

# Linear Algebra Comprehensive Worksheet

Student Working Copy

Yolymatics Tutorials

## Instructions

This worksheet covers key topics in linear algebra. Show all your working clearly in the spaces provided. Use additional paper if needed.

### 1 Cross Product (Section 3.5)

**Problem 1.** Calculate the cross product  $\vec{a} \times \vec{b}$  for the following vectors:

(a)  $\vec{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$

**Answer:**

*Working:*

(b)  $\vec{a} = \begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix}, \vec{b} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$

**Answer:**

*Working:*

**Problem 2.** Given vectors  $\vec{u} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ :

- (a) Find  $\vec{u} \times \vec{v}$

**Answer:**

*Working:*

- (b) Verify that  $\vec{u} \times \vec{v}$  is orthogonal to both  $\vec{u}$  and  $\vec{v}$

**Answer:**

*Working:*

- (c) Find the area of the parallelogram spanned by  $\vec{u}$  and  $\vec{v}$

**Answer:**

*Working:*

**Problem 3.** Prove that for any vectors  $\vec{a}$  and  $\vec{b}$  in  $\mathbb{R}^3$ :

- (a)  $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$  (anti-commutativity)

*Working:*

- (b)  $\vec{a} \times \vec{a} = \vec{0}$

*Working:*

**Problem 4.** Find the volume of the parallelepiped determined by the vectors:

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

**Answer:**

*Working:*

## 2 Eigenvalues and Eigenvectors (Section 5.1)

**Problem 5.** Find the eigenvalues and corresponding eigenvectors for each matrix:

(a)  $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

**Answer:**

*Working:*



(b)  $B = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

**Answer:**

*Working:*

(c)  $C = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$

**Answer:**

*Working:*



**Problem 6.** Consider the matrix  $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{pmatrix}$

- (a) Find the characteristic polynomial of  $A$

**Answer:**

*Working:*

- (b) Find all eigenvalues of  $A$

**Answer:**

*Working:*

- (c) Find an eigenvector for each eigenvalue

**Answer:**

*Working:*

**Problem 7.** Let  $\lambda = 5$  be an eigenvalue of  $A = \begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix}$  with eigenvector  $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ .

- (a) Verify that  $\vec{v}$  is indeed an eigenvector with eigenvalue  $\lambda = 5$

*Working:*

- (b) Find the other eigenvalue and eigenvector

**Answer:**

*Working:*

- (c) What is the trace and determinant of  $A$ ? How do they relate to the eigenvalues?

**Answer:**

*Working:*

**Problem 8.** Prove that if  $\lambda$  is an eigenvalue of an invertible matrix  $A$ , then  $\frac{1}{\lambda}$  is an eigenvalue of  $A^{-1}$ .

*Working:*

### 3 Diagonalization (Section 5.2)

**Problem 9.** Determine whether each matrix is diagonalizable. If so, find matrices  $P$  and  $D$  such that  $A = PDP^{-1}$ .

(a)  $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

**Answer:**

*Working:*

(b)  $B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

**Answer:**

*Working:*

(c)  $C = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$

**Answer:**

*Working:*

**Problem 10.** Given  $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$  has eigenvalues  $\lambda_1 = 6$  and  $\lambda_2 = 1$  with corresponding eigenvectors  $\vec{v}_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$  and  $\vec{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ :

- (a) Write  $A$  in the form  $PDP^{-1}$

**Answer:**

*Working:*

(b) Use this to compute  $A^5$

**Answer:**

*Working:*

(c) Find  $A^{10}$

**Answer:**

*Working:*



**Problem 11.** Let  $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$ .

- (a) Find the eigenvalues of  $A$

**Answer:**

*Working:*

- (b) Determine the geometric multiplicity of each eigenvalue

**Answer:**

*Working:*



(c) Is  $A$  diagonalizable? Justify your answer

**Answer:**

*Working:*

## 4 Differential Equations (Section 5.4)

**Problem 12.** Solve the system of differential equations:

$$\frac{d\vec{x}}{dt} = A\vec{x}, \quad \text{where } A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \quad \vec{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

**Answer:**

*Working:*



**Problem 13.** Consider the system  $\vec{x}' = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix} \vec{x}$  with initial condition  $\vec{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ .

- (a) Find the general solution

**Answer:**

*Working:*

- (b) Find the particular solution satisfying the initial condition

**Answer:**

*Working:*

- (c) Sketch the phase portrait

*Working:*

**Problem 14.** Solve the decoupled system:

$$\begin{cases} \frac{dy_1}{dt} = 3y_1 \\ \frac{dy_2}{dt} = -2y_2 \end{cases}$$

with initial conditions  $y_1(0) = 2$  and  $y_2(0) = 4$ .

