Evaluating the Approximation of the Affinity Laws and Improving the Efficiency Estimate for Variable Speed Pumps

Article *in* Journal of Water Resources Planning and Management · December 2013

DOI: 10.1061/(ASCE)HY.1943-7900.0000776.

2 authors:

Angela Marchi
University of Adelaide

Angus R. Simpson
University of Adelaide

Evaluating the approximation of the affinity laws and improving the estimate of the efficiency for variable speed pumping

by

Marchi, A. and Simpson, A.R.

Journal of Water Resources Planning and Management

Citation:
Marchi, A. and Simpson, A.R. (2013). "Evaluating the approximation of the affinity laws and improving the estimate of the efficiency for variable speed pumping" *Journal of Water Resources Planning and Management*, Dec., Vol. 139, No. 12, 1314-1317

For further information about this paper please email Angus Simpson at angus.simpson@adelaide.edu.au

Evaluating the approximation of the affinity laws and improving the estimate of the efficiency for variable speed pumping

Angus R. Simpson

Professor, M.ASCE, School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, 5005, Australia

angus.simpson@adelaide.edu.au

Angela Marchi

Research Associate, School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, 5005, Australia

angela.marchi@adelaide.edu.au

Abstract

Affinity laws relate to the characteristics of pumps operating at different speeds, and in a water distribution context are usually used to predict the pump curve of variable speed pumps (VSPs). VSPs can adjust the pump curve so as to meet the network requirements more efficiently with resultant savings of energy. The estimation of the effectiveness of a VSP is based on hydraulic simulations, where the behaviour of VSPs is described using the affinity laws. The affinity laws, however, contain approximations, as they do not take into account factors that do not scale with velocity. In particular, the approximation inherent in the affinity law that computes power and efficiency can produce a misleading result, especially for small size pumps. The research reported in this paper estimates the error in efficiency for a wide range of pump sizes and tests the use of the formula proposed by Sarbu and Borza (1998) as

an alternative to the affinity law. Results show that a better estimation can be achieved for the efficiency of small and medium size pumps. Moreover the formula can be easily implemented in hydraulic solvers.

Keywords: efficiency; variable speed pump; water distribution system; EPANET.

Introduction

In water distribution systems (WDSs), the component that consumes the most energy is the pump (Carlson & Walburger, 2007), and a reduction in energy use in the system can only be achieved by reducing pump inefficiencies and power demands. System designers wanting to reduce power consumption and costs, therefore, often evaluate the possibility of replacing fixed speed pumps with variable speed pumps (VSPs). VSPs have motors linked to a variable speed drive so that the motor can be run at different speeds, resulting in a change of the characteristic pump curve. Therefore, with VSPs, the pump operating point can be more efficiently adapted to the system requirements than with standard fixed speed pumps (Lingireddy and Wood 1998).

The estimate of the energy savings achievable with VSPs is usually accomplished by using hydraulic solvers to simulate WDS and pumping system behavior. Although the energy consumption has to be estimated by taking into account the efficiency of all VSP components (pump, motor and variable speed drive) (Bernier and Bourret, 1999; Walski, 2001; Walski et al. 2003; Walski, 2005; Marchi et al. 2012a, Marchi et al. 2012b) , in the initial assessment, only the pump power is usually considered, because of the difficulty in finding motor and variable speed drive data and because of the lower efficiency of pumps compared to the other VSP components (motors and variable speed drives often have efficiencies as high as or greater than 95%). Note that, despite this, motor and variable speed drive efficiencies affect the wire to water efficiency, and cannot be neglected in the computation of the total power and energy required. Moreover, the effectiveness of VSPs has to be assessed for each specific WDS, taking into account water distribution system requirements, costs and savings throughout the design life of the intervention.

This paper focuses on the efficiency of the pump in a VSP system. When the speed is changed, the pump efficiency is usually estimated using the efficiency-flow curve at the nominal pump speed and the affinity laws, which describe the relationship between the variables involved in pump performance, such as flow, total head and power, and the pump speed. However, affinity laws contain approximations that can affect the estimation of pump efficiency. This paper highlights the magnitude of this approximation and proposes the use of a formula presented by Sarbu and Borza (1998) as an alternative method for estimating the efficiency of VSPs.

