Response to the referees and summary of changes

Date: March 15, 2019

Dear editor,

We would like to resubmit our revised manuscript entitled

Ordinal partition transition network based complexity measures for inferring coupling direction and delay from time series

by Y. J. Ruan, R.V. Donner, S. G. Guan, and Y. Zou,

originally submitted for publication to *Chaos* with the manuscript number **#181384**.

Thank you very much for the comments and suggestions of the two referees. All these comments helped us to improve the manuscript. We have marked all changes in the manuscript in red color to facilitate the reading.

Please find below a detailed response to all specific remarks and questions raised by the referees. We hope that our revised manuscript is now acceptable for publication in *Chaos*.

Sincerely yours,

Y. J. Ruan, R.V. Donner, S. G. Guan, and Y. Zou

Response to the first referee

The authors are claiming to have proposed three measures for inferring coupling directions and delays from time series based on ordinal patterns. The first one, based on the standard deviation for conditional probabilities, might be new. But, the second one and the third one have been defined previously as far as I searched: The second one seems to be used at least in Li and Ouyang, NeuroImage 52, 497 (2010), while the third one was used in Staniek and Lehnertz, Phys. Rev. Lett. 100 158101 (2008). Thus, the novelty for the manuscript should be compromised and the authors should cite these preceding papers and allocating their contribution in the existing literature more appropriately. Their results shown in the figures look promising. Especially, the coupling direction and delay for the daily surface air temperature record from Oxford to that of Vienna sound reasonable. Therefore, the authors should give the credits for the related previous papers properly for the publication in Chaos.

> We thank the referee for pointing these references, which have been properly cited in the revision. We agree that our measures do have certain similarities with those that have been pointed by the referee. Yet, at the same time, there are also important conceptual differences that have been explained in detail in our revised manuscript.
>
> In a nutshell, regarding their general form, both $H_{X\to Y}(\tau)$ and KLD resemble the pattern conditional mutual information (PCMI) proposed by Li et al, as well as the more general symbolic transfer entropy (STE) as proposed by Staniek and Lehnertz. However, the latter two measures are designed to quantify causality in the predictive (Granger) sense and therefore involve conditions on the previous patterns in both, X and Y. In turn, our measures condition only on the pattern exhibited by the second variable and, hence, do not quantify predictive causality, but simply co-occurrence. Another difference is the interpretation of the parameter τ, which is used in our measures as a variable to perform explorative analyses as opposed to a fixed algorithmic parameter in PCMI and STE. We have clarified these points in great detail in a new dedicated subsection of our revised manuscript.

In the abstract and the main text, OPTN is defined as an abbreviation for two different phrases. In the abstract, it comes from ordinal pattern transition networks, while in the main text, it comes from ordinal partition transition networks. Please define the abbreviation by one of them.

> In the revision, we use of the acronym for OPTN as ordinal partition transition network, which keeps the consistency with the literature. This has been corrected in both, the abstract and the main text.

Response to the second referee

The main purpose of the manuscript is to show that ordinal pattern transition networks (OPTNs) from time series can provide an approach to improve our understanding of the underlying dynamical system (responsible for the data). The authors conclude that OPTNs can be used as complementary tools for causal inference tasks.

The text is a well-written and scientifically sound manuscript that adds understanding to the important area of time series analysis. Before its publication, I believe some questions must be better answered by the authors.

> We thank the referee for the overall positive evaluation on our manuscript.

1- On figures 4 - 8 and 11. The authors claim the results are "based on averages over 20 independent realizations of the considered processes". So the dispersion of the mean values (plotted by error bar or any other way) must be included. A discussion about dispersion values will bring more information about the validity of the results.

> This is indeed a relevant comment. In the very beginning, we had planned to include the error bars in all figures. However, as we show in this reply letter in Figs. 1 - 4, the resulting error bars (i.e., standard deviations among ensemble members) of all considered measures are practically invisible. Therefore, we have decided not to include the error bars in the manuscript since they do not provide additional information. We accordingly suggest not to include the plots with error bars in the main manuscript, and have instead added a corresponding note during our revision.

