```
% ----------------------------------------
% Assignment for linear equations
% LiXin
% 2022/3/18
% ----------------------------------------
```

Problem 1

(a)

```
clear; close all; clc;
A=[2 1; 3 -9];
b=[5; 7];
rankA=rank(A)
```

```
rankA = 2
```

```
rank(A)==rank([A b])
```

```
ans = logical
   1
```

```
sol=A\b;
fprintf("x=%f y=%f", sol(1), sol(2))
```

```
x=2.476190 y=0.047619
```

(b)

```
clear; close all; clc;
A=[-8 -5; -2 7];
b=[4; 10];
rankA=rank(A)
```

```
rankA = 2
```

```
rank(A)==rank([A b])
```

```
ans = logical
   1
```

```
sol=A\b;
fprintf("x=%f y=%f", sol(1), sol(2))
```

```
x=-1.181818 y=1.090909
```

(c)

```
clear; close all; clc;
A=[12 -5 0; -3 4 7; 6 2 3];
b=[11; -3; 22];
rankA=rank(A)
```

```
rankA = 3
```

```
rank(A)==rank([A b])
```

```
ans = logical
   1
```

```
sol=A\b;
fprintf("x=%f y=%f x3=%f",sol(1), sol(2), sol(3))
```

```
x=3.000000 y=5.000000 x3=-2.000000
```

(d)

```
clear; close all; clc;
A=[6 -3 4; 12 5 -7; -5 2 6];
b=[41; -26; 16];
rankA=rank(A)
```

```
rankA = 3
```

```
rank(A)==rank([A b])
```

```
ans = logical
   1
```

```
sol=A\b;
fprintf("x=%f y=%f x3=%f", sol(1), sol(2), sol(3))
```

```
x=2.003503 y=-2.684764 x3=5.231173
```

Problem 2

(a)

```
clear; close all; clc;
```

C=inv(B)inv(A)B-inv(B)A

(b)

```
A=[7 9; -2 4];
B=[4 -3; 7 6];
C=inv(B)*inv(A)*B-inv(B)*A
```

```
C = 2×2
   -0.8536   -1.6058
    1.5357    1.3372
```

Problem 3

(a)

```
clear; close all; clc;
A=[-2 1; -2 1];
b=[-5; 3];
rank_A=rank(A)
```

```
rank_A = 1
```

```
rank([A b])
```

```
ans = 2
```

因为 A 的秩小于[A b]的秩，所以方程无解

用最小二乘法解

```
sol=lsqr(A,b)
```

```
lsqr converged at iteration 1 to a solution with relative residual 0.97.
sol = 2×1
    0.4000
   -0.2000
```

```
fprintf("x'=%f y'=%f",sol(1),sol(2))
```

```
x'=0.400000 y'=-0.200000
```

(b)

```
clear; close all; clc;
A=[-2 1; -8 4];
b=[3; 12];
rank_A=rank(A)
```

```
rank_A = 1
```

```
rank([A b])
```

```
ans = 1
```

方乘有无限解

用 pinv()

```
sol=pinv(A)*b
```

```
sol = 2×1
   -1.2000
    0.6000
```

用'\'

```
sol=A\b
```

```
Warning: Matrix is singular to working precision.
sol = 2×1
   NaN
   NaN
```

用 rref()

```
rref([A b])
```

```
ans = 2×3
    1.0000   -0.5000   -1.5000
         0         0         0
```

x=-1.5+0.5y

(c)

```
clear; close all; clc;
A=[-2 1; -2 1];
b=[-5; -5.00001];
rankA=rank(A)
```

```
rankA = 1
```

```
rank([A b])
```

```
ans = 2
```

方程无解，使用最小二乘法

```
sol=lsqr(A,b)
```

```
lsqr converged at iteration 1 to a solution with relative residual 1e-06.
sol = 2×1
    2.0000
   -1.0000
```

(d)

```
clear; close all; clc;
A=[1 5 -1 6; 2 -1 1 -2; -1 4 -1 3; 3 -7 -2 1];
b=[19; 7; 30; -75];
rank_A=rank(A)
```

```
rank_A = 4
```

```
rank([A b])
```

```
ans = 4
```

```
x=A\b
```

```
x = 4×1
    5.0000
   14.6250
  -12.1250
  -11.8750
```

Problem 4

(a)

```
syms x y z c
eqns=[x-5*y-2*z==11*c; 6*x+3*y+z==13*c; 7*x+3*y-5*z==10*c];
sol=solve(eqns,[x y z]);
x=sol.x
```

```
x = 3 c
```

```
y=sol.y
```

