0)\ ,(\ c\,,0)$$

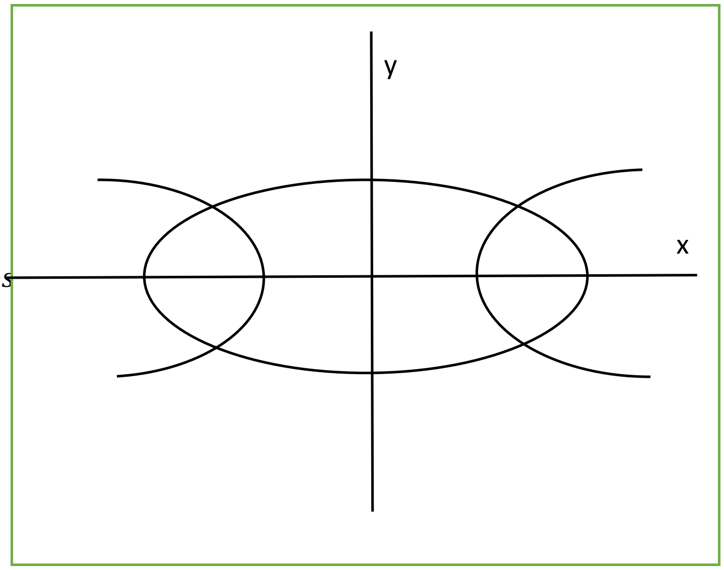
$$(-\sqrt{3}\,,0\,),(\sqrt{3}\,,0\,),$$

$$center(\,0\,,0\,)$$

$$\Rightarrow focus\ of\ hyperbolla = vertices\ of\ the\ ellips$$

$$v = (-a\,,0\,)\,,(\,a\,,0)$$

$$(-\sqrt{3}\,,0\,),(\sqrt{3}\,,0\,)$$



$$\Rightarrow e_{\ hyp} = \frac{1}{e_{\ ell}}$$

$$\frac{c}{a} = \frac{a}{c}$$

$$\frac{c}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\left.\begin{array}{l} c = 2 \\ a = \sqrt{3} \end{array}\right\}\quad \begin{array}{c} c^2 = a^2 + b^2 \\ 4 = 3 + b^2 \end{array}\qquad b = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{3} - \frac{y^2}{1} = 1$$

$$x^2 - 3y^2 = 3$$

$Answer\ B$

Grade 11 unit three – coordinate geometry

2010 E.C

1. Which one of the following is an equation of the circle whose end points of a diameter are (0, -2) and (2, 2)?

A. $X^2+y^2=4$ B. $X^2+y^2-2x-4=0$ C. $(x-1)^2+y^2=4$ D. $X^2+y^2-2y-4=0$

Solution

$$\text{Center Midpoind} = \left(\frac{0+2}{2},\frac{-2+2}{2}\right) = \left(\frac{2}{2},\frac{0}{2}\right) = (1,0) = (\mathrm{h},\mathrm{k})$$

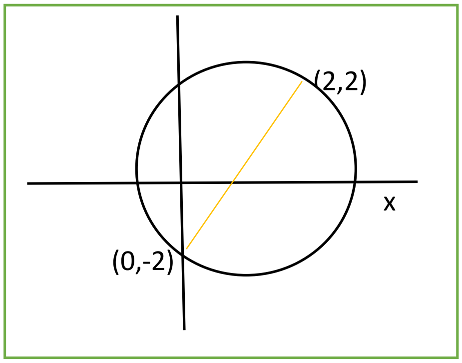
$$\text{radius} = \frac{\text{dia}}{2} = \frac{\sqrt{(2-0)^2 + (2-(-2))^2}}{2} = \frac{\sqrt{20}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$$

$$\Rightarrow (\mathrm{x}-\mathrm{h})^2 + (\mathrm{y}-\mathrm{k})^2 = \mathrm{r}^2$$

$$(\mathrm{x}-1)^2 + \mathrm{y}^2 = \left(\sqrt{5}\right)^2$$

$$\mathrm{x}^2 - 2\mathrm{x} + 1 + \mathrm{y}^2 - 5 = 0$$

$$\mathrm{x}^2 - 2\mathrm{x} + \mathrm{y}^2 - 4 = 0,\quad \text{Answer B}$$



2. What is the equation of the line that passes through (1, 1) and is parallel to the line 3y-x=1?

A. 3x-y=2 B. X+3y=4 C. x-3y+2=0 D. 13y-x+2=0

Solution

$$(1,1)$$
$$(x_1, y_1)$$
$$\text{Parallel to } 3y - x = 1$$
$$3y = x + 1$$
$$y = \frac{1}{3}x + \frac{1}{3}$$
$$m_1 = \frac{1}{3} = m_2, \text{ parallel lines have the same slope}$$
$$\frac{y - y_1}{x - x_1} = m$$
$$\frac{y - 1}{x - 1} = \frac{1}{3}$$
$$3y - 3 = x - 1$$
$$3y - x - 3 + 1 = 0$$
$$3y - x - 2 = 0$$
$$x - 3y + 2 = 0$$

Answer C

3. What is the area of triangle (in sq. units) formed by the vertex of the parabola x^2=-36y to the end points of the latus rectum?

A. 162 B. 126 C. 216 D. 261

Solution

Distance from v to F is p, latus rectum =4p

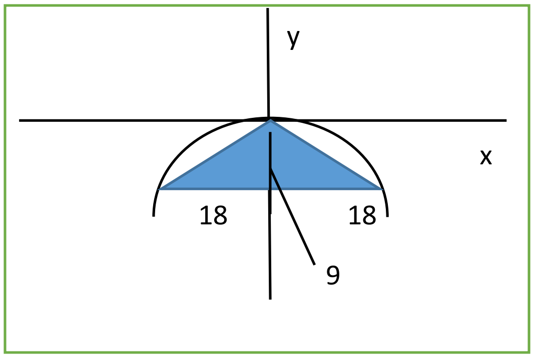
$x^2 = -36y$, -4p= - 36 , 4p=36, p=9, 2p=18

$A = 2(\frac{1}{2} * p * 2p)$, Two same size triangles

$=2\left(\frac{1}{2} * 9 * 18\right)$

$=162$,

Answer A



4. A man running a race-course noted that the sum of the distance between the flag posts from him is always 10meters and the distance between the flag posts is 8meters. What is the equation of the path traced by the man?

A. $\frac{x^2}{9} + \frac{y^2}{25} = 1$ B. $\frac{x^2}{64} + \frac{y^2}{100} = 1$ C. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ D. $\frac{x^2}{100} + \frac{y^2}{64} = 1$

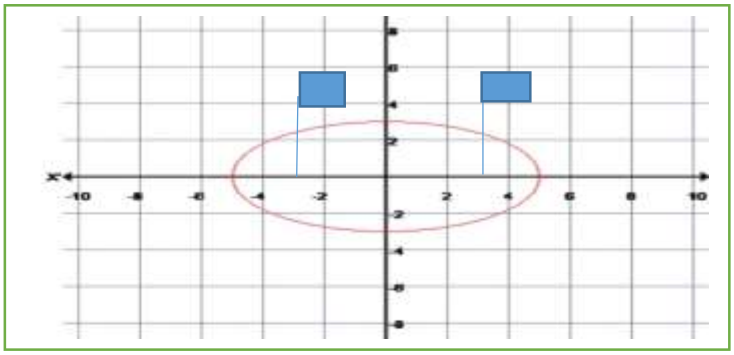
Solution

$2a = 10,$ $2c = 8$, the flags are on the focus of the ellipse

$a = 5$ $c = 4$

$$c^2 = a^2 - b^2$$
$$b^2 = a^2 - c^2$$
$$b = \sqrt{25 - 16}$$
$$= \sqrt{9} = 3$$
$$\frac{x}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Answer C



Grade 11 unit three – coordinate geometry

2011 E.C

1. The earth's orbit has a semi-major axis a ≈ 149.6 Gm (gigameters) and an eccentricity of e≈0.017. What is the approximate value of the semi-minor axis?

   A. 152.14Gm B. 145.32Gm C. 149.58Gm D. 149.06Gm

Solution

$$a = 149.6,\ e = 0.017,\ e = \frac{c}{a},\ c^2 = a^2 - b^2$$

$$C = \sqrt{a^2 - b^2},\ e = \frac{c}{a} = \frac{\sqrt{a^2-b^2}}{a}$$

$$ea = \sqrt{a^2 - b^2},$$

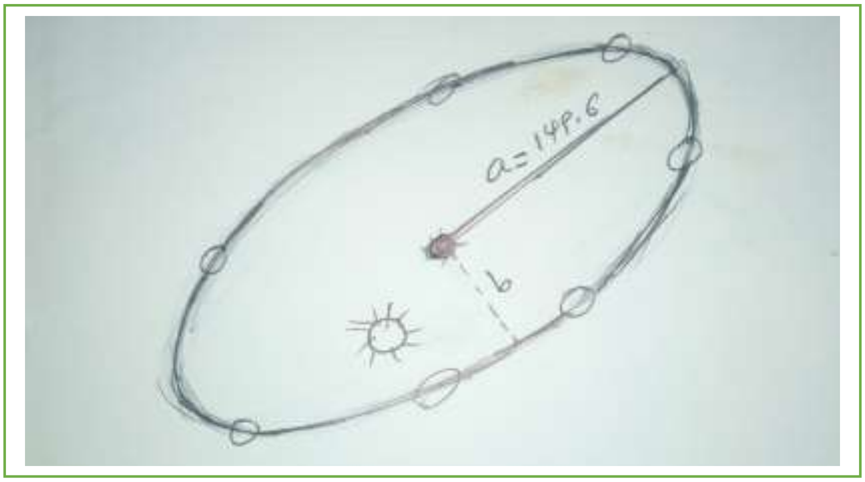
$$(ea)^2 = a^2 - b^2$$

$$b^2 = a^2 - (ea)^2 = a^2(1 - e^2)$$

$$b = \sqrt{a^2(1 - e^2)} = a\sqrt{1 - e^2} = 149.6\sqrt{1 - (0.017)^2}$$

$$149.6\sqrt{1 - 0.000289} = 149.6\sqrt{0.999711} = 149.6 * 0.999855 = 149.58$$

$$answer\ C$$



2. The center of a circle is on the line y=2x and the line x=1 is tangent to the circle at (1, 6). How long is the radius of the circle?

   A. 5 B. 2 C. 3 D. 4

Solution

$y = 2x$, $y = 6$ the center is at a point in which y=2x and y=6 intersects

$6 = 2x$

$\frac{6}{2} = \frac{2x}{2}$

$x = 3$

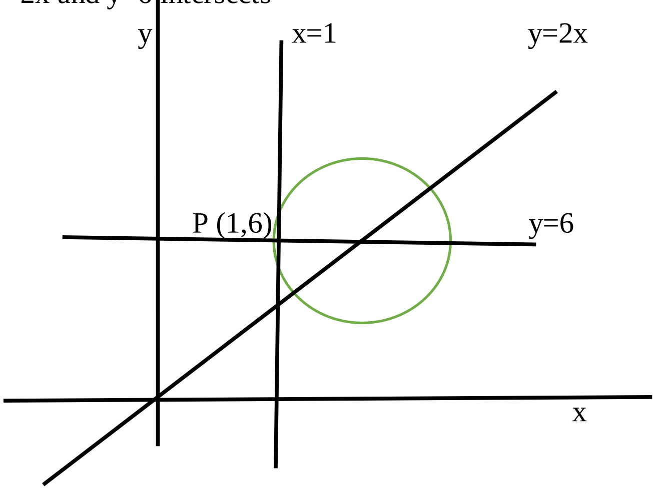
$center\ C(3{,}6)$

$r \Rightarrow C(3{,}6) - P(1{,}6)$

$r^2 = (6-6)^2 + (1-3)^2$

$r = \sqrt{0 + (-2)^2} = \sqrt{4} = 2$

Answer B

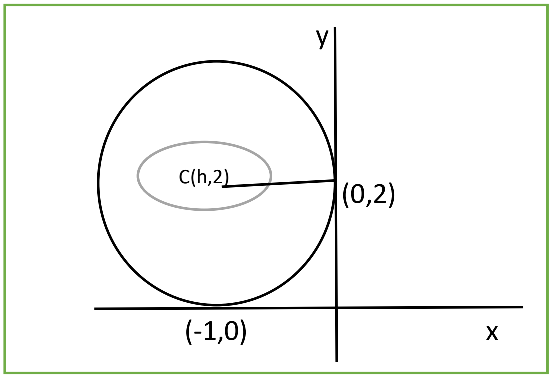


3. If the circle passing through the point (-1, 0) touches the y-axis at (0, 2) , then what is the equation of the circle?

   A. $X^2 + y^2+5x-4y+4=0$ C. $X^2 + y^2-5x-4y+4=0$

   B. $X^2 + y^2-5x-4y+4=0$ D. $X^2 + y^2+5x+4y+4=0$

<u>Solution</u>



$r\ is\ perpendicular\ to\ y-axis$

$\Rightarrow (h,k) = (h,2)$

$(h,2)\underline{\quad r \quad}(0,2) \quad and\ (h,2)\underline{\quad r \quad}(-1,0)$

$\sqrt{(2-2)^2+(0-h)^2} = \sqrt{(0-2)^2+(-1-h)^2}$

$\sqrt{(0)^2+(-h)^2} = \sqrt{(-2)^2+(-1-h)^2}$

$h^2 = 4+(-1-h)^2$

$h^2 = 4+1+2h+h^2$

$h^2 = 5+2h+h^2$

$0 = 5+2h$

$h = \frac{-5}{2} \Rightarrow center\ (h,k) = \left(\frac{-5}{2},2\right)$

$$\Rightarrow \left(\frac{-5}{2},2\right)\underline{\qquad r \qquad}(0,2)$$

$$r = \sqrt{\left(0-\left(-\frac{5}{2}\right)^2\right)+(2-2)^2} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

$$(x-h)^2+(y-k)^2 = r^2$$

$$\left(x+\frac{5}{2}\right)^2+(y-2)^2 = \left(\frac{5}{2}\right)^2$$

$$x^2+5x+\frac{25}{4}+y^2-4y+4 = \frac{25}{4}$$

$$x^2+5x+y^2-4y+4 = 0$$

Answer A

4. Which one of the following is true about the pair of lines 3x+9y-24=0 and 4x+12y+32=0?

   A. Perpendicular lines
   B. Parallel and distinct line
   C. Intersecting line
   D. Representing the same line

<u>Solution</u>

$$3x+9y-24 = 0$$

$$9y = -3x+24$$

$$\frac{9\,y}{y} = \frac{-3x}{9}+\frac{24}{9}$$

$$y = \frac{-1}{3}x+\frac{24}{9},\ \mathrm{m}=\frac{-1}{3}$$

$$4x+12y+32 = 0$$

$$12y = -4\,x-32$$

$$\frac{12y}{12} = \frac{-4x}{12}-\frac{32}{12}$$

$$y = -\frac{1}{3}x-\frac{32}{12},$$

$$\mathrm{m} = \frac{-1}{3}$$

$They\ have\ the\ same\ slope\ thus, they\ are\ parallel$

Answer B

Grade 11 unit three – coordinate geometry

2012 E.C

1. The distance between (4, -3) and the line ℓ: x + y-7=0 is

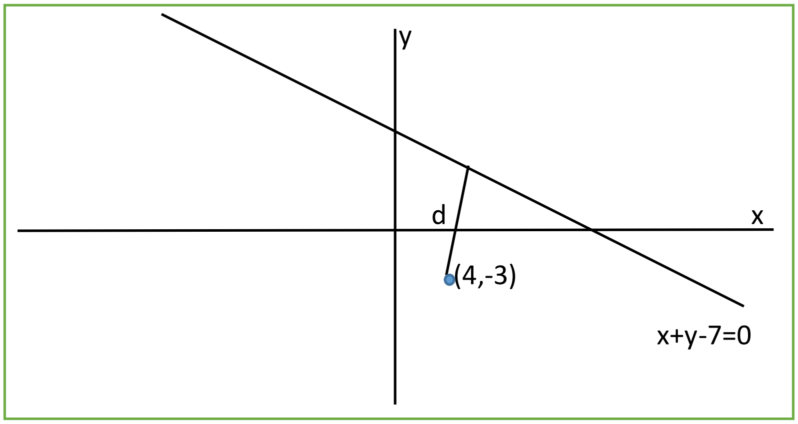
A. $4\sqrt{2}$ B. $2\sqrt{2}$ C. $\frac{3\sqrt{2}}{2}$ D. $3\sqrt{2}$

Solution

$$(4,-3),\ l: x+y-7=0$$

$$Ax+By+c=0$$

$$d=\frac{|Ah+Bk+c|}{\sqrt{A^2+B^2}}=\frac{|1(4)+1(-3)-7|}{\sqrt{1^2+1^2}}=\frac{|-6|}{\sqrt{2}}=\frac{6}{\sqrt{2}}=\frac{6}{\sqrt{2}}x\frac{\sqrt{2}}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=3\sqrt{2}$$



Answer D

2. Which one of the following is the standard form of equation of an ellipse with vertices: (3, 1), (3, 9): and minor axis length 6?

A. $\frac{(x-3)2}{9}+\frac{(y-5)2}{16}=1$ C. $\frac{(x+2)2}{6}+\frac{(y-3)2}{36}=1$

B. $\frac{(x+2)2}{4}+\frac{(y-3)2}{4}=1$ D. $\frac{(x-1)2}{16}+\frac{(y-9)2}{64}=1$

Solution

$h = \text{fixed} = \text{vertical ellipse}$

$C = M\left(\frac{3+3}{2},\frac{9+1}{2}\right)=(3,5)=(h,k)$

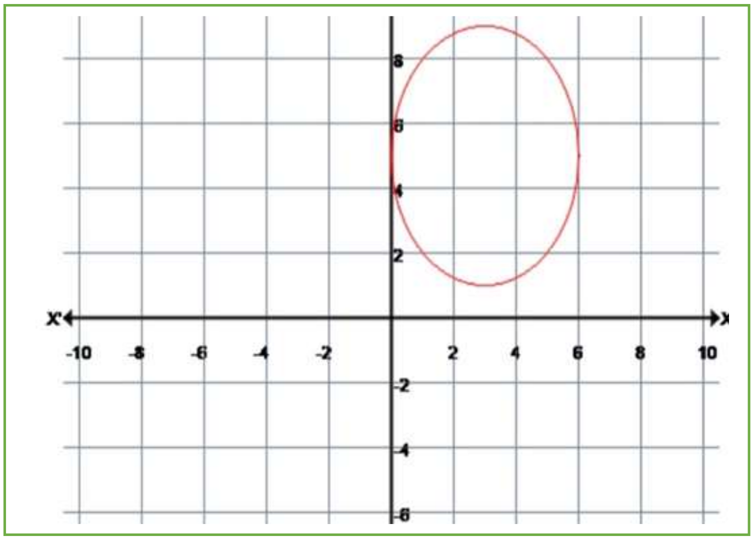
$\text{major axis} = 9-1=8=2a$

$a=4, 2b=6, \quad b=3$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\frac{(x-3)^3}{9}+\frac{(y-5)^2}{16}=1$$

Answer A



3. Let $x^2+y^2+kx=1$ be equation of a circle for some k ∈ ℝ. What is the radius of the circle if its center is at (2, 0)?

   A. 5 B. $\sqrt{5}$ C. $\sqrt{2}$ D. 2

Solution

$$x^2 + y^2 + kx = 1$$
$$x^2 + y^2 = -kx + 1 \ldots\ldots\ldots\ldots(1)$$
$$(x-2)^2 + (y-0)^2 = r^2$$
$$x^2 - 4x + 4 + y^2 = r^2$$
$$x^2 + y^2 = r^2 + 4x - 4 \ldots\ldots\ldots\ldots(2)$$
$$\text{Combine }(1)\text{and }(2)$$
$$-kx + 1 = r^2 + 4x - 4$$
$$(-kx) + (1) = (4x) + (r^2 - 4)$$
$$-k = 4\,, 1 = r^2 - 4$$
$$k = -4 \qquad r^2 = 5$$
$$r = \sqrt{5}$$

Answer B

4. The equation of a parabola is given by $y^2+6y+8x+25=0$. Which one of the following is NOT true about this parabola?

   A. Its vertex is (-2, 3) C .It is open to the right.

   B. Its directrix is the y-axis D. Its focus lies at (-4 3)

Solution

$y^2 + 6y + 8x + 25 = 0$

$y^2 + 6y = -8x - 25$

$y^2 + 6y + 9 = 8x - 25 + 9$

$(y+3)^2 = -8x - 16$

$(y+3)^2 = -8(x+2)$

A) − vertex $(h,k) = (-2,-3)$ True

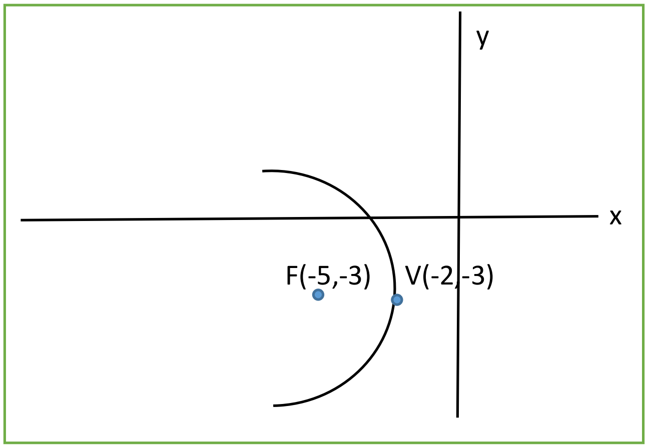
B) directrix, $x = h + p = -2 + 2 = 0$, true

C) The parabola 0pens to the left, true

D) focus $(h-p,k) = (-3-2,-3)$

$= (-5,-3)$, false

Answer D



Grade 11 unit three – coordinate geometry

2013 E.C

1. The equation of a circle is given by $(x-3)^2 + (y+2)^2 = 9$. What are the center and the radius of the circle respectively?

   A. (3, 2) and 3 B. (3, -2) and 3 C. (3, 2) and 9 D. (3, -2) and 9

Solution

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + (y+2)^2 = 9$$
$$\Rightarrow C(h,k) = (3,-2),$$
$$r^2 = 9$$
$$r = 3$$

Answer B

2. Which one of the following is the standard form of the equation of the circle centered at (2, -3) and radius 5?
   A. $(x-2)^2 + (y+3)^2 = 25$
   B. $(x-2)^2 + (y-3)^2 = 25$
   C. $(x+2)^2 + (y-3)^2 = 25$
   D. $(x-2)^2 +(y+3)^2 =5$

Solution

$$(h, k) = (2, -3), r = 5$$

General form of equation of a circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y+3)^2 = 25$$

Answer A

3. The equation of the line is given by 3x – 2y +6 = 0. Then which of the ff is true about the line?
   A. Its slope is $\frac{3}{2}$
   B. Its y – intercept is -3.
   C. Its x- intercept is 2.
   D. (2, 5) lies on the line.

Solution

3x-2y+6=0

-2y=-3x-6

y=$\frac{3}{2}x + 3$

A) $m = \frac{3}{2}$ True

B) $\text{y intercept } x = 0, y = 3$ False

C) X-interest y=0, x=-2, false

D. 3(2)-2(5)+6=0 ⟹6-10+6=0⟹2=0 False,

Answer A

4. Which of the following is true about the hyperbola given by $\frac{x^2}{4} - y^2 = 1$?
   A. Its vertices are at (-2, 0) and (2,0).
   B. Its foci are at (-5, 0) and /95,0).
   C. An asymptote to the hyperbola is y = 2x
   D. Its center is (2, 1).

Solution

$$\frac{x^2}{4} - \frac{y^2}{1} = 1, c(0,0) = (h, k) \qquad a^2 = 4, \qquad b^2 = 1 \qquad c^2 = a^2 + b^2$$

$$C^2 = 4 + 1, C = \sqrt{5}$$

A) $V(\pm a, k)$

$V(\pm 2,0)$

$V(-2,0), V(2,0)$, True

B) $F(h \pm C, k)$

$F(0 \pm \sqrt{5},0)$

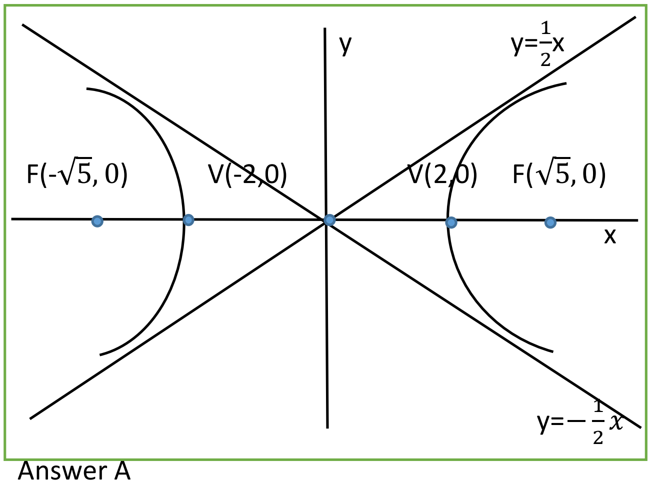
$F(\pm\sqrt{5}, 0)$

$F(-\sqrt{5}, 0)$, $F(\sqrt{5}, 0)$, False

C) Asymptote ⇒ y=$\pm\frac{b}{a}x$

=$\pm\frac{1}{2}x$, False

D) Centre (0,0), False,



Answer A

5. Which of the following is the standard form of the equation of the parabola with vertex (4, 6) and focus (-8, 6)?
   - A. $(x-4)^2=12(y-6)$
   - B. $(x-4)^2= -12(y-6)$
   - C. $(y-6)^2=-48(x-4)$
   - D. $(y-6)^2=48(x-4)$

<u>*Solution*</u>

$$Vertex, V(h,k) = (4,6), Focus,\ F(-8,6), \Longrightarrow \text{The parabola opens to the left.}$$

Thus the focus is F(h-p , k)

h-p=-8, 4-p=-8 , -p=-8 -4 , P=12

$$(\mathrm{y}-\mathrm{k})^2 = -4\mathrm{p}(\mathrm{x}-\mathrm{h})$$
$$(\mathrm{y}-6)^2 = -4(12)(\mathrm{x}-4)$$
$${}_{=}(\mathrm{y}-6)^2 = -48(\mathrm{x}-4)$$

Answer C

Grade 11 unit four – mathematical reasoning

2009 E.C

1. Let p and q stands for the statement ''Nejat is intelligent'' and ''Almaz is hardworking'', respectively. Which of the following represent the statement ''almaz is hard working if Nejat is intelligent''?
   - A. $\neg p \wedge q$
   - B. $\neg p \vee q$
   - C. $p \wedge q$
   - D. $\neg q \vee p$

<u>Solution</u>

$$p = Nejat\ is\ intellegent$$
$$q = Almaz\ is\ hard\ working$$
$$''\ If\ Nejat\ is\ intellegent\ , then\ Almaz\ is\ hard\ working\ ''$$
$$p \Rightarrow q$$
$$p \Rightarrow q \equiv \neg p\ v\ q$$

Answer B

2. Which of the following is a valid argument?
   - A. $\neg p \Rightarrow \neg q, q \dashv \neg p$
   - B. $p \Rightarrow \neg q, p, r \dashv \neg r$
   - C. $\neg p \vee q, r \vee\!\Rightarrow p, r \dashv \neg q$
   - D. $\neg p\ , p\ v\ q\ , r\ \Rightarrow q\ \dashv\ \neg r$

<u>Solution</u>

A. $\neg p \Rightarrow \neg q, q \dashv \neg p$, invalid				
p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

B. $p \Rightarrow \neg q, p, r \Rightarrow q \dashv \neg r$, valid						
p	q	r	$\neg q$	$\neg r$	$p \Rightarrow \neg q$	$r \Rightarrow q$
T	T	T	F	F	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	F
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	T	F	F	T	T	T
F	F	T	T	F	T	F
F	F	F	T	T	T	T

C. $\neg p \vee q, r \Rightarrow p, r \dashv \neg q$, invalid						
p	q	r	¬ p	¬ q	¬p v q	r ⇒p
T	T	T	F	F	T	T
T	T	F	F	F	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	T	F	T	F	T	T
F	F	T	T	T	T	F
F	F	F	T	T	T	T

D. $\neg p\ , p\ v\ q\ , r\ \Rightarrow q\ \dashv\ \neg r, invalid$						
p	q	r	¬ p	¬ r	¬p v q	r ⇒q p
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	F
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	F	T

Answer B

3. Consider the following open propositions: p(x) ≡ x is a prime number, c(x) ≡ x is a composite number, and E(x) ≡ x is an even number, which one of the following has a truth of True in the set of positive integers

A. $\forall_x[p(x) \Rightarrow \neg E(x)]$ C. $(\exists_x)[\neg p(x) \wedge \neg C(X)]$

B. $\neg(A_x)[C(x) \Rightarrow \neg p(x)]$ D. $\neg(\exists x)[E(x) \wedge \neg C(x)]$

solution

$$P(x) = \{x : x \text{ is prime Number}\}$$
$$C(x) = \{x : x \text{ is composite Number}\}$$
$$E(x) = \{x : x \text{ is an Even Number}\}$$

$A)\ (\forall x)[P(x) \Rightarrow \neg E(x)]$ C) $[\exists x]\ [\neg P(x) \wedge \neg C(x)]$

$false,\ eg\ x = 2$ $True\ \ eg\ x = 1\ \ \neg F\ \wedge \neg F$

$2\ is\ prime\ and\ even,$ $T\ \wedge T = T$

$B) \neg (\forall x)[C(x) \Rightarrow \neg P(x)]$ D) $\neg (\exists x)\big(\in (x) \wedge \neg C\ (x)\big)$

$(\exists x)\neg \big(C(x) \Rightarrow\ \neg p(x)\big)$ $(\forall x) \neg [\in (x) \wedge \neg Cx)]$

$\neg T\ \wedge\ F$ $\neg T\ v\ F$

$F\ \wedge T$ $F\ v\ F$

F $F,$ $Answer\ C$

Grade 11 unit four – mathematical reasoning

2010 E.C

1. If the truth value of $(p \wedge \neg q) \Leftrightarrow [(q \vee \neg q \Longrightarrow r]$ is true, then which one of the ff must be true?

A. $\neg r$ B. q C. $\neg q$ D. p

<u>Solution</u>

$[(p\wedge \neg p)\Leftrightarrow (q\ v \neg q)\Rightarrow r] = T$ $\Big\}$ $p\wedge \neg p$ is always F

$F \Leftrightarrow (T \Rightarrow r),\ \ F \Leftrightarrow F\ \equiv T$ $q\ v \neg q$ is always T

$T\Rightarrow r \equiv F$

$T\Rightarrow F \equiv F$

$r = F\ \ \ , \neg r = T,$

Answer A

2. Suppose the following are premises of an argument

He is healthy and he is not angry.
He is angry or his plan fails.
His plan doesn't fail if he does not travel abroad.
Given that the premises are true, which of the following can be a conclusion that makes the argument valid?

A. His plan fails and he is angry. C. His plan does not fall.

B. He travels abroad. D. His plan fails and he does not travel abroad.

<u>Solution</u>

p = he is healthy
¬q = he is not angry
q = he is angry
r = his plan fails
s = he traveled abroad

he is healty and he not angry $p\wedge\neg q = T$ $\Rightarrow p = T, q = F$
he is angry or his plan fails $qvr = T$ $\Rightarrow q = F, r = T,$
his plan doesn't fail if he does not travel abroad $\neg s\Rightarrow\neg r = T$ $\Rightarrow$r=T, s=T
A) his plan fails and he is angry $r\wedge q = T\wedge F = F$
B) He traveled abroad $= s = T$
C) His plan dpes not fail $= \neg r = F$
D) his plan fails and he does not travel abroad $r\wedge\neg s \equiv T\wedge F = F$ Answer B

3. Let U= ℕ (the set of natural numbers) be a universe. Which one of the ff proposition is true?
   A. $(\exists_x)(x+x=x)$ B. $(\forall_x)(\exists_y)(x\div y = y \div x)$ C. $(\forall x)(\exists y)(y < x)$ D. $(\forall_x)(\exists_y)(x\text{-}y=x)$

Solution

A) $\exists x(x + x = x)$ False, no x which makes $x + x = x$

B) $(\forall x)(\exists y)\left(\frac{x}{y} = \frac{y}{x}\right)$, True when $x = y$, eg. $x = 3, y = 3$

C) $(\forall x)(\exists\ y)(y < x)$, False Eg. x=1, no natural no <1

$D)(\forall x)(\exists\ y)(x - y = x)$ False eg. $x = 2, y = 0 \notin$ N,

Answer B

Grade 11 unit four – mathematical reasoning

2011 E.C

1. Which one of the following is a valid argument?
   A. If I am literate, then I can read and write. I can read but I can't write. Thus, I am not literate.
   B. If I don't change my oil regulatory, my engine will die. My engine died. Thus, I didn't change my oil regulatory
   C. If you do every problem in the book, then you will learn the subject. You learned the subject. Thus, you do every problem in the book.
   D. If it rains or snows, then my roof leaks. My roof is leaking. Thus it is raining and snowing.

Solution

Premises, P= I am literate=T

q=I can read=T

r= I can $write$=T

I can read but I can't write. Thus, I am not literate

$(q \wedge \neg r) \Rightarrow \neg q$

$(T \wedge F) \Rightarrow F$

$F \Rightarrow F \equiv T$

Answer A

2. For real numbers x and y, which of the following is NOT true?
   A. $(\forall x)(\forall y)(y^2+x \geq -1)$ C. $(\exists x)(\exists y)(y \geq x^2+1)$
   B. $(\forall x)(\exists y)(y \geq x^2+1)$ D. $(\exists x)(\exists y)(y \geq x^2+1)$

Solution

A $(\forall x)\ (\forall y)(x^2 + y^2 \geq -1)$ True, take any two numbers, eg. x=2, y=3

B) $(\forall x)\ (\exists y)(y \geq x^2 + 1)$ True, every x has its own y, eg. y=3, x=1

C) $(\exists x)\ (\exists y)(y \geq x^2 + 1)$ True, eg. x=0, y=3

D) $(\exists x)\ (\forall y)(y \geq x^2 + 1)$ False, No x for every y, eg. x=2, y=3

Answer D

3. If is $\neg p\ \Rightarrow r$ False and p⇔q is True. Which of the following is true?
   A. $\neg p \wedge (q \Rightarrow r)$ B. $p \vee (\neg q \wedge r)$ C. $\neg p \Rightarrow (q \vee r)$ D. p⇔ $(\neg q \vee r)$

Solution

$\neg p \Rightarrow r\ \equiv F$ $p \Leftrightarrow q = T,$

$\neg p = T\ \ ,r = F$ P and q are $equivalent$

$p = F\ \ ,r = F\ ,$ $q = F$

$A.\ \neg P \wedge (\ q \Rightarrow r)$

$T \wedge (\ F \Rightarrow F)$

$T \wedge T$

T

B. $p\ \ v\ (\neg\ q \wedge r)$

$F\ v\ (T \wedge F)$

$FvF\ = F$

C. $\neg p \Rightarrow (q\ v\ r)$

$T \Rightarrow (FvF)$

$T \Rightarrow F$

F

$D. p \Leftrightarrow (\neg\ q\ v\ r)$

$F \Leftrightarrow (T\ v\ F)$

$F \Leftrightarrow\ T = F$

Answer A

Grade 11 unit four – mathematical reasoning

2012 E.C

1. If p: ″the rainy season is very good this year″ and q: ″rivers are rising″, then the statement ″it is not true that neither the rainy season is very good this year nor rivers are rising″ is denoted by:
   A. $(\neg p \Rightarrow \neg q)\ v \neg q$
   C. $\neg p\ \Lambda\ \neg q$
   B. $(p \Rightarrow q)\ v(\neg p \Rightarrow \neg q)$
   D. $\neg(\neg p\ v\ \neg q)$

Solution

P= rain is very good this year

q = rivers are rising

If is not true that neither the rain season is very good or rivers are rising

$\neg(\neg p\ v\ \neg q)$

Answer D

2. Let p and q be propositions with truth value True and False respectively. Then, which one of the following is correct?
   A. $(p \vee \neg q) \Rightarrow q$ is True
   B. $\neg p \Rightarrow q$ is False
   C. $P\ v\ (q\ \Lambda\ \neg p)$ is true
   D. $(p \Rightarrow q)\ \Lambda\ p$ is true.

Solution

$$P = T, q = F$$

A) $(PV\neg q) \Rightarrow q$	B) $\neg p \Rightarrow q$	C) $PV(q \wedge \neg p)$	D)$(p \Rightarrow q) \wedge P$
$(TVT) \Rightarrow F$	$\neg T \Rightarrow F$	$TV(F \wedge F)$	$(T \Rightarrow F) \wedge T$
$T \Rightarrow F$	$F \Rightarrow F$	TVF	$F \wedge T$
=F	=T	= T	=F

Answer C

3. If p and q are propositions, then which one of the following pairs of compound propositions are equivalent?
   A. $p \Rightarrow \neg$ and $\neg p\ \Lambda\ q$
   B. $\neg(p\ \Lambda\ q)$ and $\neg p\ v\ q$
   C. $P \Rightarrow q$ and $\neg p \Leftrightarrow \neg q$
   D. $\neg p \Rightarrow q$ and $q \Rightarrow \neg p$

Solution

A. $p \Rightarrow \neg q$ and $\neg p\ \Lambda\ q$					
p	q	¬p	¬q	$p \Rightarrow \neg q$	$\neg p \wedge q$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	F

C. $p \Leftrightarrow q$ and $\neg p \Leftrightarrow \neg q$					
p	q	¬p	¬q	$p \Leftrightarrow q$	$\neg q \Leftrightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

B. $\neg(p\ \Lambda\ q)$ and $\neg p\ v\ q$					
p	q	¬p	$q \wedge q$	$\neg(p \wedge q)$	$\neg p \vee q$
T	T	F	T	F	T
T	F	F	F	T	F
F	T	T	F	T	T
F	F	T	F	T	T

D. $\neg p \Rightarrow q$ and $q \Rightarrow \neg p$					
p	q	¬p	¬q	$\neg p \Rightarrow q$	$q \Rightarrow \neg p$
T	T	F	F	T	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

Answer C

Grade 11 unit four – mathematical reasoning

2013 E.C

1. Which one of the following has a truth value?

   A. We shall win! C. Tomorrow is a beautiful day.

   B. Man is selfish. D. Man is mortal.

Solution

Man is mortal =T

The other statements are not propositions.

Answer D

2. Which one of the following is the contrapositive of the statement: " If 21 is a prime number, then 4 is an odd number''?
   - A. If 4 is an odd number, then 21 is a prime number.
   - B. If 4 is an odd number, then 21 is not a prime number.
   - C. If 21 is not a prime number, then 4 is an even number.
   - D. If 4 is not an odd number, then 21 is not a prime number.

Solution

$$\underbrace{\text{If 21 is a prime number}}_{p} \xRightarrow{\text{then}} \underbrace{\text{4 is an odd number}}_{q}$$

Contrapositive, ¬q ⇒ ¬q = If 4 is not an odd number, then 21 is not a prime number

Answer D

3. Two statements p and q by:

   P: $\frac{22}{7}$ is an irrational number.

   q: the sum of two odd integers is odd.

   Then which of the following has Truth value T?

   A. P V ¬q B. P ^ ¬q C. P ^ q D. ¬p ^ q

Solution

$$P = F, q = F$$

A) $P \vee \neg q$	B) $p \wedge \neg q$	C) $P \wedge q$	D) $\neg P \wedge q$
$F \vee T$	$F \wedge T$	$F \wedge T$	$T \wedge F$
T	F	F	F

Answer A

4. Which of the following statements has a truth value T?
   - A. $(p \wedge q) \Rightarrow \neg(p \vee q)$
   - B. $(p \wedge q) \Rightarrow (p \vee q)$
   - C. $(p \Rightarrow q) \Rightarrow (p \vee q)$
   - D. $(p \vee q) \Rightarrow (p \wedge q)$

Solution

A) $(p \wedge q) \Rightarrow \neg(p \vee q)$

If p=q=T

$T \Rightarrow F = F$

B) $(p \wedge q) \Rightarrow (p \vee q)$

i. If p=q=T

$T \Rightarrow T = T$

ii. If p=q=F

$F \Rightarrow F = T$

iii. if one of p or q, $F \Rightarrow (p \vee q) = T$

C) $(p \Rightarrow q) \Rightarrow (p \vee q)$

If p=q=F

$T \Rightarrow F = F$

D) $(p \vee q) \Rightarrow (p \wedge q)$

If p=T, q=F

$(T \vee F) \Rightarrow (T \wedge F)$

$T \Rightarrow F = F$

Answer B

5. Which of the following arguments is valid?
   A. ¬p, q ⊦ p ⇒ q
   B. p, ¬ q ⊦ p ⇒ q
   C. p, ¬q ⊦ p ^ q
   D. p, q ⊦ p ⇒ ¬q

Solution

A) ¬p, q⊦p⇒ q

If ¬p = T and q = T, The premises are true

then F⇒T is always True

Answer A

Grade 11 unit five – statistics and probability

2009 E.C

1. A team of 10 researchers consists 4 biologists and 6 chemists. If 3 persons are chosen randomly from the team, what is the probability that at least one is a biologists?
   A. 2/3 B. 2/5 C. 5/6 D. 7/10

Solution

$$n = 10\,, number\ of\ \ biologist = n(B) = 4$$

$$number\ of\ chemistry = n(c) = 6$$

$$Choose\ 3\ out\ \ of\ 10\,, atleast\ one\ is\ B\,, n(B) \geq 1$$

$$P\big(n(B)\big) \geq 1 = P(1B, 2C) + P(2B, C) + P(3B, 0C)$$

$$= \frac{C(4,1)C(6,2)}{C(10,3)} + \frac{C(4,2)C(6,1)}{C(10,3)} + \frac{C(4,3)C(6,0)}{C(10,3)}$$

$$= \frac{\frac{4!}{3!}\cdot\frac{6!}{4!\,2!}}{\frac{10!}{7!\,3!}} + \frac{\frac{4!}{2!\,2!}\cdot\frac{6!}{5!\,1!}}{\frac{10!}{7!\,3!}} + \frac{\frac{4!}{1!\,3!}\cdot\frac{6!}{6!}}{\frac{10!}{7!\,3!}}$$

$$= \frac{\frac{4*3!}{3!}\cdot\frac{6*5*4!}{4!\,2}}{\frac{10*9*8*7!}{7!*3*2}} + \frac{\frac{4*3*2}{2*2}\cdot\frac{6*5!}{5!}}{\frac{10*9*8*7!}{7!*3*2}} + \frac{\frac{4*3*2}{3!}\cdot\frac{6!}{4!\,2!}}{\frac{10*9*8*7!}{7!*3*2}}$$

$$= \frac{4*15}{120} + \frac{6*6}{120} + \frac{4*1}{120}$$

$$= \frac{60}{120} + \frac{36}{120} + \frac{4}{120} = \frac{60+36+4}{120} = \frac{100}{120} = \frac{5}{6}$$

$$Answer\ C$$

2. The probability that an electronic device produced by a company does not function properly is equal to 0.1. if 2 devices are bought, then what is the probability that at least1 device function properly?
   A. 0.81 B. 0.09 C. 0.18 D. 0.99

Solution

$$let\ Doesnot\ function, D \quad ,\ P(D) + P(F) = 1$$

$$function\,, F \quad ,\ 0.1 + P(f) = 1$$

$$P(F) = 1 - 0.1 = 0.9$$

$$P(F)\,, atleast\ one$$

$$P(1F) + P(2F) = C(2,1) * (0.9)^1(0.1)^1 + C(2,2)(0.9)^2(0.1)^o$$

$$= \frac{2!}{(2-1)!\,1!} * 0.9 * 0.1\ +\ \frac{2!}{0\,!\,2!} * 0.81 * 1$$

$$= 2 * 0.9 * 01 + 0.81 = 0.18 + 0.81 = 0.99$$

$$Answer\ D$$

3. In how many ways can a committee of 3 members be formed from 7 candidates?

A. 7 B. 21 C. 28 D. 35

Solution

$$\text{Committee of 3 out of 7} = c(7,3) = \frac{7!}{(7-3)!\,3!} = \frac{7!}{4!\,3!}$$
$$= \frac{7*6*5*4!}{4!*3*2} = 7*5$$
$$= 35$$

Answer D

4. Two machines A and B produce respectively 60% and 40% of the total number of items of a factory. The percentages of defective output of these machines are 2% and 5% respectively. If an item is selected at random, then what is the probability that the item is defective?

A. 0.032 B. 0.07 C. 0.32 D. 0.0426

Solution

$$p(A) = 60\% - 2\% = 58\% = 0.58$$
$$p(B) = 40\% - 5\% = 35\% = 0.35 \quad ’$$
$$p(\text{defective}) = p(\text{defective in A}) + p(\text{defective in B})$$
$$= C(1,1)(0.02)^1(0.58)^0 + C(1,1)(0.05)^1(0.35)^0$$
$$= 0.02 + 0.05 = 0.07$$

Answer B

5. The expenditure of 100 families is given below.
The mode of the data is 23.5, what are the values of f_1 and f_2.

Expenditure	0-9	10-19	20-29	30-39	40-49
*f*requency	14	23	f_1	21	f_2

A. F_1= 27, f_2= 15
B. F_1= 15, f_2= 27
C. F_1= 25, f_2= 17
D. F_1= 17, f_2= 25

Solution

$$m_o = 23.5 \text{ is in the } 3^{rd} \text{ class}, B_l = 19.5\,, \text{class width}, i = 10$$
$$d_1 = f_1 - 23, d_2 = f_1 - 21$$

x	f	fc	Class boundary
0-9	14	14	-0.5-9.9
10-19	23	37	9.9-19.5
20-29	f_1	$37+f_1$	19.5-29.5
30-39	21	$58+f_1$	29.5-39.5
40-49	f_2	$58+f_1+f_2$	39.5-49.5

$$mode = Bl + \left(\frac{d_1}{d_1+d_2}\right)i$$
$$23.5 = 19.5 + \left(\frac{f_1 - 23}{f_1 - 23 + f_1 - 21}\right)10\cdot$$
$$23.5 - 19.5 = \frac{10f_1 - 230}{2f_1 - 44}$$
$$4 = \frac{10f_1 - 230}{2f_1 - 44}$$
$$8f_1 - 176 = 10f_1 - 230$$
$$-176 + 230 = 10f_1 - 8f_1, 54 = 2f,$$
$$27 = f_1$$
$$14 + 23 + f_1 + 21 + f_2 = 100$$
$$14 + 23 + 27 + 21 + f_2 = 100$$
$$85 + f_2 = 100$$
$$f_2 = 100 - 85$$
$$f_2 = 15$$

answer A

6. The following is a frequency distribution table of grouped data with variable X.

X	3-7	8-12	13-17	18-22
f	4	6	8	2

What is the mean (x) and the variance (s^2) of the data respectively?

A. X= 12,s^2= 21 B. X= 12, s^2=25 C. X= 13, s^2=9 D. X= 13, s^2=16

Solution

x	f	x_c	fxc	$x_c - \bar{x}$	$(x_c-\bar{x})^2$	$f(x_c-\bar{x})^2$
3-7	*4*	*5*	*20*	*-7*	*49*	*196*
8-12	*6*	*10*	*60*	*-2*	*4*	*24*
13-17	*8*	*15*	*120*	*3*	*9*	*72*
18-22	*2*	*20*	*40*	*8*	*64*	*128*

$\sum x = 20$ $\sum fx_c = 240$ $\sum f(x_c-\overline{x})^2 = 420$

$\bar{x} = \frac{\sum fx_c}{\sum f} = \frac{240}{20} = 12$, $\delta^2 = \frac{\sum f\,(x_c-\overline{x})^2}{\sum f} = \frac{420}{20} = 21$

Answer A

7. The first group of 10 children has a mean weight of 15.6 kg, the second group of another 10 children has a mean weight of 16kg and the third group of children has a mean weight of 20 kg. If the mean weight of all the children is 17kg. What is the total number of children in all of the three groups?

A. 28 B. 29 C. 30 D. 32

Solution

$$G_1 \Rightarrow n = 10, \overline{x_1} = 15.6$$
$$G_2 \Rightarrow n = 10, \overline{x}_2 = 16$$
$$G_3 \Rightarrow x, \overline{x_3} = 20$$
$$\overline{x} = \frac{10*15.6+10*16+20x}{10+10+x}, 17 = \frac{316+20x}{20+x}$$
$$340+17x = 316+20x$$
$$340-316 = 20x-17x$$
$$24 = 3x$$
$$8 = x$$
$$total = 10+10+x$$
$$= 10+10+8$$
$$= 28$$
$$Answer\ \ A$$

Grade 11 unit five – statistics and probability

2010 E.C

1. Let A and B be two events. Suppose that the probability that neither event occurs is 3/8. What is the probability that at least one of the events occur?

A. 1/8 B. ¼ C. ¾ D. 5/8

Solution

$$\text{P(at least one)} = 1 - \text{P(neither)}$$
$$= 1 - \tfrac{3}{8}$$
$$= 5/8$$

Answer D

2. The age distribution of students in a certain class is given below:

Age	10-14	15-19	20-24	25-29
No. of students	2	10	6	7

What is the modal value of the distribution?

A. 17.38 B. 18.33 C. 17.83 D. 18.73

Solution

Age	of	clas boundary
10-14	2	9.5-14.5
15-19	10	14.5-19.5
20-24	6	19.5-24.5
25-29	7	24.5-29.5

$\text{modal clas} = 2^{\text{nd}}$ class, high frequency

$d_1 = 10 - 2 = 8$

$d_2 = 10 - 6 = 4$

$i = 25 - 20 = 5$

$B_L = 14.5,\qquad m_o = B_L + \left(\frac{d1}{d1+d2}\right)i$

$$= 14.5 + \left(\frac{8}{8+4}\right)5$$

$$= 14.5 + 3.33$$

$$= 17.83$$

Answer C

3. The time added to type a sample of 8 business letters in an office is 7, 8, 6, 8, 9, 7, 5, 6 minutes. What are the mean (x) and the standard deviation (s) of the data in munities?

A. X=7, s=√2 B. X=8, s=√1.5 C. X=8, s=√2 D. X=7, s=√1.5

Solution

$$mean, \bar{x} = \frac{7+8+6+8+9+7+5+6}{8} = \frac{56}{8} = 7$$

$$\text{var} = \frac{(7-7)^2 + (8-7)^2 + (6-7)^2 + (8-7)^2 + (9-7)^2 + (7-7)^2 + (5-7)^2 + (6-7)^2}{8}$$

$$\text{variance} = \frac{0^2 + (-1)^2 + (-1)^2 + (1)^2 + (2)^2 + 0^2 + (-2)^2 + (-1)^2}{8}$$

$$= \frac{1+1+1+4+4+1}{8} = \frac{12}{8} = 1.5,$$

$$\text{standard deviation } \delta = \sqrt{1.5},$$

Answer D

4. Let A= {1, 2, 3, 4, 5, 6,7} B={7, 8, 9} and C={8, 9, 10}. If one of the numbers is deleted randomly from each of these sets, what is the probability that all the three deleted numbers are even or all are multiples of 3?

A. 8/63 B. 2/21 C. 1/9 D. 4/5

Solution

Deleting a number randomly from each of these sets

P (3 even) +P(three multiples of 3)=$\frac{3}{7} * \frac{1}{3} * \frac{2}{3} + \frac{2}{7} * \frac{1}{3} * \frac{1}{3}$

$$= \frac{2}{21} + \frac{2}{63}$$

$$= \frac{8}{63}$$

Answer A

5. A private college has 1000 students. 60% of these students are males, 45% of the students pay their payment by credit card including 175 females. What is the probability the students is a made or a credit or a card user?
   A. 0.775 B. 0.225 C. 0.675 D. 0.325

Solution

$$\mathrm{m} = 60\% \text{ of } 1000 = 600 \qquad \mathrm{creditcard} = 45\% \text{ of } 1000 = 450$$

$$\mathrm{F} = 1000 - 600 = 400 \qquad \text{male and c. card user} = 450 - 175 = 275$$

$$\mathrm{creditcard(F)} = 175$$

$$\mathrm{p(male\ or\ credit\ card\ user)} = \mathrm{p(m)} + \mathrm{p(c.\,card\ user)} - \mathrm{p(male\ and\ credit\ card\ user)}$$

$$= \frac{600}{1000} + \frac{450}{1000} - \frac{275}{1000} = \frac{775}{1000} = 0.775$$

Answer A

6. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the variance of the resulting observations
   A. 5 B. 20 C. 10 D. 40

Solution

$$\text{Old variance} = \delta^2$$

$$\text{if each data is multipled by n, new variance} = \mathrm{n}^2\delta^2$$

$$\Rightarrow \text{old varoance} = 5, \text{each data multiplated by } 2$$

$$\Rightarrow \text{new variance} = 2^2 * 5 = 4 * 5 = 20$$

Answer B

Grade 11 unit five – statistics and probability

2011 E.C

1. Suppose 2,500 items are produced by a machine and 2% of the product are randomly selected and tested. If 5 of the tested items have defect, then what is the probability that an item produced by the machine has NO defect?
   A. 0.90 B. 0.85 C. 0.80 D. 0.95

Solution

$$2\%\ of\ 2500 = 2500 * 0.02 = 50$$

$$P(defective) = \frac{5}{50} = 0.1$$

$$P(not\ defective) = 1 - p(defective)$$

$$=1 - 0.1$$

$$= 0.9$$

Answer A

2. There are three children in a room with ages, four, five and six. If five years old child enters the room, then which of the following statement is correct?
   A. Mean age and standard deviation will increase.
   B. Mean age will stay the same but the standard deviation will increase.
   C. Mean age will stay the same but the standard deviation will decrease.
   D. Mean age and standard deviation will stay the same.

Solution

$$\overline{x} = \frac{4+5+6}{3} = \frac{15}{3} = 5, \qquad \bar{x}_{new} \overset{=}{} \frac{4+5+6+5}{4} = \frac{20}{4} = 5$$

$$\delta^2 = \frac{(4-5)^2+(5-5)^2+(6-5)^2}{3} = \frac{(-1)^2+(0)^2+(1)^2}{3} = \frac{1+1}{3} = \frac{2}{3}$$

$$\underset{new}{\delta^2} = \frac{(4-5)^2+(5-5)^2+(6-5)^2+(5-5)^2}{3} = \frac{(-1)^2+0^{2+}1^2+0^2}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\overline{X} = \overline{X}_{new} \text{ and } \delta^2 > \delta^2{}_{new},$$

Answer C

3. The marks of 50 students are given below.

x	0-10	10-20	20-30	30-40	40-50
f	5	8	f1	10	f2

The median of the data is 26, what are the values of f1 and f2?

A. f1=7, f2=20 B. f_1=20, f_2=7 C. f1=15, f=12 D. f1=12, f=15

Solution

$X(class\ boundary)$	f	Cf
0 – 10	5	5
10-20	8	13
20-30	f_1	$13+f_1$
30-40	10	$23+f_1$
40-50	f_2	$23+f_1+f_2$

$$median = 26\ \ ,\ median\ class = 3^{rd}\ class$$

$$md = B_l + \left(\frac{\frac{n}{2} - Cf_b}{f_c}\right)i$$

$$26 = 20 + \left(\frac{\frac{50}{2} - 13}{f_1}\right)10$$

$$26 - 20 = \left(\frac{25-13}{f_1}\right)10$$

$$6 = \frac{120}{f_1}$$

$$6f_1 = 120$$

$$\frac{6f_1}{6} = \frac{120}{6}, f_1 = 20$$

$$5 + 8 + f_1 + 10 + f_2 = 50$$
$$5 + 8 + 20 + 10 + f_2 = 50$$
$$43 + f_2 = 50$$
$$f_2 = 50 - 43$$
$$f_2 = 7$$

Answer B

4. Fatuma can solve 90% of the problems given in book and Mesfin can solve 70%. What is the probability that at least one of them will solve the problem selected at random from the book?

A. 0.97 B. 0.87 C. 0.77 D. 0.67

Solution

$$P(M) = 0.7,\quad P(F) = 0.9$$
$$P(FUM) = P(F) + P(M) - P(F) * \mathrm{P}(M)$$
$$= 0.9 + 0.7 - 0.63 = 1.6 - 0.63 = 0.97$$
$$Answer\ A$$

5. Among 2000 students who took a regional exam, the percentile of certain student's score is 90. Which one of the following correct about the student's score?

A. The students have answered 90% of the question correctly.
B. The student's score is the same as the top 10% of the scores.
C. The student's score is greater than or equal to that of 1800 students.
D. The score of the students is as good as that 90% of the students.

Solution

$$90\%\ of\ 2000 = 2000 * 0.9$$
$$= 1800$$

Answer C

6. If there are two children in a family. What is the probability that there is at least one girl in the family?
   A. ¾ B. ¼ C. ½ D. 2/3

Solution

$$S = \{GG, GB, BG, BB\} \quad , n(s) = 4$$
$$\in = \{GG, GB, BG\} \quad , n(\in) = 3$$
$$p(\in) = \frac{n(\in)}{n(s)} = \frac{3}{4}$$

Answer A

7. If A is 3×3 matrix and det(A)=5, then how much is det(2A^T A)?
   A. 100 B. 200 C. 50 D. 20

Solution

$$\det(A) = |A| = 5 \qquad , \qquad \begin{array}{c} |A| = |A^T| \\ |kA| = k^n|A| \end{array}$$
$$|2AA^T| = 2^3|A||A^T|$$
$$= 8 * 5 * 5$$
$$= 200$$

Answer B

Grade 11 unit five – statistics and probability

2012 E.C

1. Which one of the following statements is NOT true?
   A. When a variance of a data is close to zero, the data shows less variability.
   B. Variance indicates the variability of a set of numerical data items.
   C. If two data have equal mean, the data with less variance shows less variability.
   D. The large value of variance shows that the data values are closer to the mean.

Solution

Variance is positive number

If the values are closer to the mean, then we will get smaller variance since mean is subtracted from each data

Answer D

2. The following is the frequency distribution of a grouped data

Class intervals	Frequency (f)
3-7	2
8-12	2
13-17	10
18-22	6

What is the mean of the data?

A. 15 B. 14 C. 12.5 D. 13

Solution

x	f	x_c	fx_c
3-7	2	5	10
8-12	2	10	20
13-17	10	15	150
18-22	6	20	120

$$\sum f = 20 \qquad \sum fxc = 300$$
$$\bar{x} = \frac{300}{20}$$
$$= 15$$

Answer A

3. The table shown below is a simple frequency distribution of data with variable x.

X	1	3	4	5	7
frequency	2	5	6	5	2

What is the variance of the data?

A. 3 B. √3 C. 3.2 D. 2.3

Solution

x	f	fx	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
1	2	2	-3	9	18
3	5	15	-1	1	5
4	6	24	0	0	0
5	5	25	1	1	5
7	2	14	2	4	8

$$\sum f = 10\ ,\sum f\,x = 80 \qquad \sum f(x-\bar{x})^2 = 46$$

$$\bar{x} = \frac{80}{20} = 4, \qquad \text{variance} = \frac{46}{20} = 2.3$$

Answer D

4. Consider the experiment of rolling a die whose sample space (1, 2, 3, 4, 5, 6). If two dies are rolled simultaneously, what is the probability that a prime number turns up on one of the dice and a composite number on the other?

A. 1/3 B. 2/3 C. 5/6 D. 1/6

Solution

$$S = \{1, 2, 3, 4, 5, 6\}$$

	1	2	3	4	5	6
1	1,1	1.2	1,3	1,4	1,5	6,1
2	2.1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	3,4	4,4	4,5	4,6
5	5,1	5.2	3,5	5.4	5,5	5,6
6	6,1	6,2	4,6	6.4	6,5	6,6

$$n(S) = 36$$

$$\in = \{(2,4), (2,6), (3,4), (3,6)(5,4), (5,6)\}\,, n(\in) = 6$$

$$p(\in) = \frac{n(\in)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Answer D

5. Consider the following frequency distribution of grouped data.

Which one of the following statements is true about this frequency distribution?

