Understanding NP:

- **P:** A class of problems that can be solved by an algorithm in polynomial time.
- **NP:** Stands for "nondeterministic polynomial time." It includes problems for which a solution can be verified in polynomial time, even if finding the solution might take longer.

NP-Hard Problems:

A problem is NP-hard if solving it efficiently would also allow us to solve all NP problems efficiently. In other words, every NP problem can be reduced to an NP-hard problem in polynomial time.

Proving a Problem is NP:

1. **Show the Problem is in NP:**
    - To show a problem is in NP, demonstrate that any proposed solution can be verified in polynomial time.
    - For example, for the **Traveling Salesman Problem (TSP)**, given a route, it can be checked in polynomial time whether the route visits each city exactly once and returns to the starting city.
2. **Reduction to an NP-Complete Problem:**
    - A problem is **NP-complete** if it is both in NP and every problem in NP can be reduced to it in polynomial time.
    - To show a problem is NP-complete, show how an existing NP-complete problem can be reduced to it.
    - For example, reducing the **3-SAT** problem (which is NP-complete) to another problem can be used to demonstrate that the latter is also NP-complete.
3. **NP-Hard without being in NP:**
    - Some problems might be NP-hard but not in NP, especially if they can't be verified in polynomial time.
    - For example, the **Halting Problem** is NP-hard but not in NP.

Practical Examples:

1. **3-SAT:** A boolean satisfiability problem where you determine if there is an assignment that satisfies a boolean expression in conjunctive normal form with 3 literals per clause. It's an NP-complete problem.
2. **Subset Sum:** Given a set of integers and a target sum, determine if there is a subset of integers that sums to the target. It's NP-complete.

How to Prove NP-Completeness:

1. **Problem Verification:** Show that any proposed solution can be verified in polynomial time.
2. **Reduction:** Choose an existing NP-complete problem and show how it can be transformed into the new problem in polynomial time. This demonstrates that solving the new problem would also solve all NP problems.