# Practice Problems

## Real-World Applications

z-tests • t-tests • Mean Comparisons

**Yolymatics Tutorials**

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# Z-Tests

# Problem 1: Quality Control — Battery Life

## Scenario

A battery manufacturer claims their AA batteries last 50 hours on average. A quality control inspector tests 100 batteries and finds a mean life of 48.5 hours. The population standard deviation is known to be $\sigma = 6$ hours.

At the 5% significance level, test whether the actual mean battery life is less than claimed.

**Tasks:**

1. State $H_0$ and $H_a$
2. Calculate the test statistic
3. Find the critical value or p-value
4. Make your decision
5. Write a conclusion in context

# Problem 1: Workspace

# Problem 1: Workspace (continued)

# Problem 2: Healthcare — Weight Loss Program

## Scenario

A weight loss clinic claims their program results in an average weight loss of 15 pounds over 3 months. A researcher samples 80 participants and finds they lost an average of 16.2 pounds. The population standard deviation is $\sigma = 5.5$ pounds.

At the 1% significance level, test if the actual mean weight loss differs from 15 pounds.

**Tasks:**

1. State $H_0$ and $H_a$ (two-tailed test)
2. Calculate the test statistic
3. Find the critical values or p-value
4. Make your decision
5. Interpret in context

# Problem 2: Workspace

# Problem 2: Workspace (continued)

# Problem 3: Education — Student Performance

## Scenario

The national average score on a standardized math test is 520 with a standard deviation of 95. A new teaching method is implemented in a school district. After one year, 150 students take the test and achieve a mean score of 542.

At the 5% significance level, does the new teaching method improve test scores?

**Tasks:**

1. Identify why this is a z-test
2. State hypotheses
3. Calculate z-statistic
4. Find p-value
5. Conclusion

# Problem 3: Workspace

# Problem 3: Workspace (continued)

# Problem 4: Business — Customer Satisfaction

## Scenario

A retail chain's customer satisfaction score has historically averaged 72 (out of 100) with $\sigma = 12$. After implementing a new customer service training program, they survey 200 customers and get a mean score of 74.8.

Test at $\alpha = 0.10$ whether satisfaction has increased.

**Tasks:**

1. Set up one-tailed test
2. Compute test statistic
3. Determine critical region
4. Decision and interpretation

# Problem 4: Workspace

# Problem 4: Workspace (continued)

# Problem 5: Manufacturing — Product Dimensions

## Scenario

A factory produces metal rods that should be 10 cm long. The production process has a known standard deviation of 0.15 cm. A quality inspector measures 64 rods and finds a mean length of 10.04 cm.

At the 5% level, test if the production process is meeting the 10 cm specification.

**Tasks:**

1. Choose appropriate hypotheses (two-tailed)
2. Calculate z-statistic
3. Find rejection region
4. Make decision
5. What should the factory do?

# Problem 5: Workspace

# Problem 5: Workspace (continued)

# Problem 6: Environmental Science — Water Quality

## Scenario

Safe drinking water should have a pH of 7.0. Historical data shows water pH in a region has $\sigma = 0.4$. After industrial development, 100 water samples are tested with a mean pH of 6.85.

At the 1% significance level, test if the water pH has decreased from 7.0.

**Tasks:**

1. State hypotheses
2. Compute z-statistic
3. Find p-value
4. Decision at $\alpha = 0.01$
5. Practical interpretation

# Problem 6: Workspace

# Problem 6: Workspace (continued)

# T-Tests

# Problem 7: Medical — Blood Pressure Medication

## Scenario

A pharmaceutical company tests a new blood pressure medication on 25 patients. Before treatment, patients had an average systolic BP of 145 mmHg. After 3 months, the mean is 138 mmHg with a sample standard deviation of 12 mmHg.

At the 5% level, test if the medication significantly reduces blood pressure.

**Tasks:**

1. Explain why this is a t-test (not z-test)
2. State $H_0$ and $H_a$
3. Calculate t-statistic
4. Find critical value (df $= 24$)
5. Conclusion

# Problem 7: Workspace

# Problem 7: Workspace (continued)

# Problem 8: Agriculture — Crop Yield

## Scenario

A farmer claims that using a new fertilizer increases corn yield from the historical average of 180 bushels per acre. He tests it on 16 plots and gets a mean of 192 bushels with sample standard deviation $s = 22$ bushels.

At the 5% significance level, test the farmer's claim.

**Tasks:**

1. Set up one-tailed test
2. Calculate t-statistic
3. Find p-value or critical value (df $= 15$)
4. Decision
5. Is the fertilizer effective?

# Problem 8: Workspace

# Problem 8: Workspace (continued)

# Problem 9: Psychology — Sleep Study

## Scenario

Adults typically sleep 7 hours per night. A psychologist suspects college students sleep less. She surveys 30 students and finds they sleep an average of 6.2 hours with $s = 1.5$ hours.

