

Gauging the Contribution of X-ray Sources to Reionization Through the Kinetic Sunyaev-Zel'dovich Effect

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Abstract

Measurements of the kinetic Sunyaev-Zel'dovich (kSZ) effect from instruments such as the South Pole Telescope (SPT) and the Atacama Cosmology Telescope (ACT) will soon put improved constraints on reionization. Popular models assume that UV photons alone are responsible for reionization of the intergalactic medium. We explore the effects of a significant contribution of X-rays to reionization on the kSZ signal. Because X-rays have a large mean free path through the neutral intergalactic medium, they introduce partial ionization in between the sharp-edged bubbles created by UV photons. This smooth ionization component changes the power spectrum of the cosmic microwave background (CMB) temperature anisotropies. We quantify this effect by running semi-numerical simulations of reionization. We test a number of different models of reionization without X-rays that have varying physical parameters, but which are constrained to have similar total optical depths to electron scattering. These are then compared to models with varying levels of contribution to reionization from X-rays. We find that models with more than a 10% contribution from X-rays produce a significantly lower power spectrum of temperature anisotropies than all the UV-only models tested. The expected sensitivity of SPT and ACT may be insufficient to distinguish between our models, however, a non-detection of the kSZ signal from the epoch of reionization could result from the contribution of X-rays. It will be important for future missions with improved sensitivity to consider the impact of X-ray sources on reionization.

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I. INTRODUCTION

The epoch of reionization (EoR) is an important milestone in the history of our universe [1]. There has been much theoretical work predicting the details of reionization, however many of the predictions have yet to be verified observationally. An important probe of this process is the small scale temperature fluctuations in the cosmic microwave background (CMB). In particular, secondary anisotropies are imprinted on top of the primordial CMB from the Doppler shift associated with CMB photons scattering off free electrons in between the observer and the surface of last scattering at $z \approx 1100$. This is known as the kinetic Sunyaev-Zel'dovich (kSZ) effect. Because only ionized regions can contribute to this effect, the power spectrum of kSZ fluctuations is sensitive to details of the reionization history [2–6].

The most popular models of reionization assume that the UV photons from early stars drive the ionization of the intergalactic medium (IGM). Due to the short mean free path (MFP) of UV photons through the neutral IGM, the resulting topology of reionization is a two phase IGM consisting of ionized bubbles surrounded by regions of neutral gas [7].

Another possibility is that X-ray photons make a significant contribution to reionization (e.g. [8–12]). These X-rays could originate from accretion onto massive black holes in galactic nuclei, X-ray binaries, or inverse Compton scattering in supernova remnants [13]. Because the MFP of X-rays through the IGM is much longer than UV photons this will result in a more uniform background of ionizing photons. Thus, a significant contribution of X-rays can qualitatively change reionization by creating a partially ionized IGM in between the bubbles produced by UV photons.

In this paper we use semi-numerical simulations to gauge the imprint of X-ray sources during reionization on the kSZ signal. Because of the smooth ionization caused by X-rays, the so called “patchy” component of the kSZ power spectrum can be substantially reduced. We find that models with more than a 10% contribution from X-rays produce a significantly lower power spectrum of temperature anisotropies than all the UV-only models tested.

The paper is structured as follows. In §2, we describe our method of estimating the kSZ signal including a description of our semi-numerical simulations of reionization with and without a contribution from X-rays. In §3, we report our results, which include changes in the CMB power spectrum due to the contribution of X-ray sources to reionization. In §4 we summarize our main conclusions. Throughout, we assume a Λ CDM cosmology with $\Omega_\Lambda = 0.73$, $\Omega_m = 0.27$, $\Omega_b = 0.0456$, $h = 0.7$, and $\sigma_8 = 0.81$ [14].

II. METHOD

A. The Kinetic SZ Effect

The kSZ effect is the contribution to fluctuations in the CMB temperature associated with the Doppler shift of CMB photons Thompson scattered off free electrons moving at some radial velocity relative to the observer. The kSZ temperature fluctuation in a particular direction is given by

$$\frac{\Delta T}{T} = \sigma_T \int d\eta e^{-\tau(\eta)} a n_e v_{||}, \quad (1)$$

where σ_T is the Thomson cross section, $\tau(\eta)$ is the optical depth from the observer to the position corresponding to conformal time η , a is the cosmic scale factor, n_e is the number

density of free electrons, and v_{\parallel} is the component of the velocity along the line of sight.