2- Figures 9 and 10 show results only for KLD measure. Why not for all quantifiers? Figures 9 and 10 also show that KLD is a reliable quantifier only for time series larger than 10000. Such a result is consistent for both systems considered in the figures. Fig. 11 shows results for an experimental situation where the available data size is on the same order (10000). So the results must be corroborated at least using other quantifiers. A sentence discussing it will be also appreciated.

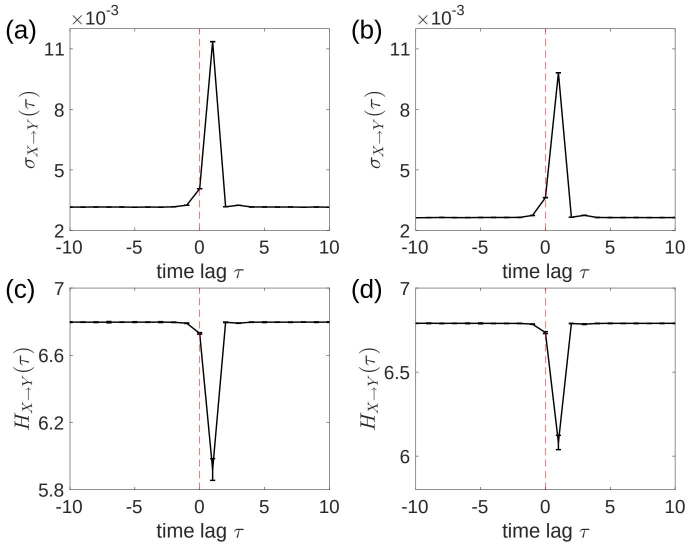

FIG. 1: Values of three ordinal pattern co-occurrence complexity measures in dependence on the mutual delay τ for realizations of Eq. (1): (a,b) $\sigma_{X\rightarrow Y}(\tau)$, (c,d) $H_{X\rightarrow Y}(\tau)$, (e,f) KLD(τ). (a,c,e) show the results for realizations from the linear model, while (b,d,f) correspond to the nonlinear transformation of the first subsystem as described in the text. The vertical red dashed lines indicate the values for vanishing mutual delay ($\tau = 0$). Note that the error bars have been magnified by a factor of 1,000 to become visible at all.

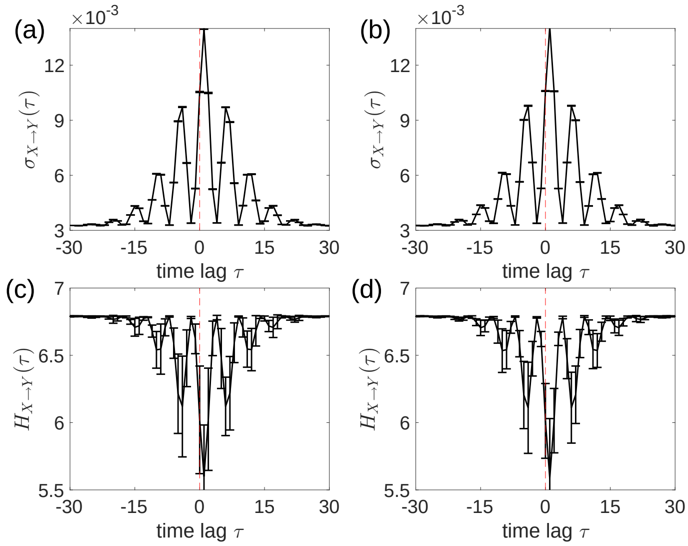

FIG. 2: Same as in Fig. 1 but for Eq. (10). Note that the error bars have been magnified by a factor of 1,000 to become visible at all.

