Pertemuan 1 - Juni

Wednesday, 05 June 2024
19.31

x=0

5. Dapatkan luas daerah yang dibatasi oleh kurva $y = x^2$, sumbu-y, garis $y = 1$ dan $y = 4$ pada kuadran I dan sketsa grafiknya. *(ETS 2022/2023, Senin 27 Maret 2023)*

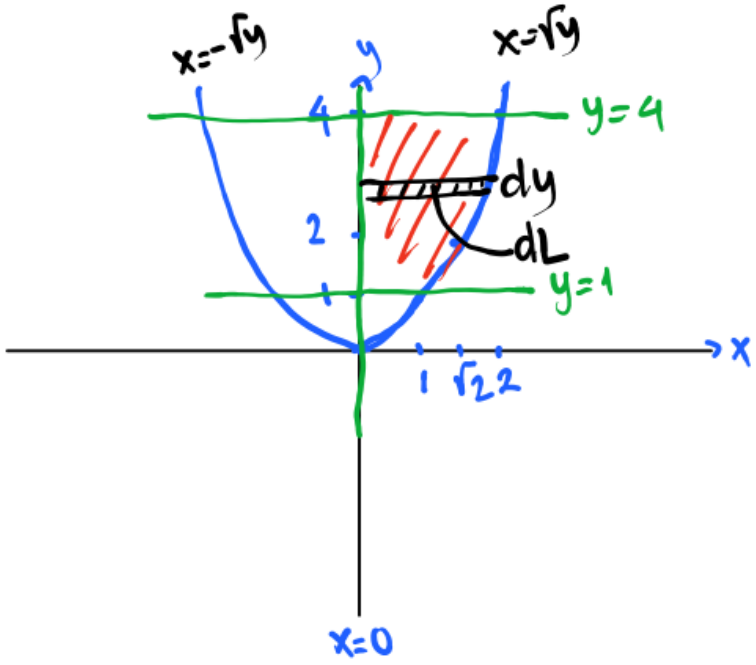
Jawab:

Gambar

$y = x^2$

$x = \sqrt{y}$ (kw 1)

x	y
1	1
$\sqrt{2}$	2
2	4



$dL = \sqrt{y}\, dy$

$L = \int_1^4 \sqrt{y}\, dy$

$$= \int_1^4 y^{1/2}\, dy$$

$$= \frac{2}{3} y^{\frac{3}{2}} \Big|_1^4 = \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3} = \cdots \text{ satuan Luas.}$$

5. Sketsa daerah yang dibatasi oleh $\overset{y_1}{y = x}$, $\overset{y_2}{y = \frac{1}{x}}$, $x = 2$, $\overset{y_3}{y = 0}$ dan dapatkan luasnya. *(ETS 2021/2022, Senin 28 Maret 2022)*

Sbx

Jawab:

