

Pertemuan 4 - Juni

Tuesday, 11 June 2024

19.32

3. Diberikan partikel bergerak sepanjang kurva: $\begin{cases} x = 1 - t \\ y = \sqrt{8 + 2t - t^2} \end{cases}$ dengan $-2 \leq t \leq 1$. (EAS 2020/2021, Rabu 30 Juni 2021)
- (a) Nyatakan dalam persamaan kutub $r = f(\theta)$.
 - (b) Tentukan panjang lintasan kurva tersebut.
 - (c) Sketsa persamaan kurva tersebut dan arah lintasan-nya.

Jawab :

a. Ubah ke $r = f(\theta)$

• $x = 1 - t$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$t = 1 - x \quad \dots (1)$$

$$y = \sqrt{8 + 2t - t^2} \quad \dots (2)$$

Substitusi (1) ke (2)

$$y = \sqrt{8 + 2(1-x) - (1-x)^2}$$

$$y = \sqrt{8 + 2 - 2x - (1 - 2x + x^2)}$$

$$y = \sqrt{-x^2 + 9}$$

menentukan batas

$$t = -2 \rightarrow x = 1 - (-2) = 3$$

$$t = 1 \rightarrow x = 1 - 1 = 0$$

$$y = \sqrt{-x^2 + 9} \quad 0 \leq x \leq 3$$

• $y^2 = -x^2 + 9$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 9$$

$$r^2 = 9$$

$$r = 3$$

menentukan batas

$$x = r \cos \theta$$

$$x = 3 \cos \theta$$

$$x = 0 \rightarrow 0 = 3 \cos \theta \rightarrow \theta = \frac{\pi}{2}$$

$$x = 3 \rightarrow 1 = \cos \theta \rightarrow \theta = 0$$

sehingga.

$$r = 3, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

b. Panjang lintasan (S)

$$r = 3, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$\frac{dr}{d\theta} = 0$$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

$$S = \int_0^{\frac{\pi}{2}} \sqrt{3^2 + (0)^2} d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{9} d\theta$$

$$= \int_0^{\frac{\pi}{2}} 3 d\theta$$

$$= 3\theta \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{3\pi}{2} - 0 = \frac{3\pi}{2} \text{ satuan panjang}$$

c. Sketsa dan arah

$$r = 3, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

Lingkeran dgn $r=3$ di kw I.

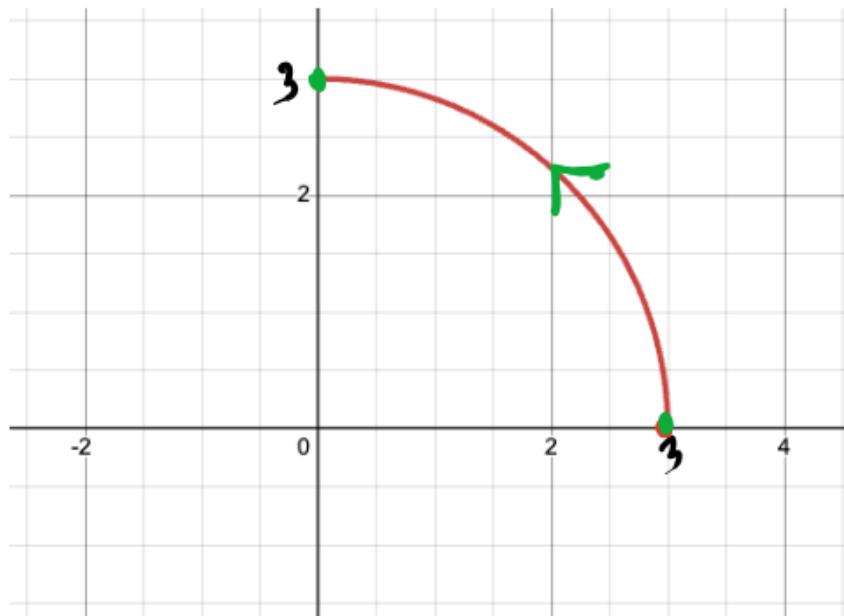
arah lintasan $\left\{ \begin{array}{l} x = 1-t \\ y = \sqrt{8+2t-t^2}, \quad -2 \leq t \leq 1 \end{array} \right.$

$$t = -2 \rightarrow x = 1+2 = 3$$

$$y = \sqrt{8+2(-2)-(-2)^2} = 0 \quad (3,0)$$

$$t = 1 \rightarrow x = 1-1 = 0$$

$$y = \sqrt{8+2-1} = 3 \quad (0,3)$$



3. The parametric equations for the trajectory of the particle motion are $x = \ln t - 1$ and $y = \frac{t}{t-1}$. (EAS 2021/2022, Rabu 8 Juni 2022)
- Find the trajectory function as $y(x)$.
 - Sketch the graph of the trajectory over the time interval $t \geq 2$.