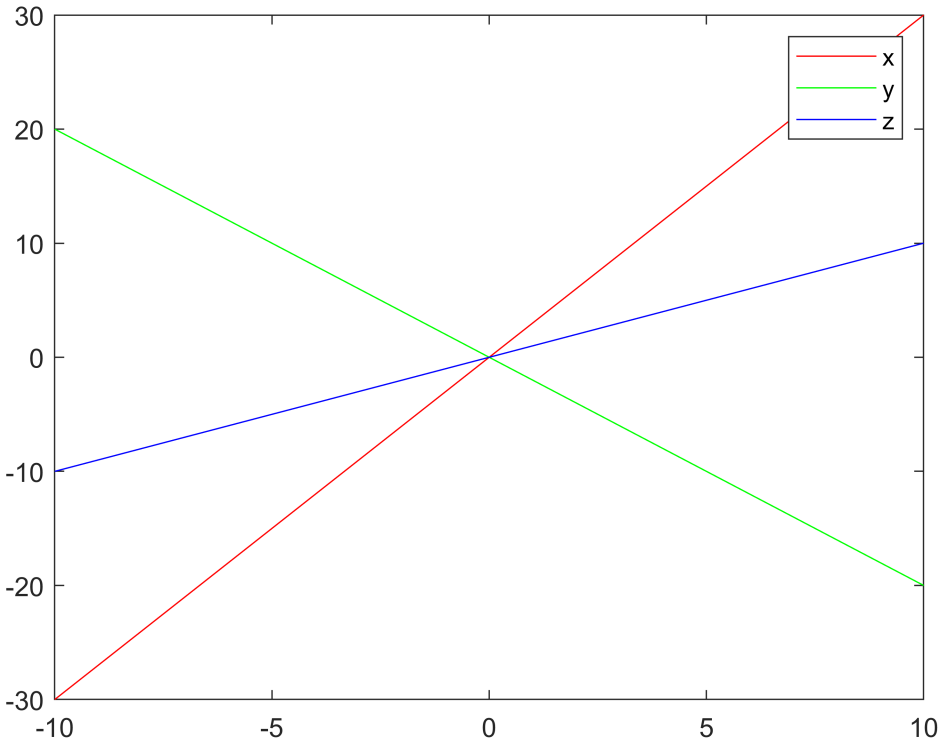
$y = -2c$

```
z=sol.z
```

$z = c$

(b)

```
c=-10:0.1:10;
sol=subs(sol);
plot(c,sol.x,'r',c,sol.y,'g',c,sol.z,'b')
legend('x','y','z')
```



Problem 5

(a)

```
clear; close; clc;
% for coeffcients a1, a2, a3
syms theta1_0 theta1_tf tf a1 a2 a3 rhs1 a
A1=[tf^3 tf^4 tf^5;3*tf^2 4*tf^3 5*tf^4; 6*tf 12*tf^2 20*tf^3];
a=[a1;a2;a3];
rhs1=[theta1_tf-theta1_0;0;0];
eqn1=A1*a==rhs1
```

```
eqn1 =
```

$$\begin{pmatrix} a_3\,\mathrm{tf}^5 + a_2\,\mathrm{tf}^4 + a_1\,\mathrm{tf}^3 = \theta_{1,\mathrm{tf}} - \theta_{1,0} \\ 5\,a_3\,\mathrm{tf}^4 + 4\,a_2\,\mathrm{tf}^3 + 3\,a_1\,\mathrm{tf}^2 = 0 \\ 20\,a_3\,\mathrm{tf}^3 + 12\,a_2\,\mathrm{tf}^2 + 6\,a_1\,\mathrm{tf} = 0 \end{pmatrix}$$

```
% for coefficients b1, b2, b3
syms theta2_0 theta2_tf b1 b2 b3 rhs2 b
A2=[tf^3 tf^4 tf^5; 3*tf^2 4*tf^3 5*tf^4; 6*tf 12*tf^2 20*tf^3];
b=[b1;b2;b3];
rhs2=[theta2_tf-theta2_0;0;0];
eqn2=A2*b==rhs2
```

```
eqn2 =
```

$$\begin{pmatrix} b_3\,\mathrm{tf}^5 + b_2\,\mathrm{tf}^4 + b_1\,\mathrm{tf}^3 = \theta_{2,\mathrm{tf}} - \theta_{2,0} \\ 5\,b_3\,\mathrm{tf}^4 + 4\,b_2\,\mathrm{tf}^3 + 3\,b_1\,\mathrm{tf}^2 = 0 \\ 20\,b_3\,\mathrm{tf}^3 + 12\,b_2\,\mathrm{tf}^2 + 6\,b_1\,\mathrm{tf} = 0 \end{pmatrix}$$

