# Numerical Methods for Differential Equations: Homework 5

*Due by 2pm Tues 27th November.* ***Please submit your hardcopy at the start of lecture.*** *The answers you submit should be attractive, brief, complete, and should include program listings and plots where appropriate. The use of "publish" in* MATLAB*/Octave is one option; another is* `ubc.syzygy.ca`.

1. **Software.** Install or otherwise find a way to use one of Chebfun *or* FEniCS *or* Clawpack. Run, and modify in some way, an interesting example from their documentation. Submit a plot or output or whatever seems appropriate.

   Notes: Chebfun does not yet run correctly on Octave—it requires MATLAB. In principle, FEniCS and Clawpack should work on `https://ubc.syzygy.ca`.

2. **Digital image processing with PDEs.** A digital image is usually represented using an rectangular array of pixels. In a gray-scale image, each pixel has an intensity value in $[0, 1]$ where 0 is black and 1 is white (often in practice, these values are "quantized" into integers in $[0, 255]$). The human eye is very sensitive to edges in an image. We're going to look at "sharpening" an image, by making the edges more obvious; this can make the image appear to be higher resolution than it actually is (technically this is known as "acutance"). This is, at best, the sort of thing televisions do when they claim to "upscale" low-rez signals to full HD or 4K.[1]

   Given an image $u$, an "unsharp mask" first computes a diffused or blurry copy of $u$. This blurred copy is the subtracted from $u$ to give an edge map. The edge map should be zero in regions where the image is smooth. Finally the edge map is added to the original image.

   (a) Download the file `unsharp_images.zip` from the course website. Run the example code given.

   (b) Write a code to "blur" the image by using it as the initial condition for the heat equation $u_t = \nabla^2 2u = u_{xx} + u_{yy}$. Use $h = 1$ (one pixel) and a time-step of $k = 0.1$. Use 10 steps.

   (c) Produce a close-up picture showing the result of `testpat_noblur.png`.

   (d) Produce another picture using $k = 0.5$: what just happened and why?

3. **The Unsharp Mask.**

   (a) Modify your code in the previous problem to perform an unsharp mask based on the algorithm described above. You can use the images in the `.zip` file to test.

   (b) Using parameter values as shown above, display the results (zoomed in) for `testpar_blur2.png`. What are the minimum and maximum values of $u$ (i.e., the lightest and darkest pixel values for the result).

   (c) If the image is not blurry to begin with (e.g., `textpat_noblur.png`), what does the unsharp mask do locally around edges? Your solution should show a "zoom-in" of the results. The number of blurring steps is a tuneable parameter in this algorithm: what effect does it have?

   (d) Choose an image of your own or from the internet and play with the unsharp mask. Show us something interesting. . .

   The tutorial at `http://www.cambridgeincolour.com/tutorials/unsharp-mask.htm` gives some more details and also discusses the biological reasons of why this works. It also cautions photographers, quite rightly, against overusing this technique.

4. **Group challenge problem.** Consider the domain $(x, y) \in \Omega$ where $\Omega = [-4, 4] \times [-3, 3]$. Let "the city" be $C \subset \Omega$, $C = [-3, -1] \times [-2, 1]$. Let the "the woods" be $W \subset \Omega$, $W = \{(x, y) : \frac{4}{9}(x - \frac{3}{2})^2 + (y - 1)^2 \leq 1\}$. Suppose $S = \Omega \setminus W \setminus C$ is "the savannah".

   We are looking for a continuous function $u(x, y)$ which solves $\|\nabla u\|_1 = 1$ in the city, $\|\nabla u\|_2 = 1/10$ in the woods, and $\|\nabla u\|_2 = 1$ in the savannah.

   Produce a contour plot of $u$ and explain a physical interpretation of $u$.

[1] Caution, learning too much about digital image processing can really spoil television for you! Ingrid Daubechies (of "Wavelets" fame) tells a story of watching football and recognizing from the compression artifacts that the broadcaster was (over-)using one of the wavelet techniques that she invented.