Jawab

a. $y(x)$

• $x > \ln t - 1$

$$e^{\ln t} = t$$

$$\ln t = x + 1$$

$$e^{\ln t} = e^{x+1}$$

$$t = e^{x+1} \dots (1)$$

• $y = \frac{t}{t-1} \dots (2)$

Substitusi pers (1) ke (2)

$$y = \frac{e^{x+1}}{e^{x+1}-1}$$

b. $y = \frac{e^{x+1}}{e^{x+1}-1}$

Mencari batas

$$t = 2 \rightarrow x = \ln 2 - 1$$

$$t = 10 \rightarrow x = \ln 10 - 1$$

$$t = \infty \rightarrow x = \infty$$

$$y = \frac{e^{x+1}}{e^{x+1}-1} ; x \geq \ln 2 - 1$$

$-0,3 \dots$ $\ln 2 - 1$

$$\frac{e^{\ln 2 - 1 + 1}}{e^{\ln 2 - 1 + 1} - 1} = \frac{e^{\ln 2}}{e^{\ln 2} - 1} = \frac{2}{2 - 1} = 2$$

$0 \rightarrow \frac{e^1}{e^1 - 1} = 1,58$



3. Suatu partikel bergerak dengan lintasan mengikuti persamaan $y = 2e^{-3t} - 1$ dan $x = e^{-2t}$. (EAS 2021/2022, Rabu 8 Juni 2022)
- (a) Dapatkan $\frac{dy}{dx}$.
- (b) Dapatkan persamaan garis singgung kurva lintasan tersebut di $t = \ln 5$.

$$(i) \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$$

$$(iii) \frac{d}{dt}[e^{at}] = ae^{at}$$

$$(ii) \text{ Persamaan garis singgung } y - y_1 = m(x - x_1)$$

$$(iii) \frac{x^m}{x^n} = x^{m-n}$$

$$3.a \quad y = 2e^{-3t} - 1$$

$$x = e^{-2t}$$

$$\frac{dy}{dx} \dots ?$$

$$\bullet \frac{dy}{dt} = -6e^{-3t}$$

$$\bullet \frac{dx}{dt} = -2e^{-2t}$$

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{-6e^{-3t}}{-2e^{-2t}} = 3e^{-t} = \frac{3}{e^t}$$

$$3.b \text{ Persamaan garis singgung di } t = \ln 5$$

$$x_1 = x(\ln 5) = e^{-2 \ln 5} = e^{\ln 5^{-2}} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$y_1 = y(\ln 5) = 2e^{-3 \ln 5} - 1 = 2e^{\ln 5^{-3}} - 1 = 2(5^{-3}) - 1 = \frac{2}{5^3} - 1$$

$$m = \left. \frac{dy}{dx} \right|_{t=\ln 5} = \frac{3}{e^{\ln 5}} = \frac{3}{5}$$

Jadi, pers

$$y - y_1 = m(x - x_1)$$

$$y - \left(\frac{2}{5^3} - 1\right) = \frac{3}{5} \left(x - \frac{1}{25}\right)$$

3. Dapatkan panjang busur dari kurva $x = t^2$ dan $y = \frac{1}{3}t^3$ sepanjang interval $0 \leq t \leq 1$. (EAS 2022/2023, Senin 12 Juni 2023)

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$S = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Jawab:

$$x = t^2, \quad y = \frac{1}{3}t^3; \quad 0 \leq t \leq 1$$

$$\bullet \frac{dx}{dt} = 2t \quad \bullet \frac{dy}{dt} = t^2$$

$$ds = \sqrt{(2t)^2 + (t^2)^2} dt$$

$$S = \int_0^1 \sqrt{(2t)^2 + (t^2)^2} dt$$

$$= \int_0^1 \sqrt{4t^2 + t^4} dt$$

$$= \int_0^1 \sqrt{t^2(4 + t^2)} dt$$

$$= \int_0^1 t \sqrt{4 + t^2} dt$$

Misal

$$u = 4 + t^2$$

$$du = 2t dt$$

Batas

$$t=0 \rightarrow u=4$$

$$t=1 \rightarrow u=5$$

$$\frac{du}{2} = t \, dt$$

$$\begin{aligned} S &= \int_0^1 \sqrt{4+t^2} \, t \, dt \\ &= \int_4^5 \sqrt{u} \, \frac{du}{2} \\ &= \frac{1}{2} \int_4^5 u^{\frac{1}{2}} \, du \\ &= \frac{1}{2} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_4^5 \\ &= \frac{1}{2} \left[\frac{2}{3} (5)^{\frac{3}{2}} \right] - \frac{1}{2} \left[\frac{2}{3} (4)^{\frac{3}{2}} \right] \\ &= \dots \text{Satuan panjang} \end{aligned}$$

3. Diberikan persamaan parametrik $x = 1 - \cos 2t$ dan $y = 1 - \cos^2 2t$ pada $0 \leq t \leq \frac{\pi}{4}$. Dapatkan kemiringan garis singgung kurva pada $t = \frac{\pi}{6}$ serta sketsa grafiknya. (EAS 2022/2023, Senin 12 Juni 2023)

Jawab: $\frac{d}{dt} [\cos at] = -a \sin at$

$$x = 1 - \cos 2t \quad ; \quad y = 1 - \cos^2 2t \quad ; \quad 0 \leq t \leq \frac{\pi}{4}$$

