Pertemuan 8 - Juni

Wednesday, 19 June 2024
17.46

5. Dapatkan luas daerah yang dibatasi oleh kurva $y = \sin x$, $y = \frac{1}{2}\sqrt{3}$, $x = 0$ dan $x = \frac{3}{4}\pi$. Sketsa grafiknya. *(ETS 2022/2023, Selasa 28 Maret 2023)*

Jawab:

$y = \sin x$, $y = \frac{1}{2}\sqrt{3}$, $x = 0$, $x = \frac{3}{4}\pi$

① Tipot

$y_1 = y_2$

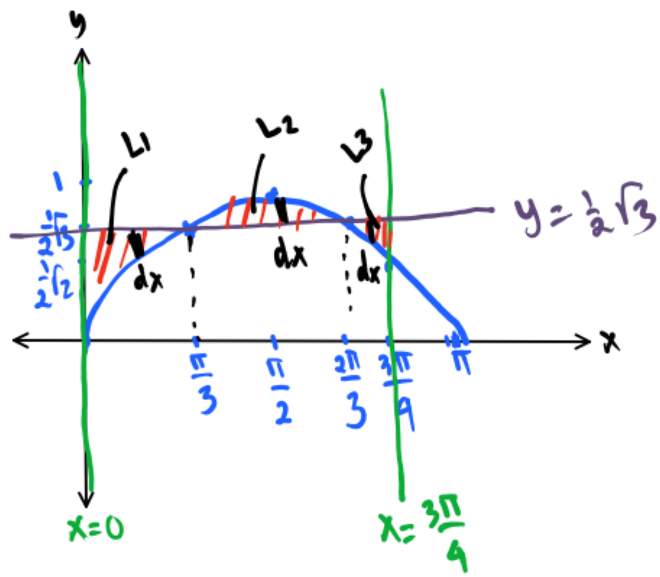
$\sin x = \frac{1}{2}\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

② Gambar

$y = \sin x$

x	y
0	0
$\frac{\pi}{3}$	$\frac{1}{2}\sqrt{3}$
$\frac{2\pi}{3}$	$\frac{1}{2}\sqrt{3}$
$\frac{3\pi}{4}$	$\frac{1}{2}\sqrt{2}$
$\frac{\pi}{2}$	1



③ Luas

$dL = dL_1 + dL_2 + dL_3$

$dL = \left(\frac{1}{2}\sqrt{3} - \sin x\right)dx + \left(\sin x - \frac{1}{2}\sqrt{3}\right)dx + \left(\frac{1}{2}\sqrt{3} - \sin x\right)dx$

$$L = \int_0^{\frac{\pi}{3}} \frac{1}{2}\sqrt{3} - \sin x \, dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sin x - \frac{1}{2}\sqrt{3} \, dx + \int_{\frac{2\pi}{3}}^{\frac{3\pi}{4}} \frac{1}{2}\sqrt{3} - \sin x \, dx$$

$$= \left[\frac{1}{2}\sqrt{3}\,x + \cos x\right]\Big|_0^{\frac{\pi}{3}} + \left[-\cos x - \frac{1}{2}\sqrt{3}x\right]\Big|_{\frac{\pi}{3}}^{\frac{2\pi}{3}} + \left[\frac{1}{2}\sqrt{3}x + \cos x\right]\Big|_{\frac{2\pi}{3}}^{\frac{3\pi}{4}}$$

$= \ldots$ Satuan Luas

5. Dapatkan volume benda padat yang terjadi bila daerah yang dibatasi oleh $y = (x-2)^2, x + y = 4$ diputar pada $x = 4$. *(ETS 2021/2022, Rabu 30 Maret 2022)*

Jawab

$(a-b)^2 = a^2 - 2ab + b^2$

$y = (x-2)^2$, $x + y = 4$ ↺ $x = 4$

↳ $y = 4 - x$

① Tipot

$y_1 = y_2$

$(x-2)^2 = 4 - x$

$x^2 - 4x + 4 = 4 - x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \vee x = 3$