Affinity laws for VSPs

The affinity laws are commonly used to describe pump behaviour (flow, head and power) when pump speed is changed. The laws reflect the fact that dimensionless characteristics (dimensionless flow, C_Q, dimensionless head, C_H, and dimensionless power, C_P) are constant for similar pumps. Dimensionless pump characteristics presented in Eq. 1 directly relate flow, Q (m^3/s), head, H (m), and power, P (W) to the speed, N (rpm) and to the impeller diameter, D (m), of the pump.

a) $$C_Q = \frac{Q}{N \cdot D^3}$$ b) $$C_H = \frac{gH}{N^2 \cdot D^2}$$ c) $$C_P = \frac{P}{\rho \cdot N^3 \cdot D^5}$$ (Eq. 1) where ρ is the liquid density (kg/m^3) and g is the acceleration of gravity (m/s^2).

The efficiency, η, is indirectly described by Eq. 1c as the power, P, is

$$P = \frac{\rho \cdot g \cdot Q \cdot H}{\eta} \qquad \text{(Eq. 2)}$$

For variable speed pumps, only the pump speed (and not the impeller diameter) is modified, thus the following equations, known as affinity laws, can be derived:

$$\text{a)} \quad \frac{Q_1}{Q_2} = \frac{N_1}{N_2} \qquad \text{b)} \quad \frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \qquad \text{c)} \quad \frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3 \qquad \text{(Eq. 3)}$$

where subscripts 1 and 2 refer to two different pump speeds.

The affinity laws state that the change in pump flow, head and power is a linear, squared and cubic function of the change in pump speed, respectively. In particular, equations 1c and 3c assume that a point on the pump curve maintains the same efficiency when the pump is run at different speeds as shown in Figure 1. Hence the efficiency of the best efficiency point (BEP) is constant. Figure 1 also shows that the difference between the use of Eq. 1c or 3c when the pump speed is changed is that in a graph η-C_Q the efficiency points of a pump operated at different speeds lie on the same curve, while in a graph η-Q the curve is scaled according to Eq. 3a.

Approximations in the affinity laws

As mentioned, the affinity laws relate flow, head and power (and hence efficiency) to the speed of the pump. However, they cannot take into account the factors that do not scale with velocity and whose magnitude depends on the machine size, and must, therefore, rely on approximations. For centrifugal pumps, Eqs. 3a and 3b are a good approximation of real pump behavior for a wide range of speeds, and the impact of the factors affected by the machine size can be easily neglected. In contrast, the approximation in the power and efficiency relationships is larger, especially for smaller pumps.

Table 1 shows the differences between the measured best efficiency point (BEP) and the efficiency predicted by the affinity laws for pumps with different power specifications. As can be seen, for the large HPL 54-30-20 Sulzer pump (data from Ulanicki et al. 2008) reducing the speed from N_1 = 1525 rpm to N_2 = 1182 rpm (relative speed=0.775) does not decrease appreciably the efficiency (BEP at N_1 = 83.6%, while BEP at N_2 = 83.5%). However, this is a large pump that reaches 556 kW of power at the higher flow. If a small pump is considered, the decrease in efficiency is greater. For example, for the small pump 50x32-160H.T. (5.5 kW), the affinity laws predict a BEP equal to 56% instead of the 52% measured for the BEP at 2000 rpm.

Improving the efficiency estimation

The decrease in efficiency when the pump speed is lowered has been reported by several authors. Morton (1975) proposed a graphical method to identify the new curve, while Sarbu and Borza (1998) related the new efficiency to the original efficiency and the speed. More general approaches for tackling the problem of efficiency scaling previously considered that the efficiency is a function of the Reynolds number ($Re = VD/v$, where V is the velocity of the fluid (m/s), D is the diameter (m) and v is the kinematic viscosity (m^2/s)) and the interior roughness of the pipe. As reported by Gűlich (2003), where a comprehensive review of the past works can be found, one of the earliest methods to scale the efficiency of similar pumps was formulated by noting that only a part of the energy losses, K, is a function of the Reynolds number:

$$\frac{1-\eta_2}{1-\eta_1} = K + (1-K)\left(\frac{Re_1}{Re_2}\right)^m \tag{4}$$

where η_1 and Re_1 are the efficiency and the Reynolds number at the speed N_1 and η_2 and Re_2 are the efficiency and the Reynolds number at the reduced speed N_2. By assuming that all losses are independent of Re (K=0) and the value of m is equal to 0.1, Eq. 4 can be rewritten as the Sarbu and Borza formula (Eq. 5). Note that the ratio of Reynolds numbers in Eq. 4 (Re_1/Re_2) is replaced by the ratio of the pump speeds (N_1/N_2) as the velocity of the fluid is proportional to the pump speed, while diameter and viscosity do not change.

$$\eta_2 = 1 - (1-\eta_1)\cdot\left(\frac{N_1}{N_2}\right)^{0.1} \tag{5}$$

According to Eq. 5, the efficiency decrease is greater for a large reduction of speed and further decreases the efficiency at already low efficiency points on the pump curve. Although the approach presented in Eq. 4 and 5 is limited by the fact that the exponent m depends on the Reynolds number and the roughness (Osterwalder, 1978; Gűlich, 2003), the Sarbu and Borza formula can provide reasonably good results if the pump speed is not reduced below 70% of the nominal speed.