A. The class interval is 4

B. 95.5 is the upper limit of the second class.

C. 85.5-90.5 is one of the class boundaries.

D. 82 is the class midpoint of the fourth class.

values	Frequency
95-99	4
90-94	6
85-89	10
80-84	16
75-79	9
70-74	5

Solution

X	f	class vondany	xc
95-99	4	94.5-99.5	94
90-94	6	89.5-94.5	92
85-89	10	84.5-89.5	87
80-84	16	79.5-84.5	82
75-79	9	74.5-79.5	77
70-74	5	69-5-74-5	72

$$-\text{class interval} = 90 - 95 = 5, - \text{upper limit of } 2^{\text{nd}} \text{ classs} = 90$$

$$-\text{class mid point of } 4^{\text{th}}\text{class} = \frac{80 + 84}{2} = 82$$

Answer D

6. Suppose the measurement of height (in meter) of twelve students is: 1.72, 1.65, 1.70, 1.56, 1.72, 1.70, 1. 65, 1.70, 1.65, 1.65, 1.70, 1.72. Which of the following is true about the mode of the data?
   A. The data is unimodal with mode 1.65.
   B. The data is unimodal with mode 1.65.
   C. The data is bimodal with modes 1.65 and 1.70
   D. The data is multi modal.

Solution

x	1.56	1.65	1.7	1.72
f	1	4	4	3

$$\text{mode} = 1.65 \text{ and } 1.7$$

Answer C

Grade 11 unit five – statistics and probability

2013 E.C

1. Consider the following frequency distribution table, what is the sixth decile?

ν	5	3	7	6	8
f	3	4	5	3	5

A. 5 B. 6.5 C. 7 D. 7.5

Solution

ν	5	3	7	6	8
f	3	4	5	3	5

$$D_6 = \left(\frac{\frac{6n}{10}+\frac{6n}{10}+1}{2}\right)^{tn} V$$

$$= \left(\frac{\frac{6(20)}{10} + (\frac{6(20)}{10} + 1)}{2}\right)^{th} \text{Value}$$

$$= \left(\frac{12+(12+1)}{2}\right)^{th} \text{Value} = \left(\frac{25}{2}\right)^{th} \text{ value} = 12.5^{tn} \text{ value}$$

$$= \frac{12^{tn}\text{ v}+13^{th}\text{ v}}{2} = \frac{7+7}{2} = 7$$

Answer C

2. In a certain health center there are 3 doctors, 8 nurses and 2 physicians. In how many ways one can form a group of 5 members consisting of 1 doctor, 3 nurses and 1 physician?
   A. 112 B. 168 C. 330 D. 336

Solution

3 doctors------------------------------------ 1
8 nurses ------------------------------------ 3
2 physicians --------------------------------- 1

⇒ C(3,1)* C(8,3) *(2,1)

$$= \frac{3!}{(3-1)!} * \frac{8!}{(8-3)!} * \frac{2!}{(2-1)!}$$

$$= \frac{3*2!}{2!} * \frac{8*7*6*5!}{5!*3!} * 2 = 3*8*7*2 \quad = \underline{\underline{336}}$$

Answer D

3. Which one of the following forms represents an exhaustive set of events if a die is thrown once?
   A. {1, 2, }, {4, 5}, {6}
   C. {1, 2, 3}, {3, 4} ,{2,6}
   B. {1, 2}, {4}, {4, 5, 6}
   D. {1, 2}, {2}, {4,5}, {5, 6}

Solution

A die thrown once, s={1, 2,3, 4,5 ,6}, each number exists oce

Answer A

4. A bag contains 2 red, 3 yellow and 5 green balls.one ball is drawn random. What is the probability that the drawn ball is not yellow?

A. $\frac{1}{5}$ B. $\frac{3}{10}$ C. $\frac{7}{10}$ D. $\frac{4}{5}$

Solution

n(R)= 2, n(y)=3, n(G)=5,

Sample space, n(S)= 2+3+5

=10

P (not yellow)= 1-p(y)

$= 1-\frac{3}{10}$

$= \frac{7}{10}$

Or

P (not yellow) = p(R)+p(G)

$= \frac{2}{10} + \frac{5}{10}$

$= \frac{7}{10}$

Answer C

5. Math test sores distribution of 20 students is given as in Table 2 below?

Scores	1-5	6-10	11-15	16-20
Frequency	4	6	8	2

What is the standard deviation of the scores?

A. 4 B. 5 C. $\sqrt{20}$ D. $\sqrt{21}$

Solution

X	**F**	**x_c**	Fxc	xc-$\bar{x}$	$(xc-\bar{x})^2$	$f(x_{c-\bar{x}})^2$
1-5	4	3	12	-7	49	196
6-10	6	8	48	-2	4	24
11-15	8	13	104	3	9	72
16-20	2	18	36	8	64	128

$\sum f = 20$ $\sum fx_{c=200}$ $\sum f(x_c - \bar{x})^2 = 420$

$\bar{x}=\frac{200}{20}=10$ $variance = \frac{420}{20} = 21$

$s.d = \sqrt{21}$

Answer D

6. What is the coefficient of x^2 in the binomial expansion of $\left(2x + \frac{3}{x^3}\right)^8$?

A. 0 B. 6 C. 500 D. 576

Solution

$$\left(2x + \frac{3}{x^2}\right)^8 = c(8,0)(2x)^8 + c(8,1)(2x)^7\left(\frac{3}{x^3}\right) + c(8,2)(8,2)^6\left(\frac{3}{x^3}\right)^2$$

$$+c(8,3)(2x)^5\left(\frac{3}{x^3}\right)^3 + c(8,4)(2x)^4\left(\frac{3}{x^3}\right)^4$$

$$+ c(8,5)(2x)^3\left(\frac{3}{3^x}\right)^5 + c(3,6)(2x)^2\left(\frac{3}{x^2}\right)^6$$

$$+ c(8,7)(2x)\left(\frac{3}{x^3}\right)^7 + (8.8)\left(\frac{3}{x^3}\right)^8$$

No x^2 in the expression, coefficient of x^2 is zero,

Answer A

Grade 11 unit six – matrices and determinants

2009 E.C

1. If A= $(a_{ij})_3 \times_3$ is square matrix with A^{-1}== $\begin{pmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & 5 \end{pmatrix}$ then what is the cofactor of a_{23}?

A. -3/14 B. 2/7 C. 2/7 D. -3/7

<u>Solution</u>

$$Given\ A^{-1} = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & 5 \end{pmatrix}, Find\ Cofactor\ of\ a_{33}\ of\ A = (aij)_{3*3}$$

$$note - i)\ (A^{-1})^{-1} = A \quad ii)\ A^{-1} = \frac{1}{|A|} adjA \quad iii)\ A = \frac{1}{A^{-1}} adjA^{-1}$$

$$|A^{-1}| = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & 5 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ 4 & 5 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} + 0\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= 1(5-12) - 1(15-8) = 0(9-2)$$

$$= 1(-7) - 1(7)$$

$$= -14$$

$$Cofactors\ of\ A^{-1} = \begin{vmatrix} \begin{vmatrix} 1 & 3 \\ 4 & 5 \end{vmatrix} & -\left\|\begin{matrix} 1 & 3 \\ 0 & 5 \end{matrix}\right\| & \begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} \\ -\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} \\ \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} \end{vmatrix}$$

$$= \begin{vmatrix} (5-12) & -(5-0) & (4-0) \\ -(15-8) & (5-0) & -(4-0) \\ (0-2) & -(3-2) & (1-3) \end{vmatrix}$$

.

$$= \begin{vmatrix} -7 & -5 & 4 \\ -7 & 5 & -4 \\ 7 & -1 & -2 \end{vmatrix}$$

$$adj\ \ A^{-1} = transpose\ of\ cofactors\ of\ A^{-1} = \begin{vmatrix} -7 & -7 & 7 \\ -5 & 5 & -1 \\ 4 & -4 & -2 \end{vmatrix}$$

$$A = \frac{1}{|A^{-1}|} adjA^{-1} = \frac{1}{14} \begin{vmatrix} -7 & -7 & 7 \\ -5 & -5 & -1 \\ 4 & -4 & -2 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{5}{14} & -\frac{5}{14} & \frac{1}{14} \\ -\frac{2}{7} & \frac{2}{7} & \frac{1}{7} \end{vmatrix}$$

$$Cofactor\ of\ a_{23} = -\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{2}{7} & \frac{2}{7} \end{vmatrix} = -\left(\frac{1}{2} * \frac{2}{7} - \left(-\frac{2}{7} * \frac{1}{2}\right)\right)$$

$$= -\left(\frac{1}{7} + \frac{1}{7}\right) = -\left(\frac{2}{7}\right) = -\ \frac{2}{7}$$

$$Answer\ B$$

2. What are the value of λ and μ so that the system $\begin{cases} x+y+z=6 \\ x+2y+3z=10 \\ x+2y+\lambda z=\mu \end{cases}$ has infinite solution?

A. $\lambda \neq 3 \; and \; \mu \in \mathbb{R}$

B. $\lambda = 3 \; and \; \mu \neq 10$

C. $\lambda = 3 \; and \; \mu = 10$

D. $\lambda \in \mathbb{R} \; and \; \mu = 10$

Solution

$$\begin{cases} x+y+z=6 \\ x+2y+3z=10 \\ x+2y+\lambda z=\mu \end{cases}$$

$$coefficicient \; matrix$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{pmatrix} \quad \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 - R_1 \end{matrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{pmatrix} \quad R_3 = R_3 - R_2$$

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{pmatrix} \quad must \; be \; zero \; if \; it \; has \; infinite \; solution$$

$$\lambda - 3 = 0 \;, \quad \mu - 10 = 0$$

$$\lambda = 3 \qquad \mu = 10$$

Answer C

3. When $\begin{vmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{vmatrix} = a^3$ then what is the solution set of $\begin{vmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$

A. X=0, y=-2a, z=2a

B. X=1/a, y=-2a, z=2a

C. X=1/a, y=2/a, z=2/a

D. X=0, y=2/a, z=2/a

Solution

$$\begin{vmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{vmatrix} = a^3 \;\; \Rightarrow D \; = a^3$$

$$\begin{vmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} \quad = \quad \begin{cases} ax \;\; by \quad cz = 1 \\ ax - ay \;\; az = 0 \\ ax + ay - az = 0 \end{cases}$$

$$Dx \;\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & -a & a \\ 0 & a & -a \end{vmatrix} = 1 \begin{vmatrix} -a & a \\ a & -a \end{vmatrix} - 0 \begin{vmatrix} b & c \\ a & -a \end{vmatrix} + 0 \begin{vmatrix} b & c \\ -a & a \end{vmatrix}$$

$$= 1(a^2 - a^{2)} \;= 0$$

$$x = \frac{Dx}{D} = \frac{0}{a^3} = 0$$

$$Dy \;\Rightarrow \begin{vmatrix} a & 1 & c \\ a & 0 & a \\ a & 0 & -a \end{vmatrix} = a \begin{vmatrix} 0 & a \\ 0 & -a \end{vmatrix} - a \begin{vmatrix} 1 & c \\ 0 & -a \end{vmatrix} + a \begin{vmatrix} 1 & c \\ 0 & a \end{vmatrix}$$

$$= a(0) - a(-a) + a(a) = \; a^2 + a^2 \;\; = 2a^2$$

$$y = \frac{Dy}{D} = \frac{2a^2}{a^3} = \frac{2}{a}$$

$$Dz \;\Rightarrow \begin{vmatrix} a & b & 1 \\ a & -a & 0 \\ a & a & 0 \end{vmatrix} = a \begin{vmatrix} -a & 0 \\ a & 0 \end{vmatrix} - a \begin{vmatrix} b & 1 \\ a & 0 \end{vmatrix} + a \begin{vmatrix} b & 1 \\ -a & 0 \end{vmatrix}$$

$$= a\;(\,0) - a\;(-a) + a(a) = 2a^2$$

$$z = \frac{Dz}{D} = \frac{2a^2}{a^3} = \frac{2}{a}$$

Answer D

Grade 11 unit six – matrices and determinants

2010 E.C

1. A salesman sold items x_1,x_2 and x_3 with different rates of commissions as shown in the table

Months	Sales of unit			Total commission (In birr)
	X1	X2	X3	
February	90	100	20	800
March	130	50	40	900
April	60	100	30	850

What are the rate of commissions on items $X_{1,}$ X_2, X_3

A. 4, 2, and 11 B. 4, 11 and 12 C. 11, 2 and 4 D. 2, 4 and 11

Solution

$$\begin{cases} 90x_1 + 100x_2 + 20x_3 = 800 \\ 130x_1 + 50x_2 + 40x_3 = 900 \qquad \text{use Cramer's rule} \\ 60x_1 + 100x_2 + 30x_3 = 580 \end{cases}$$

$$D = \begin{vmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{vmatrix} = 90\begin{vmatrix} 50 & 40 \\ 100 & 30 \end{vmatrix} - 100\begin{vmatrix} 130 & 40 \\ 60 & 30 \end{vmatrix} + 20\begin{vmatrix} 130 & 50 \\ 60 & 100 \end{vmatrix}$$

$$= 9091500 - 4000) - 100(3900 - 240) + 20(13000 - 30000 = -175000$$

$$D_x = \begin{vmatrix} 800 & 100 & 20 \\ 900 & 50 & 40 \\ 850 & 100 & 30 \end{vmatrix} = 800\begin{vmatrix} 50 & 40 \\ 100 & 30 \end{vmatrix} - 100\begin{vmatrix} 900 & 40 \\ 850 & 30 \end{vmatrix} + 20\begin{vmatrix} 900 & 50 \\ 850 & 100 \end{vmatrix}$$

$$= 800(1500 - 4000) - 100(27000 - 34000) + 20(90000 - 42500) = -350000$$

$$D_y = \begin{vmatrix} 90 & 800 & 20 \\ 130 & 900 & 40 \\ 60 & 850 & 30 \end{vmatrix} = 90\begin{vmatrix} 900 & 40 \\ 850 & 30 \end{vmatrix} - 800\begin{vmatrix} 130 & 40 \\ 60 & 30 \end{vmatrix} + 20\begin{vmatrix} 130 & 900 \\ 60 & 850 \end{vmatrix}$$

$$= 90(27000 - 38000) - 80093900 - 2400) + 20(1125 - 5600) = -700000$$

$$D_z = \begin{vmatrix} 90 & 100 & 800 \\ 130 & 50 & 900 \\ 60 & 100 & 850 \end{vmatrix} = 90\begin{vmatrix} 50 & 900 \\ 100 & 850 \end{vmatrix} - 100\begin{vmatrix} 130 & 900 \\ 60 & 850 \end{vmatrix} + 800\begin{vmatrix} 130 & 50 \\ 60 & 100 \end{vmatrix}$$

$$= 90(42500 - 90000) - 100(3900 - 2400) + 800(1125 - 56000) = -1925000$$

$$x = \frac{D_x}{D} = \frac{-35000}{-175000} = 2$$

$$y = \frac{D_y}{D} = \frac{-700000}{-175000} = 4$$

$$z = \frac{D_z}{D} = \frac{-1925000}{-175000} = 11$$

Answer D

2. If A is a square matrix of order 3 and det (A) =5, then what is the value of det (A.adj (A))?

A. 125 B. 5 C. 25 D. 3

Solution

$$|AB| = |A||B|$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A \quad \Rightarrow \quad \text{adj } A = |A|A^{-1},\ |\text{adj } A| = |A||A^{-1}|$$

$$|kA| = K^n|A| \quad , |A| = \frac{1}{|A^{-1}|}$$

$$|A\text{adj}A| = |A||\text{adj}A|$$

$$= |A||A||A^{-1}|$$

$$= 5 * |5A^{-1}| = 5 * 5^3|A^{-1}|$$

$$= 5 * 5^3 * \frac{1}{|A|} = 5 * 5^3 * \frac{1}{5} = 125,$$

Answer A

3. Let A be 3× 3 matrix and |A|= - 2. Then what is the value of |adj|(A)?

A. – 8 B. – 2 C. -1/2 D. 4

Solution

$$|A| = -2, |adA| = ?$$

$$A^{-1} = \frac{1}{|A|}\text{ adjA}$$

$$|A^{-1}| = \frac{|\text{adjA}|}{|A|}$$

$$|\text{adjA}| = |A||A^{-1}|$$

$$= -8 * \frac{1}{-2} = 4$$

$$or\ |\text{adjA}| = |A||A^{-1}|$$

$$= |-2A^{-1}|$$

$$= -2^3\frac{1}{|A|}$$

$$= \frac{-8}{-2} = 4$$

Answer D

4. If the sum of the first three consequence terms of an arithmetic progression $\{A_n\}$, with $A_n>0$ for all n, is 9 and the sum of their squares is 35, then what is the sum s_n of the first n terms?

A. n^2+1 B. n^2 C. $-2n^2+1$ D. n^2-1

Solution

let the three terms be a_1, a_2, a_3

$a_1 + a_2 + a_3 = 9,$

$a_1 + (a_1 + d) + (a_2 + d) = 9$

$a_1 + (a_1 + d) + (a_1 + d + d) = 9$

$a_1 + a_1 + d + a_1 + 2d = 9$

$3a_1 + 3d = 9$

$a1 + d = 3$

$a_1 = 3 - d \ldots\ldots\ldots\ldots\ldots\ldots\ldots i$

$\Rightarrow sn = \frac{n}{2}(2a_1 + (n-1)d)$

$= \frac{n}{2}(2 + 2n - 2)$

$= \frac{n}{2}(2 + (n-1)2)$

$= \frac{n}{2}(2n) = n^2,$

${a_1}^2 + {a_2}^2 + {a_3}^2 = 35$

${a_1}^2 + (a_1 + d)^2 + (a_1 + 2d)^2 = 35\ldots\ldots\ldots ii$

$(3 - d)^2 + (3 - d + d)^2 + (3 - d + 2d)^2 = 35$

$9 - 6d + d^2 + 9 + 9 + 6b + d^2 = 35$

$9 + 9 + 9 + 2d^2 = 35$

$2d^2 = 35 - 27$

$d^2 = 4,$

$d = 2$

from (i), $a_1 = 3 - 2 = 3 - 2 = 1$

Answer B

5. Let A be a 3× 3 invertible matrix and B be any 3× 3 matrix. If |A|= a and |B|= b, then which of the following is NOT true?

A. $|A^T A| = a^2$

B. $|A^{-1} B| = ab$

C. $|KA| = K^3|A|$, for any K∈ ℝ

D. If b=0, then B is not invertible

Solution

$|A| = a$

$[B] = b$

A) $|A^TA| = |A^T||A| = axa = a^2$ True, $|A| = |A^T|$

B) $|A^{-1}B| = \frac{1}{|A|}|B| = \frac{b}{a}$ False, $|A| = \left|\frac{1}{A^{-1}}\right|$

C) $|KA| = k^3|A|$ True

D) if $|B| = 0$, then B is not inverible, True

Answer B

Grade 11 unit six – matrices and determinants

2011 E.C

1. If $2\begin{pmatrix}2x & X\\ -5 & -3\end{pmatrix}^{-1} = \begin{pmatrix}3 & 2\\ -5 & -4\end{pmatrix}$, then what is the value of x?

   A. -2 B. ½ C. 2 D. -1/2

Solution

$$2\begin{pmatrix}2x & X\\ -5 & -3\end{pmatrix}^{-1} = \begin{pmatrix}3 & 2\\ -5 & -4\end{pmatrix},$$

$$\textit{multiply both sides of the equation by } \begin{pmatrix}2x & x\\ -5 & -3\end{pmatrix}$$

$$2\begin{pmatrix}2x & x\\ -5 & -3\end{pmatrix}^{-1}\begin{pmatrix}2x & x\\ -5 & -3\end{pmatrix} = \begin{pmatrix}3 & 2\\ -5 & -4\end{pmatrix}\begin{pmatrix}2x & x\\ -5 & -3\end{pmatrix} \ because\ AA^{-1} = I$$

$$2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}6x-10 & 3x-6\\ -10x+20 & -5x+12\end{pmatrix}$$

$$\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix} = \begin{pmatrix}6x-10 & 3x-6\\ -10x+20 & -5x+12\end{pmatrix}$$

$$0 = -10X + 20$$

$$10x = 20, x = 2$$

Answer C

2. Consider the following system of equations: $\begin{cases} x - 2y + z = 1\\ -x + y + z = 3\\ 3x - 5y + z = k\end{cases}$, How much must be the value of k so that the system has a solution?

   A. 7 B. – 1 C. 0 D. 1

Solution

$$Coefficent\ matrix$$

$$\begin{pmatrix}1 & -2 & 1\\ -1 & 1 & 1\\ 3 & -5 & 1\end{pmatrix}\begin{pmatrix}1\\ 3\\ k\end{pmatrix} \quad \begin{matrix}R_2 = R_2 + R_1\\ R_3 = R_3 - 3R_1\end{matrix}$$

$$\begin{pmatrix}1 & -2 & 1\\ 0 & -1 & 2\\ 0 & 1 & -2\end{pmatrix}\begin{pmatrix}1\\ 4\\ k-3\end{pmatrix} \quad R_3 = R_3 + R_2$$

$$\begin{pmatrix}1 & -2 & 1\\ 0 & -1 & 2\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}1\\ 4\\ k+1\end{pmatrix}$$

$$The\ system\ has\ solution \Rightarrow\ k + 1 = 0, K = -1$$

Answer B

3. If $\begin{vmatrix}-1 & 1 & 2\\ 3 & 2 & x\\ 2 & 4 & 1\end{vmatrix} = \begin{vmatrix}-x & 3 & 2\\ 2 & 2 & 3\\ 1 & -1 & -2\end{vmatrix}$, then what is the value of x?

   A. 2/3 B. -2/5 C. 2/5 D. -2/3

Solution

$$\begin{vmatrix}-1 & 1 & 2\\ 3 & 2 & x\\ 2 & 4 & 1\end{vmatrix} = \begin{vmatrix}-x & 3 & 2\\ 2 & 2 & 3\\ 1 & -1 & -2\end{vmatrix}$$

$$-1\begin{bmatrix}2 & x\\ 4 & 1\end{bmatrix} - 1\begin{bmatrix}3 & x\\ 2 & 1\end{bmatrix} + 2\begin{bmatrix}3 & 2\\ 2 & 4\end{bmatrix} = -x\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix} - 3\begin{bmatrix}2 & 3\\ 1 & -2\end{bmatrix} + 2\begin{bmatrix}2 & 2\\ 1 & -1\end{bmatrix}$$

$$-(2-4x) - (3-2x) + 2(12-4) = -x(-4+3) - 3(-4-3) + 2(-2-2)$$

$$-2 + 4x - 3 + 2x + 24 - 8 = 4x - 3x + 12 + 9 - 4 - 4$$

$$6x + 11 = x + 13$$

$$5x = 2$$

$$x = \frac{2}{5}$$

Answer C

Grade 11 unit six – matrices and determinants

2012 E.C

1. If the inverse of the matrix $\begin{pmatrix} x & 2x \\ -x & x \end{pmatrix}$ is $\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$, then which one of the following is the value of x?

   A. -1 B. 1 C. -1/3 D. 1/4

Solution

$$\begin{pmatrix} x & 2x \\ -x & x \end{pmatrix}\begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -x + 2x & & -2x & +2x \\ x + x & . & 2x & +x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x & 0 \\ 2x & 3x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x = 1, \quad 2x = 0, \quad 3x = 1$$

$$x = -1, x = 0, x = \frac{1}{3}$$

Answer B

2. What is the set of values of k so that the system $\begin{cases} x - y + z = 3 \\ 2x + y - z = 4 \\ kx - 2y - z = 5 \end{cases}$ has a solution?

   A. $\mathbb{R}$ B. $\mathbb{R} \setminus \{1\}$ C. $\mathbb{R} \setminus \{-1\}$ D. $\mathbb{R} \setminus \{-1, 1\}$

Solution

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ k & -2 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$

$$\text{If A has solution } |A| \neq 0$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ k & -2 & -1 \end{vmatrix} = \left(1\begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ k & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ k & -2 \end{vmatrix} \neq 0\right)$$

$$= (-1 - 2) + (-2 + k) + (-4 - k)$$

$$= -3 - 2 + k - 4 - k$$

$$= -9 \neq 0$$

$$\text{k is an element of } \Re$$

Answer A

3. Let A=$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$. Be two matrices, then which one of the following is equal to 3A+2B?

   A. $\begin{pmatrix} 7 & 4 \\ 7 & 18 \end{pmatrix}$ B. $\begin{pmatrix} 7 & 1 \\ 7 & 7 \end{pmatrix}$ C. $\begin{pmatrix} 3 & 1 \\ 2 & 7 \end{pmatrix}$ D. $\begin{pmatrix} 3 & 4 \\ 2 & 18 \end{pmatrix}$

Solution

$$3A + 2B$$

$$= 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 \\ -2 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 4 \\ 7 & 18 \end{pmatrix}$$

Answer A

4. Which one of the following is a row reduced echelon form of the matrix $\begin{pmatrix}1 & 1 & 1\\ 2 & 1 & 3\\ 4 & -1 & 4\end{pmatrix}$?

A. $\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$ B. $\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 3\\ 0 & 0 & 1\end{pmatrix}$ C. $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}$ D. $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

<u>Solution</u>

$$\begin{pmatrix}1 & 1 & 1\\ 2 & 1 & 3\\ 4 & -1 & 4\end{pmatrix} \qquad \begin{matrix}R_2 = R_{2-}2R_1\\ R_3 = R_3 - 4R_1\end{matrix}$$

.

$$\begin{pmatrix}1 & 1 & 1\\ 0 & -1 & 1\\ 0 & -5 & 0\end{pmatrix} \qquad R_3 = -\tfrac{1}{5}R_3$$

$$\begin{pmatrix}1 & 1 & 1\\ 0 & -1 & 1\\ 0 & 1 & 0\end{pmatrix} \qquad \text{interchange R1 and R2}$$

$$\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ 0 & -1 & 1\end{pmatrix} \qquad R_3 = R_2 + R_3$$

$$\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad R_1 = R_1 - R_2$$

$$\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad R_1 = R_1 - R_2$$

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Answer D

5. For what value of k does the system $\begin{cases}2x - y + 4z = 6\\ kx - y - z = 2\\ x + y + 2z = 8\end{cases}$ has unique solution

A. k≠ $\frac{-1}{2}$ B. k≠ $-\frac{1}{3}$, k $\neq \frac{1}{2}$ C. k≠ $-\frac{1}{6}$ D. k≠ $\frac{1}{2}$

<u>Solution</u>

$$\begin{cases}2x - y + 4z = 6\\ kx - y - z = 2\\ x + y + 2z = 8\end{cases}$$

A matrix have solution if the determinant of the coefficient matrix is different from zero

$$\begin{vmatrix}2 & -1 & 4\\ k & -1 & -1\\ 1 & 1 & 2\end{vmatrix} = 2\begin{vmatrix}-1 & -1\\ 1 & 2\end{vmatrix} + 1\begin{vmatrix}k & -1\\ 1 & 2\end{vmatrix} + 4\begin{vmatrix}4 & -1\\ 1 & 1\end{vmatrix}$$

=2(-2+1)+(2k+1)+4(k+1) =-2+2k+1+4k+4=6k+3

6k+3≠0

6k≠ -3, k≠ $-\frac{1}{2}$

Answer A

6. Let A=$\begin{pmatrix}2 & 3\\ 2 & -1\end{pmatrix}$ and $\begin{pmatrix}2 & 5 & -4\\ 3 & 2 & 5\end{pmatrix}$. Then, AB= --------

A. $\begin{pmatrix}8 & 13 & -18\\ -1 & 8 & -2\end{pmatrix}$ B. $\begin{pmatrix}13 & 1\\ 16 & 8\\ 10 & 14\end{pmatrix}$ C. $\begin{pmatrix}8 & -1\\ 13 & 8\\ 18 & -2\end{pmatrix}$ D. $\begin{pmatrix}13 & 16 & 10\\ 1 & 8 & -14\end{pmatrix}$

<u>Solution</u>

$$AB = \begin{pmatrix}2 & 3\\ 2 & -1\end{pmatrix}\begin{pmatrix}2 & 5 & -4\\ 3 & 2 & 6\end{pmatrix} = \begin{pmatrix}4+9 & 10+6 & -8+18\\ 4-3 & 10-2 & -8-6\end{pmatrix} = \begin{pmatrix}13 & 16 & 10\\ 1 & 8 & -14\end{pmatrix}$$

Answer D

Grade 11 unit six – matrices and determinants

2013 E.C

1. What is the determinant of the matrix $\begin{bmatrix}1 & -7\\6 & 5\end{bmatrix}$?

   A. -7 B. -37 C. 37 D. 47

Solution

$$\begin{vmatrix}1 & -7\\6 & 5\end{vmatrix} = 5-(-42) \; = 5+42 = 47$$

Answer D

2. Which of the following is NOT true about the determinate of a given square matrix A?
   A. If A has identical rows, then its determinate is zero.
   B. If A contains a row of zeros, then its determinate is zero.
   C. Interchanging two rows of A gives the same determinant.
   D. The determinant of A is equal to determinant of its transpose.

Solution

A) $\begin{pmatrix}1 & 2\\1 & 2\end{pmatrix}$, |2 – 2|=0 true

B) A= $\begin{pmatrix}0 & 0 & 0\\3 & 2 & 1\\4 & 3 & 4\end{pmatrix}$, D=0 true

C) A= $\begin{pmatrix}4 & 2 & 2\\1 & -3 & 2\\0 & 5 & 0\end{pmatrix}$, B= $\begin{pmatrix}1 & -3 & 2\\4 & 2 & 2\\0 & 5 & 0\end{pmatrix}$

|A|=4(0-10)-2(0-0)+2(5-0) =4(-10)+0+2(5)=-40+10=-30

|B|=1(0-10)+3(0-0)+2(20-0) =-10+0+40=30, false

D)$|A|=|A^T|$ True

Answer C

3. Which of the following is the inverse of the matrix $\begin{bmatrix}2 & 3\\3 & 4\end{bmatrix}$

   A. $\begin{bmatrix}-2 & 3\\3 & -4\end{bmatrix}$ B. $\begin{bmatrix}-4 & 3\\3 & -2\end{bmatrix}$ C. $\begin{bmatrix}-4 & -3\\-3 & -2\end{bmatrix}$ D. $\begin{bmatrix}4 & -3\\3 & 2\end{bmatrix}$

Solution

A=$\begin{pmatrix}2 & 3\\3 & 4\end{pmatrix}$ let A^{-1} =$\begin{pmatrix}a & c\\b & d\end{pmatrix}$

$AA^{-1} = I$

$$\begin{pmatrix}2 & 3\\3 & 4\end{pmatrix}\begin{pmatrix}a & b\\c & d\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$\begin{pmatrix}2a+3c & 2b+3d\\31+4c & 3b+4d\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$A^{-1}=\begin{pmatrix}a & b\\c & d\end{pmatrix} = \begin{pmatrix}-4 & 3\\3 & -2\end{pmatrix}$$

$$\begin{matrix}3\\2\end{matrix}\begin{Bmatrix}2a+3c=1\\3a+4c=0\end{Bmatrix}$$

____

$6a+9c=3$

$-6a-8c=0$

____

C=3

$21 = 1-3c$

$2a = 1-9$

$2a = -8$

a=4

- A^{-1} = $\begin{pmatrix}a & c\\b & d\end{pmatrix}$ =$\begin{pmatrix}-4 & 3\\3 & -2\end{pmatrix}$

$$\begin{matrix}3\\-2\end{matrix}\begin{Bmatrix}2b+3d=0\\3b+4d=1\end{Bmatrix}$$

____

$6b+96=0$

$-6b-8d=-2$

____

d=-2

$2b=-3d$

$2b=-3(2)$

$2b=6$

b=3

Answer B

4. Consider the system of linear equation $\begin{cases} 3x - ay = 1 \\ 6x + 4y = 5 \end{cases}$, then which of the following is true?
   A. If a = -2, then the system has a unique solution.
   B. If a> -2, then the system has a unique solution.
   C. If a>2, then the system has infinitely many solutions
   D. If a=2, then the system has no solution.

Solution

A. $\begin{Bmatrix} 3x + 2x = 1 \\ 6x + 4y = 5 \end{Bmatrix}$	B. $\begin{Bmatrix} 3x = 1 \\ 6x + 4y = 5 \end{Bmatrix}$	$\begin{Bmatrix} 3x - 4y = 1 \\ 6x + 4y = 5 \end{Bmatrix}$	$\begin{Bmatrix} 3x - 2y = 1 \\ 6x + 4y = 5 \end{Bmatrix}$
a= -2	a=0	a=4	a=2
0=3	x=1/3, y=3/4	x=2/3, y=1/4	x=7/12, y=9/8
No solution	one solution	one solution	one solution

Answer B

5. Which of the following is the product AB of the following two matrices $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$?
   A. $\begin{bmatrix} -11 & 10 \\ 11 & 5 \end{bmatrix}$ B. $\begin{bmatrix} -11 & -10 \\ 11 & 5 \end{bmatrix}$ C. $\begin{bmatrix} -11 & 10 \\ 11 & -5 \end{bmatrix}$ D. $\begin{bmatrix} 11 & -10 \\ 11 & 5 \end{bmatrix}$

Solution

$$AB= \begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -1 & 5 \\ 3 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 2+9 & -10+0 \\ -1+12 & 5+0 \end{pmatrix}$$
$$= \begin{pmatrix} 11 & -10 \\ 11 & 5 \end{pmatrix}$$

Answer D

Grade 11 unit seven – the set of complex numbers

2009 E.C

1. If $z_1=\frac{2-i}{1+i}$ and $z_2=\frac{1+i}{1-i}$, then what is the value of z_1+2z_2?
   A. 1+i B. $\frac{1+i}{2}$ C. $\frac{1-i}{2}$ D. 1-i

Solution

$$z_1 = \frac{2-i}{1+i} = \frac{2-i}{1+i} * \frac{1-i}{1-i} = \frac{2-2i-i-1}{1^2+1^2} = \frac{1-3i}{2}$$
$$z_2 = \frac{1+i}{1-i} = \frac{1+i}{1-i} * \frac{1+i}{1+i} = \frac{1+i+i-1}{1^2+1^2} = \frac{2i}{2} = i$$
$$z_1 + 2z_2 = \frac{1-3i}{2} + 2(i) = \frac{1-3i+4i}{2} = \frac{1+i}{2}$$

Answer B

2. Which one of the following is the conjugate of Z= $|3+\ 4i| - \frac{25\,i}{3+4i}$?
   A. 5+3i B. 1+3i C. 3-5i D. 1-3i

Solution

$$z = |3+\ 4i| - \frac{25\,i}{3+4i} = \sqrt{3^2+\ 4^2} - \frac{25\,i}{3+4i} * \frac{3-4i}{3-4i} = \sqrt{9+16} - \frac{75i+100}{3^2+(-4)^2}$$
$$= \sqrt{25} - \frac{75i+100}{9+16} = 5 - \frac{75i+100}{25} = 5-(3i+4) = 5\ -3i-4 = 1-3i$$
$$conjugate\ of\ z = 1+3i,$$

Answer B

3. If $z = \sqrt{2}\cos\frac{\pi}{12} + i\sqrt{2}\sin\frac{\pi}{12}$, then what is the value of $(z)^3$?

A. 2+2i B. $\sqrt{2}+\sqrt{2}i$ C. $2\sqrt{2}+2\sqrt{2}i$ D. $3\sqrt{2}+3\sqrt{2}i$

<u>Solution</u>

$$z = \sqrt{2}\cos\frac{\pi}{12} + i\sqrt{2}\sin\frac{\pi}{12} \quad , \qquad z^n = r^n\,(\cos n\emptyset + i\ sinn\emptyset)$$

$$(z)^3 = \left(\sqrt{2}\right)^3\cos\frac{3\pi}{12} + i\left(\sqrt{2}\right)^3\sin\frac{4\pi}{12}$$

$$= 2\sqrt{2}\cos\frac{\pi}{4} + i2\sqrt{2}\sin\frac{\pi}{4} = 2\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 2 + 2i$$

Answer A

Grade 11 unit seven – the set of complex numbers

2010 E.C

1. If z= -3+4i and w= 1+2i, then what is the value of $\frac{2z}{w}+\overline{w}$?

A. 2+3i B. 3-2i C. 3+5i D. 3+2i

<u>Solution</u>

$$z = -3+4i \ , w = 1+2i\, , \frac{2z}{w}+\overline{w}$$

$$\frac{2z}{w}+\overline{w} = \frac{2(-3+4i)}{1+2i} + \overline{(1+2i)}$$

$$= \frac{-6+8i}{1+2i} + 1 - 2i = \frac{-6+8i}{1+2i} * \frac{1-2i}{1-2i} + 1 - 2i$$

$$= \frac{-6+12i+8i+16}{1^2+2^2} + 1 - 2i$$

$$= \frac{10+20i}{5} + 1 - 2i = 2 + 4i + 1 - 2i = 3 + 2i$$

Answer D

2. If z= (1+$\sqrt{3}$i) (1+i), then which one of the following is the polar represents of z?

A. 4(cos 105^0+ isin105^0) C. 4(cos 75^0 + isin75^0)

B. $2\sqrt{2}$ (cos 15^0+ isin 15^0) D. $2\sqrt{2}$ (cos105^0+isin105^0)

<u>Solution</u>

$$z = (1+\sqrt{3}i)\ (1+i)$$

$$= 1 + i + \sqrt{3}\,i - \sqrt{3}$$

$$= \left(1-\sqrt{3}\right) + (1+\sqrt{3})\ i$$

$$r = \sqrt{x^2+y^2} = \sqrt{\left(1-\sqrt{3}\right)^2 + \left(1+\sqrt{3}\right)^2}$$

$$= \sqrt{1-2\sqrt{3}+3+1+2\sqrt{3}+3} = \sqrt{8} = 2\sqrt{2}$$

$$\tan(\theta+B) = \frac{\tan\theta + \tan B}{1-\tan\theta\tan B}$$

$$\tan(105)^0 = \tan(60^0+45)^0$$

$$= \frac{\tan 60^0 + \tan 45^0}{1-\tan 60^0\tan 45^0} = \frac{\sqrt{3}+1}{1-\sqrt{3}}$$

$$\tan\theta = \frac{y}{x} = \frac{1+\sqrt{3}}{1-\sqrt{3}}\ , \theta = \tan -1\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right) = 105^0$$

$$z = r(\cos\theta + i\sin\theta)\ ,$$

$$= 2\sqrt{2}(\cos 105^0 + i\sin 105^0)\ ,$$

Answer D

3. Let $Z = \left(\frac{1-i}{1+i}\right)^{18}$ Then, what is the value of z?

A. i B. – i C. – 1 D. 1 – i

Solution

$$Z = \left(\frac{1-i}{1+i}\right)^{18}, \frac{1-i}{1+i} = \frac{1-i}{1+i} x \frac{1-i}{1-i} = \frac{1-i-1}{1^2+(-1)^2} = \frac{-2i}{2} = -i$$

$$= (-i)^{18}$$

$$= (-1)^{18}i^{18} = i^{18} = i^{16+2} = i^{16}\ i^2 = (i^4)^4 i^2 = 1xi^2 = -1$$

Answer C

Grade 11 unit seven – the set of complex numbers

2011 E.C

1. What is the polar form of $\frac{7-i}{3-4i}$?

A. $\sqrt{2}(\cos\frac{\pi}{2} + i\ \sin\frac{\pi}{2})$ B. $\sqrt{2}(cos\frac{\pi}{4} + i\ sin\frac{\pi}{4})$ C. $2(\cos\frac{\pi}{2} + i\ sin\frac{\pi}{2}$ D. $2(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

solution

$$Z = \frac{7-i}{3-4i} = \frac{7-i}{34i} * \frac{3+4i}{3+4i}$$

$$= \frac{21+28i-3i+4}{3^2+4^2}$$

$$= \frac{25+25i}{25}$$

$$Z = 1+i \quad , \quad r^2 = x^2 + y^2$$

$$r = \sqrt{1^2+1^2}$$

$$r = \sqrt{2}$$

$$\tan\emptyset = \frac{y}{x} = \frac{1}{1} = 1$$

$$\emptyset = tan^{-1}(1) = 45^0 = \frac{\pi}{4}$$

$$Z = r(cos\emptyset + sin\emptyset) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Answer B

2. Let z be a complex number and w=3+4i. If, $\frac{z^2+1}{z+i} = |w|z - \frac{\overline{w}}{i}$ then what is the value of z?

A. -4-2i B. 4-2i C. -i-1 D. -1+i

Solution

$$w = 3+4i$$

$$\frac{z^2+1}{z+i} = |w|z - \frac{\overline{w}}{i}$$

$$\frac{z^2+1}{z+i} * \frac{z-i}{z-i} = \left(\sqrt{3^2+4^2}\right)z - \frac{(3-4i)}{i}$$

$$\frac{(z^2+1)(z-i)}{(z^2+1)} = 5z - \frac{(3-4i)}{i} * \frac{-i}{-i}$$

$$z - i = 5z - (-3i-4)$$

$$z - i = 5z + 3i + 4$$

$$5z - z = -3i - i - 4$$

$$4z = -4i - 4$$

$$\frac{4z}{4} = \frac{4(-i-1)}{4}$$

$$z = -i - 1$$

Answer C

3. which one of the following is the multiplicative inverse of z=$\frac{3+4i}{4-5i}$?

   A. $\frac{8}{25}-\frac{31}{25}i$ B. $\frac{-8}{25}+\frac{31}{25}i$ C. $\frac{8}{25}+\frac{31}{25}i$ D. $\frac{-8}{25}-\frac{31}{25}i$

solution

$$z=\frac{3+4i}{4-5i}\quad ,multiplicative\ inverse\ of\ z\ is\ \frac{1}{z}\Rightarrow z*\frac{1}{z}=1$$

$$\Rightarrow\frac{1}{z}=\frac{1}{\frac{3+4i}{4-5i}}=\frac{4-5i}{3+4i}=\frac{4-5i}{3+4i}*\frac{3-4i}{3-4i}$$

$$=\frac{12-16i-15i-20}{3^2+4^2}=\frac{-8-31i}{25}=\frac{-8}{25}-\frac{-31}{25}i$$

Answer D

Grade 11 unit seven – the set of complex numbers

2012 E.C

1. Let z_1= 2+i, z_2= 3+i and $z_{3=}$ 1-i. what are the real and Imaginary parts of $z_1z_2z_3$ respectively?

   A. 6 and 3 B. -6 and -3 C. -1 and 1 D. 10 and 0

Solution

$$Z_1=2+i,Z_2=3+i,Z_3=1-i$$

$$Z_{1.}Z_2=(2+i)(3+i)=6+2i+3i-1=5+5i$$

$$Z_1Z_2Z_3=(5+5i)(1-i)=5-5i+5i+5=10=10+0i$$

10= real part

0= imaginary nary part

Answer D

2. What are the values of a and b if $\frac{a+2i}{3i}=\frac{b+i}{4}$?

   A. a=3/4, b=-8/3 B. a=3/4, b=8/3 C. a=-3/4, b=8/3 D. a=-3/4, b=-8/3

Solution

$$\frac{a+2i}{3i}=\frac{b+i}{4}$$

$$4a+8i=3bi-3$$

$$4a=-3,8=3b$$

$$a=\frac{-3}{4}\quad b=\frac{8}{3}$$

Answer C

3. If z=-1+i and w=1-3i, what is the simplified form of $\frac{5z}{w}+|z|^2\overline{w}$?

   A. 4i B. 5i C. 4+5i D. 5+4i

Solution

Z=-1+i ,$w=1-3i$

$$\frac{5z}{w}+|z|^2\,\overline{w}$$

$$\frac{5(-1+i)}{1-3i}+\left(\sqrt{(-1)^2+1^2}\right)^2\overline{1-3\imath}$$

$$=\frac{-5+5i}{1-3i}+\sqrt{2}^2\,\overline{(1-3i)}$$

$$=\frac{-5+5i}{1-3i}*\frac{1+3i}{1+3i}+2(1+3i)=\frac{5-15i+5i-15}{1^2+3^2}+2+6i$$

$$=\frac{-20-10i}{10}+2+6i$$

$$=-2-i+2+6i$$

$$=5i$$

Answer B

Grade 11 unit seven – the set of complex numbers

2013 E.C

1. The additive inverse of the complex number 3 - 4i is

A. -3 - 4i B. 3 + 4i C. -3 + 4i D. 3 - 4i

Solution

If z+(-z)=0, then –z is additive inverse of z

$$z = 3 - 4i,$$
$$-z = -3 + 4i$$

Answer C

2. What is the modulus of the complex number -6 + 8i?

A. 2 B. 5 C. 8 D. 10

Solution

$$z = -6 + 8i,$$
$$|z| = \sqrt{(-6)^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10,$$

Answer D

3. What are the modulus and argument of the complex number 2 + 2i respectively?

A. $2 \text{ and } \frac{\pi}{4}$ B. $2\sqrt{2} \text{ and } \frac{\pi}{4}$ C. $4 \text{ and } \frac{\pi}{4}$ D. $2\sqrt{2} \text{ and } -\frac{\pi}{4}$

Solution

$$z = 2+2i$$
$$|z| = \sqrt{2^2 + 2^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2},$$
$$\Theta = \tan^{-1}\left(\frac{Y}{X}\right)$$
$$= \tan^{-1}\left(\frac{2}{2}\right)$$
$$= \tan^{-1}(1)$$
$$= {}^{\pi}/_{4}$$

Answer B

Grade 11 unit eight – vectors and transformation of the plane

2009 E.C

1. Let PQ be a vector with initial point p=(1, 5) and terminal point Q=(4, 0). If V=xi +2j is parallel to PQ, then what is the value of x?

   A. -6/5 B. -2/5 C. -3 D. 3

<u>Solution</u>

$$p-(1,5)\quad ,Q-(4,0)$$
$$PQ=(4-1,0-5)=(3,-5)$$
$$v=xi+2j$$
$$PQ=3i-5j$$
$$if\ V\ parallel\ to\ PQ\ ,then\quad v\ .pQ=-|v||PQ|,\cos 180=-1$$
$$3x-2(5)=-1\sqrt{x^2+2^2}\ \sqrt{3^2+(-5)^2}$$
$$3x-10=-1\sqrt{(x^2+4)(9+25)}$$
$$3x-10=-\sqrt{(x^2+4)(34)}$$
$$(3x-10)^2=34x^2+136$$
$$9x^2-60x+100=34x^2+136$$
$$25x^2+60x+36=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-60\pm\sqrt{(60)^2-4(25)(36)}}{2(25)}$$
$$=\frac{-60\pm\sqrt{3600-3600}}{50}=-\frac{60}{50}$$
$$=-\frac{6}{5}$$

Answer A

2. A line given by the vector equation (x,y)= (-t,6+2t), t ∈ ℝ, is tangent to a circle at a point (1, 4). What is the radius of the circle if its center is on y-axis?

   A. $\sqrt{5}$ B. $\frac{\sqrt{5}}{2}$ C. $2\sqrt{5}$ D. $\sqrt{10}$

<u>*solution*</u>

$$(x,y)=(-t,6+2t)$$
$$=(0,6)+t(-1,2)$$
$$center\ is\ on\ y\ axis\ -(0,K)$$
$$Direction\ vector\ of\ l\ is\ U(-1,2)$$
$$\text{points }(1,4),(0,K),v=(0-1,K-4)$$
$$v\ and\ U(-1,2)are\ perpendicular\ ,U.V=0$$
$$\Rightarrow(0-1,K-4)\ .\ (-1,2)=0$$
$$1+2(k-4)=0$$
$$1+2k-8=0,\qquad (1,4)\qquad r\qquad (0,7/2)$$
$$k=\frac{7}{2},r=\sqrt{(1-0)^2+\left(4-\frac{7}{2}\right)^2}=\frac{\sqrt{5}}{2}$$

Answer B

3. What is the translation vector u(h, k) so that the equation $x^2+2y^2+6x-8y+15=0$ is transformed to an equation of the form $x^2+2y^2+d=0$, where d is constant?
   A. $u=(-3, 2)$ B. $u=(3, -2)$ C. $u=(-2, 3)$ D. $u= (2, -3)$

<u>*solution*</u>

$$x^2+2y^2+6x-8y+15=0$$
$$x^2+6x+2y^2-8y=-15$$
$$x^2+6x+9+2(y^2-4y+4-4)=-15+9$$
$$(x+3)^2+2(y^2-4y+4)-8=-15+9$$
$$(x+3)^2+2(y-2)^2=-15+9+8$$
$$(x+3)^2+2(y-2)^2=2$$
$$\frac{(x+3)^2}{2}+\frac{(y-2)^2}{1}=1$$
$$x^2+2y^2+d=0$$
$$Center\ (-3,2) translated\ in\ to\ center\ (0,0)$$
$$translation\ vector, (3,-2)$$

Answer B

Grade 11 unit eight – vectors and transformation of the plane

2010 E.C

1. Let P=(1, α, α) and Q= (α − 1, 1, 1) be two points in space the distance between P and Q is 3. Then what is the value of(s) α of?
   A. $\alpha=1, \alpha=-9$ B. $\alpha=3, \alpha-1/3$ C. $\alpha=-1, \alpha=9$ D. $\alpha=-3, \alpha=1/3$

<u>Solution</u>

$$p=(1,\propto,\propto), Q=(\propto-1,1,1) \qquad \overline{pQ}=3$$
$$\sqrt{(\propto-1-1)^2+(1-\propto)^2+(1-\propto)^2}=3$$
$$(\propto-2)^2+(1-\propto)^2+(1-\propto)^2=9$$
$$\propto^{2-}4\propto+4+1-2\propto+\propto^2+1-2\propto+\propto^2=9$$
$$3\propto^2-8\propto+6=9 \qquad \left.\begin{matrix} sum=b=-8 \\ pr=ac=-9 \end{matrix}\right\} \begin{matrix} -9 \\ 1 \end{matrix}$$
$$3\propto^2-8\propto-3=0$$
$$3\propto^2-9\propto+\propto-3=0$$
$$3\propto(\propto-3)+(\propto-3)=0$$
$$(3\propto+1)(\propto-3)=0$$
$$3\propto+1=0, \propto-3=0$$
$$3\propto=-1, \quad \propto=\frac{-1}{3} \text{ or } \propto-3=0, \quad \propto=3,$$
$$or\ \propto=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{8\pm\sqrt{(-8)^2-4(3)(-3)}}{2*3}$$
$$=\frac{8\pm\sqrt{64+36}}{6}$$
$$=\frac{8\pm\sqrt{100}}{6}$$
$$=\frac{8\pm10}{6}=\frac{8+10}{6}\ or\ \frac{8-10}{6}$$
$$=3\ or =\frac{-1}{3}$$

Answer B

2. What is the standard equation of the line passing through point (2, 3) and parallel to the line given by (x= 1+2λ , y= -2- λ,) λ ∈ ℝ

A. $\frac{y-2}{2}=\frac{x-3}{1}$ B. $\frac{y-3}{-1}=\frac{x-2}{2}$ C. $\frac{x-1}{-1}=\frac{y-2}{2}$ D. $\frac{x-1}{1}=\frac{y-3}{2}$

Solution

$$\begin{cases} x = 1+2\lambda \\ y=-2-\lambda' \end{cases} \quad x = 1+2\lambda, \quad y=-2-\lambda$$

$$\frac{x-1}{2}=\lambda, \qquad y=-2-\lambda$$

$$\frac{x-1}{2}=\lambda \qquad 2-y=\lambda$$

$$\frac{x-1}{2}= 2-y$$

$$x-1=4-2y$$

$$2y=-x+5$$

$y=\frac{-1}{2}x+\frac{5}{2}$, $m=\frac{-1}{2}$, parallel lines have the same slope

Equation of a line passing through (2,3) with slope -1/3 is

$$\frac{y-y1}{x-y1}=m, \frac{y-3}{x-2}=\frac{-1}{2}, \ 2y-6=-x+2, \ \frac{2y-6}{2}=\frac{-x+2}{2}$$

$$\frac{y-3}{1}=\frac{-x+2}{2} \text{ or } \frac{y-3}{-1}=\frac{x-2}{2},$$

Answer B

3. If the image of the line 2x-3y=7 under a translation is 2x-3y=0, which of the following is a translation vector of the translation line?

A. U= (2, -1) B. U= (1,-2) C. U= (-1, 2) D. U= (-2, 1)

Solution

$$2x-3y=7 \text{ translated into } 2x-3y=0$$

A) $\cup=(2,-1)=(h,k)$

$$T(2x-3y=7)=2(x-h)-3(y-k)=7,$$
$$2(x-2)-3(y+1)=7$$
$$2x-4-3y-3=7$$
$$2x-3y=14 \ \text{ False}$$

B) $\cup=(1,-2)$

$$T(2x-3y=7)$$
$$\Rightarrow 2(x-1)-3(y+2)=7$$
$$\Rightarrow 2x-2-3y-6=7$$
$$2x-3y=15 \ \text{ False}$$

C) $\cup=(-1,2)$

$$T(2x-3y=7)=2(x+1)-3(y+2)=7$$
$$2x+2-3y-6=7$$
$$2x-3y=11 \quad \text{False}$$

D) U=(-2,1)

$$T(2x-3y=7)$$
$$\Rightarrow 2(x+2)-3(y-1)=7$$
$$\Rightarrow 2x+4-3y+3=7$$
$$2x-3y=7-7$$
$$2x-3y=0 \qquad \text{True,}$$

Answer D

Grade 11 unit eight – vectors and transformation of the plane

2011 E.C

1. When the plane is rotated 45^0 about the point (1, -2), then what is the image of the point (2, 4)?

   A. $\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$ B. $(1-\frac{\sqrt{2}}{2}, -2+\frac{\sqrt{2}}{2})$ C. $(\frac{7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2})$ D. $(1-\frac{5\sqrt{2}}{2}, -2+\frac{7\sqrt{2}}{2})$

   solution

$$(x, y)\ ,\quad (x_0, y_0)$$
$$(2,4)\ ,\quad (1, -2)$$
$$\emptyset = \frac{\pi}{4}$$
$$x^l = x_0 + (x - x_0)cos\emptyset - (y - y_0)\sin\emptyset$$
$$= 1 + (2-1)cos45^0 - (4 - (-2))sin45^0$$
$$= 1 + \frac{\sqrt{2}}{2} - \frac{6\sqrt{2}}{2} = 1 - \frac{5\sqrt{2}}{2}$$
$$y^l = y_0 + (x - x_0)sin\emptyset - (y - y_0)\cos\emptyset$$
$$= -2 + (2-1)sin45^0 - \big(45 - (-2)\big)\cos 45^0$$
$$= -2 + \frac{\sqrt{2}}{2} + \frac{6\sqrt{2}}{2} = -2 + \frac{7\sqrt{2}}{2}$$

Answer D

2. What is the image of the circle $x^2+y^2-4x^2-6y-11=0$, when the origin is shifted to the point (1, 1) after translation of axes?

   A. $X^2+y^2+6x+8y-23=0$
   B. $X^2+y^2+4x+6y+3=0$
   C. $X^2+y^2-6x-8y+23=0$
   D. $X^2+y^2-4x-2y=0$

   <u>Solution</u>

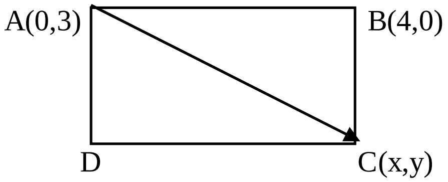
$$x^2 + y^2 - 4x - 6y + 1 = 0 \qquad \cup = (1,1)$$
$$(x-1)^2 + (y-1)^2 - 4(x-1) - 6(y-1) + 11 = 0$$
$$x^2 - 2x + 1 + y^2 - 2y + 1 + 4x + 4 - 6y + 6 + 11 = 0$$
$$x^2 - 2x + y^2 - 2y - 4x - 6y + 23 = 0$$
$$x^2 + y^2 - 6x - 8y + 23 = 0$$

Answer C

3. Consider a rectangle ABCD with base vertices A= (0, 3) and B= (4, 0) and the other vertices, C and D, in the first quadrant of the coordinate plane, if its height BC is half of the length of the base, then which of the following indicates the coordinates of the vertex C?

A. (11/ 2, 2) B. .(6, 3/2) C. (5/2, 2) D. (4/5, 2)

Solution



$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-0)^2 + (0-3)^2}$$
$$= \sqrt{4^2 + (-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25}$$
$$= 5$$

$$BC = \frac{1}{2}AB = \frac{5}{2} \qquad diagonal\ (AC)^2 = (AB)^2 + (BC)^2$$

$$AC = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}}$$

$$BC = \sqrt{(x-4)^2 + (y-0)^2}$$
$$\frac{5}{2} = \sqrt{(x-4)^2 + y^2}$$
$$\frac{25}{4} = (x-4)^2 + y^2$$
$$\frac{25}{4} = x^2 - 8x + 16 + y^2 \ldots\ldots\ldots\ldots\ldots\text{i}$$
$$AC = \sqrt{(x-0)^2 + (x-3)^2}$$
$$\frac{125}{4} = x^2 + (y-3)^2$$
$$\frac{125}{4} = x^2 + y^2 - 6y + 9 \ldots\ldots\ldots\ldots\ldots\text{ii}$$

Combine i and ii

$$-\begin{cases} \dfrac{25}{4} = x^2 - 8x + 16 + y^2 \\ \dfrac{125}{4} = x^2 - 6y + 9 + y^2 \end{cases}$$
$$\frac{100}{4} = 8x - 6y - 7$$
$$25 = 8x - 6y - 7$$
$$25 + 7 = 8x - 6y$$
$$32 = 8x - 6y$$
$$16 = 4x - 3y$$
$$\Rightarrow \left(\frac{11}{2}, 2)satisties\ the\ equetion\right)$$

Answer A

4. Let ℓ be the line given by the vector equation (x, y)= (1, 1) + t(√3, 1), t∈ ℝ. What is the equation of the image of L after being rotated 15^0 about (1, 1) and then translated by vector u= (-1, 1)?

   A. $\sqrt{3} - y = 2$　　B. x-y=2　　C. -x+y=2　　D. -x+$\sqrt{3y} = 1$

<u>Solution</u>

$$(x, y) = (1,1) + t(\sqrt{3}, 1)$$
$$(x, y) = (1,1) + t\left(\sqrt{3}, 1\right)$$
$$(x, y) = (1 + \sqrt{3}t)(1 + t)$$
$x = 1 + \sqrt{3}t$, y=1+t

$t = \frac{x-1}{\sqrt{3}}$, t=y-1

$$\frac{x - 1}{\sqrt{3}} = y - 1$$
$$\frac{x}{\sqrt{3}} - \frac{1}{\sqrt{3}} = y - 1$$
$$m = \frac{1}{\sqrt{3}}$$
$$rotated\ 15^0 about\ (1,1)\ translated\ by\ \cup = (-1,1)$$
$$\tan(x - y) = \frac{m_2 - m_1}{1 + m_1 m_2}$$
$$\tan 15^0 = \tan(45^0 - 30^0) = \tan 15^0 = \frac{\frac{1}{\sqrt{3}} - m_1}{1 + m_1\frac{1}{\sqrt{3}}} = \frac{\tan 45^0 - \tan 30^0}{1 + \tan 45^0 \tan 30^0}$$
$$\frac{\frac{1}{\sqrt{3}} - m_1}{1 + m_1\frac{1}{\sqrt{3}}} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$
$$\left(\frac{1 - \sqrt{3}m_1}{\sqrt{3} + m_1}\right) = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \quad ,$$

$m_1 = 1$, if (1,1) is translated to (-1,1), the equation is y=x

$$y - k = x - h$$
$$y - 1 = x + 1$$
$$y = x + 2$$
$$y - x = 2$$

Answer C

Grade 11 unit eight – vectors and transformation of the plane

2012 E.C

1. Let u= (2, 0) and v= (-1 3) be two vectors in a plane and w= -3u + 2v. Which one of the following is the unit vector in the direction of w, in terms of the standard unit vectors I and j?