**Answer:**

*Working:*

**Problem 15.** A dynamical system is described by  $\vec{x}' = A\vec{x}$  where  $A = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$ .

- (a) Find the eigenvalues of  $A$

**Answer:**

*Working:*



- (b) Classify the equilibrium point at the origin (stable/unstable, node/spiral/saddle)

**Answer:**

*Working:*

- (c) Describe the long-term behavior of solutions

**Answer:**

*Working:*

## 5 Inner Product (Section 6.1)

**Problem 16.** Compute the inner product  $\langle \vec{u}, \vec{v} \rangle$  for:

(a)  $\vec{u} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ ,  $\vec{v} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$  (standard inner product)

**Answer:**

*Working:*

(b)  $\vec{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $\vec{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$  with  $\langle \vec{u}, \vec{v} \rangle = 2u_1v_1 + 3u_2v_2$

**Answer:**

*Working:*

**Problem 17.** Let  $V = P_2$  be the space of polynomials of degree at most 2, with inner product  $\langle p, q \rangle = \int_0^1 p(x)q(x) dx$ .

- (a) Compute  $\langle 1, x \rangle$ ,  $\langle 1, x^2 \rangle$ , and  $\langle x, x^2 \rangle$

**Answer:**

*Working:*

- (b) Find  $\|x\|$  and  $\|x^2\|$

**Answer:**

*Working:*



(c) Are  $\{1, x, x^2\}$  orthogonal?

**Answer:**

*Working:*

**Problem 18.** Verify that the following defines an inner product on  $\mathbb{R}^2$ :

$$\langle \vec{u}, \vec{v} \rangle = 3u_1v_1 + 2u_2v_2$$

by checking all four axioms of an inner product.

*Working:*

**Problem 19.** Given  $\vec{u} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ :

- (a) Find  $\|\vec{u}\|$

**Answer:**

*Working:*

- (b) Find a unit vector in the direction of  $\vec{u}$

**Answer:**

*Working:*

- (c) Find all vectors orthogonal to  $\vec{u}$

**Answer:**

*Working:*



## 6 Angle and Orthogonality (Section 6.2)

**Problem 20.** Find the angle between the vectors:

(a)  $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

**Answer:**

*Working:*

(b)  $\vec{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ ,  $\vec{v} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

**Answer:**

*Working:*

(c)  $\vec{u} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}, \vec{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

**Answer:**

*Working:*

**Problem 21.** Determine which of the following pairs of vectors are orthogonal:

(a)  $\vec{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 1 \\ -2 \\ -4 \end{pmatrix}$

**Answer:**

*Working:*

(b)  $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

**Answer:**

*Working:*

(c)  $\vec{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

**Answer:**

*Working:*

**Problem 22.** Find the orthogonal projection of  $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$  onto  $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ .

(a) Find  $\text{proj}_{\vec{a}} \vec{b}$

**Answer:**

*Working:*

(b) Find the component of  $\vec{b}$  orthogonal to  $\vec{a}$

**Answer:**

*Working:*

- (c) Verify that these two components are orthogonal

*Working:*

**Problem 23.** Use the Cauchy-Schwarz inequality to prove that for any vectors  $\vec{u}$  and  $\vec{v}$  in an inner product space:

$$|\langle \vec{u}, \vec{v} \rangle| \leq \|\vec{u}\| \cdot \|\vec{v}\|$$

Then verify this inequality for  $\vec{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  and  $\vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ .

*Working:*

## 7 Gram-Schmidt Process (Section 6.3)

**Problem 24.** Apply the Gram-Schmidt process to find an orthogonal basis for the subspace spanned by:

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Then normalize to obtain an orthonormal basis.

**Answer:**

*Working:*

**Problem 25.** Find an orthonormal basis for the column space of:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

**Answer:**

*Working:*

**Problem 26.** Consider the polynomials  $p_1(x) = 1$ ,  $p_2(x) = x$ ,  $p_3(x) = x^2$  on the interval  $[0, 1]$  with inner product  $\langle p, q \rangle = \int_0^1 p(x)q(x) dx$ .

