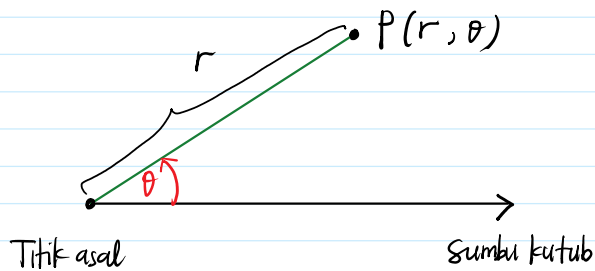
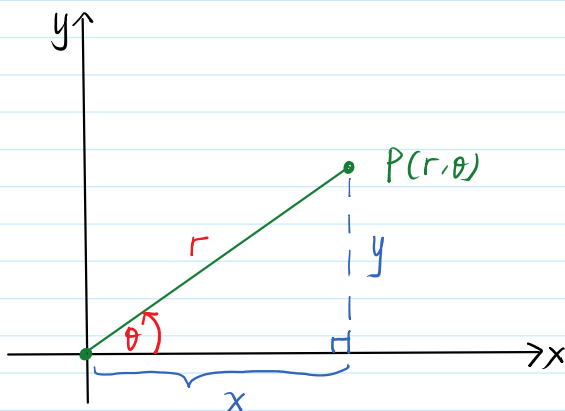


## Materi : Koordinat Kutub (Grafik, Luasan, Panjang Busur & Garis Singgung)

### A. Koordinat Kutub



### B. Hubungan Koordinat Kutub dengan Koordinat Siku-siku



•> Menentukan  $x$

$$\cos \theta = \frac{x}{r} \longrightarrow \boxed{x = r \cdot \cos \theta}$$

•> Menentukan  $y$

$$\sin \theta = \frac{y}{r} \longrightarrow \boxed{y = r \cdot \sin \theta}$$

$$\begin{aligned} \bullet \rightarrow x^2 + y^2 &= (r \cdot \cos \theta)^2 + (r \cdot \sin \theta)^2 \\ &= r^2 \cdot \cos^2 \theta + r^2 \cdot \sin^2 \theta \\ &= r^2 (\cos^2 \theta + \sin^2 \theta) \\ &= r^2 \end{aligned}$$

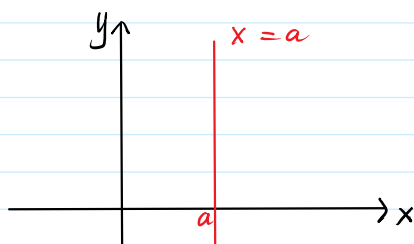
$$\boxed{x^2 + y^2 = r^2}$$

$$\bullet \rightarrow \tan \theta = \frac{y}{x} \longrightarrow \boxed{\theta = \tan^{-1} \frac{y}{x}}$$

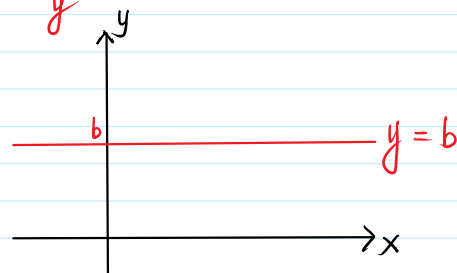
### C. Grafik dalam Koordinat Kutub

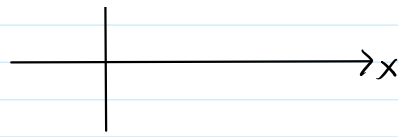
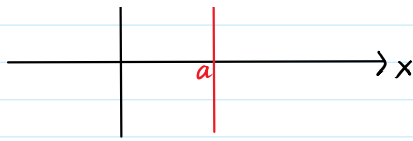
#### (I) Garis

(i)  $\underbrace{r \cos \theta}_x = a$



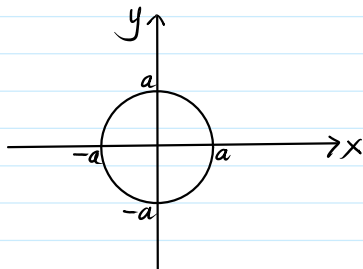
(ii)  $\underbrace{r \sin \theta}_y = b$