   A. $\frac{4}{5}$i $-$ $\frac{3}{5}$j　　B. $-\frac{4}{5}$i + $\frac{3}{5}$j　　C. $\frac{3}{5}$i - $\frac{4}{5}$j　　D. $\frac{-3}{5}$ + $\frac{4}{5}$j

<u>Solution</u>

$$\mathrm{U} = (2,0), \mathrm{V} = (-1,3)$$
$$\mathrm{W} = -3 \cup + 2\mathrm{V} = -3(2,0) + 2(-1,3) = (-6,0) + (-2,6) = (-8,6) = -8\mathrm{i} + 6\mathrm{j}$$
$$\text{unit vector, U} = \frac{1}{|\mathrm{w}|}\mathrm{w} = \frac{-8\mathrm{i} + 6\mathrm{j}}{\sqrt{(-8)^2 + (6)^2}}$$
$$= \frac{8\mathrm{i} + 6\mathrm{i}}{\sqrt{64 + 36}} = \frac{-8\mathrm{i} + 6\mathrm{j}}{\sqrt{100}}$$
$$= \frac{-8\mathrm{i}}{10} + \frac{6\mathrm{j}}{10} = \frac{-4}{5}\mathrm{i} + \frac{3}{5}\mathrm{j},$$

Answer B

2. What is the image of the ellipse $\frac{(x-3)2}{6}+\frac{(y-2)2}{4}=1$ when it is rotated through 90^0 about the origin?
   A. $\frac{(x-2)2}{9}+\frac{(y-5)2}{16}=1$
   B. $4(x+3)^2\ 6y(y-2)^2=24$
   C. $\frac{(x+2)2}{6}+\frac{(y-3)2}{4}=1$
   D. $6(x+2)^2+4(y-3)^2=24$

Solution

$$\frac{(x-3)^2}{6}+\frac{(y-2)^2}{4}=1$$

The center $(3,2)$is rotated about 90^0

$R\ (3,2)=(-2,3)$, if the angle is 90^0, (x,y) is rotated to (-y,x).

image $\frac{(x+2)^2}{6}+\frac{(y-3)^2}{4}=1,$

Answer C

3. Which one of the following is NOT true about the scalar product of vectors?
   A. The scalar product is commutative.
   B. The scalar product of non-zero parallel vectors is zero.
   C. The scalar product is distributive over addition of vectors.
   D. The scalar product of perpendicular vectors is zero.

Solution

If U and V are parallel (non zero vectors)

$$U.V=|U||V|\cos 180^0$$

$$U.V=-|U||V| \text{ which is non zero}$$

Answer B

4. Which one of the following is NOT true about transformation?
   A. Rotation moves triangles in to congruent angles.
   B. Translations move angles to congruent angles.
   C. Rotation is a grid motion.
   D. Reflection is not a rigid motion.

Solution

Reflaction is a rigid motion

Rotation is rigid ratio

Translation is a rigid motion

Answer D

Grade 11 unit eight – vectors and transformation of the plane

2013 E.C

1. Given two vectors u = (1 3) and v = (-3, 5) in the plane. Then 6u + 2v is equal to
   A. (3, 3) B. (0, 18) C. (0, 28) D. (2, 28)

Solution

$$U=(1,3), V=(-3,5)$$

$$6U+2V=6(1,3)+2(-3,5)$$

$$=(6,18)+(-6,10)$$

$$=(0,28)$$

Answer C

2. Which is the magnitude of the vector whose initial point is at (1, -3) and terminal point is at (4, 1)?
   A. 6 B. 5 C. 4 D. 1

Solution

$$\text{U}=(1,-3),\text{V}(4,1)$$

$$|\text{U V}|=\sqrt{(4-1)+(1-(-3))^2}$$

$$=\sqrt{(4-1)^2+(1+3)^2}=\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$$

Answer B

3. Let T be a translation that takes the origin into (2, 3). Then which of the following is true about the translation T?
   A. $(x-5)^2 + (y-6)^2 = 9$ is the translation of $(x-3)^2 + (y-3)^2=9$
   B. $(x-4)^2 + (y-3)^2 = 9$ is the translation of $(x-2)^2 + (y+6)^2=9$
   C. $(x+4)^2 + (y-3)^2 = 9$ is the translation of $(x-2)^2 + (y+6)^2=12$
   D. $(x-4)^2 + (y+3)^2 = 12$ is the translation of $(x-2)^2 + (y+6)^2=12$

Solution

$$\text{U}=(2,3)$$

A) $(x-5)^2 +(y-6)^2=9$, C (5,6), C (5-2,6-3)=(3,3), $(x-3)^2 +(y-3)^2=9$ True
B) $(x-4)^2 +(y-3)^2=9$,C (4,3)= C (4-2, 3-3)=(2,0), $(x-2)^2 +y^2=9$ False
C) False, the radius cannot be translated
D) False the radius cannot be translated

Answer A

4. What is the image of the circle $x^2+y^2-4x-6y+12=0$ when it is reflected with respect to the line y=-x?
   A. $(x-2)^2 + (y-3)^2=1$ B. $(x+2) + (y+3)^2=1$ C. $(x+3)^2 + (y+2)^2=1$ D. $(y-3)^2 + (y-2)^2$

Solution

$$x^2+y^2-4x-6y+12=0$$

$x^2-4+y^2-6y+12=0$
$x^2-4x+4+y^2-6y+9=-12+9+4$
$(x-2)^2+(y-3)^2=1$
The center (2,3)is reflected trough $y=-x$
$m(2,3)=(-3,-2)$ whrn $\theta=135^0$
$=$ image $((x-h)^2+(y-k)^2=r^2,\ (x+3)^2+(y+2)^2=1,$

Answer C

Grade 11 unit nine – further on trigonometric functions

2009 E.C

1. If cot ∅= √8 and ∅ is first quadrant angle, then what is the value of csc θ?
   A. 1/3 B. 3 C. √8/3 D. 1/√8

Solution

$$\cot\emptyset=\sqrt{8}$$

$$\sin^2\emptyset+\cos^2\emptyset=1$$

$$1+\cot^2 1=\csc^2\emptyset$$

$$\left(\sqrt{8}\right)^2+1=\csc^2\emptyset$$

$$8+1=\csc^2\emptyset$$

$$\csc^2\emptyset=9$$

$$\csc^2\emptyset=\sqrt{9}$$

$$\csc\emptyset=\pm3\,,\qquad \emptyset\ is\ in\ the\ first\ quadrant\ and\ \csc\emptyset\ is\ positive\ in\ the\ first\ quadrant$$

$$\csc\emptyset=3$$

Answer B

2. If Ø=arctan (2), then what is the value of sin(2Ø)?

A. $\frac{2}{5}$

B. $\frac{4}{5}$

C. $\frac{4}{\sqrt{5}}$

D. $\frac{2}{\sqrt{5}}$

Solution

$$\emptyset = arc\tan 2 \quad , \quad sin\, 2\emptyset = ?$$

$$\Rightarrow \tan\emptyset = 2 = \frac{opp}{adj} = \frac{2}{1}$$

$$(hyp)^2 = (\,opp\,)^2 + (adj)^2$$

$$(hyp)^2 = 2^2 + 1^2$$

$$hyp = \sqrt{4+1} = \sqrt{5}$$

$$\sin\, 2\emptyset = \sin(\emptyset + \emptyset\,)$$

$$= \sin\emptyset cos\emptyset + cos\emptyset sin\emptyset = 2sin\emptyset\, cos\emptyset = 2\left(\frac{opp}{hyp}\right)\left(\frac{adj}{hyp}\right) = 2\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right) = \frac{4}{5}$$

Answer B

.

3. A patrol boat on a sea sailed from its station 7km to the north; and changed its course & sailed $4\sqrt{2}$in the direction of 45^0 south-east. What is the shortest (straight) distance the boat should travel in order to return to its station?

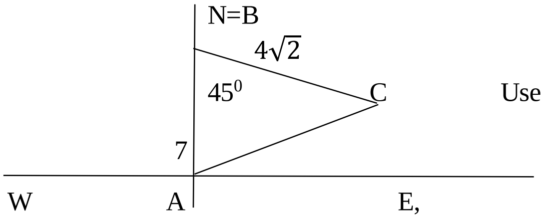
A. 5km

B. 7km

C. $5\sqrt{2}$km

D. $5+\sqrt{2}$km

Solution



$AB = 7, BC = 4\sqrt{2}$, m(< ABC)=45^0

Use cosine law, AC=$\sqrt{(AB)^2 + (BC)^2 - 2AB * BC * cos\theta}$,

$$=\sqrt{(7)^2 + (4\sqrt{2})^2 - 2 * 7 * 4\sqrt{2} * \frac{\sqrt{2}}{2}}$$

$$=\sqrt{49+32-56}=\sqrt{25}=5,$$

Answer A

4. Which one of the following is true?

A. The amplitude of f(x)=sin3x is 3.

B. The period of f(x)=2sin 4x is π

C. The period of f(x)= 3cos(1/2x- $\pi/3$) is4 π

D. The amplitude of f(x)=-5cos(3x+2)-2 is 7.

Solution

$$A)\ f\ (x) = \sin(3x) \qquad ,|A| = 1 \quad False$$

$$B)\ f(x) = 2sin4x \quad ,p = \frac{2\pi}{k} = \frac{2\pi}{4} = \frac{\pi}{2} \quad False$$

$$C)\ f(x) = 3\cos\left(\frac{1}{2}x - \frac{\pi}{3}\right) \quad , \quad p = \frac{2\pi}{k} = \frac{2\pi}{\frac{1}{2}} = 4\pi \quad True$$

$$D)\ f(x) = -5\cos(3x+2) - 2\,, \quad |A| = |-5| = 5 \quad False$$

Answer C

Grade 11 unit nine – further on trigonometric functions

2010 E.C

1. What is the period (P) and the range of f(x) $= 5\sin\left(\frac{1}{3}x + 2\right) + 3$?

A. P=6π, ranges= [-5, 5]

C. P=6π, range=[-2, 8]

B. P=2/3π, range=[-5, 5]

D. P=2/3π, range=[-2, 8]

Solution

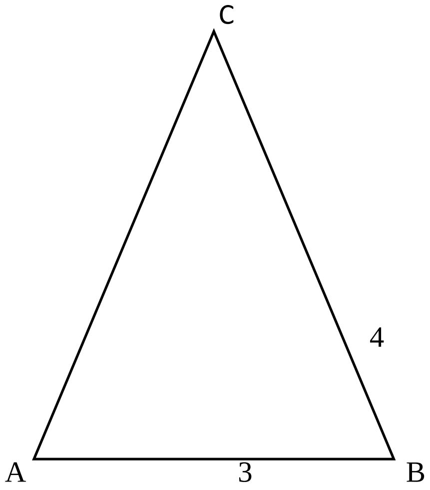
$$f(x) = 5\sin\left(\frac{1}{3}x + 2\right) + 3$$

$$p = \frac{2\pi}{k} = \frac{2\pi}{\frac{1}{3}} = 6\pi,\ \text{range}\ (\pm 5{+}3){=}(-5+3, 5+3) = [-2, 8]$$

Answer C

2. If, in ΔABC, AB= 3, BC= 4, and m(∠B) =60^0, then what are the lengths of AC and the cosine of ∠A respectively?
   - A. $\sqrt{13}$ and $6/5\sqrt{13}$
   - B. $\sqrt{13}$ and $-1/\sqrt{13}$
   - C. $\sqrt{13}$ and $1/\sqrt{13}$
   - D. $\sqrt{13}$ and $-6/5\sqrt{13}$

Solution



AC =?

< A =?

cosine low

$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos(< B)$

$\Rightarrow (AC)^2 = 4^2 + 3^2 - 2(3)(4)\cos 60^0$

$(AC)^2 = 16 + 9 - 24 * \frac{1}{2} = 25 - 12 = 13$

$AC = \sqrt{13}$

$\Rightarrow (BC)^2 = (AC)^2 + (AB)^2 - 2(Ac)(AB)\cos A$

$4^2 = \left(\sqrt{13}\right)^2 + (3)^2 - 2\left(\sqrt{13}\right)(3)\cos A$

$16 = 13 + 9 - 6\sqrt{13}\cos A$

$16 - 22 = -6\sqrt{13}\cos A$

$-6 = -6\sqrt{13}\cos A$

$\frac{1}{\sqrt{13}} = \cos A$

Answer C

3. If θ is fourth quadrant angle and sec θ=$\sqrt{2}$, then what is csc θ equals to?

A. $-\sqrt{2}$ B. $-1/\sqrt{-2}$, C. $1/\sqrt{2}$, D. $\sqrt{2}$

Solution

$\theta = 4^{\text{th}}\text{quaddrant}$

$\sec\theta = \sqrt{2}, \cos\theta = \frac{1}{\sec\theta}$, cos is positive

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{\sqrt{2}}{2}\right)^2 = 1 - \frac{2}{4} = 1 - \frac{1}{2} = \frac{1}{2}$

$\sin\theta = {}^{\pm}\sqrt{\frac{1}{2}}$, sin θ is negative in the 4^{th} quadrant ⇒ $\sin\theta = {}^{-}\sqrt{\frac{1}{2}} = \frac{-1}{\sqrt{2}}$,

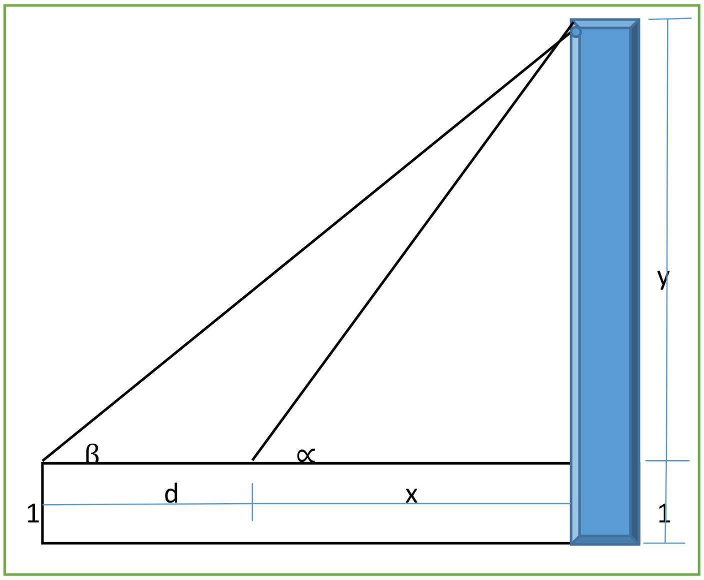
$\csc\theta = \frac{1}{\sin\theta} = -\sqrt{2}$,

Answer A

4. In order to measure the height of a tower, suppose a surveyor takes two sighting from a transit 1 meter high which are positioned d meters apart on the same ground level as in the figure below. If the first measured angle of elevation is α and the second is β (see, to figure), then what is the height of the tower (in meter) in terms of α, β, and d?

A. $\left(\frac{\tan\propto \tan\beta}{\tan\propto \tan\beta}\right) d + 1$
C. $\left(\frac{\tan\propto - \tan\beta}{\tan\propto \tan\beta}\right) d + 1$
B. $\left(\frac{\tan\propto \tan\beta}{tana(\tan\propto - \tan\beta)}\right) d + 1$
D. $\left(\frac{\tan\propto \tan\beta}{\tan\propto - \tan\beta}\right) d + 1$

Solution



$$\tan\propto = \frac{y}{x}, \quad x = \frac{y}{\tan\propto}$$

$$\tan\beta = \frac{y}{x + d}$$

$$d\tan\beta + x\tan\beta = y$$

$$d\tan\beta + \frac{y}{\tan\propto}\tan\beta = y$$

$$d\tan\beta + \frac{\tan\beta}{\tan\propto} y = y$$

$$d\tan\beta = y - \frac{\tan\beta}{\tan\propto} y$$

$$d\tan\beta = y\left(1 - \frac{\tan\beta}{\tan\propto}\right)$$

$$d\tan\beta = y\left(\frac{\tan\propto - \tan\beta}{\tan\propto}\right)$$

$$\frac{d\tan\beta}{\frac{\tan\propto - \tan\beta}{\tan\propto}} = y$$

$$\left(\frac{\tan\propto\tan\beta}{\tan\propto - \tan\beta}\right)d = y,$$

The height of the tower is y+1=$\left(\frac{\tan\propto\tan\beta}{\tan\propto - \tan\beta}\right)d + 1$,

Answer D

Grade 11 unit nine – further on trigonometric functions

2011 E.C

1. What is the solution set of $\sin^2 x - \sin x \cos x = 0$ over $[0, 2\pi]$

A. $\{0, \pi, \frac{5\pi}{4,}, 2\pi\}$
B. $\{0, \frac{\pi}{4}, \frac{5\pi}{4,}, 2\pi\}$
C. $\{0, \frac{\pi}{4}, \pi, 2\pi\}$
D. $\{0, \frac{\pi}{4}, \pi, \frac{\pi}{4}\}$

Solution

$$sin^2x - sinxcosx = 0 \quad ,[0\,,2\pi]$$

$$sinx(sinx - cosx) = 0$$

$$sinx = 0 \qquad or \qquad \sin x - \cos x = 0$$

$$x = sin^{-1}(0)\,, \qquad sinx = \cos x,$$

$$x = 0\,, \pi\,, 2\,\pi \qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$$

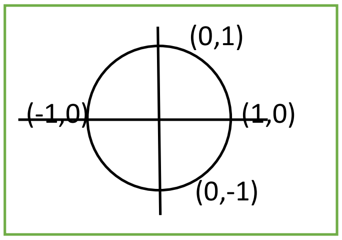
Answer B

2. What is the value of cot 270^0 + 2cos 90^0 +4sec 180^0

A. -2
B. 4
C. 8
D. 7

Solution

Use a unit circle to find the trigonometric values



$$\text{Cot}\,270^0 + cos90^0 + 4sec^2 180$$

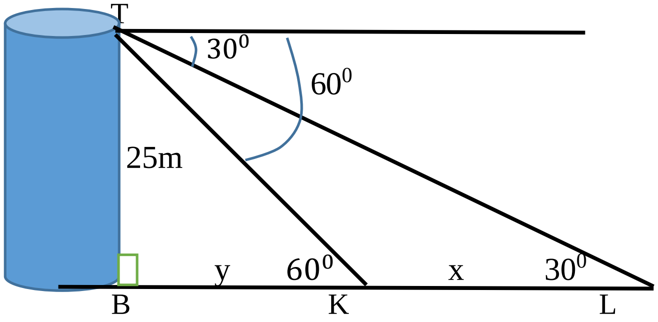
$$= 0 + 2(0) + 4(-1)^2$$

$$= 0 + 0 + 4$$

$$= 4$$

Answer B

3. The diagram below is a representation of a 25m vertical observation tower TB and two cars K and L on a road. The angle of depression from T to car L is 30^0. The angle of elevation from car K to the top of the tower is 60^0. B, K, L, lie in a straight and lie on the same horizontal plane is the base of the tower. What is the distance between the two cars?



A. $50+\sqrt{3}m$ B. $50+\sqrt{3}m$ C. $\frac{50\sqrt{3}}{2}m$ D. $\frac{50\sqrt{3}}{3}m$

Solution

$$\tan 60^0 = \frac{25}{y}, \qquad \sqrt{3} = \frac{25}{y},\ y = \frac{25}{\sqrt{3}}$$

$$\tan 30^0 = \frac{25}{x+y}$$

$$\frac{\sqrt{3}}{3} = \frac{25}{x+y}$$

$$\sqrt{3}x + \sqrt{3}y = 75,$$

$$x = \frac{75-\sqrt{3}y}{\sqrt{3}}$$

$$= \frac{75-\sqrt{3}*\frac{25}{\sqrt{3}}}{\sqrt{3}} = \frac{75-25}{\sqrt{3}} = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3},$$

Answer D

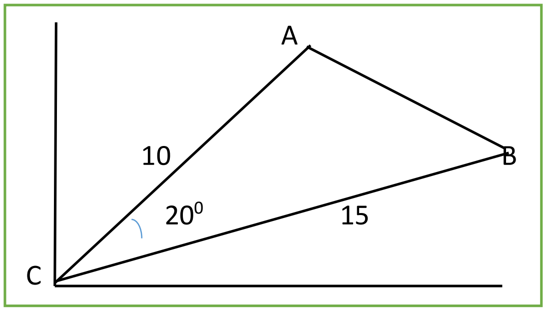
4. Ship A and B depart from the same point at the same time on the course $N60^0E$ and $N40^0E$, respectively. If the speed of ship A is 20 km per hour and the speed of ship B is 30 km per hour, what is the distance between the two ships just after 30 minutes of their departure? You may like: [cos 40^0=0.77, cos 20^0=0.94, tan 20^0=0.34]

A. $\sqrt{40}km$ B. $\sqrt{50}km$ C. $\sqrt{43}km$ D. $\sqrt{53}km$

solution

$$A\ traveled = 0.5\ hr\left(20\frac{kh}{hr}\right) = 10km$$

$$B\ traveled = 0.5\ hr\left(\frac{30km}{hr}\right) = 15km$$



Use cosine law

$(AB)^2 = (BC)^2 + (AC)^2 - 2(BC)(AC)\cos 20^0$

$x^2 = (15)^2 + (10)^2 - 2(15)(10)\cos 20^0$

$x^2 = 225 + 100 - 300x0.94$

$x^2 = 325 - 182 = 43$

$x = \sqrt{43}$

Answer C

Grade 11 unit nine – further on trigonometric functions

2012 E.C

1. Which one of the following is equal to sin^{-1}(sin$\frac{5\pi}{4}$)?

A. $-\frac{\pi}{4}$ B. $\frac{5\pi}{4}$ C. $\frac{-\sqrt{2}}{2}$ D. $\frac{\sqrt{2}}{2}$

Solution

$\sin\left(\frac{5}{4}\pi\right)$

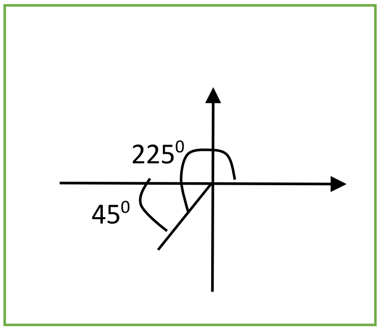
$\frac{5}{4}\pi = \frac{5}{4}\pi * \frac{180}{\pi} = 225^0$

$\sin 225^0 = -\sin 45^0$, sine is negative in the 3rd quadrant

$\sin 225^0 = \frac{-\sqrt{2}}{2}$

$\sin^{-1}\left(\sin\frac{5}{4}\pi\right) = \sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

$=\frac{-\pi}{4}$



Answer A

2. Which one of the following is true about f(x)= -2+3sin($\frac{1}{2}\times +3$)

A. The period is 2 π C. The range is [-5, 1]

B. The amplitude is 1 D. The phase shift is 6

Solution

$$f(x) = -2 + 3\sin\left(\frac{x}{2} + 3\right)$$

$$-\text{ period} = \frac{2\pi}{a} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$-\text{ amplitude} = 3$$

$$-\text{range }(c - |a|, c + |a|)$$

$$=(-2 - 3, -2 + 3)$$

$$=((-5,1)$$

$$\text{phase shift} = \frac{-b}{k} = \frac{-3}{\frac{1}{2}} = -6,$$

Answer C

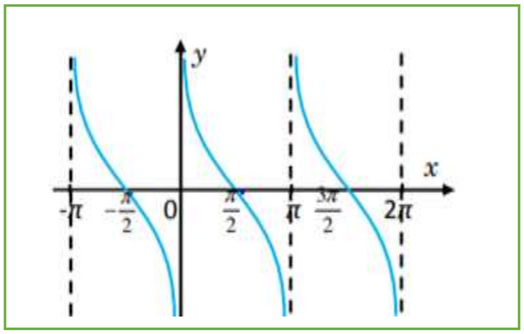
3. Which one of the following is correct about f(x)= cot x?

A. Its range is (-∞, 1) ∪ (1, ∞)

B. Its domain is $\{x \in \mathbb{R}: x \neq n\pi, n \in \mathbb{Z}$

C. Its domain is $\{x\in \mathbb{R}: x \neq \frac{n}{2}\pi, n \in \mathbb{Z}\}$

D. Its period is 2 π



Solution

Range R

Period π

$f(x) = \cot x$

Cot x is undefined on

------, -2 π,- π, π, 2 π,----------

The domain does not include these angles

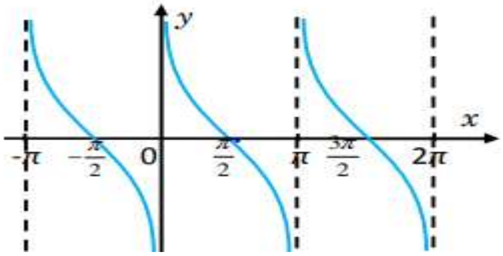
Its domain is $\{x \in \mathbb{R}: x \neq n\pi, n \in \mathbb{Z})$

Answer A

Grade 11 unit nine – further on trigonometric functions

2013 E.C

1. Which of the following functions represents, the graph shown in Figure 2?



A. f(x) = sec x B. f(x) = tan x C. f(x) = cot x D. f(x) = csc

Solution

$$y = \cot x$$

Answer C

2. Which of the following is rue about the graph of $y = -4\cos\left(\frac{2x}{3}\right)$?

A. Its amplitude is -4 C. The graph completes one cycle on the interval $\left[0,\frac{2\pi}{3}\right]$

B. Its period is 3π D. The graph completes one cycle on the interval $\left[0,\frac{3\pi}{3}\right]$

Solution

$$Y= -4\cos\left(\frac{2x}{3}\right), \quad a = |-4| = 4$$

$$p = \frac{2\pi}{k} = \frac{2\pi}{2/3} = 3\pi$$

Answer B

3. The angle of elevation of top of a tree is found to be 60^0as measured from a point on a level ground. What is the height of the tree if the angle of elevation of a point on the tree that is 8m below the top is 30^0 as measured from the same point on the ground?

A. 4m B. 8m C. 12m D. 14m

Solution

$$\tan 30^0 = \frac{y}{x} \qquad \tan 60^0 = \frac{8+y}{x}$$

$$x = \frac{y}{\tan 30^0} = \frac{8+y}{\tan 60^0}$$

$$\frac{y}{\tan 30^0} = \frac{8+y}{\tan 60^0}$$

$$y\ \tan 60^0 = (8+y) = \tan 30^0$$

$$\sqrt{3}\, y = (8+y)\frac{\sqrt{3}}{3}$$

$$3y = \frac{(8+y)}{3}$$

$$3y = 8 + y$$

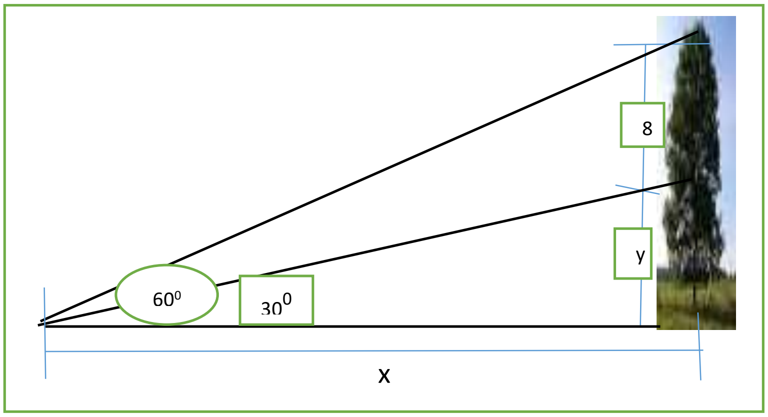
$$3y - y = 8$$

$$2y = 8$$

$$y = 4$$

height of the tree is

$$= 8 + y = 8 + 4 = 12$$



Answer C

4. Which of the following is true?
   A. The domain of y = $\cos^{-1}$ x is (1, 1)   C. $\tan^{-1}(0) = 1$
   B. The domain of y = $\tan^{-1}$x is (-∞, ∞)   D. $\text{Sin}^{-1} = (0) = \frac{\pi}{2}$

Solution

A) $y = \cos^{-1}(x)$,Doman $[-1,1]$, False

B) $y = \tan^{-1}(x) = \text{domain} = \Re = \text{rage of } y = \tan x$, True

C) $y = \tan^{-1}(0) = o^0 \text{ or } \pi \text{ or } 2\pi - - -$ Fasle

D) $\sin^{-1}(0) = 0^0 \text{ or } \pi \text{ o r}2\pi, - - -$ False, .

Answer B

Grade 11 unit ten – introduction to linear programming

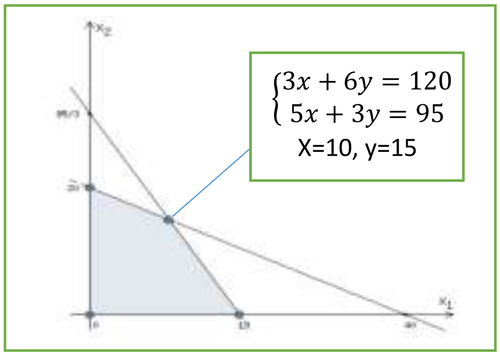
2009 E.C

1. Furniture manufacturing company produced two different types of chairs A and B. to produce chair A it takes 3 hours for cutting and 5 hours for assembling. To produce chair B it takes 6 hours for cutting and 3 hours for assembling. The company has at most 120 hours for cutting labour and 95 hours of assembly labour per day. The company's profit is 40 birr for ach chair A produced and 60 birr for each chair B produced. Let x and y represents number of chair A and chair B produced respectively. If the company wants to maximize the profit P, how many of each type should be made daily?
   A. 10 type A and 12 type B   C. 10 type A and 15 type B
   B. 12 type A and 10 type B   D. 15 type A and 10 type B

Solution

$$\text{Let type } A = x \text{ , type } B = y$$
$$\text{Cutting}: 3x + 6y \le 120$$
$$\text{Assamb}: 5x + 3y \le 95$$
$$p = 40x + 60y$$
$$(0,0), p = 40(0) + 60(0) = 0$$
$$(19,0), p = 40(19) + 60(0) = 760$$
$$(0,20), p = 40(0) + 60(20) = 1200$$
$$(10,15), p = 40(10) + 60(15) = 1300, max$$

Answer C



$$\begin{cases}3x + 6y = 120\\ 5x + 3y = 95\end{cases}$$
X=10, y=15

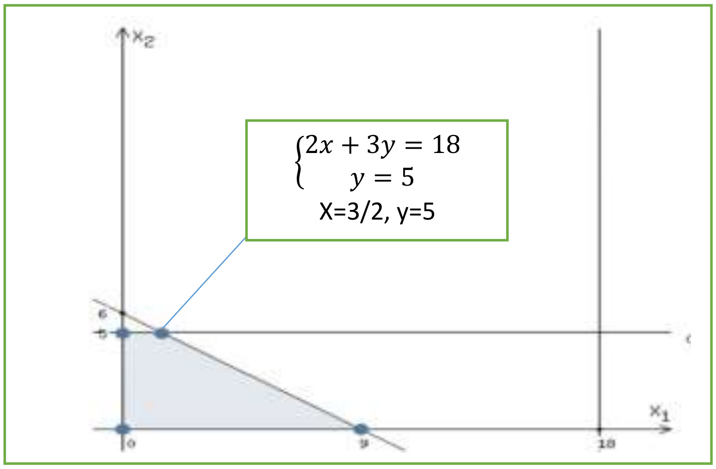
2. What is the maximum value of $z = 6x + 3y$ subject to $\begin{matrix} 2x + 3y \le 18 \\ 0 \le x \le 18 \\ 0 \le y \le 5 \end{matrix}$
   A. 30   B. 28   C. 42   D 54

Solution

$$\left.\begin{matrix} 2x + 3y = 18 \\ x = 0 \text{ , } y = 6 \\ y = o \text{ , } x = 9 \end{matrix}\right|$$
$$(0,0), z = 6(0) + 3(0) = 0$$
$$(9,0), z = 6(0) + 3(0) = 54$$
$$\left(\frac{3}{2}, 5\right), z = 6\left(\frac{3}{2}\right) + 3(5) = 0$$
$$(0,5), z = 6(0) + 3(5) = 15,$$
$$maximum = 54,$$

.

Answer D



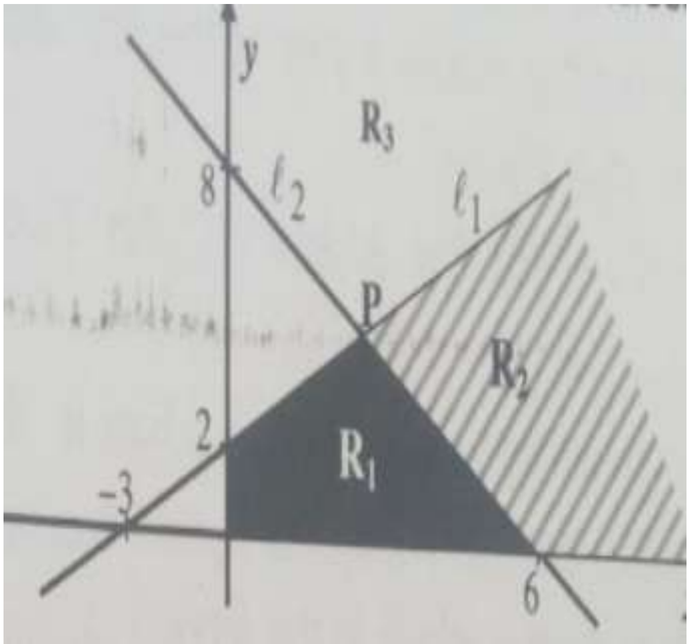
$$\begin{cases}2x + 3y = 18\\ y = 5\end{cases}$$
X=3/2, y=5

3. In the figure below l_1 and l_2 are straight lines given by $-2x + 3y = 6$ and $4x + 3y = 24$ respectively and p is the intersection point. Which one of the following is true about the parts of the figure?
   A. The dark shaded region R₁, is the solution region for $-2x + 3y \leq 6, 4x + 3y \leq 24, y \geq 0$
   B. The light shaded region R₂, is the solution region for $-2x + 3y \leq 6, 4x + 3y \leq 24, y \geq 0$
   C. The solution region for $-2x + 3y \leq 6, 4x + 3y \leq 24, y \geq 0$ is bound.
   D. The solution region for $-2x + 3y \leq 6, 4x + 3y \leq 24, y \geq 0$ is unbounded.

Solution

$$R1 = \begin{cases} 4x + 3y \leq 24 \\ -2x + 3y \leq 6 \end{cases}$$

$$R2 = \begin{cases} 4x + 3y \geq 24, \\ -2x + 3y \leq 6 \end{cases}$$



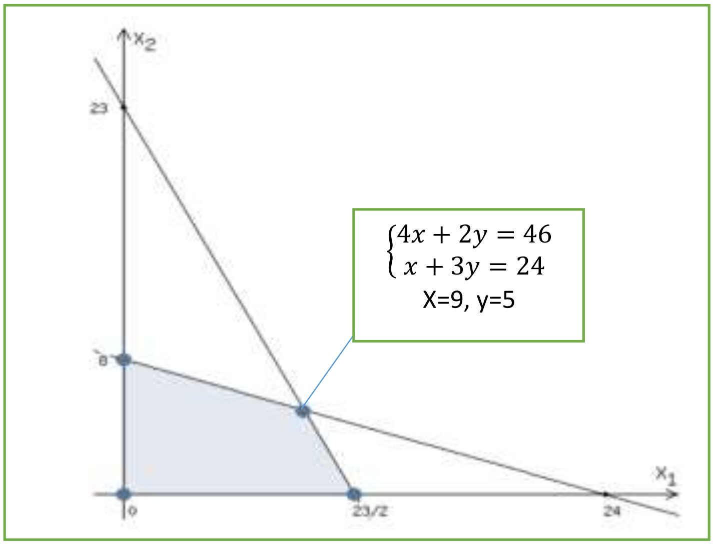
Answer A

Grade 11 unit ten – introduction to linear programming

2010 E.C

1. A firm deal with two kids of fruit juices pineapple and orange juice. These are mixed and two types of mixtures are obtained which are sold as soft drinks A and B. One tin of A needs 4 liters pineapple juice and 1 liter of orange juice. One tin of B needs 2 liters of pineapple and 3 liters of orange juice. The firm needs atmost 46 liters of pineaaple juice and atmose 24 liters of orange juice. Each tin of and B sold at a profit of birr 4 and birr 3 respectively. How many tins of A and B should the firm produce to maximize its profit?
   A. 9 tins of A and 5 tins of B
   B. 9 tins of A and 6 tins of B
   C. 5 tins of A and 9 tins of B
   D. 6 tins of A and 9 tins of B

Solution



$$\begin{cases} 4x + 2y = 46 \\ x + 3y = 24 \end{cases}$$

X=9, y=5

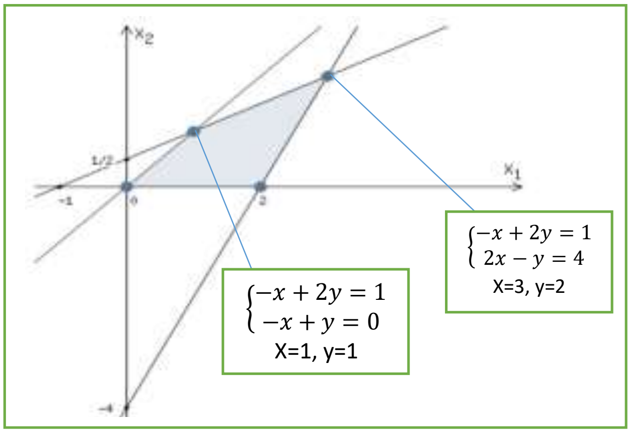
$$let\ type = x\ and\ type\ B = y$$
$$pineapple: 4x + 2y \le 46$$
$$orange : x + 3y \le 24$$
$$x \ge 0, y \ge 0$$
$$p = z = 4x + 3y$$
$$(0,0), Z = 4(0) + 3(0) = 0$$
$$\left(\frac{23}{2}, 0\right), Z = 4\left(\frac{23}{2}\right) + 3(0) = 46$$
$$(9,5), Z = 4(9) + 3(5) = 51$$
$$(0,8), Z = 4(0) + 3(8) = 24$$

Answer A

2. What is the maximum value of $z = 3x - 2y$

   Subject to

   $-x + y \le 0$
   $-x + 2y \le 1$
   $2x - y \le 4$
   $x \ge 0,$
   $y \ge 0$



A. 5 B. 8 C. 6 D. 10

Solution

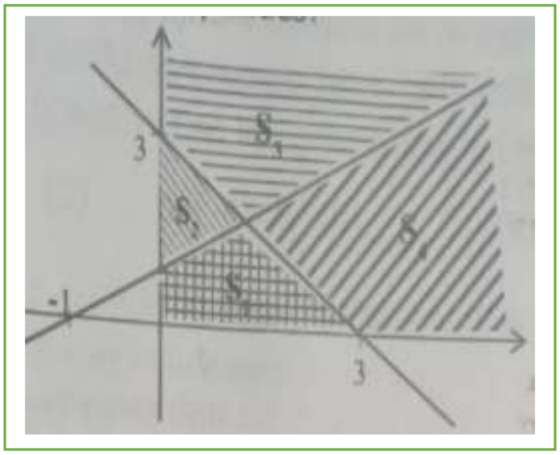
$-x + y = 0$	$-x + 2y = 1$	$2x - y = 4$
$x = 0, y = 0$	$x = 0, y = 1/2$	$x = 0, y = -4$
$y = 0, x = 0$	$y = 0, x = -1$	$y = 0, x = 2$

$$z = 3x - 2y$$
$$(2,0), z = 3(2) - 2(0) = 6 \qquad (0,0), z = 3(0) - 2(0) = 0$$
$$(3,2), z = 3(3) - 2(2) = 5 \qquad (1,1), z = 3(1) - 2(1) = 1$$

Answer C

3. Which one of regions shaded in the figure below is the solution region of the following system of inequalities? $\begin{cases} x + y \le 3 \\ -x + 3y \le 1 \\ x \ge 0, y \ge 0 \end{cases}$

   A. S_1 B. S_4 C. S_3 D. S_2



Solution

$$\begin{cases} x + y \le 3 \\ -x + 3y \le 1 \\ x \ge 0, y \ge 0 \end{cases}$$

S_1 is the regon which can satisfy the inequalities

Answer A

Grade 11 unit ten – introduction to linear programming

2011 E.C

$\begin{cases}-x + y = 1\\ x + y = 3\end{cases}$
X=1, y=2

1. What is the maximum valve of $z = -3x + 2y$

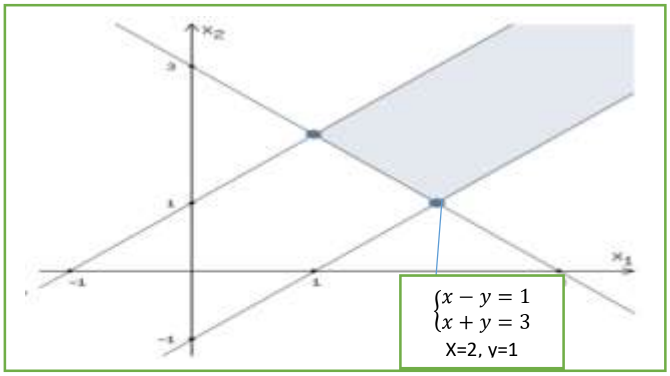
   Subject to $-x + y \le 1$
   $X - y \le 1$
   $X + y \ge 3$

A. No maximum B. 4 C. 6 D. 1

Solution

$$z = -3x + 2y$$
$$(1, 0) \Rightarrow z = -3 + 0 = -3$$
$$(3, 0) \Rightarrow z = -9 + 0 = -9$$
$$(1, 2) \Rightarrow z = -3 + 4 = 1$$
$$(2, 1) \Rightarrow z = -6 + 2 = -4$$

No maximum *Answer A*



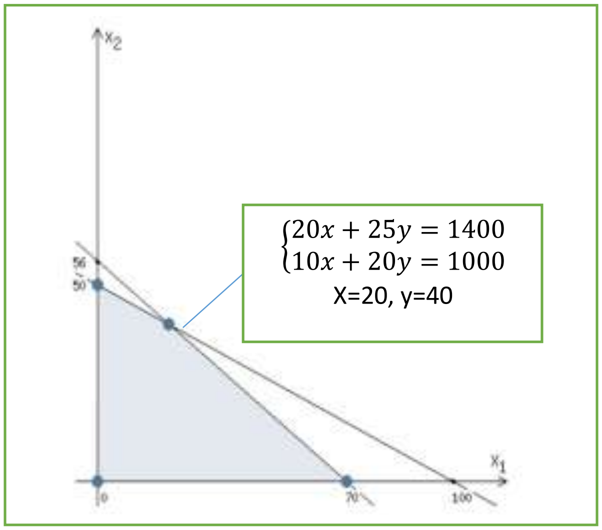
$$\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$$
X=2, y=1

2. A man owns a field of area 1000 m^2. HE wants to plant fruit trees in it. He has a sum of birr 1400 to purchase young trees. Type A requires 10 m^2 of ground per tree and cost birr 20 per tree and type B requires 20 m^2 ground per tree and cost birr 25 per tree. When fully grown, type A produces an average of 20 Kg which can be sold at a profit of birr 2 and type B produces an average of 40 Kg of fruit which can be sold at a profit of 1.5 birr per Kg. How many of each type should he plant to achieve maximum profit when the trees are fully grown?
   - A. 40 trees of type A and 20 trees of type B
   - B. 20 Tree of type A and 40 trees of type B
   - C. 30 trees of type A and 30 trees of type B
   - D. 35 trees of type A and 35 trees of type B

Solution

$$Let\ no\ of\ tree\ A = x$$
$$no\ of\ tree\ B = y$$
$$Area: 10\,x + 20y\ \le 1000$$
$$Birr: 20x + 25y\ \le 1400$$
$$P = 2(20)x + 1.5(40)y$$
$$= 40x + 60y$$
$$(0,0), P = 40(0) + 60(0) = 0$$
$$(0,50), P = 40(0) + 60(50) = 3000$$
$$(20,40), P = 40(20) + 60(40) = 3200$$
$$(70,0), P = 40(70) + 60(0) = 2800$$

Answer B



$$\begin{cases} 20x + 25y = 1400 \\ 10x + 20y = 1000 \end{cases}$$
X=20, y=40

3. Which one of the following points is in solution region of the linear in equalities

$$-x + 3y \le 10$$
$$-x + y \le 6$$
$$X - y \le 2$$
$$X, y \ge 0?$$

A. (1, 1) B. (3, 4) C. (3, 0) D. (0, 4)

Solution

$-x + 3y \le 10 \quad \Rightarrow -1+3 \le 10$, *true*
$-x + y \le 6 \quad \Rightarrow -1+1 \le 6$, *true*
$X - y \le 2 \quad \Rightarrow 1-1 \le 2$, *true*

(1,1), satisfies all the inequalities
$(1, 1)$ *is on the region*
Answer A

Grade 11 unit ten – introduction to linear programming

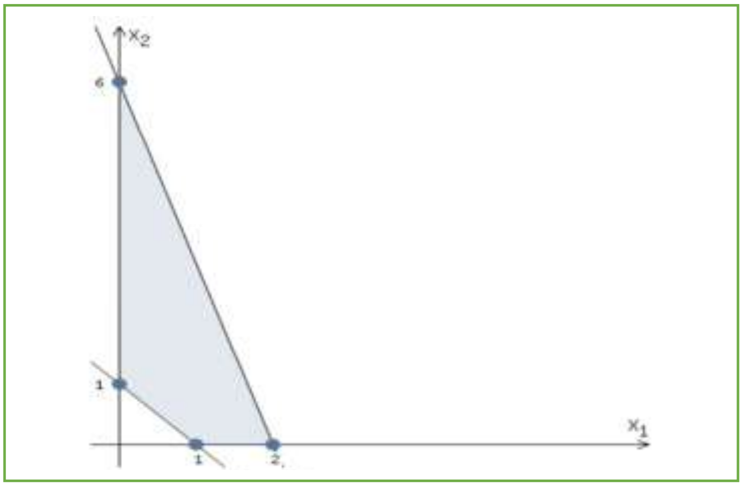
2012 E.C

1. Consider the following linear programming model
   - Objective function *$z= 3x +5y$*, Subject to *$3x+ y \leq 6$*, *$x + y \geq 1$*, *$x \geq 0, y \geq 0$*
   Which one of the following is the minimum value of z?
   A. 3 B. 0 C. It has number minimum value D. 5

   Answer

$Z= 3x +5y$, $3x + y \leq 6$
$x + y \geq 1$
$3x + y \leq 6$
$x\geq 0$, $y \geq 0$,
$3x + y = 6$ $x+y = 1$ $x+ 2y = 6$
$x= 0, y = 6$ $x = 0, y = 1$ $x = 0, y =3$
$y = 0, x = 2$ $y =0, x = 1$ $y=0, x = 6$
$z = 3x + y$
(1,0), $z = 3(1) +5(0) = 3$
(2,0) , = z = 392) +5(0) = 6
(0,1), $z = 3(0) +5(1) = 5$
(0,6), $z = 3(0) + 5(6) = 30$

Answer A



2. A problem to be solved using linear programing method is given as follow:
   "A coffee packer blends coffee from Harar and from Sidama to prepare two kinds of products, viz "Super" and "Deluxe" brands. Each kilogram of super coffee contains 0.5 Kg of Harar coffee and 0.5 Kg of Sidama coffee, whereas each kilogram Deluxe coffee contains 0.25 Kg of Harar coffee and 0.75 kg of Sidama coffee. The packer has 120 kg of Harar coffee and 160 kg of Sidama coffee in the store. Moreover, the plan is to get 20-birr profit on each kilogram of upper coffee and profit of 30birr on each kilogram of Deluxe coffee. In order to maximize profit the packer needs to know the amount (in kg) of each brand of coffee that should be blended" If
   x = The amount of super coffee in kg.
   Y = The amount of Deluxe coffee in Kg
   Then which one of the following standard forms of linear programming models should be used to solve the above problem?

   A. Maximize $P = 20x + 30y$
   Subject to $0.75x + 0.25y \leq 160$
   $0.5x + 0.5y \leq 160$
   $x\geq 0, y\geq 0$

   B. Maximize $P = 30x + 20y$
   Subject to $0.5x + 0.5y \leq 120$
   $0.25x + 0.75 \leq 160$
   $x \geq 0, y \geq 0$

   C. Maximize $P = 30x +20y$
   Subject to $0.5x + 0.5y \leq 120$
   $0.5x + 0.25y \leq 160$
   $x \geq 0, y \geq 0$

   D. Maximize $P = 20x +30y$
   Subject to $0.5x + 0.25y \leq 120$
   $0.5x + 0.75y \leq 160$
   $x \geq 0, y \geq 0$

Solution

Let x = amount of super coffee in kg.
Y = amount of Deluxe coffee in Kg
Harar: $0.5x + 0.25 y \leq 120$
Sidama : $0.5x + 0.75y \leq 160$
$x\geq 0, y \geq 0$
$P =20x + 30y$

Answer D

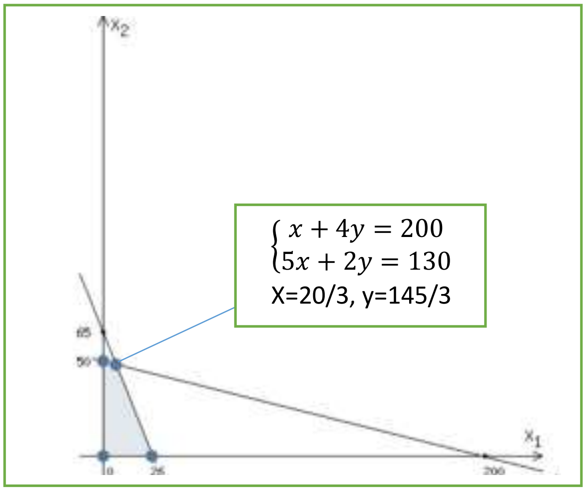
Grade 11 unit ten – introduction to linear programming

2013 E.C

1. A cottage industry produces good either by capital of labour intensive, method according to
   I -Capital intensive using 1 unit of labour and 4 units of capital.
   II -Labour intensive using 5 units of labour and 2 units of capital.
   The industry managed to arrange up to 200 units of labour and 130 units of capital. If the owner can sell goods at a constant price and get a profit of 100 birr per unit of the two quantities of goods produced by the processes I and ii, what will be the maximum profit generated.
   A. 5,500 birrs B. 5,000 birr C. 4,500 birr D. 4,000 birr

Solution

$$x = labour$$
$$y = Capital$$
$$I: x + 4y \ \leq 200$$
$$II: 5x + 2y \ \leq 130$$
$$x \ \geq 0\,, y \geq 0$$
$$P = 100x + 100y$$
$$(0,0), p = 100(0) + 100(0) = 0$$
$$(26,0), p = 100(26) + 100(0) = 2600$$
$$(0,50), p = 100(0) + 100(50) = 5000$$
$$\left(\frac{20}{3}, \frac{145}{3}\right), p = 100\left(\frac{20}{3}\right) + 100\left(\frac{145}{3}\right)$$
$$= \frac{2000 + 14500}{3}$$
$$= \frac{165000}{3} = 5500$$
$$Answer\ A$$



$$\begin{cases} x + 4y = 200 \\ 5x + 2y = 130 \end{cases}$$
X=20/3, y=145/3

2. Consider the system of linear equations $\begin{cases} 3x - ay = 1 \\ 6x + 4y = 5 \end{cases}$, then which of the following is true?
   A. If a > 2, the system has infinitely many solutions
   B. If a = 2 -, then system has no solution
   C. If a = -2, the system has a unique solution
   D. If a > -2, the system has a unique solution

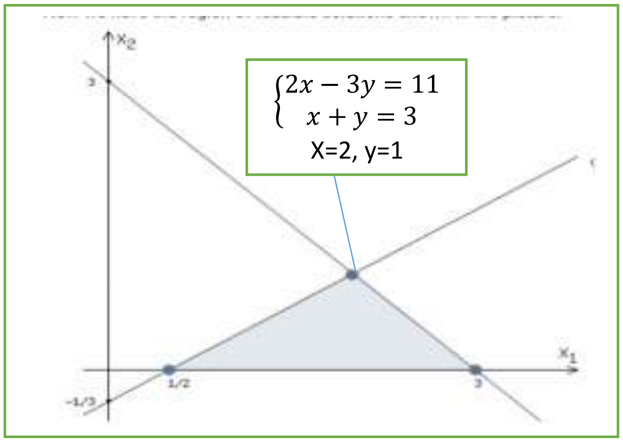
Solution

$$\begin{cases} 3x - ay = 1 \\ 6x + 4y = 5 \end{cases}$$
$$if\ a\ \neq\ -2\ it\ has\ unique\ solution$$
$$\frac{a1}{a2} \neq \frac{b1}{b2}$$
$$Answer\ D$$

3. What is the maximum value of the function
   z = 3x + y subject to *2x – 3y ≥ 1*
   x + y ≤ 3
   x ≥ 0, y ≥ 0
   A. 9/2 B. 3/2 C. 9 D. 7

Solution

z = 3x + y, (1/2, 0), z=3/2
(0,3), z=3
(2,1), z=7,
Answer D



$$\begin{cases} 2x - 3y = 11 \\ x + y = 3 \end{cases}$$
X=2, y=1

Grade 11 unit eleven – Mathematical Applications in Business

2009 E.C

1. There are 900 students in grade 4, 5 and 6 of school. 300 of the students are in grade 4, 330 of them are in grade 5. If a sample of size 150 is to be taken randomly from all the students for a survey proportionally, how many of grade 6 students should be taken in the sample?

   A. 50 B. 48 C. 45 D. 42

   Solution

$$G4 + G5 + G6 = 900$$
$$300 + 330 + G6 = 900$$
$$G6 = 900 - 300 - 330$$
$$G6 = 270$$
$$G4 : G5 : G6$$
$$10 : 11 : 9 - ratio$$
$$\frac{150}{10 + 11 + 9} = \frac{150}{30} = 5$$
$$sample\ G4 = 10 * 5 = 50$$
$$G5 = 11 * 5 = 55$$
$$G6 = 9 * 5 = 45$$
$$Answer\ C$$

2. Mr. Ali invested birr 52000 in a share company buy with he bought share each of with costs birr 500 if the company is to pay divide birr 70 per share for shareholders and the dividend is subjected to 10% income tax which will be withhold by the company.

   What is the amount of the dividend that Mr. Ali will receive after tax?

   A. Birr 882 B. Birr 6300 C. Birr 6552 D. Birr 7280

   Solution

$$Number\ of\ share = \frac{52,000}{500}$$
$$= 104$$
$$Paymetn = 104 * 70$$
$$= 7280$$
$$tax = 7280 * 10\%$$
$$= 7280 * 0.1$$
$$= 728$$
$$Net = 7280 - 728$$
$$= 6552\ Birr$$
$$Answer\ C$$

3. An investment of Birr 3000 was made over 3 years of an interest rate of 5% with interest compounding annually. What is the principal at the start of the second year and the interest earned during the second year (in Birr), respectively?

   A. 3150, 152.5 B. 3100.8, 152.5 C. 3150, 157.5 D. 3307.5, 157.5

   Solution

$$P2 = P(1 + r)^t$$
$$p2 = 3000\ (\ 1 + 0.05)^2$$
$$= 3000\ (\ 1.05)^2 \quad , \qquad I=3307.5 * 0.05=157.5$$
$$= 3000(1.1025) = 3307.5$$
$$Answer\ D$$

4. Birr 10,000 is deposited in an account that pays 8% annual interest compounded semiannually another 9,184 birr is deposited in the same account after exactly 1 year of the first deposit how much is in the account after 2 years?

   A. Birr 20,824 B. Birr 20,864 C. Birr 21,632 D. Birr 21,864

Solution

$$A_n = A_0(1+\frac{r}{n})^{nt}$$
$$= 10{,}000(1+\frac{0.08}{2})^2$$
$$= 10{,}000(1.04)^2$$
$$= 10{,}000 * 1.0816$$
$$= 10816$$
$$After\ One\ Year$$
$$10{,}816 + 9184 = 20{,}000$$
$$After\ Two\ Years\ (\ 1\ year\ from\ 20{,}000)$$
$$= 20{,}000\ ((1+\frac{0.08}{2})^2$$
$$= 20{,}000(1.04)^2$$
$$= 10{,}000 * 1.0816$$
$$= 21{,}632$$
$$Answer\ C$$

5. The ratio of students to teachers in a school is 39:2, If the number of students is 819, and the number of female teachers is 25, then what is the number of male teachers in the school?

   A. 17 B. 25 C. 32 D. 35

Solution

$$39:2, \quad S = 819$$
$$S = \frac{819}{39} = 21$$
$$Students = 39 * 21 = 819$$
$$Teachers = 2 * 21 = 42$$
$$Teachers = m + F = 42$$
$$m + 25 = 42$$
$$m = 42 - 25$$
$$m = 17$$
$$Answer\ A$$

Grade 11 unit eleven – Mathematical Applications in Business

2010 E.C

1. The population of certain city is increasing at a rate of 3% per year. If the population was 100.000 in 2010 E.C. then what will be the population In 2020 E.C?

   (Given: $(1.03)^9$=1.30, $(1.03)^{10}$=1.34, $(1.3)^9$=10.60, $(1.3)^{10}$=13.78

   A. 130,000 B. 1,060,000 C. 134,000 D. 1,378,000

Solution

$$p_t = p_0\ (1+r)^t$$
$$= 100{,}000\ (\ 1 + 0.03)^{10}$$
$$= 100{,}000 * 1.34$$
$$= 134000,$$
$$\text{Answer C}$$

2. In the previous month, the regular price of sugar and salt were birr 12 and birr 8 per Kg respectively. This month the price of sugar increased by 20% and the price of salt decreased by 30%. If a family must buy 5 Kg of sugar and 3 Kg of salt every month, how much more or less does the family need t pay for the two items this month compared to their previous month bill?
   A. Birr 4.8 more
   B. Birr 4.8 less
   C. Birr 8.4 more
   D. the same as in previous month

Solution

$$Sugar = 12\ birr$$
$$now = 12 + 12(20\%)$$
$$= 12 + 12(0.2)$$
$$= 12 + 2.4 = 14.4$$
$$Difference = 2.4\ per\ Kg$$
$$salt = 8\ birr$$
$$now = 8 - 8(30\%)$$
$$= 8 - 8(0.3)$$
$$= 8 - 2.4 = 5.6$$
$$Difference = 2.4\ per\ Kg$$
$$5\ Kg\ of\ sugar = more\ 5 * 2.4 = 12$$
$$3\ Kg\ of\ salt = less\ 3 * 2.4 = 7.2$$
$$-\ Family\ must\ add\ more$$
$$= 12 - 7.2 = 4.8$$
$$Answer\ A$$

3. A hospital wants to buy three X ray machines from a supplier. Suppose the price of each machine including 15% VAT is birr 161,000. If the hospital wants to subtract a 2% withholding tax before VAT, what is the amount the hospital should pay (in birr) to the supplier after withholding 2% is subtracted?
   A. 473,340 B. 483,340 C. 475,340 D. 474,600

$$solution$$
$$Total\ price\ including$$
$$VAT = 161{,}000 * 3$$
$$= 483{,}000$$
$$Before\ VAT$$
$$x + 0.15x = 483{,}000$$
$$x(1.15) = 483{,}000$$
$$x = \frac{483{,}000}{1.15}$$
$$x = 420{,}000$$
$$With\ holding = 420{,}000 * 0.02 = 8400$$
$$A = 483{,}000 - 8400$$
$$= 474{,}600$$
$$Answer\ D$$

4. A jacket discounted by 20% for holyday has a price tag of birr 576 what is the amount of discount?
   A. 154 Birr B. 135.8 Birr C. 144Birr D. 115.2 Birr

<u>Solution</u>

$$Price\ before$$
$$Discount = x$$
$$x - 20\%x = 576$$
$$x(1 - 0.2) = 576$$
$$0.8x = 576$$
$$x = \frac{576}{0.8}$$
$$x = 144$$
$$Answer\ C$$

5. Birr 1,000 is deposited in a saving account that pays 6% annual interest compounded monthly. Which one of the following is the amount (in birr) that will be in the account by the end of the 3 years?
   A. $1000(1.005)^{36}$ C. $1000(1.05)^{36}$
   B. $1000 + 1000(1.05)^{3}$ D. $1000 + 1000(1.005)^{36}$

<u>Solution</u>

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 1000\left(1 + \frac{0.06}{12}\right)^{12(3)}$$
$$= 1000(1 + 0.005)^{36}$$
$$= 1000(1.005)^{36}$$
$$Answer\ A$$

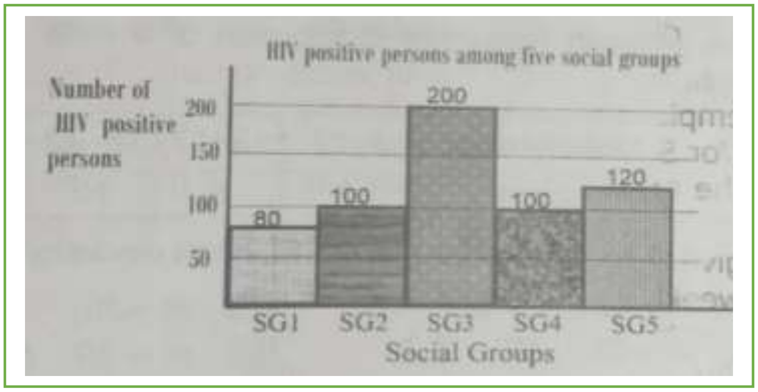
6. The following histogram presents the number of HIV positive persons among 5 social groups coded as SG1, SG2, SG3, SG4 and SG5 in certain town

   Moreover, the number of persons in each of those social groups SG1, SG2, SG3, SG4 and SG5 are 500, 2500, 4000, 500 and 1200 respectively. If so, which one of the following can be deduced from the data?