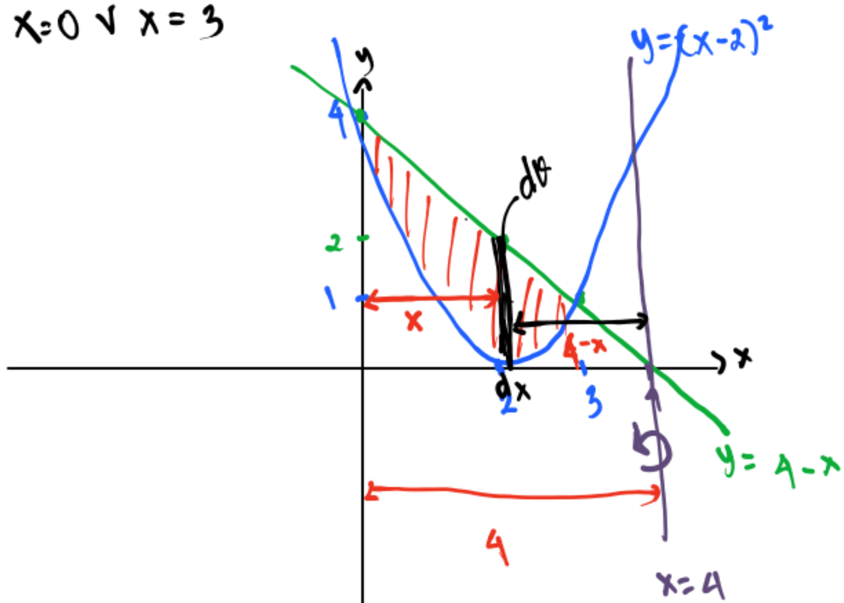
② Gambar

$y = (x-2)^2$

x	y
0	4
2	0
3	1

$y = 4 - x$

x	y
0	4
2	2
3	1



Cincin

$$dV = 2\pi \underbrace{(4-x)}_{\text{jarak partisi ke sb putar}} \Big(\underbrace{4-x}_{\text{atas}} - \underbrace{(x-2)^2}_{\text{bawah}} \Big) \, dx$$

$$V = \int_0^3 2\pi \, (4-x)(4-x-(x-2)^2) \, dx$$

$$\boxed{\int ax^n \, dx = \frac{a}{n+1} x^{n+1} + C}$$

$$= 2\pi \int_0^3 (4-x)(4-x-(x^2-4x+4))\,dx$$

$$= 2\pi \int_0^3 (4-x)(-x^2+3x)\,dx$$

$$= 2\pi \int_0^3 -4x^2+12x+x^3-3x^2\,dx$$

$$= 2\pi \left[-\frac{4}{3}x^3+6x^2+\frac{1}{4}x^4-x^3\right]\Big|_0^3$$

$$= 2\pi \left[-\frac{4}{3}(27)+6(9)+\frac{1}{4}(81)-27\right]-0$$

$= \ldots$ satuan volume

2. Dapatkan titik berat keping datar homogen yang dibatasi kurva $y = \sqrt{1-(2-x)^2}$ dan sumbu-x. *(EAS 2022/2023, Senin 12 Juni 2023)*

jawab

$y = \sqrt{1-(2-x)^2}$ dan sb x

① Gambar

$y = \sqrt{1-(2-x)^2}$

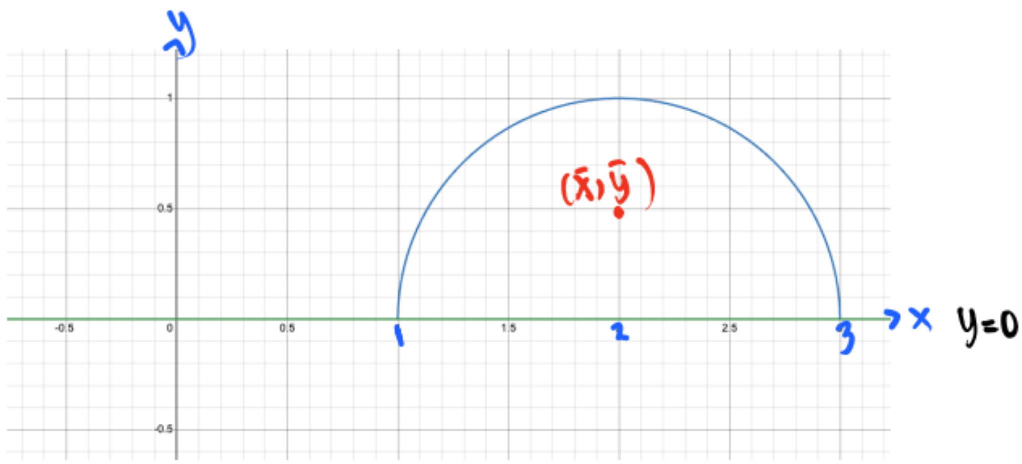
$y^2 = 1-(2-x)^2$

$(2-x)^2 + y^2 = 1$

$(-(x-2))^2 + y^2 = 1$

$(x-2)^2 + y^2 = 1$ (Lingkaran P(2,0) dan r = 1)