(i) Tipot Pertama

$y_1 = y_2$

$x = \frac{1}{x}$

$x^2 = 1$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x = -1 \vee x = 1$

Tipot kedua

$y_1 = y_3$

$x = 0$

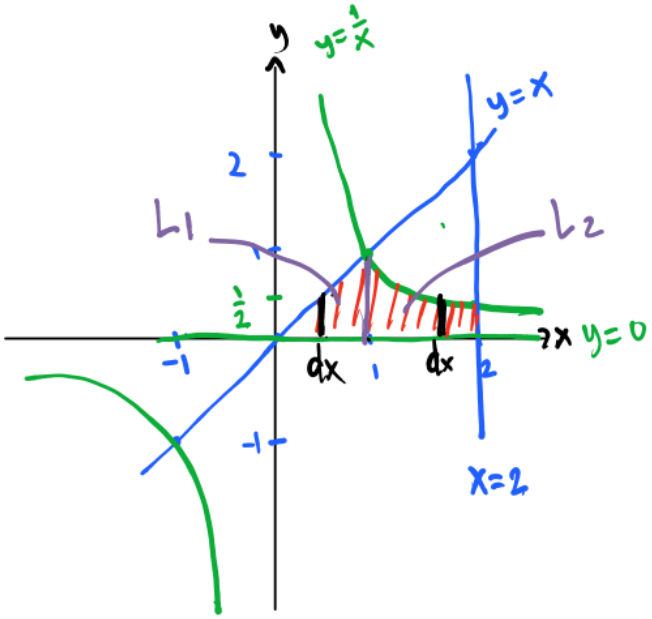
(ii) Gambar

- $y = x$

x	y
-1	-1
0	0
1	1
2	2

- $y = \frac{1}{x}$

x	y
-1	-1
1	1
2	$\frac{1}{2}$



(iii) $dL = dL_1 + dL_2$

$dL = x\,dx + \frac{1}{x}\,dx$

$L = \int_0^1 x\,dx + \int_1^2 \frac{1}{x}\,dx$

$= \frac{1}{2}x^2\Big|_0^1 + \ln|x|\Big|_1^2$

$= \left[\frac{1}{2} - 0\right] + \left[\ln 2 - \ln 1\right]$ ($\ln 1 \sim 0$)

$= \frac{1}{2} + \ln 2$

$\int x^n dx = \frac{1}{n+1} x^{n+1} + C; \quad n \neq -1$

$\int \frac{1}{x} dx = \ln |x| + C$

$\ln 1 = 0 \rightarrow e^0 = 1$

5. Dapatkan luas daerah yang dibatasi oleh $y = \sqrt{x+2}, y = \sqrt{2-x}, y = 0$ dan sketsa daerahnya. *(ETS 2021/2022, Rabu 30 Maret 2022)*

y_1 y_2 y_3

jawab :

