

0-1 Knapsack

- a thief is robbing a store with n items
- each item i is worth z_i dollars and has weight w_i where z_i, w_i are integers
- the thief can only carry W weight and can't take fractional amounts of items (i.e. 0-1 or "leave" or "take")
- what items should the thief take to maximize his haul's value?

Trying Greedy Choice

Using a greedy algorithm will not work for 0-1 knapsack. You'll need DP.

Max value first

This has easy counter examples.

Min weight first

This has easy counter examples.

Max value per weight first

- this is a.k.a. the unit price
- this has easy counter examples
- however, the *fractional* knapsack problem can be solved using this greedy choice

Proving the greedy unit price choice works for *fractional* knapsack

- assume we have an optimal solution A to S that doesn't have our greedy choice
- take a that has the maximum unit price in A and replace it with s that has the maximum unit price in S
- this gives us 3 possible cases:
 1. s 's weight is equal to a 's weight in A giving a solution with a greater value
 2. s 's weight is less allowing us to fill the rest of missing weight with a fraction of a 's again giving a solution with a greater value
 3. s 's weight is more allowing us to take a fraction of s 's again giving a solution with a greater value
- all cases lead to a contradiction that A is an optimal solution