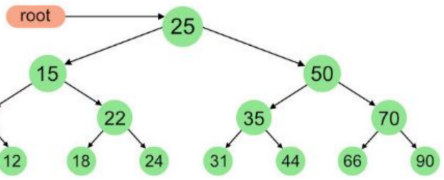
Binary Search Tree (BST)

- each node contains a quintuple
    - an index
    - a key
    - pointers to its left, right child, and parent
- all keys in the left subtree of x should be less than or equal to that of x
    - and all in right subtree should greater than or equal to that of x
- search, insert, delete, predecessor, successor, minimum, maximum operations are all $O(h)$ where h is the height of the tree
- in a standard BST, h is determined by the order of inserting n items
    - the best case $h = n \lg n$
    - the worst case $h = n$

Tree Traversals



In Order

```
4, 10, 12, 15, 18, 22, 24, ...
```

1. left subtree
2. root
3. right subtree

Pre-Order

```
25, 15, 10, 4, 12, 22, 50, 35, 31, 44, 70, 66, 90
```

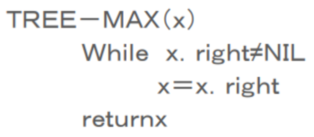
1. root
2. left subtree
3. right subtree

Post-Order

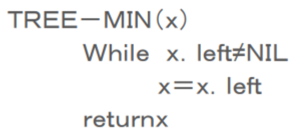
```
4, 12, 10, 18, 24, 22, 15, 32, 44, 35, 66, 90, 70, 50, 25
```

1. left subtree
2. right subtree
3. root

Searching

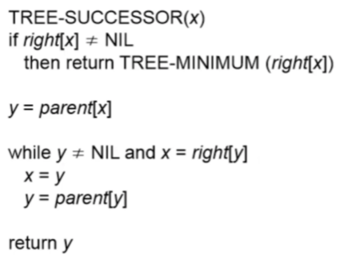


```
TREE−MAX(x)
        While  x. right≠NIL
                x=x. right
        returnx
```



```
TREE−MIN(x)
        While  x. left≠NIL
                x=x. left
        returnx
```

Successor



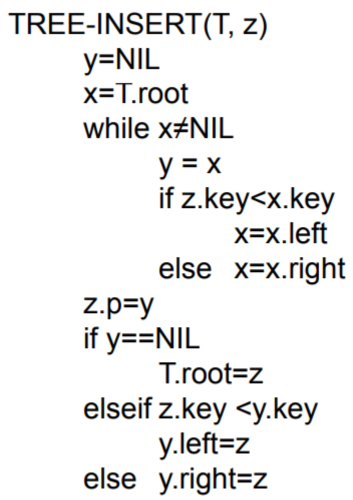
```
TREE-SUCCESSOR(x)
if right[x] ≠ NIL
   then return TREE-MINIMUM (right[x])

y = parent[x]

while y ≠ NIL and x = right[y]
   x = y
   y = parent[y]

return y
```

Insert

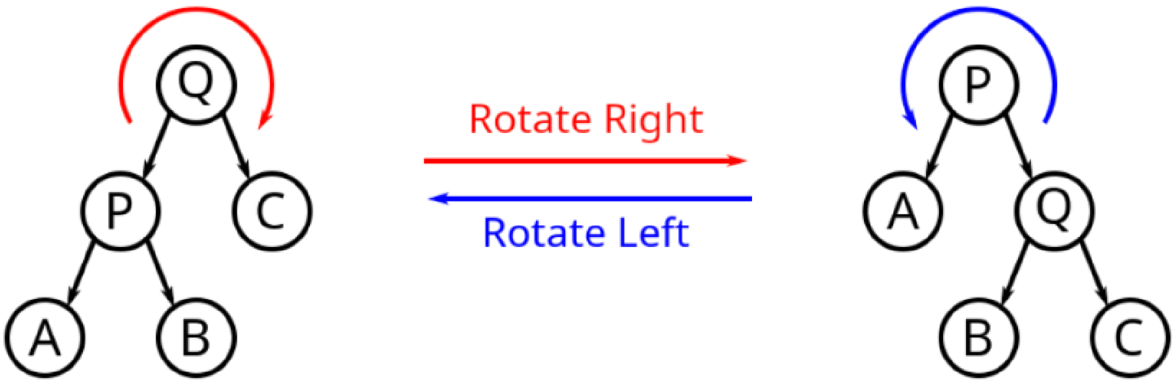


```
TREE-INSERT(T, z)
        y=NIL
        x=T.root
        while x≠NIL
                y = x
                if z.key<x.key
                        x=x.left
                else   x=x.right
        z.p=y
        if y==NIL
                T.root=z
        elseif z.key <y.key
                y.left=z
        else   y.right=z
```

Delete

1. `z` has not children
   - just remove `z`
2. `z` has 1 child
   - replace `z` with its child
3. `z` has 2 children
   - replace `z` with its successor

Rotation



```
# Right rotation pseudocode
function rightRotate(y):
    x = y.left
    T = x.right
    # Perform rotation
    x.right = y
    y.left = T
    return x

# Left rotation pseudocode
function leftRotate(x):
    y = x.right
    T = y.left
    # Perform rotation
    y.left = x
    x.right = T
    return y
```