- (a) Apply Gram-Schmidt to obtain an orthogonal set  $\{q_1, q_2, q_3\}$

**Answer:**

*Working:*

- (b) Normalize to get an orthonormal set

**Answer:**

*Working:*

**Problem 27.** Given vectors  $\vec{u}_1 = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$  and  $\vec{u}_2 = \begin{pmatrix} -1 \\ 0 \\ 7 \end{pmatrix}$ :

- (a) Find an orthogonal basis  $\{\vec{v}_1, \vec{v}_2\}$  for  $\text{span}\{\vec{u}_1, \vec{u}_2\}$

**Answer:**

*Working:*



- (b) Extend this to an orthogonal basis for  $\mathbb{R}^3$

**Answer:**

*Working:*

## 8 Orthogonal Matrices (Section 7.1)

**Problem 28.** Determine which of the following matrices are orthogonal:

(a)  $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

**Answer:**

*Working:*

(b)  $B = \frac{1}{3} \begin{pmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{pmatrix}$

**Answer:**

*Working:*



(c)  $C = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$

**Answer:**

*Working:*

**Problem 29.** Let  $Q = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$

- (a) Verify that  $Q$  is orthogonal

*Working:*

(b) Find  $Q^{-1}$

**Answer:**

*Working:*

(c) Show that  $Q$  preserves the length of vectors

*Working:*

**Problem 30.** Prove that if  $Q$  is an orthogonal matrix, then:

- (a)  $\det(Q) = \pm 1$

*Working:*

- (b)  $Q^T$  is also orthogonal

*Working:*

- (c) The rows of  $Q$  form an orthonormal set

*Working:*

**Problem 31.** Find an orthogonal matrix  $Q$  whose first column is  $\frac{1}{3} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ .

**Answer:**

*Working:*



## 9 Orthogonal Diagonalization (Section 7.2)

**Problem 32.** Determine whether each symmetric matrix is orthogonally diagonalizable. If so, find an orthogonal matrix  $P$  and diagonal matrix  $D$  such that  $A = PDP^T$ .

(a)  $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

**Answer:**

*Working:*

(b)  $B = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

**Answer:**

*Working:*

**Problem 33.** Let  $A = \begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$ .

- (a) Show that  $A$  is symmetric

*Working:*

- (b) Find the eigenvalues and eigenvectors

**Answer:**

*Working:*

- (c) Find an orthogonal matrix  $P$  that diagonalizes  $A$

**Answer:**

*Working:*

- (d) Verify that  $P^TAP$  is diagonal

*Working:*

**Problem 34.** Prove the Spectral Theorem: Every symmetric matrix is orthogonally diagonalizable.

*Working:*

**Problem 35.** Given  $A = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ :

- (a) Find an orthogonal diagonalization of  $A$

**Answer:**

*Working:*

(b) Use this to compute  $A^{10}$

**Answer:**

*Working:*

## 10 Quadratic Forms (Section 7.3)

**Problem 36.** Write each quadratic form in matrix notation  $\vec{x}^T A \vec{x}$ :

(a)  $Q(x_1, x_2) = 3x_1^2 + 4x_1x_2 + 5x_2^2$

**Answer:**

*Working:*

(b)  $Q(x_1, x_2, x_3) = x_1^2 - 2x_2^2 + 3x_3^2 + 4x_1x_2 - 6x_2x_3$

**Answer:**

*Working:*



**Problem 37.** For the quadratic form  $Q(x_1, x_2) = 5x_1^2 + 8x_1x_2 + 5x_2^2$ :

- (a) Write  $Q$  in the form  $\vec{x}^T A \vec{x}$

**Answer:**

*Working:*

- (b) Find an orthogonal change of variables that eliminates the cross-product term

**Answer:**

*Working:*



- (c) Classify the quadratic form (positive definite, negative definite, or indefinite)

**Answer:**

*Working:*

**Problem 38.** Determine whether each quadratic form is positive definite, negative definite, indefinite, or positive/negative semidefinite:

(a)  $Q(\vec{x}) = 2x_1^2 + 4x_1x_2 + 3x_2^2$

**Answer:**

*Working:*



(b)  $Q(\vec{x}) = -x_1^2 + 2x_1x_2 - x_2^2$

**Answer:**

*Working:*

(c)  $Q(\vec{x}) = x_1^2 + x_2^2 + x_3^2$

**Answer:**

*Working:*

**Problem 39.** Find the principal axes and classify the conic section:

$$5x^2 - 4xy + 2y^2 = 6$$

**Answer:**

*Working:*

## 11 Optimization using Quadratic Forms (Section 7.4)

**Problem 40.** Find the maximum and minimum values of  $Q(\vec{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2$  subject to  $\|\vec{x}\| = 1$ .