Karena $y = \sqrt{1-(2-x)^2}$, maka ambil lingkaran atas



② Titik berat $\int_a^b xy\, dx$

$\bar{x} = \frac{M_y}{M}$; $\bar{y} = \frac{M_x}{M}$

- $\bar{x} = 2$ (karena simetri thd $x = 2$)

- $M = \int_a^b y \, dx$

$= L$ setengah Lingkaran

$= \frac{1}{2}\pi(r)^2$

$= \frac{1}{2}\pi(1)^2$

$= \frac{1}{2}\pi$

- $M_x = \frac{1}{2}\int_a^b y^2 \, dx$

$= \frac{1}{2}\int_1^3 \left(\sqrt{1-(2-x)^2}\right)^2 dx$

$= \frac{1}{2}\int_1^3 1-(2-x)^2 \, dx$

$= \frac{1}{2}\int_1^3 1-(4-4x+x^2) \, dx$

$= \frac{1}{2}\int_1^3 -x^2+4x-3 \, dx$

$= \frac{1}{2}\left[-\frac{1}{3}x^3+2x^2-3x\right]\Big|_1^3$

$= \frac{1}{2}\left[-\frac{1}{3}(27)+2(9)-3(3)\right]-\frac{1}{2}\left[-\frac{1}{3}+2-3\right]$

$$= \frac{1}{2}\left[-9+18-9\right] - \frac{1}{2}\left[\frac{-1+6-9}{3}\right]$$

$$= 0 - \frac{1}{2}\left[-\frac{4}{3}\right]$$

$$= \frac{2}{3}$$

Jadi, $\bar{x} = 2$, $\bar{y} = \frac{M_x}{M} = \frac{2/3}{\frac{1}{2}\pi} = \frac{2}{3} \times \frac{2}{\pi} = \frac{4}{3\pi}$

Titik berat $\left(2, \frac{4}{3\pi}\right)$

3. Partikel bergerak sepanjang kurva $x = t$ dan $y = \sqrt{4t - t^2}$ pada $0 \leq t \leq 4$. Dapatkan panjang kurva dan sketsa kurva beserta arah lintasannya. *(EAS 2022/2023, Selasa 13 Juni 2023)*

① Gambar

$x = t \quad \dots (1)$

$y = \sqrt{4t - t^2} \quad \dots (2)$

Substitusi pers 1 ke 2

$x^2 - 4x = (x-2)^2 - 2^2$

$x^2 - 4x - 4$

$y = \sqrt{4x - x^2} \quad ; \quad 0 \le x \le 4$

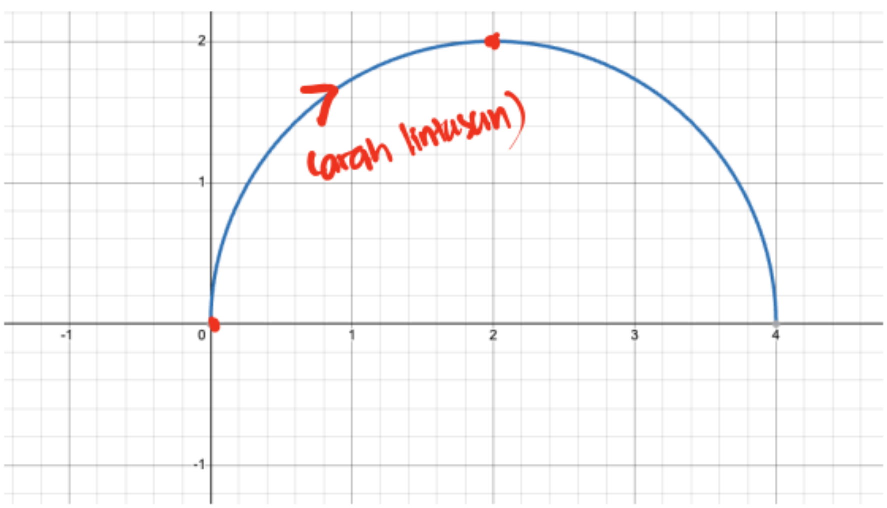
$y^2 = 4x - x^2$

$x^2 - 4x + y^2 = 0$

$(x-2)^2 - 2^2 + y^2 = 0$

$(x-2)^2 + y^2 = 2^2$ (lingkaran $P(2,0)$ dan $r = 2$)