Figure 2 shows the efficiency prediction using Eq. 5 for a wide range of efficiency points and speeds. It can be seen that the predicted efficiency can be nearly equal to the original one if the point has a high efficiency and the relative speed is above 0.7 as suggested by Sarbu and Borza (1998). In contrast, if the efficiency is below 0.5 already, the formula predicts a lower efficiency even for a small reduction in speed. The formula can also produce negative efficiencies. However, these results are located in low efficiency regions of the pump ($\eta<0.3$) and for small relative speeds ($N_1/N_2<0.40$) that are usually avoided in practice.

Figure 3 compares the data for the small pump 50x32-160H.T. (5.5 kW), the curves of which were taken from the Thompsons Kelly and Lewis catalogue (1989), and the prediction using the affinity laws and the Sarbu and Borza (1998) relation. A reduced speed of 2000 rpm (0.56 of the nominal speed equal to 3600 rpm) has been chosen to better visualize the differences. As can be seen, the affinity laws can predict well the *Q*-*H* relation and the small differences can be attributed to errors in retrieving the actual data. However, the prediction of efficiency is appreciably inaccurate. The pump does not maintain the same efficiency. In particular, the BEP at 2000 rpm is 52% instead of the 56% obtained with the affinity laws. Efficiency results using Eq. 5 are unequivocally a better approximation than the affinity law.

Although the prediction of efficiency still contains some approximation, the use of the Sarbu and Borza formula (Eq. 5) instead of the affinity law related to power, enables a more accurate estimation of efficiency for a wide range of pump speeds. The formula overestimates the efficiency only for small pumps. For medium-large pumps, it can predict well the BEP (see Table 1, 125x80-250 Hydro-Titan 90 kW) and, only for very large pumps, does it underestimate the BEP and therefore estimate a larger pump power.

Finally it should be noted that Eq. 5 can be easily implemented in hydraulic solvers, as it requires the same input data as those used by the affinity laws: the efficiency at the nominal speed, η_1, and the inverse of the pump relative speed. Appendix A shows how to modify the popular hydraulic solver EPANET 2 source code (Rossman, 2000) to compute VSP efficiency at the reduced speeds using the Sarbu and Borza relationship.

Conclusion

Assessing the cost effectiveness of variable speed pumps requires, amongst other factors (e.g. estimating motor and variable speed drive efficiency), the estimate of the efficiency of the pump at reduced speeds. This task is usually accomplished by using the affinity laws. However, the affinity laws cannot take into account the effects of factors that do not scale with velocity, the efficiency computed using the affinity laws unavoidably contains an approximation. The use of the Sarbu and Borza (1998) relationship to estimate the pump efficiency of VSPs is proposed in order to decrease the error in the efficiency estimation, especially for small pumps. As this formula requires the same input data as that used by the affinity laws, it can be easily implemented in hydraulic solvers: the modification of the EPANET 2 solver has been presented.

References

Bernier, M. A., and Bourret, B. (1999) "Pumping energy and variable frequency drives," *ASHRAE Journal*, December, 37-40.

Carlson, S. and Walburger, A. (2007) "Energy Index Development for Benchmarking Water and Wastewater Utilities", AwwaRF, Denver.

Gűlich, J.F. (2003). "Effect of Reynolds number and surface roughenss on teh efficiency of centrifugal pumps", Journal of Fluid Engineering, 125(4), 670-679.

Lingireddy, S., and Wood, D. J. (1998) "Improved operation of water distribution systems using variable-speed pumps," *Journal of Energy Engineering*, 124(3), 90-102.

Marchi, A., Simpson, A.R., and Ertugrul, N. (2012a) "Assessing variable speed pump efficiency in water distribution systems", *Drink. Water Eng. Sci.*, 5, 15–21.

Marchi, A., Simpson, A.R. (2012b) "Correction of the EPANET inaccuracy in computing the efficiency of variable speed pumps," accepted in *J. Water Resour. Plann. Manage.*, doi:10.1061/(ASCE)WR.1943-5452.0000273

Morton, W. R. (1975). "Economics of AC Adjustable Speed Drives on Pumps", *IEEE Transactions on Industry Applications*, IA-11(3), 282-286.