(b)

```
tf=2;
theta1_0=-19;
theta2_0=44;
theta1_tf=43;
theta2_tf=151;
A1=subs(A1);
rhs1=subs(rhs1);
p1=A1\rhs1;
fprintf("a1=%f a2=%f a3=%f",p1(1),p1(2),p1(3))
```

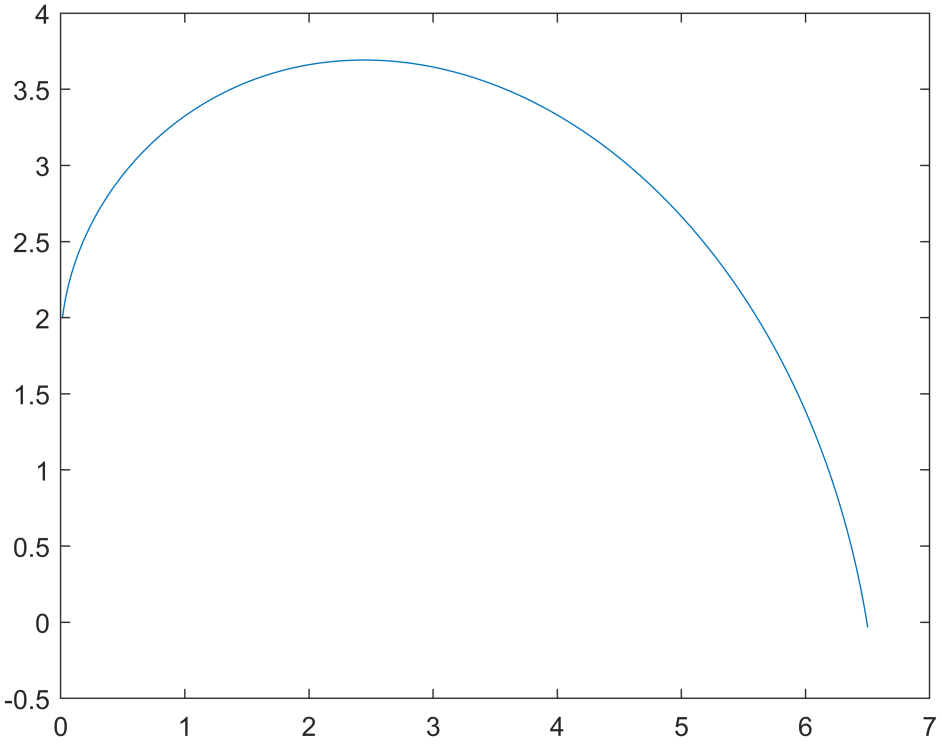
```
a1=77.500000 a2=-58.125000 a3=11.625000
```

```
A2=subs(A2);
rhs2=subs(rhs2);
p2=A2\rhs2;
fprintf("b1=%f b2=%f b3=%f",p2(1),p2(2),p2(3))
```

```
b1=133.750000 b2=-100.312500 b3=20.062500
```

(c)

```
t=0:0.01:2;
t=t';
A=[t.^3 t.^4 t.^5];
theta1=theta1_0+A*p1;
theta2=theta2_0+A*p2;
L1=4; L2=3;
x=L1*cos(deg2rad(theta1))+L2*cos(deg2rad(theta1+theta2));
y=L1*sin(deg2rad(theta1))+L2*sin(deg2rad(theta1+theta2));
plot(x,y)
```



Problem 6

```
clear; close; clc;
Ti=20;
To=-10;
R1=0.036*10;
R2=4.01*10;
R3=0.408*10;
R4=0.038*10;
syms q T1 T2 T3
eqns=[q==1/R1*(Ti-T1); q==1/R2*(T1-T2); q==1/R3*(T2-T3); q==1/R4*(T3-To)];
S=solve(eqns,[q T1 T2 T3]);
fprintf("q=%f T1=%f T2=%f T3=%f", S.q, S.T1, S.T2, S.T3)
```

```
q=0.667854 T1=19.759573 T2=-7.021371 T3=-9.746215
```