We wish to calculate the impact on the kSZ effect from an X-ray contribution to reionization using semi-numerical simulations of the EoR. These simulations will examine varying levels of X-ray contribution to reionization. We generate a number of simulation cubes that each span a solid angle of one square degree on the sky (corresponding to a comoving length of $L \approx 150 - 200$ Mpc from $z = 6 - 20$) and have a resolution of 256^3 separate volume elements. In each 3D element of the simulation we calculate the matter over-density, velocity, and ionization fraction. Cubes are produced at appropriate cosmic times and stacked along the line of sight to give continuous coverage from a redshift of $z = 6$ to $z = 20$, corresponding to the EoR. We then perform the integral in Eq. (1) along each line of sight to produce a 2D map of CMB temperature fluctuations.

We compute the density and velocity fields of our simulations using the 21cmFAST software package [15]. Non-linearities are included using leading order perturbation theory (Zel'dovich approximation). We expect this to be a reasonable approximation, as the power spectra of the density field using this approach agree well with that from detailed hydrodynamic simulations at the redshifts and scales relevant for our calculations [15]. We generate our ionization fields with the model described below. Note that we do not produce data cubes at redshifts below $z = 6$. The kSZ fluctuations from these redshifts will be independent of reionization history and we incorporate their effect analytically.

B. Reionization Model

To simulate the reionization history we use a semi-numerical approach similar to that described in [11]. The basic assumption in our prescription is that there is a fixed number of ionizing photons, N_{ion} , released into the IGM for each baryon which is incorporated into galaxies. From extended Press-Schechter theory it is possible to calculate the rate at which matter will collapse into dark matter halos [16]. In this way one can calculate the ionized fraction in a region of a given size and density analytically. For the case without X-rays where all ionizing photons are assumed to be UV, the evolution of the ionization in a spherical region of comoving radius R with over-density δ relative to the cosmic mean is given by [17]

$$\frac{dQ_{R,\delta}}{dt} = \frac{N_{\text{ion}}}{0.76} \left(Q_{R,\delta} \frac{dF(M_{\text{ion}})}{dt} + (1 - Q_{R,\delta}) \frac{dF(M_{\text{min}})}{dt} \right) - B, \quad (2)$$

where the 0.76 accounts for the cosmic mass fraction of hydrogen, $Q_{R,\delta}$ is the mass averaged ionized fraction of hydrogen, and $F(M)$ (which depends implicitly of δ and R) is the fraction of mass which has collapsed into dark matter halos above mass M ,

$$F(M) = \text{erfc} \left(\frac{\delta_c(z) - \delta(z=0)}{\sqrt{2(\sigma(M)^2 - \sigma(M_R)^2)}} \right), \quad (3)$$

where $\delta_c = 1.686/D(z)$ is the threshold for spherical collapse at $z \gg 1$ extrapolated to redshift zero with linear theory, $D(z)$ is the linear growth factor of density fluctuations, $\delta(z=0)$ is the linear over-density of the region at the current time, and the σ 's are the root mean squared density fluctuations on scales corresponding to mass M and radius R today. The minimum mass of dark matter halos that can host galaxies is denoted by M_{min} in the

neutral IGM and by M_{ion} in the ionized and photo-heated IGM. We parameterize this mass in terms of the virial temperature which for $z \gg 1$ is given by

$$T_{\text{vir}} = 10^4 \left(\frac{M}{10^8 M_{\odot}} \right)^{1/3} \left(\frac{1+z}{10} \right) \text{ K.} \quad (4)$$

The recombination rate is given by

$$B = \alpha_B C n_{\text{H}}^0 (1+z)^3 (1+\delta) Q_{R,\delta}, \quad (5)$$

where $\alpha_B = 2.6 \times 10^{-13} \text{ cm}^3/\text{s}$ is the case B recombination coefficient for $T = 10^4 \text{ K}$ [18], $C = \langle n_e^2 \rangle / \langle n_e \rangle^2$ is the clumpiness of the ionized gas in the IGM and n_{H}^0 is the density of hydrogen gas today.

In order to produce the 3D simulation of the ionization fraction we smooth a random realization of the linear density field in our simulation in spheres of varying radius and solve Eq. (3) as a function of the size and over-density at each point. At this stage, regions for which $Q_{R,\delta} > 1$ are taken to be bubbles and the ionization fraction is set to one while other regions are taken to be completely neutral. Next, the equation is solved on the scale of each spatial pixel and then used to determine the ionization fraction in regions not encompassed by ionization bubbles.