(i) Titpot

$y_1 = y_2$

$\sqrt{x+2} = \sqrt{2-x}$

$x + 2 = 2 - x$

$2x = 0$

$x = 0$

$y_1 = y_3$

$\sqrt{x+2} = 0$

$x + 2 = 0$

$x = -2$

$y_2 = y_3$

$\sqrt{2-x} = 0$

$2 - x = 0$

$x = 2$

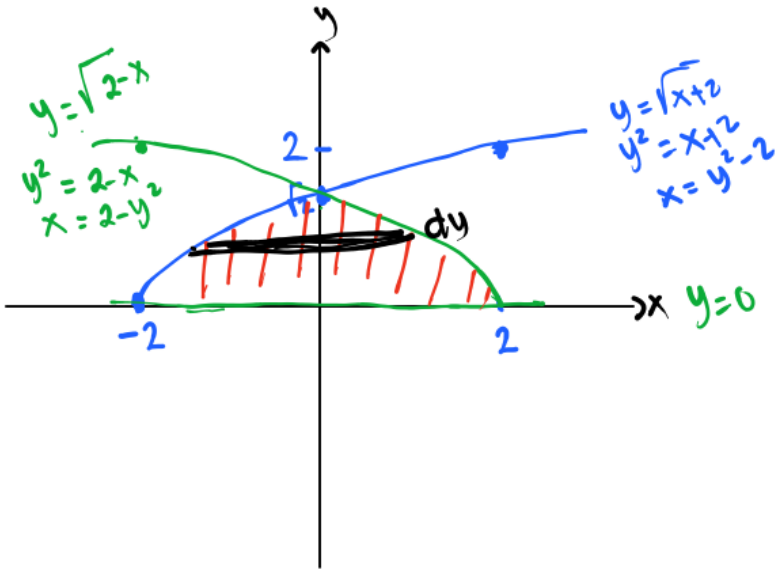
(ii) Gambar

- $y = \sqrt{x+2}$

x	y
-2	0
0	$\sqrt{2}$
2	2

- $y = \sqrt{2-x}$

x	y
-2	2
0	$\sqrt{2}$
2	0



$$dL = (2-y^2) - (y^2-2)\,dy$$

$$L = \int_0^{\sqrt{2}} (2-y^2) - (y^2-2)\,dy$$

$$= \int_0^{\sqrt{2}} 4 - 2y^2\,dy$$

$$= \left[4y - \frac{2}{3}y^3\right]\Big|_0^{\sqrt{2}}$$

$$= \left(4\sqrt{2} - \frac{2}{3}(\sqrt{2})^3\right) - 0$$

$$= 4\sqrt{2} - \frac{4}{3}\sqrt{2} \quad \text{satuan luas.}$$

5. Diberikan daerah yang dibatasi oleh $y = 4x - x^2, y = 4 - x$. *(ETS 2021/2022, Kamis 31 Maret 2022)*
   a. Sketsa daerah tersebut!
   b. Dapatkan luas daerah tersebut!