Osterwalder, J. (1978). "Efficiency scale-up for hydraulic turbo-machines with due consideration of surface roughness", Journal of Hydraulic Research, 16(1), 55-76.

Rossman, L. A. (2000). "EPANET2." EPA/600/R- 00/057, Water Supply and Water Resources Division, National Risk Management Research Laboratory, Office of research and Development, USEPA, Cincinnati, Ohio, USA.

Sârbu, I., and Borza, I. (1998) “Energetic optimization of water pumping in distribution systems,” *Periodica Polytechnica Ser. Mech. Eng.*, vol. 42, no. 2, 1998, pp. 141-152.

Ulanicki, B., Kahler, J., and Coulbeck, B. (2008) “Modelling the efficiency and power characteristics of a pump group,” *J. Water Resour. Plann. Manage.*, 134(1), 88-93.

Walski, T. M. (2001) “Don’t forget energy cost when selecting pumps,” *ACE2001 – 2001 Annual Conference Proceedings*, 1-16.

Walski, T. M. (2005) “The tortoise and the hare,” *Water Envir. Technol.*, 6, 57-60.

Walski, T., Zimmerman, K., Dudinyak, M., and Dileepkumar, P. (2003) “Some surprises in estimating the efficiency of variable speed pumps with the pump affinity laws”, in *Proc. World Water and Environmental Resources Congress 2003*, Philadelphia, PA: ASCE 2003, pp. 1-10.

Appendix A: How to compute VSP efficiency in EPANET 2 using Sarbu and Borza equation

This appendix shows how to modify EPANET 2 to compute VSP efficiency using Sarbu and Borza (1998) equation. Only the function *getenergy()* needs to be modified: the few lines of code added are shown in italics. The Sarbu and Borza formula only requires the knowledge of the efficiency at the nominal speed, *i.e.* η_1 in Eq. 3 and *eN* in the following source code, and of the relative speed, *i.e.* the inverse of N_1/N_2 in Eq. 3 and *patMult* in the source code. Note that *eN* is initially a percentage because of the way the efficiency curve is inserted in EPANET and has to be converted in a rate to be used in Sarbu and Borza formula. Afterwards, *eN* has to be reconverted in a percentage to continue the energy computation. Note that the variable *eN* is introduced for clarity, but can be omitted and substituted with the variable *e* of EPANET 2.

```
void  getenergy(int k, double *kw, double *eff)
{
  int   i,j;
  double dh, q, e;
  double q4eff, patMult; //corresponding flow at the nominal speed, relative speed factor
  long p, temp; //variables used to compute the relative speed if it is inserted as a pattern
  double eN; //efficiency at the nominal speed
[...]
  /* For pumps, find effic. at current flow */
  if (Link[k].Type == PUMP)
  {
    j = PUMPINDEX(k);
    e = Epump;
```

```
      if ( (i = Pump[j].Ecurve) > 0)
     { //compute the relative speed patMult for the specific time step
         if ( Pump[j].Upat > 0 ) {
              p = (Htime+Pstart)/Pstep;
             temp = p % (long) Pattern[Pump[j].Upat].Length;
             patMult = Pattern[Pump[j].Upat].F[temp];
         }
        else patMult = K[k];
       q4eff = q/patMult;
       eN = interp(Curve[i].Npts,Curve[i].X,Curve[i].Y,q4eff*Ucf[FLOW]);
       eN = eN/100; // convert to use in Sarbu and Borza formula
       e = 1- ((1-eN)*pow((1/patMult),0.1)));  // Sarbu and Borza formula
       e = e*100; // convert in percentage to use with EPANET     }

      e = MIN(e, 100.0);
[...]
```

Table 1. Comparison of data and predictions of efficiency.

Pump	Power (kW)	Data		Efficiency Prediction		Difference (%)	
		BEP at N_1	BEP at N_2	BEP Affinity laws	BEP Sarbu & Borza	BEP Affinity laws	BEP Sarbu & Borza
HPL 54-30-20 (i)	556	83.6	83.5	83.6	83.2	0.12	-0.36
Hydro-Tytan 125x80-250 (ii)	90	80.5	79.1	80.5	79.3	1.77	0.25
50x32-160H.T. (ii)	5.5	56.0	52.0	56.0	53.3	7.69	2.50

(i) Sulzer; data reported by Ulanicki et al. (2008). N_1 = 1525 rpm, N_2 = 1182 rpm;

(ii) Thompsons Kelly and Lewis (1989) N_1 = 3600 rpm, N_2 = 2000 rpm.