We use a similar approach to calculate the ionization fraction when X-rays make a significant contribution to reionization. This approach has been adapted from [17]. The key simplifying assumption we make is that X-rays have a fixed comoving MFP and that X-ray photons uniformly ionize regions of that size. The comoving MFP of X-rays of energy E_X through a neutral medium is given by [13, 19],

$$\lambda \approx 180 \left(\frac{1+z}{15} \right)^{-2} \left(\frac{E_X}{1 \text{ keV}} \right)^3 \text{ Mpc.} \quad (6)$$

We assume a constant MFP of $\lambda = 20 \text{ Mpc}$, but find that the measured angular power spectrum of temperature anisotropies is essentially unchanged if we set $\lambda = 50 \text{ Mpc}$. UV photons have a very short MFP compared to the size of the spatial pixels in our simulation. Thus, when considering a region smaller than the MFP of X-rays one must consider the X-ray sources from a larger surrounding region with a radius equal to the X-ray MFP, λ , while the ionization from UV photons will only depend on the local over-density. We capture this behavior by solving a system of 6 coupled ordinary differential equations (ODEs) which compute the ionization history (including the relative amounts from X-rays versus UV photons) on various scales. As in the UV only case, we smooth the density field on all scales to identify ionized bubbles, but in addition simultaneously track the ionization history and production of X-ray photons from surrounding regions with radius equal to the X-ray MFP (λ). The system of 6 ODEs (adapted from [17]) is

$$\frac{dQ_R}{dt} = \frac{dQ_{R,UV}}{dt} + \frac{dQ_{R,X}}{dt} \quad (7)$$

$$\frac{dQ_{\lambda}}{dt} = \frac{dQ_{\lambda,UV}}{dt} + \frac{dQ_{\lambda,X}}{dt} \quad (8)$$

$$\frac{dQ_{R,UV}}{dt} = (1 - X_{\text{frac}}) \frac{N_{\text{ion}}}{0.76} \left(Q_R \frac{dF(M_{\text{ion}})}{dt} + (1 - Q_R) \frac{dF(M_{\text{min}})}{dt} \right) - B_3 \quad (9)$$

$$\frac{dQ_{\lambda,UV}}{dt} = (1 - X_{\text{frac}}) \frac{N_{\text{ion}}}{0.76} \left(Q_{\lambda} \frac{dF(M_{\text{ion}})}{dt} + (1 - Q_{\lambda}) \frac{dF(M_{\text{min}})}{dt} \right) - B_4 \quad (10)$$

$$\frac{dQ_{R,X}}{dt} = X_{\text{frac}} \frac{N_{\text{ion}}}{0.76} \left(Q_R \frac{dF(M_{\text{ion}})}{dt} + (1 - Q_R) \frac{dF(M_{\text{min}})}{dt} \right) - B_5 \quad \text{if } R > \lambda \quad (11)$$

$$\frac{dQ_{R,X}}{dt} = X_{\text{frac}} \frac{N_{\text{ion}}}{0.76} \left(\frac{1 + \delta_{\lambda}}{1 + \delta_R} \right) \left(Q_{\lambda} \frac{dF(M_{\text{ion}})}{dt} + (1 - Q_{\lambda}) \frac{dF(M_{\text{min}})}{dt} \right) - B_5 \quad \text{if } R < \lambda \quad (12)$$

$$\frac{dQ_{\lambda,X}}{dt} = X_{\text{frac}} \frac{N_{\text{ion}}}{0.76} \left(Q_{\lambda} \frac{dF(M_{\text{ion}})}{dt} + (1 - Q_{\lambda}) \frac{dF(M_{\text{min}})}{dt} \right) - B_6, \quad (13)$$

where Q_R and Q_{λ} are the mass averaged ionization fractions on scales of R and λ respectively. $Q_{R,UV}$ is the ionization on scale R due to UV photons and $Q_{R,X}$, $Q_{\lambda,X}$, and $Q_{\lambda,UV}$ are the various ionization fractions from either X-rays or UV photons ionization on the two scales of interest. The fraction of ionizing photons which are X-rays is denoted by X_{frac} . Note that the time derivative of the collapsed fraction in each equation is evaluated on the same scale as the Q value it is multiplied with.