karena $y = \sqrt{4x - x^2}$, maka ambil setengah lingkaran atas



arah lintasan

$t = 0 \rightarrow x = 0$

$y = \sqrt{0 - 0} = 0 \quad (0,0)$

$t = 2 \rightarrow x = 2$

$y = \sqrt{4(2) - 2^2} = 2 \quad (2,2)$

(2) Panjang busur

S = K setengah lingkaran

$= \frac{1}{2}(2\pi r)$

$= \frac{1}{2}(2\pi \cdot 2)$

$= 2\pi$ satuan panjang.

Integral taknagar

$$\int_1^2 \frac{1}{x-2} = \lim_{t \to 2^-} \int_1^t \frac{1}{x-2}\, dx$$

4. Sketsa grafik daerah di dalam kurva kutub $r = 2 - 2\sin\theta$ dan diluar kurva kutub $r = 2 + 2\sin\theta$, selanjutnya hitung luas daerah tersebut. *(EAS 2021/2022, Rabu 8 Juni 2022)*

Jawab

$r = 2 - 2\sin\theta \quad , r = 2 + 2\sin\theta$

$dL = \frac{1}{2} r^2 \, d\theta$

$L = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta$

① Tipot

$r_1 = r_2$

$2 - 2\sin\theta = 2 + 2\sin\theta$

$-4\sin\theta = 0$

$\sin\theta = 0$

$\theta = \{0, \pi\}$

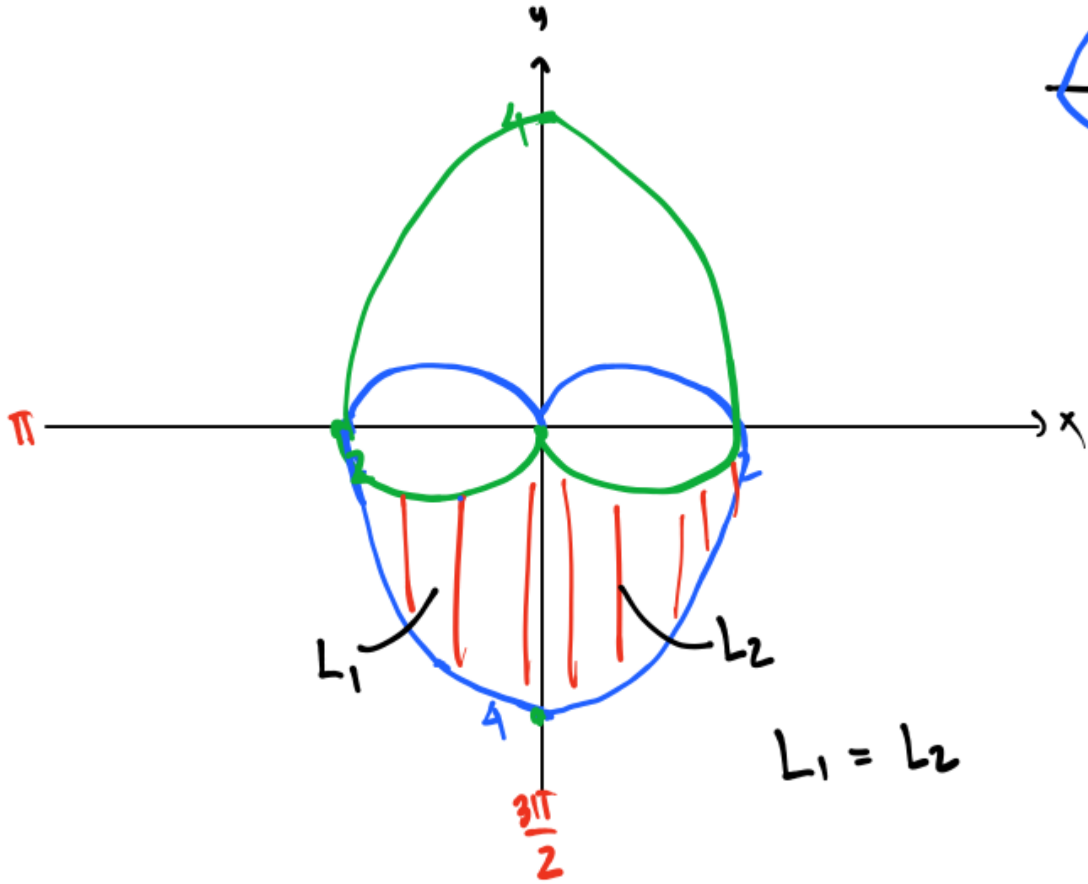
② Gambar

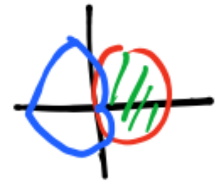
- $r = 2 - 2\sin\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	2	0	2	4	2

