

## Pertemuan 7 - Juni

Saturday, 15 June 2024  
19.29

5. Dapatkan luas daerah yang dibatasi oleh kurva  $y = x$ ,  $y = 4x$ , dan  $y = 2 - x$ . Sketsa grafiknya. (ETS 2022/2023, Senin 27 Maret 2023)

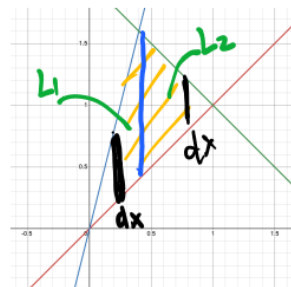
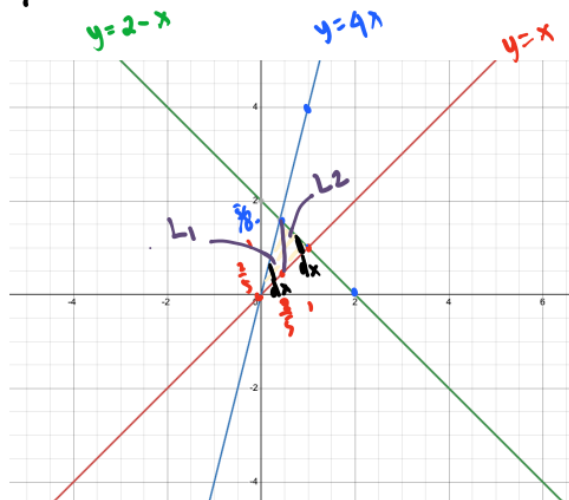
Jawab

① Mencari titik potong

• $y_1 = y_2$	• $y_1 = y_3$	• $y_2 = y_3$
$x = 4x$	$x = 2 - x$	$4x = 2 - x$
$0 = 3x$	$2x = 2$	$5x = 2$
$0 = x$	$x = 1$	$x = \frac{2}{5}$

② Gambar

• $y = x$	• $y = 4x$	• $y = 2 - x$
$x   y$	$x   y$	$x   y$
0   0	0   0	0   2
$\frac{2}{5}   \frac{2}{5}$	$\frac{2}{5}   \frac{8}{5}$	$\frac{2}{5}   \frac{8}{5}$
1   1	1   4	1   1



③. Luas

$$dL = dL_1 + dL_2$$

$$dL = (4x - x) dx + (2 - x - x) dx$$

$$L = \int_0^{\frac{2}{5}} 4x - x \, dx + \int_{\frac{2}{5}}^1 (2 - x - x) \, dx$$

$$= \int_0^{\frac{2}{5}} 3x \, dx + \int_{\frac{2}{5}}^1 2 - 2x \, dx$$

$$= \frac{3}{2} x^2 \Big|_0^{\frac{2}{5}} + [2x - x^2] \Big|_{\frac{2}{5}}^1$$

$$= \frac{3}{2} \left(\frac{2}{5}\right)^2 - 0 + [2 \cdot 1 - 1^2] - \left[2 \cdot \frac{2}{5} - \left(\frac{2}{5}\right)^2\right]$$

$$= \dots \text{ satuan Luas}$$

5. Dapatkan volume benda padat yang terjadi bila daerah yang dibatasi oleh  $y = \sqrt{x}$ ,  $y = x^2$  diputar terhadap garis  $y = 1$ . (ETS 2021/2022, Selasa 29 Maret 2022)

Jawab:

① Titik

$$y_1 = y_2$$

$$\sqrt{x} = x^2$$

$$x = x^4$$

$$0 = x^4 - x$$

$$0 = x(x^3 - 1)$$

$$x=0 \vee x=1$$

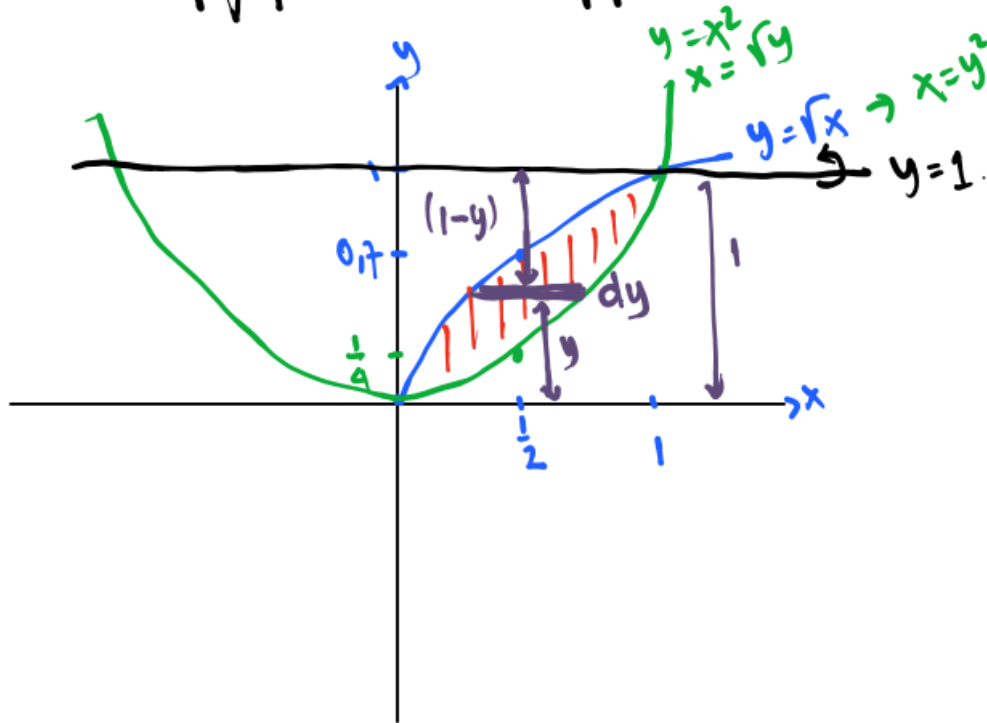
② Gambar

$$\bullet y = \sqrt{x}$$

$$\bullet y = x^2$$

x	y
0	0
$\frac{1}{2}$	$\sqrt{\frac{1}{2}} = 0,707$
1	1

x	y
0	0
$\frac{1}{2}$	$\frac{1}{4} = 0,25$
1	1



$dV = 2\pi \cdot \text{jarak partisi ke sumbu} \cdot (\text{K. Kanan} - \text{K. Kiri}) dy$

$$dV = 2\pi (1-y) (\sqrt{y} - y^2) dy$$

$$V = \int_0^1 2\pi (1-y) (\sqrt{y} - y^2) dy$$

$$\begin{aligned}
&= \int_0^1 2\pi (1-y) (y^{1/2} - y^2) dy \\
&= 2\pi \int_0^1 y^{1/2} - y^2 - y^{3/2} + y^3 dy \\
&= 2\pi \left[ \frac{2}{3} y^{3/2} - \frac{y^3}{3} - \frac{2}{5} y^{5/2} + \frac{1}{4} y^4 \right] \Big|_0^1 \\
&= 2\pi \left[ \frac{2}{3} - \frac{1}{3} - \frac{2}{5} + \frac{1}{4} \right] - 0
\end{aligned}$$

= ... Satuan Volume.

1. Dapatkan panjang busur kurva  $y = 3x^{\frac{3}{2}} - 1$  dari  $x = 0$  ke  $x = 1$ .

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$S = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \text{atau}$$

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$S = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

jawab:

$$y = 3x^{\frac{3}{2}} - 1, \quad x=0 \text{ ke } x=1$$

$$\frac{dy}{dx} = 3 \cdot \frac{3}{2} x^{\frac{1}{2}} = \frac{9}{2} x^{\frac{1}{2}}$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$ds = \sqrt{1 + \left(\frac{9}{2} x^{\frac{1}{2}}\right)^2} dx$$

$$S = \int_0^1 \sqrt{1 + \left(\frac{9}{2} x^{\frac{1}{2}}\right)^2} dx$$

$$= \int_0^1 \sqrt{1 + \frac{81}{4} x} \quad \text{dx}$$

Misal

$$u = 1 + \frac{81}{4} x$$

Batas

$$x=0 \rightarrow u = 1 + \frac{81}{4}(0)$$

$$du = \frac{81}{4} dx$$

$$u = 1$$

$$x=1 \rightarrow u = 1 + \frac{81}{4}$$

$$\frac{4}{81} du = dx$$

$$= \frac{85}{4}$$

$$S = \int_1^{\frac{85}{4}} \sqrt{u} \cdot \frac{4}{81} du$$

$$= \frac{4}{81} \int_1^{\frac{85}{4}} u^{1/2} du$$

$$= \frac{4}{81} \left[ \frac{2}{3} u^{3/2} \right]_1^{\frac{85}{4}}$$

$$= \frac{4}{81} \left[ \frac{2}{3} \left[ \frac{85}{4} \right]^{3/2} \right] - \frac{4}{81} \left[ \frac{2}{3} (1)^{3/2} \right]$$

