

Materi : Fungsi Parametrik (Grafik, Panjang Busur, Garis Singgung) & Barisan

[A] Persamaan Parametrik

→ fungsi : $y = f(x)$ → koordinat kartesian

$r = f(\theta)$ → koordinat kutub.

Persamaan parametrik

→ $x = f(t)$

$y = g(t)$

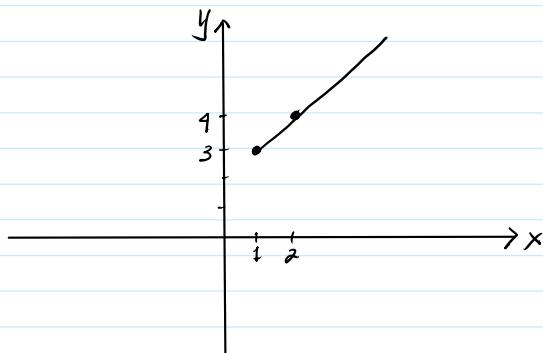
→ $x = f(s)$

$y = g(s)$

[B] Grafik Fungsi Parametrik dalam Koordinat Kartesian

Contoh : ① $x = t$
 $y = t + 2$

| t | x | y | (x, y) |
|-----|---|---|----------|
| 1 | 1 | 3 | (1, 3) ✓ |
| 2 | 2 | 4 | (2, 4) ✓ |
| ⋮ | | | |
| dst | | | |



* Mengeliminasi Parameter

→ $x = f(t)$ & $y = g(t)$

⇓

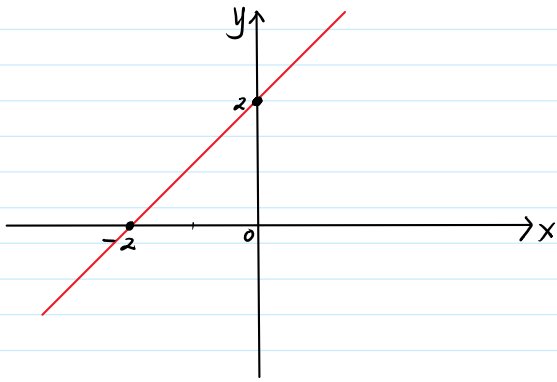
$y = f(x)$

Contoh : ① $x = t$
 $y = t + 2$ } Sketsa grafiknya.

$x = t$, substitusi ke $y = t + 2$

$y = x + 2$

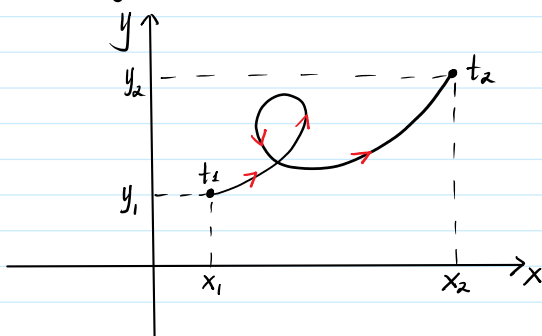
| | | |
|---|---|----|
| x | 0 | -2 |
| y | 2 | 0 |



C Orientasi Kurva Parametrik

$$\Rightarrow x = f(t) \quad ; \quad y = g(t) \quad ; \quad a \leq t \leq b \quad ; \quad a, b \geq 0$$

Sketsa grafik \rightarrow arah pertambahan parameter.



$$t = a \rightarrow t_1$$

$$t = b \rightarrow t_2$$

Contoh soal :

Diberikan persamaan parametrik : $x = 2 \cos t$; $y = 5 \sin t$; $0 \leq t \leq 2\pi$.

Sketsa kurva dgn mengeliminasi parameter t ; tunjukkan arah pertambahan t .

$$\Rightarrow x = 2 \cos t$$

$$x^2 = 4 \cos^2 t \rightarrow \cos^2 t = \frac{x^2}{4}$$

$$\Rightarrow y = 5 \sin t$$

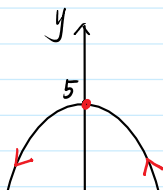
$$y^2 = 25 \sin^2 t \rightarrow \sin^2 t = \frac{y^2}{25}$$