   A. 3% of these in SG4 are HIV positive
   B. SG1 consists of the highest percentage of HIV positive persons followed by SG5.
   C. SG3 consists of the highest percentage of HIV positive persons.
   D. 7.4% of the entire population of the five social groups of HIV positive.

<u>Solution</u>

$$Percentage\ of\ HIV\ positive$$
$$SG_1 = \frac{80}{500} * 100\% = 16\%$$
$$SG_2 = \frac{100}{2500} * 100\% = 4\%$$
$$SG_3 = \frac{200}{4000} * 100\% = 5\%$$
$$SG_4 = \frac{100}{500} * 100\% = 2\%$$
$$SG_5 = \frac{120}{1200} * 100\% = 10\%$$
$$Answer\ B$$



7. A person is planning to make a regular monthly saving over the next five years in account that pays no interest. If his monthly income is birr 4000 then what percentage of his income should he save monthly so that his total saving will be birr 26400 by the end of 5 years?

A. 11% B. 10% C. 9% D. 12%

Solution

$$Income$$
$$= 4000 * 5 * 12$$
$$= 240{,}000$$
$$\% = \frac{26400}{240{,}000} * 100\%$$
$$= \frac{264}{24} = 11\%$$
$$Answer\ A$$

Grade 11 unit eleven – Mathematical Applications in Business

2011 E.C

1. A young student borrows a certain amount of money from a money lender. The money lender chares interest at a rate of 20% compounded yearly. If the money lender sends a letter pf demand of 400,000 birr after 2 years, how much did the students borrow?

A. 277,7778 Birr B. 40,000 Birr C. 84,467.5 Birr D. 320,000 Birr

Solution

$$400{,}000 = x(1 + r)^t$$
$$400{,}000 = x\ (1 + 0.2)^2$$
$$400{,}000 = x(1.2)^2$$
$$400{,}000 = 1.44x$$
$$x = \frac{400{,}000}{1.44}$$
$$x = 277{,}777.8$$
$$Answer\ A$$

2. A company wants to buy two cars from marathon motors. The price of each car including VAT (15%) is birr 402,500. If the company wants to subtract a 2% withholding tax before VAT. What is the amount to be subtracted?

A. 14,000 Birr B. 14,250 Birr C. 16,000 Birr D. 16,250 Birr

Solution

$$including\ tax$$
$$= 402{,}500 * 2$$
$$= 805{,}000$$
$$Befor\ VAT$$
$$x + 0.15x = 805{,}000$$
$$1.15x = 805{,}000$$
$$x = \frac{805{,}000}{1.15}$$
$$x = 700{,}000$$
$$with\ holding = 2\%\ of\ 700{,}000$$
$$= 0.02 * 700{,}000$$
$$= 14{,}000$$
$$Answer\ A$$

3. A sweatshirt with a regular price of 340 Birr is first discounted by 20% and then by 10%. What is the final selling price of the sweatshirt after the discount?

   A. 238 Birr B. 244.8 Birr C. 102 Birr D. 95.2 Birr

Solution

$$P = 340$$
$$First\ discount$$
$$P = 340 - 340(0.2)$$
$$= 340 - 68$$
$$= 272$$
$$Seconf\ discount$$
$$P = 272 - 272(0.1)$$
$$= 272 - 27.2$$
$$= 244.8$$
$$Answer\ B$$

4. If birr 2000 is deposited in a bank with interest rate 10 % per annum compounded semiannually, then how much birr will it be by the end of one year?

   A. 21000 B. 2250 C. 2205 D. 2210

Solution

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 2000\left(1 + \frac{0.1}{2}\right)^{2(1)}$$
$$= 2000\ (1.05)^2$$
$$= 2000(1.1025)$$
$$= 2{,}205\ birr$$
$$Answer\ C$$

Grade 11 unit eleven – Mathematical Applications in Business

2012 E.C

1. Which one of the following is not principle of Taxation?
   A. Taxation should seek to be neutral and equitable between forms of business activities
   B. Exemption of taxes are made to ensure revenue to cover government cost of administration
   C. Tax rules should be clear and simple to understand so that taxpayers know where they stand.
   D. Compliance costs to business and administration costs for governments should be minimized as far as possible.

Solution

Compliance costs to business and administration costs for governments should be minimized as far as possible.

Answer D

2. If 20,000 birr is divided among three persons in the ratio 5: 2: 3 then which one of the following is the maximum share?

   A. 3000 Birr B. 6000 Birr C. 10,000 Birr D. 4000 Birr

Solution

20,000

5: 2: 3 $\Longrightarrow \frac{20{,}000}{10}$ = 2000

5shares = 5 × 2000 = 10,000

2 shares = 2 × 2000 = 4000

3 shares = 3 × 2000 = 6000

Answer C

3. A Sum of money is inverted at a compound interest calculation annually which amounts to 21,632 birr at the end of the second year and to 22,497.28 birr at the end of the third year. Which one of the following pairs of values gives the rate of interest and the principle respectively?

A. 0.04 and 20,000 Birr B. 0.03 and 18,550 Birr C. 0.0 and 21,550 Birr D. 0.05 and 21,00 Birr

Solution

$$G2 = G(1+r)^2$$
$$G3 = G(1+r)^3$$
$$\frac{G3}{G2} = \frac{(1+r)^3}{(1+r)^2}$$
$$G3 / G2 = 1+r$$
$$\frac{22497.28}{21,632} = 1+r$$
$$1.04 = 1 + r$$
$$1.04 - 1 = r$$
$$r = 0.04$$
$$G2 = G(1+r)^2$$
$$21632 = G\,(1+0.04)^2$$
$$21632 = G\,(1.04)^2$$
$$21632 = 1.0816G$$
$$\frac{21632}{1.0816} = G$$
$$G = 20{,}000$$

Answer A

Grade 11 unit eleven – Mathematical Applications in Business

2013 E.C

1. In leasing a building tax is required from rental income. The following is the tax rate to a total amount of rent per year in birr

Amount of taxable income	Up to 1,800	1,801 – 7,800	7,801 – 16,800	16.801 – 28,200
Tax rate	0%	10%	15%	20%
Deducted amount	0	180	570	1410

If a shop is rented at 2,350 birr per month for one year, what amount of tax is payable in birr?

A. 4,230 B. 5,460 C. 2,420 D. 3,750

Solution

$$Rent\ per\ year$$
$$= 2350 * 12$$
$$= 28200$$
$$Tax = 20\%$$
$$Deduct\ 1410$$
$$Tax = 28200 * 0.2 - 1410$$
$$= 5840 - 1410$$
$$= 4230$$
$$Answer\ A$$

2. On the average, which one of the following is less significant as a factor to save money in a saving institution in our context?

A. Save for medical cases
B. Save for retirement
C. Save for emergency needs
D. Save for charity and donations.

Solution

Save for retirement

Answer B

Grade 12 unit one – sequences and series

2009 E.C

1. Which of the following relations holds for the sequences: - 10, -3, 4, 11…?
   A. $a_n=a_n-1-8$
   B. $a_n=a_n+7$
   C. $a_n=a_{n-1}-7$
   D. $a_n=a_{n-1}+8$

Solution

$$-10\,,-3\,,4\,,11$$
$$a_n = a_{n-1}+7 \;\;, a_1 = -10$$
$$a_2 = a_1+7 = -10+7 = -3$$
$$a_3 = a_2+7 = -3+7 = 4$$
$$a_4 = a_3 \;+7 = 4+7 = 11$$
$$Answer\ B$$

2. Which of the following is the sum of the series $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\cdots$….?
   A. – 5 B. – 3 C. 3 D. 5

Solution

$$5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+\cdots$$
$$Geometric\,, r = -\frac{2}{3}\;\;, G_1 = 5$$
$$S_n = \frac{G_1}{1-r} = \frac{5}{1-\left(-\frac{2}{3}\right)} = \frac{5}{1+\frac{2}{3}} = \frac{5}{\frac{5}{3}} = 5*\frac{3}{5} = 3$$
$$Answer\ C$$

3. Suppose a radioactive material loses one-third of its mass per year. If its current mass is 81g, then how much will mass be just after 7 years?
   A. 27g B. 1/27g C. 128/27g D. 128/81g

Solution

$$G_0 = 81$$
$$G_1 = 81-\frac{1}{3}*81 = 81\left(1-\frac{1}{3}\right) = 81*\frac{2}{3} \quad ,r = \frac{G_1}{G_0} = \frac{81*\frac{2}{3}}{81} = \frac{2}{3}$$
$$G_7 = G_1 r^n = 81*\left(\frac{2}{3}\right)^7 = 81*\frac{2^7}{3^7}$$
$$= 81*\frac{2^7}{3^4*3^3} = \frac{81*2^7}{81*3^3} = \frac{2^7}{3^3} = \frac{128}{27}$$
$$Answer\ C$$

4. Which of the following is lower bounded for the sequence $\left\{\frac{n}{n-1}\right\}_{n=2}^{\infty}$
   A. 9/5 B. 1 C. 7/4 D. 5/3

Solution

$\left\{\frac{n}{n-1}\right\}_{n=2}^{\infty}$ has terms 2, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, − − − − 1

Or $\left\{\frac{n}{n-1}\right\}_{n=2}^{\infty}$ converges to 1,

Lower bound ≤ 1 glb $= 1$
upper bpund ≥ 2 ’ lub $= 2$

Answer B

Grade 12 unit one – sequences and series

2010 E.C

1. What are the greatest lower bound and least upper bound of the sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$ respectively

   A. -2 and 2 B. -2 and -3/2 C. -3/2 and 2 D. -2 and 3/2

   Solution

$$\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}=\left\{-2,\frac{3}{2},\frac{-4}{3},\frac{5}{4},---\right\}$$

$$\begin{cases} -2,\frac{-4}{3},\frac{-6}{5},,,---,=-1, \\ \frac{3}{2},\frac{5}{4},\frac{7}{6},---=1 \end{cases}$$

lower bound ≤ -2, glb= - 2

upper bound $\geq {}^{3}/_{2}$, lub=${}^{3}/_{2}$

Answer D

2. Which one of the following is a convergent sequence?

   A. $\left\{\frac{(-1)^n}{2}\right\}$ B. $\left\{\frac{1}{n}+\sin n\right\}$ C. $\left\{\frac{1+2^n}{3^n}\right\}$ D. $\left\{\frac{1+3^n}{2^n}\right\}$

   Solution

A. $\left\{\frac{(-1)^n}{2}\right\}=\left\{-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\ldots\right\}$divergent $\left(\pm\frac{1}{2}\right)$

B) $\left\{\frac{1}{n}+\sin n\right\}=\frac{1}{\infty}+\sin(\infty)=0+\sin(\infty)=\text{between}-1\text{ and }1\text{ , divergent}$

C) $\left\{\frac{1+2^n}{3^n}\right\}=\left(\frac{1}{3^n}\right)+\left(\frac{2}{3}\right)^n=\frac{1}{\infty}+\left(\frac{2}{3}\right)^\infty=0+0=0,\text{Convergent}$

D) $\left\{\frac{1+3^n}{2^n}\right\}=\left(\frac{1}{2^n}\right)-\left(\frac{3}{2}\right)^n=0-\infty=-\infty,$ Divergent

Answer C

3. What is the value of $\sum_{n\to 2}^{20}\left(\frac{1}{n-1}-\frac{1}{n}\right)$?

   A. 17/20 **B.** 19/20 C. 21/20 D. 23/20

   Solution

$$\sum_{n=2}^{20}\left(\frac{1}{n-1}-\frac{1}{n}\right)=\left\{1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{19}-\frac{1}{20}\right\}$$
$$=1-\frac{1}{20}=\frac{20-1}{20}=\frac{19}{20},$$

Answer B

4. What is the sum of the series $\sum_{n=1}^{\infty}3^n4^{-n}$?

   A. ∞ B. 3 C. 4 D. 3/16

   Solution

$$\sum_{n=1}^{\infty}3^n4^{-n}=\sum_{n=1}^{\infty}\frac{3^n}{4^n}$$
$$=\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n=\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+---$$

Geomatrie sequence with $G_1=\frac{3}{4},\quad r=\frac{3}{4}$

$$s_n=\frac{G_1}{1-r}$$
$$=\frac{\frac{3}{4}}{1-3/4}=\frac{\frac{3}{4}}{\frac{1}{4}}=3$$

Answer B

5. What is the value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(\frac{2k}{n}\right)\Big)^3 + 5\left(\frac{2k}{n}\right)\Big)$?

A. 4   B. 10   C. 14   D. 18

Solution

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(\frac{2k}{n}\right)\Big)^3 + 5\left(\frac{2k}{n}\right)\Big)$$

$$\sum_{k=1}^{n} \left(\frac{2}{n}\right)\left(\left(\frac{2k}{n}\right)^3 + 5\left(\frac{2k}{n}\right)\right) = \frac{2}{n}\left[\sum_{k=1}^{n} \left(\frac{2k}{n}\right)^3 + 5\sum_{k=1}^{n} \frac{2k}{n}\right] = \frac{2}{n}\left(\sum_{k=1}^{n} \frac{8k^3}{n^3} + \sum_{k=1}^{n} \frac{10}{n}\right)$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)}{4}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{2}{n}, \frac{8}{n}\sum_{k=1}^{n} k^3 + \frac{2}{n} x \frac{10}{n}\sum_{k=1}^{n} k = \frac{16}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) + \frac{20}{n}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{4(n+1)^2}{n^2} + \frac{10n\,(n+1)}{n} = \frac{4n^2+8n+4}{n^2} + \frac{10n^2+10^n}{n^2} = \frac{4n^2+8n+4+10n^2+10^n}{n^2}$$

$$= \frac{14n^2+8n+4}{n^2}, \lim_{n\to\infty}\left(\frac{14n^2+8n+4}{n^2}\right) = \lim_{n\to\infty}\left(14 + \frac{18}{n} + \frac{4}{n^2}\right) = 14$$

Answer C

Grade 12 unit one – sequences and series

2011 E.C

1. If the second and fifth terms of a geometric progression are - ½ and 1/16, respectively, what is the sum of the first eight terms of the sequence?

A. 256/255   B. 255/256   C. 128/85   D. 85/128

Solution

$$G_2 = \frac{-1}{2}, G_5 = \frac{1}{16}$$

$$G_2 = G_1 r \quad , G_5 = G_1 r^4$$

$$\frac{-1}{2} = G_1 r \qquad \frac{1}{`16} = G_1 r^4$$

$$\frac{G_5}{G_2} = \frac{\frac{1}{16}}{\frac{-1}{2}}$$

$$\frac{G_1 r^4}{G_1 r} = \frac{-1}{8}$$

$$r^3 = \frac{-1}{8}$$

$$r = \frac{-1}{2}$$

$$G_2 = G_1 r$$

$$\frac{-1}{2} = G_1\left(\frac{-1}{2}\right)$$

$$1 = G_1$$

$$S_8 = \frac{G_1\left(1-r^8\right)}{1-r}$$

$$= \frac{1(1-\left(-\frac{1}{2}\right)^8)}{1-\left(\frac{-1}{2}\right)}$$

$$= \frac{1-\frac{1}{256}}{1+\frac{1}{2}} = \frac{\frac{256-1}{256}}{\frac{3}{2}}$$

$$= \frac{255}{256} x \frac{2}{3} = \frac{85}{128},$$

Answer D

2. What is the sum of the infinite series $\sum_{k=0}^{\infty} 5\left(\frac{2^k+5^k}{10^k}\right)$

A. 65/4   B. 25/4   C. 5   D. 15

Solution

$$\sum_{k=0}^{\infty} 5\left(\frac{2^k+5^k}{10^k}\right) = 5\sum_{k=0}^{\infty}\left(\frac{2^k}{10^k} + \frac{5^k}{10^k}\right)$$

$$= 5\sum_{k=0}^{\infty}\left(\left(\frac{1}{5}\right)^k + \left(\frac{1}{2}\right)^k\right) = 5\left(1 + \frac{1}{5} + \frac{1}{25} + \cdots + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\ldots\right)$$

$$= 5\left(\frac{G_1}{1-r} + \frac{G_1}{1-r}\right) = 5\left(\frac{1}{1-\frac{1}{5}} + \frac{1}{1-\frac{1}{2}}\right) = 5\left(\frac{1}{4/5} + \frac{1}{1/2}\right)$$

$$= 5\left(\frac{5}{4} + 2\right) = 5\left(\frac{13}{4}\right) = \frac{65}{4}$$

Answer A

3. If a_1=2, a_2=6, a_3=10, a_4=14,…, then$\sum_{n=1}^{100} an$ =…
   A. 20,020 B. 20,200 C. 22,000 D. 20,000

Solution

$a_1 = 2\ \ , a_2 = 6 \,, a_3 = 10\ ...,\ \ d = 4$

$S_{100} = \frac{n}{2}((2A_l) + (n-1)d)$

$= \frac{100}{2}(2 * 2 + (100-1)4)$

$= 50(4 + 99 * 4)$

$= 50(4 + 396)$

$= 50(400) = 20000,$

Answer D

4. Everyday a person saves 5 cents more than the amount he saved on the previous day. His target is to save the total amount of 3225 cents by the end of 30 days. How much must be the starting saving to meet the target?
   A. 25 cents B. 35 cents C. 50 cents D. 60 cents

Solution

$$d = 5\ \ ,\ n = 30\,,\ \ \ S_{30} = 3225$$
$$s_{30} = \frac{n}{2}(2A_1 + (n-1)d) = 3225$$
$$\frac{30}{2}(2A_1 + (30-1)5) = 3225$$
$$15(2A_1 + 145) = 3225$$
$$30A_1 + 2175 = 3225$$
$$30A_1 = 3225 - 2175$$
$$30A_1 = 1050$$
$$A_1 = \frac{1050}{30}$$
$$A_1 = 35$$

Answer B

Grade 12 unit one – sequences and series

2012 E.C

1. A geometric series with first term a and common ratio r is convergent if
   A. r ≥ 1 B. r> 1 C. r ≤ 1 D. r < 1

Solution

$$\mathrm{s_n} = \sum_{k=1}^{n} \mathrm{Gn} = \frac{\mathrm{G_{1(1-r^n)}}}{1-\mathrm{r}}\text{, if r<1 eg r=}\frac{1}{2}, (\frac{1}{2})^{\infty} = 0$$
$$\mathrm{s_n} = \frac{\mathrm{G_1(1-0)}}{1-\mathrm{r}},\ \text{then it conveges to}\frac{\mathrm{G1}}{1-\mathrm{r}}\ \text{if } \mathrm{r} < 1,$$

Answer D

2. Which one of the following is an upper bound of the sequence $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$?
   A. 1 B. 0 C. 2 D. 4/3

Solution

$$\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty} = \left\{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4} - - - 1\right\}$$
$$\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty} \mathit{converges\ in\ to}\ 1$$
$$\text{upper bounds } \geq 2, \text{lup} = 2$$
$$\text{lower bonds } \leq 1, \text{glb} = 1$$

Answer C

3. The general term of a sequence is given by $a_n = (\frac{1}{2})^{1-n}$, then which one of the following is true about this sequence?
   A. 0 is one of its term
   B. Its terms are non-positive.
   C. Its terms are odd numbers
   D. Its terms are natural numbers

Solution

$$a_n = \left(\frac{1}{2}\right)^{1-n} = \left(\frac{1}{2}\right)^{0}, \left(\frac{1}{2}\right)^{-1}, \left(\frac{1}{2}\right)^{-2}, \left(\frac{1}{2}\right)^{-3}, ---$$

$$= 1, 2, 4, 8 - - - - - - - - -$$

= natural numbers,

Answer D

4. If the 1st and 4th terms of a geometric progression are 3 and 81 respectively, what is the common ratio of this progression?
   A. 1/3 B. 9 C. 1/9 D. 3

Solution

$$G_1 = 3$$

$$G_4 = 81$$

$$G_4 = G_1 r^{n-1}$$

$$81 = 3r^{4-1}$$

$$81 = 3r^3$$

$$27 = r^3$$

$$\sqrt[3]{27} = r$$

$$3 = r$$

Answer D

5. Consider a sequence whose terms are $a_1=6$ and $a_{n+1} = \frac{-1}{3} a_n$ for $n \in \mathbb{N}$. then, which one of the following is the sum of a_n
   A. $9(1-(-\frac{1}{3})10)$ C. $9(1-\frac{1}{3})11)$
   B. $\frac{9}{2}\left(1-\left(\frac{-1}{3}\right)^{10}\right)$ D. $\frac{9}{2}(1--\frac{1}{3})11)$

solution

$$a_1 = 6, a_{n+1} = \frac{-1}{3} a_n$$

$$a_1 = 6, a_2 = \frac{-1}{3} * 6 = -2$$

$$r = \frac{a_2}{a_1} = \frac{-2}{6} = \frac{-1}{3}$$

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}$$

$$s_{10} = \frac{6\left(1-\left(\frac{-1}{3}\right)^{10}\right)}{1-\left(\frac{-1}{3}\right)} = \frac{6\left(1-(\frac{-1}{3})^{10}\right)}{\frac{3+1}{3}}$$

$$\frac{6}{\frac{4}{3}}\left(1-(\frac{-1}{3})^{10}\right) = \frac{18}{4}\left(1-(\frac{-1}{3})^{10}\right)$$

$$= \frac{9}{2}\left(1-\left(\frac{-1}{3}\right)^{10}\right)$$

Answer B

6. A real number a is the limit of a sequence $\{a_n\}$ if
   - A. a is closer to every term of the sequence.
   - B. a ≤ $|an|$ for all n.
   - C. a is very close to all terms a_n from some arbitrary big n onwards.
   - D. $|a_n| \leq a_n \leq |a|$ for all

Solution

A real number a is the limit of a sequence $\{a_n\}$ if a_n gets closer and closer to a as n goes to infinity from some arbitrary big n onwards,

Answer C

7. Let $\{a_n\}$ be a sequence of real numbers. Then which one of the following is true about this sequence?
   - A. M is an upper bound of $\{a_n\}$ if $M \leq$ an for all a_n
   - B. M is a lower bound of $\{a_n\}$ if $a_n \leq M$ for all a_n
   - C. The sequence $\{a_n\}$is bounded if it has both an upper and a lower bound.
   - D. The sequence $\{a_n\}$is bounded if there exist k > 0 such that $k \leq |an|$ for all a_n.

Solution

If a sequere has both upper and lower bounds then it is bounded sequence,

Answer C

Grade 12 unit one – sequences and series

2013 E.C

1. Which one of the following is the fourth term of sequence $\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$?

A. -1 B. 0 C. 0.5 D. 1

Solution

$$a_{4=\cos\frac{4\pi}{2}}$$
$$=\cos(2\pi)$$
$$=\cos(360^0) = 1$$

Answer D

2. Which of the following is a geometric sequence?

A. 01, 0.01, 0.0011, 0.000111, … C. 0.1, 0.11, 0.12, 0.13, …

B. 1, 1.5, 2.5, 3.5, … D. 8, -4, 2, -1, …

Solution

$$\frac{-1}{2} = \frac{2}{-4} = \frac{-4}{8},$$
$$r = -\frac{1}{2}$$

Answer D

3. What is the least upper bound of the sequence $\left\{2+\frac{n}{n+1}\right\}_{n=1}^{\infty}$?

A. 0 B. 1 C. 2 D. 3

Solution

$$\{a_n\} = \left\{2+\frac{n}{n+1}\right\}_{n=1}^{\infty}$$
$$= 2+\frac{1}{2}, 2+\frac{2}{3}, 2+\frac{3}{4}, 2+\frac{4}{5} ---$$
$$= \frac{5}{2}, \frac{8}{3}, \frac{11}{4}, \frac{14}{5}, --------- 3$$
$$= 2.5, 2.66 --, ------- 3$$

Upper bond ≥ 3,

lub = 3

Answer D

4. A factory that produces electric cable had sales Birr 1,000,000 the first day. During each successive days the sales increased by Birr 10,000 every day. What is the total sales of the factory in the first 10 days?
   A. Birr 1,090,000 B. Birr 10,000,000 C. Birr 10,450,000 D. Birr 20,900,000

Solution

$A_{1=}$ 1,000,000
d= 10,000
$A_{10}=A_1+$ (n-1)d=1,000,000 (10-1)x 10,000
= 1,000,000 (9) (10,000) = 1,000,000 (90,000) = $\underline{\underline{9{,}000{,}000{,}000}}$

$$S_{10} = \frac{n}{2}(2A_1+(n-1)d) = \frac{10}{2}(2(1{,}000{,}000) + (10-1)10{,}000)$$
$$= 5(2{,}000{,}000 + 90{,}000) = 5(2{,}090{,}000) = 10{,}450{,}000$$

Answer C

5. Which of the following series is convergent?
   A. $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^{-n}$ B. $\sum_{n=1}^{\infty}5^{11-2n}$ C. $\sum_{n=1}^{\infty}2^{2n-5}$ D. $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{n}$

Solution

$$\text{A) } \left(\frac{2}{5}\right)^{-n} = \left[\left(\frac{5}{2}\right)^{n}\right]_{n=1}^{\infty} = \qquad = \infty - \text{divegent}$$
$$\text{B) } (5^{11-2n})_{n=1}^{\infty} = \frac{5^{11}}{5^{2n}} = \frac{5^{11}}{\infty} \qquad = 0 - \text{convegent}$$
$$\text{C) } (2^{2n-5})_{n=1}^{\infty} = \frac{2^{2n}}{2^{5}} = \frac{\infty}{32} \qquad = \infty - \text{divergent}$$
$$\text{D) } \left(\left(\frac{3}{2}\right)^{n}\right)^{\infty} \qquad = \infty - \text{divergent}$$

Answer B

Grade 12 unit two – introduction to limits and continuity

2009 E.C

1. Suppose f is continuous on [2, 6] and the only solutions of the equation f(x) =7 are x=2 and x=5. If f(3)=9, then one of the following **CAN NOT** be the value of f(4)?
   A. 5 B. 7.5 C. 8 D. 9

Solution

$$f \text{ is continues on } [2,6], f(3) = 9, f(2) = f(5) = 7$$
$$f(x) > 7 \text{ for } (2,5) \text{since } f(x) = 7 \text{ only in } x = 2,5 \text{ and } f(3) = 9$$
$$\Rightarrow f(4) > 7, 5 \text{ can not be } f(4)$$

Answer A

2. what is the value of k so that f(x)= $\begin{cases} \frac{\tan 2x}{x} & if\ x > 0 \\ k - e^{x} & if\ x \le 0 \end{cases}$ is continues
   A. 2 B. 3 C. 1 D. 0

Solution

$$f(x) = \begin{cases} \frac{\tan 2x}{x} & if\ x > 0 \\ k - e^{x} & if\ x \le 0 \end{cases}$$

If f is continues, then both sides limit at x = 0 are equal

$$\lim_{x\to 0^+}\left(\frac{\tan 2x}{x}\right) = \lim_{x\to 0^+}(k - e^{x})$$
$$2 = k - e^{0}$$
$$2 = k - 1$$
$$k = 3$$

Answer B

$$\lim_{x\to 0^+}\left(\frac{\tan 2x}{x}\right)$$
$$= \lim_{x\to 0^+}\frac{2}{2}\left(\frac{\tan 2x}{x}\right)$$
$$=2 \lim_{x\to 0^+}\left(\frac{\tan 2x}{2x}\right)$$
$$=2*1=2$$

3. If f is continuous at x=0 and g(x)= $\sqrt{x}\left(2f(x)+\frac{3}{\sqrt{x}}\right)$for all x>0, then find the value $\lim_{x\to 0^+} g(x)$?

A. 0 B. 3 C. 1 D. 5

Solution

$$g(x) = \sqrt{x}\left(2f(x)+\frac{3}{\sqrt{x}}\right) = 3$$

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+}\sqrt{x}\left(2f(x)+\frac{3}{\sqrt{x}}\right) = \lim_{x\to 0^+}\left(2\sqrt{x}f(x)+3\right) = (2(0)f(0)+3) = 0+3 = 3$$

Answer C

4. What is the value of $\lim_{x\to 0^+}\frac{sinx\ cos2x}{x^2+3x}$?

A. 1/3 B. 2/3 C. 1 D. 2

Solution

$$\lim_{x\to 0^+}\frac{sinx\ cos2x}{x^2+3x} = \lim_{x\to 0^+}\frac{sinx\cos 2x}{x(x+3)} = \lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)\frac{\cos 2x}{(x+3)}$$

$$= \lim_{x\to 0^+}\frac{\sin x}{x} * \lim_{x\to 0^+}\frac{cos\ 2x}{x+3} = 1 * \frac{1}{0+3} = 1 * \frac{1}{3} = \frac{1}{3} \quad ,\ cos0^0 = 1,$$

Answer A

5. . If $a \neq 0$, then what is the value of $\lim_{x\to 0^a}\frac{x^2-a^2}{x^4-a^4}$

A. $1/a^2$ B. $1/2a^2$ C. $1/6a^2$ D. 0

Solution

$$\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4} = \lim_{x\to a}\frac{x^2-a^2}{(x^2)^2-(a^4)^2} \quad , \qquad a^2-b^2 = (a+b)(a-b)$$

$$\lim_{x\to a}\frac{x^2-a^2}{(x^2-a^2)(x^2+a^2)} = \lim_{x\to a}\frac{1}{x^2+a^2} = \frac{1}{a^2+a^2} = \frac{1}{2a^2}$$

Answer B

6. What is the sum of $\sum_{n=1}^{30}(-1)^n\left(\frac{1}{n}+\frac{1}{1+n}\right)$?

A. -29/30 B. 29/30 C. 30/31 D. -30/31

Solution

$$\sum_{n=1}^{30}(-1)^n\left(\frac{1}{n}+\frac{1}{1+n}\right)$$

$$= -\left(1+\frac{1}{2}\right)+\left(\frac{1}{2}+\frac{1}{3}\right)-\left(\frac{1}{3}+\frac{1}{4}\right)+\cdots+\left(\frac{1}{29}+\frac{1}{30}\right)+\left(\frac{1}{30}+\frac{1}{31}\right)$$

$$= -1-\frac{1}{2}+\frac{1}{2}+\frac{1}{3}-\frac{1}{3}+\frac{1}{4}+\cdots-\frac{1}{29}-\frac{1}{30}+\frac{1}{30}+\frac{1}{31} = -1+\frac{1}{31} = \frac{-31+1}{31} = -\frac{30}{31}$$

Answer D

7. Which of the following is a convergent sequence?

A. $\left(\frac{5}{3}\right)^n$ B. $\left(\frac{2n}{n+1}\right)$ C. $\frac{n^2}{n+1}$ D. $\frac{(-1)^n}{3}$

Solution

$$\left(\frac{5}{3}\right)^n \Longrightarrow \lim_{n\to\infty}\left(\frac{5}{3}\right)^n = \infty, divergent$$

$$\left(\frac{2n}{n+1}\right) \Longrightarrow \lim_{n\to\infty}\left(\frac{2n}{n+1}\right) = \lim_{n\to\infty}\left(\frac{\frac{2n}{n}}{\frac{n}{n}+\frac{1}{n}}\right) = \lim_{n\to\infty}\left(\frac{2}{1+\frac{1}{n}}\right) = \frac{2}{1+0} = 2 \quad , \quad Convergent$$

$$\frac{n^2}{n+1} \Longrightarrow \lim_{n\to\infty}\left(\frac{n^2}{n+1}\right) = \lim_{n\to\infty}\left(\frac{\frac{n^2}{n^2}}{\frac{n}{n^2}+\frac{1}{n^2}}\right) = \lim_{n\to\infty}\left(\frac{1}{\frac{1}{n}+\frac{1}{n^2}}\right) = \frac{1}{0} = \infty \quad Divergent$$

$$\frac{(-1)^n}{3} \Longrightarrow \lim_{n\to\infty}\left(\frac{(-1)^n}{3}\right) = \pm\frac{1}{3} \quad Divergent$$

Answer B

Grade 12 unit two – introduction to limits and continuity

2010 E.C

1. What is the value of $\lim_{x \to 0} \to \frac{1}{x^2}\sin^2(x/2)$?

   A. 4 B. ½ C. 2 D. ¼

Solution

$$\lim_{x \to 0} \to \frac{1}{x^2}\sin^2(x/2) = \lim_{x \to 0} \frac{1}{4}\frac{\sin^2(x/2)}{\frac{1}{4}x^2} = \frac{1}{4}\lim_{x\to 0}\left(\frac{\sin(\frac{x}{2})}{\frac{x}{2}}\right)^2 = \frac{1}{4} * 1 = \frac{1}{4}$$

Answer D

2. If $f(x) = \frac{|x|}{x}$ and $g(x) = \frac{x+2}{x^3+4x}$, then what is the value of $\lim_{x\to -2}(f(x) + g(x)))$

   A. -7/8 B. -9/8 C. 9/8 D. ∞

Solution

$$f(x) = \frac{|x|}{x}, \qquad \lim_{x\to -2}\frac{|x|}{x} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$$

$$g(x) = \frac{x+2}{x^3-4x}, \quad \lim_{x\to -2}\frac{(x+2)}{x^3-4x} \quad \lim_{x\to -2}\frac{(x+2)}{x(x^2-4)} \quad \lim_{x\to -2}\frac{(x+2)}{x(x+2)(x-2)}$$

$$= \lim_{x\to -2}\frac{1}{x(x-2)} = \frac{1}{-2(-2-2)} = \frac{1}{-2(-4)} = \frac{1}{8}$$

$$\lim_{x\to -2}(f(x) + g(x)) = \lim_{x\to -2} f(x) + \lim_{x\to -2} g(x) = -1 + \frac{1}{8} = \frac{-7}{8}$$

Answer A

3. Let $\{a_n\}$ be a sequence with $a_1 = a$, $a_2 = f(a_1) = f(a)$, $a_3 = f(a_2) = f(f(a))$,….,$a_{n+1} = f(a_n)$, where f is continuous function. If $\lim_{n\to\infty} a_n = 5$, what is f (5)?

   A. 5^{n+1} B. 5^n C. 5 D. 1

Solution

$$a_1 = a, a_2 = f(a_1) = f(a)$$

$$a_3 = f(a_2) = f(f(a))$$

$$a_{n+1} = (f(a_n))$$

$$f(5) = f\left(\lim_{n\to\infty} a_n\right) = \lim_{n\to\infty} a_n = 5$$

Answer C

4. Find a and b so that the function $f(x) = \begin{cases} x+1 \text{ if } x < 1 \\ ax+b \text{ if } 1 \le x < 2 \\ 3x \text{ if } x \ge 2 \end{cases}$ is continuous everywhere?

   A. a=4, b=2 B. a=-4, b=-2 C. a=4, b= -2 D. a=-4, b=2

Solution

$$f(x) = \begin{cases} x+1 \text{ if } x < 1 \\ ax+b \text{ if } 1 \le x < 2 \\ 3x \text{ if } x \ge 2 \end{cases}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x)$$

$$\lim_{x\to 1}(ax+b) = \lim_{x\to 1}(x+1)$$

$$a + b = 2 \ldots\ldots i$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$$

$$\lim_{x\to 2}(3x) = \lim_{x\to 2}(ax+b)$$

$$6 = 2a + b \ldots\ldots ii$$

$$-\begin{cases} a+b=2 \\ \underline{2a+b=6} \end{cases} \Rightarrow \begin{cases} -a-b=-2 \\ \underline{2a+b=6} \end{cases}$$

$$a = 4 \quad , \quad a + b = 2$$

$$4 + b = 2$$

$$b = -2$$

Answer C

Grade 12 unit two – introduction to limits and continuity

2011 E.C

1. What is the limit of the sequence 1, $\frac{2}{2^2-1^2}, \frac{3}{3^2-2^2}, \frac{4}{4^2-3^2}$,…..?

   A. ½ B. 1 C. 0 D. 3/2

Solution

$$a_1 = 1 \quad , \quad a_2 = \frac{2}{2^2-1^2} \quad , \quad a_3 = \frac{3}{3^2-2^2}$$

$$a_n = \frac{n}{n^2-(n-1)^2} = \frac{n}{n^2-n^2+2n-1} = \frac{n}{2n-1}$$

$$\lim_{n\to\infty}\left(\frac{n}{2n-1}\right) = \lim_{n\to\infty}\left(\frac{\frac{n}{n}}{\frac{2n}{n}-\frac{1}{n}}\right) = \lim_{n\to\infty}\left(\frac{1}{2-\frac{1}{n}}\right) = \frac{1}{2},$$

Answer A

2. What is the greatest lower bound of the sequence $\left\{(-1)^n\left(\frac{1}{n+1}\right)\right\}^{\infty}$

   A. -1 B. 0 C. ½ D. -1/2

Solution

$$\left\{(-1)^n\left(\frac{1}{n+1}\right)\right\}_{n=0}^{\infty} = 1, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \dots\dots,$$

$$Upper\ boundary\ \geq 1\ ,$$

$$Lower\ boundary\ \leq -\frac{1}{2},\ g\ lb = -\frac{1}{2},$$

Answer D

3. What is the value of $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$

   A. 2 B. -2 C. 3 D. The limit doesn't exist.

Solution

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{X\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$$

$$= \lim_{x\to 2}\frac{x^2+2x+4}{x+2} = \frac{2^2+2(2)+4}{2+2} = \frac{4+4+4}{4} = \frac{12}{4} = 3$$

Answer C

4. What is the value of the left hand side limit $\lim_{x\to 0^-}\frac{sinx}{x^2}\sqrt{4x^3+9x^2}$

   A. 2 B. 3 C. – 3 D. Does not exit

Solution

$$\lim_{x\to 0^-}\frac{sinx}{x^2}\sqrt{4x^3+9x^2} = \lim_{x\to 0^-}\frac{sinx}{x^2}\sqrt{x^2(4x+9)}$$

$$\lim_{x\to 0^-}\frac{sinx}{x^2}|x|\sqrt{(4x+9)} = \lim_{x\to 0^-}\left(\frac{sinx}{x}\right)\left(\frac{|x|}{x}\right)\left(\sqrt{4x+9}\right)$$

$$= 1 * -1 * \sqrt{0+9} = 1 * -1 * 3 = -3$$

Answer C

5. What is the value of $\lim_{h\to 0}\frac{8\left(\frac{1}{2}+h\right)^2-8\left(\frac{1}{2}\right)^2}{h}$?

   A. 0 B. 4 C. The limit does not exist D. 8

Solution

$$\lim_{h\to 0}\frac{8\left(\frac{1}{2}+h\right)^2-8\left(\frac{1}{2}\right)^2}{h} = \lim_{h\to 0}\frac{8\left(\frac{1}{4}+h+h^2\right)-8\left(\frac{1}{4}\right)}{h}$$

$$= \lim_{h\to 0}\frac{8\left(\frac{1}{4}\right)+8h+8h^2-8\left(\frac{1}{4}\right)}{h} = \lim_{h\to 0}\frac{8h+8h^2}{h} = \lim_{h\to 0}\frac{h(8+8h)}{h} = \lim_{h\to 0}(8+8h)$$

$$= 8 + 8(0) = 8$$

Answer D

6. What is the value of $\lim\limits_{n\to\infty}\frac{(3^n+2^n)}{6^n}$?

A. 0 B. 1 C. ∞ D. 5/6

Solution

$$\lim_{n\to\infty}\frac{(3^n+2^n)}{6^n} = \lim_{n\to\infty}\left(\frac{3^n}{6^n}+\frac{2^n}{6^n}\right) = \lim_{n\to\infty}\left(\left(\frac{3}{6}\right)^n+\left(\frac{2}{6}\right)^n\right) = \lim_{n\to\infty}\left(\frac{1}{2}\right)^n + \lim_{n\to\infty}\left(\frac{1}{3}\right)^n = 0+0=0$$

Answer A

Grade 12 unit two – introduction to limits and continuity

2012 E.C

1. $\lim\limits_{x\to\infty}\left(\frac{x-2}{x}\right)^{-2x+1}$ = ------------

A. e^4 B. e^{-4} C. e^2 D. e^{-2}

Solution

$$\lim_{x\to\infty}\left(\frac{x-2}{x}\right)^{-2x+1} = \lim_{x\to\infty}\left(1-\frac{2}{x}\right)^{-2x+1} = (e^{-2})^{-2} = e^4$$

$$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x+n} = e}$$

Answer A

2. Given 0 < a_n < 1/n for each n= 1, 2, 3,…. Which one of the following is true?

A. $\lim\limits_{n\to\infty}$ an < 0 B. $\lim\limits_{n\to\infty}\frac{1}{n}=0$ C. $\lim\limits_{n\to\infty}$ a_n does not exist. D. $\lim\limits_{n\to\infty}\frac{1}{n}a_n > 0$

Solution

$$0 < a_n < \frac{1}{n}$$

$$\lim_{n\to\infty}\frac{1}{n}=0$$

Answer B

3. If f(x)= $f(x) = \begin{Bmatrix} k2^x, x>2 \\ x^2-x+1, x\le 2\end{Bmatrix}$ then for what value of k is f continuous at x=2?

A. 7/4 B. 4 C. 2 D. 7/2

Solution

$$f(x) = \begin{Bmatrix} k2^x, x>2 \\ x^2-x+1, x\le 2\end{Bmatrix}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\lim_{x\to 2} k2^x = \lim_{x\to 2}(x^3-x+1)$$

$$k2^2 = 2^3-2+1$$

$$4k = 8-2+1$$

$$4k = 7, k=\frac{7}{4}$$

Answer A

4. $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^x$ is equal to

A. 1/e B. e^5 C. 5e D. e^{-5}

Solution

$$\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^x$$

$$\text{Let } \frac{5}{x}=\frac{1}{y} \quad \text{as} \begin{matrix} x\to\infty \\ y-\infty\end{matrix}$$

$$\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^{5y} = \lim_{y\to\infty}\left(\left(1+\frac{1}{y}\right)^y\right)^5 = e^5$$

Answer B

5. Which one of the following is true about limit of a function?
   A. Limit of a constant function is always zero.
   B. Right and left hands limits are the same for every function.
   C. Initiatively, the limit of a function f is a y-value L to which f(x) approaches as x approaches to some specified number.
   D. Whenever the limit exists for a function it is the same as functional value.

Solution

A. $-$ limit of a constant function is the constant number itself eg $\lim_{x\to5}(3) = 3$,false .

B. Both sides limit may or may not be equal, it they are equal the limit exists, false .

C. $\lim_{x\to a} f(x) = L$, true .

D. $\lim_{x\to1}\frac{x^2-1}{x-1} = \lim_{x\to1}\frac{(x-1)(x+1)}{x-1} = \lim_{x\to1}(x+1) = 2 \neq f(1)$,f(1)=∄ , $\lim_{x\to a} f(x) \neq f(a)$false

Answer C

6. Which one of the following is equal to $\lim_{n\to\infty}\frac{\sin n}{n^2}$?

A. -1 B. 1 C. π D. 0

Solution

$$\lim_{n\to\infty}\frac{\sin n}{n^2} = \lim_{n\to\infty}\left(\frac{1}{n}\right)\left(\frac{\sin n}{n}\right)$$

$$= \frac{1}{\infty}\lim_{n\to\infty}\frac{\sin n}{n} = 0 * \left(\lim_{n\to\infty}\frac{\sin n}{n}\right)$$

$$= 0$$

Answer D

7. If the functions f and g are continuous at x=c, then which of the following combinations s continues at x=c?

A. $\sqrt{g}$ B. f/g C. f^{-1} D. f^2

Solution

$-$If f is continues at C , then f^2 is continues, $\lim_{x\to a} f^2 = \lim_{x\to a} f \lim_{x\to a} f$

Answer D

8. What is the value of $\lim_{x\to0}\frac{\sin 4x}{\sin 6x}$?

A. 3/2 B. 2/3 C. 24 D. 6

Solution

$$\lim_{x\to0}\frac{\sin 4x}{\sin 6x} = \frac{\lim_{x\to0}\frac{\sin 4x}{x}}{\lim_{x\to0}\frac{\sin 6x}{x}} = \frac{\lim_{x\to0}\frac{4\sin 4x}{4x}}{\lim_{x\to0}\frac{6\sin 6x}{6x}} = \frac{4\lim_{x\to0}\frac{\sin 4x}{4x}}{6\lim_{x\to0}\frac{\sin 6x}{6x}} = \frac{4*1}{6*1} = \frac{2}{3}$$

Answer B

Grade 12 unit two – introduction to limits and continuity

2013 E.C

1. Which one of the following is a null sequence?

A. $\left\{1-\frac{1}{n}\right\}$ B. $\left\{n\sin\left(\frac{1}{n}\right)\right\}$ C. $\left\{\frac{\cos n}{n}\right\}$ D. $\left\{\frac{n}{n+1}\right\}$

Solution

A) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right) = 1$

C) $\lim_{n\to\infty}\left(\frac{\cos n}{n}\right) = 0$

B) $\operatorname{lin}_{n\to\infty} n\sin\left(\frac{1}{n}\right) = 1$

D)) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$

Answer C

2. Suppose {a_n} and {b_n} are sequences with $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$, then, Then which of the following is NOT necessarily true?
   - A. $\lim_{n\to\infty}(a_{n-}b_n) = L - M$
   - B. $\lim_{n\to\infty}(3\,a_{n+}2b_n) = 3L + 2M$
   - C. $\lim_{n\to\infty} a_n b_n = LM$
   - D. $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{L}{M}$

<u>Solution</u>

$$\underset{n\to\infty}{\lim a_n} = L\ ,\ \underset{n\to\infty}{\text{lin}}\ b_n\text{=m}$$

$$\text{A)}\underset{n\to\infty}{\lim a_n}(a_n - b_n) = L - m\ \text{True}$$

$$\text{B)}\underset{n\to\infty}{\lim a_n}(3a_n + 2b_n) = 3L + 2m\ \text{True}$$

$$\text{C)}\ \underset{n\to\infty}{\lim a_n}(a_{nb_n}) = LM\ \text{True}$$

$$\text{D)}\ \underset{n\to\infty}{\lim a_n}\left(\frac{a_n}{b_n}\right) = \frac{L}{M}\text{, not necessarily true it is ∄ if m=0}$$

Answer D

3. $\lim_{n\to\infty}\frac{3x^2+2x}{5x-x^2}$ is equal to
   - A. -3
   - B. -1
   - C. 0
   - D. 1

<u>Solution</u>

$$\lim_{x\to\infty}\frac{3x^2 + 2x}{5x - x^2}$$

take coefficients of x^2 which is $= \frac{3}{-1} = -3$

Answer A

4. Let f be a function defined on a closed bounded interval [a, b]. Which of the following describes that f is continuous on [a, b]?
   - A. f is continuous at each point in (a, b) and at both a and b.
   - B. f is continuous on (a, b), f is continuous from the right at a and from the left at b.
   - C. f is continuous on (a, b), f is continuous from the left at a and from the left at b.
   - D. f is continuous (a, b), f is continuous from the left at a and from the right at b.

<u>Solution</u>

f is contonues [a, b]

f is continues on (a, b), at a and at b,

Answer A

Grade 12 unit three – introduction to differential calculus

2009 E.C

1. Let f be differentiable function with $f(1) = -1$, $f^l(1) = 1$, $g(x) = (f(2x+1)+2)^2$, then what is the value of $g^1(0)$?
   - A. 4
   - B. 2
   - C. – 2
   - D. – 4

<u>Solution</u>

$$f(1) = -1\quad ,f^l(1) = 1\qquad\qquad g(x) = (f(2x+1)+2)^2\quad ,g^l(0) = ?$$

$$g(x) = (f(2x+1)+2)^2$$

$$g^l(x) = 2(\,f(2x+1)+2\,) * f^l(2x+1) * (2x+1)^l$$

$$g^l(x) = 2(\,f(2x+1)+2\,) * f^l(2x+1) * 2$$

$$g^l(0) = 2(f(1)+2) * f^l(1) * 2$$

$$g^l(0) = 2\ (-1+2) * 1 * 2$$

$$g^l(0) = 2\ (1) * 1 * 2$$

$$g^l(0) = 4$$

Answer A

2. If f(x)= ln(x^2+2), then, what is the value of $f^{ll}(x)$)?
   A. 3/2 B. 5/9 C. 2/3 D. 2/9

Solution

$$f(x) = \ln(x^2+2)$$
$$f^{l}(x) = \frac{2x}{x^2+2}$$
$$f^{ll}(x) = \frac{2\,(x^2+2) - 2x\,(2x)}{(x^2+2)^2} = \frac{2x^2+4-4x^2}{(x^2+2)^2} = \frac{4-2x^2}{(x^2+2)^2}$$
$$f^{ll}(1) = \frac{4-2}{3^2} = \frac{2}{9}$$

Answer D

3. If x^2+xy=10, then what is the value of $\frac{dy}{dx}\ at\ x = 2\quad, y = 3$
   A. -7/2 B. 2/7 C. 3/2 D. 7/2

Solution

$$x^2+xy = 10\quad,\frac{dy}{dx}\ at\ x = 2\quad, y = 3$$
$$x^2+xy = 10, \frac{d}{dx}(x^2+xy = 10)$$
$$(\,2x+y\,) + x\frac{dy}{dx} = 0$$
$$x\frac{dy}{dx} = -(2x+y), \frac{dy}{dx} = -\frac{(\,2x+y\,)}{x}$$
$$\frac{dy}{dx} = -\frac{(4+3)}{2} = -\frac{7}{2}$$

Answer A

.

4. What is the equation of the tangent into the graph of f(x)=3x^2+4x-5 at (1,2)
   A. 10x-y-8=0
   B. -10x+y-8=0
   C. -10x-y-8=0
   D. 10x+y-8=0

Solution

$$f(x) = 3x^2+4x-5\quad,(1\,,2)$$
$$f^{l}(x) = 6x+4$$
$$slope\ of\ the\ tangent\ line, m = f^{l}(1) = 6+4 = 10$$
$$equation\ of\ the\ tangent\ line, \frac{y-2}{x-1} = 10$$
$$10x-10 = y-2$$
$$10x-y-8 = 0$$

Answer A

5. If f(x) =π^2+1, then what is the value of f'(x)?
   A. 5 B. 2π C. 4 D. 0

Solution

$$f(x) = \pi^2+1\,, note\ that\ \pi^2+1\ \ is\ constant\ number\ and\ derivative\ of\ constant\ number\ is\ zero$$
$$f^{l}(x) = 0$$

Answer D

Grade 12 unit three – introduction to differential calculus

2010 E.C

1. If a function f is differentiable at a, then what is the value of $\lim_{x\to a} x$

A. $f^{l}(a)$ B. 0 C. f(a) D. $f^{l}(a)$-f(a)

Solution

If f is differentiable at a, then it is continues at a.

If f is continues at a, then $\lim_{x\to a} x = f(a)$

Answer C

2. What is the slope of the tangent line to graph of $f(x)=2e^x + \sin x + 2$ at (0, 5)

A. 4 B. 3 C. 2 D. 5

Solution

$$f(x)=2e^x + \sin x+ 2 \text{ at } (0, 5)$$

$$f^l(x) = 2e^x + \cos x$$

$$m = f^l(0) = 2e^0 + \cos 0 = 2 + 1 = 3, \cos 0=1 \text{ and } e^0=1$$

Answer B

3. Let $f(x)= \ln x\sqrt{x}$. Then what is $f^{l}(x)$ equal to

A. 3/2x B. $\sqrt{x}/2$ C. 2x/3 D. $2/x\sqrt{x}$

Solution

$$f(x) = \ell n(x\sqrt{x}) = \ell n\left(x^{3/2}\right),$$

$$f^l(x)=\frac{1}{x^{3/2}}*(\frac{3}{2}x^{1/2})= \frac{3}{2x^{2/2}}=\frac{3}{2x}$$

Answer A

4. Which one of the following is equal to $\frac{d}{dx}\log_2\sqrt{6x}$?

A. 3x/2 ln 2 B. 3/2x ln 2 C. $\frac{1}{2x\ell n2}$ D. 1/6x ln 2

Solution

$$\frac{d}{dx}\log_2\sqrt{6x} = \frac{1}{\sqrt{6x}\,\ell n2} * \left(\sqrt{6x}\right)^{l}$$

$$= \frac{1}{\sqrt{6x}\,\ell n2} * \frac{1}{2\sqrt{6x}} * 6 = \frac{6}{2(6x)\ell n2} = \frac{1}{2x\ell n2}$$

Answer C

5. If $f(x) = k\ell nx + e^{\sin x}$ and $f^{ll}(\pi)=-1$, then what is the value of k?

A. Π B. π^2 C. $2\pi^2$ D. 2π

Solution

$$\text{If } f(x) = k\ell nx + e^{\sin x}, f^{11}(\pi) = -1, k =?$$

$$f^1(x) = \frac{k}{x} + \cos x.e^{\sin x}$$

$$f^{11}(x) = \frac{-k}{x^2} + (\cos x)^1 e^{\sin x} + \left(e^{\sin}\right)^1 \cos x \qquad ,\left(\frac{1}{x}\right)^1 = \frac{-1}{x^2}$$

$$f^{11}(x) = \frac{-k}{x^2} - \sin x\, e^{\sin x} + \cos^2 x e^{\sin x}$$

$$f^{11}(\pi) = -1$$

$$-1 = \frac{-k}{\pi^2} - \sin\pi e^{\sin\pi} + \cos^2\pi e^{\sin\pi}, \begin{matrix}\sin\pi = 0\\ \cos\pi = -1\end{matrix}$$

$$-1 = \frac{-k}{\pi^2} - 0 + (-1)^2 e^0$$

$$-1 = \frac{-k}{\pi^2} + 1,$$

$$-2 = \frac{-k}{\pi^2}, 2\pi^2 = k$$

Answer C

Grade 12 unit three – introduction to differential calculus

2011 E.C

1. If $f(x) = \frac{1}{x}$, then what is the value of $f^{(n)}(x)$?

   A. $f^n(x) = \frac{(-1)^n n!}{x^n}$ C. $f^n(x) = \frac{(-1)^n n!}{x^{n-1}}$

   B. $f^n(x) = \frac{(-1)^n n!}{x^{n+1}}$ D. $f^n(x) = \frac{(-1)^n (n+1)!}{x^{n+1}}$

Solution

$$f(x) = \frac{1}{x}$$

$$f^l(x) = (x^{-1})^l = -1x^{-2} = -\frac{1}{x^2} ------ \frac{(-1)(1!)}{x^{1+1}}$$

$$f^{ll}(x) = \left(-\frac{1}{x^2}\right)^l = (-1x^{-2})^l = 2x^{-3} = \frac{2}{x^3} --- \frac{(-1)^2 2!}{x^{2+1}}$$

$$f^{lll}(x) = \left(\frac{2}{x^3}\right)^l = (2x^{-3})^l = -6x^{-4} = \frac{-6}{x^4} --- \frac{(-1)^3\ 3!}{x^{3+1}}$$

$$f^n(x) = \frac{(-1)^n n!}{x^{n+1}},$$

Answer B

2. If f(x) =ln($2^{\tan x}$), then what is the value of $f^l(0)$?

   A. -2ln2 B. ln2 C. $\frac{ln2}{2}$ D. 1

Solution

$$f(x) = ln(2^{\tan x})$$

$$f^1(x) = \frac{1}{2^{\tan x}} * (2^{\tan x})^l = \frac{1}{2^{\tan x}} * 2^{\tan x} \ln 2 * (\tan x)^l$$

$$f^1(x) = \frac{1}{2^{tanx}} * 2^{tanx}\, ln2 * sec^2 x = ln2sec^2 x$$

$$f^l(0) = ln2sec^2\, 0 = ln2(1) = ln2, \quad \text{sec0=}\frac{1}{cos0} = 1$$

Answer B

3. Let f(x)=2e^x-k sin x + 1. If the equation of the tangent line to the graph f at (0, 3) is y= 5x+3, then what is the value of k?

   A. 2 B. – 2 C. – 5 D. – 3

Solution

$$f(x) = 2e^x - ksinx + 1 \quad , \ y = 5x + 3\, , m = 5 \quad , at\ (0{,}3)$$

$$f^l(0) = m \quad , \ f^l(x) = 2e^x - kcosx$$

$$2e^0 - kcos0 = 5, \text{ note } e^0 = 1, \cos 0=1$$

$$2 - k = 5$$

$$k = 2 - 5$$

$$k = -3$$

Answer D

4. If $h(x) = \sqrt{1 + \sqrt{x}}$, then which one of the following is equal to $h^l(x)$?

   A. $\frac{1}{2\sqrt{1}+\sqrt{x}}$ B. $\frac{x}{4\sqrt{x}+x\sqrt{x}}$ C. $\frac{x}{2\sqrt{x}+x\sqrt{x}}$ D. $\frac{1}{4\sqrt{x+x\sqrt{x}}}$

Solution

$$h(x) = \sqrt{1 + \sqrt{x}}$$

$$h^l(x) = \frac{1}{2\sqrt{1+\sqrt{x}}} * \frac{1}{2\sqrt{x}} = \frac{1}{4\sqrt{x+x\sqrt{x}}}$$

Answer D

Grade 12 unit three – introduction to differential calculus

2012 E.C

1. Let f(x)= x^2+2x+3. For what a number c ∈(1, 3), f'(c) =$\frac{f(3)-f(1)}{2}$?

   A. -6 B. 6 C. -2 D. 2

   Solution

$$f(x) = x^2 + 2 + 3 \ \ f^1(c) = \frac{f(3)-f(1)}{2} \quad , \qquad f^1(x) = 2x + 2$$

$$2(c) + 2 = \frac{(3^2+2(3)+3)-(1^2+2(1)+3)}{2}$$

$$2(2c+2) = (9+6+3) - (1^2+2(1)+3)$$

$$4c + 4 = 18 - 6$$

$$4c = 18 - 6 - 4$$

$$4c = 8$$

$$c = 2$$

Answer D

2. For what value of c the conclusion of Rolle's Theorem is satisfied for the function f(x)= $2x-x^2-x^3$ on [-2, 1]?

   A. $\frac{-1\pm\sqrt{5}}{3}$ B. $\frac{1\pm\sqrt{7}}{3}$ C. $\frac{1\pm\sqrt{5}}{3}$ D. $\frac{-1\pm\sqrt{7}}{3}$

   Solution

$$f(x) = 2x - x^2 - x^3$$

$$f^l\,(c) = 0\,, f^l(x) = 2 - 2x - 3x^2$$

$$2 - 2c - 3c^2 = 0$$

$$3c^2 + 2c - 2 = 0$$

$$c = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-2\pm\sqrt{2^2-4(3)(-2)}}{2(3)} = \frac{-2\pm\sqrt{4+24}}{6}$$

$$= \frac{-2\pm\sqrt{28}}{6} = \frac{-2\pm2\sqrt{7}}{6} = \frac{-1\pm\sqrt{7}}{3},$$

Answer D

3. Which one of the following is an interval at which the function $f(x) = \frac{1}{x-1} - \sqrt{4-x^2}$ is differentiable?