jawab

a) Cari titik

$y_1 = y_2$

$4x - x^2 = 4 - x$

$0 = x^2 - 5x + 4$

$0 = (x-1)(x-4)$

$x = 1 \vee x = 4$

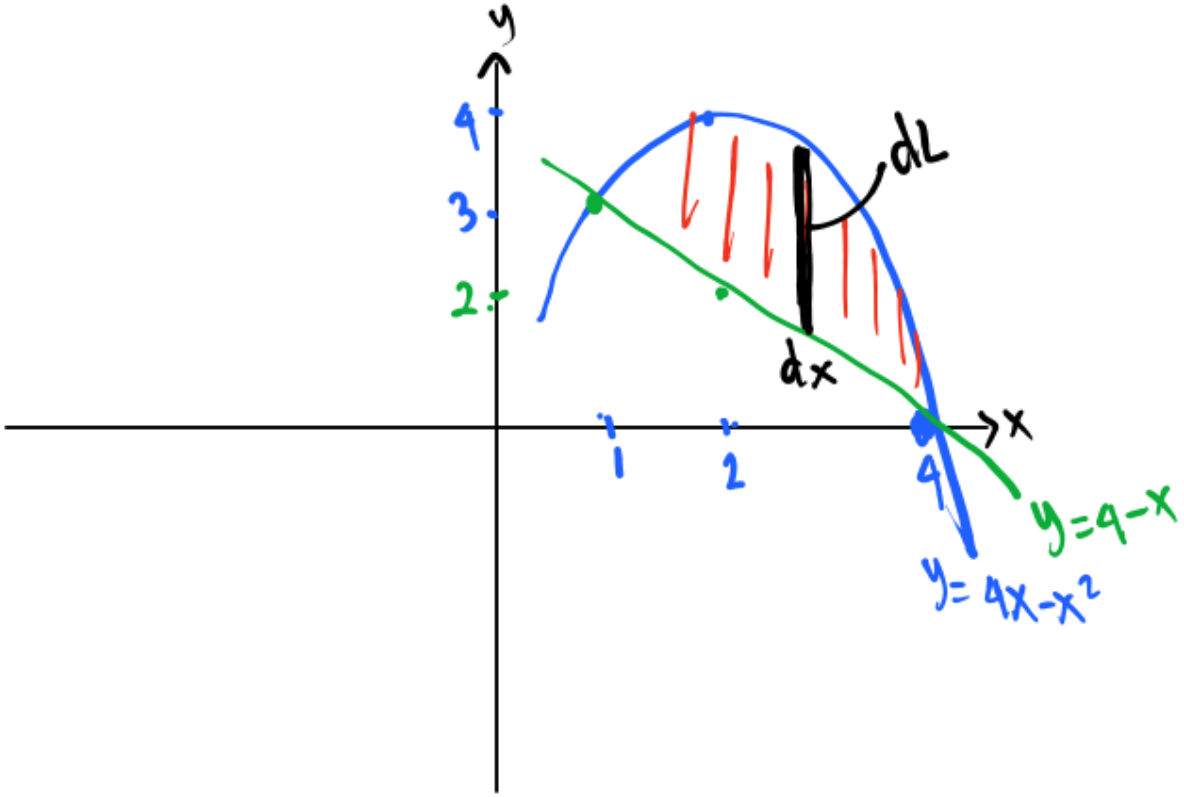
Gambar

- $y = 4x - x^2$

x	y
1	3
2	4
4	0

- $y = 4 - x$

x	y
1	3
2	2
4	0



b) Luas

$$dL = 4x - x^2 - (4-x)\,dx$$

$$L = \int_1^4 4x - x^2 - (4-x)\,dx$$

$$= \int_1^4 5x - x^2 - 4\,dx$$

$$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3 - 4x \right] \Big|_1^4$$

$$= \cdots \text{ Satuan Luas}$$

$x=y$

5. Dapatkan luas daerah yang dibatasi oleh kurva $y = x$, $x = \frac{1}{y^2}$, dan garis $y = 2$. Sketsa grafiknya. *(ETS 2022/2023, Senin 27 Maret 2023)*

Jawab :

(i) Tipot

$x_1 = x_2$

$y = \frac{1}{y^2}$

$y^3 = 1$

$y = 1$

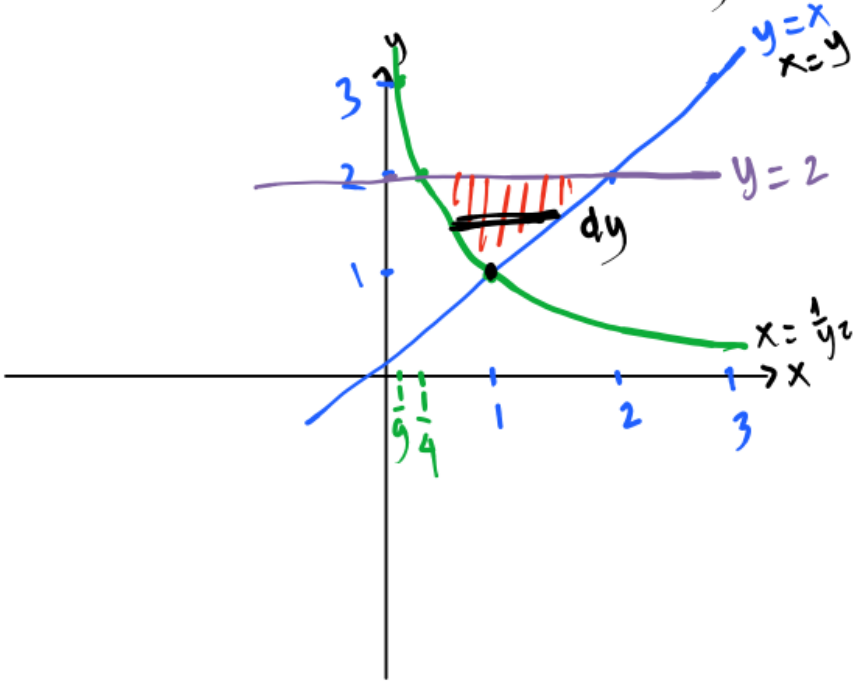
(ii) Gambar

- $x = y$

x	y
1	1
2	2
3	3

- $x = \frac{1}{y^2}$

x	y
1	1
1/4	2
1/9	3



(ii) Luas

$dL = y - \frac{1}{y^2}\,dy$

$L = \int_1^2 y - y^{-2}\,dy$

$= \left[\frac{1}{2}y^2 + y^{-1}\right]\Big|_1^2$

$= \left[\frac{1}{2}y^2 + \frac{1}{y}\right]\Big|_1^2$

$= \ldots$ Lanjutkan.