$$\cos^2 t + \sin^2 t = \frac{x^2}{4} + \frac{y^2}{25}$$

$$1 = \frac{x^2}{4} + \frac{y^2}{25}$$

Ellips

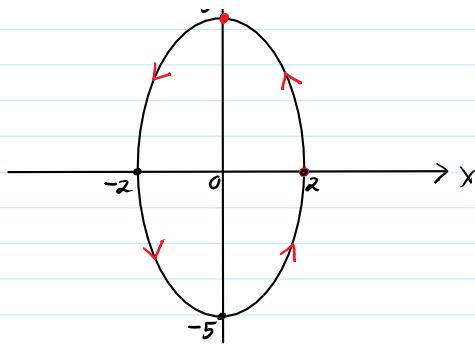
Grafik



* Arah pertambahan t

$$0 \leq t \leq 2\pi$$

$$\Rightarrow x = 2 \cos t$$



$$0 \leq t \leq 2\pi$$

$$\Rightarrow x = 2 \cos t$$

$$y = 5 \sin t$$

$$\Rightarrow t = 0 \rightarrow \left. \begin{matrix} x = 2 \\ y = 0 \end{matrix} \right\} (2,0)$$

$$\Rightarrow t = \frac{\pi}{2} \rightarrow \left. \begin{matrix} x = 0 \\ y = 5 \end{matrix} \right\} (0,5)$$

D Menyajikan Grafik Fungsi secara Parametrik.

$$\Rightarrow y = f(x) \rightarrow \text{diubah menjadi} \quad \begin{matrix} x = f(t) \\ y = g(t) \end{matrix}$$

Contoh :

$$\left. \begin{matrix} y = 4x^2 - 1 \\ \Rightarrow x = t \\ y = 4t^2 - 1 \end{matrix} \right\}$$

Jawaban 1

$$\Rightarrow x = t + 1 \quad \checkmark$$

$$\begin{aligned} y &= 4(t+1)^2 - 1 \\ &= 4(t^2 + 2t + 1) - 1 \\ &= 4t^2 + 8t + 3 \quad \checkmark \end{aligned}$$

Jawaban 2

E Garis Singgung Kurva Parametrik

$$\text{Persamaan parametrik : } \begin{matrix} x = f(t) \\ y = g(t) \end{matrix}$$

$$\Rightarrow \text{Kemiringan (Gradien) : } m = \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}}$$



$$\Rightarrow \text{Garis singgung horizontal : } \frac{dy}{dx} = 0$$

$$\Rightarrow \text{Garis singgung vertikal : } \frac{dx}{dt} = 0 \quad \& \quad \frac{dy}{dt} \neq 0$$

Contoh soal :

Dapatkan persamaan garis singgung pada kurva $x = 2t - 4$; $y = 9t^2 - 2t + 4$; $t = 1$.

① Gradien di $t = 1$.

$$\begin{aligned} \Rightarrow x &= 2t - 4 \\ \frac{dx}{dt} &= 2 \end{aligned}$$

$$\begin{aligned} \Rightarrow y &= 9t^2 - 2t + 4 \\ \frac{dy}{dt} &= 18t - 2 \end{aligned}$$

$$\Rightarrow x = 2t - 4$$

$$\frac{dx}{dt} = 2$$

$$\Rightarrow y = 9t - 2t + 4$$

$$\frac{dy}{dt} = 18t - 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{18t - 2}{2} = 9t - 1$$

$$m = \left. \frac{dy}{dx} \right|_{t=1}$$

$$= 9t - 1 \Big|_{t=1}$$

$$= 9 - 1$$

$$m = 8$$

$$t = 1$$

$$\Rightarrow x = 2 - 4 = -2$$

$$\Rightarrow y = 9 - 2 + 4 = 11$$

$(-2, 11)$

Persamaan garis singgung dititik $(-2, 11)$ dan $m = 8$:

$$y - y_0 = m(x - x_0) \quad \checkmark$$

$$y - 11 = 8(x + 2)$$

$$y - 11 = 8x + 16$$

$$y = 8x + 27$$

[F] Turunan Kedua pada Fungsi Parametrik.

$$\Rightarrow x = f(t)$$

$$y = g(t)$$

$$\text{Turunan pertama} \rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\text{Turunan kedua} \rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{dy'}{dx}$$

$$= \frac{dy'/dt}{dx/dt}$$

Contoh Soal :

Dapatkan $\frac{d^2y}{dx^2}$ dari : $x = \sqrt{t}$, $y = 2t + 4$

$$\Rightarrow \text{Menentukan } \frac{dy}{dx}$$

$$\Rightarrow x = t^{1/2} \rightarrow \frac{dx}{dt} = \frac{1}{2} \cdot t^{-1/2} = \frac{1}{2t^{1/2}} = \frac{1}{2\sqrt{t}}$$

$$\Rightarrow y = 2t + 4 \rightarrow \frac{dy}{dt} = 2$$

()

$$\rightarrow y = 2t + 4 \rightarrow \frac{dy}{dt} = 2$$

$$y' \leftarrow \left(\frac{dy}{dx} \right) = \frac{dy/dt}{dx/dt} = \frac{2}{\frac{1}{2\sqrt{t}}} = 4\sqrt{t}$$