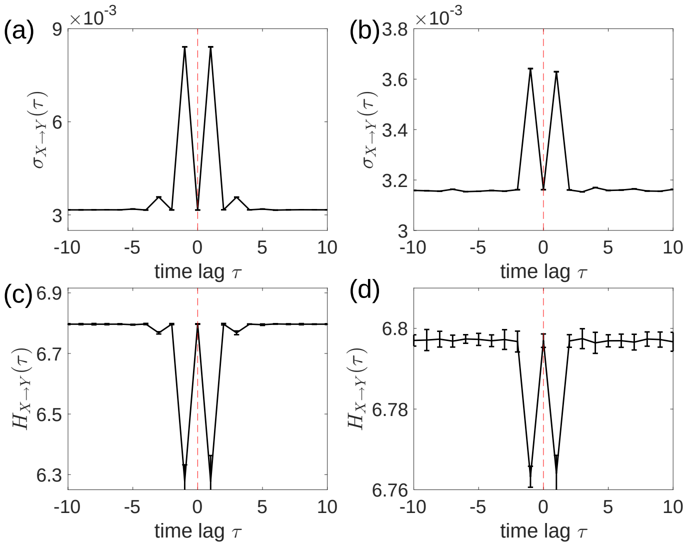

FIG. 3: Same as in Fig. 1 but for Eq. (11). Note that the error bars have been magnified by a factor of 1,000 to become visible at all.

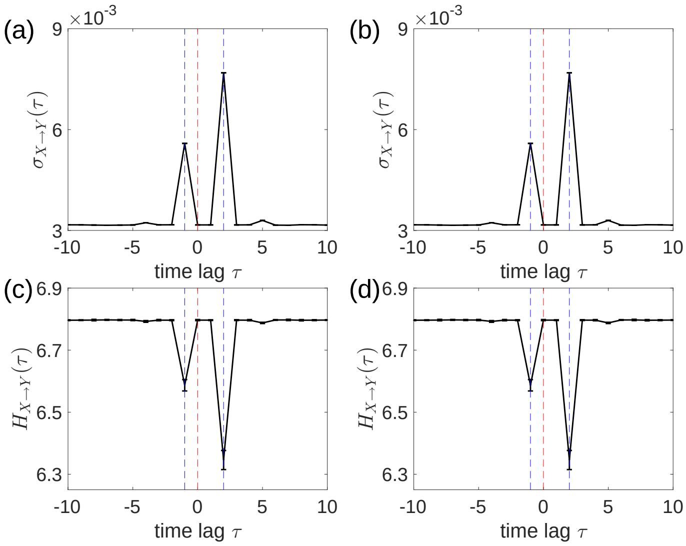

FIG. 4: Same as in Fig. 3 but for Eq. (12). The two employed delays of $\tau = -1$ and $\tau = 2$ are additionally highlighted by vertical blue dashed lines. Note that the error bars have been magnified by a factor of 1,000 to become visible at all.

We have clarified the motivations of KLD measures in the main body of the manuscript in the revision. The main reason for focusing on KLD only has been that this measure is by construction not affected by non-uniform marginal distributions of ordinal pattern frequencies. However, the other two measures $\sigma_{X\to Y}$ and $H_{X\to Y}$ may differ from theor theoretical limit values for uncoupled processes in case of non-uniform distributions.

In our numerical studies, we have obtained consistent results for all three complexity measures, which show asymptotic convergences towards their expected values for uncoupled systems. Along with our revised manuscript, we now provide some supplementary material file containing additional results on the finite size effects of the two other measures. Moreover, we have included a corresponding discussion as suggested, in particular on the fact that the obtained numerical results appear reliable only if time series of lengths larger than 10,000 are considered. This rather slow convergence can be understood by the very large number of ordinal patterns considered, while lower pattern orders would lead to a considerably faster convergence.

3- Please, provide References for the sentences:

" In general, temperature records like those used here are characterized by some irregular alternation between colder and warmer phases due to large-scale atmospheric patterns (corresponding to high and low pressure centers, respectively), which in the case of western-to- central Europe travel in the majority of situations in eastward direction."

"This coupling delay is reasonable since temperature variations in Oxford arising from general atmospheric circulation patterns are typically propagated eastward and reach Vienna about 1to 2 days later, which corresponds to the average propagation speed of frontal systems in the northern mid-latitudes west-wind zone."

In the revision, we have included some further discussion and references along with these two sentences.