   A. (-2, 1) U (1, 2) B. (-2, 2) C. (-∞, −2 U (2, ∞) D. [-2, 1] U (1, 2)

   Solution

$$f(x) = \frac{1}{x-1} - \sqrt{4-x^2}$$

$$f^l(x) = \frac{-1}{(x-1)^2} + \frac{2x}{2\sqrt{4-x^2}}$$

$$-\text{not differentable at } x = -2,1,2,>2,<-2$$

$$-\text{differentible at}(-2,1)\text{U}(1,2)$$

Answer A

4. The function f(x)= $(x-1)^{2/3}$ is differentiable on:

   A. (1, ∞) B. (−∞, 1) C. (−∞, −1) U (-1, , ∞) D. (−∞, 1) U (1 , ∞)

   Solution

$$f(x) = (x-1)^{2/3}$$

$$f(x) = \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3(x-1)^{1/3}}$$

$$-\text{Not differentiable at } x = 1$$

$$-\text{differentiable } (-\infty, 1)\ \text{U}\ (1, \infty)$$

Answer D

5. What is the slope of the line tangent to the graph of $f(x)= x^2 + \tan x$ at a point (π,f(π))?

A. 2 π B. 2 C. 2 π + 1 D. 2 π − 1

Solution

$$f(x) = x^2 + \tan x$$
$$f^1(x) = 2x + \sec^2 x$$
$$m = f^1(\pi)$$
$$= 2\pi + \sec^2\pi = 2\pi + (-1)^2$$
$$= 2\pi + 1,$$

Answer C

6. If $f(x)=4-x^2 \sin x$, then which one of the following expressions describes $f^{l}(x)$?

A. -x(x cos x+2sinx) C. X(x cos x – 2sinx)

B. 1 + cosx + 2sinx D. 1-cos x – 2sinx)

Solution

$$f(x) = 4 - x^2 \sin x$$
$$f^1(x) = (4)^l - (x^2 \sin x)^1$$
$$= -(2x \sin x + x^2 \cos x) = -x(2\sin x + x\cos x),$$

Answer A

7. Let $f(x) = \frac{1}{(1-\sqrt{x})^{2}}$ Then $f^{l}(4)$ is equal to -----

A. ½ B. -1/2 C. 1/3 D. -1

Solution

$$f(x) = \frac{1}{(1-\sqrt{x})^{2}}$$
$$f(x) = \frac{(1)^1(1-\sqrt{x})^2-((1-\sqrt{x})^2)^1}{(1-\sqrt{x})^4} = \frac{-2(1-\sqrt{x})\frac{1}{-2\sqrt{x}}}{(1-\sqrt{x})^{4}} = \frac{2(1-\sqrt{x})}{2\sqrt{x}(1-\sqrt{x})^{4}} = \frac{1}{\sqrt{x}(1-\sqrt{x})^{3}}$$
$$f^1(4) = \frac{1}{\sqrt{4}\,(1-\sqrt{3})^{3}}$$
$$= \frac{1}{2(1-2)^3} = \frac{1}{2(-1)} = -\frac{1}{2}$$

Answer B

8. Given the function $f(x)= 2x^2-3x+1$, what is the slope of the line passing through the point (-1, f(-1)) and (2, f(2))?

A. -1 B. -1/3 C. 1/3 D. -1

Solution

$$f(x) = 2x^2 - 3x + 1$$
$$f(-1) = 2(-1)^2 - 3(-1) + 1 = 2 + 3 + 1 = 6 \Rightarrow (-1,6)=(x_2,y_2)$$
$$f(2) = 2(2)^2 - 3(2) + 1 = 8 - 6 + 1 = 3 \Rightarrow (2,3)=(x_1,y_1)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{-1-2} = \frac{3}{-3} = -1$$

Answer D

9. If $f(x) = \frac{\ln 2}{x^2}$, then $f'(x)$ is equal to

A. $\frac{1}{2x2}$ B. $\frac{-2\ln 2}{x^3}$ C. $\frac{-2}{x4}$ D. $\frac{1+2\ln x}{x3}$

Solution

$$f(x) = \frac{\ln 2}{x^2} = \ln 2x^{-2},$$
$$f^1(x) = -2\ln 2x^{-3} = \frac{-2\ln 2}{x^3}$$

Answer B

10. What is the slope of the line tangent to the graph of $f(x)=x^2 + \tan x$ at $(\pi, f(\pi))$?
    A. 2π B. 2 C. $2\pi + 1$ D. $2\pi - 1$

Solution

$$f(x) = x^2 + \tan x$$
$$f^1(x) = 2x + \sec^2 x$$
$$m = f^1(\pi) = 2\pi + \sec^2\pi$$
$$=2\pi + (-1)^2$$
$$= 2\pi + 1$$

Answer C

11. The second derivative of the function $f(x)= xe^{-x}$ is -------------
    A. $-xe^{-x}$ B. $(2-x)e^{-x}$ C. $(x-2)e^{-x}$ D. $(x + 2)^2e^{-x}$

Solution

$$f(x) = xe^{-x}$$
$$f^l(x) = (x)^1e^{-x} + (e^{-x})^1x$$
$$f^l(x) = e^{-x} - e^{-x}x = e^{-x}(1 - x)$$
$$f^{ll}(x) = (e^{-x})^l(1 - x)+(1 - x)^le^{-x}$$
$$= -e^{-x}(1 - x) - 1(e^{-x})$$
$$= e^{-x}(-1 + x - 1)$$
$$= e^{-x}(x - 2),$$

Answer C

Grade 12 unit three – introduction to differential calculus

2013 E.C

1. Which of the following is the same as the derivative of a function f at a point P, where P is on the graph of f?
   A. The slope of a secant line through P. C. The graph of f has no jump at P.
   B. The continuity of the function at P. D. The slope of the tangent line trough P.

Solution

$$f^l(n) = \text{slope of the tangent line},$$

Answer D

2. What is the derivative of the function, $f(x) = 3x^2 + 2\sqrt{x} - 4x$ at $x = 1$?
   A. -3 B. -1 C. 1 D. 3

Solution

$$f(x) = 3x^2 + 2\sqrt{x} - 4x$$
$$f^l(x) = 6x + \frac{1}{\sqrt{x}} - 4$$
$$f^l(1) = 6 + 1 - 4 = 3$$

Answer D

3. What are the 2nd and the nth derivative of $f(x) = e^{3x}$ respectively?
   A. 2 f(x) and n f(x) B. 4 f(x) and 2^n f(x) C. 9 f(x) and n^2f(x) D. 9 f(x) and 3^nf(x)

Solution

$$f(x) = e^{3x},$$
$$f(x)^l = 3e^{3x}=3f(x)$$
$$f(x)^{ll} = 9e^{3x}=9f(x)=3^2f(x)$$
$$f(x)^{lll} = 27e^{3x}=27f(x)=3^3f(x)$$
$$f(x)^n = 3^ne^{3x}$$
$$=3^nf(x)$$

Answer D

4. A position of a particle is given by the equation S(t) = $4t^4 - 4t + 1$ where t is measured ins second and S in meters. Which of the following is NOT true about the motion of the particle?
   A. The velocity v at any time t is given by v(t) = $16t^3 - 4$
   B. The velocity at 2 seconds is given by 124 m/sec.
   C. The particle is at rest at t=0.5 seconds.
   D. The acceleration of the particle at 1 sec is 48 m/sec^2.

Solution

$$s(t) = 4t^4 - 4t + 1$$

A) $V(t) = s^l(t) = 16t^3 - 4$ True

B) $V(t) = 16t^3 - 4$, $V(2) = 16(8) - 4 = 128 - 4 = 124$ True

C) $V(0.5) = 16\left(\frac{1}{2}\right)^3 - 4 = \frac{16}{8} - 4 = 2 - 4 = -2$ False

D) $a(t) = V^l(t) = 48t^2$, $a(1) = 48(1)^2 = 48$ True

Answer C

5. If $f^{\,l}(1) = 4$, f (1) = 3 and $g^{\,l}(3) = 5$, then $(gof)^{\,l}(1)$ is equal to
   A. 3 B. 5 C. 15 D. 20

Solution

$$f^l(1) = 4, \quad f(1) = 3, \quad g^l(3) = 5$$

$$(gof)^l(1) = g^l(f(1)) * f^l(1)$$

$$= g^l(3) * f^l(1)$$

= 5x4

= 20

Answer D

Grade 12 unit four – applications of differential calculus

2009 E.C

1. Which one of the following is the set of critical numbers of $f(x) = \frac{4}{3}x^3 + |x|$?
   A. {1/2} B. {0, 1/2} C. {0, -1/2} D. { -1/2, 0, 1/2}

Solution

$$f(x) = \frac{4}{3}x^3 + |x| \quad , \quad f^l(x) = 0$$

$$= \frac{4}{3}x^3 + \begin{cases} x \ if \ x \geq 0 \\ -x \ if \ x \ \leq 0 \end{cases}$$

$i) f^l(x) = 0$

$$4x^2 \pm 1 = 0$$

$4x^2 - 1 = 0$ $4x^2 + 1 = 0$

$4x^2 = 1$ *no critical Number*

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$ *are critical numbers*

ii) f is not differentiable at x = o

$f^{\,l}(0^+) \neq f^{\,l}(0^-)$, *0 is critial number*

Answer D

2. Air is being pumped into a spherical balloon so that its volume increases at a rate of 50m/s. how fast is the radius of the balloon increasing when the diameter is 5cm?

   A. 1/50π cm/s B. 1/25 π cm/s C. 5/ π cm/s D. 2/ π cm/s

   Solution

$$\frac{dv}{dt} = 50 \quad , d = 5 \ , r = \frac{5}{2} \quad , \frac{dr}{dt}, ?$$
$$v = \frac{4}{3}\pi r^3$$
$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$
$$50 = 4\pi\left(\frac{5}{2}\right)^2 \frac{dr}{dt}$$
$$50 = \frac{4\pi(25)}{4}\frac{dr}{dt},$$
$$50 = 25\pi\frac{dr}{dt}$$
$$\frac{50}{25\pi} = \frac{25\pi}{25\pi}\frac{dr}{dt}$$
$$\frac{2}{\pi} = \frac{dr}{dt}$$
Answer D

3. A tin can of volume 54π cm^3 is to be made in the form of a right circular cylinder that has both flat top and flat bottom. What is the base radius of the tin if it is to be made of the least common of metal?

   A. 2cm B. 3cm. C. 4cm. D. 6cm

   Solution

$$let\ h = height, r = radius, A = area, V = volume$$

V= 54π

V=π r^{2}h =54π, h=$\frac{54}{r^2}$……..i

A=A$_l$+2A$_b$=$2\pi r^2 + 2\pi rh$

A=$2\pi r^2 + 2\pi r\frac{54}{r^2}$

=$2\pi r^2 + \frac{108\pi}{r}$

A^1=0

$4\pi r - \frac{108\pi}{r^2}$=0,

$\frac{4\pi r^3 - 108\pi}{r^2} = 0,$

$4\pi r^3 - 108\pi = 0,$

r^3=27, r=3

Answer B

Grade 12 unit four – applications of differential calculus

2010 E.C

1. What is the maximum value of the function f(x)=x^4-2x^2 on [-2, 1]

   A. 24 B. 12 C. 8 D. 40

   Solution

$$f(x)=x^4 - 2x^2 \text{ on } [-2,1]$$
$$f^1(x) = 4x^3 - 4(x), f^1(x) = 0, 4x(x^2 - 1)=0$$
$$4x(x-1)(x+1) = 0 \Rightarrow \text{Critical number are } -1, 0, \text{and } 1$$
$$f(-1) = (-1)^4 - 2(-1)^2 = 1 - 2 = -1$$
$$f(1) = (1)^4 - 2(1)^2 = 1 - 2 = -1$$
$$f(0) = (0)^4 - 2(0)^2 = 0$$
$$f(-2) = (-2)^4 - 2(-2)^2 = 16 - 8 = 8 \text{ max},$$
Answer C

2. A cylindrical tank whose inner diameter is 2 m contains 4 πcm^3 oil. If the oil is discharging from the tank at the rate of $\frac{2\pi}{3}\frac{m3}{min}$, then how long (in min) does it take for the tank to be empty?

A. 6　B. 4　C. 4/3　D. 12

Solution

d=2, r=1, v= 4π , dv/dt=2π/3 ,t=?

$$3dv = 2\pi dt$$

$$\int 3dv = \int 2\pi dt$$

$$3v = 2\pi t$$

$$3 * 4\pi = 2\pi t,\ 12\pi = 2\pi t,\ 12 = 2t,\ 6 = t$$

Answer A

3. If $(x) = \frac{1}{3}x^3 + cx^2 + ax + 5$ has a local minimum value at x=1, then which one of the following is true about the possible values of a and c?

A. a=-2c-1, c>-1　C. a=2c-1, c<-1

B. a=3, c=-2　D. a=-2c-1, c any real number

Solution

$$f(x) = \frac{1}{3}x^3 + cx^2 + ax + 5, \qquad \text{local min at } x = 1$$

$f^1(x) = 0$

$x^2 + 2cx + a = 0$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2c\pm\sqrt{(2c)^2-4a}}{2}$$

$$= \frac{-2c\pm\sqrt{4(c^2-a)}}{2}$$

$$= -c \pm \sqrt{c^2 - a}$$

$x = -c \pm \sqrt{c^2 - a}$ Are critical numbers

1 is one of the critical numbers

$$-c \pm \sqrt{c^2 - a} = 1$$

$$\pm\sqrt{c^2 - a} = 1 + c$$

$$c^2 - a = 1 + 2c + c^2$$

$$-a = 1 + 2c$$

$$a = -2c - 1,\ c \in \Re, \qquad .$$

Answer D

4. What is the maximum possible area of a rectangle in square units with diagonal of length 16 units?

A. 48　B. 64　C. 256　D. 128

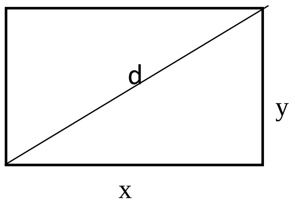
Solution

$$d^2 = x^2 + y^2$$

$256 = x^2 + y^2,$　$y^2 = 256 - x^2$

$\sqrt{x^2} = \sqrt{256 - y^2}$　$y = \sqrt{256 - x^2}$

$x = \sqrt{256 - y^2}$



$$A(x, y) = xy,\ A(x, y) = x\left(\sqrt{256 - x^2}\right)$$

$$A^l(x, y) = 0,$$

$$(x)^l\left(\sqrt{256 - x^2}\right) + \left(\sqrt{256 - x^2}\right)^l x = \sqrt{256 - x^2} - \frac{2x^2}{2\sqrt{256-x^2}} = 0$$

$$\sqrt{256 - x^2} - \frac{x^2}{\sqrt{256-x^2}} = 0,\ \frac{\left(\sqrt{256-x^2}\right)^2 - x^2}{\sqrt{256-x^2}} = 0$$

$$256 - x^2 - x^2 = 0$$

$$256 - 2x^2 = 0,\ 2x^2 = 256,\ x^2 = 128,\ x = \pm\sqrt{128} = \sqrt{64x2} = 8\sqrt{2}$$

$$y = \sqrt{256 - x^2} = \sqrt{256 - 128} = 8\sqrt{2}$$

$$A(x, y) = xy = 8\sqrt{2} * 8\sqrt{2} = 64 * 2 = 128$$

Answer D

5. Which one of the following is NOT true about the function f(x) =$3x^4$-$4x^3$?
   A. (0, 0) is point of inflection of f
   B. 0 and 1are critical number of f
   C. f is concave downward on (0, 2/3) and concave upward on (-∞, 0) and on (2/3, ∞)
   D. f is increasing on (-∞, 1)

Solution

$$f(x) = 3x^4 - 4x^3$$

$$f^l(x) = 12x^3 - 12x^2 = 12x^2(x-1),\ f^l(x) = 0,\ 12x^2(x-1)=0$$

⇒x = 0, x = 1 are critical numbers

⇒strictly increasing on $[1,\infty)$

strictly decreasing on (-∞,1]

$f^l(x) = 12x^3 - 12x^2$

$f^{ll}(x) = 36x^2 - 24x$

$= 12x(3x-2)$

⇒ concave up word or $(-\infty,0)$and $\left(\frac{2}{3},\infty\right)$

⇒convave down ward on $\left(0,\frac{2}{3}\right)$,

⇒ infection points $-\left(0,f(0)\right) = (0,0)$

$-\left(\frac{2}{3},f(\frac{2}{3})\right) = \left(\frac{2}{3},\frac{-16}{27}\right)$

Answer D

$f^l(x) = 12x^3 - 12x^2$=$12x^2(x-1)$

```
                      0              1
12x²+++++++++0+++++++++++++++++++++++++++++++++
(x – 1)--------------------------------0++++++++++++++
f¹(x)-------------0----------------0++++++++++++++
```

$f^{ll}(x) = 12x(3x-2)$

```
                      0                    2/3
12x-------------------0++++++++++++++++++++++++.
(3x-2)---------------------------------------0+++++++++.
fˡˡ(x)++++++++++0-------------------0+++++++++++
```

Grade 12 unit four – applications of differential calculus

2011 E.C

1. At what vales of x does the function $f(x) = \frac{4x^3}{3} - x^4$ Attains its maximum value?

   A. – 1 B. 1 C. 0 D. 4/3

Solution

$$f(x) = \frac{4x^3}{3} - x^4$$

$$f^l(x) = 0$$

$$\frac{12x^2}{3} - 4x^3 = 0$$

$4x^2 - 4x^3 = 0$

$4x^2(1 - x) = 0$

$4x^2 = 0, 1 - x = 0$

$x = 0 , \quad 1 = x, are\ critical\ numbers$

$f^l(x) = 4x^2 - 4x^3$

$f^{ll}(x) = 8x - 12x^2$

$f^{ll}(0) = 0$

$f^{ll}(1) = 8 - 12 = -4 < 0$

$\Rightarrow$ max $value\ is\ at\ x = 1$

OR

$f^l(x) = 4x^2(1 - x)$

0 and 1 are critical numbers

0 1

$4x^2$ ++++++++++++++0+++++++++++++++++++

(1-x)++++++++++++++++++++++++0---------------

$f^l(x)$ =++++++++++++0++++++++0-----------------

The derivative changes sign from positive to negative at x=1, maximum at 1

Answer B

2. A water tank is a rectangular parallelepiped with base length 3m, width 2m, and height 2.5m. If water is flowing in to the tank at rate of 0.12m^3/sec, then how fast does the level of the water rise up in the tank?

   A. 0.03m/sec B. 0.02m/sec C. 0.04m/sec D. 0.06m/sec

Solution

$$\frac{dv}{dt} = 0.12$$

$$V = lwh$$

$$V = 2 * 3 * h$$

$$\frac{dv}{dt} = \frac{6dh}{dt}$$

$$0.12 = \frac{6dh}{dt}$$

$$\frac{0.12}{6} = \frac{dh}{dt}$$

$$0.02 = \frac{dh}{dt}$$

Answer B

3. If f(x)=x^3-2x+1, then which of the following is NOT true?
   A. f(c)=0 for some c $\in [-2, 0]$
   B. f(c)=1/2 for some c $\in [0, 1]$
   C. f(c)=3 for some c $\in [1, 2]$
   D. f(c)=1/4 for some c $\in [-1, 0]$

Solution

F(x)=$x^3 - 2x + 1$

A. If $f(a) and\ f(b) have\ opposite\ sign\ these\ is\ at\ least\ one\ root\ between\ a\ and\ b$
   f(-2)=$(-2)^3$-2(-2)+1=-8+4+1= -3<0
   f(0)=0^3-2(0)+1=1>0,
   Intermediate value theorem - 3≤ $0 \leq 1, true$

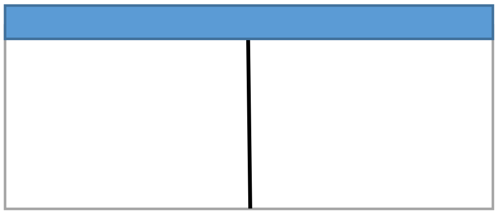
B. f(1)=1^3-2(1)+1=2+(-2)=0
   f(0)=0^3-2(0)+1=1>0
   Intermediate value theorem 0≤ $\frac{1}{2} \leq 1, true$

C. f(1)=$(1)^3$-2(1)+1=0
   f(2)=2^3-2(2)+1=5
   Intermediate value theorem 0≤ $3 \leq 5, true$

D. f(-1)=$(-1)^3$ -2(-1)+1=-1+2+1=2> 0
   f(0)=0^3-2(0)+1=1>0 ,
   Does not satisfy Intermediate value theorem ,false

Answer D

4. A man wants to fence two identical rectangular enclosures in a field a long side a straight river as shown in the following figure. What is the maximum area of each enclosure that he can make with 192 meter fencing material if the side along the river does not need a fence?



A. 1530m^2 B. 1536m^2 C. 1164m^2 D. 1564m^2

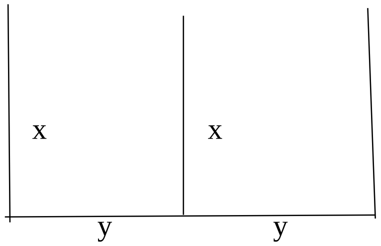
Solution

x+x+y+y+y=192

3x+2y=192

2y=192-3x

y=$\frac{192-3x}{2}$



A=2xy

A=2x$\left(\frac{192-3x}{2}\right)$

A=x(192-3x) = 192x-3x^2

A^1=0

$(192x-3x^2)^1$=0

192-6x=0

192=6x

$\frac{192}{6} = x$, $32 = x, y = \frac{192-3x}{2} = \frac{192-3(32)}{2} = \frac{192-96}{2} = \frac{96}{2} = 48$

$A = xy$ =each area =(48)(32)=(1536)

Answer B

5. Which one of the following is the set of critical numbers of $f(x) = \frac{3}{8}x^{\frac{8}{3}} - 6x^{\frac{2}{3}}$?

A. {-2, 2} B. {-2, 0, 2} C. {-1, 0, 1} D. {-1, 1}

Solution

$$f(x) = \frac{3}{8}x^{\frac{8}{3}} - 6x^{\frac{2}{3}}$$

$$f^l(x) = x^{\frac{5}{3}} - 4x^{\frac{-1}{3}} = x^{\frac{5}{3}} - \frac{4}{x^{\frac{1}{3}}}\ , f\ is\ not\ differentiable\ at\ x = 0, 0\ is\ critical\ number$$

$$f^l(x) = 0$$

$$x^{\frac{5}{3}} - \frac{4}{x^{\frac{1}{3}}} = 0$$

$$\frac{x^{\frac{6}{3}} - 4}{x^{\frac{1}{3}}} = 0$$

$$x^{\frac{6}{3}} - 4 = 0$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$Critical\ numbers\ are\ \{-2\ , 0\ , 2\}$$

Answer B

6. If $f(x) = ax^3 + \frac{b}{x} + 5$ has local minimum at (2, -3) what are the values of a and b?

A. a=-1/4, b=12

B. a=1/4, b= -12

C. a=1/4, b=12

D. a= -1/4, b= -12

Solution

$f(x) = ax^3 + \frac{b}{x} + 5$, *local* min *at* $(2, -3)$, 2 is critiacal number

$$\Rightarrow f(2) = -3$$

$$a(2)^3 + \frac{b}{2} + 5 = -3$$

$$8a + \frac{b}{2} = -3 - 5$$

$$8a + \frac{b}{2} = -8$$

$$16a + b = -16,$$

$$f^l(2) = 0$$

$$3\ ax^2 - \frac{b}{x^2} = 0$$

$$3\ a(2)^2 - \frac{b}{(2)^2} = 0$$

$$12a - \frac{b}{4} = 0$$

b=48a

$$16a + 48a = -16$$

$$64a = -16$$

$$a = \frac{-1}{4}$$

$$b = 48a = 48\left(\frac{-1}{4}\right)$$

$$b = -12$$

Answer D

Grade 12 unit four – applications of differential calculus

2012 E.C

1. The volume of a cube is increasing at a rate of 9cm^3/sec. how fast is the surface area increasing when the length of the edge is 10cm?

   A. 3.6cm^2/sec   C. 90 cm^2/sec

   B. 36cm^2/sec   D. 6 cm^2/sec

Solution

$$\frac{dv}{dt} = 9 \ , \frac{dA}{dt}?$$

$$V = lwh$$

$$v = x^3$$

$$\frac{dv}{dt} = 3x^2\frac{dx}{dt}$$

$$\frac{1}{3x^2}\frac{dv}{dt} = \frac{dx}{dt} - - - -(1)$$

$$A_b = x^2$$

$$A_s = 6x^2$$

$$\frac{dA}{dt} = 12x\frac{dx}{dt}$$

$$\frac{1}{12x}\frac{dA}{dt} = \frac{dx}{dt} - - - (2)$$

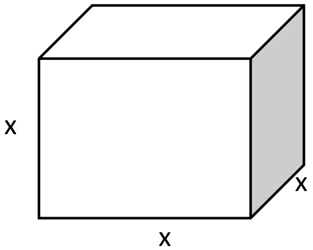
$$\frac{1}{3x^2}\frac{dv}{dt} = \frac{1}{12x}\frac{dA}{dt}$$

$$\frac{1}{x}\frac{dv}{dt} = \frac{1}{4}\frac{dA}{dt}$$

$$\frac{4}{x}\frac{dv}{dt} = \frac{dA}{dt}$$

$$\frac{dA}{dt} = \frac{4}{x}\frac{dv}{dt}$$

$$= \frac{4}{10} * 9 = 3.6$$



Answer A

2. The product of two positive numbers is 100 such that the sum of two times the first number and eight times the second number is minimum. Which one of the following pairs of numbers are the first and the second numbers respectively

   A. 25 and 4   B. 20 and 5   C. 1 and 99   D. 50 and 2

Solution

$$xy = 100$$

$$y = \frac{100}{x}$$

$$2x + 8y = s(x, y)$$

s(x,y)=2x+84

$$s(x) = 2x + 8\left(\frac{100}{x}\right)$$

$$s(x) = 2x + \frac{800}{x}$$

minimum values are in the critical numbers ·

$$S^l(x) = 0$$

$$2 - \frac{800}{x^2} = 0$$

$$\frac{2x^2 - 800}{x^2} = 0$$

$$2x^2 - 800 = 0$$

$$2x^2 = 80,$$

$$x^2 = 400$$

$$x = \sqrt{400} = 20$$

$$y = \frac{100}{x} = \frac{100}{20} = 5$$

Answer B

3. The maximum profit that a company can make if the profit function is given by p(x)=36+72x-18x^2 is -----

A. 124 B. 31 C. 2232 D. 108

Solution

$$p(x) = 36 + 72x - 18x^2$$

$$p^l(x) = 0$$
$$72 - 36(x) = 0$$
$$72 = 36x$$
$$2 = x$$

$$p = p(2)$$
$$= 36 + 72(2) - 18(2)^2$$
$$= 36 + 144 - 18x4$$
= 36+144-72= 108, is max

Answer D

Grade 12 unit four – applications of differential calculus

2013 E.C

1. Which of the following is true about the zero of a function?

A. It is the x-intercept of the function.
B. It is the y-intercept of the function.
C. It is the critical pint of the function.
D. It is a concavity point of the function.

Solution

Zero of function is the point at which the graph crosses or touches the x-axis

Answer A

2. Which of the following is true about the function defined by f(x) = x^3 – 3x^2 on [-1, 3]?

A. Its critical numbers are 0, 2, 3 and maximum valve is 0.
B. Its critical numbers are 0 and 2 and maximum value is 15.
C. Its critical numbers are 0 and 2 and its maximum value is -4.
D. It has critical numbers at 0 and 2 and its maximum value is 0.

Solution

$$f(x) = x^3 - 3x^2 , [-1,3]$$
$$(f(x))^l = 0$$
$$3x^2 - 6x = 0$$
$$3x(x - 2) = 0$$
$$x = 0, x = 2 \text{ are critical numbers}$$
$$f(0) = 0 \text{ max}$$
$$f(2) = 8 - 12 = -4 \text{ min}$$
$$f(-1) = -1 - 3 = -4 \text{ min}$$
$$f(3) = 27 - 27 = 0 \text{ max}$$

Answer D

3. What is the number that satisfies Rolle's Theorem for f(x) = x^2 – 3x +1 on [0, 3].

A. $-\frac{3}{2}$ B. 1 C. $\frac{3}{2}$ D. 2

solution

$$f(x) = x^2 - 3x + 1$$
f^l(x)=2x-3
Rolles theorem, $f^1(C) = 0$
$$2c - 3 = 0$$
2c=3
c=3/2

Answer C

Rolle's theorem

1. F is continues on [0,3]
2. F is differentiable on (0,3)
3. F(0)=1=F(3)
F satisfies the three conditions of Rolle's theorem, thus there exist c such that f^l(c)=0

4. What are the length L and width W of a rectangle with perimeter 10,000m that maximizes the area?
   A. L = 2,500 m and W = 2,500 m
   B. L = 2,500 m and W = 3,000 m
   C. L = 3,000 m and W = 2,500 m
   D. L = 5,000 m and W = 5,000 m

<u>Solution</u>

P=2l+2w

2l+2w=10,000

L+w=5,000

L=5,000-w

A=lw

A= (5000-w)w

A=5000w-w^2

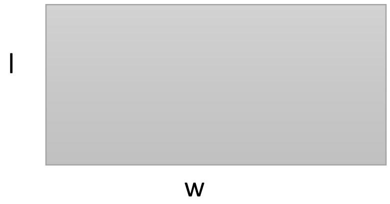
A^{l}(w)=0

5000-2w=0

5000=2w , l=5000-w

$\underline{\underline{2500=w}}$ l=5000-2500$\underline{\underline{=2500}}$

Answer A



5. If the radius r of a sphere is increasing at the rate of 2 cm/min, then the rate of change of the volume when r=1 cm is

   A. $4\pi\ {}^{cm^3}/_{min}$ B. $6\pi\ {}^{cm^3}/_{min}$ C. $8\pi\ {}^{cm^3}/_{min}$ D. $12\pi\ {}^{cm^3}/_{min}$

<u>Solution</u>

$$\frac{dr}{dt} = 2$$

$$V = \frac{4}{3}\pi r^3$$

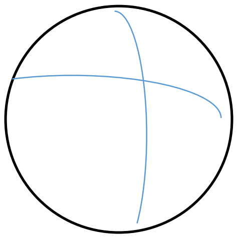
$$\frac{dV}{dt} = \frac{12\pi r^2}{3}\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(1^2)2)$$

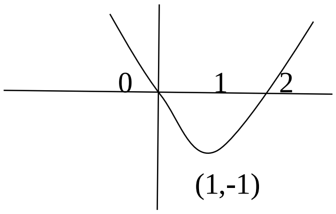
$$\frac{dV}{dt} = 8\pi$$

Answer C



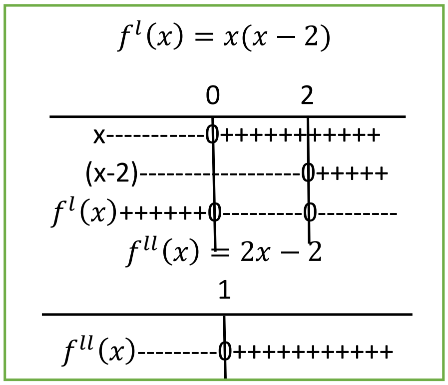
Volume of a sphere is given by$V = \frac{4}{3}\pi r^3$

6. Suppose f:(-∞, ∞) → ℝ is differentiate & the graph of its derivative y=$f^{l}(x)$ is as shown in the figure below, which one of the following is true about f(x)?
   A. f is increasing on (1, ∞).
   B. f is concave upward on (0, ∞).
   C. f has no relative maximum value.
   D. f has a relative minimum value at x=2



Solution

$$The\ equation\ of\ the\ graph\ is\ f^{l}(x) = (x-1)^2 - 1$$
$$f^{l}(x) = x^2 - 2x + 1 - 1$$
$$f^{l}(x) = x^2 - 2x$$
$$f^{l}(x) = x(x-2)$$
$$A)\ f\ is\ increasing\ on\ (-\infty, 0]\ and\ [2, \infty)\ \ False$$
$$B)\ f\ is\ concave\ up\ on\ (1, \infty)\ \ False$$
$$C)\ f\ has\ local\max at\ x = 0\ False$$
$$D)\ f\ has\ local\ min\ at\ x = 2\ \ True$$
$$Answer\ D$$



Grade 12 unit five – introduction to integral calculus

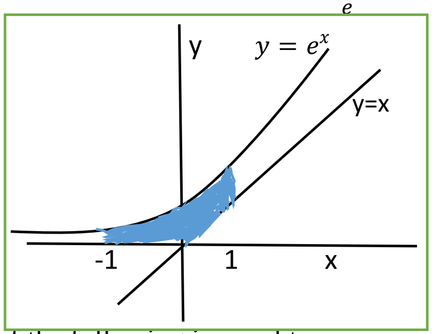
2009 E.C

1. What is the value of the area of the region enclosed by the graph of f(x)= e^x , g(x)= x and the x axis between the lines x=-1 and x=1?

A. $\frac{e^2-1}{e}$ B. $e^2 - \frac{1}{e}$ C. $e^2 - \frac{1}{e} + 2$ D. $e - \frac{1}{e} + 2$

Solution

$$f(x) = e^x\ \ , g(x) = x\ \ \ between\ x = -1\ and\ 1$$
$$A = \int_{-1}^{1} (e^x - x)dx = (e^x - \frac{x^2}{2})/_{-1}^{1}$$
$$= (e^x - \frac{1}{2}) - (e^{-1} - \frac{1}{2})$$
$$= e - \frac{1}{2} - \frac{1}{e} + \frac{1}{2} = e - \frac{1}{e} = \frac{e^2-1}{e}$$
$$Answer\ A$$



2. If f(0)=-1, f(1)=2 and f'(x) is continuous on [0, 1], then which of the following is equal to $\int_0^1 f^{l}(x)\sqrt{2+f(x)}\,dx$?

A. 16/3 B. 14/3 C. 8/3 D. 4/3

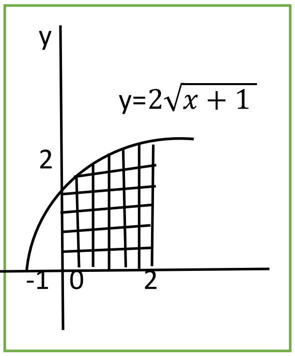
Solution

$$f(0) = -1\ \ , f(1) = 2$$
$$\int_0^1 f^{l}(x)\sqrt{2+f(x)}\,dx,\ let\ 2 + f(x) = u, f^{l}(x)dx = du$$
$$\int_0^1 \sqrt{u}\ du = \int_0^1 u^{\frac{1}{2}}\ du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}u^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}\left((2+f(x))^{\frac{3}{2}}\right)\Big|_0^1$$
$$= \frac{2}{3}(2+f(1))^{\frac{3}{2}} - \frac{2}{3}(2+f(0))^{\frac{3}{2}} = \frac{2}{3}(2+2)^{\frac{3}{2}} - \frac{2}{3}(2-1)^{\frac{3}{2}}$$
$$= \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(1)^{\frac{3}{2}} = \frac{2*\sqrt{64}}{3} - \frac{2}{3}$$
$$= \frac{16-2}{3} = \frac{14}{3}$$
$$Answer\ B$$

3. Which of the following is equal to the volume of a solid generated when the region bounded by the graph of y=$2\sqrt{x+1}$ and x-axis, when $0\le x \le 2$, rotates about the x-axis?

A. 8π B. $8\pi^2$ C. 16π D. $16\pi^2$

Solution



$$y = 2\sqrt{x+1} \quad , \quad 0 \le x \le 2$$

$$f(x) = 2\sqrt{x+1} \quad , \quad g(x) = x\ axis, \ g(x) = 0$$

$$v = \int_0^2 (f(x)^2 - g(x)^2)\pi dx$$

$$= \int_0^2 \left(2\sqrt{x+1}\right)^2 \pi dx = \int_0^2 (4(x+1))\pi dx = \int_0^2 (4x+4)\pi dx$$

$$= (2x^2+4x)\pi \Big|_0^2 = \left(2(2)^2 + 4(2) - (0^2+0)\right)\pi = (2(4)+8)\pi$$

$$= (8+8)\pi = 16\pi$$

Answer C

4. A particle moves along the x-axis with velocity given by v(t)=$3t^2$ +6t for time t≥ 0.if the particle is at position x=2 at a time t=0, what is the position of the particle at t=1

A. 6 B. 9 C. 11 D. 12

Solution

$$v(t) = 3t^2 + 6t \quad , x(t) = \int v(t)dt$$

$$\int v(t)dt = \int (3t^2 + 6t)dt$$

$$x(t) = t^3 + 3t^2 + c \quad , x(0) = 2$$

$$0 + 0 + c = 2$$

$$c = 2$$

$$x(t) = t^3 + 3t^2 + c$$

$$x(t) = t^3 + 3t^2 + 2$$

$$x(1) = 1^3 + 3(1^2) + 2$$

$$= 1 + 3 + 2$$

$$= 6$$

Answer A

5. Which of the following is equal to $\int x(e^x + \sin(x^2)\,dx$?

A. $e^x(x+1)+1/2\sin(x^2)+C$

B. $e^x(x-1)-1/2\sin(x^2)+C$

C. $e^x(x-1) - \frac{1}{2}\cos(x^2) + c$

D. $e^x(x+1)+1/2\cos(x^2)+C$

Solution

$$\int x(e^x + \sin(x^2)\,dx = \int xe^x\,dx \qquad + \qquad \int x \sin x^2\,dx$$

Let x=u, $e^x dx = dv$, $\qquad$ $let\ x^2 = u, 2xdx = du$

$dx = du,\ e^x = v,$ $\qquad$ $xdx = \frac{du}{2}$

$$\int x(e^x + \sin(x^2)\,dx = \int xe^x\,dx \qquad + \qquad \int x \sin x^2\,dx$$

$$= uv - \int vdu \qquad + \qquad \int \sin u \frac{du}{2}$$

$$= xe^x - \int e^x dx \qquad + \qquad \frac{(-\cos u)}{2} + c$$

$$= xe^x - e^x - \frac{\cos(x^2)}{2} + c$$

$$= e^x(x-1) - \frac{1}{2}\cos(x^2) + c$$

Answer C

6. If $2 \leq f^{l}(x) \leq 4$ for all value of x, then the value of f(8)-f(2) is between which of the following numbers?
   A. 14 and 24 C. 12 and 18
   B. 12 and 24 D. 8 and 10

Solution

$$2 \leq f^{l}(x) \leq 4 \quad , \quad f(8) - f(2)$$
$$\int 2dx \leq \int f^{l}(x)dx \leq \int 4dx$$
$$2x + c_l \leq f(x) \leq 4x + c_2$$
$$f(8) - f(2)$$
$$(2(8) + c) - (2(2) + c_1) \leq f(8) - f(2) \leq (4(8) + c_2) - (4(2) + c_2)$$
$$16 + c_1 - 4 + c_1 \leq f(8) - f(2) \leq 32 + c_2 - 8 - c_2$$
$$12 \leq f(8) - f(2) \leq 24$$
$$Answer\ B$$

7. What is the value of $\int_0^{ln\sqrt{3}} \left(\frac{e^x}{e^{-x} + e^x}\right) dx$?
   A. $\frac{1}{2}\ln 2$ B. ln4 C. ln2 D. 1

Solution

$$\int_0^{ln\sqrt{3}} \left(\frac{e^x}{e^{-x} + e^x}\right) dx \quad , \quad \frac{e^x}{e^{-x} + e^x} = \frac{e^x}{\frac{1}{e^x} + e^x} = \frac{e^x}{\frac{1 + e^{2x}}{e^x}} = \frac{e^{2x}}{1 + e^{2x}}$$
$$\int_0^{ln\sqrt{3}} \left(\frac{e^{2x}}{1 + e^{2x}}\right) dx$$
$$let\ 1 + e^{2x} = u$$
$$2e^{2x}\ dx = du$$
$$e^{2x}\ dx = \frac{du}{2}$$
$$\int_0^{ln\sqrt{3}} \left(\frac{1}{u}\right) \frac{du}{2}$$
$$= \frac{1}{2}\ln|u| \Big|_0^{ln\sqrt{3}} = \frac{1}{2}\ln(1 + e^{2x}) \Big\|_0^{ln\sqrt{3}} = \frac{1}{2}\ln\left(1 + e^{2ln\sqrt{3}}\right) - \frac{1}{2}\ln(1 + e^{0})$$
$$= \frac{1}{2}\ln(1 + e^{ln3}) - \frac{1}{2}\ln(1 + 1) = \frac{1}{2}\ln(1 + 3) - \frac{1}{2}\ ln2 = \frac{1}{2}\ ln4 - \frac{1}{2}\ln 2$$
$$= \frac{1}{2}\ln\left(\frac{4}{2}\right) = \frac{1}{2}\ln 2 \quad , \quad note\ ; - e^{ln3} = e^{\log_e 3} = 3$$
$$Answer\ A$$

Grade 12 unit five – introduction to integral calculus

2010 E.C

1. What is the value of $\int_1^2 \frac{x+4}{x(x+2)}$
   A. 2 B. ln 2 C. ln 4 – ln 3 D. ln 3

Solution

$$\int_1^2 \frac{x+4}{x(x+2)} = \int \left(\frac{A}{x} + \frac{B}{x+2}\right) dx$$

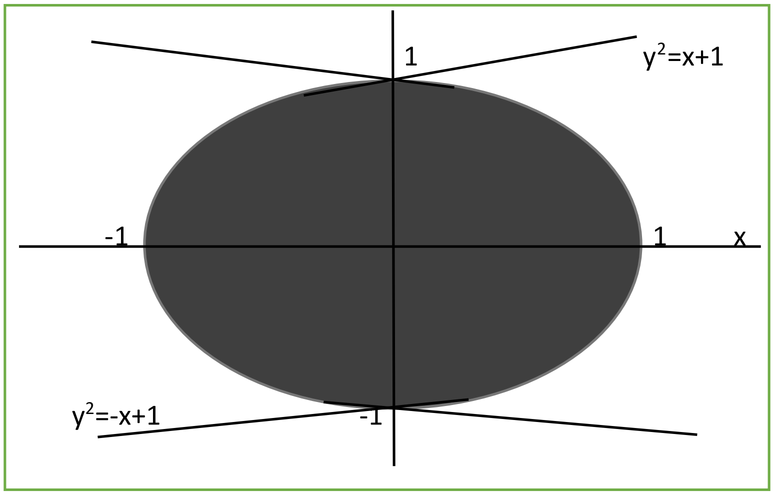
$$x + 4 = A(x + 2) + Bx$$
$$x + 4 = Ax + 2A + Bx$$
$$1 = A + B, 4 = 2A, A = 2, B = -1$$

$$= \int_1^2 \left(\frac{2}{X} - \frac{1}{X+2}\right) dX = (2\ell n|x| - \ell n|x + 2|)\Big/_{1}^{2} = (2\ell n2 - 1n4) - (2\ell n1 - \ell n\ 3)$$
$$= 2\ell n2 - \ell n4 + \ell n3 = \ell n4 - \ell n4 + \ell n3 = \ell n3, \text{ note ln 1=0}$$

Answer D

2. What is the area of the region enclosed by the graph of $y^2=x+1$ and $y^2= -x+1$?
   A. 4/3sq. units B. 8/3sq. units C. 3/8sq. units D. 3/4sq. units

Solution



$$y^2 = x+1$$
$$y^2 = -x+1$$
$$\Rightarrow x = y^2 - 1$$
$$x = -y^2 + 1$$
$$A = \int_{-1}^{1} ((-y^2+1)-(y^2-1))dy$$
$$A = \int_{-1}^{1} (-y^2-y^2+1+1)dy$$
$$= \int_{-1}^{1} (-2y^2+2)dy$$
$$= \left(-\frac{2}{3}y^3+2y\right)\Big|_{-1}^{1}$$
$$= \left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right) = -\frac{2}{3}+2-\frac{2}{3}+2 = -\frac{4}{3}+4 = \frac{8}{3},$$

Answer B

3. What is the value of $\int_1^9 \frac{e\sqrt{x}}{\sqrt{x}dx}$?
   A. $2(e^3\text{-}e)$ B. $e^3/3\text{-}e$ C. $e^3 – e$ D. $e(e^2\text{-}1/3)$

Solution

$$\int_2^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx \qquad , \quad \text{let } \sqrt{x} = U,$$
$$\frac{1}{2\sqrt{x}}\, dx = d\,U$$
$$\frac{1}{\sqrt{x}}\, dx = 2d\,U$$

$$\int_2^9 e^u(2du) = 2e^u\Big/_{1}^{9} = 2e^{\sqrt{x}}\Big/_{1}^{9}$$
$$= 2e^{\sqrt{9}} - 2e^1$$
$$= 2e^3 - 2e$$
$$= 2(e^3 - e),$$

Answer A

4. What is the value of $\int x\sqrt{1-x^2}\,dx$?
   A. $\frac{1}{2}(1-x^2)^{\frac{3}{2}}+c$ B. $2(1-x^2)^{\frac{3}{2}}+c$ C. $\frac{-1}{3}(1-x^2)^{\frac{3}{2}}+c$ D. $(1-x^2)^{\frac{3}{2}}+c$

Solution

$$\int x\sqrt{1-x^2}\,dx \qquad , \text{Let } 1-x^2 = U$$
$$= \int \sqrt{U}\left(\frac{-du}{2}\right) \qquad -2x\,dx = du$$
$$= -\frac{1}{2}\int U^{\frac{1}{2}}du = \frac{-1}{2}\frac{U^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{-1}{3}U^{\frac{3}{2}} + c$$
$$= \frac{-1}{3}(1-x^2)^{\frac{3}{2}} + c$$

Answer C

5. What is the value of $\int \frac{1}{x}(\ell nx + x^2e^{-x})dx$?
   A. $=\frac{1}{2}(\ell nx)^2 - x(x+1) + c$
   B. $=\frac{1}{2}(\ell nx)^2 - e^{-x}(2+x) + c$
   C. $=\frac{1}{2}(\ell nx)^2 - e^{-x}(2-x) + c$
   D. $=\frac{1}{2}(\ell nx)^2 - e^{-x}(x+1) + c$

Solution

$$\int \frac{1}{x}(\ell nx + x^2e^{-x})dx = \int \frac{1}{x}\ell nxdx + \int \frac{x^2e^{-x}}{x}dx$$

$$= \int \frac{1}{x}\ell nxdx \qquad + \qquad \int xe^{-x}dx$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\text{Let } \ell nx = U \qquad \text{Let } x = U, e^{-x}dx = dv$$

$$\frac{1}{x}dx = dU \qquad dx = dU, -e^{-x} = V$$

$$= \int UdU \quad + \quad Uv - \int(vdU) = \int UdU + -xe^{-x} - \int -e^{x}dx \quad = \int UdU - xe^{-x} + \int e^{-x}dx$$

$$= \frac{U^2}{2} - xe^{-x} - e^{-x} + c = \frac{1}{2}(\ell nx)^2 - e^{-x}(x+1) + c,$$

Answer D

Grade 12 unit five – introduction to integral calculus

2011 E.C

1. If the region enclosed by the graph of f(x)= x^2 and g(x)= x^3 from x=0 to x=1 rotates about the x-axis, what is the volume of the solid of revolution?
   A. $2\pi/27$ cubic units
   B. $2\,\pi/25$ cubic units
   C. $2\pi/35$ cubic units
   D. $2\,\pi/5$ cubic units

Solution

$$f(x) = x^2, g(x) = x^3$$

$$v = \int_0^1 \pi((f(x))^2 - (g(x))^2)dx$$

$$= \int_0^1 \pi\,((x^2)^2 - (x^3)^2)dx$$

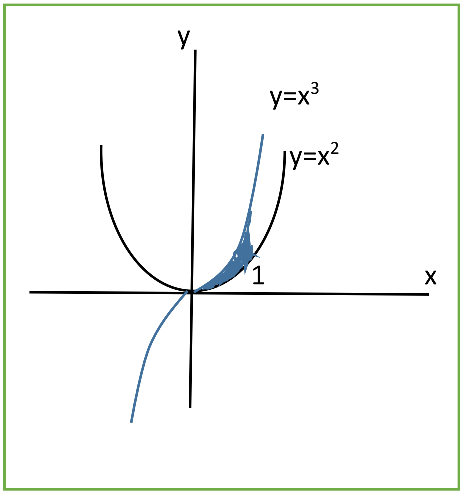
$$= \int_0^1 (\pi x^4 - \pi x^6)\,dx$$

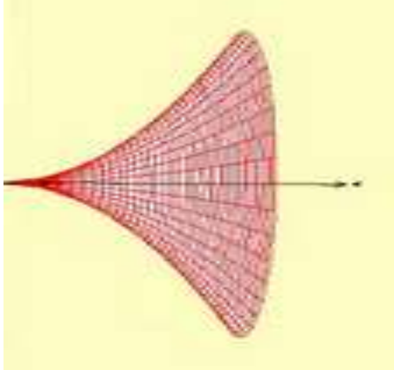
$$= \frac{\pi x^5}{5} - \frac{\pi x^7}{7}\Big|_0^1$$

$$= \frac{1}{5}\pi - \frac{1}{7}\pi$$

$$= \frac{7\pi - 5\pi}{35}$$

$$= \frac{2}{35}\pi$$





Answer C

2. What is the value of $\int \frac{x^{e-1}+e^{x-1}}{x^e+x^x}\, dx$?

A. $\frac{1}{e}$ln $|x^e + x^e|$+C

C. $\frac{1}{e}\, ln|x^e + e^x|$+ C

B. $\frac{1}{e}\, ln|(x+1)^e + e^{x+1}|$+ C

D. $\frac{1}{e}\, ln|(x^{e+1} + e^{x+1}|$+ C

Solution

$$\int \frac{x^{e-1}+e^{x-1}}{x^e+x^x}\, dx = \frac{e}{e}\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x}\, dx = \frac{1}{e}\int \frac{ex^{e-1}+e,e^{x-1}}{x^e+e^x}\, dx = \frac{1}{e}\int \frac{ex^{e-1}+e^x}{x^e+e^x}\, dx,$$

$$Let\ x^e + e^x = \cup$$

$$(ex^{e-1} + e^x)dx = du$$

$$=\frac{1}{e}\int \frac{1}{\cup}\, du = \frac{1}{e}\, ln \cup + c \quad = \frac{1}{e} ln(x^e + e^x) + C$$

Answer C

3. What is the value $\int 4x\left(lnx + \left(\frac{1}{x^2}\right)\right) dx$

A. $4x^2$(ln x+1)-$2x^2$+C

C. $X^2(2lnx - 1) + 4ln|x| + c$

B. X^2(4ln x-1) + $2x^2$ + C

D. X^2(2ln x + 1) +4ln x + C

Solution

$$\int 4x\left(lnx + \left(\frac{1}{x^2}\right)\right) dx$$

$$=\int \left(4xlnx + 4x\left(\frac{1}{x^2}\right)\right) dx$$

$$=\int 4xlnxdx + \int 4\frac{1}{x}dx$$

$=4\int xlnxdx + 4lnx + c$, let lnx =u, $\frac{1}{X}dx$=du, xdx=dv, $\frac{X^2}{2}$=v

$$=4(uv - \int vdu) + 4ln|x| + c,$$

$$= 4\frac{X^2}{2}lnX - 4\int \frac{X^2}{2} * \frac{1}{X}dx + 4ln|x| + c$$

$$= 2X^2lnx - 4\int \frac{1}{2}X\, dx + 4ln\,|x| + c$$

$$= 2X^2lnx - \frac{4X^2}{4} + 4ln|x| + c$$

$$= 2X^2lnx - X^2 + 4ln|x| + c$$

$$= X^2(2lnx - 1) + 4ln|x| + c$$

Answer C

4. What is the area of the region bounded by the lines x=0, x=2 and y=1 and the curve y=e^{2x}?

A. $\frac{e4}{2} + \frac{5}{2}$

B. $\frac{e2}{2} - \frac{1}{2}$

C. $\frac{e3}{3} - \frac{1}{3}$

D. $\frac{e^4}{2} - \frac{5}{2}$

- Solution

$$Y = e^{2X} \quad ,Y = 1$$

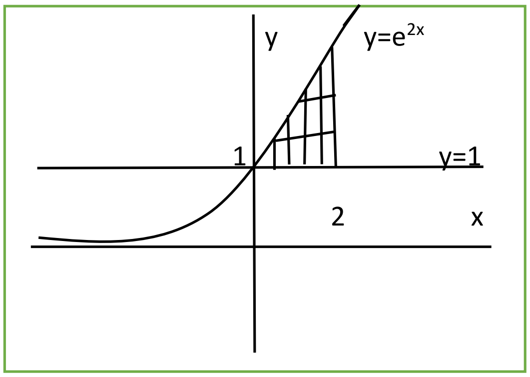
$$A=\int_0^2 (e^{2X} - 1)\, dx = \left(\frac{e^{2X}}{2} - X\right)\Big|_0^2$$

$$= \left(\frac{e^4}{2} - 2\right) - \left(\frac{e^0}{2} - 0\right)$$

$$= \frac{e^4}{2} - 2 - \frac{1}{2}$$

$$= \frac{e^4}{2} - 2 - \frac{1}{2}$$

$$= \frac{e^4}{2} - \frac{5}{2}$$



Answer D

5. What is the value of $\int_1^2 \frac{lnx}{x^2}dx$

A. $\frac{-\ln 2}{2}-\frac{1}{2}$ B. $\frac{-\ln 2}{2}+\frac{1}{2}$ C. $\frac{ln2}{2}-\frac{1}{2}$ D. $\frac{ln2}{2}+\frac{1}{2}$

Solution

$$\int_1^2 \frac{lnx}{x^2}dx \qquad let \quad \begin{matrix} lnx = u \\ \frac{1}{x}dx = du \end{matrix} \quad , \quad \begin{matrix} \frac{1}{x^2}dx = dv \\ -\frac{1}{x} = v \end{matrix}$$

$$= (uv - \int vdv\,)\Big|_1^2$$

$$= \left(\frac{-1}{x}lnx - \int \frac{-1}{x} * \frac{1}{x}dx\right)\Big|_1^2$$

$$= \left(\frac{-1}{x}lnx - \int \frac{-1}{x^2}dx\right)\Big|_1^2$$

$$= \left(\frac{-1}{x}lnx - \int -x^{-2}dx\right)\Big|_1^2$$

$$= \left(\frac{-1}{x}lnx - \frac{-x^{-1}}{-1}\right)\Big|_1^2$$

$$= \left(\frac{-1}{x}lnx - \frac{1}{x}\right)\Big|_1^2$$

$$= \left(\frac{-1}{2}ln2 - \frac{1}{2}\right) - (-1ln1 - 1)$$

$$= \frac{-1}{2}ln2 - \frac{1}{2} - (0 - 1)$$

$$= \frac{-1}{2}ln2 - \frac{1}{2} + 1$$

$$= \frac{-1}{2}ln2 + \frac{1}{2}$$

Answer B

6. If f(x)= $\int_0^x e^{-t}$ dt, then what is the value of $f^l(x)$?

A. $-e^{-x}$ B. $-e^{-x}-1$ C. $\frac{e-x+1}{-x+1}$ D. e^{-x}

Solution

$f(x) = \int_0^x e^{-t}\,dt \quad , \quad f^l(x) = ?$

$f^l(x) = g^l(x)f(g(x))$

$f^l(x) = (x)^l e^{-x}$

$\Rightarrow f^l(x) = e^{-x}$

Answer D

$$if\ f(x) = \int_0^{g(x)} f(t)dt$$

$$then, f^l(x) = g^l(x)f(g(x))$$

Grade 12 unit five – introduction to integral calculus

2013 E.C

1. If f is a continuous function on [a, b] and F is its anti-derivative then which of the following is true?

A. $\int_a^b F(x)dx = f(b) - f(a)$ C. $\int_a^b f^l(x)dx = F(b) - F(a)$

B. $\int_a^b F^l(x)dx = f(b) - f(a)$ D. $\int_a^b f(x)dx = F(b) - F(a)$

Solution

$$\int_a^b f^l\,(x)dx = F(x)/_a^b = F(b) - F(a)$$

Answer C

2. Which of the following is equal to $\int (\cos x - 2x)\,dx$ where c is a constant?

A. $-\sin x - x^2 + c$ B. $\sin x + x^2 + c$ C. $-\sin x + x^2 + c$ D. $\sin x - x^2 + c$

Solution

$$\int (\cos x - 2x)\,dx = \sin x - x^2 + c$$

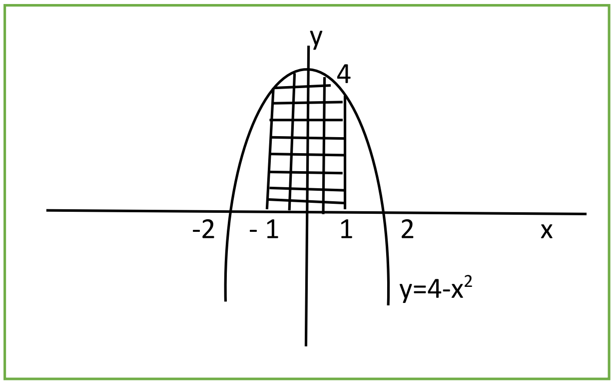
Answer D

3. What is the area of the region bounded by y = 0 and the graph of $f(x) = 4 - x^2$ on the interval [-1, 1]?

A. $\frac{2}{3}$ B. $\frac{22}{3}$ C. 8 D. $\frac{26}{3}$

Solution

$$A=\int_{-1}^{1}(4-x^2)dx$$
$$=\left(4x-\frac{x^3}{3}\right)/_{-1}^{1}$$
$$=\left(4-\frac{1}{3}\right)-\left(-4+\frac{1}{3}\right)$$
$$=4-\frac{1}{3}+4-\frac{1}{3}$$
$$=8-\frac{2}{3}$$
$$=\frac{24-2}{3}=\frac{22}{3}$$



Answer B

4. Suppose $\int_1^3 f(x)dx = 6$ and $\int_1^2 f(x)dx = 10$, what is the value of $\int_2^3 f(x)dx$?

A. -4 B. 2 C. 4 D. 16

Solution

$$\int_1^3 f(x)dx = 6 \quad \int_1^2 f(x)dx = 10$$
$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x),\ a \le c \le b$$
$$\int_2^3 f(x)dx = \int_2^1 f(x)dx + \int_1^3 f(x)dx$$
$$=-\int_1^2 f(x)dx + \int_1^3 f(x)dx$$
$$= -10 + 6$$
$$= -4$$

Answer A

5. Which of the following is true where c is constant?

A. $\int (3x+5)^6\,dx = \frac{(3x+5)^7}{7} + c$ C. $\int xe^x dx = \frac{x^2}{2}e^x + c$

B. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = -2\cos\sqrt{x} + c$ D. $\int \ln x dx = \frac{1}{x} + c$

Solution

A. $\int (3x+5)^6\ dx = \frac{(3x+5)^7}{21} + c$, False

B. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$, let $\sqrt{x} = \cup$, $\frac{1}{2\sqrt{x}}\,dx = du$

$$\int \sin u\,du(2) = -2\cos\ \cup + c = -2\cos\sqrt{x}\ + c\ ,\text{True}$$

$C.\int xe^x\,dx$ let $\begin{matrix} x = \cup \\ dx = d\cup \end{matrix}$, $\begin{matrix} e^x dx = dv \\ e^x = v \end{matrix}$

$$= \cup v - \int v d\cup = xe^x - \int e^x\,dx = xe^x - e^x + c\,,\text{False}$$

D. $\int \ln x\,dx$ let $1nx = \cup$, dx=dv

$\frac{1}{x}dx = d\cup$, x= v

$$=UV-\int vdU = x\ln x - \int dx$$
$$= x\ln x - \ x + c\,,\text{False}$$

Answer B

6. The acceleration of a body at any time t is given by a(t) = $4 - 2t + 3t^2$ with initial velocity v (0) = 5. Then what is the velocity v of the body as a function of time?
   A. $v(t) = 5 + 4t - t^2 + t^3$
   B. $v(t)=1+4t-t^2+t^3$
   C. $v(t)=9 - t^2+t^3$
   D. $v(t)=-2+6t$

Solution

$$a(t) = 4 - 2t + 3t^2, v(0) = 5$$
$$v(t) = \int a(t)dt$$
$$= \int (4 - 2t + 3t^2)\, dt$$
$$=4t-t^2 + t^3 + c \ , \quad v(0) = 5$$
$$4(0) - 0^2 + 0^3 + c = 5, c =5$$
$$v(t)=4t - t^2 + t^3 + 5$$

Answer A

Grade 12 unit six – three Dimensional Geometry and Vectors in Space

2009 E.C

1. If P (2, √5, 1) and Q= (3, 0, 9) are points on a sphere whose center is on z-axis, then which of the following point is outside of the sphere?
   A. (-4, 3, 5) B. (2, -2, 1) C. (3, 1, 1) D. (0, 0, 0)

Solution

$$P(2,\sqrt{5},1)\ ,Q\ (3,0,9)\ ,center\ (0,0,K)$$
$$PC = r = \sqrt{(2-0)^2 + (\sqrt{5}-0)^2 + (1-k)^2} = QC = \sqrt{(3-0)^2 + (0-0)^2 + (9-k)^2}$$
$$2^2 + \left(\sqrt{5}\right)^2 + (1-k)^2 = 3^2 + 0^2 + (9-k)^2$$
$$4 + 5 + 1 - 2k + k^2 = 9 + 81 - 18k + k^2$$
$$10 - 2k = 90 - 18k$$
$$18k - 2k = 90 - 10$$
$$16k = 80$$
$$\frac{16k}{16} = \frac{80}{16}$$
$$k = 5$$
$$Center\ (0,0,5) \qquad r \qquad (2,\sqrt{5},1)$$
$$PC = r = \sqrt{(2-0)^2 + (\sqrt{5}-0)^2 + (1-5)^2}$$
$$r = \sqrt{4+5+16}$$
$$r = \sqrt{25}$$
$$r = 5$$
$$A)\ (-4,3,-5) \qquad r \qquad (0,0,5)$$
$$r = \sqrt{(-4-0)^2 + (3-0)^2 + (-5-5)^2} = \sqrt{25} = 5\ \ldots on$$
$$B)\ (2,-2,1) \qquad r \qquad (0,0,5)$$
$$r = \sqrt{(2-0)^2 + (-2-0)^2 + (1-5)^2} = \sqrt{24} = 2\sqrt{6}\ \ldots on$$
$$C)\ (3,1,1) \qquad r \qquad (0,0,5)$$
$$r = \sqrt{(3-0)^2 + (1-0)^2 + (1-5)^2} = \sqrt{26} > 5\ \ldots off$$
$$D)\ (0,0,0) \qquad r \qquad (0,0,5)$$
$$r = \sqrt{(0-0)^2 + (0-0)^2 + (0-5)^2} = \sqrt{25} = 5\ \ldots on$$

Answer C

2. Let A and B be vectors in space such that A.B= -2 and B=6i-7j + $\sqrt{15}K$ if θ is the angle between A and B, then what is the value of |A|?A

   A. 1/2cos θ B. 1/5cos θ C. -1/5cos θ D. $|A| = \frac{-1}{5cos\theta}$

Solution

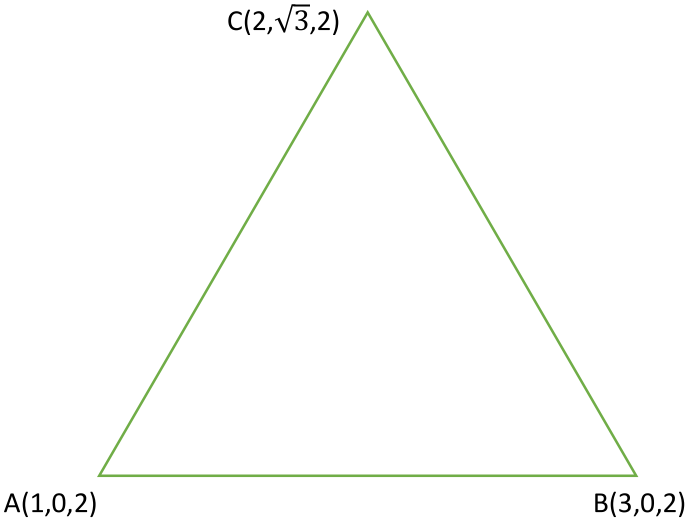
$$\vec{A}.\vec{B} = -2, \quad \vec{B} = 6i - 7i + \sqrt{15}\,k$$
$$A.B = |A||B|\cos\emptyset$$
$$-2 = |A|\sqrt{6^2 + (-7)^2 + (\sqrt{15})^2}\ cos\theta$$
$$-2 = |A|\sqrt{36 + 49 + 15}\,cos\theta$$
$$-2 = |A|10\cos\theta$$
$$\frac{-2}{10\cos\theta} = |A|$$
$$|A| = \frac{-1}{5cos\theta}$$
$$Answer\ D$$

3. If A(x, 0, 2), B(3, 0, 2) and C(2, √3, 2) are vertices of an equatorial triangle in space, then what is the value of x?