① Kemiringan $t = \frac{\pi}{6}$ (m)

- $x = 1 - \cos 2t$

$$\frac{dx}{dt} = 2 \sin 2t$$

- $y = 1 - \cos^2 2t \rightarrow y = 1 - u^2$

Misal

$$u = \cos 2t$$

$$\frac{du}{dt} = -2 \sin 2t$$

$$\frac{dy}{dt} = -2u \cdot \frac{du}{dt}$$

$$= -2 \cos 2t \cdot (-2 \sin 2t)$$

$$= 4 \cos 2t \sin 2t$$

- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4 \cos 2t \sin 2t}{2 \sin 2t} = 2 \cos 2t$

- $m = \left. \frac{dy}{dx} \right|_{t=\frac{\pi}{6}} = 2 \cos(2 \cdot \frac{\pi}{6}) = 2 \cos \frac{\pi}{3} = 2 \cdot (\frac{1}{2}) = 1$

b Sketsa grafik

$$X = 1 - \cos 2t$$

$$0 \leq t \leq \frac{\pi}{4}$$

$$\cos 2t = 1 - x \quad \dots (1)$$

$$y = 1 - \cos^2 2t \quad \dots (2)$$

Subs (1) ke (2)

$$y = 1 - (1 - x)^2$$

$$y = 1 - (1 - 2x + x^2)$$

$$y = -x^2 + 2x$$

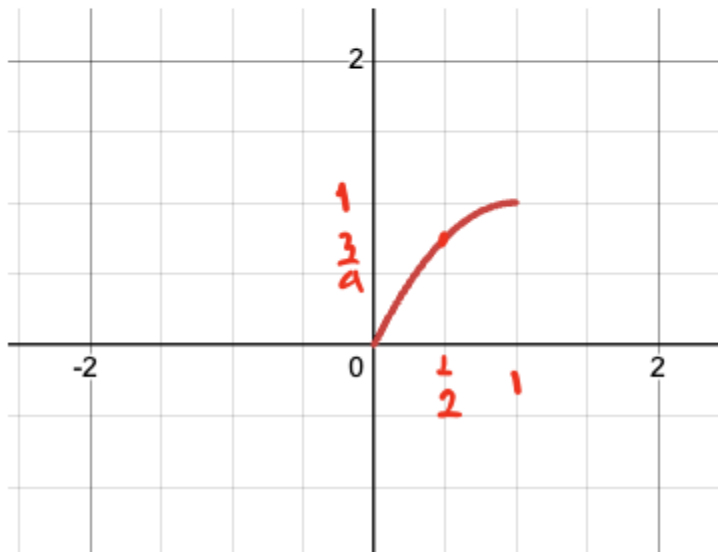
Batas-batas

$$t=0 \rightarrow x = 1 - \cos 0 = 0$$

$$t = \frac{\pi}{4} \rightarrow x = 1 - \cos \frac{\pi}{2} = 1$$

$$y = -x^2 + 2x \quad ; \quad 0 \leq x \leq 1$$

x	y
0	0
$\frac{1}{2}$	$-\frac{1}{4} + 1 = \frac{3}{4}$
1	$-1 + 2 = 1$



3. Dapatkan persamaan garis singgung di $t = \frac{\pi}{6}$ pada kurva $x = 2 \sin t$ dan $y = t + \cos t$. (EAS 2022/2023, Senin 12 Juni 2023)

Pgs.

$$y - y_1 = m(x - x_1)$$

Jawab

$$x = 2 \sin t, \quad y = t + \cos t, \quad t = \frac{\pi}{6}$$

$$\frac{dx}{dt} = 2 \cos t \quad ; \quad \frac{dy}{dt} = 1 - \sin t$$

$$\begin{aligned} \bullet \quad \frac{dy}{dx} &= \frac{dy/dt}{dx/dt} \\ &= \frac{1 - \sin t}{2 \cos t} \end{aligned}$$

$$\begin{aligned} m &= \left. \frac{dy}{dx} \right|_{t=\frac{\pi}{6}} = \frac{1 - \sin \frac{\pi}{6}}{2 \cos \frac{\pi}{6}} \\ &= \frac{1 - \frac{1}{2}}{2 \cdot \frac{1}{2} \sqrt{3}} \\ &= \frac{\frac{1}{2}}{\sqrt{3}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{6} \end{aligned}$$

$$\bullet \quad x_1 = x\left(\frac{\pi}{6}\right) = 2 \sin \frac{\pi}{6} = 2 \cdot \frac{1}{2} = 1$$

$$\begin{aligned} y_1 &= y\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \cos \frac{\pi}{6} \\ &= \frac{\pi}{6} + \frac{1}{2} \sqrt{3} \end{aligned}$$

jadi, pgs

$$y - y_1 = m(x - x_1)$$

$$\underline{\underline{y - \left(\frac{\pi}{6} + \frac{1}{2}\sqrt{3}\right) = \frac{\sqrt{3}}{6}(x - 1)}}$$