

Exercise 2

- ① $\lambda a. a$
- ② $\lambda a. (\lambda b. b a)$
- ③ $\lambda z. (\lambda x. x z)$

Exercise 3

- ① $(\lambda x. x) y$
 $\llbracket x := y \rrbracket. x$
 $y \rightarrow$ Don't diverge

- ② $\lambda x. x x$
 $\llbracket x := x \rrbracket. x$
 $x \rightarrow$ Don't diverge

- ③ $(\lambda z. z z) (\lambda y. y y)$
 $\llbracket z := \lambda y. y y \rrbracket. z z$
 $(\lambda y. y y) (\lambda y. y y) \rightarrow$ Diverge

- ④ $(\lambda x. x x) y$
 $\llbracket x := y \rrbracket. x x \rightarrow$ Don't diverge
 $y y$

Exercise 4

- i) $(\lambda y. z y) a$
 $\llbracket y := a \rrbracket. z y$
 $z a \neq$
- ii) $(\lambda x. x) (\lambda x. x)$
 $\llbracket x := \lambda x. x \rrbracket. x$
 $(\lambda x. x) (\lambda x. x) \neq$
- iii) $(\lambda x. x y) (\lambda x. x x)$
 $\llbracket x := \lambda x. x x \rrbracket x y$
 $(\lambda x. x x) y$
 $\llbracket x := y \rrbracket. x x \quad y y \neq$

- iv) $(\lambda z. z) (\lambda a. a a) (\lambda z. z b)$
 $\llbracket z := \lambda a. a a \rrbracket (\lambda z. z) (\lambda z. z b)$
 $(\lambda a. a a) (\lambda z. z) (\lambda z. z b)$
 $\llbracket a := z z \rrbracket a a (\lambda z. z b)$
 $(\lambda z. z b) (\lambda z. z b) \cdot z z$
 $\llbracket z := z \rrbracket z b (\lambda z. z)$
 $z b$

- iv) $(\lambda z. z) (\lambda a. a a) (\lambda z. z b)$
 $\llbracket z := \lambda a. a a \rrbracket z (\lambda z. z b)$
 $(\lambda a. a a) (\lambda z. z b)$
 $\llbracket a := \lambda z. z b \rrbracket a a$
 $(\lambda z. z b) (\lambda z. z b)$
 $\llbracket z := \lambda z. z b \rrbracket z b$
 $(\lambda z. z b) b$

$\llbracket z := b \rrbracket. z b$
 $b b \neq$

Exercise 1

$I = \lambda x. x x$

$\hookrightarrow I$ Combinator is an anonymous fⁿ that takes a parameter x and return x
 \hookrightarrow it is used to wrap a value in fⁿ that can be passed to other fⁿ as other fⁿ may want it as an input

travelling in test

Exercise 5

i) $\lambda x. 2 \cdot x$
 $= 2$

ii) $\lambda x. x \cdot 2$
 \rightarrow cannot be reduced by eta

iii) $(\lambda x. bx) (\lambda y. ay)$
 $[x := \lambda y. ay] \cdot bx$
 $(b) (\lambda y. ay)$
 $[y := b] \cdot ya$
 ba

Exercise 6

i) $\lambda x. xx$
 $[x := x] \cdot xx$
 xx

\rightarrow combinator

ii) $\lambda xy. 2x$
 ~~$\lambda x \lambda y. 2x$~~
 \rightarrow not a combinator

iii) $\lambda xy 2 \cdot xy (2x)$
 $(\lambda x \lambda y \lambda 2 \cdot xy) (2x)$
 $[x := 2x] \lambda y \lambda 2 \cdot xy$
 $\lambda y \lambda 2 \cdot 2xy$
 \rightarrow not a combinator

iv) $\lambda xy 2 \cdot xy (2xy)$
 $(\lambda x \lambda y \lambda 2 \cdot xy) (2xy)$
 $[x := 2xy] \lambda y \lambda 2 \cdot xy$
 $\lambda y \lambda 2 \cdot 2xy$
 \rightarrow not a combinator