- $r = 2 + 2\sin\theta$

θ	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
r	2	4	2	0	2





$L_1 = L_2$

③ Luas

$dL = dL_1 + dL_2$

$dL = 2\,dL_1$

$dL = \cancel{2}\left[\frac{1}{\cancel{2}}\left[(2-2\sin\theta)^2-(2+2\sin\theta)^2\right]\right]d\theta$

$$L = \int_{\pi}^{\frac{3\pi}{2}} (2-2\sin\theta)^2 - (2+2\sin\theta)^2 \, d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} 4 - 8\sin\theta + 4\sin^2\theta - (4 + 8\sin\theta + 4\sin^2\theta) \, d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} -16\sin\theta \, d\theta$$

$$= 16\cos\theta \Big|_{\pi}^{3\pi/2}$$

$$= 16\cos\frac{3\pi}{2} - 16\cos\pi$$

$$= 0 - (-16)$$

$= 16$ Satuan Luas.

5. Diberikan fungsi $f(x) = \ln(2-3x)$. *(EAS 2022/2023, Senin 12 Juni 2023)*
   (a) Dapatkan polinomial Maclaurin derajat 4 dari fungsi tersebut.
   (b) Dapatkan deret Maclaurin fungsi tersebut dan nyatakan dalam notasi sigma.

Polinomial Maclaurin

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

Deret dan notasi sigma

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}x^k = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(k)}(0)}{k!}x^k + \cdots$$

5a) $f(x) = \ln(2-3x)$, $P_4(x) = ?$

$$\boxed{\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}}$$

- $f(x) = \ln(2-3x) \rightarrow f(0) = \ln 2$

- $f'(x) = \frac{1}{2-3x} \cdot -3 = -3(2-3x)^{-1} \rightarrow f'(0) = -3(2)^{-1} = -\frac{3}{2}$

- $f''(x) = 3 \cdot (2-3x)^{-2} \cdot -3 = 3 \cdot -3 \; (2-3x)^{-2}$

  $f''(0) = 3 \cdot -3 \; (2)^{-2} = \frac{3 \cdot -3}{2^2} = -\frac{3^2}{2^2} = \frac{-9}{4}$

- $f'''(x) = 3 \cdot 3 \cdot -2 \; (2-3x)^{-3} \cdot -3 = -3^3 \cdot 2 \; (2-3x)^{-3}$

  $f'''(0) = -3 \;\; 2 \; (2)^{-3} = \frac{-3^3 \cdot 2}{2^3} = -\frac{54}{8}$

- $f^{(4)}(x) = -3^3 \cdot 2 \cdot - \;\; (2-3x)^{-4} \cdot -3 = -3^5 \cdot 2 \; (2-3x)^{-4}$

  $f^{(4)}(0) = -3^5 \cdot 2 \; (2)^{-3} = \frac{\;^5 \cdot 2}{2} = \frac{-486}{16}$

Sehingga.

$P_4(x) = \ln 2 - \frac{3}{2}x - \frac{9}{4} \cdot \frac{1}{2!} x^2 - \frac{54}{8} \cdot \frac{1}{3!} x^3 - \frac{486}{16} \cdot \frac{1}{4!} x^4$

$P_4(x) = \ln 2 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{9}{8}x^3 - \frac{486}{16}x^4$

(5b) Deret dan notasi sigma

Perhatikan

$$\ln(1+x) = -\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Perhatikan juga

$$(2-3x) = (1 + (1-3x))$$

Sehingga, gantilah x menjadi $1-3x$

$$\ln(2-3x) = -\sum_{k=1}^{\infty} \frac{(-1)^k (1-3x)^k}{k}$$

$$= (1-3x) - \frac{(1-3x)^2}{k} + \frac{(1-3x)^3}{k} + \cdots$$