$e = 2{,}7\ldots$

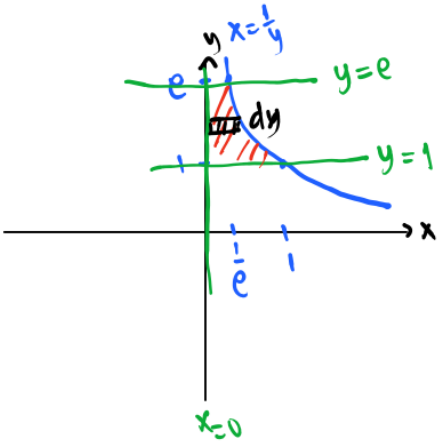
5. Dapatkan luas daerah yang dibatasi oleh kurva $x = \frac{1}{y}, x = 0, y = 1$, dan garis $y = e$. Sketsa grafiknya. *(ETS 2022/2023, Senin 27 Maret 2023)*

Jawab.

(i) Gambar

- $x = \frac{1}{y}$

x	y
1	1
$\frac{1}{e}$	e



(ii) Luas

$dL = \frac{1}{y}\,dy$

$L = \int_1^e \frac{1}{y}\,dy$

$= \ln|y|\Big|_1^e$

$= \ln e - \ln 1$

$\ln e^x = x$

$\ln e^1 = 1$

$\ln 1 = 0$

= 1 - 0

= 1 satuan luas

SAINS

1. Dapatkan volume benda putar jika daerah yang dibatasi oleh $y = \sqrt{4 - x^2}$, sumbu-$x = 0$ dengan $-1 \leq x \leq 2$ diputar pada sumbu-x, serta sketsa daerahnya. *(EAS 2022/2023, Senin 12 Juni 2023)*

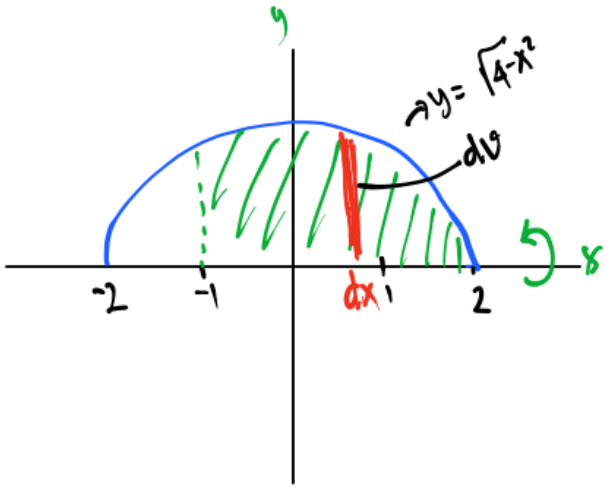
(i) Jawab

Gambar

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 2^2$ (lingkaran $P(0,0)$ $r = 2$)



(ii) Volume (Cakram)

$dv = \pi \left(\sqrt{4-x^2}\right)^2 dx$

$V = \int_{-1}^{2} \pi \left(\sqrt{4-x^2}\right)^2 dx$

$= \int_{-1}^{2} \pi (4 - x^2)\, dx$

$= \pi \left[4x - \frac{1}{3}x^3\right]\Big|_{-1}^{2}$

= ... Satuan Volume

y_1 y_2

1. Dapatkan volume benda putar jika daerah yang dibatasi oleh $y = \frac{1}{x}, y = 2$, dan garis $x = 2$ diputar terhadap garis $y = 2$, serta sketsa daerahnya. *(EAS 2022/2023, Senin 12 Juni 2023)*

Jawab:

(i) Titot

$y_1 = y_2$

$\frac{1}{x} = 2$

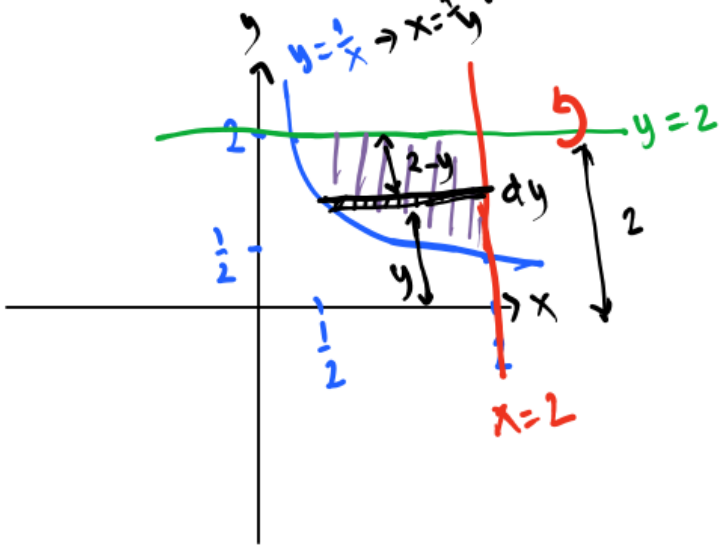
$1 = 2x$

$x = \frac{1}{2}$

(ii) Gambar

$y = \frac{1}{x}$

x	y
$\frac{1}{2}$	2
2	$\frac{1}{2}$



(iii) Volume (Cincin)

$$dV = 2\pi (2-y)\left(2 - \frac{1}{y}\right) dy$$

$$V = \int_{\frac{1}{2}}^{2} 2\pi (2-y)\left(2 - \frac{1}{y}\right) dy$$

$$= 2\pi \int_{\frac{1}{2}}^{2} 4 - \frac{2}{y} - 2y + 1 \, dy$$

$$= 2\pi \left[4y - 2\ln y - y^2 + y\right]\Big|_{\frac{1}{2}}^{2}$$

$= \ldots$ satuan volume

$y = \sqrt{25 - x^2}$

1. Find the volume of the solid generated when the region enclosed by $x^2 + y^2 = 25$, and $y = 3$ is revolved about the x-axis. *(EAS 2022/2023, Senin 12 Juni 2023)*

reorsi : harusnya hanya setengah lingkaran

(i) Gambar

$x^2 + y^2 = 25$

Lingkaran P(0,0) dan $r = 5$

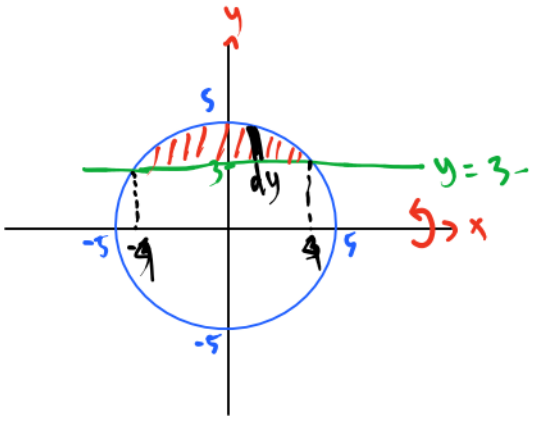
Namun harusnya $\frac{1}{2}$ lingkaran atas saja

$y = \sqrt{25 - x^2}$

Tipot

$y_1 = y_2$

$\sqrt{25-x^2} = 3 \rightarrow 25 - x^2 = 9 \rightarrow 16 = x^2 \rightarrow x = \pm 4$



(ii) Volume (cakram)

$dV = \pi \left(\left(\sqrt{25-x^2} \right)^2 - (3)^2 \right) dx$

$V = \int_{-4}^{4} \pi \left(\left(\sqrt{25-x^2} \right)^2 - 3^2 \right) dx$

$= \pi \int_{-4}^{4} 25 - x^2 - 9 \, dx$

$= \pi \int_{-4}^{4} 16 - x^2 \, dx$

$= \pi \left[16x - \frac{1}{3}x^3 \right] \Big|_{-4}^{4}$

$= \ldots$ Satuan Volume

Note!

soal yg belum terbahas diakhir, nanti dibahas pada pertemuan all chapter ya