

```
% -----
% ICE Linear Equations
% LiXin
% 2022/3/18
% -----
```

## Problem 1

```
clear all; close all; clc;
A=[7 9 -9; 3 2 -4; 1 5 -1]; % coefficient matrix
b=[22; 12; -2];
rankA=rank(A)
```

```
rankA = 2
```

```
rank(A)==rank([A b])
```

```
ans = logical
      1
```

The linear equations have infinite solutions

using pinv()

```
x=pinv(A)*b
```

```
x = 3x1
     1.6437
    -1.2024
    -2.3684
```

using '\'

```
x=A\b
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND = 6.745286e-18.
```

```
x = 3x1
    -1.1940
    -1.0448
    -4.4179
```

using rref

```
rref_sol=rref([A b])
```

```
rref_sol = 3x4
     1.0000         0    -1.3846     4.9231
         0     1.0000     0.0769    -1.3846
         0         0         0         0
```

$x_1 = 4.9231 + 1.3846x_3$ ,  $x_2 = -1.3846 - 0.0769x_3$

## Problem 2

(a)

```
clear all; close all; clc;
x=[1; 4; 5];
A=[x.^2 x ones(3,1)];
b=[4; 73; 120];
rankA=rank(A)
```

```
rankA = 3
```

```
coeff=A\b; % calculate a, b, c
fprintf("a: %f b: %f c: %f", coeff(1), coeff(2), coeff(3))
```

```
a: 6.000000 b: -7.000000 c: 5.000000
```

(b)

```
A=[x.^3 x.^2 x ones(3,1)];
rankA=rank(A)
```

```
rankA = 3
```

The linear equations have infinite solutions

using pinv()

```
coeff=pinv(A)*b
```

```
coeff = 4x1
    0.2705
    3.2951
    0.8443
   -0.4098
```

using '\'

```
coeff=A\b
```

```
coeff = 4x1
    0.2414
    3.5862
         0
    0.1724
```

using rref

```
rref_sol=rref([A b])
```

```
rref_sol = 3x5
    1.0000         0         0    0.0500    0.2500
         0    1.0000         0   -0.5000    3.5000
         0         0    1.0000    1.4500    0.2500
```

$a=0.25-0.05d$ ,  $b=3.5+0.05d$ ,  $c=0.25-1.45d$

Problem 3

```
clear all; close all; clc;
Ta=150; Tb=20;
```

```
syms T1 T2 T3 T4
eqns = [T1==1/3*(Ta+T2+T3);T2==1/2*(T1+T4);T3==1/2*(T1+T4);T4==1/3*(T2+T3+Tb)];
S=solve(eqns,[T1 T2 T3 T4]);
fprintf("T1: %f T2: %f T3: %f T4: %f", S.T1, S.T2, S.T3, S.T4)
```

```
T1: 106.666667 T2: 85.000000 T3: 85.000000 T4: 63.333333
```

```
A=[1 -1/3 -1/3 0; -1/2 1 0 -1/2; -1/2 0 1 -1/2; 0 -1/3 -1/3 1];
b=[1/3*Ta; 0; 0; 1/3*Tb];
rankA=rank(A)
```

```
rankA = 4
```

```
rank([A b])
```

```
ans = 4
```

```
Ts=A\b
```

```
Ts = 4x1
    106.6667
     85.0000
     85.0000
     63.3333
```

#### Problem 4

```
clear all; close all; clc;
Ta=150; Tb=20;
syms T1 T2 T3 T4 T5 T6 T7 T8 T9
eqns=[T1==1/3*(Ta+T2+T4); T2==1/3*(T1+T3+T5); T3==1/2*(T2+T6); T4==1/3*(T1+T5+T7);
      T5==1/4*(T2+T4+T6+T8);T6==1/3*(T3+T5+T9);T7==1/2*(T4+T8);
      T8==1/3*(T7+T5+T9); T9==1/3*(Tb+T8+T8)];
s=solve(eqns,[T1 T2 T3 T4 T5 T6 T7 T8 T9]);
fprintf("T1: %f T2: %f T3: %f T4: %f T5: %f T6: %f " + ...
        "\nT7: %f T8: %f T9: %f",s.T1,s.T2,s.T3,s.T4,s.T5,s.T6,s.T7, ...
        s.T8, s.T9)
```

```
T1: 112.857143 T2: 94.285714 T3: 85.000000 T4: 94.285714 T5: 85.000000 T6: 75.714286
T7: 85.000000 T8: 75.714286 T9: 57.142857
```

#### Problem 5

```
clear all; close all; clc;
```

(a)

```
syms x y z
eqns=[6*x+2*y+10*z==35; 3*x+5*y+2*z==40]
```

```
eqns =
```

$$\begin{pmatrix} 6x + 2y + 10z = 35 \\ 3x + 5y + 2z = 40 \end{pmatrix}$$

```
A=[6 2 10; 3 5 2];
```

```
b=[35; 40];
rankA=rank(A)
```

```
rankA = 2
```

```
rank(A)==rank([A b])
```

```
ans = logical
      1
```

```
fprintf("doesn't exit a unique solution")
```

```
doesn't exit a unique solution
```

```
rref_sol=rref([A b])
```

```
rref_sol = 2x4
    1.0000         0    1.9167    3.9583
         0    1.0000   -0.7500    5.6250
```

