Pertemuan 2 - Juni

Friday, 07 June 2024
19.19

2. Dapatkan panjang kurva $y = 75\cosh\frac{x}{75}$ dari $x = -150$ ke $x = 150$. *(EAS 2020/2021, Rabu 30 Juni 2021)*

Jawab

$$y = 75\cosh\frac{x}{75}$$

$$\frac{dy}{dx} = \cancel{75}\sinh\frac{x}{75}\cdot\frac{1}{\cancel{75}}$$

$$\frac{dy}{dx} = \sinh\frac{x}{75}$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_{-150}^{150} \sqrt{1+\left(\sinh\frac{x}{75}\right)^2}\,dx$$

$$= \int_{-150}^{150} \sqrt{1+\sinh^2\frac{x}{75}}\,dx$$

Misal

$u = \frac{x}{75}$

$du = \frac{1}{75}\,dx$

$75\,du = dx$

Batas

$x = -150 \rightarrow u = -2$

$x = 150 \rightarrow u = 2$

$\sinh x = \frac{e^{x} - e^{-x}}{2}$

$$S = \int_{-2}^{2} \sqrt{1 + \sinh^2 u}\; 75\,du$$

$$= 75 \int_{-2}^{2} \sqrt{\cosh^2 u}\; du$$

$$= 75 \int_{-2}^{2} \cosh u \; du$$

$$= 75 \left[\sinh u\right]\Big|_{-2}^{2}$$

$$= 75 \sinh 2 - 75 \sinh(-2)$$

$$= 75\left(\frac{e^{2} - e^{-2}}{2}\right) - 75\left(\frac{e^{-2} - e^{2}}{2}\right)$$

$$= \frac{75e^{2}}{2} - \frac{75e^{-2}}{2} - \frac{75e^{-2}}{2} + \frac{75e^{2}}{2}$$

$$= 75e^{2} - 75e^{-2}$$ S. panjang.

1. Dapatkan panjang busur kurva $24xy = x^4 + 48$ dari $x = 2$ sampai dengan $x = 4$. *(EAS 2021/2022, Rabu 8 Juni 2022)*

Jawab

$$24xy = x^4 + 48 \quad , \quad 2 \le x \le 4$$

$$y = \frac{x^4}{24x} + \frac{48}{24x}$$

$$y = \frac{x^3}{24} + 2x^{-1}$$

$$\frac{dy}{dx} = \frac{x^2}{8} - 2x^{-2} = \frac{x^2}{8} - \frac{2}{x^2}$$

$$ds = \sqrt{1 + \left(\frac{x^2}{8} - \frac{2}{x^2}\right)^2}\, dx$$

$(a-b)^2 = a^2 - 2ab + b^2$

$$S = \int_2^4 \sqrt{1 + \left(\frac{x^2}{8} - \frac{2}{x^2}\right)^2}\, dx$$

$$= \int_2^4 \sqrt{1 + \frac{x^4}{64} - \frac{1}{2} + \frac{4}{x^4}}\, dx$$

$$= \int_2^4 \sqrt{\frac{1}{2} + \frac{x^4}{64} + \frac{4}{x^4}}\, dy$$

$$= \int_2^4 \sqrt{\frac{32x^4 + x^8 + 256}{64x^4}}\, dx$$

$$= \int_2^4 \sqrt{\frac{(x^4 + 16)^2}{(8x^2)^2}}$$

$$= \int_2^4 \frac{x^4 + 16}{8x^2}\, dx$$

$$= \int_2^4 \frac{x^2}{8} + \frac{2}{x^2}\, dx$$

$$= \int_2^4 \frac{x^2}{8} + 2x^{-2}\, dx$$

$$= \left[\frac{x^3}{24} - \frac{2}{x}\right]\Bigg|_2^4$$

$$= \left[\frac{64}{24} - \frac{2}{4}\right) - \left[\frac{8}{24} - \frac{2}{2}\right]$$

$= \dots$ S. panjang.

2. Diberikan persamaan kurva $x = 2\sqrt{1-y}$; $-1 \le y \le 0$. *(EAS 2020/2021, Rabu 30 Juni 2021)*
   (a) Buatlah sketsa grafik persamaan kurvanya.
   (b) Dapatkan luas permukaan benda putar jika kurva diputar terhadap sumbu-y.