$$= \dots \text{Saklan panjang.}$$

2. Dapatkan volume benda putar dengan dalil Guldin I dari daerah yang dibatasi  $y = x^2 + 4$  dan  $x + y = 6$  jika diputar terhadap garis  $y = 6 - x$ . (EAS 2022/2023, Senin 12 Juni 2023)

$$y = 6 - x$$

$$\downarrow$$

$$x + y - 6 = 0$$

Volume Guldin 1

$$V = 2\pi \cdot d \cdot L$$

$d$  = jarak titik berat ke sb putar

$L = \text{Luas}$

Jawab

① Titik dan gambar

$$y_1 = y_2$$

$$\bullet y = x^2 + 4$$

$$\bullet y = 6 - x$$

$$x^2 + 4 = 6 - x$$

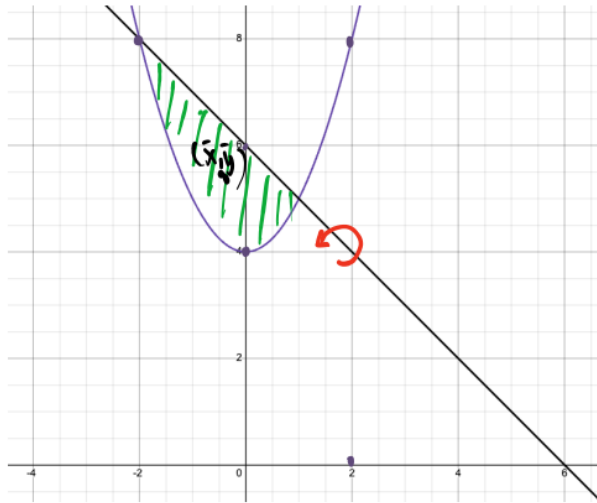
$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2 \vee x = 1$$

x	y
-2	8
0	4
1	5

x	y
-2	8
0	6
1	5



② Titik berat

$$\bar{x} = \frac{My}{M} ; \bar{y} = \frac{Mx}{M}$$

$$\bullet M = \int_a^b y_1 - y_2 \, dx$$

$$= \int_{-2}^1 6 - x - (x^2 + 4) \, dx$$

$$= \int_{-2}^1 -x^2 - x + 2 \, dx$$

$$= \left[ -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \right]_{-2}^1$$

$$= \frac{9}{2} \text{ (hitung rinciannya ya)}$$

$$\bullet M_y = \int_a^b x(y_1 - y_2)^2 dx$$

$$= \int_{-2}^1 x(6-x-(x^2+4)) dx$$

$$= \int_{-2}^1 x(-x^2-x+2) dx$$

$$= \int_{-2}^1 -x^3 - x^2 + 2x dx$$

$$= \left[ -\frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2 \right]_{-2}^1$$

$$= -\frac{9}{4}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\bullet M_x = \frac{1}{2} \int_a^b y_1^2 - y_2^2 dx$$

$$= \frac{1}{2} \int_{-2}^1 (6-x)^2 - (x^2+4)^2 dx$$

$$= \frac{1}{2} \int_{-2}^1 36 - 12x + x^2 - (x^4 + 8x^2 + 16) dx$$



$$\begin{aligned}
 &= \frac{1}{2} \int_{-2}^1 -x^4 - 7x^2 - 12x + 20 \, dx \\
 &= \frac{1}{2} \left[ -\frac{1}{5}x^5 - \frac{7}{3}x^3 - 6x^2 + 20x \right]_{-2}^1 \\
 &= \frac{1}{2} \left[ \frac{252}{5} \right] \\
 &= \frac{126}{5}
 \end{aligned}$$