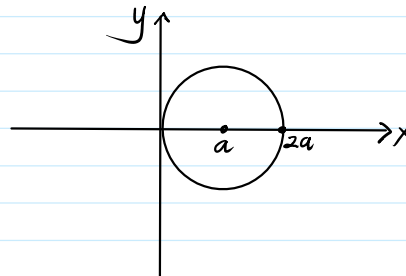


## II Lingkaran

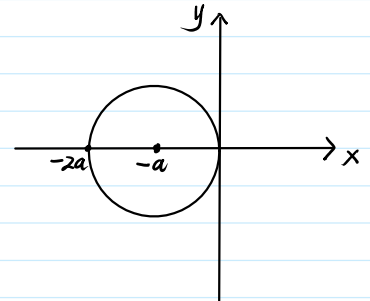
(i)  $r = a$



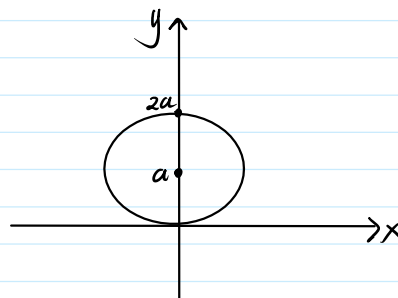
(ii)  $r = 2a \cos \theta$



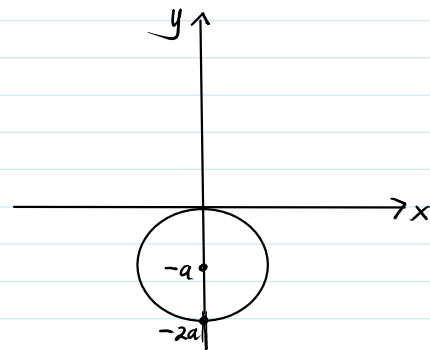
(iii)  $r = -2a \cos \theta$



(iv)  $r = 2a \sin \theta$



(v)  $r = -2a \sin \theta$



## III Limacon

$\rightarrow r = a \pm b \sin \theta$

$\rightarrow r = a \pm b \cos \theta$

(i)  $\frac{a}{b} < 1$  ✓

Limacon dgn bundaran dalam



(ii)  $\frac{a}{b} = 1$  ✓

Kardioida



(iii)  $1 < \frac{a}{b} < 2$

Limacon cekung



(iv)  $\frac{a}{b} > 2$

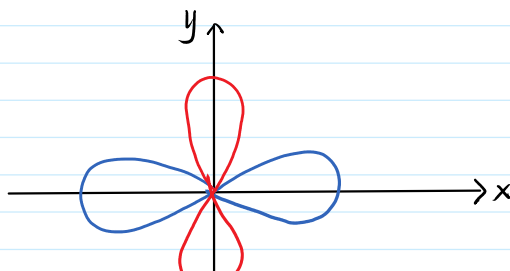
Limacon cembung

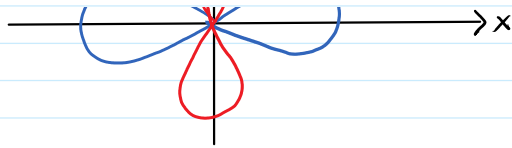
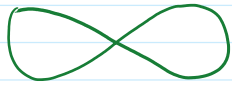