   A. 5 B. 3 C. 2 D. 1

Solution

$$A(x,0,2), B(3,0,2), C\left(2,\sqrt{3},2\right)$$
$$AB = BC \quad ,$$
$$BC = \sqrt{(x-2)^2 + (0-\sqrt{3})^2 + (2-2)^2} = \sqrt{1^2 + \sqrt{3}^2 + 0^2} = \sqrt{1+3} = \sqrt{4} = 2$$
$$AB = BC$$
$$\sqrt{(x-2)^2 + (0-0)^2 + (2-2)^2} = 2$$
$$\sqrt{(x-3)^2} = 2$$
$$(x-3)^2 = 4$$
$$(x-3) = \pm 2$$
$$x = 2 + 3 = 5, does\ not\ satisfy$$
$$x = -2 + 3 = 1$$
$$Answer\ D$$



Grade 12 unit six – three Dimensional Geometry and Vectors in Space

2010 E.C

1. Let A(0, 2, 5) for a>0, be a point on a sphere $x^2+y^2+z^2-6z=0$ and C be the center of the sphere. If P(k, 2, 4) is a point in space such that PA is perpendicular to CA, what is the cosine of the angle between PA and PC?

   A. $5/\sqrt{70}$ B. $7/\sqrt{35}$ C. $5/\sqrt{35}$ D. $7/\sqrt{70}$

Solution

$$A(0,2{,}5), x^2+y^2+z^2-6z=0, a>0, \qquad \overline{PA}\perp C$$

$\propto^2+2^2+5^2-6(5)=0$ since $(\propto,2{,}5)$ is on the sphere

$\propto^2+4+25-30=0$

$\propto^2-1=0$

$\propto=\pm1 \Rightarrow A(\propto,2{,}5)=(1{,}2{,}5)$

$x^2+y^2+z^2-6z+9-9=0$

$x^2+y^2+(z-3)^2-9=0$, center $C(0,0,3)$

$PA\perp CA$ $\qquad CA=(0{,}0{,}3)(1{,}2{,}5)=(k,2,2)$

$PA, CA=0$, $\qquad PA=(k,2{,}4)(1{,}2{,}5)=(1-k,0{,}1)$

$(1-k,2-2{,}5-4)*(1-0{,}2-0{,}5-3)=(1-k,0{,}1)*(1{,}2{,}2)=0$

$1-k+0+1=0$

$3-k=0$

$3=k\Rightarrow P(3{,}2{,}4)$, p=(k,2,4)=(3,2,4)

$PA=(3{,}2{,}4)(1{,}2{,}5)=(-2{,}0{,}1)$

$PC=(3{,}2{,}4)(0{,}0{,}3)=(-3,-2,-1)$

$$\cos(p)=\frac{\overline{PA}\,.\overline{PC}}{|PA||PC|}=\frac{6+0-1}{\sqrt{(-2)^2+0^2+1^2}\sqrt{(3)^2+(-2)^2+(-1)^2}}=\frac{5}{\sqrt{5}\sqrt{14}}=\frac{5}{\sqrt{70}}$$

Answer A

2. If the dot product of a vector A with vectors i –j +k, 2i+i-3k and i +j + k are 4, 0, 2, respectively. What is A?

   A. A= (2, -1, 1) B. A= (-2, 1, -1) C. A = (-2, -1, 1) D. A= (2, 1, 1)

Solution

let $\overline{A}=ai+bj+ck$

$(ai+bj+ck).(i-j+k)=4,$ $\quad\Rightarrow a-b+c=4$

$(ai+bj+ck).(2i+j\ +-3k)=0$ $\quad\Rightarrow 2a+b-3c=0$

$(ai+bj+ck).(i+j+k)=2$ $\quad\Rightarrow a+b+c=2$

$$\begin{cases} a-b+c=4 \\ 2a+b-3c=0, \text{ systems of linear equations, use cramers rule} \\ a+b+c=2 \end{cases}$$

$$D=\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{vmatrix}=1\begin{vmatrix}1 & -3\\ 1 & 1\end{vmatrix}+1\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}+1\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=4+5+1=10$$

$$Dx=\begin{vmatrix}4 & -1 & 1\\ 0 & 1 & -3\\ 2 & 1 & 1\end{vmatrix}=4\begin{vmatrix}1 & -3\\ 1 & 1\end{vmatrix}+0\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}+2\begin{vmatrix}-1 & 1\\ 1 & -3\end{vmatrix}=4(4)+2(2)=20$$

$$Dy=\begin{vmatrix}1 & 4 & 1\\ 2 & 0 & -3\\ 1 & 2 & 1\end{vmatrix}=1\begin{vmatrix}0 & -3\\ 2 & 1\end{vmatrix}-4\begin{vmatrix}2 & -3\\ 1 & 1\end{vmatrix}+1\begin{vmatrix}2 & 0\\ 1 & 2\end{vmatrix}=6-20+4=-10$$

$$Dz=\begin{vmatrix}1 & -1 & 4\\ 2 & 1 & 0\\ 1 & 1 & 2\end{vmatrix}=1\begin{vmatrix}1 & 0\\ 1 & 2\end{vmatrix}+1\begin{vmatrix}2 & 0\\ 1 & 2\end{vmatrix}+4\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=2+4+4=10$$

$x=\frac{Dx}{D}=\frac{20}{10}=2,$ $\qquad y=\frac{Dy}{D}=\frac{-10}{10}=-1,$ $\qquad z=\frac{Dz}{D}=\frac{10}{10}=1$, s.s=$\{(2,-1{,}1),\}$

Answer A

Grade 12 unit six – three Dimensional Geometry and Vectors in Space

2011 E.C

1. If the angle between the vector A= (2,-1, 1) and B= (1, 1, α) is $\frac{\pi}{3}$, then what is the value of α?

   A. 1 B. – 1 C. – 2 D. 2

Solution

$$\bar{A} = (2,-1,1) \quad \overline{B} = (1,1,\alpha), \quad \cos\theta = cos\,\frac{\pi}{3} = cos\,60^0 = \frac{1}{2}$$

$$A.B = |A||B|\cos\theta$$

$$(2,-1,1)(1,1,\alpha) = \sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+1^2+\alpha^2} * cos\,60^0$$

$$2(1)-1(1)+1(\alpha) = \sqrt{6}\sqrt{2+\alpha^2} * \frac{1}{2}$$

$$1+\alpha = \frac{\sqrt{6(2+\alpha^2)}}{2}$$

$$2+2\alpha = \sqrt{12+6\alpha^2}$$

$$4+8\alpha+4\alpha^2 = 12+6\alpha^2$$

$$2\alpha^2-8\alpha+8 = 0$$

$$\alpha^2-4\alpha+4 = 0$$

$$(\alpha-2)^2 = 0, \alpha = 2$$

Answer D

2. If u=2j-k and v=i-8j+3k, then what is the unit vector in the direction of 5u + v?

   A. i+2j-2k B. $\frac{1}{3}i+\frac{2}{3}j-\frac{2}{3}k$ C. 2/3i+2/3i-2/3k D. 1/3i+2/3i+2/3k

Solution

$$\text{U}= 2j-k, v = i-8j+3k$$

$$5\,\text{U} + v = 5(2j-k)+i-8j+3k$$

$$= 10j-5k+i-8j+3k$$

$$= i+2j-2k$$

$$\text{U}= \frac{i+2j-2k}{\sqrt{1^2+2^2+(-2)^2}}$$

$$= \frac{i+2j-2k}{\sqrt{9}}$$

$$= \frac{i+2j-2k}{3}$$

$$= \frac{1}{3}i+\frac{2}{3}j-\frac{2}{3}k$$

Answer B

3. If the point $(\alpha, 0, 3)$ is on the sphere centered at (1, 2, 3) with radius 2,what is the value of α

   A. 1 B. 0 C. 2 D. – 3

Solution

$$(\alpha,0,3) \quad (1,2,3)$$

$$r = \sqrt{(1-\alpha)^2+(2-0)^2+(3-3)^2}$$

$$2 = \sqrt{(1-\alpha)^2+2^2}$$

$$4 = (1-\alpha)^2+4$$

$$(1-\alpha)^2 = 0$$

$$1-\alpha = 0$$

$$\alpha = 1$$

Answer A

Grade 12 unit six – three Dimensional Geometry and Vectors in Space

2013 E.C

1. Which of the following defines the equation of a sphere whose center is at (0, -1 2) and radius 6 units?
   A. $x^2 + (y-1)^2 + (1-2)^2 = 36$
   B. $x^2 + (y+1)^2 + (z-2)^2 = 36$
   C. $x^2 + (y+1)^2 + (z+2)^2 = 36$
   D. $x^2 + (y+1)^2 + (z-2)^2 = 12$

Solution

$$(x-a)^2 + (y-b)^2 + (z-C)^2 = 6^2, \qquad (a,b,c) = (0,-1,2), r = 6$$
$$(x\text{-}a)^2 + (y-b)^2 + (z-c)^2 = 6^2$$
$$x^2 + (y+1)^2 + (z-2)^2 = 36$$

Answer B

2. Which of the following represents the vector (3, -5, 7) in the space using the standard unit vector i, j and K?
   A. 3i + 5j + 7K B. 3i – 5j – 7K C. -3i -5j -7K D. 3i – 5j + 7K

Solution

$$(3,-5,7), V = 3i - 5i + 7k,$$

Answer D

3. Which one of the following is the scalar (dot) product of the vector (2, -4, 6) and (-2, -1,0)?
   A. – 8 B. 0 C. 6 D. 8

Solution

(2,-4, 6)* (-2,-1, 0) = 2(-2) + (-4) (-1)+ 6(0)

= - 4 + 4 + 0

= 0

Answer B

Grade 12 unit seven – mathematical proofs

2009 E.C

1. Which one of the following is a valid assertion that can be proved by principle of mathematical induction?
   A. The sums of any two positive rational numbers are positive
   B. $r^2 \geq 1$, for every real number $r \geq 1$
   C. $n^2 \geq 4n$, for every integer $n \geq 4$
   D. $2n \leq 2^n$, for every integer $n \leq 100$

Solution

$$Principal\ matihematical\ induction\ \Rightarrow use\ only\ positive\ integers$$
$$Answer\ C$$

2. Consider the assertion: ''the sum of positive irrational numbers is positive irrational number''. Which one of the following is correct about the assertion?
   A. Taking irrational numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$, and so on. If we add any two of them, the sum is irrational. Therefore, the assertion is true.
   B. The sum of $1+\sqrt{2}$ and $1-\sqrt{2}$ is 2, which is rational. This is a counter example that disproves the assertion
   C. The sum of $\sqrt{7}$ & $\sqrt{2}$ is a counter example that shows the assertion is false.
   D. The assertion can be disproved by taking the sum of $1+\sqrt{2}$ and $2-\sqrt{2}$ as a counter example

Solution

$$use\ counter\ example, disproved\ using\ counter\ example$$
$$Answer\ D$$

3. Suppose if $'x \in A$, then $y \in B'$, is a true statement. Then, which one of the following necessary true?
   A. $y \in B$ B. If $x \notin A$, then $y \in B$ C. If $y \in B$, then $x \in A$ D. If $y \notin B$, then $x \notin A$

Solution

$$If\ x\ \in A\ , then\ y\ \in B\ \equiv T, p\ \Rightarrow q\ \equiv T$$

   A. $q\ can\ be\ T\ if\ p = T and\ F\ if\ p = F$
   B. $\neg\, p\ \Rightarrow\ \neg\, q\ can\ be\ F\ if\ \neg\, p = T\ and\ \neg\, q = F$
   C. $q\ \Rightarrow p\ can\ be\ F\ if\ q = T\ and\ p = F$
   D. $\neg\, q\ \Rightarrow\ \neg p\ is\ contrapositive\ of\ p\ \Rightarrow q\ \Rightarrow True,$

$Answer\ D$

Grade 12 unit seven – mathematical proofs

2010 E.C

1. Which one of the following is correct application of the principle of mathematical induction?
   A. If P(10) is true; and assuming p(n) is true for any n>10 if it follows that p(n+1) is true, then P(n) is true for all n≥ 10
   B. If P(1) is true for n=1; and both P(n) and P(n+1) are true for certain n∈ ℕ, then P(n) is true for all n∈ ℕ is true, then P(n) is true for all n∈ ℕ
   C. If P(10) is true; and if P(n) is true implies that P(n+1) is true, P(n) is true for n∈ ℕ
   D. If P(1) is true; and P(n)⇒ P(n+1) is true for any n∈ ℕ, then P(n) is true for all n∈ ℕ.

Solution

Inductive

$$\text{If p(n)is T, then p}(n+1)\text{is T}$$

Answer A

2. Consider the following assertion:

   P+2^n is an odd number for any prime P and any n∈ ℕ.

   Let P(n) be an open proposition on the set of natural numbers(ℕ). Which of the following is correct about prove or disprove of the assertion?
   A. The assertion can be proved by direct method: because P is odd and $2^n = 2(2^{n-1})$ is even imply that P+2^n is odd since the sum of odd and even is odd.
   B. The assertion can be proved by indirect method: because if n∈ ℕ, then $2^n \nexists$ ℕ and hence p +2^n is not odd.
   C. The assertion can be disproved by the method of contradiction.
   D. There is counter example that disproves the assertion

Solution

$$\text{Let } p = 2, n = 1, p + 2^n = 2 + 2 = 4, not\ an\ odd\ number$$

disprove by counter example,

Answer D

Grade 12 unit seven – mathematical proofs

2011 E.C

1. (∀n) 3^n -2 is prime that can be proved or disproved by which one of the following mathematical proofs?
   A. Direct proof C. Proof by contradiction
   B. Proof by Exhaustion D. Disprove by counter Example

Solution

$$3^n - 2\ is\ prime$$

$$dis - proof\ by\ counter\ example$$

$$eg\ n = 1, 3^n - 2 = 3^1 - 2 = 3 - 2$$

$$= 1\ is\ not\ prime$$

Answer D

2. Which of the following is a correct assertion that can be proved by the principle of mathematical induction?
   A. K!≥ 2^k, for each integer k≥4
   B. m!≤ 4^m, for each positive integer m
   C. 2^p-1 is prime for each integer m.
   D. $\frac{1}{n+1} \leq 1$, for each real number n≥ 1

solution

$$k! \geq 2^k \ True$$

Answer A

3. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?
   A. 3n+25<3^n,n≥3
   B. n^3-n is divisible by 6, for every integer n≥ 1
   C. 2^n>n+20, for every integer n≤ 2^n, for every integer n≥ 4
   D. $n^2 \leq 2^n$, for every integer n≥1

Solution

$$A)\ 3n + 25 < 3^n \ for\ n \geq 3$$
$$eg\ \ n = 3\ \ , 3(3) + 25 < 3^{\,3}$$
$$9 + 25 < 27$$
$$34 < 27 \quad False$$
$$B)\ n^3 - n\ \ is\ divisible\ by\ 6\ \ , n\ \geq 1$$
$$if\ n = 1 \Rightarrow 1^3 - 1 = 0\ \ is\ \ divisible\ by\ 6$$
$$n\,\epsilon\,N\ , \qquad n^3 - n\ = 6d\ \ , d\,\epsilon\,N\ \Rightarrow\ n^3 = n + 6d$$
$$n + 1\,\epsilon\,N\ , \qquad (n + 1)^3 - (n + 1)\ \ = n^3 + 3n^2 + 2n$$
$$n + 6d + 3n^2 + 2n$$
$$3n^2 + 3n + 6d$$
$$If\ n\ is\ odd \Rightarrow n = 2m - 1$$
$$(n + 1)^3 - (n + 1) = 3n + 6d + 3n^2$$
$$= 3\ (2m - 1) + 6d + 3(2m - 1)^2$$
$$= 6m - 3 + 6d + 3(4m^2 - 4 + 1)$$
$$= 6m - 3 + 6d + 12m^2 - 12m + 3$$
$$= 6(2m^2 - m + d) \quad , divisible\ by\ 6$$
$$If\ n\ is\ even\ \Rightarrow\ \ n = 2m$$
$$(n + 1)^3\ (n + 1) = 3(2m) + 6d + 3(2m)^2$$
$$= 6m + 6d + 12m^2$$
$$= 6(2m^2 + m + d\ ) \quad , divisible\ by\ 6$$
$$Answer\ B$$

Grade 12 unit seven – mathematical proofs

2013 E.C

1. Which of the following proof is more suitable to prove the assertion: 1 + 2^3 + 3^3 + … + n^3 = (1 + 2+ 3 + … + n$)^2$
   A. Proof by the principle of mathematical induction.
   B. Proof by counter example.
   C. Proof by indirect proof.
   D. Proof by contradiction.

Solution

1+2^3+3^3+--------n^3 = (1+2+3+----+n$)^2$

Proof by the principle of mathematical induction,

Answer A

Grade 12 unit eight – further on statistics

2009 E.C

1. Ten different terms played football over the summer, after the summer the top goal scorers from each team scored 10, 5, 18, 2, x, 4, 10, 15, 11, 4. If the mean number of goals scored is 9, then what is the mean deviation of the data?

   A. 2.4 B. 3.4 C. 4.2 D. 4.3

Solution

$$\frac{10+5+18+2+x+4+10+15+11+4}{10}=9$$

$$\frac{79+x}{10}=9$$

$$79+x=90$$

$$x=90-79$$

$$x=11$$

$$MD=\frac{|10-9|+|5-9|+|18-9|+|2-9|+|11-9|+|4-9|+|10-9|+|15-9|+|11-9|+|4-9|}{10}$$

$$=\frac{42}{10}=4.2$$

$$Answer\ C$$

2. Ten students took two tests, Test I and Test II. The mean mark in test I is 5 with standard deviation 2, and the mean mark in test II is 6 with standard deviation 2. Which one of the following is true?
   A. The coefficient of variation of test I is 25%.
   B. The coefficient of variation of test II is 30%.
   C. The students mark in test II is more consistent than in test I
   D. The student mark in test I and test II have the same degree of variability.

Solution

$$I,\overline{X}=5\,,\delta=2$$

$$II,\overline{X}\ =6\,,\delta=2$$

$$CV\ (I)=\frac{\delta}{\overline{X}}*100=\frac{2}{5}*100=40\%$$

$$CV\ (II)=\frac{\delta}{\overline{X}}*100=\frac{2}{6}*100=33.3\%$$

$$CV(II)>CV(I)$$

$$I\ is\ more\ variable$$

$$II\ is\ more\ consistant$$

$$Answer\ C$$

Grade 12 unit eight – further on statistics

2010 E.C

1. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the variance of the resulting observations

   A. 5 B. 20 C. 10 D. 40

Solution

$$\text{Old variance}=\delta^2$$

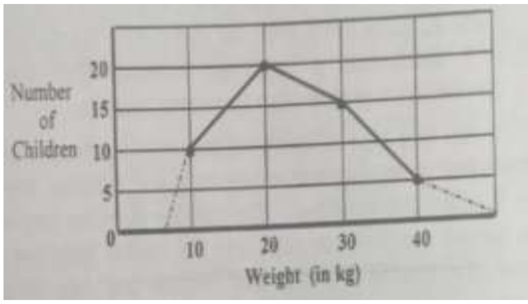
$$\text{if each data is multipled by n, new var}=n^2\delta^2$$

$$\Rightarrow\text{old varoance}=5,\text{each data multiplated by 2}$$

$$\Rightarrow\text{new variance}=2^2x5=4x5=20$$

Answer B

2. The weight (in Kg) of children in a certain nursery are grouped in to four class intervals of equal width and represented by the following frequency polygon. Which one of the following is true about the data?
   A. The mode of the weight is 20Kg.
   B. The median weight is 20Kg.
   C. The distribution of the weight is positively skewed.
   D. The mean weight is 25Kg.



V	10	20	30	40	Total
f	10	20	15	5	50

<u>Solution</u>

$$mode = 20\ high\ frequent$$

$$\overline{x} = \frac{10(10) + 20(20) + 30(15) + 40(5)}{10 + 20 + 15 + 5}$$

$$= \frac{100 + 400 + 450 + 200}{50} = \frac{1150}{50} = 23$$

$$md = \frac{25^{th}v + 26^{th}v}{2} = \frac{20+20}{2} = 20$$

$$Answer\ A$$

3. Which one of the following is true?
   A. If the mean, mode and median of a distribution are 7, 5.5 and 6 respectively, then the distribution is negatively skewed.
   B. If Q1, Q2 and Q3 of a distribution are 7, 10 and 13 respectively, then the distribution is symmetrical.
   C. If the mean, median and standard deviation of a distribution are 4, 6 and 2 respectively, then the distribution is positively skewed
   D. If a distribution is negatively skewed, then the mean pf the distribution is greater than its second quartile.

<u>Solution</u>

$$A)\ \overline{x} = 7$$

$$md = 5.5$$

$$mo = 6$$

$$\overline{x}\ > mode\ , positively\ skewed\ False$$

$$B)\ Symmetric \begin{cases} Q1 = 7 \\ Q2 = 10\ , true \\ Q3 = 13 \end{cases}$$

$$C)\ \overline{x} = 4$$

$$md = 6$$

$$\delta = 2$$

$$\overline{x} < md - negatively\ skewed, \qquad False$$

$$D)\ \overline{x} > Q2$$

$$\overline{x} > md$$

$$Because\ median = Q2$$

$$Positively\ Skewed, false$$

$$Answer\ B$$

4. The grouped frequency distribution of a data is given by

Class interval	8-12	13-17		18-22	23-27	28-32
frequency	4	8		10	5	3

What is the median (m) and mean deviation (MD) about the median?

A. $m = 20, Md = 4.2$ C. $m = 20, MD = 4.7,$

B. $m = 19, MD = 4.2$ D. $m = 19, MD = 4.7$

Solution

x	*xc*	*f*	*Cf*	*Class boundary*	*CX*
8-12	*10*	*4*	*4*	*7.5 - 12.5*	*10*
13-17	*15*	*8*	*12*	*12.5 – 17.5*	*15*
18-22	*20*	*10*	*22*	*17.5 – 22.5*	*20*
23-27	*25*	*5*	*27*	*22.5 – 27.5*	*25*
28-32	*30*	*3*	*30*	*27.5 – 32.5*	*30*

$$median\ class = \left(\frac{15^{th} + 16^{th}}{2}\right)\ item$$

$$= 3^{rd}\ class$$

$$md = B_l + \left(\frac{\frac{n}{2} - Cfb}{f}\right)i = 17.5 + \left(\frac{\frac{30}{2} - 12}{10}\right)5 = 17.5 + \left(\frac{3}{2}\right) = 17.5 + 1.5 = 19$$

$$MD = \frac{4|10 - 19| + 8|15 - 19| + 10|20 - 19| + |25 - 19| + 3|30 - 19|}{30}$$

$$= \frac{4(9) + 8(4) + 10(1) + 6(5) + 3(11)}{30} = \frac{36 + 32 + 10 + 30 + 33}{30} = \frac{141}{30} = 4.7$$

Answer D

Grade 12 unit eight – further on statistics

2011 E.C

1. Given a data score of students

X	3	4	5	6	7	8	9	10
f	2	3	5	4	8	6	6	6

Which one of the following is true?

A. The 2nd quartile of the mark is

B. The 75th percent of the mark is 8.

C. The 25th percent of mark is 5.

D. The top 30% of the scores are above 8.

Solution

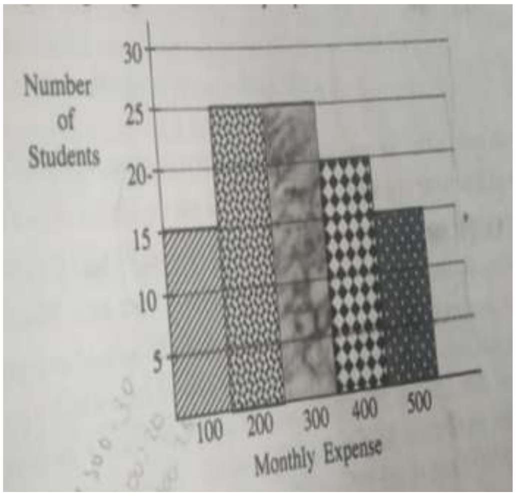
$$above\ 8 = \left(\frac{6 + 6}{40}\right) * 100\%$$

$$= \frac{12}{40} * 100\%$$

$$= 30\%$$

Answer D

2. The following histogram gives the monthly expenses of 100 students
   Which one of the following is true?
   A. Only 40 students have monthly expense lees than birr 400.
   B. 60% of the students have monthly expense less than 400.
   C. 35% of the students have monthly expense more than 300.
   D. SO students have monthly expense more than 200.



Solution

X	100	200	300	400	500
f	15	25	25	20	15

$$A) < 400 = \frac{(15+25+25)}{100} * 100\% = 65\%$$

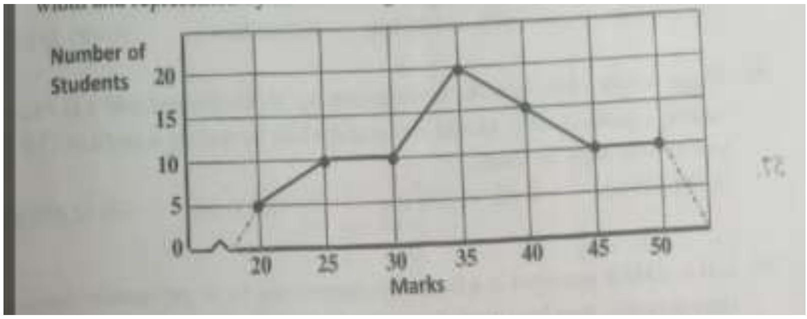
$$B) < 400 = 65\%$$

$$C) \ > 300 = \frac{(20+15)}{100} * 100\% = 35\%$$

$$D) \ > 200 = \frac{(25+20+15)}{100} * 100\% = 60\%$$

Answer C

3. Marks scored by students in exam re grouped in to seven class intervals of equal width and represented by the following frequency distribution.



   Which one of the following is true about the marks?
   A. The distribution of the mark is positively skewed.
   B. The mean and the mode of the marks are equal
   C. The mode of the mark is 35
   D. 75% of the students scored greater than 30

Solution

X	20	25	30	35	40	45	50
f	5	10	10	20	15	10	10

$$\bar{x} = \frac{100+250+300+700+600+450+500}{5+10+10+20+15+10+10}$$

$$= \frac{3000}{80} = 37.5, mean\ is\ greater\ than\ mode\ and\ median$$

$$md = \frac{40th + 45th}{2} = \frac{35+35}{2} = 35$$

$$mo = 35, most\ frequent$$

$$> 30 = \frac{35}{80} * 100\% = 43\ \%$$

Answer C

4. The following table presents the mean weight (in Kg) of four group of students

Group	I	II	III	IV
Mean weight	72	80	75	80
Number of Students	10	12	8	10

What is the mean weight (in Kg) of all the students?

A. 77 B. 72 C. 87 D.80

Solution

$$mean = \frac{72*10+80*12+75*8+80*10}{10+12+8+10}$$

$$= \frac{3080}{40}$$

$$= 77$$

Answer A

Grade 12 unit eight – further on statistics

2013 E.C

1. Which one of the following is an advantage of using the techniques of random sampling in statistical methods?
   A. The sample collected may not be good representative
   B. The investigator personally cannot be a source of bias.
   C. A person with limited know how can apply the techniques.
   D. Each technique of sampling is applicable to any type of population.

Solution

Advantages of random sampling

- It is free from any personal bias of the investigator.
- The sample is a better representative.

Limitations of random sampling

- It needs skill and experience.
- It requires time to plan and carry out

Answer B

2. On the graph of the cumulative frequency distribution of a certain data, it is observed that, the mean, median, mode and standard deviation are 9.8, 10, 10.4 and 2.1 respectively. Which one of the following is correct conclusion about this distribution?
   A. The distribution is skewed to the positive
   B. The data value are systematically distributed
   C. Most of the data value lie to the left of the modal valve.
   D. Most of the data valves lie to the right of the modal valve.

Solution

$$\bar{x} = 9.8$$

$$md = 10$$

$$Mo = 10.4$$

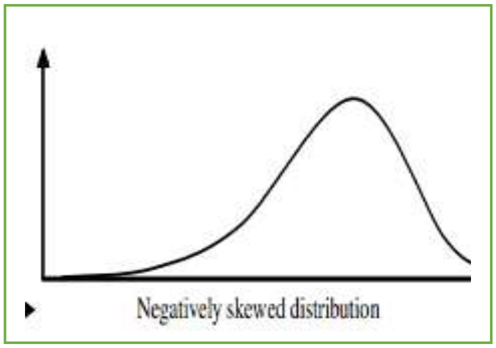
$$S.d = 2.1$$

$$x < md < mo$$

$$Negetively\ skewed$$

$$most\ of\ the\ data\ lies\ to\ the\ right$$

Answer D



3. Consider the data given below

   10, 12, 10, 15, 12, 16, 13, 11, 12, 15

   The mean deviation about the mode of this data is equal to: -

   A. 1.6 B. 1.5 C. 2 D. 1.8

Solution

$$10, 10, 11, 12,12,12, 13, 15, 15, 16 - \ mode\ 12$$

$$MD = \frac{|10-12|+|10-12|+|11-12|+|12-12|+|12-12|+|12-12|+|13-12|+|15-12|+|15-12|+|16-12|}{10}$$

$$= \frac{2+2+1+0+0+0+1+3+3+4}{10}$$

$$= \frac{16}{10} = 1.6$$

$$Answe\ A$$

4. The following table shows the mean and standard deviation of expenditures in birr of three factories F1, F2 and F3 in a week

	F1	F2	F3
Mean	12,500	12,500	2,500
Standard Deviation	3.6	3.56	3.6

Which factory of factories show more consistent in their expenditures than the other(s)?

A. Factor 2 B. Factor 1 C. Factories 1 and 2 D. Factories 1 and 3

Solution

$$F1, CV = \frac{S}{x} * 100 = \frac{3.6}{12{,}500} * 100 = 0.0289$$

$$F2,\ \ CV = \frac{S}{x} * 100 = \frac{3.56}{12{,}500} * 100 = 0.0284$$

$$F3,\ \ CV = \frac{S}{x} * 100 = \frac{3.6}{2{,}500} * 100 = 0.144$$

$$F2\ s\ more\ consistent$$

$$F3\ is\ more\ variable$$

$$Answer\ A$$

Grade 12 unit nine – Mathematical Applications for Business

2009 E.C

1. In a shop, the marked price of an item was birr 260 with 30% markup based on its cost price. Later, the shop owner sold the item with 30% discount. What is the profit or lost form the item?

   A. NO profit and NO loss B. Birr 78 loss C. Birr 18 profit D. Birr 18 loss

Solution

$$260 = x + 30\%\ x$$

$$260 = 1.3x$$

$$x = Price = 200$$

$$Sale = 260 - 260(30\%) = 260 - 260(0.3) = 260 - 78 = 182$$

$$200 - 182 = 18\ loss$$

$$Answer\ D$$

2. If a loan of Birr 21,000 with 12% annual interest compounded monthly is amortized over 10 years, how much is the monthly payment?

   (Given $(1.012)^{120} = 0.24$, $(1.01)^{-120} = 0.3$, $(1.01)^{120} = 3.3$, $(1.012)^{120} = 4.2$*)*

   A. Birr 280 B. Birr 300 C. Birr 333 D. Birr 350

Solution

$$PP = P\left(\frac{i}{1-(1+i)^{-n}}\right) = 21{,}000\left(\frac{0.01}{1-(1+0.01)^{-10(12)}}\right) = 21{,}000\left(\frac{0.01}{1-(10.01)^{-120}}\right)$$

$$\boxed{i=\frac{0.12}{12}=0.01}$$

$$= 21{,}000\left(\frac{0.01}{1-0.3}\right) = 21{,}000\left(\frac{0.01}{0.07}\right) = 21{,}000\ (0.014285) = 300, Answer\ B$$

3. In a company the regular work time is 40 hours per week, and the over-time period is 1.5 times the regular hourly period rate. If an employ whose regular hourly period is 12 birr received Birr 750 for his work of a week, how many hours did he work overtime in the week?

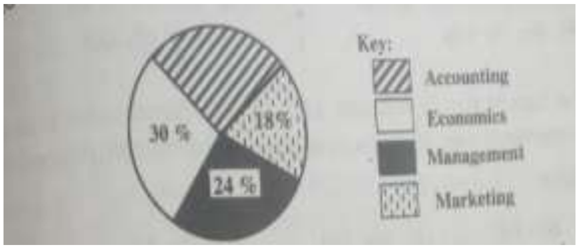
A. 12 B. 15 C. 18 D. 20

$$\underline{Solution}$$
$$regular = 12\ Birr$$
$$Over\ time = 1.5\ (12)$$
$$= 18\ BIrr$$
$$12(40) + 18x = 750$$
$$480 + 18x = 750$$
$$18x = 750 - 480$$
$$18x = 270,$$
$$x = \frac{270}{18} = 15\ hour$$
$$Answer\ B$$

4. If the number of students enrolled in an accounting department is 140, how many students rolled in economics department?

A. 152 B. 150 C. 145 D. 142

Solution



$$Let\ x = total\ number\ of\ students$$
$$Accounting = 100\% - 24\% - 18\% - 30\%$$
$$= 100\% - 72\%$$
$$= 28\%$$
$$28\%\ X = 140$$
$$\frac{28}{360}x = 140$$
$$x = \frac{140 * 360}{28} = 1800$$
$$eco = 30\%\ of\ 1800$$
$$\frac{30}{360} * 1800 = 150$$
$$Answer\ B$$

Grade 12 unit nine – Mathematical Applications for Business and consumers 2010 E.C

1. A man bought a one-bedroom house for birr 250,000 and paid a down payment of 20%. The remaining is a mortgage to be paid monthly for 15 years with 6% annual interest. What is the monthly payment in birr?

(Given $(1.005)^{15} = 0.93$, $(1.005)^{-180}=0.41$, $(1.006)^{15}=0.91$, $(1.006)^{180}=0.34$)

A. 1333 B. 1694 C. 1428 D. 1818

Solution

$$p = 250{,}000 - 250{,}000 * 0.2 = 250{,}000 - 50{,}000 = 200{,}000$$
$$P.P = P * \frac{i}{1 - (1+i)^{-n}} \quad where\ i = \frac{6\%}{12} = \frac{0.06}{12} = 0.005$$
$$= 200{,}000\ \frac{0.005}{1 - (1 + 0.005)^{-180}} = 200{,}000\left(\frac{0.005}{1 - 0.41}\right)$$
$$= 200{,}000\left(\frac{0.005}{0.59}\right) = 200{,}000\ (0.00847) = 1694$$
$$Answer\ B$$

2. A firm has two type of overtime pay rates (payment per hour) for its employees; normal overtime pay rate and holyday overtime pay rate (for Sundays and holydays). The holyday overtime pay rate is 1.5 times the normal pay rate and the normal overtime pay rate is 1.5 times the regular pay rate. The regular pay rate for an employee is given by his weekly basic wage divided by 40. If an employee whose weekly basic wage is 2400 has additionally, 6 hours of normal overtime rate and 3 hours of holyday overtime in a week, how much (in birr) is his payment for the week?

A. 3345 B. 3300 C. 3210 D. 2525

Solution

$$\frac{2400}{40} = 60$$
$$regular\ pay = 60$$
$$Over\ time = 1.5 * 60 * 6 = 540$$
$$Holy = 1.5 * 60 * 1.5 * 3 = 405$$
$$Weekly = 2400 + 540 + 405 = 3345$$
$$Answer\ A$$

Grade 12 unit nine – Mathematical Applications for Business and consumers 2011 E.C

1. You are in the process of buying a house, and you need a mortgage loan of birr 200,000. You got approved for a 15-year fixed rate loan at a rate of 4.5%. How much in total will you pay for this house? Given $\begin{bmatrix}(1.00375)^{180} = 1.96 \quad , \quad (100375)^{-180} = 0.51 \\ \frac{0.00375}{0.49} = 7.65 * 10^{-3}\end{bmatrix}$

A. Birr 257,000 B. Birr 250,000 C. Birr 275,400 D. Birr 250,450

Solution

$$P = P\left(\frac{i}{1-(1+i)^{-n}}\right), i = \frac{0.045}{12} = 0.00375$$
$$= 200{,}000\left(\frac{0.00375}{1-(1+0.00375)^{-180}}\right) = 200{,}000\left(\frac{0.00375}{1-(1.00375)^{-180}}\right)$$
$$= 200{,}000\left(\frac{0.00375}{1-0.51}\right) = 200{,}000\left(\frac{0.00375}{0.49}\right)$$
$$= 200{,}000\,(7.65 * 10^{-3})$$
$$= 1{,}530\ per\ month$$
$$total = 18{,}360 * 12 * 15 = 275{,}400$$
$$Answer\ C$$

2. IN a company, a regular work time is 40 hours per week. For a service above the regular is 1.5 times the regular rate and the holyday overtime rate is twice of the regular rate. If a employee whose regular pay is birr 50 per hour, has worked 55 hours in a week among which 5 hour are holyday overtime, how much (in birr) is his payment for 1the week?

A. 2500 B. 3215 C. 3500 D. 3250

Solution

$$55 hours$$
$$40(reg) + 10(\ overtime) + 5(holiday)$$
$$= 40(50) + 10(1.5)(50) + 5(2)(50)$$
$$= 2000 + 10(75) + 5(100)$$
$$= 2000 + 750 + 500$$
$$= 3250\ birr$$
$$Answer\ D$$

3. HAGOS bought a bike from the manufacture by 10,000 birr and sold it to ABRAHA by making a profit of 10%. ABRAHA also sold it by making a profit of 15%. What was the sale price of ABRAHA?

A. 15,500 Birr B. 11,500 Birr C. 11,500 Birr D. 12,650 Birr

Solution

$$Price = 10{,}000$$

$$Sold\ to\ Abraha$$

$$= 10{,}000 + 10\%\ of\ 10{,}000 = 10{,}000 + 0.1\ (10{,}000) = 10{,}000 + 1000 = 11{,}000\ BIrr$$

$$Abraha\ Sold\ it\ by\ 15\%$$

$$=11{,}000 + 15\%\ of\ 11{,}000 = 11{,}000 + 0.15\ (11{,}000) = 11{,}000 + 1650 = 12{,}650$$

$$Answer\ D$$

Grade 12 unit nine – Mathematical Applications for Business and consumers 2013 E.C

1. Store I sell a dozen of pens for birr 54 and store II sells 5 pens of the same brand for birr 23. If clerk wants to buy too pens from economical sales, then how much can be saved in birr?

A. 9 B. 8.5 C. 10 D. 12.5

Solution

$$I: Each\ price = \frac{54}{12} = 4.5, \qquad II: Each\ price = \frac{23}{5} = 4.6$$

$$difference = 4.6 - 4.5 = 0.1$$

$$Save = 100 * 0.1 = 10$$

$$Answer\ C$$

2. In a book shop, 15 copies of an algebra book cost 105 birr and 20 copies of a geometry book costs 125 birr. A customer needs to buy 6 algebra and 4 geometry books from the respective copies. What is the total amount that he has to pay?

A. 69 birr B. 67 birr C. 77 birr D. 72 birr

Solution

$$Price\ of\ each\ algebraBook = \frac{105}{15} = 7, Price\ of\ each\ gometeryBook = \frac{125}{20} = 6.25$$

$$total\ price = 6 * 7 + 4 * 6.25 = 42 + 25 = 67$$

$$Answer\ B$$

3. A painter agreed to work 40 hours per week at on hourly rate 60 birr including Saturdays and earns an overtime f time and half. If he is committed to work for 9 hours each day from Monday to Friday and 5 hours on Saturday, how much (in birr) will be earn after if weeks?

A. 12,400 B. 12,200 C. 13,200 D. 12,800

Solution

$$9 * 5 + 5 = 45 + 5 = 50\ hours,$$

$$40\ hours + 10\ overtime = 40(60) + 10(60)(1.5) = 2400 + 900 = 3300\ per\ week$$

$$In\ 4\ weeks = 3300 * 4 = 13{,}200$$

$$Answer\ B$$

4. A shop sale a park of 20 articles for 5,500 birr. It planned to clear the article during a holiday. If the shop sells each item for 220 birr. What percent discount has it made?

A. 18% B. 15% C. 22% D. 20%

Solution

$$Price = \frac{5500}{20} = 275, Sale = 220$$

$$\%\ of\ discount = \left(\frac{275 - 220}{275}\right) * 100\% = \frac{55}{275} * 100\% = 0.2 * 100 = 20\%$$

$$Answer\ D$$

SAT – QUANTITATIVE RESONING (2009 E.C – 2013 E.C)

SAT (quantitative reasoning)-2009

2009 E.C

1. When a number is divided by 2 the result is equal to the result when the same number is divided by 4. What is the number?

   A. -2 B. 0 C. 2 D. 4

   Solution

   $$\frac{x}{2} = \frac{x}{4}$$
   $$4x = 2x$$
   $$4x - 2x = 0$$, Answer B
   $$2x = 0$$
   $$x = 0$$

2. If $\frac{6}{x} = \frac{18}{x+6}$, what is the value of $\frac{x}{3}$?

   A. 1 B. 2 C. 3 D. 4

   Solution

   $$\frac{6}{x} = \frac{18}{x+6}$$
   $$18x = 6x + 36$$
   $$18x - 6x = 36$$
   $$12x = 36$$, Answer A
   $$\frac{12x}{36} = \frac{36}{36}$$
   $$\frac{x}{3} = 1$$

3. What is the value of x + 10, if $(x-5) = (\frac{11}{x+5})$, for x ≠-5

   A. -10 B. 4 C. 10 D. 25

   Solution

   $$x - 5 = \frac{11}{x+5}$$
   $$(\mathrm{x}-5)(\mathrm{x}+5) = 11,\ x^2 - 25 = 11$$
   $$x^2 = 36 \Rightarrow x = \pm\sqrt{36}$$
   $$x = \pm 6$$,
   $$x + 10 = 6 + 10 = 16$$
   $$x + 10 = -6 + 10 = 4$$
   Answer B

4. Which of the following is less than z if the number is between 0 and 1?

   A. z^2 B. $\sqrt{z}$ C. 2z D. $\frac{1}{z}$

   Solution

   $0 < Z < 1$, let Z=$\frac{1}{4}$

   $A. Z^2 = (\frac{1}{4})^2 = \frac{1}{16}$< z=$\frac{1}{4}$

   C. $2z = 2(\frac{1}{4})$=$\frac{1}{2}$>z=$\frac{1}{4}$

   $B.\ \sqrt{z} = \sqrt{\frac{1}{4}}$=$\frac{1}{2}$> z=$\frac{1}{4}$

   D. $\frac{1}{z} = \frac{1}{\frac{1}{4}}$=4 >z=$\frac{1}{4}$,

   Answer A

5. What is the sum of the following values (1/4 + 0.25 + 25/100+10/40+25/50+50/100?

   A. 1 B. 2 C. 3 D. 4

   Solution

   $$\frac{1}{4} + 0.25 + \frac{25}{100} + \frac{10}{40} + \frac{25}{50} + \frac{50}{100}$$
   $$= 0.25 + 0.25 + 0.25 + 0.25 + 0.5 + 0.5$$,
   $$= (0.25 + 0.25 + 0.25 + 0.25) + (0.5 + 0.5)$$
   $$= 1 + 1 = 2,\quad \mathit{Answer\ B}$$

6. What is the value of (0.0001 x 20^4) ÷ 16/10?

A. 2 B. 4 C. 6 D. 10

Solution

$$0.0001 * \; 20^4 \div \frac{16}{10}$$
$$= 1x10^{-4} * 20^4 * \frac{10}{16}$$
$$= 10^{-4} * 10 * \frac{20^4}{2^4}$$
$$= 10^{-4+1} * \left(\frac{20}{2}\right)^4$$
$$= 10^{-3} * 10^4$$
$$= 10^{-3+4} = 10, a^m * a^n = a^{m+n}$$

, Answer D

.

7. If 75 percent of x ix equal to p percent of 25, where p>0, what is the value of $\frac{x}{p}$?

A. 1/3 B. 3/16 C. ¾ D. 3

Solution

$$75\% \; of \; x = p \,\% \; of \; (25)$$
$$\frac{75x}{75p} = \frac{25p}{75p}$$
$$\frac{x}{p} = \frac{25}{75} = \frac{1}{3}$$

, Answer A

8. If 15 + x is 5 more than 20, what is the value of x?

A. 5 B. 10 C. 25 D. 50

Solution

$$15 + x = 20 + 5$$
$$15 + \mathrm{x} = 25$$
$$x = 25 - 15$$
$$x = 10$$

, Answer B

9. When 80,000 is written as 8 x 10^n, what is the value of n?

A. 2 B. 3 c. 4 D. 5

Solution

$$80000 = 8x10^4$$
$$n = 4,$$

Answer C

10. The price of one quintal of sugar is birr 5800 and it decreases at a rate of birr 2 every three months. The price of one quintal of coffee is birr 4200 and it increases at a rate of birr 8 every four months. How many years will take of the two commodities to become equal in price?

A. 10 B. 20 C. 50 D. 60

Solution

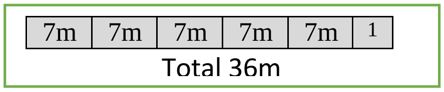
$$P \; of \; suger = mt + p_0 = \frac{-2}{3}t + 5800$$
$$p \; of \; coffee = mt + p_o = \frac{8}{4}t + 4250$$
$$\frac{-2}{3}t + 5800 = \frac{8}{4}t + 4200$$
$$5800 - 4200 = 2t + \frac{2}{3}t$$
$$1600 = \frac{8}{3}t$$
$$\frac{3(1600)}{8} = t$$
$$600 \; months = t, \quad \frac{600}{12} = 50 \; years = t$$
$$Answer \;\; C$$

11. If as many 7 meter pieces of wire as possible are cut from a wire that is 36 meters long, what is the total length of the wire that is over?

A. 1 meter B. 2 meters C. 3 meters D. 4 meters

Solution

36÷7=5+$\frac{1}{7}$, remainder is 1, Answer A



12. Hawa used ¼ of her monthly salary for buying teff and ½ more than the price of teff for house rent. What fraction of her monthly salary did Hawa use that month for teff and rent combined?

A. 1/3 B. 7/12 C. 5/8 D. ¾

Solution

$$teff = \frac{1}{4}x$$

$$rent = \frac{1}{4}x + \frac{1}{2}\left(\frac{1}{4}x\right) = \frac{1}{4}x + \frac{1}{8}x = \frac{3}{8}x$$

$$total\ ratio$$

$$\Rightarrow \frac{\frac{1}{4}x + \frac{3}{8}x}{x} = \frac{\frac{5}{8}x}{x} = \frac{5}{8}$$

$$Answer\ \ C$$

13. When 5 times the number x is added to 12, the result is 32. What number results when 2 times x added to 7?

A. – 15 B. – 5 C. 5 D. 15

Solution

$$5x + 12 = 32$$
$$5x = 32 - 12$$
$$5x = 20$$
$$x = 4$$
$$result\ when\ \text{2 times x added to 7} = 2(4) + 7$$
$$= 8 + 7$$
$$= 15$$

$$Answer\ \ D$$

14. If three students have score X, Y, and 80, with their sum equal to 180, what is the average (arithmetic mean) of the three scores?

A. 60 B. 70 C. 80 D. 90

Solution

$$\bar{x} = \frac{x + y + 80}{3} = \frac{180}{3} = 60$$

$$Answer\ \ A$$

15. If the sum of three consecutive integers is 33. What is the middle integer?

A. 9 B. 10 C. 11 D. 12

Solution

$$let\ the\ numbers\ be\ x, x+1, and\ x+2$$
$$x + x + 1 + x + 2 = 33$$
$$3x + 3 = 33$$
$$X = 10$$
$$middle\ term = x + 1 = 10 + 1 = 11$$

$$Answer\ \ C$$

16. What is the value of E, if ABCDE is a five digit positive number divisible by 5 and 10?

A. 0 B. 1 C. 2 D. 5

Solution

$$if\ the\ last\ digit\ is\ zero\ then\ the\ number\ is\ divisible\ by\ 5\ and\ 10$$

$$Answer\ \ A$$

17. The arithmetic mean P and q is 15, and the average of y and x is 15. What is the average of p, y, x and q?

A. 7.5 B. 15 C. 22.5 D. 30

Solution

$$\frac{p+q}{2} = 15\,, \qquad \frac{y+x}{2} = 15$$
$$p+q = 30 \qquad y+x = 30$$
$$\bar{x} = \frac{p+q+x+y}{4} = \frac{30+30}{4} = \frac{60}{4} = 15$$
$$Answer\ \ B$$

18. Which of the following is the average of the first 10 odd numbers?

A. 10 B. 15 C. 20 D. 25

Solution

$$S_{10} = \frac{n}{2}(2a_1 + (n-1)d), \quad a_1 = 1, d = 2$$
$$= \frac{10}{2}(2(1) + (10-1)(2)) = 5(2 + 9(2))$$
$$= 5(2+18) = 5(20) = 100$$
$$average = \frac{S_{10}}{10} = \frac{100}{10} = 10$$
$$Answer\ \ A$$

19. A number is called ''even-odd'' if it is halfway between an even integer and an odd integer. If x is an even-odd number, which of the following must be true?

I. 2x is an integer.

II. 2x is an even-odd.

III. x is halfway between two even integers

A. I only B. II only C. I and II only D. II and III only

Solution

$$x = even\ odd = halfway\ between\ even\ and\ odd\ integers$$
$$x = \frac{2m+2n-1}{2} = m+n-\frac{1}{2}, 2m\ is\ even\ and\ 2n-1\ is\ odd$$
$$I)\ 2x = 2(m+n-\frac{1}{2}) = 2m+2n-1, integer$$
$$II)\ 2x = 2\left(m+n-\frac{1}{2}\right) = 2m+2n-1, not\ even\ odd$$
$$III)\ x = m+n-\frac{1}{2} not\ half\ way\ between\ two\ even\ nombers$$
$$m+n-\frac{1}{2} \neq \frac{2m+2n}{2}$$
$$Answer\ \ A$$

Data interpretation questions (20-25)

DIRECTIONS: Questions 20-**25** below are based on the hypothetical data given in table below. Answer the questions based on the data and **blacken** the letter of your choice on the separate answer sheet provided.

Questions **20-25** are based on the hypothetical data given below

Annual income and Expenditure of Five Schools.

schools	**Income (in ETB)**	**Expenditure (in ETB)**
A	1,200,000	1,000,000
B	1,400,000	1,200,000
C	1,000,000	1,100,000
D	2,000,000	1,600,000
E	1,600,000	1,100,000
sum	**7,200,000**	**6,000,000**

20. How many schools had higher rate of saving compared to the rate of the saving from the total income of all schools

A. 1 B. 2 C. 3 D. 4

Solution

school	income	exp	Saving /total saving	Saving= income-exp
A	1,200,000	1,000,000	1/6	200,000
B	1,400,000	1,200,000	1/6	200,000
C	1,000,000	1,100,000	-1/12	-100,000
D	2,000,000	1,000,000	1/3	400,000
E	1,600,000	1,100,000	5/12	500,000
Total	7,200,000	6,000,000		1,200,000

$$\text{only schools } D \text{ and } E \text{ have highest rate of saving}$$
$$\text{Answer } B$$

21. If each school was expected to save at least 10% of its income, how many schools have met the expectation?

A. 1 B. 2 C. 3 D. all

Solution

school	income	10% of income	Saving=inco-ex	Met expectation
A	1,200,000	120,000	200,000	Yes
B	1,400,000	140,000	200,000	Yes
C	1,000,000	100,000	-100,000	No
D	2,000,000	200,000	400,000	Yes
E	1,600,000	160,000	500,000	Yes
sum	7,200,000		1,200,000	Yes
Answer A				

22. Assume that one school will be awarded based on the proportion of saving to the income generated. Which school do you think deserve the award?