Test at the 1% level whether college students sleep significantly less than 7 hours.

**Tasks:**

1. Hypotheses for one-tailed test
2. Compute t-statistic
3. Find critical value (df $= 29$) at $\alpha = 0.01$
4. Make decision
5. Interpret for the psychologist

# Problem 9: Workspace

# Problem 9: Workspace (continued)

# Problem 10: Finance — Investment Returns

## Scenario

An investment advisor claims a portfolio will generate 8% annual returns. You analyze 12 years of data and find a mean return of 6.5% with sample standard deviation 3.2%.

At the 5% level, test if the actual return is different from 8%.

**Tasks:**

1. Two-tailed hypotheses
2. Calculate t-statistic
3. Find critical values (df $= 11$)
4. Decision
5. Should you trust the advisor's claim?

# Problem 10: Workspace

# Problem 10: Workspace (continued)

# Problem 11: Education — Reading Speed

## Scenario

A reading program claims to increase reading speed from an average of 250 words per minute. After completing the program, 20 students are tested with mean speed 275 wpm and $s = 45$ wpm.

At the 10% level, test if the program increases reading speed.

**Tasks:**

1. State hypotheses
2. Calculate t-statistic
3. Determine p-value or critical value (df $= 19$)
4. Decision at $\alpha = 0.10$
5. Program effectiveness?

# Problem 11: Workspace

# Problem 11: Workspace (continued)

# Problem 12: Retail — Store Foot Traffic

## Scenario

A store typically has 500 customers per day. After a renovation, they track daily customer counts for 18 days: mean $= 535$, $s = 68$ customers.

Test at the 5% level if foot traffic has increased.

**Tasks:**

1. Explain why t-test is appropriate
2. Set up test (one-tailed)
3. Calculate t-statistic
4. Find critical value (df $= 17$)
5. Was renovation successful?

# Problem 12: Workspace

# Problem 12: Workspace (continued)

# Mean Comparisons (Two-Sample Tests)

# Problem 13: Education — Teaching Methods Comparison

## Scenario

Two teaching methods are compared. Method A is used with 35 students (mean score $= 78$, $s_1 = 12$). Method B is used with 40 students (mean score $= 82$, $s_2 = 10$).

At the 5% level, test if there's a significant difference between the methods.

**Tasks:**

1. State $H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 \neq \mu_2$
2. Calculate pooled standard deviation
3. Compute t-statistic for two samples
4. Find critical value
5. Which method is better?

# Problem 13: Workspace

# Problem 13: Workspace (continued)

# Problem 14: Healthcare — Drug Comparison

## Scenario

Drug A (n=50): reduces pain score by mean of 4.2 points, $s_1 = 1.8$. Drug B (n=45): reduces pain score by mean of 3.5 points, $s_2 = 1.5$.

Test at the 1% level if Drug A is more effective than Drug B.

**Tasks:**

1. One-tailed hypotheses
2. Calculate two-sample t-statistic
3. Determine degrees of freedom
4. Find critical value at $\alpha = 0.01$
5. Decision and recommendation

# Problem 14: Workspace

# Problem 14: Workspace (continued)

# Problem 15: Business — Marketing Campaign

## Scenario

Company tests two ad campaigns. Campaign 1 reaches 60 people with mean engagement time 45 seconds ($s_1 = 12$). Campaign 2 reaches 55 people with mean 52 seconds ($s_2 = 15$).

At the 5% level, test if Campaign 2 generates more engagement.

**Tasks:**

1. Set up appropriate test
2. Calculate test statistic
3. Find p-value or critical value
4. Decision
5. Which campaign should they use?

# Problem 15: Workspace

# Problem 15: Workspace (continued)

# Problem 16: Sports Science — Training Programs

## Scenario

Program A: 28 athletes improve 100m sprint time by mean 0.8 sec ($s_1 = 0.3$). Program B: 32 athletes improve by mean 0.6 sec ($s_2 = 0.25$).

Test at 5% level if there's a difference in program effectiveness.

**Tasks:**

1. Two-tailed test setup
2. Compute pooled variance
3. Calculate t-statistic
4. Degrees of freedom and critical values
5. Conclusion

# Problem 16: Workspace

# Problem 16: Workspace (continued)

# Problem 17: Manufacturing — Assembly Lines

## Scenario

Line A: 40 products made with mean time 15.2 min ($s_1 = 2.1$). Line B: 38 products made with mean time 14.5 min ($s_2 = 2.4$).

At 10% level, test if Line B is faster than Line A.

**Tasks:**

1. One-tailed hypotheses
2. Calculate test statistic
3. Find critical value at $\alpha = 0.10$
4. Make decision
5. Should they switch to Line B?