## IV Lemniscate

$\rightarrow r^2 = \pm a^2 \cos 2\theta$  ∞

$\rightarrow r^2 = \pm a^2 \sin 2\theta$  8

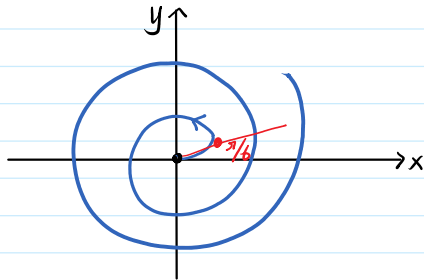




## (V) Spiral

→  $r = a \cdot \theta$  ( $\theta \geq 0$ ) → berlawanan arah jarum jam

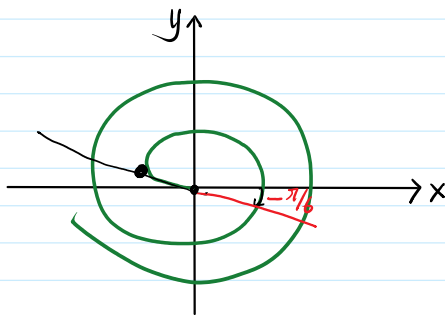
Contoh:  $r = \theta$  ( $\theta \geq 0$ )



$\theta$	0	$\frac{\pi}{6}$
$r$	0	$\frac{\pi}{6}$

→  $r = a \cdot \theta$  ( $\theta \leq 0$ ) → searah jarum jam

Contoh:  $r = \theta$  ( $\theta \leq 0$ )



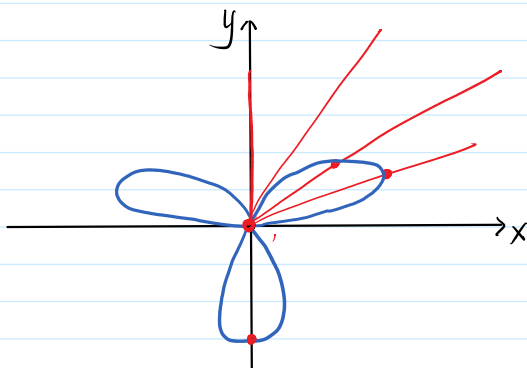
$\theta$	0	$-\frac{\pi}{6}$
$r$	0	$-\frac{\pi}{6}$

## (VI) Rose

→  $r = a \cdot \sin(n\theta)$  ;  $n = \text{ganjil}$  → jumlah daun =  $n$

→  $r = a \cdot \cos(n\theta)$  ;  $n = \text{genap}$  → jumlah daun =  $2n$

Contoh:  $r = 2 \sin 3\theta$  ;  $n = 3$  (ganjil) → jumlah daun = 3



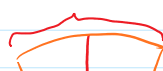
$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r$	0	2	$\sqrt{2}$	0	-2

## [D] Luasan dalam koordinat kutub

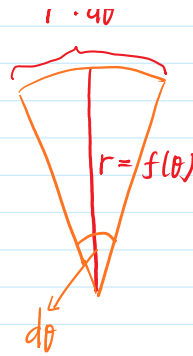
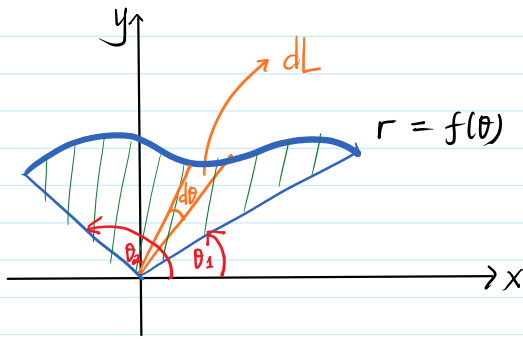


→  $dL$

$r \cdot d\theta$



$$dL = \frac{1}{2} r \cdot d\theta \cdot r$$



$$dL = \frac{1}{2} r \cdot d\theta \cdot r$$

$$dL = \frac{1}{2} r^2 d\theta$$

$$dL = \frac{1}{2} [f(\theta)]^2 d\theta$$

$$L = \int_{\theta_1}^{\theta_2} \frac{1}{2} [f(\theta)]^2 d\theta$$

**E** Garis Singgung dalam Koordinat Kutub

$$\rightarrow x = r \cdot \cos \theta$$

$$\rightarrow y = r \cdot \sin \theta$$

$r$  adalah fungsi dalam variabel  $\theta \rightarrow r = f(\theta)$ .