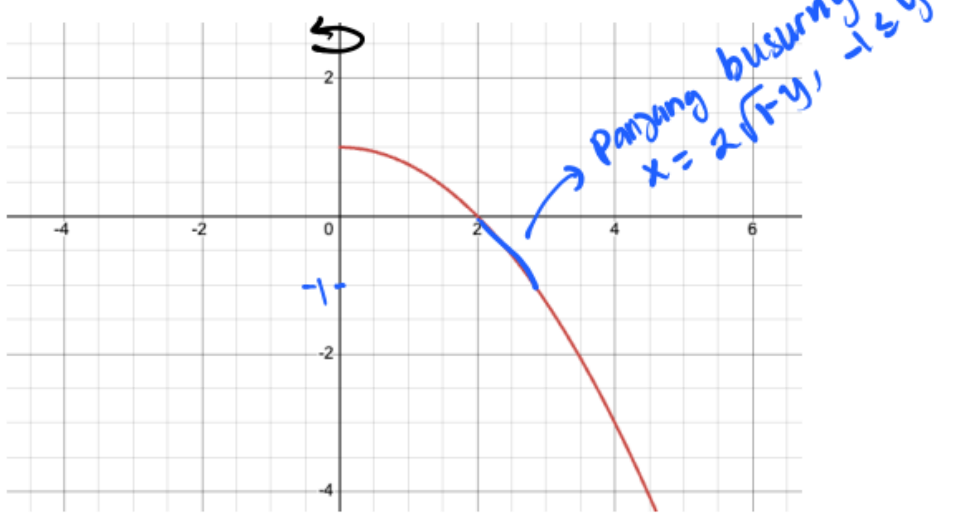
Jawab:

2.a $x = 2\sqrt{1-y}$, $-1 \le y \le 0$

$y = -1 \to x = 2\sqrt{2}$

$y = -\frac{1}{2} \to x = 2\sqrt{\frac{3}{2}}$

$y = 0 \to x = 2$



2b. Luas permukaan

$$dL = 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$x = 2\sqrt{1-y} = 2(1-y)^{1/2}$$

$$\frac{dx}{dy} = \cancel{2} \cdot \frac{1}{\cancel{2}} (1-y)^{-1/2} \cdot (-1)$$

$$\frac{dx}{dy} = \frac{-1}{\sqrt{1-y}}$$

$$\boxed{L = \int_c^d 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy}$$

$$= \int_{-1}^{0} 2\pi\left(2\sqrt{1-y}\right) \sqrt{1+\left(\frac{-1}{\sqrt{1-y}}\right)^2}\, dy$$

$$= \int_{-1}^{0} 4\pi\sqrt{1-y} \sqrt{1 + \frac{1}{1-y}}\, dy$$

$$= \int_{-1}^{0} 4\pi \sqrt{1-y} \sqrt{\frac{1-y+1}{1-y}}\, dy$$

$$= \int_{-1}^{0} 4\pi\cancel{\sqrt{1-y}} \frac{\sqrt{2-y}}{\cancel{\sqrt{1-y}}}\, dy$$

$$= \int_{-1}^{0} 4\pi \sqrt{2-y}\, dy$$

Misal

$u = 2 - y$

$du = -dy$

$-du = dy$

Batas

$y = -1 \rightarrow u = 3$

$y = 0 \rightarrow u = 2$

$$L = \int_3^2 4\pi \sqrt{u}\,(-du)$$

$$= -4\pi \int_3^2 u^{1/2}\,du$$

$$= -4\pi \left[\frac{2}{3} u^{3/2} \right] \Big|_3^2$$

$$= -4\pi \left[\frac{2}{3}(2)^{3/2} \right] - \left[-4\pi \left(\frac{2}{3}(3)^{3/2} \right) \right]$$

$= \ldots$ Satuan Luas

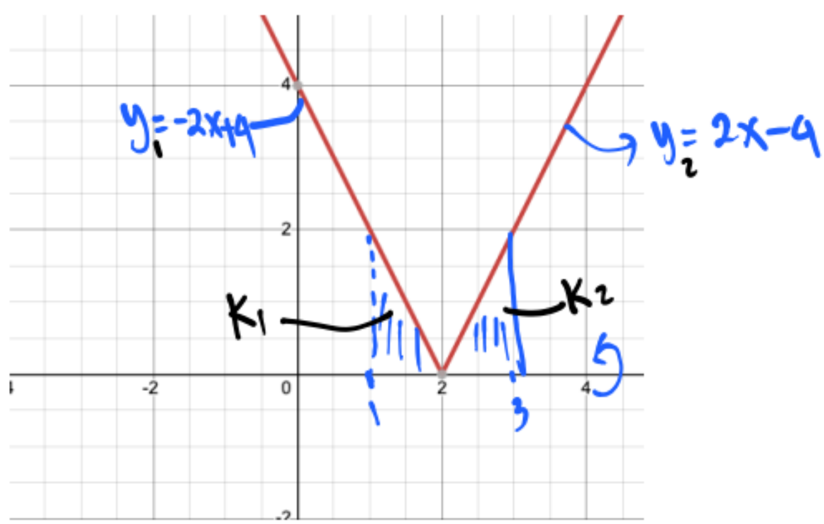
1. Dapatkan luas permukaan dari kurva $y = |2x - 4|$, $1 \le x \le 3$ diputar terhadap sumbu-x.
(EAS 2021/2022, Rabu 8 Juni 2022)