Jadi

$$\bar{x} = \frac{M_y}{M} = \frac{-9/4}{9/2} = -\frac{9}{4} \times \frac{2}{9} = -\frac{1}{2}$$

$$\bar{y} = \frac{M_x}{M} = \frac{126/5}{9/2} = \frac{126}{5} \times \frac{2}{9} = \frac{252}{45} = \frac{28}{5}$$

Titik berat  $(\bar{x}, \bar{y}) = (-\frac{1}{2}, \frac{28}{5})$

- ③ mencari d (titik) garis  
jarak titik  $(\bar{x}, \bar{y})$  ke sb pth  $ax+by+c=0$

adalah

$$d = \frac{|a\bar{x} + b\bar{y} + c|}{\sqrt{a^2 + b^2}}$$

Jarak titik berat  $(\bar{x}, \bar{y}) = (-\frac{1}{2}, \frac{28}{5})$  ke  $x+y-6=0$

$$a=1, b=1, c=-6$$

$$d = \frac{|a\bar{x} + b\bar{y} + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-\frac{1}{2} + \frac{28}{5} - 6|}{\sqrt{1^2 + 1^2}}$$

$$= \frac{|-\frac{5}{10} + \frac{56}{10} - \frac{60}{10}|}{\sqrt{2}}$$

$$= \frac{9}{10\sqrt{2}}$$

④ Luas

$$L = M = \frac{9}{2}$$

$$\textcircled{5} V = 2\pi \cdot d \cdot L$$

$$= 2\pi \cdot \frac{9}{10\sqrt{2}} \cdot \frac{9}{2}$$

= ... Satuan Volume.

3. Given a parametric  $x = t(t+1)$  and  $y = 2t$  on the interval  $0 \leq t \leq 3$ . Find the equation of tangent line when  $t = 2$ . Sketch the graph! (EAS 2022/2023, Senin 12 Juni 2023)

jawab

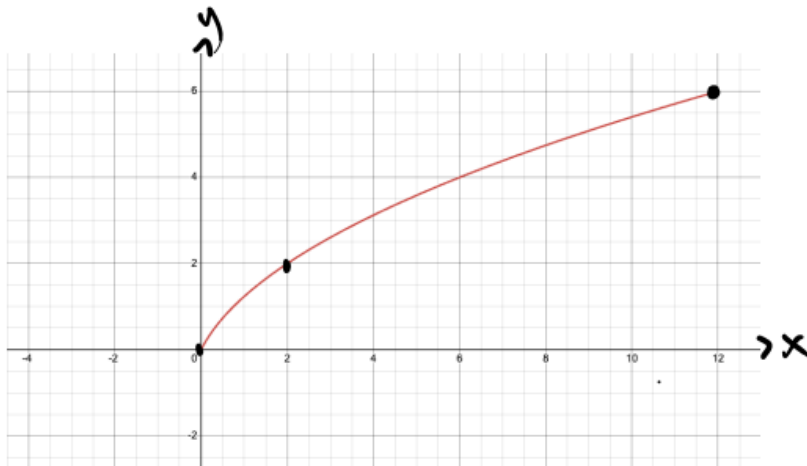
① Gambar

$$\begin{cases} x = t(t+1) \\ y = 2t \end{cases} \quad 0 \leq t \leq 3$$

$$t=0 \rightarrow \begin{aligned} x &= 0(0+1) = 0 \\ y &= 2(0) = 0 \end{aligned} \quad (0,0)$$

$$t=1 \rightarrow \begin{aligned} x &= 1(1+1) = 2 \\ y &= 2(1) = 2 \end{aligned} \quad (2,2)$$

$$t=3 \rightarrow \begin{aligned} x &= 3(3+1) = 12 \\ y &= 2(3) = 6 \end{aligned} \quad (12,6)$$



② Persamaan garis singgung di  $t = 2$

a. mencari  $\frac{dy}{dx}$

$$x = t(t+1) = t^2 + t \rightarrow \frac{dx}{dt} = 2t+1$$

$$y = 2t \rightarrow \frac{dy}{dt} = 2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2}{2t+1}$$

b. mencari m

$$m = \left. \frac{dy}{dx} \right|_{t=2} = \frac{2}{2(2)+1} = \frac{2}{5}$$

c. mencari  $x_1$  dan  $y_1$

$$x_1 = x(2) = 2(2+1) = 6$$

$$y_1 = y(2) = 2(2) = 4$$

d. Pers. garis singgung

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{5}(x - 6)$$