Contoh :  $r = 2 \sin \theta$   
atau

$r = 3 \cos 2\theta$  ; dst

$$\rightarrow \text{Cari : } \frac{dx}{d\theta}$$

$$x = r \cdot \cos \theta \rightarrow \frac{dx}{d\theta} = \frac{dr}{d\theta} \cdot \cos \theta + r \cdot -\sin \theta$$

$u \cdot v = u'v + uv'$

$$\rightarrow \text{Cari : } \frac{dy}{d\theta}$$

$$y = r \cdot \sin \theta \rightarrow \frac{dy}{d\theta} = \frac{dr}{d\theta} \cdot \sin \theta + r \cdot \cos \theta$$

$u \cdot v = u'v + uv'$

$$\rightarrow \text{Cari : } \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \rightarrow \frac{dy}{dx} = \frac{\sin \theta \cdot \frac{dr}{d\theta} + r \cdot \cos \theta}{\cos \theta \cdot \frac{dr}{d\theta} - r \cdot \sin \theta}$$

$$\text{Nilai gradien} \rightarrow m = \left. \frac{dy}{dx} \right|_{\theta = \alpha}$$

$$\text{Dari } r = f(\theta) \rightarrow \text{cari } x \text{ \& } y \rightarrow \text{Titik } (x, y)$$

$x = r \cos \theta$        $y = r \sin \theta$

Persamaan garis singgung dgn gradien  $m$  & melalui titik  $(x_0, y_0)$

|| - ||      || - ||      || - ||      || - ||

Persamaan garis singgung dgn gradien  $m$  & melalui titik  $(x_0, y_0)$

$$y - y_0 = m(x - x_0)$$

**F** Panjang Busur dalam Koordinat Kutub

$$r = f(\theta) ; \quad \theta_1 \leq \theta \leq \theta_2$$

$$S = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Latihan soal

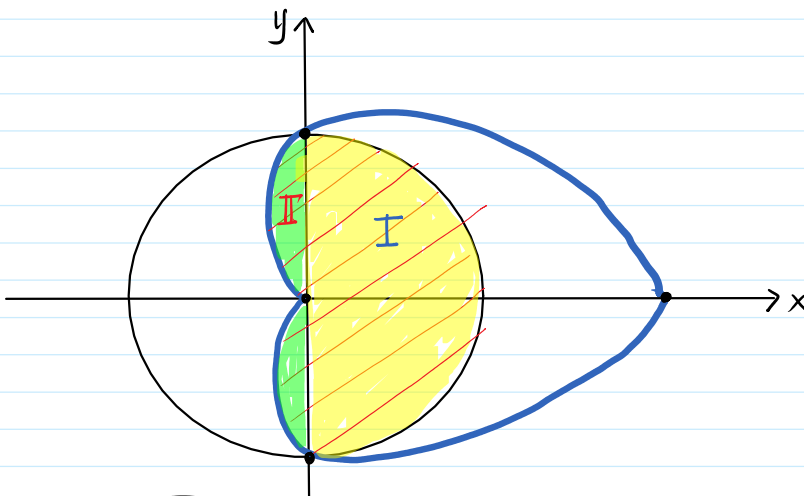
① Dapatkan luas daerah irisan  $r = 4$  &  $r = 4 + 4 \cos \theta$ .

lingkaran  
pusat  $(0,0)$   
jari-jari = 4

$$r = a + b \cos \theta$$

$$\frac{a}{b} = \frac{4}{4} = 1$$

Kardioida



Kardioida :  $r = 4 + 4 \cos \theta$

$\theta$	0	$\pi/2$	$\pi$	$3\pi/2$
$r$	8	4	0	4

Titik potong :

$$r = r$$

$$4 = 4 + 4 \cos \theta$$

$$0 = 4 \cos \theta$$

$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2} \quad \& \quad \theta = \frac{3\pi}{2}$$