**Answer:**

*Working:*

**Problem 41.** Consider the function  $f(x, y) = 4x^2 + 4xy + y^2$ .

- (a) Find the maximum value of  $f$  on the unit circle  $x^2 + y^2 = 1$

**Answer:**

*Working:*

- (b) Find the minimum value of  $f$  on the unit circle

**Answer:**

*Working:*



(c) At what points are these extreme values attained?

**Answer:**

*Working:*

**Problem 42.** Use quadratic forms to find the extreme values of:

$$f(x, y, z) = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$$

subject to the constraint  $x^2 + y^2 + z^2 = 1$ .

**Answer:**

*Working:*



**Problem 43.** A company's profit function is given by  $P(x_1, x_2) = -2x_1^2 - x_2^2 + x_1x_2 + 8x_1 + 6x_2$ .

- (a) Find the critical points

**Answer:**

*Working:*

- (b) Use the second derivative test (Hessian matrix) to classify each critical point

**Answer:**

*Working:*



(c) Find the maximum profit

**Answer:**

*Working:*

## 12 Rank, Nullity and Matrix Spaces (Section 4.9)

**Problem 44.** For each matrix, find the rank, nullity, a basis for the column space, and a basis for the null space:

(a)  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 2 \end{pmatrix}$

**Answer:**

*Working:*

$$(b) \quad B = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 5 \\ -1 & -2 & 1 & 1 \end{pmatrix}$$

**Answer:**

*Working:*

**Problem 45.** Verify the Rank-Nullity Theorem for the matrix:

$$A = \begin{pmatrix} 1 & 3 & 4 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & -4 & 0 \end{pmatrix}$$

**Answer:**

*Working:*

**Problem 46.** Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be a linear transformation with matrix representation:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- (a) Find  $\text{rank}(A)$  and  $\text{nullity}(A)$

**Answer:**

*Working:*

- (b) Find a basis for  $\text{Col}(A)$  and  $\text{Nul}(A)$

**Answer:**

*Working:*



(c) Is  $T$  one-to-one? Is  $T$  onto?

**Answer:**

*Working:*

**Problem 47.** Find the dimension of the row space, column space, and null space of:

$$A = \begin{pmatrix} 2 & 4 & -2 & 1 \\ 1 & 2 & -1 & 0 \\ 3 & 6 & -3 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then verify that  $\dim(\text{Row}(A)) = \dim(\text{Col}(A))$ .

**Answer:**

*Working:*



**Problem 48.** Show that the set of all  $2 \times 2$  symmetric matrices forms a subspace of  $M_{2 \times 2}$ . What is its dimension? Find a basis.

**Answer:**

*Working:*

## 13 Geometry of Matrix Operators (Section 8.6)

**Problem 49.** Describe the geometric action of each linear transformation:

(a)  $T(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \vec{x}$

**Answer:**

*Working:*

(b)  $T(\vec{x}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \vec{x}$

**Answer:**

*Working:*

(c)  $T(\vec{x}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \vec{x}$

**Answer:**

*Working:*

**Problem 50.** Let  $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$  where  $\theta = \frac{\pi}{4}$ .

- (a) Describe the geometric transformation represented by  $A$

**Answer:**

*Working:*

- (b) Find the image of  $\vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  under this transformation

**Answer:**

*Working:*

- (c) Find  $A^4$ . What transformation does it represent?

**Answer:**

*Working:*

**Problem 51.** Consider the shear transformation  $S = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ .

- (a) Describe how  $S$  transforms the unit square with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$ ,  $(0, 1)$  for  $k = 2$

*Working:*

- (b) Find the eigenvalues and eigenvectors of  $S$

**Answer:**

*Working:*

- (c) Does a shear transformation preserve area? Justify your answer

**Answer:**

*Working:*



**Problem 52.** The matrix  $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$  has eigenvalues  $\lambda_1 = 4$  and  $\lambda_2 = 2$  with eigenvectors  $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and  $\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ .

- (a) Describe geometrically what  $A$  does to vectors in the directions of  $\vec{v}_1$  and  $\vec{v}_2$

**Answer:**

*Working:*

- (b) Sketch the image of the unit circle under transformation by  $A$

*Working:*

- (c) What is the area magnification factor of this transformation?

**Answer:**

*Working:*

**Problem 53.** Decompose the transformation  $T(\vec{x}) = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix} \vec{x}$  into:

- (a) A rotation followed by scaling along coordinate axes

**Answer:**

*Working:*



- (b) Describe the geometric effect on the unit circle

**Answer:**

*Working:*

**End of Worksheet**

*Well done for completing this comprehensive linear algebra worksheet!  
Review your work and check your answers with your instructor.*