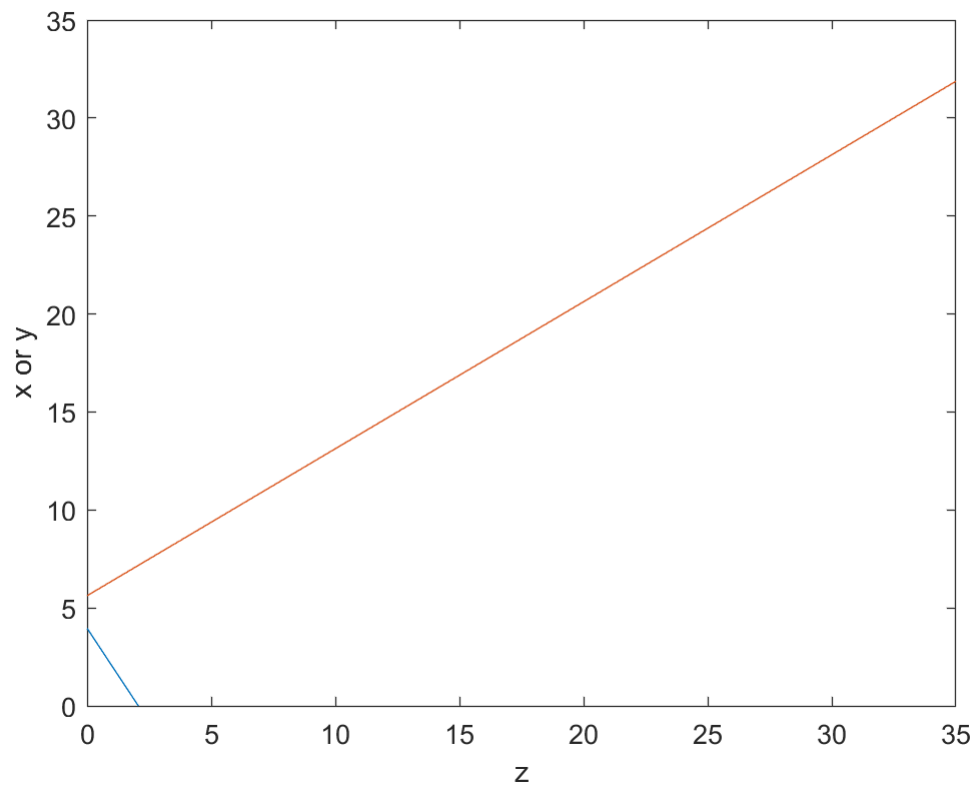
$x=3.9583-1.9167z$ ,  $y=5.625+0.75z$

(b)

```
syms x1 z1 y1
eqns=[x1*6+y1*2+z1*10<35 x1*6+y1*2+z1*10>0 x1*3+y1*5+z1*2<40 x1*3+y1*5+z1*2>0]
```

```
eqns = (6 x_1 + 2 y_1 + 10 z_1 < 35  0 < 6 x_1 + 2 y_1 + 10 z_1  3 x_1 + 5 y_1 + 2 z_1 < 40  0 < 3 x_1 + 5 y_1 + 2 z_1)
```

```
S=solve(eqns, [x1 z1 y1], 'ReturnConditions',true);
z2=0:0.01:35;
x2=3.9583-1.9167*z2;
y2=5.625+0.75*z2;
plot(z2,x2)
hold on
plot(z2,y2)
xlabel('z'), ylabel('x or y'), ylim([0 35])
```



by solving the equations

$$0 \leq z \leq 2.065$$

$$0 \leq x \leq 3.9583$$

$$5.625 \leq y \leq 7.1738$$

(c)

```
max=0;
max_x=0; max_y=0; max_z=0;
for z = 0:0.001:2.065
    x=3.9583-1.9167*z;
    y=5.625+0.75*z;
    tempt=200*x+300*y+100*z;
    if tempt > max
        max_x=x;
        max_y=y;
        max_z=z;
        max=tempt;
    end
end
fprintf("the values to maximize profit, x:%.4f y:%.4f z:%.4f",max_x,max_y,max_z)
```

the values to maximize profit, x:3.9583 y:5.6250 z:0.0000

(d)

```

max=0;
max_x=0; max_y=0; max_z=0;
for z = 0:0.001:2.065
    x=3.9583-1.9167*z;
    y=5.625+0.75*z;
    tempt=200*x+500*y+100*z;
    if tempt > max
        max_x=x;
        max_y=y;
        max_z=z;
        max=tempt;
    end
end
fprintf("the values to maximize profit, x:%.3f y:%.3f z:%.3f",max_x,max_y,max_z)

```

the values to maximize profit, x:0.000 y:7.174 z:2.065

## Problem 6

```

clear all; close all; clc;
syms f1 f2 f3 f4 f5 f6 f7
A=[1 0 1 0 0 0 0; 1 -1 0 -1 0 0 0;
    0 1 0 0 1 0 0; 0 0 1 0 0 1 0;
    0 0 0 1 0 -1 1; 0 0 0 0 1 0 1];
b=[300; -300; 600; 400; 200; 600];
rankA=rank(A)

```

rankA = 5

```
rank([A b])
```

ans = 5

```
rref_sol=rref([A b])
```

```

rref_sol = 6x8
    1     0     0     0     0    -1     0   -100
    0     1     0     0     0     0    -1     0
    0     0     1     0     0     1     0    400
    0     0     0     1     0    -1     1    200
    0     0     0     0     1     0     1    600
    0     0     0     0     0     0     0     0

```

f1=-100+f6, f2=f7, f3=400-f5, f4=200+f6-f7, f5=600-f7