A. B B. C C. D D. E

Solution

school	income	saving	Saving/income
A	1,200,000	200,000	1/6
B	1,400,000	200,000	1/7
C	1,000,000	-100,000	-1/10
D	2,000,000	400,000	1/5
E	1,600,000	500.000	5/16, max
sum	7,200,000	1,200,000	

$$\text{Answer } D$$

23. Which one of the following is **NOT** true about the income-expenditure data given above?

A. The difference between income and expenditure was the highest for school E

B. All schools have managed to keep their expenditure lower than their income

C. Most of the schools had annual income which was larger than the average annual income of the five schools combined

D. Only one school had an annual expenditure of the five schools combined

Solution

$$\text{from question number 22, the difference between income and expenditute}$$
$$(\text{high saving } E)$$
$$\text{Answer } A$$

24. The difference between the average annual income and average annual expenditure of the five schools combined

A. 100,000 B. 240,000 C. 200,000 D. 1,200,000

Solution

$$\bar{x}\ income = \frac{total\ income}{5} = \frac{7,200,000}{5} = 1,440,000$$

$$\bar{x}\ expe = \frac{totoal\ expenditure}{5} = \frac{6,000,000}{5} = 1,200,000$$

$$difference = \bar{x}in - \bar{x}ex = 1,440,000 - 1,200,000$$

$$= \quad 240,000$$

$$Answer\ \ B$$

25. The average annual income of the five schools in ETB is

A. 1,200,000 B. 1,400,000 C. 1,300,000 D. 1,440,000

Solution

$$\bar{x}income = \frac{total\ income}{5} = \frac{7,200,000}{5} = 1,440,000$$

$$Answer\ \ D$$

SAT (quantitative reasoning)-2010

2010 E.C

1. The difference between a positive proper fraction and its reciprocal is 9/20. What is the value of the fraction?

   A. 5/4 B. 3/5 C. 4/3 D. 4/2

   Solution

$$x-\frac{1}{x}=\frac{9}{20}$$
$$\frac{x^2-1}{x}=\frac{9}{20}$$
$$20x^2-20=9x$$
$$20x^2-9x-20=0, quadratic\ equation$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-9)\pm\sqrt{(-9)^2-4(20)(-20)}}{2(20)}$$
$$=\frac{9\pm\sqrt{81+1600}}{40}$$
$$=\frac{9\pm\sqrt{1681}}{40}$$
$$=\frac{9\pm 41}{40},$$
$$=\frac{9+41}{40}=\frac{50}{40}=\frac{5}{4}\ positive$$
$$or=\frac{9-41}{40}=\frac{-32}{40}=\frac{-4}{5} negative\ Answer\ \ A$$

2. 1÷0.1÷0.001 is equal to

   A. 0.00001 B. 0.001 C. 1,000,000 D. 100

   Solution

$$1÷0.1÷0.01÷0.001$$
$$\frac{1}{0.1}÷0.01÷0{,}001$$
$$10÷0.01÷0.001$$
$$\frac{10}{0.01}÷0.001$$
$$1000÷0{,}001$$
$$\frac{1000}{0.001}=1{,}000{,}000$$
$$Answer\ C$$

3. If b≠0 and $\frac{a}{5}=\frac{b}{7}$, what is the value of $\frac{a}{b}$?

   A. 7/5 B. 1/5 C. 1/7 D. 5/7

   Solution

$$\frac{a}{5}=\frac{b}{7}, 7a=5b$$
$$\frac{7a}{7b}=\frac{5b}{7b}, \frac{a}{b}=\frac{5}{7}$$
$$Answer\ D$$

4. If s is a number between 0 &1, which of the following is **NOT** more than s?
   A. √s B. 1/s C. S^2 D. 2s

Solution

$$0 < s < 1 \ \ take \ \ s = \frac{1}{4}$$

$$A) \sqrt{s} = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow \sqrt{s} > s$$

$$B) \frac{1}{s} = \frac{1}{\frac{1}{4}} = 4 \Rightarrow \frac{1}{s} > s$$

$$C) s^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} \Rightarrow s^2 < s$$

$$D) 2s = 2\left(\frac{1}{4}\right) = \frac{1}{2} \Rightarrow 2s > s$$

Answer C

5. What is the simplified form of($1\div10^{-1}$) $(-1)^{10}$?
   A. 10 B. 1 C. 0.1 D. 100

Solution

$$(1\div10^{-1}) * (-1)^{-10}$$

$$\frac{1}{10^{-1}} * \frac{1}{(-1)^{10}} \quad , (-1)^{10} = 1 \ and \ \frac{1}{10^{-1}} = 10,$$

$$10 * 1 = 10$$

Answer A

6. Three professors contested an election for a university president and received 1136, 7636 and 11628 votes, respectively. What percentage of the total votes did the winning professor get?
   A. 65% B. 57% C. 59% D. 61%

Solution

$$total \ vote = 1136 + 7636 + 11628 = 20400$$

$$winner \ is \ votted \ by \ 11{,}628$$

$$\% \ of \ winner = \frac{11{,}628}{20{,}400} * 100\%$$

$$= 57\%$$

Answer B

7. What number increased by 3 is equal to 3 less than twice the number?
   A. 3 B. 6 C. 9 D. 12

Solution

$$X + 3 = 2x - 3$$

$$3 + 3 = 2x - x$$

$$6 = x$$

Answer B

8. If one-fourth of one-third of a number is 15, then what is three-tenth of that numbers?
   A. 54 B. 45 C. 39 D. 12

Solution

$$\frac{1}{4}\left(\frac{1}{3}x\right) = 15, \frac{1}{12}x = 15$$

$$\frac{36}{10}\left(\frac{1}{12}x\right) = (15)\left(\frac{36}{10}\right)$$

$$\frac{3}{10}x = 54$$

Answer A

9. The sum of ages of 5 children born at the interval of 3 years each is 50 years. What is the age of the youngest child?

A. 10 years B. 6 years C. 8 years D. 4 years

$$\underline{solution}$$
$$x, x+3, x+6, x+9, x+12$$
$$x+x+3+x+6+x+9+x+12=50$$
$$5x+30=50$$
$$5x=50-30$$
$$5x=20$$
$$x=4, youngest$$
$$Answer\ \ D$$

10. If y≠0 and $xy = \frac{y}{6}$, what is the value of x?

A. 1/8 B. ¼ C. 1/6 D. ½

Solution

$$xy=\frac{y}{6}, \quad \frac{xy}{y}=\frac{y}{6y}$$
$$x=\frac{1}{6}$$
$$Answer\ \ C$$

11. An athlete completes a journey in 10 hours. He travels first half of the journey at a rate of 21 km/hr and second half at rate of 24 km/hr. what is the total journey in km?

A. 224 km B. 218 km C. 216 km D. 230 km

Solution

$$let\ distance = x, t = time$$
$$first\ half\ \frac{x}{2}=21t \ \Rightarrow \ \frac{x}{42}=t$$
$$second\ half\ \frac{x}{2}=24(10-t)$$
$$\Rightarrow \frac{x}{48}=10-t \Rightarrow 10-\frac{x}{48}=t$$
$$\frac{x}{42}=10-\frac{x}{48} \qquad Answer\ A$$
$$\frac{x}{42}+\frac{x}{48}=10$$
$$\frac{48x+42x}{2016}=10$$
$$90x=20160, x=\frac{20160}{90}=224$$

12. What is the product of$(\sqrt{7}-2)(\sqrt{7}+2)$?

A. 5 B. 4 C. 2 D. 3

Solution

$$(\sqrt{7}-2)(\sqrt{7}+2)$$
$$note, (a+b)(a-b)a^2-b^2$$
$$(\sqrt{7}-2)(\sqrt{7}+2)$$
$$=(\sqrt{7})^2-(2)^2$$
$$=7-4$$
$$=3$$
$$Answer\ D$$

13. A library has an average of 510 visitors on Sunday and 240 on other days. What is the average number of visitors per day in a month of 30 days beginning with Sunday?

A. 285 B. 283 C. 275 D. 290

Solution

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number	5	5	4	4	4	4	4
Visitors	510	240	240	240	240	240	240

$$\bar{x} = \frac{(5(510) \ + 5(240) + \ 4(240) \ +4(240) + 4(240) + 4(240) + 4(240))}{5+5+4+4+4+4+4}$$

$$\bar{x} = \frac{2550 + 1200 + 960 + 960 + 960 + 960 + 960}{5+5+4+4+4+4+4}$$

$$= \frac{8550}{30} = 285$$

s	m	t	w	t	f	s
s	m	t	w	t	f	s
s	m	t	w	t	f	s
s	m	t	w	t	f	s
s	m					
5	5	4	4	4	4	4

Answer A

14. Look at the pattern carefully: 10 20 25 35 40 50 55, then which of the following pairs of numbers comes next?

A. 60,75 B. 60,65 C. 65,70 D. 60,70

Solution

$$10 \underbrace{\quad}_{+10} 20 \underbrace{\quad}_{+5} 25 \underbrace{\quad}_{+10} 35 \underbrace{\quad}_{+5} 40 \underbrace{\quad}_{+10} 50 \underbrace{\quad}_{+5} 55 \underbrace{\quad}_{+10} 65 \underbrace{\quad}_{+5} 70$$

Answer C

15. What number should come next in the series: 7, 10, 8, 11, 9, 12….?

A. 12 B. 10 C. 13 D. 7

Solution

$$7, \quad 10, \quad 8, \quad 11, \quad 9, \quad 12, \quad 10$$
$$+3 \quad -2 \quad +3 \quad -2 \quad +3 \quad -2$$

Answer B

16. What number should come next in the series: 58, 52, 46, 40, 34….?

A. 34 B. 30 C. 32 D. 28

Solution

$$58, \quad 52, \quad 46, \quad 40, \quad 34, \quad 28$$
$$d = -6, substruct\ 6, 34 - 6 = 28$$

Answer D

17. What number should come next in the series: 544, 509, 474, 439….?

A. 445 B. 420 C. 404 D. 414

Solution

$$544, \quad 509, \quad 474, \quad 439, \quad 404 - - -$$
$$d = -35, Substruct\ 35, 439 - 35 = 404$$

Answer C

18. Look at the pattern carefully: 32 31 32 29 32 27 32, then which of the following pairs of numbers come next?

A. 29, 32 B. 25 ,32 C. 25 ,30 D. 29 ,30

Solution

$$32, \quad 31, \quad 32, \quad 29, \quad 32, \quad 27, \quad 32, \quad 25, \quad 32$$
$$odd\ numbers\ are\ next\ to\ 32, which\ are\ 31,29,27,25,23, \dots \dots .$$

Answer B

19. Compare the two quantities and answer the questions by choosing the alternative that best explain the relationship between ‘’column A’’ and ‘’column B’’ X > Y

Quantity A	Quantity B
30% of x	50% of y

A. The two quantities are equal
B. The relationship cannot be determined from the information given
C. Quantity is less than quantity B
D. Quantity A is greater than B

Solution

$QuaA$	$QuaB$
$30\%\ of\ x$	$50\% of\ y$

Let x=100, y=80,

$30\%\ of\ x$ =0.3*100=30,

$50\% of\ y$ =0.5*80=40, B>A

Let x=150, y=80.

$30\%\ of\ x$ =0.3*150=45,

$50\% of\ y$ =0.5*80=40, A>B

$Can'tbe\ determind$

$\underline{Answer\ \ B}$

20. Compare the two quantities and answer the questions by choosing the alternative that best explain the relationship between ‘’column A’’ and ‘’column B

Quantity A	Quantity B
$(-21)^{12}$	$(-31)^{13}$

A. Quantity A is greater than quantity B
B. The quantities are equal
C. Quantity A is less than B\
D. The relationship cannot be determined from the information given

Solution

A	B
$(-21)^{12}$	$(-31)^{13}$

$negative\ number\ the\ power\ of\ even\ number\ is\ positive$

$(-21)^{12}, is\ positive$

$negative\ number\ the\ power\ of\ odd\ number\ is\ negative$

$(-31)^{13}, is\ negative$ ’

A>B

Answer A

Data interpretation questions (21-25)

Table: Number of candidates that took and qualified in a license examination from different regions over the years.

region	**year**									
	1997-2000		1998-2001		1999-2002		2000-2003		2001-2004	
	Took	Qual	Took	Qual	Took	Qual	Took	Qual	Took	Qual
AA	5200	720	8500	980	7400	850	6800	775	9500	1125
AM	7500	840	9200	1050	8450	920	9200	980	8800	1020
OR	6400	780	8800	1020	7800	890	8750	1010	9750	1250
TI	8100	950	9500	1240	8700	980	9700	1200	8950	995
SP	7800	870	7600	940	9800	1350	7600	945	7900	885

21. What is the percentage of candidates qualified from region AM for all the years together, over the candidates took the exam from region AM during the years together?
A. 11.75% B. 11.46% C. 11.57% D. 11.15%

Solution

$Region$	1997-2000		1998-2001		1999-2008		2000-2013		2001-2004	
	took	Qua	took	Que	took	Qua	took	Qua	took	Qua
AM	7500	840	9200	1050	8450	920	9200	980	8800	1020

$$\%\ 0f\ region\ AM = \frac{qua}{took} * 100\% = \frac{840 + 1050 + 920 + 980 + 1020}{7500 + 9200 + 8450 + 9200 + 8800} x100\%$$

$$= \frac{4810}{43150} x100\% = 11.15\%$$

$$Answer\ \ D$$

22. What percent is the total number of qualified candidates in 1997-2000 to the total number of those qualified in 2001-2004?
A. 80% B. 78% C. 84% D. 76%

Solution

$Qualified\ in\ an\ region\ 1997 - 2000\ and\ 2001 - 2004$

$Region$	1997-2000	2001-2004
	$qualified$	$qualified$
AA	720	1125
AM	840	1020
OR	780	1250
TI	950	995
SP	870	885

$\%\ of\ No\ of\ qualified\ in\ 1997 - 2000\ to\ 2001 - 2004$

$$= \frac{qualified\ in\ 1997 - 2000}{qualified\ in\ 2001 - 2004} x100\%$$

$$= \frac{720 + 840 + 780 + 950 + 870}{1125 + 1020 + 1250 + 995 + 885} x100\%$$

$$= \frac{4160}{5275} x100\% = 78.86\%$$

$$Answer\ \ B$$

23. What is the percentage of total number of qualified candidates to the total number of candidates took the exam among all the five regions in 2002?
A. 11.84% B. 11.52% C. 11.48% D. 11.89%

Solution

$Region$	1999-2002	
	$took$	$qualified$
AA	7400	850
AM	8450	920
OR	7800	890
TI	8700	980
SP	9800	1350

$\%\ of\ qual\ in\ 2002\ to\ took\ exam$

$$= \frac{qual\ in 2002}{canddates\ in\ 2002} x100\%$$

$$\frac{850 + 920 + 890 + 980 + 1350}{7400 + 8450 + 7800 + 8700 + 9800} x100\%$$

$$\frac{4990}{42150} x100\% = 11.84\%$$

$$Answer\ \ A$$

24. In which of the given years the number of candidates took the exam from region OR has maximum percentage of qualified candidates?

A. 2000 B. 2003 C. 2001 D. 2004

Solution

$Region$	1997-2000		1998-2001		1999-2008		2000-2003		2001-2004	
	took	Qua	took	Que	took	Qua	took	Qua	took	Qua
OR	6400	780	8800	1020	7800	890	8750	1010	9750	1250
$\%of\ qual$	$\frac{780}{6400}x100\%$ 12.19		11.6		11.41		11.54		12.82	
Answer D										

25. What is the average number of candidates who took the exam from region TI during the given years?

A. 8990 B. 8900 C. 8860 D. 8800

Solution

$Region$	1997-2000	1998-2001	1999-2002	2000-2003	2001-2004
	$took$	$took$	$took$	$took$	$took$
IT	8100	9500	8700	9700	8900

$$av = \frac{total\ No\ of\ candidates}{5\ years}$$

$$= \frac{8100 + 9500 + 8700 + 9700 + 8900}{5}$$

$$= \frac{44950}{5}$$

$$= 8990$$

$Answer\ \ A$

SAT (quantitative reasoning)-2011

2011 E.C

1. Find the value of X in the following sequence 3, 12, X, 48, 75

A. 15 B. 21 C. 36 D. 27

Solution

$$\begin{array}{ccccc} 3, & 12, & x, & 48, & 75 \\ 3*1, & 3*4, & x, & 3*16, & 3*25 \\ 3*1^2, & 3*2^2, & x, & 3*4^2, & 3*5^2 \end{array}$$

$$x = 3*3^2 = 27$$

$$Answer\ D$$

2. What is the value of X in the following sequence? 10, 15, 22, X, 42, 55, 70

A. 28 B. 31 C. 23 D. 29

Solution

$$\begin{array}{ccccccc} 10, & 15, & 22, & x, & 42, & 55, & 70 \\ 10, & 10+5, & 15+7, & x, & 42 & ,55 & ,70 \end{array}$$

$$thus, x = 22 + 9$$
$$= 31$$

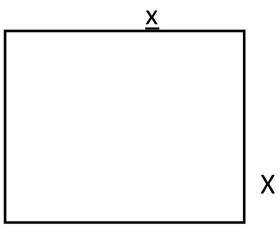
$$Answer\ B$$

3. What is the length of a side of a square with area C?

A. 4c B. 2c C. $\sqrt{c}$ D. C^2

Solution

$$A = x * x = x^2$$
$$c = x^2$$
$$\sqrt{c} = x$$
$$Answer\ C$$



4. If ABCD is a four digit positive number divisible by 5 and 10, what is the value of D?

A. 0 B. 1 C. 2 D. It can't be determined

Solution

IF the last digit is 0, then the number is divisible by 5 and 10.

Answer A

5. If one-third of one fourth of a number is 15, then what is the three-tenth of that number?

A. 35 B. 36 C. 54 D. 45

Solution

$$\frac{1}{3}\left(\frac{1}{4}x\right) = 15, \frac{1}{12}x = 15$$

$$\frac{36}{10}\left(\frac{1}{12}x\right) = 15\left(\frac{36}{10}\right)$$

$$\frac{3}{10}x = 54$$

$$Answer\ C$$

6. Which of the following numbers pair is different from the others in the group?

A. 7, 3 B. 3, 5 C. 6, 2 D. 1, 7

Solution

$$\left.\begin{array}{l} 7,\ 3 \\ 3,\ 5 \\ 1,\ 7 \end{array}\right\} Pairs\ of\ odd\ numbers$$

$$6,\ 2\}\ Pair\ of\ even\ numbers$$

$$Answer\ C$$

7. Which of the following numbers is different from the others in the group?

A. 37 B. 29 C. 17 D. 27

Solution

$$\left.\begin{matrix}37\\29\\17\end{matrix}\right\} \quad Prime\ numbers$$

$$27\}\ Composite\ number,$$

$$Answer\ D$$

A natural number is prime if it is divisible by only 1 and itself

8. Find the value of X in the following sequence? 16, 8, 4, 2, X, ½

A. 2 B. 1 C. ¼ D. 1/8

Solution

$$16,\ 8,\ 4,\ 2,\ x,\ \frac{1}{2},\dots$$

$$r=\frac{1}{2}\Rightarrow r=\frac{\frac{1}{2}}{x}=\frac{x}{2}\Rightarrow\frac{x}{2}=\frac{1}{2}\Rightarrow\ 2x=2\Rightarrow x=2$$

$$Answer\ B$$

9. Find the value of X in the following sequence? 6, 9, 7, 10, 8, 11, 9, 12, X

A. 15 B. 14 C. 9 D. 10

Solution

$$6,9,7,10,8,11,9,12,x,\dots$$

$$6+3=9,9-2=7,7+3=10,10-2=8,8+3=11,11-2=9,9+3=12,12-2=x$$

$$\Rightarrow 12-2=x\Rightarrow 10=x\ Answer\ D$$

10. What is the sum of the first five prime numbers?

A. 11 B. 26 C. 18 D. 28

Solution

$$2,3,5,7,11\ are\ first\ 5$$

$$prime\ numbers$$

$$\Rightarrow 2+3+5+7+11=28$$

$$Answer\ D$$

11. It takes two and half an hour from Addis Ababa to Adama traveling at a constant rate of speed. What part of the distance is travelled by 4/3 of an hour, approximately?

A. 1/3 B. ½ C. 2/3 D. ¾

Solution

$$x=\frac{4/3}{2+1/2}=\frac{4/3}{5/2}=\frac{8}{15}=0.5333\dots\ \approx 0.5=\frac{1}{2}$$

$$Answer\ B$$

12. Assume that Ujulu saves ¼ of his income and his father had been saving half of his income. If the income of his father is 5000 Birr,which is doubled of his son, how much is saved by his son?

A. 500 B. 1250 C. 625 D. 2500

Solution

$$Iu=Ujulu\ income$$

$$Su=Ujulu\ Saving$$

$$If=Father\ income$$

$$Sf=Father\ saving$$

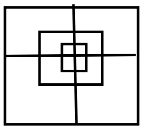
$$Su=\frac{1}{4}Iu\ and\ Sf=\frac{1}{2}If,If=5000$$

$$Iu=\frac{1}{2}If=\frac{1}{2}*5000=2500$$

$$Su=\frac{1}{4}\ Iu=\frac{1}{4}\ (2500)=625$$

$$Answer\ C$$

13. What is the number of squares in the figure below?



A. 18 B. 15 C. 12 D. 9

Solution

$$\Longrightarrow 3\ Mian\ Squres$$
$$\Longrightarrow Each\ Squares\ has\ 4\ Squres$$
$$\Longrightarrow total\ =\ 3+\ 3(4)=3+12=15$$
$$Answer\ A$$

14. A bag contains 28 yellow, 44 black and 36 green marbles. Ali takes out one marble from the bag at random. What is the probability that he will take out a green marble?

A. 1/3 B. 36 C. 1/36 D. 1/9

Solution

$$Total=n(S)=28+44+36=108$$
$$Green, n(G)=36$$
$$P(G)=\frac{n(G)}{n(S)}=\frac{36}{108}=\frac{1}{3}$$
$$Answer\ C$$

15. How many irrational numbers are there between 2 and 7?

A. 10 B. 20 C. Infinitely many D. 30

Solution

$$There\ are\ infinte\ irrational\ numbers\ between\ 2\ and\ 7, eg\ \sqrt{5},\sqrt{7},\sqrt{11}\ \ldots.$$
$$Answer\ C$$

16. What number should replace X in the following series? -1, 0, 2, 6, X, 30, 62

A. 14 B. 24 C. 12 D. 10

Solution

$$-1,0\ ,2\ ,6\ ,x\ ,30,62,\ldots$$
$$a_1=\ -1$$
$$a_2=0=\ a_1+2^{2-2}$$
$$a_3=2=\ a_2+2^{3-2}$$
$$a_4=6=\ a_3+2^{4-2}$$
$$a_5=x=\ a_4+2^{5-2}$$
$$=6+2^3$$
$$=6+8=14$$
$$Answer\ A$$

17. The square positive number is greater than the number itself when the number is?

A. Greater than 1 C. Between zero and one

B. Greater than 0 D. Impossible to determine

Solution

$$let\ the\ number\ be\ x^2$$
$$x^2>x$$
$$this\ is\ true\ if\ x>1$$
$$example\ 2^2>2$$
$$Answer\ A$$

Quantitative comparison

Directions: Questions 18-20 consists of two quantities, one in column A and one in column B. compare the two quantities and answer the questions by choosing the alternative that best explains the relationship between ‘’column A and column B. give the answer by blackening the letter of your choice on the separate answer sheet provided

18. Suppose that 0<y<5. Compare the following two quantities

Quantity A	Quantity B
Y^2	Y^3

A. Quantity A is greater than quantity B
B. The relationship cannot be determined from the information given
C. Quantity A is equal quantity B
D. Quantity A is less than quantity

Solution

0<y<5

If y=1, then A=B

If y>1, then B>A, eg. y=3

If y<1, then A>B, eg. y=$\frac{1}{2}$,

Can not be determined

Answer B

19. Compare the two quantities

Quantity A	Quantity B
The percent increase from 10 meters to 15 meters	The percent increase from 50 meters to 65 meters

A. Quantity A is equal quantity B
B. Quantity A is less than quantity B
C. Quantity A is greater than quantity B
D. The relationship cannot be determined from the information given

Solution

$$A)\ \frac{15-10}{10} * 100\% = 50\%$$

$$B)\ \frac{65-50}{50} * 100\% = 30\%$$

$$Qua\ A > Qua\ B$$

Answer C

20. Compare two quantities

Quantity A	Quantity B
5/3	13/8

A. Quantity A is equal quantity B
B. Quantity A is less than quantity B
C. Quantity A is greater than quantity B
D. The relationship cannot be determined from the information given

Solution

$$\frac{5}{3} : \frac{13}{8}$$

$$\frac{5}{3}\left(\frac{8}{8}\right) : \frac{13}{8}\left(\frac{3}{3}\right), make\ same\ denomenators$$

$$\frac{40}{24} > \frac{38}{24}$$

Answer C

Data interpretation

Directions: Question 21-25 are based on the hypothetical data in the table below. It is about the total population, primary school age population and school Enrolment by sex of seven regions of Ethiopia.

Regions	Total population		School age population		School enrolment	
	Male	Female	Boys	Girls	Boys	Girls
A	1,387,000	1,500,000	162,000	166,000	220,000	267,000
B	756,000	603,000	141,000	114,000	34,000	22,000
C	9,309,000	9,317,000	1,913,000	1,887,000	1,711,000	1,572,000
D	12,883,000	12,934,000	2,709,000	2,670,000	2,734,000	2,099,000
E	7,203,000	7,287,000	1,522,000	1,500,000	1,474,000	1,113,000
F	2,268,000	1,950,000	445,000	389,000	157,000	95,000
G	2,080,000	2,143,000	443,000	431,000	447,000	436,000
Total	35,886,000	35,734,000	7,335,000	7,157,000	6,777,000	5,604,000

21. Which region is least in enrolling girls to school compared to the boys?

A. C B. F C. E D. D

Solution

Region	School Enrolment		
	Boys	Girls	Boys-Girls
A	220,000	267,000	-47,000
B	34,000	22,000	12,000
C	1,711,000	1,572,000	139,000
D	2,734,000	2,099,000	635,000
E	1,474,000	1,113,000	361,000
F	157,000	95,000	2,000
G	447,000	436,000	11,000
Total	6,777,000	5,604,000	1,173,000

Difference in D is maximum, enrolment of girls is less

Answer D

22. Total male population, in million, in seven regions is nearly

A. 35 B. 13 C. 20 D. 26

Solution

$$Tota\ male\ population$$
$$= 35{,}886{,}000$$
$$\approx 35\ million$$
$$Answer\ A$$

23. Assuming that the total population of the country at the time the above data is taken is 73 million, what is the share in percent of the population in C and D together?

A. 40 B. 50 C. 70 D. 60

Solution

$$\%\ of\ C\ and\ D\ togather$$
$$= \frac{P(C)+P(D)}{total} * 100\% = \frac{(9309000+9317000)+(12883000+12934000)}{73{,}000{,}000} * 100$$
$$= \frac{44{,}443{,}000}{73{,}000{,}000} * 100\% = 60\%$$
$$Answer\ D$$

24. The proportion of girls enrolled in schools in region D to the female

A. 1:2 B. 2:13 C. 1:17 D. 5:14

Solution

$$PoP = \frac{Girls\ in\ D}{Total\ girls} = \frac{2{,}099{,}000}{35{,}734{,}000} = 0.058 \approx \frac{1}{17}$$
$$Ratio\ \ 1:17$$
$$Answer\ C$$

25. The three least populous regions in order are:

A. A, D, G B. B, A, E C. A, G, F D. B, A, F

Solution

Region	Population			Least
	Male	Female	Sum	
A	1,387,000	1,500,000	2,887,000	2
B	756,000	603,000	1,359,000	1
C	9,309,000	9,317,000	18,626,000	
D	12,883,000	12,934,000	25,817,000	
E	7,203,000	7,287,000	14,490,000	
F	2,268,000	1,950,000	4,218,000	3
G	2,080,000	2,143,000	4,223,000	

Least population B, A, F

Answer D

SAT (quantitative reasoning)-2012

2012 E.C

For Questions 1-5 use the Following Information. The following table give the expenditure (in millions of Birr) of a certain company per year over five years.

Year	Items of Expenditure				
	salary	Fuel and Transport	Bon	Interest on Loan	Taxes
2008	288	98	3.1	23.4	83
2009	342	112	2.52	32.5	105
2010	324	101	3.84	41.2	74
2011	336	133	3.68	36.4	88
2012	422	142	3.96	49.4	98

1. What is the average amount of interest on Loan per year that the company has to pay during the five year period?

   A. 36.58 B. 34.18 C. 33.72 D. 33.43

<u>Solution</u>

$$Av\ of\ interest\ in\ Loan = \frac{23.4+32.5+41.2+36.4+49.4}{5}$$

$$= \frac{182.9}{5} = 36.58$$

Answer A

2. What percent of the total amount of salary was paid a bonus by the company during the given period?

   A. 1.25% B. 1.00% C. 0.5% D. 0.10%

<u>Solution</u>

Bonus =3.1+2.52+3.84+3.68+3.96=17.1, salary=288+342+324+336+422=1712

% of bonus= $\frac{17.1}{1712}$* 100 %= 0.998….=1%

Answer B

3. In which year did the company have the highest expenditure?

   A. 2010 B. 2012 C. 2009 D. 2008

<u>Solution</u>

Year	Items of Expenditure					total
	salary	Fuel and Transport	Bon	Interest on Loan	Taxes	
2008	288	98	3.1	23.4	83	495.5
2009	342	112	2.52	32.5	105	594.02
2010	324	101	3.84	41.2	74	544.04
2011	336	133	3.68	36.4	88	597.08
2012	422	142	3.96	49.4	98	715.36

Spent in 2012 is high expenditure = 715.36

Answer D

4. What is the total expenditure of the company in the year 2012?

   A. 456.36 B. 586 C. 715.36 D. 1710

<u>Solution</u>

Year	Items of Expenditure					Total
	salary	Fuel and Transport	Bon	Interest on Loan	Taxes	
2012	422	142	3.96	49.4	98	715.36

Total expenditure in 2012 = 715.36

Answer C

5. For what item or purpose was the company's expenditure the last during the given period?
   A. Taxes B. Transport and Fuel C. Bonus D. Salary

Solution

Year	salary	Fuel and Transport	Bon	Interest on Loan	Taxes
2008	288	98	3.1	23.4	83
2009	342	112	2.52	32.5	105
2010	324	101	3.84	41.2	74
2011	336	133	3.68	36.4	88
2012	422	142	3.96	49.4	98
total	1712	586	17.1	182.9	448

Least expenditure = 17.1

Answer C

6. If x= 15% of 30 and y=30% of 15, then which of the following is true?
   A. The relationship between x and y cannot be determined C. Y is less than x
   B. Y is less than x D. X and y are equal

Solution

x = 15% of 30

*= 0.15 * 30 = 4.5*

Y = 30% of 15

*= 0.3 * 15 = 4.5*

They are equal.

Answer B

7. Student X and student Y have Birr 2000. If $\frac{4}{15}$ of X's share is equal to $\frac{2}{5}$ of Y's share. How much amounts does Y have?
   A. Birr 1200 B. Birr 1100 C. Birr 800 D. Birr 1500

Solution

$$\frac{4}{15}x = \frac{2}{5}y \Longrightarrow \ 20x = 30y$$

$$\Longrightarrow x = \frac{3}{2}y$$

$$x + y = 2000$$

$$\frac{3}{2}y + y = 2000$$

$$\frac{5}{2}y = 2000$$

$$5y = 4000$$

$$y = 800$$

$$x = \frac{3}{2}y = \frac{3}{2}(800) = 12000$$

$$Answer\ C$$

8. What is the simplified form of (2x+3)(x+6)-(2x-5)(x+10)?
   A. 72 B. 68 C. 16 D. 27

Solution

$$(2x + 3)(x + 6) - (2x - 5)(x + 10)$$

$$= 2x^2 + 12x + 3x + 18 - (2x^2 + 20x - 5x - 50)$$

$$= 2x^2 + 12x + 3x + 18 - 2x^2 - 20x + 5x + 50$$

$$= 18 + 50 = 68$$

$$Answer\ B$$

9. If the square of a number increased by two times the number is equal to six times the number increased by twelve, which one of the following is the possible value of the number?

A. -4 B. -6 C. 4 D. 6

Solution

$$x^2+2x=6x+12$$
$$x^2+2x-6x-12=0$$
$$x^2-4x-12=0, sum=b=-4, pro=ac=-12, are\ -6\ and\ 2$$
$$x^2-6x+2x-12=0$$
$$x(x-6)+2(x-6)=0$$
$$(x-6)(x+2)=0$$
$$x=6 \quad x=-2$$
$$Answer\ D$$

10. The monthly telephone bill of Getachew was Birr 400 and it increases by 10% after 12 months and another 20% increase is applied six months later. What is the price of Get chew's telephone bill after 18 months?

A. Birr 489 B. Birr 430 C. Birr 520 D. Birr 528

Solution

$$i)\ 400+10\%\ (400)$$
$$=4000+400*0.1=400+40=440$$
$$ii)\ 440+20\%(440)$$
$$=440+0.2(440)=440+88=528$$
$$Answer\ D$$

11. The sum of the ages of the father and his son is equal to sixty years. Six years ago the age of the father was five times the age of the son. What will be the age of the son six years from now?

A. 22 years B. 20 years C. 19 years D. 23 years

Solution

$$let\ x=son\ age\ , y=Father\ age$$
$$now, x+y=60,\ \ y=60-x$$
$$6\ years\ ago, y-6=5(x-6)$$
$$y-6=5x-30,$$
$$60-x-6=5x+30$$
$$60+30-6=6x$$
$$84=6x, 14=x$$
$$son\ age\ six\ years\ fom\ now=x+6=14+6=20\ year$$
$$Answer\ B$$

12. The triangular number sequence T_n can be defined recursively as $T_1=1$, $T_n=T_{n-1}+n$ for n>1. What is the eleventh term of sequence?

A. 78 B. 55 C. 66 D. 45

Solution

$$T_1=1, \quad T_n=T_{n-1}+n$$
$$T_2=1+2=3$$
$$T_3=3+3=6$$
$$T_4=6+4=10$$
$$T_5=10+5=15$$
$$T_6=15+6=21$$
$$T_7=21+7=28$$
$$T_8=28+8=36$$
$$T_9=36+9=45$$
$$T_{10}=45+10=55$$
$$T_{11}=55+11=66$$
$$Answer\ C$$

13. What is the sum of the following infinite series? $s = 2 - \frac{1}{3} + 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{27} + \frac{1}{4} - \frac{1}{81} + \cdots$

A. 41/9 B. 9/2 C. 7/2 D. 32/9

Solution

$$S = 2 - \frac{1}{3} + 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{27} \ldots$$

$$S_1 = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots positive\ terms$$

$$G_1 = 2 \quad , r = \frac{1}{2}$$

$$S_1 = \frac{G_1}{1-r} = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$$

$$S_2 = -\frac{1}{3}, -\frac{1}{9}, -\frac{1}{27}, \ldots negative\ terms$$

$$= -\left(\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\right)$$

$$= G_1 = \frac{-1}{3}, r = \frac{1}{3}$$

$$S_2 = \left(\frac{G_1}{1-r}\right) = \frac{-\frac{1}{3}}{1-\frac{1}{3}} = \frac{-\frac{1}{3}}{\frac{2}{3}} = -\frac{1}{2}$$

$$S = S_1 + S_2$$

$$= 4 - \frac{1}{2} = \frac{7}{2}$$

Answer C

14. The next term of the sequence 162, 54, 18, 6,… equal to

A. 1/3 B. 3 C. 2 D. 1

Solution

162, 54, 18, 6, …, geometric progression with $G_1 = 162$, and

$$r = \frac{1}{3}, next\ term = 6 * \frac{1}{3}$$

$$= 2$$

Answer C

15. The first four terms of an arithmetic sequence are 39, 33, 27 and 21. What is the tenth term of the sequence?

A. -33 B. -27 C. -21 D. -15

Solution

39, 33, 27, 21,………..

Arithmetic sequence with, d = -6 and A_1*=39*

Tenth term, $A_{10} = A_1 + (n - 1) d$

$= 39 + (10 - 1) (- 6)$

$=39+9(-6)$

$= 39-54$

$=-15$

Answer D

16. The sum of the first n terms of arithmetic progression is $2n^2 + n$. what is the tenth term of sequence

A. 38 B. 41 C. 39 D. 40

Solution

$$S_n = 2n^2 + n$$
$$a_1 = 2(1)^2 + 1 = 3$$
$$S_{10} = \frac{n}{2}(A_1 + A_{10})$$
$$2(10)^2 + 10 = \frac{10}{2}(3 + a_{10})$$
$$200 + 10 = 5(3 + a_{10})$$
$$210 = 15 + 5a_{10}$$
$$210 - 15 = 5a_{10}$$
$$\frac{195}{5} = \frac{5a_{10}}{5}$$
$$a_{10} = 39$$

Answer C

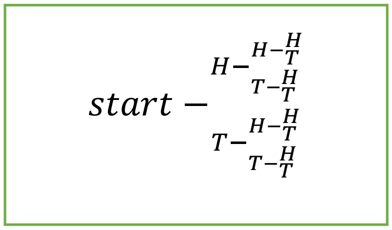
17. What is the probability of getting exactly two tails when three coins are tossed at the same time?

A. ¾ B. 3/8 C. ¼ D. 1/8

Solution

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$
$$\epsilon = \{HTT, THT, THT\}$$
$$n(s) = 8, \quad n(\epsilon) = 3$$
$$P(\epsilon) = \frac{n(\epsilon)}{n(s)} = \frac{3}{8}$$

Answer B



18. What is the probability that a number selected at random from the first 50 natural numbers is prime and odd number?

A. 8/25 B. 3/5 C. 3/10 D. 7/25

Solution

$$First\ 50\ natural\ numbers$$
$$sample\ space, n(s) = 50$$
$$event, E = prime\ and\ odd$$
$$= \{3,5,7,11,13,17,19,23,29,31,37,41,43,47\}$$
$$n(\epsilon) = 14$$
$$p(\epsilon) = \frac{n(\epsilon)}{n(s)} = \frac{14}{50} = \frac{7}{25}$$

Answer D

19. If 3x+7y= 80 and x-y=10, then what is the value of x+y?

A. 25 B. 20 C. 10 D. 15

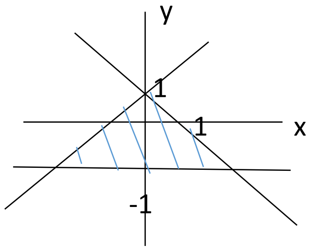
Solution

x - y = 10
x= y +10
y = x – 10

3x + 7y =80
3x +7(x – 10) = 80
3x + 7x -70 = 80
3x+7x = 80 +70
10x = 150
X = 15
Y = x -10
= 15 - 10 = 5
x +y = 15+5= 20,

Answer B

20. Which of the following systems of linear inequalities is represented by the graph given below?



If the sign is ≤, then the region will be shaded below the graphe of the equations.

A. $\begin{cases} y \le x+1 \\ y \le x-1 \\ y \ge -1 \end{cases}$ B. $\begin{cases} y \le x+1 \\ y \le -x+1 \\ y \ge -1 \end{cases}$ C. $\begin{cases} y \le x+1 \\ y \le x-1 \\ y \le -1 \end{cases}$ D. $\begin{cases} y \le x+1 \\ y \le -x+1 \\ y \le -1 \end{cases}$

Solution

See the shaded region on the graph

Answer B

21. The square of the 7th term of an arithmetic progression with positive common difference is equal to the product of the 3rd and 17th terms. What is the ratio of the first term to the common difference?

A. 4:3 B. 3:2 C. 3:4 D. 2:3

Solution

$$n^{th}\ term\ of\ arithimetic\ sequence, a_n = a_1 + (n-1)d, \qquad a_3 = a_1 + 2d$$

$$a_7 = a_1 + 6d$$

$$(a_7)^2 = a_3 a_{17}$$

$$(a_1 + 6d)^2 = (a_1 + 2d)(a_1 + 16d)$$

$$a_1{}^2 + 12a_1 d + 36d^2 = a_1{}^2 + 16a_1 d + 2a_1 d + 32d^2$$

$$12a_1 d + 36d^2 = 18a_1 d + 32d^2$$

$$36d^2 - 32d^2 = 18a_1 d - 12a_1 d$$

$$4d^2 = 6a_1 d$$

$$4d = 6a_1$$

$$\frac{4d}{6d} = \frac{6a_1}{6d}$$

$$\frac{4}{6} = \frac{a_1}{d}$$

$$\frac{2}{3} = \frac{a_1}{d} \quad ratio, 2:3$$

Answer D

22. If $1^2 + 2^2 + 3^2 + \ldots + 10^2 = a$, then what is the value of $2^2 + 4^2 + 6^2 + \ldots + 20^2$

A. a+4 B. a+2 C. 4a D. 2a

Solution

$$1^2 + 2^2 + 3^2 + \cdots + 10^2 = a$$

$$2^2 + 4^2 + 6^2 + \cdots + 20^2 = 4 + 16 + 36 \ldots + 400$$

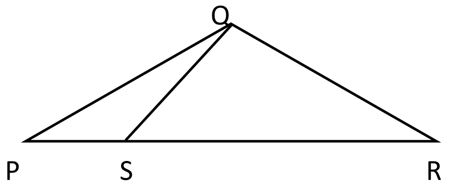
$$= 4(1 + 4 + 9 + \cdots + 100)$$

$$= 4(1^2 + 2^2 + 3^2 + \cdots + 10^2)$$

$$= 4a$$

Answer C

23. Consider the figure below It is given that PQ=RQ.



Compare the following two quantities X and Y whose information are given below

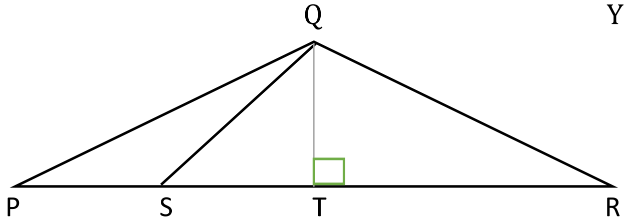
Quantity X	Quantity Y
PS	SR

A. The relation cannot be determined
B. The two quantities are equal
C. Quantity X is greater than quantity Y
D. Quantity Y is greater than quantity X

Solution

Since PQ=RQ, the triangle is an isosceles triangle and the altitude is perpendicular bisector of side PR, thus, $PT = RT$ and $SR > PS$

$Y > X$



Answer C

24. IF Y>2 , compare the following two quantities X and Y whose information are given below

Quantity X	Quantity Y
$\frac{3Y-2}{5}$	y

A. The relation cannot be determined with the information provided
B. The two quantities are equal
C. Quantity Y is greater than quantity X
D. Quantity X is greater than quantity Y

Solution

If y = 0, quantity Y= 0 $>$ quantity X= - 2/5
If y>0, eg y=1, quantity X= 1/5 $<$ quantity Y=1
If y<0, eg y= - 2, quantity X= -8/5 $>$ quantity Y= -2
Cannot be determined,

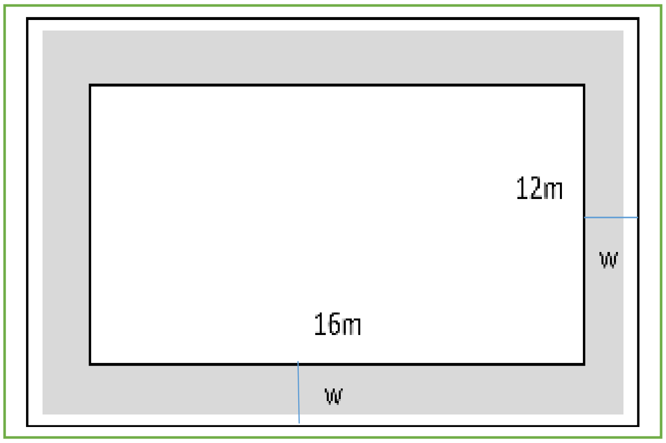
Answer A

25. The rectangular garden measuring 12 meters by 16 meters is to have a pedestrian pathway that is w meters wide installed all way around so that the total area including the pedestrian pathway is 252 m^2. What is the width of the pathway?

A. 1m B. 15m C. 12m D. 1.5m

Solution

$$(16+2w)(12+2w)=252$$
$$192+32w+24w+4w^2=252$$
$$192+56w+4w^2=252$$
$$.$$
$$4w^2+56w+192-252=0$$
$$4w^2+56w-60=0$$
$$\frac{4w^2+56w-60}{4}=\frac{0}{4}$$
$$w^2+14w-15=0$$
$$w^2+15w-w-15=0$$
$$w(w+15)-(w+15)=0$$
$$(w+15)(w-1)=0$$
$$w+15=0\,, \qquad w-1=0$$
$$w=-15 \qquad w=1$$
$$\textit{take the positive}$$
$$\textit{Answer A}$$



SAT – Quantitative reasoning

SAT (quantitative reasoning)-2013

2013 E.C

1. Which one of the following is the value of x in the sequence: 1, 5, 14, 30, x, 91, …?
   A. 65 B. 85 C. 75 D. 55

Solution

$$1, 5, 14, 30, x, 90, \dots ?$$

$$1 \underset{+2^2=4}{\underline{\qquad\qquad}} 5 \underset{+3^2=9}{\underline{\qquad\qquad}} 14 \underset{+4^2=16}{\underline{\qquad\qquad}} 30 \underset{+5^2=25}{\underline{\qquad\qquad}} x \underset{+6^2=36}{\underline{\qquad\qquad}} 9$$

$$x = 30 + 25 = 55$$

Answer D

2. Which of the following is the sum of the series: $3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$?
   A. $\frac{3}{2}$ B. $\frac{9}{2}$ C. $\frac{1}{3}$ D. $\frac{2}{9}$

Solution

$$3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$$

$$\text{Geometric Sequence with } G_1 = 3\,, r = \frac{1}{3}$$

$$S = \frac{G_1}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{2}{3}}$$

$$= \frac{9}{2}$$

Answer B

3. Which of the following is the quadrant that the common solution of the equation $2x + 3y = 2$ and $3x + 2y = 2$ is found?
   A. III quadrant B. I quadrant C. IV quadrant D. II quadrant

Solution

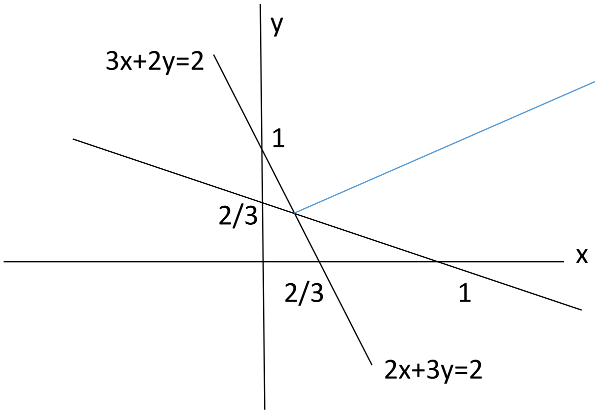
$$2x + 3y = 2 \qquad\qquad 3x + 2y = 2$$

$$x = 0, y = 2/3 \qquad\qquad x = 0, y = 1$$

$$y = 0\,, x = 1 \qquad\qquad y = 0, x = 2/3$$

$$\text{They intersect in the first quadrant}$$

Answer B



OR

$$\overset{3}{\cdot -2}\begin{cases}2x + 3y = 2\\ 3x + 2y = 2\end{cases}$$

$$\begin{cases}6x + 9y = 6\\ -6x - 4y = -4\end{cases}$$

5y=2

Y=2/5, x=2/5

X and y are positive numbers, they belong to the second quadrant

4. Given the inequality $y \leq 2x + 2$, which of the following ordered pairs of numbers is not a solution for the given inequality.

A. (0, 2) B. (2, 3) C. (-1, 0) D. (-3, 3)

Solution

$$y \leq 2x + 2$$
$$A)\ (0,2), 2 \leq 2(0) + 2$$
$$2\ \leq 2\ true$$
$$B)\ (2,3), 3 \leq 2(2) + 2$$
$$3 \leq 6\ true$$
$$C)\ (-1,0), 0 \leq 2(-1) + 2$$
$$0 \leq 0\ true$$
$$D)\ (-3,3), 3 \leq 2(-3) + 2$$
$$3 \leq -6 + 2$$
$$3 \leq -4\ false$$
$$Answer\ D$$

5. The equation $T = 25 + 3c$ is used to model the number of chirps c, made by a certain species of cricket on one minute, and the temperature, T in degrees Fahrenheit. According to this model, what is the estimated increase in temperature, in degrees Fahrenheit, when the number of chirps in one minute is increased by 1?

A. 5 B. 3 C. 28 D. 25

Solution

$$T = 25 + 3C, C = 1$$
$$T = 25 + 3(1)$$
$$= 25 + 3 = 28$$
$$Answer\ C$$

Data Interpretation

DIRECTIONS: Questions 6-10 are based on a data given in the table below. Answer each of these questions based on the given data.

Year	Heavy	Light Commercial	Cars	Jeeps	Two-wheelers	Total
1996	21	32	202	146	349	750
1997	43	66	236	178	323	846
1998	74	75	232	217	402	1000
1999	86	94	289	256	418	1143
2000	103	119	261	235	476	1194
Total	327	386	1220	1032	1968	

Table: A data on the number of different categories of vehicles sold by a company over five years (in thousands)

6. In which of the following years was the number of light commercial vehicles sold was 25% of the number of two-wheelers sold on the same year?

A. 2000 B. 1996 C. 1999 D. 1998

Solution

Year	light com	Two wheel	Ratio
1996	32	349	(32/349) *100%=9.1%
1997	66	323	(66/323) *100%=30.4%
1998	75	402	(75/402) *100%=18.7%
1999	94	418	(94/418) *100%=22.5%
2000	119	476	(119/476)*100% = 25%

Answer A

7. If the same percentage increased in the number of Heavy Vehicles as in 2000 over 1999 was expected in 2000 over 2001, approximately how many heavy vehicles were sold in 2001?

A. 128 B. 138 C. 166 D. 123

Solution

Year	Heavy Vehicle	Increased %
1999 2000	86 103	$\frac{(103-86)}{86} * 100\% = 19.7$
2001	123	$103 + 19.7\%(103)$ $= 103 + 20.3 = 123.3$
Answer D		

8. Which of the following is the percentage of the number of heavy vehicles sold in 1999 to that of the total number of all vehicles sold in 1998?

A. 7.5% B. 12% C. 9% D. 8.6%

Solution

$$Heavy\ Vehicles\ in\ 1999 = 86$$

$$All\ vehicles\ in\ 1998 = 74 + 75 + 232 + 217 + 402 = 1000$$

$$\% = \frac{86}{1000} = 100\% = 8.6\%$$

Answer D

9. In which of the following years was the percentage of number of two-wheelers sold to the total of vehicles sold during the same year was highest?

A. 1996 B. 1999 C. 1997 D. 1998

Solution

Year	Two wheels	Total	%
1996	349	750	(349/750) *100%=46.5%
1997	323	846	(323/846) *100%=38.2%
1998	402	1000	(402/1000) * 100%=40.2%
1999	418	1143	(418/1143) *100%=36.6%
2000	476	1194	(476/1194) *100%=39.9
Total	1968		Answer A

10. For which of the following category of vehicles that the percentage increase in the sale in 1997 over the previous year was maximum?

A. Two-wheelers B. cars C. Light commercial D. Jeeps

Solution

Year	Two wheel	Car	Light Comm	Jeeps
1996 1997	349 323	202 236	32 66	146 178
%	$\frac{(323-349)}{349} * 100\% = -7.4\%$	$\frac{(236-202)}{202} * 100\% = 16.8\%$	$\frac{(66-32)}{32} * 100\% = 106.3\%$	$\frac{(178-146)}{146} * 100\% = 21.9\%$

Answer C

11. A container has 235 flowers in a variety of colors: red, blue, yellow, and purple. There are 45 blue flowers; one-fifth of the flowers are red, forty percent of them are purple and the rest are yellow. A flower picked at random from the container. Which one of the following is the probability that the flower picked is yellow?

    A. $\frac{2}{5}$ B. $\frac{1}{5}$ C. $\frac{9}{47}$ D. $\frac{49}{235}$

Solution

$$sample\ spce, n(s) = 235$$
$$number\ of\ blue, n(B) = 45$$
$$number\ of\ red, n(R) = \frac{1}{5}(235) = 47$$
$$number\ of\ purple, n(P) = 40\%\ of\ 235 = 94$$
$$number\ of\ yellow, n(y) = 235 - 94 - 47 - 45 = 49$$
$$p(y) = \frac{n(y)}{n(S)} = \frac{49}{235}$$
$$Answer\ D$$

12. You go to a restaurant for lunch and want to order a sandwich. There are twenty-five sandwiches on the menu and six of them are toasted. If you order a sandwich at random, then what is the probability of your ordering a sandwich is NOT toasted?

    A. $\frac{9}{25}$ B. $\frac{6}{25}$ C. $\frac{19}{25}$ D. $\frac{16}{25}$

Solution

$$number\ of\ sample\ space, n(s) = 25$$
$$number\ of\ toasted, n(t) = 6$$
$$p(t) = \frac{n(t)}{n(s)} = \frac{6}{25}$$
$$p(not\ t) = 1 - p(t), because\ p(t) + p(not\ t) = 1$$
$$= 1 - \frac{6}{25}$$
$$= \frac{25 - 6}{25}$$
$$= \frac{19}{25}$$
$$Answer\ C$$

13. 28 male loins and 172 female lions are living in 500-acre Park. If 35 more male loins are introduced into the park, how many more female lions should be introduced so that $\frac{6}{7}$ of the total number of lions in the park is female?

    A. 206 B. 378 C. 241 D. 324

Solution

$$m = 28 + 35 = 63$$
$$F = 172 + x$$
$$Total = 63 + 172 + x$$
$$= 235 + x$$
$$F = \frac{6}{7}\ Total$$
$$172 + x = \frac{6}{7}(235 + x)$$
$$7(172 + x) = 6(235 + x)$$
$$1204 + 7x = 1410 + 6x$$
$$7x - 6x = 1410 - 1204$$
$$x = 206$$
$$Answer\ A$$

14. A business is owned by 1 man and 5 women, each of whom has equal share. If one of the women sells 2/5 of her share to the man, and another woman keeps ¼ of her share and sells the rest to the man, what fraction of the total share of the business is owned by the man?

A. 37/120 B. 9/40 C. 43/120 D. 2/3

Solution

$$m = \frac{1}{6}x, \qquad w = \frac{1}{6}x\ each$$

$$sell\ by\ w_1 = \frac{2}{5}\left(\frac{1}{6}x\right) = \frac{1}{15}x$$

$$sell\ by\ w_2 = \frac{3}{4}\left(\frac{1}{6}x\right) = \frac{1}{8}x$$

$$Owned\ by\ man$$

$$= \frac{1}{6}x + \frac{1}{15}x + \frac{1}{8}x$$

$$= \frac{48x + 120x + 90x}{720} = \frac{258}{720}x = \frac{43}{120}x$$

$$Answer\ C$$

15. At certain school, if the ratio of teachers to students is 1 to 10. Which one of the following can be a total number of teachers and students in the school?

A. 121 B. 100 C. 1,011 D. 144

Solution

$$1:10, Total\ 1 + 10 = 11$$

$$A)\ \frac{121}{11} = 11\ \Rightarrow \begin{matrix} S:T \\ 1(11):10(11) \\ 11 + 110 \end{matrix}$$

$$B)\ \frac{100}{11}\ not\ divisible\ by\ 11$$

$$C)\ \frac{1011}{11}\ not\ divisible\ by\ 11$$

$$D)\ \frac{144}{11}\ not\ divisible\ by\ 11$$

$$Answer\ A$$

16. If x is 30% of 400, y is 40% of x, and z is 25% of y, which of the following is equal to $x + y + z$.

A. 180 B. 120 C. 175 D. 150

Solution

$$x = 30\%\ of\ 400$$

$$= \frac{30}{100} * 400 = 0.3 * 400 = 120$$

$$y = 40\%\ of\ x$$

$$= \frac{40}{100} * (120) = 0.4 * 120 = 48$$

$$z = 25\%\ of\ y$$

$$= \frac{25}{100} * (48) = 0.25 * 48 = 12$$

$$x + y + z = 120 + 48 + 12 = 180$$

$$Answer\ A$$

17. A radioactive substance decays at an annual rate of 17%. If the initial amount of the substance is 650 grams, which one of the following function *h(t)* models the remaining amount of the substance in grams, after *t* years.

A. $h(t) = 650(0.17)^t$
B. $h(t) = 0.17(650)^t$
C. $h(t) = 650(0.83)^t$
D. $h(t) = 0.83(650)^t$

Solution

$$17\% = \frac{17}{100} = 0.17$$
$$Decay - Decrease$$
$$h(t) = h_0(1-r)^t$$
$$= 650\ (1-0.17)^t$$
$$= 650\ (0.83)^t$$
$$Answer\ C$$

18. Which of the following is equal to the expression $\left(\frac{2y+6}{4}\right)\left(\frac{3y-36}{3y+9}\right)\ for\ y \neq -3$

A. $\frac{y-12}{2}$
B. $\frac{12y^2-126}{12y+36}$
C. $y-6$
D. $\frac{8y-30}{3y+13}$

Solution

$$\frac{2y+6}{4} * \frac{3y-36}{3y+9}$$
$$= \frac{6y^2-72y+18y-216}{12y+36}$$
$$= \frac{6y^2-54y-216}{12y+36} = \frac{6(y^2-9y-36)}{6(2y+6)}$$
$$= \frac{y^2-9y-36}{2y+6} = \frac{(y-12)(y+3)}{2(y+3)}$$
$$= \frac{y-12}{2}$$
$$Answer\ A$$

$$y^2-9y-36$$

Is quadratic equation

2 numbers, Sum=b= - 9, pro=ac= - 36

are – 12 and 3

$$y^2-9y-36 = y^2-12y+3y-36$$

=y(y-12)+3(y-12)

=(y-12)(y+3)

DIRECTIONS: Questions **19 and 20** both consist of two quantities one in column A and another one in Column B. Compare the two quantities and answer the questions by choosing the alternative that best explains the relationship between " Column A" and " Column B"

19. The diameter of a circle O is the same as the length of the side of a square S.

Column A	Column B
The Area of circle O	The Area of a Square S

A. The quantity in column B is greater
B. The quantity in column A is greater
C. The relationship cannot be determined
D. The two quantity are equal

Solution

$$\frac{dia}{2} = r \Rightarrow d = s \Rightarrow 2r = S$$

$column\ A$	$column\ B$
$A = \pi r^2$	$A = S^2$
$= \pi r^2$	$= (2r)^2$
$= 3.14r^2$	$= 4r^2$

$$4r^2 > 3.14r^2$$
$$Answer\ A$$

20. Let Y be a real number.

Column A	Column B
*(y+1) * y * (y-1)*	*y * y *y*

A. The quantity in column B is greater
B. The quantity in column A is greater
C. The relationship cannot be determined
D. The two quantities are equal

Solution

$$\begin{array}{cc} \underline{column\ A} & \underline{column\ B} \\ (y+1)(y)(y-1) & y(y)(Y) \\ (y^2+y)(y-1) & y^3 \\ y^3+y^2+y^2-y & \\ y^3-y & \end{array}$$

$$If\ y<0\,,A>B$$
$$If\ y>0\,,B>A$$
$$If\ y=0,A=B \quad Answer\ C$$

21. If $a_5 + a_9 = 72$ and $a_7 + a_{12} = 72$ then, which one of the following is equal to the arithmetic sequence?

A. 5, 1, 15, 20, 25, …
B. 6, 11, 16, 21, 26, …
C. 3, 8, 13, 18, 23, …
D. 4, 9, 14, 19, 224, …

Solution

$$a_5 + a_9 = 72$$
$$a_1 + (5-1)d + a_1 + (9-1)d = 72$$
$$a_1 + 4d + a_1 + 8d = 72$$
$$2a_1 + 12d = 72$$
$$a_1 + 6d = 36 \ldots\ldots\ldots\ldots\ldots\ldots..i$$
$$a_7 + a_{12} = 97$$
$$a_1 + (7-1)d + a_1 + (12-1)d = 97$$
$$a_1 + 6d + a_1 + 11d = 97$$
$$2a_1 + 17d = 97 \ldots\ldots\ldots\ldots\ldots\ldots.ii$$
$$-2\begin{cases} a_1 + 6d = 36 \\ \underline{2a_1 + 17d = 97} \end{cases}$$
$$-2a_1 - 12d = -72$$
$$\underline{2a_1 + 17d = 97}$$
$$5d = 25$$
$$d = 5$$
$$a_1 + 6d = 36$$
$$a_1 + 6(5) = 36$$
$$a_1 + 30 = 36$$
$$a_1 = 36 - 30$$
$$a_1 = 6$$
$$a_n = 6, \quad 6+5, \quad 6+5+5, \quad 6+5+5+5, \ldots$$
$$= 6{,}11\ {,}16, 21, \ldots$$
$$Answer\ B$$

22. Which term of the following geometric sequence 2, 8, 32, 128, … is 2048?

A. 12th B. 11th C. 7th D. 6th

Solution

$$G_1 = 2, \qquad r = 4$$

$$n^{th}\ term\ of\ geometric\ sequence, \qquad G_n = G_1 r^{n-1}$$

$$2048 = 2(4)^{n-1}$$

$$1024 = 4^{n-1}$$

$$2^{10} = 2^{2n-2}, note\ a^x = a^y \Longrightarrow x = y$$

$$10 = 2n - 2$$

$$12 = 2n$$

$$n = 6$$

$$Answer\ D$$

23. In the arithmetic sequence: *-1, n, 13, …,* which one of the following is value of n.

A. 6 B. 7 C. 8 D. 9

Solution

$$-1, n, 13, \dots$$

$$13 - n = n - (-1)$$

$$13 - n = n + 1$$

$$13 - 1 = n + n$$

$$12 = 2n$$

$$n = 6$$

$$Answer\ A$$

24. Which one of the following is a term in the sequence 6, 14, 22, …?

A. 264 B. 276 C. 258 D. 284

Solution

$$6, 14, 22, \dots \qquad a_1 = 6, \quad d = 8,$$

$$n^{th}\ term\ of\ arithimetic\ sequence\ , a_n = A_1 + (n-1)d, n \in N$$

$$A)\ 264 = 6 + (n-1)8 \qquad B)\ 246 = 6 + (n-1)8$$

$$264 = 6 + 8n - 8 \qquad 246 = 6 + 8n - 8$$

$$264 + 8 - 6 = 8n \qquad 246 + 8 - 6 = 8n$$

$$266 = 8n \qquad 248 = 8n$$

$$\frac{266}{8} = n \qquad \frac{248}{8} = n$$

$$n = 33.25 \qquad n = 31$$

$$C)\ 258 = 6(n-1)8 \qquad D) 284 = 6 + (n-1)8$$

$$258 = 6 + 8n - 8 \qquad 284 = 6 + 8n - 8$$

$$258 + 8 - 6 = 8n \qquad 284 + 8 - 6 = 8n$$

$$260 = 8n \qquad 286 = 8n$$

$$\frac{260}{8} = n \qquad \frac{286}{8} = n$$

$$n = 32.5 \qquad n = 35.75$$

$$Answer\ B$$

25. Which one of the following is equal to the series$\sum_{n=1}^{\infty}(2n-5)$?

A. (1) +(-3) +(1) +… C. (-3) +(-1) +(1) +…

B. (1) +(5) +(9) +… D. (1) +(-3) +(-7) +…

Solution

$$\sum_{n=1}^{\infty}(2n-5)$$

$$[2(1) - 5] + [2(2) - 5] + [2(3) - 5] + [2(4) - 5] + \cdots$$

$$= (2 - 5) + (4 - 5) + (6 - 5) + (8 - 5) + \cdots$$

$$(-3) + (-1) + (1) + (3) + \cdots$$

$$Answer\ C$$

Additional questions

1. If $f(x) = \sqrt[3]{1+e^{-x}}$, which of the following is equal to $f^{-1}(x)$?

   A. $\ln\frac{1}{x^3-1}$ B. $\frac{1}{\ln(x^3+1)}$ C. $\ln(x^3-1)$ D. $(1+e^{-x})^3$

2. Which of the following functions is a one-to-one correspondence?

   A. F: $\Re \to \Re$,f(x)=tan x, where R' is the domain of f

   B. t: $\Re \to \Re$, g(x) = 2^x

   C. h:[0, ∞) $\to$ [0, ∞), h(x)=x^2

   D. r:[0, ∞) $\to$ [0, ∞), r(x)=x+5

3. The inverse of the function defined by g(x)=$\frac{2x}{x+3}$ is equal to:

   A. $g^{-1}(x) = \frac{-2x}{x-3}$ C. $g^{-1}(x) = \frac{-(x+3)}{2x}$

   B. $g^{-1}(x) = \frac{-3x}{x-2}$ D. $g^{-1}(x) = \frac{-(x+2)}{3x}$

4. Which one of the following is a one-to one correspondence function from A = $[0,1]$ to B= $[1,2]$?

   A. f(x)=x B. f(x)= $\frac{1}{3}x^3$ C. f(x)=2x+1 D. f(x)=x^2+1

5. If the point (3, -2) is on the graph of y= f(x), which point is on the graph of y = f^{-1}(x)?

   A. $\left(\frac{1}{3},-2\right)$ B. (3, -1) C. (-2,3) D. $\left(3,-\frac{1}{2}\right)$

6. If $f(x) = \frac{x+1}{x-1}$ and f(a)=5 then f(2a) is equal to:

   A. 2 B. 4 C. 6 D. 8

7. Which one of the following is true?

   A. A polynomial can have infinitely many vertical asymptotes.

   B. The graph of a rational function can never cross its horizontal asymptote.

   C. The graph of $f(x)\frac{3x-1}{x-1}$ has no horizontal asymptote.

   D. The graph of $f(x)\frac{x^3-x}{x^2-x}$ has not vertical asymptote.

8. Which one of the following is true about the graph of $f(x) = \frac{x^2-1}{x-x^2}$?

   A. x = 0 and x = 1 are its vertical asymptotes.

   B. Y = 1 is its horizontal asymptote.

   C. Y = x – 1 is its oblique asymptote.

   D. It is almost the same as the horizontal line y = -1 as $x \to \pm\infty$.

9. The value(x) of x where the graph of the function $y = \frac{x^2-1}{x^3}$ crosses its horizontal asymptote is (are):

   A. x = -2 C. x = 0

   B. x = -1 and x = 1 D. x = -$\sqrt{2}$ and x = 1 + $\sqrt{2}$

10. If p(x) = $3x^2$ and q(x) = x^2 + x, then what is the solution set of $\frac{p(x)}{3q(x)} - \frac{1}{x} = \frac{1}{q(x)}$?