$$L = \int_{\theta_1}^{\theta_2} \frac{1}{2} [f(\theta)]^2 d\theta$$

$$L_{\text{total}} = 2 (L_I + L_{II})$$

$$\begin{aligned} \Rightarrow L_I &= \int_0^{\pi/2} \frac{1}{2} \cdot (4^2) d\theta \\ &= \int_0^{\pi/2} 8 d\theta \\ &= 8\theta \Big|_0^{\pi/2} \\ &= 8 \left( \frac{\pi}{2} - 0 \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow L_{II} &= \int_{\pi/2}^{\pi} \frac{1}{2} (4 + 4 \cos \theta)^2 d\theta \\ &= \int_{\pi/2}^{\pi} \frac{1}{2} (16 + 32 \cos \theta + 16 \cos^2 \theta) d\theta \\ &= \int_{\pi/2}^{\pi} 8 + 16 \cos \theta + 8 \cos^2 \theta d\theta \\ &= 8 \left( \frac{1}{2} + \frac{1}{2} \cos 2\theta \right) \end{aligned}$$

$$\begin{aligned}
 &= 8 \left( \frac{\pi}{2} - 0 \right) \\
 &= 4\pi
 \end{aligned}$$

$$\begin{aligned}
 &\int_{\pi/2}^{\pi} 0 + 16 \cos \theta + 4 \cos 2\theta \, d\theta \\
 &= \int_{\pi/2}^{\pi} 12 + 16 \cos \theta + 4 \cos 2\theta \, d\theta \\
 &= 12\theta + 16 \sin \theta + \frac{4}{2} \sin 2\theta \Big|_{\pi/2}^{\pi} \\
 &= (12\pi + 0 + 0) - (6\pi + 16 + 0) \\
 &= 6\pi - 16
 \end{aligned}$$

$$\begin{aligned}
 L_{\text{total}} &= 2(4\pi + 6\pi - 16) \\
 &= 2(10\pi - 16) \\
 &= 20\pi - 32 \text{ satuan luas}
 \end{aligned}$$

② Dapatkan persamaan garis singgung dari kurva  $r = 2 \cos \theta$  di  $\theta = \frac{\pi}{3}$ .

$$\begin{aligned}
 \Rightarrow x &= r \cos \theta \\
 x &= 2 \cos \theta \cdot \cos \theta \\
 x &= 2 \cos^2 \theta
 \end{aligned}$$

$$\begin{aligned}
 \frac{dx}{d\theta} &= 2 \cdot 2 \cos \theta \cdot -\sin \theta \\
 &= -4 \sin \theta \cdot \cos \theta
 \end{aligned}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2 \cos 2\theta}{-4 \sin \theta \cdot \cos \theta}$$

$$\begin{aligned}
 m &= \frac{dy}{dx} \Big|_{\theta = \frac{\pi}{3}} \\
 &= \frac{2 \cos \frac{2\pi}{3}}{-4 \sin \frac{\pi}{3} \cdot \cos \frac{\pi}{3}}
 \end{aligned}$$

$$= \frac{2(-\frac{1}{2})}{-4 \cdot \frac{1}{2} \sqrt{3} \cdot \frac{1}{2}} = \frac{-1}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \sqrt{3}$$

$$\Rightarrow x = 2 \cos^2 \theta$$

$$\Rightarrow y = 2 \sin 2\theta$$

$$\begin{aligned}
 \therefore x &= 2 \cos^2 \theta \\
 &= 2 \cos^2 \frac{\pi}{3} \\
 &= 2 \left( \frac{1}{2} \right)^2 \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore y &= \sin 2\theta \\
 &= \sin \frac{2\pi}{3} \\
 &= \frac{1}{2} \sqrt{3} \\
 &\left( \frac{1}{2}, \frac{1}{2} \sqrt{3} \right)
 \end{aligned}$$

$\therefore$  PGS dgn  $m = \frac{1}{3} \sqrt{3}$  & titik  $\left( \frac{1}{2}, \frac{1}{2} \sqrt{3} \right)$

$$y - y_0 = m(x - x_0)$$

$$y - \frac{1}{2} \sqrt{3} = \frac{1}{3} \sqrt{3} \left( x - \frac{1}{2} \right)$$

$$y - \frac{1}{2} \sqrt{3} = \frac{1}{3} \sqrt{3} x - \frac{1}{6} \sqrt{3}$$

$$y = \frac{1}{3} \sqrt{3} x + \frac{1}{3} \sqrt{3}$$