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

(i) $$y = |2x-4| = \begin{cases} 2x-4, & 2x-4 \geq 0 \\ -(2x-4), & 2x-4 < 0 \end{cases}$$

$$= \begin{cases} 2x-4; & 2x \geq 4 \\ -2x+4; & 2x < 4 \end{cases}$$

$$= \begin{cases} 2x-4, & x \geq 2 \\ -2x+4, & x < 2 \end{cases}$$



(ii) $y_1 = -2x+4$ $\qquad$ $y_2 = 2x-4$

$\frac{dy_1}{dx} = -2$ $\qquad$ $\frac{dy_2}{dx} = 2$

$$dK = dK_1 + dK_2$$

$$= 2\pi y_1 \sqrt{1+\left(\frac{dy_1}{dx}\right)^2}dx + 2\pi y_2 \sqrt{1+\left(\frac{dy_2}{dx}\right)^2}dx$$

$$K = \int_1^2 2\pi(-2x+4)\sqrt{1+(-2)^2}\,dx + \int_2^3 2\pi(2x-4)\sqrt{1+(2)^2}\,dx$$

$$= \int_1^2 2\pi\sqrt{5}\,(-2x+4)\,dx + \int_2^3 2\pi\sqrt{5}\,(2x-4)\,dx$$

$$= 2\pi\sqrt{5}\int_1^2 -2x+4\,dx + 2\pi\sqrt{5}\int_2^3 (2x-4)\,dx$$

$$= 2\pi\sqrt{5}\left[-x^2+4x\right]\Big|_1^2 + 2\pi\sqrt{5}\left[x^2-4x\right]\Big|_2^3$$

$$= 2\pi\sqrt{5}\left[-4+8\right] - 2\pi\sqrt{5}\left[-1+4\right] + 2\pi\sqrt{5}\left[9-12\right] - 2\pi\sqrt{5}\left[4-8\right]$$

$= \ldots$ satuan luas

1. Find the area of the surface that is generated by revolving the curve $y = \sqrt{9 - x^2}, -2 \leq x \leq 2$ about the x-axis. *(EAS 2021/2022, Rabu 8 Juni 2022)*

Jawab

$y = \sqrt{9 - x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = r^2$

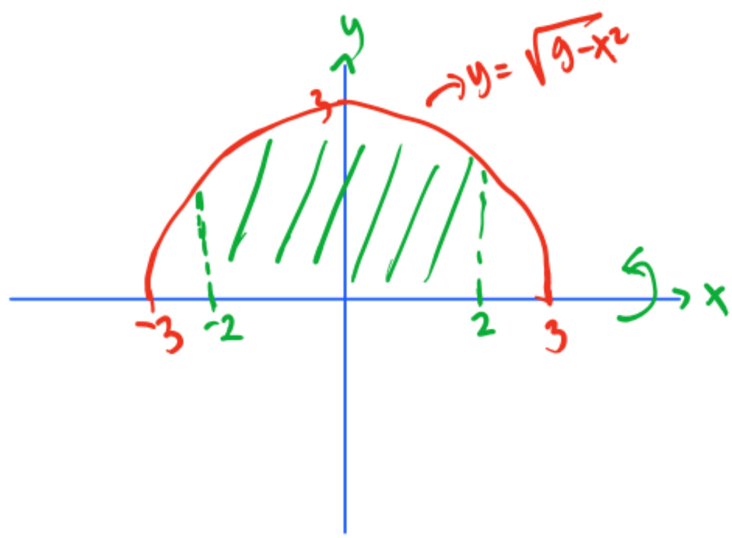
$y^2 = r^2 - x^2$

$y = \pm\sqrt{r^2 - x^2}$

$x^2 + y^2 = 9$

$x^2 + y^2 = 3^2$ (Lingkaran $P(0,0), r = 3$)

$y = \sqrt{9 - x^2}$ (Lingkaran bagian atas)



$$y = \sqrt{9-x^2}$$

$$y = (9-x^2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(9-x^2)^{-1/2} \cdot (-\cancel{2}x)$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{9-x^2}}$$

$$dK = 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$K = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} 2\pi \sqrt{9-x^2} \sqrt{1+\left(\frac{-x}{\sqrt{9-x^2}}\right)^2}\, dx$$

$$= \int_{-2}^{2} 2\pi \sqrt{9-x^2} \sqrt{1+\frac{x^2}{9-x^2}}\, dx$$

$$= \int_{-2}^{2} 2\pi \sqrt{9-x^2} \sqrt{\frac{9-x^2+x^2}{9-x^2}}\, dx$$

$$= \int_{-2}^{2} 2\pi \cancel{\sqrt{9-x^2}} \cdot \frac{\sqrt{9}}{\cancel{\sqrt{9-x^2}}}\, dx$$

$$= \int_{-2}^{2} 2\pi \cdot 3 \, dx$$

$$= \int_{-2}^{2} 6\pi \, dx$$

$$= 6\pi x \Big|_{-2}^{2}$$

$$= 6\pi(2) - 6\pi(-2)$$

$$= 12\pi + 12\pi$$

$$= \underline{\underline{24\pi}} \text{ S. Luas.}$$

$$(\underset{u}{x^2}\ \underset{v}{\sin x})$$

1. $\frac{d}{dx}[uv] = u'v + uv'$

2. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$

3. $\frac{d}{dx}[f(u)] = f'(u) \cdot \frac{du}{dx}$

$y = \sin^2 x$

$\frac{dy}{dx} = 2 \sin x \cdot (\cos x)$