Pertemuan 7 - Juni

Friday, 14 June 2024
15.04

5. Diberikan fungsi $f(x) = e^{-x}$. *(EAS 2021/2022, Rabu 8 Juni 2022)*
   (a) Dapatkan polinomial Maclaurin derajat 5 dari fungsi tersebut.
   (b) Dapatkan deret Maclaurin fungsi tersebut dan nyatakan dalam notasi sigma.

Polinomial Maclaurin derajat-n

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

Notasi sigma Deret Maclaurin

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}x^k = f(0) + f'(0) + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(k)}(0)}{k!}x + \cdots$$

5a $f(x) = e^{-x}$

$$\boxed{\frac{d}{dx}[e^{ax}] = ae^{ax}}$$

Jawab

$f(x) = e^{-x} \rightarrow f(0) = e^0 = 1$

$f'(x) = -e^{-x} \rightarrow f'(0) = -e^0 = -1$

$f''(x) = e^{-x} \rightarrow f''(0) = e^0 = 1$

$f'''(x) = -e^{-x} \rightarrow f'''(0) = -e^0 = -1$

$f^{iv}(x) = e^{-x} \rightarrow f^{iv}(0) = e^0 = 1$

$f^{(5)}(x) = -e^{-x} \rightarrow f^{(5)}(0) = -e^0 = -1$

$$P_5(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5$$

$$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$$

5b. Deret dan notasi sigma

$$f^{(k)}(x) = (-1)^k e^{-x} \rightarrow f^{(k)}(0) = (-1)^k e^0 = (-1)^k$$

Deret Maclaurin

$$e^{-x} = 1 - x + \frac{x^3}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^k x^k}{k!} + \cdots$$

Notasi sigma

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^k$$

5. Diberikan fungsi $f(x) = \sinh(6x - 12)$. *(EAS 2022/2023, Senin 12 Juni 2023)*
   (a) Dapatkan polinomial Taylor derajat 5 dari fungsi tersebut di sekitar $x = 2$.
   (b) Dapatkan deret Taylor fungsi tersebut di sekitar $x = 2$ dan nyatakan dalam notasi sigma.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Polinomial Taylor derajat-n di $x = a$

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Deret taylor dan notasi sigma di $x=a$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots+\frac{f^{(k)}(a)}{k!}(x-a)^k+\dots$$

5.a $f(x) = \sinh(6x-12)$ di $x=2$

Jawab

$f(x) = \sinh(6x-12) \rightarrow f(2) = \sinh 0 = \frac{e^0-e^0}{2} = 0$

$f'(x) = 6\cosh(6x-12) \rightarrow f'(2) = 6\cosh 0 = 6\left(\frac{e^0+e^0}{2}\right) = 6$

$f''(x) = 6^2\sinh(6x-12) \rightarrow f''(2) = 6^2\sinh 0 = 0$

$f'''(x) = 6^3\cosh(6x-12) \rightarrow f'''(2) = 6^3\cosh 0 = 6^3\left(\frac{e^0+e^0}{2}\right) = 6^3$

$f^{IV}(x) = 6^4\sinh(6x-12) \rightarrow f^{IV}(2) = 6^4\sinh 0 = 6^4\left(\frac{e^0-e^0}{2}\right) = 0$

$f^{(5)}(x) = 6^5\cosh(6x-12) \rightarrow f^{(5)}(2) = 6^5\cosh 0 = 6^5\left(\frac{e^0+e^0}{2}\right) = 6^5$

$$P_5(x) = f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\frac{f^{IV}(a)}{4!}(x-a)^4+\frac{f^{(5)}(a)}{5!}(x-a)^5$$

$= 0 + 6(x-2) + 0 + \frac{6^3}{3!}(x-2)^3 + 0 + \frac{6^5}{5!}(x-2)^5$

$= 6(x-2) + \frac{6^3}{3!}(x-2)^3 + \frac{6^5}{5!}(x-2)^5$

5b. Deret dan notasi sigma.

Dari 5.a

$P_5 = 6(x-2) + \frac{6^3}{3!}(x-2)^3 + \frac{6^5}{5!}(x-2)^5$

Sehingga deret

$\sinh(6x-12) = 6(x-2)^1 + \frac{6^3}{3!}(x-2)^3 + \frac{6^5}{5!}(x-2)^5 + \cdots + \frac{(6(x-2))^{2k+1}}{(2k+1)!} + \cdots$

Notasi sigma

$\sum_{k=0}^{\infty} \frac{[6(x-2)]^{2k+1}}{(2k+1)!}$ atau $\sum_{k=1}^{\infty} \frac{[6(x-2)]^{2k-1}}{(2k-1)!}$

sigma.

5. Diberikan fungsi $f(x) = \frac{1}{1-2x}$. *(EAS 2022/2023, Senin 12 Juni 2023)*
   (a) Dapatkan polinomial Maclaurin derajat 4 dari fungsi tersebut.
   (b) Dapatkan deret Maclaurin fungsi tersebut dan nyatakan dalam notasi sigma.

5.a. $f(x) = \frac{1}{1-2x} = (1-2x)^{-1} \rightarrow f(0) = (1)^{-1} = 1 = 0!$

$f'(x) = -1(1-2x)^{-2} \cdot -2$

$= 2(1-2x)^{-2} \rightarrow f'(0) = 2(1)^{-2} = 2 = 2 \cdot 1! = 2^1 \cdot 1!$

$f''(x) = -4(1-2x)^{-3} \cdot -2$

$= 8(1-2x)^{-3} \rightarrow f''(0) = 8(1)^{-3} = 8 = 4 \cdot 2! = 2^2 \cdot 2!$

$f'''(x) = -24(1-2x)^{-4} \cdot -2$

$= 48(1-2x)^{-4} \rightarrow f'''(0) = 48(1)^{-4} = 48 = 8 \cdot 3! = 2^3 \cdot 3!$

$f^{IV}(x) = -192(1-2x)^{-3} \cdot -2$

$= 384(1-2x)^{-5} \rightarrow f^{IV}(0) = 384(-1)^{-5} = 384 = 16 \cdot 4!$

$n! = n(n-1)(n-2) \times \cdots \times 2 \times 1$

$P_4(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{IV}(0)x^4}{4!}$

$\rightarrow f^{(k)}(0) = 2^k \cdot k!$

$P_4(x) = 1 + 2x + \frac{8x^2}{2!} + \frac{48x^3}{3!} + \frac{384x^4}{4!}$

$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$

$P_4(x) = 1 + 2x + \frac{8x^2}{2} + \frac{48x^3}{6} + \frac{384x^4}{24}$

$\sum_{k=0}^{\infty} \frac{2^k \cancel{k!}}{\cancel{k!}} x^k$

$P_4(x) = 1 + 2x + 4x^2 + 8x^3 + 16x^4$

$\sum_{k=0}^{\infty} 2^k x^k = \sum_{k=0}^{\infty} (2x)^k$

$P_4(x) = 2^0 + 2^1x + 2^2x^2 + 2^3x^3 + 2^4x^4$

$P_4(x) = 2^0 + (2x) + (2x)^2 + (2x)^3 + (2x)^4$

5.b. Deret dan Notasi sigma-

dari 5a

$P_4(x) = 2^0 + (2x) + (2x)^2 + (2x)^3 + (2x^4)$

Sehingga

Deret

$$\frac{1}{1-2x} = 2^0 + (2x) + (2x)^2 + (2x)^3 + (2x)^4 + \cdots + (2x)^k + \cdots$$

Notasi sigma

$$\sum_{k=0}^{\infty} (2x)^k$$