\rightarrow Menentukan $\frac{d^2y}{dx^2}$

$$\rightarrow y' = 4\sqrt{t} = 4t^{1/2} \rightarrow \frac{dy'}{dt} = \frac{2}{\sqrt{t}} = \frac{2}{\sqrt{t}}$$

$$\# \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{\frac{2}{\sqrt{t}}}{\frac{1}{2\sqrt{t}}}$$

$$= \frac{2}{\sqrt{t}} \times 2\sqrt{t} = 4$$

[G] Panjang Busur Kurva Parametrik

$$\rightarrow x = f(t) \quad \& \quad y = g(t) \quad ; \quad a \leq t \leq b$$

$$\text{Panjang Busur} \rightarrow S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

[H] Luas Kulit pada Kurva Parametrik

$$\text{Diberikan } x = f(t) \quad \& \quad y = g(t) \quad ; \quad a \leq t \leq b$$

Luas kulit $\rightarrow K$

① diputar pada sumbu x

$$K = \int_a^b 2\pi \cdot y \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

② diputar pada sumbu y

$$K = \int_a^b 2\pi \cdot x \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Contoh Soal :

Dapatkan panjang busur dari $x = \cos 2t$; $y = \sin 2t$; $0 \leq t \leq \frac{\pi}{2}$

Dapatkan panjang busur dari $x = \cos 2t$; $y = \sin 2t$; $0 \leq t \leq \frac{\pi}{2}$

$$\Rightarrow \frac{dx}{dt} = -2 \sin 2t \rightarrow \left(\frac{dx}{dt}\right)^2 = 4 \sin^2 2t$$

$$\Rightarrow \frac{dy}{dt} = 2 \cos 2t \rightarrow \left(\frac{dy}{dt}\right)^2 = 4 \cos^2 2t$$

$$\begin{aligned} S &= \int_0^{\pi/2} \sqrt{4 \sin^2 2t + 4 \cos^2 2t} \, dt \\ &= \int_0^{\pi/2} \sqrt{4 (\underbrace{\sin^2 2t + \cos^2 2t}_{=1})} \, dt \\ &= \int_0^{\pi/2} 2 \, dt \\ &= 2t \Big|_0^{\pi/2} \\ &= \pi - 0 \\ &= \pi \text{ satuan panjang} \end{aligned}$$

Barisan

[A] Barisan Tak Hingga

Notasi : $\{a_n\}_{n=1}^{\infty}$; $\{a_n\}$

* Menentukan rumus a_n jika diberikan barisan tak hingga

$$\textcircled{1} \quad \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots \rightarrow \{a_n\} = \dots ?$$

$$\Rightarrow n = 1 \rightarrow a_1 = \frac{1}{2} \rightarrow \frac{2n-1}{2n}$$

$$\Rightarrow n = 2 \rightarrow a_2 = \frac{3}{4} \rightarrow \frac{2n-1}{2n}$$

$$\{a_n\} = \left\{ \frac{2n-1}{2n} \right\}$$

B. Konvergensi Barisan

$$\{a_n\}_{n=1}^{\infty}$$



$$a_1, a_2, a_3, \dots, a_n, \dots$$

Menentukan $\{a_n\}$ konvergen atau tidak $\rightarrow \lim_{n \rightarrow +\infty} a_n = \dots ?$

\rightarrow Jika hasil limit adalah suatu nilai tertentu (selain $-\infty$ & $+\infty$) $\rightarrow \{a_n\}$ konvergen

\rightarrow Jika hasil limit adalah $-\infty$ atau $+\infty$ $\rightarrow \{a_n\}$ divergen

\rightarrow Sifat - Sifat Limit :

Barisan $\{a_n\}$ dan $\{b_n\}$, masing^{2x} konvergen ke limit L_1 dan L_2 . c adalah konstanta

$$(i). \lim_{n \rightarrow +\infty} c = c$$

$$(ii). \lim_{n \rightarrow +\infty} c \cdot a_n = c \cdot \lim_{n \rightarrow +\infty} a_n = c \cdot L_1$$

$$(iii). \lim_{n \rightarrow +\infty} (a_n \pm b_n) = \lim_{n \rightarrow +\infty} a_n \pm \lim_{n \rightarrow +\infty} b_n = L_1 \pm L_2$$

$$(iv). \lim_{n \rightarrow +\infty} a_n \cdot b_n = \lim_{n \rightarrow +\infty} a_n \cdot \lim_{n \rightarrow +\infty} b_n = L_1 \cdot L_2$$

$$(v). \lim_{n \rightarrow +\infty} \frac{a_n}{b_n} = \frac{\lim_{n \rightarrow +\infty} a_n}{\lim_{n \rightarrow +\infty} b_n} = \frac{L_1}{L_2} \quad ; L_2 \neq 0$$