# Problem 17: Workspace

# Problem 17: Workspace (continued)

# Problem 18: Environmental — Pollution Levels

## Scenario

City A (n=25): mean air quality index 85, $s_1 = 18$. City B (n=30): mean air quality index 92, $s_2 = 20$.

Test at 5% level if there's a significant difference in air quality between cities.

**Tasks:**

1. State hypotheses
2. Two-sample t-test calculation
3. Determine critical region
4. Decision
5. Interpretation

# Problem 18: Workspace

# Problem 18: Workspace (continued)

# Problem 19: Technology — Battery Life Comparison

## Scenario

Brand X batteries (n=50): mean life 42 hours, $s_1 = 6$. Brand Y batteries (n=48): mean life 45 hours, $s_2 = 7$.

At 1% level, test if Brand Y lasts significantly longer.

**Tasks:**

1. One-tailed test setup
2. Calculate pooled standard error
3. Compute t-statistic
4. Critical value and decision
5. Recommendation

# Problem 19: Workspace

# Problem 19: Workspace (continued)

# Problem 20: Retail — Customer Spending

## Scenario

Store location A: 65 customers spend mean \$48 ($s_1 = 12$). Store location B: 58 customers spend mean \$52 ($s_2 = 15$).

Test at 5% level if spending differs between locations.

**Tasks:**

1. Two-tailed hypotheses
2. Calculate test statistic
3. Find p-value or critical values
4. Decision
5. Business implications

# Problem 20: Workspace

# Problem 20: Workspace (continued)

# Mixed Practice

# Problem 21: Comprehensive — Restaurant Wait Times

## Scenario

A restaurant chain advertises average wait time of 20 minutes. Regional manager samples 45 customers: mean = 22.5 min, $s = 8$ min.

Also, they compare two locations: Location 1 (n=30, mean=23, $s_1 = 7$) vs Location 2 (n=35, mean=21, $s_2 = 9$).

**Tasks:**

1. Test if actual wait time exceeds 20 min (use t-test, $\alpha = 0.05$)
2. Compare the two locations (two-sample test, $\alpha = 0.05$)
3. Write recommendations

# Problem 21: Workspace

# Problem 21: Workspace (continued)

# Problem 21: Workspace (continued 2)

# Problem 22: Comprehensive — Salary Analysis

## Scenario

National average salary for a position is \$55,000 ($\sigma = \$8,000$). Company A surveys 100 employees: mean = \$57,200.

Compare Company A (above data) with Company B: 80 employees, mean = \$54,500, $s_2 = \$7,500$.

**Tasks:**

1. Test if Company A pays above national average (z-test, $\alpha = 0.01$)
2. Compare salaries between companies (appropriate test, $\alpha = 0.05$)
3. Interpret results

# Problem 22: Workspace

# Problem 22: Workspace (continued)

# Problem 22: Workspace (continued 2)

# Problem 23: Real-World — Clinical Trial

## Scenario

A new therapy is tested. Control group (n=42): improvement score mean 15 ($s_1 = 5$). Treatment group (n=38): improvement score mean 19 ($s_2 = 6$).

Additionally, compare treatment results to historical standard of 16 points.

**Tasks:**

1. Compare treatment vs control groups ($\alpha = 0.05$)
2. Test treatment group vs historical standard ($\alpha = 0.05$)
3. Is the new therapy effective?

# Problem 23: Workspace

# Problem 23: Workspace (continued)

# Problem 23: Workspace (continued 2)

# Problem 24: Real-World — Website Performance

## Scenario

Website should load in under 3 seconds. Test 200 page loads: mean $= 3.2$ sec, $\sigma = 0.8$ sec.

After optimization: Design A (n=50, mean=2.7, $s_1 = 0.6$) vs Design B (n=55, mean=2.5, $s_2 = 0.7$).

**Tasks:**

1. Test if current site exceeds 3 sec (z-test, $\alpha = 0.01$)
2. Compare optimized designs ($\alpha = 0.05$)
3. Which design should they implement?

# Problem 24: Workspace

# Problem 24: Workspace (continued)

# Problem 24: Workspace (continued 2)

# Problem 25: Challenge — Complete Analysis

## Scenario

A company wants to analyze employee productivity before and after a new policy.
**Before:** Historical data shows mean 100 units/day, $\sigma = 15$ (large dataset).
**After (small sample):** 20 employees measured: mean 108, $s = 18$.
**Departments:** Dept A (n=25, mean=110, $s_1 = 16$) vs Dept B (n=22, mean=105, $s_2 = 20$).

**Complete analysis:**

1. Test if productivity increased overall
2. Compare departments
3. Write executive summary

# Problem 25: Workspace

# Problem 25: Workspace (continued)

# Problem 25: Workspace (continued 2)

# Problem 25: Workspace (continued 3)

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Keep testing those hypotheses!