The various B_i terms are recombination rates. In the UV case, Eq. (5) was derived by assuming that all regions are either completely ionized or neutral. With a significant contribution from X-rays this will not be the case. We assume that X-rays uniformly ionize the IGM on scale λ and UV photons completely ionize bubbles in the IGM around their sources. In order to calculate the recombination terms we calculate the volume of the IGM which is uniformly ionized, V_{uniform} , and the fraction in ionized bubbles, V_{bub} . The uniform X-ray ionization also allows bubbles to grow larger than they would from the UV photons alone; we denote this additional “fringe” volume with V_{fringe} . These add up to the total volume of the spherical regions we consider, V , and are described by

$$V_{\text{bub}}/V = Q_{UV} \quad (14)$$

$$V_{\text{fringe}}/V = \frac{Q_{UV}Q_X}{1 - Q_X} \quad (15)$$

$$V_{\text{uniform}} = V - V_{\text{bub}} - V_{\text{fringe}}, \quad (16)$$

where either the UV or X-ray ionization must be evaluated on the relevant length scale. It follows that various recombination rate terms are given by

$$B_3 = \alpha_B C n_{\text{H}}^0 (1 + z)^3 (1 + \delta_R) \frac{V_{\text{bub}}}{V}, \quad (17)$$

$$B_5 = \alpha_B C n_{\text{H}}^0 (1 + z)^3 (1 + \delta_R) \left(\frac{V_{\text{fringe}}}{V} + Q_{R,X}^2 \frac{V_{\text{uniform}}}{V} \right). \quad (18)$$

B_4 and B_6 are the same as B_3 and B_5 except that the appropriate values for δ_{λ} and the volumes corresponding to scale λ must be used. Note that this treatment of the recombination rates was not included in previous work. We do not include heating due to X-rays which could further suppress the formation of low mass galaxies. This would be straightforward to incorporate in the future [17].

TABLE I: Parameters for the different reionization models used. For the minimum masses of dark matter halos which host galaxies, the corresponding virial temperature (assumed to be constant) is listed instead of the mass (see Eq. (4) in the text).

Model	N_{ion}	C	$T_{\text{vir}}(M_{\text{min}})$	$T_{\text{vir}}(M_{\text{ion}})$	X_{frac}	λ	τ
Fiducial	60	2	10^4K	10^5K	0	-	0.088
Clump	355	20	10^4K	10^5K	0	-	0.088
M_{min}	115	2	$5 \times 10^4\text{K}$	10^5K	0	-	0.088
10% X-ray	53	2	10^4K	10^5K	0.1	20 Mpc	0.088
30% X-ray	45	2	10^4K	10^5K	0.3	20 Mpc	0.088
50% X-ray	40	2	10^4K	10^5K	0.5	20 Mpc	0.088

We smooth our density field on all scales and if $Q_R > 1$ the region R is set to be completely ionized. After the entire cube has been searched on all relevant scales we solve Eqs. (7-13) on the scale of each spatial pixel to determine the ionization fraction outside of the completely ionized bubbles.

III. RESULTS

In order to test the impact of X-ray contribution to reionization on the kSZ effect we produce several different simulations using the model above. The model parameters which can be adjusted are N_{ion} , C , M_{min} , M_{ion} , X_{frac} , and λ . We produce several simulations with different parametrizations, but which have an optical depth to the surface of last scattering, τ , consistent with WMAP measurements ($\tau = 0.088 \pm 0.015$) [14]. For simplicity, we ignore the residual ionization from cosmological recombination (prior to reionization). From these models we can evaluate the effect of a varying level of X-ray contribution to reionization. The values of the various parameters for each model are listed in Table I. Due to the constraint on τ , our models have fairly similar global reionization histories. These are shown in Fig. 1.

The main difference with X-rays is that their relatively long MFP would cause some smooth ionization fraction throughout the IGM in regions which are not inside bubbles. This is illustrated clearly in Figure 2. We plot the ionization fraction through a slice of our simulation at $z = 9$ for the “fiducial” and “50% X-ray” models. In both cases we have used the same underlying density field and the global mass weighted ionization fraction is roughly the same. There is a striking difference in the character of reionization between these two models. The UV-only case is dominated by completely ionized bubbles with surrounding neutral regions. The model with X-rays has smaller bubbles surrounded by a partially ionized IGM.