    A. {-1,2} B. {2} C. {-3,2} D. {-3}

11. What is the solution set of $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}} = 3x^2 - \frac{x}{1+\frac{1}{x}}$?

A. $\left\{-1, \frac{1}{3}\right\}$ B. $\left\{\frac{1}{3}\right\}$ C. $\left\{3, \frac{-1}{3}\right\}$ D. $\left\{\frac{-1}{3}\right\}$

12. Suppose $f(x) = \frac{Q(x)}{x(x^2-1)}$ where Q(x) is a quadratic function. Which one of the following is necessarily true about the graph of f ?

A. x= 0, x = 1 and x = -1 are the vertical asymptotes of the graph of f .
B. The graph of f does not intersect with its horizontal asymptote.
C. The vertical asymptote of the graph of f is only x = -1 if Q(x) = x^2-x.
D. The vertical asymptote of the graph of f is only x = 1 if Q(x) = $2x^2$.

13. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x|+2}{x}$?

A. y = 2 is the only horizontal asymptote of the graph.
B. y = 1 and y = -1 are the horizontal asymptotes of the graph.
C. y = 2 and y =-2 are th horizontal asymptotes of the graph.
D. y = 1 is the only horizontal asymptote of the graph.

14. What is the distance from the origin to the line that passes through (1,0) and (0,1)?

A. $\frac{1}{2}\sqrt{2}$ B. 1 C. $\frac{1}{2}$ D. $\sqrt{2}$

15. What is the equation of the directrix for the parabola whose equation is $y^2 + 8x + 6y + 25 = 0$?

A. y = 3 B. x = 2 C. x = 0 D. x = 4

16. If two lines y = x and y = x – 4 are tangent to a circle at (2,2) and (4,0), respectively, then what is the equation of the circle?

A. $(x-2)^2 + y^2 = 4$ C. $(x-3)^2 + (y-1)^2 = 2$
B. $(x-4)^2 + (y-2)^2 = 4$ D. $(x-1)^2$ + ?$(y+1)^2 = 10$

17. A semi-elliptical are over a tunnel for a road through a mountain has a major axis of length 80 meters and a height of 30 meters at the center. What is the equation of the semi-elliptical are over the tunnel, if the center is considered as the origin?

A. $\frac{x^2}{6400} + \frac{y^2}{900} = 1$ C. $\frac{x^2}{900} + \frac{y^2}{6400} = 1$
B. $\frac{x^2}{1600} + \frac{y^2}{900} = 1$ D. $\frac{x^2}{8100} + \frac{y^2}{6400} = 1$

18. If the equation $(x-2)^2 = (y-2)^2 = 1$ represents a hyperbola, which one of the following represents equation of an asymptote to the hyperbola?

A. x = 4 – x B. x + y = 1 C. x = 2 – y D. x + 2y = 3

19. Two perpendicular lines ℓ_1 and ℓ_2 are intersecting at (-1,2). If the angle of inclination of ℓ_1 is 45°, then what is the equation of ℓ_2?

A. y = -x + 3 B. y = x + 3 C. y = -x + 1 D. y = x + 1

20. The equation of an ellipse with center at (1,4), vertices at (10,4) and (1,2) is:
   A. $4(x-1)^2+81(y-4)^2=324$
   C. $9(x-1)^2+4(y-4)^2=4$
   B. $(x-1)^2+9(y-4)^2=4$
   D. $2(x-1)^2+9(y-4)^2=4$

21. What is the focus of the parabola $y^2+4y+8x=4$?
   A. (1,2)
   B. (-1,-2)
   C. (3,-2)
   D. (-3,2)

22. Which one of the following is true about a conic section represented by the equation $\frac{x^2}{k}+\frac{y^2}{k-9}=1$
   A. It is a circle whose center is at origin for some $k \in \Re$.
   B. It is an ellipse whose major axis is vertical when k>9.
   C. It is a hyperbola whose foci are at (-3,0) and (3,0) when 0<k<9.
   D. It is a hyperbola whose foci are at (-3k,0) and (3k,0) when 0<k<9.

23. Which one of the following is equation of a circle whose center is on y-axis and radius is 3?
   A. $x^2 + y^2 + 6y = 0$
   B. $(x - 2)^2 + y^2 = 9$
   C. $x^2 + (y - 2)^2 = 3$
   D. $x^2 - 2x + y^2 = 8$

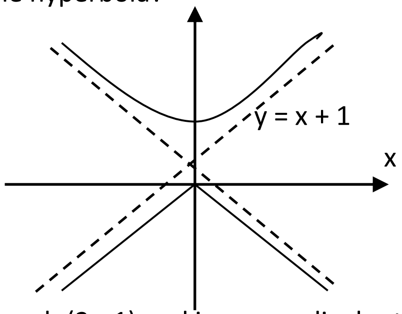
24. The planet Mercury's orbit around the sun is an ellipse with eccentricity 0.206, length of the major axis 1.16×10^8 km and the sun at one focus. What is the maximum distance from Mercury to the sun?
   A. 6.99×10^8 km
   B. 6.99×10^7 km
   C. 9.66×10^7 km
   D. 9.66×10^8 km

25. The graph of a hyperbola and the lines of its asymptotes are as shown in the following figure. Which one of the following is an equation of the hyperbola?

y

   A. $y^2 - 2y - x^2 = 0$
   B. $y^2 - 3y - x^2 = 0$
   C. $x^2 \ (y - 1)^2 = 1$
   D. $(x - 1)^2 - y^2 = 1$

y=x + 1



26. The equation of the line that passes through (2, -1) and is perpendicular to 3x + 4y = 6 is:
   A. -4x + 3y = 5
   C. 4x = 3y = 11
   B. 4x – 3y = 5
   D. -4x + 3y = -11

27. if $(p \vee q) \Rightarrow (\neg r \wedge r)$ is true, then which one of the following is necessarily true?
   A. $(p \vee q) \Rightarrow q$
   B. $\neg r \wedge r$
   C. $\neg p \Leftrightarrow r$
   D. $\neg p \vee r$

28. Which one of the following is a valid logical argument?
   A. $p \Rightarrow q, q \vdash p$
   C. $\neg p \wedge q, q \Rightarrow r \vdash r$
   B. $p \Leftrightarrow q, p \Rightarrow q \vdash q$
   D. $\neg p, p \vee q, r \Rightarrow q \vdash r$

29. For arbitrary propositions p and q, which one of the following is a valid equivalence?
   A. $\neg(p \Rightarrow q) \equiv (q \Rightarrow p)$
   C. $[p \vee \neg q] \equiv [p \Rightarrow q]$
   B. $[\neg(p \Rightarrow q) \wedge p] \equiv (p \wedge \neg q)$
   D. $[(p \vee q) \Rightarrow q] \equiv [p \Rightarrow \neg q]$

30. Suppose that p represents the statement "He missed the tournament.", q represents the statement "He got the gold medal." And r represents the statement "He took a trip abroad.". Then which of the following symbolic expression represents the statement: "If he takes a trip abroad and he does not miss the tournament, then he will get the gold medal."?
   A. $(r \Rightarrow q) \wedge \neg p$
   C. $(r \wedge \neg p) \Rightarrow q$
   B. $(r \wedge (p \Rightarrow q)$
   D. $\neg(r \vee p) \vee q$

31. Which one of the following is NOT a tautology?

A. $[p \vee (q \Rightarrow r)] \Leftrightarrow [\neg p \Rightarrow (q \Rightarrow r)]$ C. $p \Rightarrow (p \Rightarrow q) \vee q$

B. $p \vee (q \Rightarrow \neg p)$ D. $[p \Leftrightarrow (q \wedge \neg r)] \Leftrightarrow [\neg p \Leftrightarrow (\neg q \vee r)]$

32. If the truth value of a proposition p is False, then which one of the following compound proposition has a truth value True?

A. $\neg p \wedge p$ B. $\neg p \Rightarrow p$ C. $\neg(\neg p \vee p)$ D. $p \Rightarrow \neg p$

33. Which one of the following compound propositions is a tautology?

A. $(q \vee \neg q) \Rightarrow p$ B. $p \Rightarrow (q \vee \neg q)$ C. $p \vee (q \wedge \neg q)$ D. $p \Rightarrow (q \wedge \neg q)$

34. If the list of a measurement is 10, α, 5, α, 5,10,20,15,20,5 with mean $\bar{x}$, then what is the value of α in terms of $\bar{x}$?

A. $10\bar{x}$ - 90 B. $9\bar{x} - 90$ C. $5\bar{x} - 90$ D. $5\bar{x} - 45$

35. The following is the frequency distribution of a grouped data.

Class Intervals	Frequency (f)
3 – 7	2
8 – 12	2
13 – 17	10
18 - 22	6

What is the mean and standard deviation of the distribution, respectively?

A. 15, $2\sqrt{5}$ B. 15, $\sqrt{7.5}$ C. 12.5, $5\sqrt{2}$ D. 12.5, $\sqrt{15}$

36. If distinct codes (words) of eight letters are formed by rearranging the letters in the word "ABBEBAYE", how many of the codes begin with B or Y?

A. 840 B. 630 C. 1680 D. 420

37. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defect of type D_1, 60 have defect of type D_2 and 25 have both types of defects, what is the probability that a bulb produced by the company has **none** of the defects?

A. 0.95 B. 0.80 C. 0.85 D. 0.20

38. If S is a set with 10 elements and $A \subseteq S$, what is the probability that A has 3 or more elements?

A. $\frac{7}{10}$ B. $\frac{8}{11}$ C. $\frac{121}{128}$ D. $\frac{7}{128}$

39. The following is set of data representing the average mark of 13 students: 91, 89, 93, 91, 87, 94, 92, 85, 91, 90, 96, 93, and 89. Then which one of the following statements is true about the data?

A. The median is 90.5. C. The range of the marks is 11.

B. The upper quartile is 92. D. The mean is 91.5

40. Different codes, each of which consisting of five characters, are to be generated in such a way that the first two characters are any of the English capital letters (A to Z) and the remaining three are any of the digits (0,1,…,9). How many distinct codes can be generated so?

A. 468,000 B. 260 C. 676,000 D. 26! x 10!

41. A ladder 6m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate (speed) of $\frac{1}{2}m/\sec$, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ rad?

A. $\frac{\sqrt{2}}{12} rad/\sec$ B. $\frac{\sqrt{2}}{2} rad/\sec$ C. $\frac{\sqrt{2}}{6} rad/\sec$ D. $\frac{\sqrt{2}}{3} rad/\sec$

42. A city has two daily newspapers, X and Y. The following information was obtained from a survey of 100 residents of the city: 35 people subscribe to X,60 people subscribe to Y and 20 subscribe to both newspapers. Then how many of the people in the survey do not subscribe to either of the newspapers?

A. 5 B. 25 C. 40 D. 55

43. A measurement is groped into five class intervals with the following frequency distribution.

Class Interval	5 - 15	15 - 25	25 – 35	35 - 45	45 – 55
Frequency	22	40	68	50	20

What are the first quartile Q_1 and the 75th percentile P_{75} of the measurement?

A. $Q_1 = 20, P_{75} = 40$, B. $Q_1 = 22, P_{75} = 40$ C. $Q_1 = 20, P_{75} = 39$ D. $Q_1 = 22, P_{75} = 39$

44. Three persons P_1, P_2 and P_3 are firing at a target independently and have a probability 0.7, 0.5 and 0.4, respectively, of hitting the target. What is the probability that at least one of them hits the target?

A. 0.95 B. 0.85 C. 0.91 D. 0.99

45. The following is a simple frequency distribution of a data with variable X.

X	3	5	6	7
F	2	5	2	1

What are the mean $\left(\overline{x}\right)$and variance δ^2 of the data?

A. $\overline{x} = 5, \delta^2= 0.7$ B. $\overline{x} = 6, \delta^2= 1.4$ C. $\overline{x} = 6, \delta^2= 0.7$ D. $\overline{x} = 5, \delta^2= 1.4$

46. A box contains 10 items of which 3 are defective. If 2 items are randomly taken out of the box, what is the probability that both items are not defective?

A. $\frac{7}{10}$ B. $\frac{4}{7}$ C. $\frac{7}{15}$ D. $\frac{49}{100}$

47. Items produced by a certain company are subjected to two kinds of defects D1 and D2. Out of the total product, 5% have the defect D1, 10% have the defect D2, and 2% have both defects. What is the probability that a randomly selected item has neither defect D1 nor defect D2?

A. 0.13 B. 0.5 C. 0.98 D. 0.87

48. There are three children in a room, ages three, four, and five. If a four-year-old child enters the room then which one of the following is true?

A. Mean age will stay the same but the standard deviation will increase.
B. Mean age will stay the same but the standard deviation will decrease.
C. Mean age and standard deviation will increase.
D. Mean age and standard deviation will stay the same.

49. In how many more ways can 4 people be arranged in a row than if they were arranged in a circle?

A. 1 B. 6 C. 18 D. 12

50. Two machines A and B work independently. The probability that both machines A and B work is 0.4. if the conditional probability that machine B works given that machine A works is 0.5, then the conditional probability that machine A works given that machine B works is ______________.

A. 0.8 B. 0.3 C. 0.5 D. 0.7

51. If $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $(2A + B)^T = A^TA$, then which one of the following is equal to B?

A. $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ B. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ C. $\begin{pmatrix} 8 & 0 & -4 \\ 4 & 8 & 0 \\ 0 & 0 & -4 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$

52. If $M=\begin{pmatrix}0 & 1 & 2\\ 3 & -1 & 0\\ 5 & 2 & 4\end{pmatrix}$ and $A^TM=2I$, where A is a 3x3 matrix and I is the identity matrix of order 3, then what is det (A)?

    A. 0.2 B. $\frac{4}{17}$ C. 0.8 D. $\frac{1}{17}$

53. What should be the value of K so that the system of equation $\begin{cases}x-y+z=1\\ -x+5y-4z=1\\ 2x+2y-z=k\end{cases}$ has a solution?

    A. 0 B. 1 C. -4 D. 4

54. Suppose AX = b, where A is a 3 x 3 matrix, b = $(b_1,b_2,b_3)^T$ and X = $(x,y,z)^T$. which one of the following is necessarily true about this system of linear equations?

    A. The system has a solution only when det (A) $\neq$ 0.
    B. The Cramer's rule is suitable to solve the system if two rows of A are identical.
    C. If det (A) $\neq$ 0 and the second column of A is a multiple of b, then x = 0.
    D. If b = 0, then X = $(0, 0, 0)^T$ is the only solution of the system.

55. If $A=\begin{pmatrix}3 & -2 & 8\\ 0 & 6 & 7\\ 0 & 4 & 5\end{pmatrix}$, then det ($A^TA$) is equal to __________

    A. 12 B. 36 C. 30 D. 15

56. If $\begin{pmatrix}0 & x & 0\\ 1 & -1 & 1\\ 0 & y & -1\end{pmatrix}$ and $A^{-1}=\begin{pmatrix}1 & 1 & 1\\ 3 & 0 & 0\\ 2 & 0 & -1\end{pmatrix}$, then what are the values of x and y?

    A. x = 3, y = -2 B. $x=\frac{2}{3}, y=\frac{1}{3}$ C. x = -3, y = 2 D. $x=\frac{1}{3}, y=\frac{2}{3}$

57. Consider the following system of equations: $\begin{cases}ax+by=2\\ x+3y+2z=0\\ 2x+y+z=0\end{cases}$

If the determinant of the coefficient matrix is 2, then what is the solution set of the system?

    A. {(1,3,-5)} B. $\left\{\left(\frac{1}{a},\frac{1}{b},0\right)\right\}$ C. {(-2,-6,10)} D. {}

58. What is the solution set of the system $\begin{cases}x+y-z=1\\ x+2y=3z=1\\ 2x+3y-4z=2\end{cases}$?

    A. {(0,2,1)} B. {(1-k,2k,k) | k $\in \Re$} C. {(2k+1,-k,k) | k $\in \Re$} D. {}

59. If A = $\begin{pmatrix}2 & 7\\ 1 & 3\end{pmatrix}$ and B^{-1} = $\begin{pmatrix}1 & 1\\ 0 & 2\end{pmatrix}$, then $(AB)^{-1}$ is equal to:

    A. $\begin{pmatrix}4 & -3\\ 4 & -5\end{pmatrix}$ B. $\begin{pmatrix}-2 & 5\\ 2 & -4\end{pmatrix}$ C. $\begin{pmatrix}-3 & 11\\ 1 & -3\end{pmatrix}$ D. $\begin{pmatrix}4 & 0\\ 0 & 2\end{pmatrix}$

60. Let A = $\begin{pmatrix} 0 & \alpha & \beta \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}, b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$, and x= $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. If det(A) = 3, then what is the solution set of the system AX = b?

A. $\{(6,-2,-8)^T\}$ B. $\left\{\left(0,\frac{1}{a},\frac{5}{b}\right)^T\right\}$ C. $\{(-3,1,4)^T\}$ D. ϕ

61. For any $n \times n$ square matrix A, which one of the following is true?
   A. Det(A) = -det (A^T), where A^T is the transpose of A
   B. If k is a scalar, then det (kA) = K^n det (A)
   C. If B is a matrix obtained from A by interchanging of two rows of A, then det (B) = det (A)
   D. If A is invertible, then det (A) = det (A^{-1})

62. The solution of the system of linear equation of

$$\begin{cases} x - 3x - 2z = 6 \\ 2x - 4y - 3z = 8 \\ -3x + 6y + 8z = -5 \end{cases}$$ is:

A. X = -1, y = -3, z = -2 B. X = -1, y = -3, z = 2 C. x = 1, y = -3, z = 2 D. x = 1, y = 3, z = -2

63. if $w = \frac{16i}{1+i} + (1-3i)^2$ and $z = |w| + \overline{w}$, which one of the following is the simplest form of z?

A. $\sqrt{2} + 2i$ B. 2 + 2i C. 4 – 2i D. 2 – 2i

64. If $z = \frac{3+i}{i-2}$ is a given complex number, then what is the conjugate, $\overline{z}$ of z?

A. $\overline{z} = \frac{-3+i}{2+i}$ B. $\overline{z} = \frac{i-3}{2-i}$ C. $\overline{z}$ = -6 - 2i D. $\overline{z}$ = -1 – i

65. What is the principal argument of $(5+5i)^{11}$?

A. $\frac{\pi}{2}$ B. $\frac{2\pi}{3}$ C. $\frac{\pi}{4}$ D. $\frac{3\pi}{4}$

66. What are the values of u and v that satisfy the equation: $\frac{u+3i}{4-2i} = \frac{2+vi}{20}$?

A. u = 2, v = 3 B. u = -6, v = 10 C. u = 2 , v = 16 D. u = -4, v = 6

67. In the set of complex numbers, what is the solution set of $x^2 + 4x + 5 = 0$?

A. ϕ B. $\{1-2i, 1+2i\}$ C. $\{2-i, 2+i\}$ D. $\{-2-i, 2+i\}$

68. If $z = (1+i)^{10}$, then which of the following is equal to z?

A. 1+32i B. 32i C. 10i D. 1+10i

69. If $Z = \frac{2-4i}{1+i}$ then the modulus of the conjugate of Z, $|\overline{z}|$ is:

A. $\sqrt{10}$ B. $3\sqrt{2}$ C. $2\sqrt{3}$ D. $2\sqrt{2}$

70. Suppose $\vec{A} = 2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{B}$ is a vector in space such that $|\vec{B}| = \vec{A}.\vec{B}$. If $\vec{U}$ is the unit vector in the direction of $\vec{B}$, then $|\vec{A} + \vec{U}|^2$ is equal to:

A. 16 B. 12 C. 10 D. 14

71. What is the image of the line given by (x,y) = (-1,0) + t(3,6),t$\in$R, under the translation that takes (1,0) to (0,1) following by the reflection about the line y = 2x?

A. y = 2x + 3 B. y = 2x – 3 C. y = 2x + 6 D. y = 2x – 5

72. Suppose $\vec{A}=3\vec{i}-4\vec{j}$ and $\vec{B}$ is a vector in the xy=plane such that the angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{3}$. If $\vec{U}$ is the unit vector in the direction of $\vec{B}$, then $\vec{A}.(\vec{A}-2\vec{U})$ is equal to:

A. 20 B. 5 C. 15 D. 30

73. Let the equation $x^2 + 2x + y^2 = 8$ represents a circle. Then which one of the following lines cut the circle at exactly two points?

A. $4x + 3y + 19 = 0$ B. $3x + 4y + 14 = 0$ C. $2y = 5x + 43$ D. $2x = y - 50$

74. If ℓ is the line passing through (0,2) and parallel to $V=\vec{i}+3\vec{j}$, which one of the following is true about ℓ and the circle $(x-2)^2 + (y-1)^2=5$?

A. ℓ is tangent to the circle at (0,2)

B. ℓ is tangent to the circle at some point p, where p $\neq$ (0,2)

C. Intersects the circle at two distinct points.

D. The distance between ℓ and the center of the circle is greater than $\sqrt{5}$

75. If a translation T take the circle $x^2 + y^2 - 2x + 6y + 3 = 0$ into the circle whose equation is $(x+2)^2 + (y-4)^2=7$, then what is the image of the origin under T?

A. (-3,7) B. (1,2) C. (1,-3) D. (-2,4)

76. If A(-2,3), B=)3,1) and C is any other point on the plane, then which one of the following is the coordinate from of $\overrightarrow{AC}-\overrightarrow{BC}$?

A. (-5,2) B. (5,-2) C. (1,4) D. (-1,-4)

77. What is the equation of a line that passes through the point (-1,2) and parallel to a the vector (1,-1)?

A. $2x - y = 1$ B. $x + y - 1 = 0$ C. $x - 2y = 3$ D. $y - 2x + 1 = 0$

78. If $\vec{A}=4\vec{i}-3\vec{j}$ and $\vec{u}$ is a unit vector such that $|\vec{A}+\vec{u}|^2=27$, then the cosine of the angle between $\vec{A}$ and $\vec{u}$ is equal to

A. 0.1 B. 0.2 C. 0.3 D. 0.4

79. If A=(1,-2), B=(-3,2) and $\vec{V}$ is a position vector such $2\vec{V}+\overrightarrow{AB}=\vec{0}$, then $\vec{V}$ is equal to _________.

A. (2,0) B. (-1,0) C. (-2,2) D. (2,-2)

80. What is the image of the ellipse $(x-1)^2+4y^2=1$ under the translation that takes (1,1) to (0,2) followed by the reflection through the x-axis?

A. $x^2+4(y-1)^2=1$ B. $4x^2+(y-1)^2=1$ C. $x^2+4(y+1)^2=1$ D. $4x^2+(y+1)^2=1$

81. If $\vec{V}=\overrightarrow{AB}+3\overrightarrow{BA}$, where A and B are distinct points in the coordinate plane, then which one of the following is equal to $3\vec{V}$?

A. $6\overrightarrow{AB}$ B. $-6\overrightarrow{AB}$ C. $12\overrightarrow{AB}$ D. $-12\overrightarrow{AB}$

82. If $\vec{u}$ is a unit vector in the direction of $\vec{A}$ and $\left|\vec{A}=4\right|$, then $\vec{A}.\vec{u}$ Is equal to ______________.

A. $\frac{1}{4}$ B. 4 C. $\frac{1}{2}$ D. 2

83. If $\vec{A}$ and $\vec{B}$ are parallel vectors with opposite directions and $\left|\vec{B}\right|=\left|2\vec{A}\right|$, is equal to:

A. $\vec{A}$ B. $-\vec{A}$ C. $-3\vec{A}$ D. $3\vec{A}$

84. The image of a figure with vertices A(1, 2), B(3, 6), C(-1, 2), and D(-2,-2) after reflection across the x-axis is:

A. A'(1,-2), B'(-3, -6), C'(1, -2), D'(2, 2) C. A'(1,-2), B'(3, -6), C'(-1, -2), D'(-2, 2)

B. A'(-1,-2), B'(-3, 6), C'(-1, -2), D'(-2, 2) D. A'(1,-2), B'(3, 6), C'(-1, 2), D'(-2, -2)

85. What is the value k, for which the two vectors $\vec{u} = \begin{pmatrix} 1 \\ k \\ -3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2k \\ -5 \\ 4 \end{pmatrix}$ are perpendicular?

A. 4 B. -4 C. 3 D. -3

86. If $\theta = 2\,\text{arc tan}\left(\frac{1}{2}\right)$, then which one of the following is equal to secθ?

A. $\frac{25}{3}$ B. $\frac{4}{5}$ C. $\frac{5}{3}$ D. $\frac{5}{4}$

87. If a point (2,5) is reflected under a line to the point (-3,1), what is the line of reflection?

A. 2x + 3y = 7 B. x + 3y =7 C. 8y + 10x = 19 D. 2x + 3y + 5 = 0

88. Suppose that an airplane is descending at a speed of 50 miles per hour at an angle of 30° below the horizontal line. What is the x- and y-components, respectively, of the velocity of the plane?

A. $50\sqrt{3},25$ B. $-25,50\sqrt{3}$ C. $25,-25\sqrt{3}$ D. $-25\sqrt{3},-25$

89. An observer on level ground is at a distance $10\sqrt{3}$ m from a building. The angles of elevation to the bottom of the windows on the second and third floors at 30° and 60°, respectively. What is the distance h between the bottoms of the windows?

[You may use the values; sin 30° = cos 60° = $\frac{1}{2}$ and sin 60° = cos 30° = $\frac{\sqrt{3}}{2}$]

A. 15m B. 20m C. 15 $\sqrt{3}$ m D. 32m

90. What is the possible value of x that solves the equation: $\sin^{-1} x + \cos^{-1}\left(\frac{3}{5}\right) = \pi$?

A. $\frac{\pi}{3}$ B. $\frac{3}{5}$ C. $\frac{5\pi}{2}$ D. $\frac{4}{5}$

91. What is the work done (in Joules) when a force of 50 Newton used to pull a crate 20 meters along a level path if the force is an angle of 60°?

[Remember that: Work = (Force) x (distance travelled)].

A. 360 B. 500 C. 760 D. 1500 $\sqrt{2}$

92. What is the amplitude and period, respectively, of the graph of f(x)=-6 sinx. cosx?

A. 3 , π B. 6 , π C.3,$\frac{\pi}{2}$

D. 6 , 2 π

93. If angle θ is an acute angle of a right triangle, what is the length of the side adjacent to θ, given that the hypotenuse has 6 units length and sec θ = 10/3?

A. 1.8 units B. 2 units C. 18 units D. 20 units

94. Two ships, one with angle of depression 60° due east and the other with 30° due west are observed from a plane 1,000 meters above a sea. If the two ships are on the same line, what is the distance between the two ships?

[You may use the values: sin 30° = cos 60° = ½ and sin 60° = cos 30°]

A. $\frac{4000}{\sqrt{3}}$ m B. 2,000 m C. $\sqrt{3}$ 500 m D. $\frac{600}{\sqrt{3}}$ m

95. If $\cos(\theta) = 2$, then which of the following is equal to $\csc(\theta)$?

A. $\sqrt{5}$ B. $\frac{2}{\sqrt{5}}$ C. $\frac{1}{\sqrt{5}}$ D. $\frac{1}{2}$

96. What is the amplitude and period, respectively, of the graph of $f(x) = 4\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)$?

A. 4, $\frac{\pi}{3}$ B. 2, 3π C. 2, $\frac{2\pi}{3}$ D. 4, 3π

97. A boat on a sea sailed from its station toward North with constant speed of 80 km/h. another boat from the same station sailed 600 NE (North East) with constant speed of 100 km/h. if the two boats started sailing at the same time, what is the straight distance between them after they have sailed for just 30 minutes?

A. $10\sqrt{42}km$ B. 90 km C. $10\sqrt{41}km$ D. $10\sqrt{21}km$

98. What is the value of arcs in $\left(-\frac{\sqrt{2}}{2}\right)$?

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $-\frac{\pi}{4}$ D. $-\frac{\pi}{2}$

99. Which one of the following represents a geometric sequence?

A. $3,1,\frac{1}{3},\frac{1}{9},\frac{1}{27},\ldots$ C. 1,3,6,10,15,...

B. $\frac{1}{2},\frac{1}{3},\frac{1}{4},-\frac{1}{5},\frac{1}{6},\ldots$ D. -3,6,-9,12,-15,...

100. What is the actual value of the sum $\sum_{n=1}^{\infty}\left(\frac{2^n+5^n}{10^n}\right)$?

A. 0.325 B. 1 C. $\frac{5}{4}$ D. $\frac{37}{9}$

101. What is the sum of the series $\sum_{n=1}^{\infty}(-1)^n 3^{-2n}$

A. $\frac{-1}{8}$ B. -0.13 C. -0.1 D. $\frac{1}{8}$

102. Which one of the following is an arithmetic sequence?

A. 3, 5, 7, 9, 11... B. 3, 6, 12, 24, 48... C. -3, 6,-9, 12,-15... D. 1, 3, 6, 10, 15, 21...

103. Which one of the following sequence is a convergent sequence?

A. $1,\frac{1}{2},1,\frac{1}{3},1,\frac{1}{4},\ldots$ B. $\{(-1)^n\}_{n=1}^{\infty}$ C. $\left\{10^{109}-\frac{1}{100}n\right\}_{n=1}^{\infty}$ D. $\left\{\sin\left(\frac{1}{n}\right)\right\}_{n=1}^{\infty}$

104. A certain meeting hall has 20 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. How many seats are there on the last (20th) row of the hall?

A. 46 B. 58 C. 760 D. 5240

105. A ball is thrown vertically from ground up to a height of 16m. Each time it drops h meters, it rebounds 0.80h m. Nothing that the ball travels every height of h twice, what is the total vertical distance travelled by the ball before it comes to rest?

A. 40m B. 80m C. 160m D. 320m

106. What is the sum of all multiples of 3 between 20 and 200?

A. 7,227 B. 6,570 C. 6,150 D. 5,166

107. If $\{A_n\}_{n=1}^{\infty}$ is an arithmetic sequence such that its 1st term A_1 = -5 and its 5th term A_5 =15, then its 11th term A_{11} is equal to:

A. 40 B. 50 C. 45 D. 55

108. What is the sum of all multiples of 4 that are between 30 and 301?

A. 12,882 B. 11,288 C. 6,288 D. 6,882

109. If $f: A\to B$ and $g: B\to C$ are functions, then which one of the following is true about the composition function?

A. Domain of $(gof)\subseteq$ Domain of f C. Domain of $(gof)\not\subset$ Domain of f

B. Range of $(gof)\not\subset$ Range of g D. Range of Range of $(gof)\subseteq f$

110. The nth term of the sequence: 1, -4, 9, -16, . . . is:

A. $a_n = (-2)^n$ B. $a_n = (-1)^n n^2$ C. $a_n = (-1)^{2n} n^2$ D. $a_n = (-1)^{n-1} n^2$

111. The sum of $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ is ____________________.

A. 0 B. 15 C. $\frac{10}{3}$ D. 5

112. Which of the following expression is a polynomial expression?

A. $x^2 - 3x + \sin x$ C. $\frac{2+\pi}{1+\pi^2}$

B. $\frac{4x^3 + 12x^2 - x}{\pi x^2}$ D. $2 - 3x^{2/3} + 7x^{5/2} + 3x^{-1}$

113. The sequence $\left\{\frac{(n-1)(2n+1)}{1-n^2}\right\}_{n=1}^{\infty}$ converges to:

A. $-\infty$ B. -2 C. 0 D. 1

114. Let $f(x) = \begin{cases} a\frac{\sin x}{x-|x|}, & if x < 0 \\ e^{-x} + \cos x, & if x \geq 0 \end{cases}$ If f continuous at x=0, then what is the value of a?

A. 4 B. 2 C. $\frac{1}{2}$ D. -4

115. Which one of the following is equal to $\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{-3x}$?

A. e^6 B. e^{-3} C. $e^{-3/2}$ D. e^{-6}

116. If $a_n = \left(\frac{n+3}{n+1}\right)^n$, then the limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to:

A. 1 B. e/2 C. e^2 D. infinity

117. Let $f(x) = \begin{cases} a\frac{\sin 2x}{x}, if\ x < 0 \\ e^{2x} - 2, if\ \geq 0 \end{cases}$ If f is continuous at x = 0, then what is the value of a?

A. $\frac{1}{2}$ B. 2 C. $\frac{-1}{2}$ D. -1

118. Which one of the following is equal to $\lim_{x\to\infty}\left(\frac{3x}{3x+2}\right)^{-3x}$?

A. e^2 B. e^{-3} C. e^{-2} D. e^3

119. The left hand side limit, $\lim_{x\to 0^-}\frac{xe^x - |x|}{x}$ is equal to ____________________.

A. 0 B. 2 C. 1 D. Does not exist

120. Which one of the following is equal to $\lim_{x\to 0}\frac{2x + \tan x}{x\sec x}$?

A. 2 B. 0 C. 1 D. 3

121. In which interval the sequence $\left\{\frac{(-1)^n}{3n}\right\}_{n=1}^{\infty}$ is bounded?

A. $\left[\frac{-1}{9},\frac{1}{12}\right]$ B. $\left[\frac{-1}{3},\frac{1}{6}\right]$ C. $\left[\frac{-1}{6},\frac{1}{3}\right]$ D. $\left[\frac{-1}{12},\frac{1}{9}\right]$

122. $\lim_{x\to\infty} x\sin\frac{1}{x}$ is equal to:

A. 0 B. 1 C. ∞ D. -1

123. If $f(x)=\frac{x^2}{1+xg(x)}$, $g(2)=1$ and g'(x)=10, then which one of the following is equal to f'(2)?

A. -8 B. $\frac{-8}{9}$ C. $\frac{4}{3}$ D. $\frac{8}{9}$

124. The simplified form of the derivative of f(x)=$\frac{1+\sin x}{\cos x}$ is

A. Sec x + tan x B. $\frac{1+\sin x}{\cos^2 x}$ C. $\frac{1}{1+\tan x}$ D. $\frac{\cos x}{sin^2 x}$

125. If f(x)=e^{2x} sinx, then f"(x) is equal to

A. $3e^{2x}$ sin – $4e^{2x}$, cos x C. e^{2x}(3 sin x + 4 cos x)

B. $3e^{2x}$ sin x + $2e^{2x}$ cos x D. e^{2x} (4sinx – 3 sin x)

126. If y=sin($3x^2$), then the simplified form of $\frac{d^2y}{dx^2}$ is:

A. -6 sin ($3x^2$) C. 6 cos ($3x^2$)-$36x^2$ sin ($3x^2$)

B. Cos (6x) – 6 sin($3x^2$) D. x^2 cos ($3x^2$)+6sin ($3x^2$)

127. If $f(x)=x^2\sqrt{2x+12}$, what is the slope of the tangent line to the graph of f at x=2?

A. -4 B. 2 C. 18 D. 17

128. If $F(x)=f(2x+2).g(1-x^2)$, with f(2)=-3, f'(2)=4, g(1)=-5, and g'(1)=1, then what is the actual value of F'(0)?

A. -40 B. -20 C. 0 D. 19

129. If $f(x)=\ell n(\sqrt{x^2+1})$, which of the following is equal to f'(x)?

A. $\frac{x}{\sqrt{x^2+3}}$ B. $\frac{x}{x^2+1}$ C. $\frac{2x}{\sqrt{x^2+1}}$ D. $\frac{2x}{x^2+1}$

130. If f(x) = 2x (x^2 + 1)^4, then which of the following is an anti-derivative of f(x)?

A. $\frac{2x}{5}(x^2+1)^5+c$ C. $\frac{x}{5}(x^2+1)^5+1$

B. $\frac{2}{5}(x^2+1)^5+c$ D. $\frac{1}{5}(x^2+1)^5-1$

131. The Ozone level (in ppb – parts per billion) on a sunny day in a metropolitan area is given by the formula: p (t) = 80 + 12t – t^2, where t is time in hours and t = 0 corresponds to 9 A.M. what is the rate of increase of the ozone level after 3 hours (i.e. at 12 A.M.)?

A. 6 ppb B. 12 ppb C. 107 ppb D. 113 ppb

132. Suppose that a function f has the property that
f(x + y) = f(x) f(y) for all value of x and y and that
f (0) = 2, f'(0) = 1. Then which one of the following represents the formula for the derivative f'(x)?

A. f'(x) = 2f'(x) + 1 B. f'(x) = f(x) + 2f'(x) C. f'(x) = f(x) + 2 D. f'(x) = 2f(x) – 1

133. if f(2)=-3, f'(2) = 4, g(1) = -5, g'(1) and
f(x) = f(2x + 2). g (1-x^2), then what is the value of F' (0)?

A. 19 B. 0 C. -20 D. -40

134. For what values of a and b is the function $\begin{cases} f(x) = 1 - 3x^2, \text{for } x \le 1 \\ \text{ax} + \text{b, for x} > 1 \end{cases}$ differentiable at x = 1?

A. a = 6, b = 0 B. a = -3, b = 1 C. a = 0, b = -2 D. a = -6, b =1

135. If $f(x) = 2x^5 - 3x$, then $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ is equal to ________________.

A. 1 B. -1 C. 7 D. ∞

136. If $f(x) = e^{3x} \cos x - \frac{x + \pi}{x^2 + 2}$, then $f'(0)$ is equal to _________________ .

A. $3 - \frac{\pi}{2}$ B. $\frac{3}{2}$ C. $\frac{7}{4}$ D. $\frac{5}{2}$

137. If $f(x) = \text{In}\left(\sqrt{x^2 - 5}\right)$, which one of the following is equal to $f'(x)$?

A. $\frac{x}{x^2 - 5}$ B. $\frac{-x}{\sqrt{x^2 - 5}}$ C. $\frac{2x}{\sqrt{x^2 - 5}}$ D. $\frac{-x}{x^2 - 5}$

138. $\frac{d}{dx}\left(Ine^{2x}\right)$ is equal to:

A. $\frac{1}{e^{2x}}$ B. $\frac{2}{e^{2x}}$ C. 2x D. 2

139. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at x= 3 is __________.

A. -1 B. does not exist C. 1 D. 2

140. The total cost (in Birr) of producing x iron sheets per day is c(x)=1,000 + 100x – 0.5x2, 0≤ x ≤ 100. What is the marginal (rate of change of) cost at a production level of 80 iron sheets?

A. 8.5 B. 20 C. 1,800 D. 5,800

141. A water tank is a circular cylinder with base radius 2m and height 3m. if the tank is empty and water is pumped into it a rate of 2m^3/min, how long does it take for the tank to be full?

A. 1.5 min B. $\frac{3}{2}\pi$ min C. 6π min D. 12 min

142. Which one of the following is the set of all critical numbers of $f(x) = \frac{1}{3}x^3 - |4x - 1|$?

A. $\left\{\frac{1}{4}, 2\right\}$ B. $\left\{-2, \frac{1}{4}, 2\right\}$ C. {-2,2} D. $\left\{\frac{1}{4}\right\}$

143. If a box with square base and open top is made from 1,200 cm^2 material, what is the largest volume of the box in cm^3?

A. 4,000 B. 8,000 C. 15,000 D. 3,000

144. An object is moving along the parabola $y = \sqrt{2x}$ in xy-plane. At what point on its path does the object becomes closest to the point (2, 0)?

A. $\left(3, \sqrt{6}\right)$ B. (1,1) C. $\left(3, \sqrt{2}\right)$ D. (2,2)

145. What is the absolute maximum value of f(x)=2x^2 - x^4 - 4 on [0,2]?

A. -3 B. 3 C. -4 D. 12

146. A leader 6 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rat (speed) of ½ m/sec, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ rad?

A. $\frac{\sqrt{2}}{12}$ rad/sec B. $\frac{\sqrt{2}}{2}$ rad/sec C. $\frac{\sqrt{2}}{6}$ rad/sec D. $\frac{\sqrt{2}}{3}$ rad/sec

147. Which one of the following is necessarily true?

A. If f'(x) = 0 for all x in an interval I, then f(x) = 0 for all x in I.

B. If f(x) = x^2 sin x + 5, then there is c $\in$ (0, π) such f'(c)=0.

C. If f (x) = ex + x^2 is increasing on ($-\infty,\infty$).

D. If f'(c) = 0, then f attains its maximum or minimum value at x = c.

148. Let $f(x)=\begin{cases}3-e^{2x}, if x<0.5\\ \dfrac{2^x-5}{x+1}, if x\geq 0.5\end{cases}$. If c is a zero of f, that is, f(c)=0, then which one of the following intervals must contain c?

A. $(-\infty,0]$ B. [0,1] C. [1,2] D. [2,3]

149. A company manufactures x computer sets per month. The monthly marginal profit (in Birr) is given by: P'(x) = 165 – 0.1x, for 0 $\leq$ x $\leq$ 400.

The company is currently manufacturing 10 sets of computers per month, but is planning to increase production. What is the total change in the monthly profit if the monthly production increased to 60 sets?

A. Birr 500. B. Birr 1,865. C. Birr 8,075. D. Birr 18,635

150. The total cost (in Birr) of producing x radio sets per day is given by the expression ¼ x^2 + 35x + 25 and the price set at which they may be sold is given by 50 – ½ x. what should be the daily output to obtain a maximum total profit?

A. 50 sets per day B. 23 sets per day C. 10 sets per day D. 7 sets per day

151. The number of shoes s that a factory can produce per day is a function of the number of hours t it operates: s(t) = 40t for 0 $\leq$ t $\leq$ 12.

The daily cost c, in Birr, to manufacture s shoes is given by the function c(s) = $0.1s^2$ + 90s + 800.

If the factory operates for 10 hours, what is the cost it incurs in production as much shoes it can within this time?

A. Birr 400 B. Birr 1,600 C. Birr 52,800 D. Birr 124,600

152. Let f be twice differentiable function on **R**. Which one of the following is necessarily true?

A. If $f'(c)=0$, at some $C\in$ **R**, then f has a relative extreme value at x = c.

B. If $f'(x)$ is increasing, the n the graph of y = $f(x)=0$ $x\in$ **R**.

C. If $f'(x)=0$ for all $x\in$**R** then $f(x)=0$ for all $x\in$ **R**.

D. If $f(x)$ is increasing, then $f''(x)\geq 0$ for $x\in$ **R**.

153. A closed cylindrical can is to be made to hold 1000 cm3 of oil. What are the dimensions (radius r and height (h) that will minimize the total surface area of the can?

A. $r=\dfrac{\sqrt[3]{50}}{\pi}$, $h=2\dfrac{\sqrt[3]{50}}{\pi}$ C. $r=\dfrac{\sqrt[3]{50}}{\pi}$, $h=2\sqrt[3]{\dfrac{50}{\pi}}$

B. $r=\dfrac{\sqrt[3]{500}}{\pi}$, $h=2\dfrac{\sqrt[3]{500}}{\pi}$ D. $r=\dfrac{\sqrt[3]{500}}{\pi}$, $h=2\sqrt[3]{\dfrac{500}{\pi}}$

154. The graph of y = 5x4 – x5 has a point of inflection at:

A. (3, 162) only B. (4, 256) only C. (0, 0) only nD. (0, 0) and (3, 162)

155. Which one of the following is true about the function f defined by $f(x)=x^2+e^{2x}$?

A. f is decreasing for $x\geq 0$ C. f has a relative minimum at $x=0$

B. f is increasing for $x\geq 0$ D. f has a relative maximum at $x=0$

156. If F(x) is an antiderivative of f(x)=$1-\dfrac{2}{x^2}$ and F(1)=0, then $F'(2)$ is equal to:

A. 0 B. $\frac{1}{2}$ C. $\frac{-1}{2}$ D. 3

157. Which one of the following is equal to $\int_0^{\pi/2} \frac{x - \sin x}{\sec x} dx$?

A. $\frac{\pi - 3}{2}$ B. $\frac{\pi - 1}{2}$ C. $\frac{3 - \pi}{2}$ D. $\frac{\pi + 3}{2}$

158. What is the area of the region between the graphs of $y=-x^2 + 2$ and $y = |x|$, where $-1 \le x \le x$?

A. $\frac{11}{6}$ B. $\frac{25}{6}$ C. $\frac{7}{6}$ D. $\frac{11}{3}$

159. If Q_i , D_i and P_i are respectively the i^{th} – quartile, decile and percentile of a data arranged in an increasing order, then which one of the following is necessarily true?

A. $Q_2 = \frac{Q_1 + Q_2}{2}$ B. $D_3 > P_{25}$ C. $P_{25} > Q_1$ D. Q_2 = mean of the data

160. Which one of the following is equal to $\int \frac{x + \ell n(x+1)}{(x+1)^2} dx$?

A. $\ell n(x+1) + \frac{x}{x+1} + c$ C. $(x+1)^2 - \frac{1}{x+1} + c$

B. $(x+1)^2 + \frac{1}{x+1} + c$ D. $\frac{x \ell n(x+1)}{x+1} + c$

161. What is the area of the region between the graph of $f(x) =-x^2+4x-3$ and the x-axis from x=0 to x=3?

A. $-\frac{2}{3}$ B. $\frac{2}{3}$ C. $\frac{4}{3}$ D. $\frac{8}{3}$

162. What is the area of the region between the graphs of y = sin x and x – axis where $0 \le x \le 2\pi$?

A. 4 B. 4π C. 2 D. 2π

163. Which one of the following is equal to $\int \frac{\ell nx + x^2 e^x}{x} dx$?

A. $\frac{1}{2} \ell n^2 x + e^x (x^2 - 1) + c$ C. $\frac{1}{x^2} \ell nx + e^x (x-1) + c$

B. $\frac{1}{2} \ell n^2 x + e^x (x \quad -1) + c$ D. $\frac{-1}{x^2} \ell nx + e^x (x-1) + c$

164. If $f(x) = 3x^2 \sqrt{x^3 - 1}$, then which of the following is an anti-derivative of f(x)?

A. $\frac{3x}{2}(x^3 - 1)^{3/2} + c$ C. $\frac{3}{2}(x^3 - 1)^{3/2}$

B. $\frac{2}{3}(x^3 - 1)^{3/2}$ D. $\frac{3}{2}(x^3 - 1)^{2/3} + c$

165. Which of the following is equal to $\int \frac{(Inx)^2 + x^2 \cos x}{x} dx$?

A. $\frac{1}{x^2} Inx + x\sin x - \cos x + c$ C. $\frac{1}{3}(Inx)^3 + x\sin x + \cos x + c$

B. $\frac{1}{3}(Inx)^3 + x\sin x - \cos x + c$ D. $\frac{1}{x^2} Inx + x\sin x + \cos x + c$

166. The volume of the solid generated when the region bounded between the graph of $y=\begin{cases}x^2, 0\le x\le 2\\ 4,\ 2\le x\le 3\end{cases}$ and x-axis is rotated about the x-axis is:

A. $\frac{32\pi}{5}$ B. $\frac{112\pi}{5}$ C. $\frac{112\pi}{3}$ D. $\frac{64\pi}{5}$

167. The value of $\int_0^3 (x+1)e^{(x^2+2x)}dx$ is:

A. $\frac{e^3}{2}$ B. $\frac{e^4-e}{2}$ C. $\frac{e^3-1}{2}$ D. $e^3 - 1$

168. $\int_0^3 (x+1)^{\frac{1}{2}} dx$ = ____________________.

A. $\frac{21}{2}$ B. $\frac{14}{3}$ C. 7 D. $\frac{16}{3}$

169. Given $f(x)=\begin{cases}x+1,\ for x<0\\ \cos\pi x,\ for x\ge 0\end{cases}$, then $\int_{-1}^{1} f(x)dx$ = ________________________.

A. $\frac{1}{2}+\frac{1}{\pi}$ B. $\frac{1}{2}-\frac{1}{\pi}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$

170. Which one of the following is equivalent to $\neg[(\forall x)(p(x)\Rightarrow Q(x))]$?

A. $(\forall x)(\neg P(x)\Rightarrow \neg Q(x))$ C. $(\exists x)(\neg P(x)\wedge Q(x))$

B. $(\exists x)(\neg P(x)\Rightarrow \neg Q(x))$ D. $(\exists x)P(x)\wedge Q(x))$

171. Suppose P and Q are points in space such that the midpoint of $\overline{PQ}$ is on the negative z-axis and the distance between P and Q is 6. If P=(2,-1,0), then what is the coordinate of Q?

A. (-2,1,4) B. (2,-1,6) C. (2,-1,-6) D. (-2,1,-4)

172. If P= (3, α - 1, α + 2) and Q = (2α + 1, 3, 3α) are points in space, what should be the value (s) of α so that the distance between the two points is 6?

A. α = -2 or α = 5 B. α = 0 or α = 5 C. α = - 1 or α = 3 D. α = - 3 or α = 2

173. If (-1, 2, 2) and (1, 0, -2) are endpoints of a diameter of a sphere, then which one of the following is true about the sphere?

A. (0,1,0) is a point on the sphere

B. The equation of the sphere is $x^2 + (y-1)^2 + z^2 = 6$

C. The equation of the sphere is $x^2 + (y-2)^2 + z^2 = 6$

D. The radius of the sphere is 6.

174. Suppose ℓ is the line through the center of the sphere $x^2 + y^2 = (z-2)^2 = 9$ and intersects sphere at (1, 2, 4). What is the cosine of the angle between ℓ and positive z – axis?

A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$

175. Suppose that the equation $x^2 + y^2 + z^2 + 2x + 8z = 6(y + 1)$. Represents a sphere. Where is the point (1, -1, 4) located relative to the sphere?

A. Inside the sphere, C. At the center of the sphere

B. On the sphere D. Outside the sphere

176. Let the angle between $\vec{V} = -2\vec{i} - \vec{j} + 2\vec{k}$ and $\overrightarrow{PQ}$ be 600, where P and Q are points in space. If $\vec{V} \bullet \overrightarrow{PQ} = 2$, then what is that distance between P and Q?

A. $\frac{3}{4}$ B. $\frac{4}{5}$ C. $\frac{4}{3}$ D. $\frac{5}{4}$

177. If one of the end point of the line segment is (3, 2, -4) and the mid-point is (4, 1, -2), then the coordinate of the other end point is:

A. (5, 0, 0) B. (2, 0, 5) C. (5, 1, 2) D. (3, 1, 0)

178. The following is an assertion of a person and his proof.

“For any natural numbers n, $n! < 10^n$.

Proof:

Step 1. Let N = 1. Since 1! = 1 and $10^1 = 10$, it is true that $1! < 10^1$.

Step 2. Let N = 2. Since 2! = 2 and $10^2 = 100$, it is true that $2! < 10^2$.

Step 3. Let N = 3. Since 3! = 6 and $10^3 = 1000$, it is true that $2! < 10^3$.

Step 4. Continuing in this manner, we can see that whenever

$K! < 10^k$ is true, then $(K + 1)! < 10^{k+1}$ is also true.

Therefore, by induction, $n! < 10^n$ for all natural numbers.”

Which one of the following is true about the proof?

A. The proof is correct by the principle of mathematical induction, though step 2 and step 3 can be omitted.

B. The proof is correct by the principle of mathematical induction, and step 2 and step 3 are necessary since they provide additional information.

C. The proof is invalid because step 4 did not justify the desired induction step.

D. The proof follows the technique of a proof by exhaustion.

179. If each of the compound propositions $P \vee Q$, $P \Rightarrow R$ and $\neg R$ is true, then which one of the following is true?

A. P B. Q C. $Q \Rightarrow P$ D. $P \wedge \neg R$

180. What is the contra positive of “If $x \in$ N, then x is integer and $x > 0$”?

A. If x is not integer or $x < 0$, then $x \notin$ N.

B. If x is integer and $x > 0$, then $x \in$ N

C. If x is not integer or $x \leq 0$, then $x \notin$ N

D. If $x \notin$ N, then x is not integer and $x \leq 0$

181. Which one of the following is a valid assertion that can be proved y the principle of mathematical induction?

A. $2^n > 10n$ for every integer n such that $n \geq 6$.

B. $r^2 > 0$ for every real number r such that $r \geq 1$

C. $n^2 > 10n > 2n^2$ for every natural number $n \geq 1$

D. $2^n > 8n$ For every integer n such that $n \geq 3$

182. Consider the following assertion of a person and his proof. “If x and y are equal positive integers, then $x+y=y$.”

Proof: the following steps and reasons are used to proof the assertion.

Step	Reason
1. $x=y$	Given hypothesis
2. $x^2=xy$	Multiply both sides of (1) by x
3. $x^2-y^2=xy-y^2$	Subtract y^2 from both sides of (2)
4. $(x-y)(x+y)=(x-y)y$	Factor both sides of (3)
5. $x+y=y$	Divide both sides of (4) by x-y

Step 5 completes the proof

A. It is a correct direct proof of the assertion.
B. It follows the technique of a proof by contradiction because the steps lead to a contradiction.
C. The proof is invalid because Step 4 does not lead to Step 5.
D. The proof is invalid because Step 4 does not follow from step 3.

183. Which one of the following types of tax does not belong to indirect tax in Ethiopia?
A. Excise tax C. Employment income tax
B. Turnover tax D. Value added tax (VAT)

184. The compulsory payment by individual and companies for the state is called…..
A. Taxation B. Loan C. Asset D. Investment

185. The decrease in usefulness in a business expense is called……
A. Depreciation B. Taxation C. Annuity D. Asset

186. Which one of the following is not long term investing strategy?
A. Bonds B. Stocks C. Certificate of Deposit (CD) D. Mutual funds

187. In a class there are 23 girls and 18 boys. What is the ratio of girls to the total number of student in the class?
A. 23:18 B.18:41 C. 23: 41 D. 18: 23

188. If A’s age to B’s age is in the ratio of 5:8. In 9 years’ time the ratio of their ages will be 8:11, the age of A is:
A. 15 years B. 24 years C. 25 years D. 14 years

189. What should be the rate of interest per year, so that invested money will double itself in 1 year compounded annually?
A. 100% B. 120% C. 60% D. 150%

190. Which types of loan represents mutual financial organizations which are owned and run by their members for their members?
A. Unsecured loan B. Money lines C. Secured loan D. Credit union loan

191. How much should be deposited now if you require Birr 110, 250 at the end of 1 year and the interest rate is 10% and payable semiannually?
A. 110, 000 birr B. 110, 000 birr C. 105, 000 birr D. 100, 000 birr

192. If markup on selling price is 35%, and the selling price is birr 10,000, then what will be the amount of the cost price?
A. Birr 3,500 B. birr 10,000 C. birr 35,000 D. birr 6,500

193. What percent of Birr 1200 is Birr 128.50?
A. 10% B. 17% C. 10.71% D. 71%

194. Barnabas buys a shoe with birr 2,500. If he wants to buy another 3 shoes of the same standard for his friends, how much should he pay?

A. Birr 5,700 B. birr 8,000 C. birr 7,500 D. birr 2,500

195. Falcon academy has donated birr 180,000 in 2007 E.C for a class trip. If the school donated birr 250,000 in the year 2012 E.C, what is the rate of change of amount donated?

A. 40% B. 38.9% C. 35.88% D. 39.8%

196. Birr 12,500 is invested with a simple interest rate 8% per month. What is the amount of future value at the end of the year?

A. 12,000 B. 13,500 C. 24,500 D. 20,000

197. Which one of the following is not categorized among the major financial institutions?

A. Credit unions C. Commercial banks

B. Ekub D. Saving and loans associations

198. A profit of Birr 12,000 is divided among four partners in the ratio 3:2:1:6, how much should the forth partner receives?

A. Birr 3,000 B. birr 3,500 C. birr 5,500 D. birr 6,000

199. The price of 10 same standard mobile phones is birr 110,000. What is the price of each mobile phone?

A. Birr 10,000 B. birr 9,500 C. birr 1,100 C. birr 2,000

200. John deposit birr 10,000 in a bank at annual interest of 10%. Which one of the following is the amount of money after 2 years?

A. Birr 12,100 B. birr 12,000 C. birr 13,000 D. birr 11,200