In order to quantify the change in the kSZ signal due to X-rays, we compute the angular power spectrum of CMB temperature fluctuations, C_l . We calculate the kSZ temperature fluctuations from the EoR ($z = 6 - 20$) by integrating Eq. (1) through our simulations. The resulting 2D temperature field is then Fourier transformed to compute the power spectrum. We include the kSZ signal from lower redshift by calculating the power spectrum analytically and adding it to the EoR result. This is necessary because at low redshifts non-linear effects become significant and they are not accurately captured with the density fields we use from 21cmFAST. We calculate the analytic power spectrum using the method described in [2] (see their Eq. (4)). To account for the primordial temperature fluctuations, which

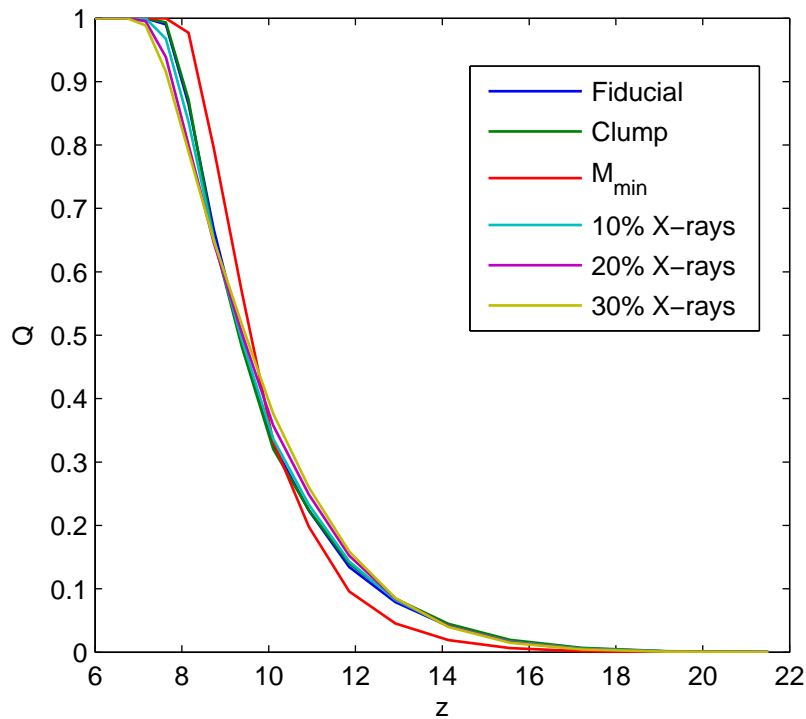


FIG. 1: The mass weighted global ionization fraction as a function of redshift for the different reionization models.

dominate on large angular scales, we add the lensed C_l 's calculated from CAMB [20]. We make the assumption that the thermal Sunyaev-Zel'dovich effect, with its distinctive spectral signature, will be removed completely.

All of the models tested were designed to have nearly identical optical depth due to electron scattering. Of course there will always be some uncertainty in value of this quantify. For example, Planck measurements will constrain τ to within ≈ 0.005 [21]. We find that this uncertainty should not strongly affect our results. As an example, for the “fiducial” model, we vary N_{ion} within the corresponding uncertainty in τ and find that the power spectrum never goes as low as the 10% X-ray model for the l values in Figure 3.

In addition to plotting the theoretical estimates, we also show the error bars of SPT in Fig. 4 [22]. SPT will only be sensitive to the kSZ signal at $l \approx 3000$ due to the high amplitude of the primordial temperature fluctuations at lower l and dusty star-forming galaxies at higher l . Detailed estimates by the SPT team project an ultimate sensitivity of $\Delta(C_l l(1+l)/(2\pi)) = 1\mu\text{K}^2/\bar{T}_{\text{CMB}}^2$. ACT [23] will be a couple times less sensitive (Matthew McQuinn, private communication 2011).

IV. DISCUSSION AND CONCLUSIONS

We have calculated the effect of an X-ray contribution to reionization on the fluctuations in the CMB from the kSZ effect. This was accomplished with semi-numerical simulations of reionization. For the case with no X-ray contribution, we tested how changes in the

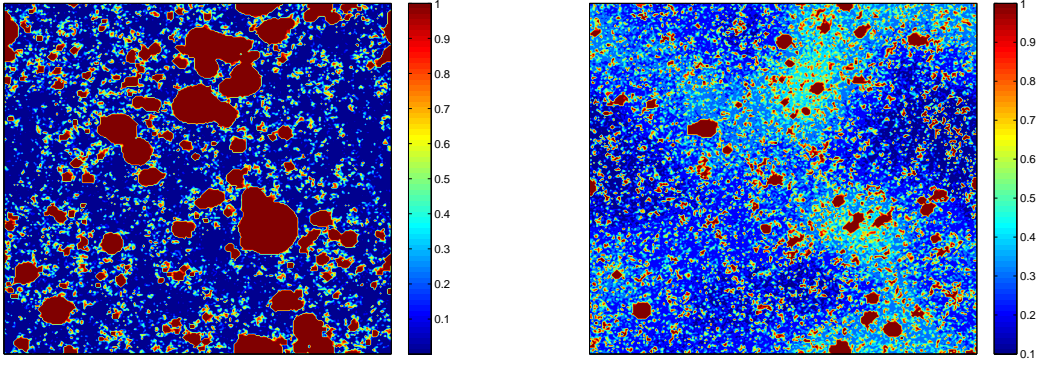


FIG. 2: The ionization fraction through a slice of our “fiducial” (left panel) and “50% X-ray” (right panel) model at $z = 9$. Note that the same underlying density field has been used for both. The slice corresponds to a $170 \times 170 \times 0.66 \text{ Mpc}^3$ volume in the simulation. The color bar gives the mass weighted neutral fraction in each pixel.

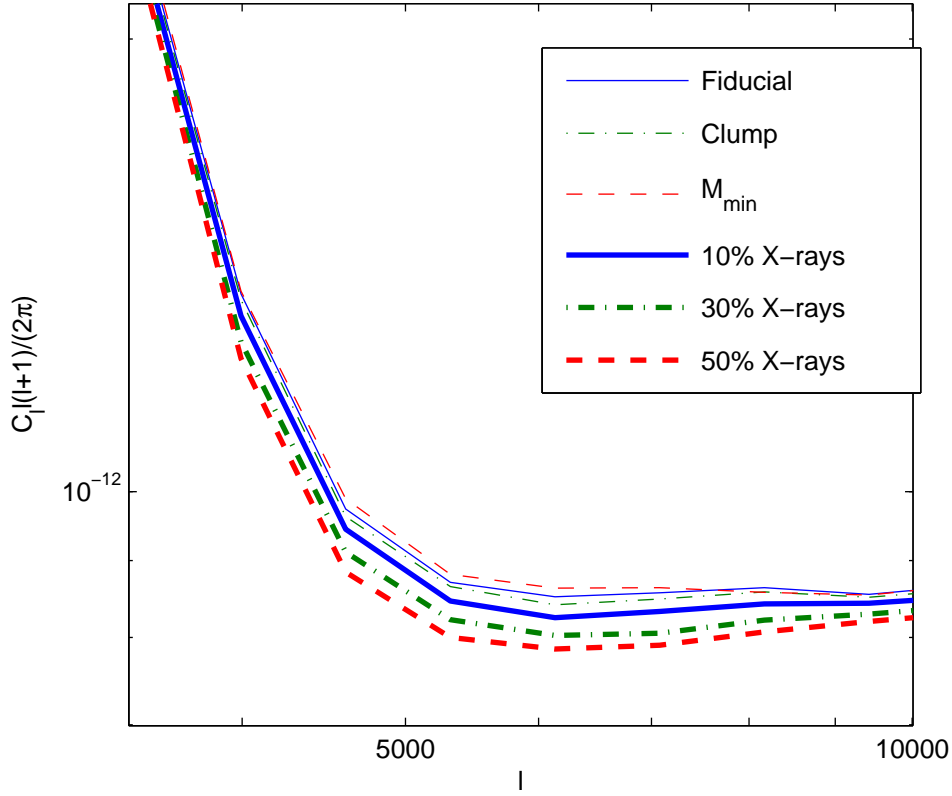


FIG. 3: The angular power spectrum of temperature fluctuations for the different reionization models tested.

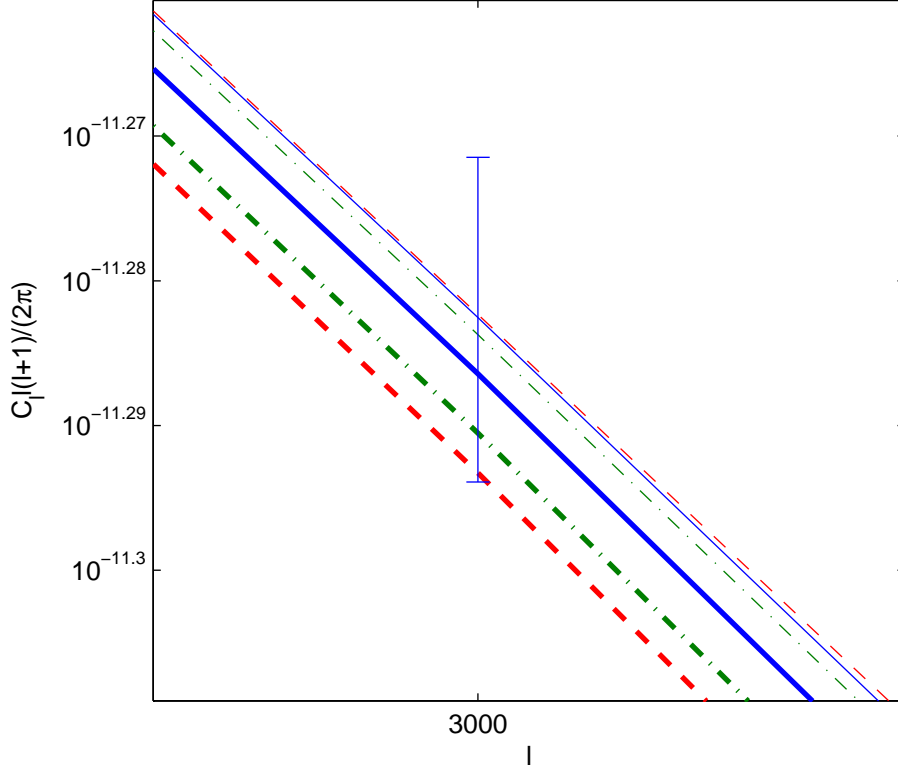


FIG. 4: The angular power spectrum of temperature fluctuations at $l = 3000$, where ACT and SPT will be sensitive to kSZ signal. The expected accuracy of SPT, $\Delta (C_l l(1+l)/(2\pi)) = 1\mu\text{K}^2/\bar{T}_{\text{CMB}}^2$, is shown as an error bar. The various reionization models are shown with the same lines as Fig. 3.

physical parameters, including the number of ionizing photons produced, the clumpiness of the IGM and the minimum mass of dark matter halos which can host galaxies affect the kSZ signal. This was then compared with various levels of X-ray contribution to reionization parametrized by the fraction of ionizing photons which are X-rays, X_{frac} .

Our reionization model assumes that a constant number of ionizing photons are produced for each baryon which is incorporated into galaxies. We then calculate how many baryons have collapsed into dark matter halos at a given redshift using the extended Press-Schechter formalism. UV photons have a MFP much smaller than the pixel scale in our simulations while X-rays are assumed to uniformly ionize regions with a fixed comoving MFP, for simplicity. Varying the MFP between $\lambda = 20$ Mpc and $\lambda = 50$ Mpc yields very similar results, suggesting that our conclusions are robust.

Due to the comparatively long MFP of X-rays, they can partially ionize the regions in between the ionized bubbles created by UV photons. For a fixed number of total ionizing photons this results in smaller bubbles surrounded by partially ionized IGM. This affects the kSZ signal by reducing the “patchy” component of the fluctuations, associated with the non-uniform topology of reionization. In this way it is possible to distinguish models which have a very similar global reionization history but different X-ray contributions.

In particular, we find that an X-ray contribution greater than $X_{\text{frac}} = 10\%$ produces

a substantially lower power spectrum than our UV-only models constrained to have equal total optical depth to electron scattering. This suggests that it may be possible to constrain the X-ray contribution to reionization with future kSZ observations. Unfortunately, the sensitivity of SPT and ACT may be insufficient to distinguish the models presented in this paper. However, a non-detection of the kSZ signal from the EoR may suggest a significant X-ray contribution. We find that our X-ray models push the kSZ power from reionization below the anticipated sensitivity of SPT and ACT. Because annihilation or decay of dark matter particles could imprint a similar uniform ionization component, such a non-detection could also be used to constrain the properties of dark matter.

V. ACKNOWLEDGMENTS

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