Supplementary Materials:
Ordinal partition transition network based complexity measures for inferring coupling direction and delay from time series

(Dated: March 11, 2019)

SM-I. SAMPLE SIZE EFFECTS ON $\sigma_{X\to Y}$

1. Stochastic models (Fig. S1)

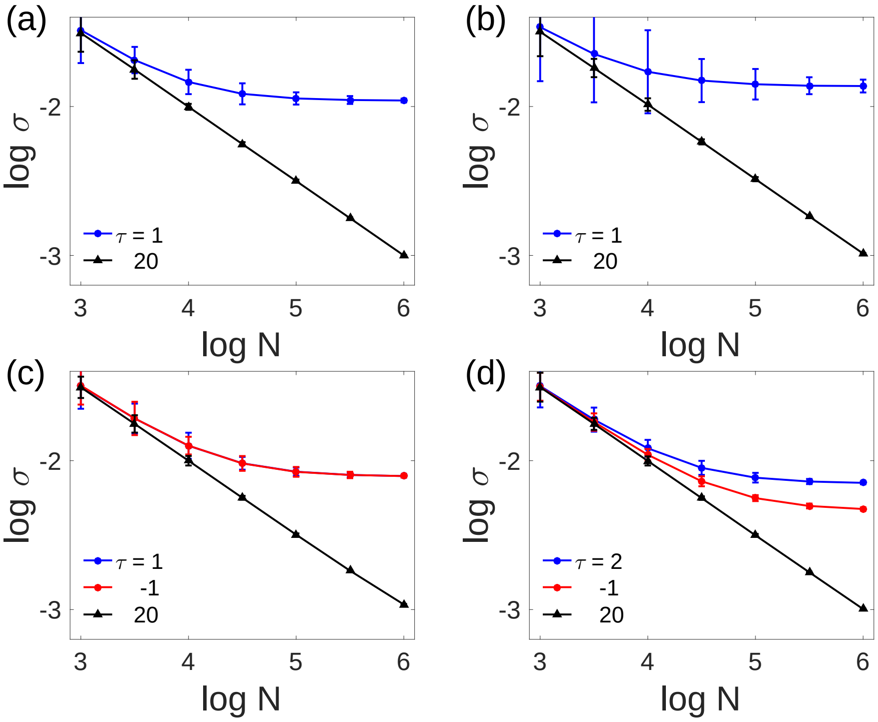

FIG. S1: (Color online) Double logarithmic plot of the dependence of $\sigma_{X\to Y}$ on the sample size N for the optimal (causal) lags (blue/red) and some non-causal lag (black) for the four cases of coupled linear-stochastic systems: (a) Eq. 1 (unidirectional), (b) Eq. 9 (unidirectional), (c) Eq. 10 (symmetric bidirectional), (d) Eq. 11 (asymmetric bidirectional). In (c,d), the values for both causal delays are shown. Errorbars correspond to the standard deviation (linear scale) over 20 independent realizations.

2. Coupled Henon maps (Fig. S2)

SM-II. SAMPLE SIZE EFFECTS ON CO-OCCURRENCE ENTROPY $H_{X\to Y}$

It is expected that the co-occurrence entropy $H_{X\to Y} \approx 6.9$ when $D = 6$ for two series of ordinal patterns that are obtained from two independent identically distributed noise processes. Considering this

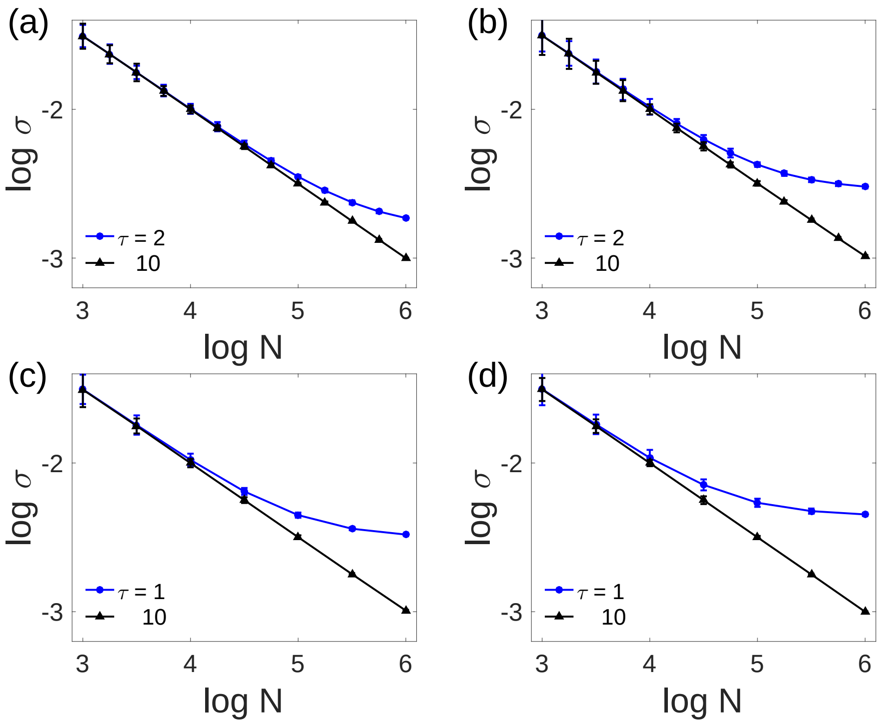

FIG. S2: (Color online) Same as in Fig. S1 but for the two coupled Hénon maps at four different coupling strengths μ: (a) $\mu = 0.2$ (b) 0.3, (c) 0.4, and (d) 0.5. The blue lines show the behavior at the delay corresponding to maximum $\sigma_{X\to Y}$, while the black ones show the values for a non-causal delay of $\tau = 10$.

fact, we calculate the distance between $H_{X\to Y}$ and the expected value of 6.9 ($D = 6$ is used), which yields similar asymptotical results as that of $\sigma_{X\to Y}$ and KLD. Namely, at positions of non-causal delays, zero values are expected asymptotically, while non-zero values appear at the positions of causal delays.

1. Stochastic models (Fig. S3)

2. Coupled Henon maps (Fig. S4)

SM-III. SAMPLE SIZE EFFECTS ON TEMPERATURE RECORDS

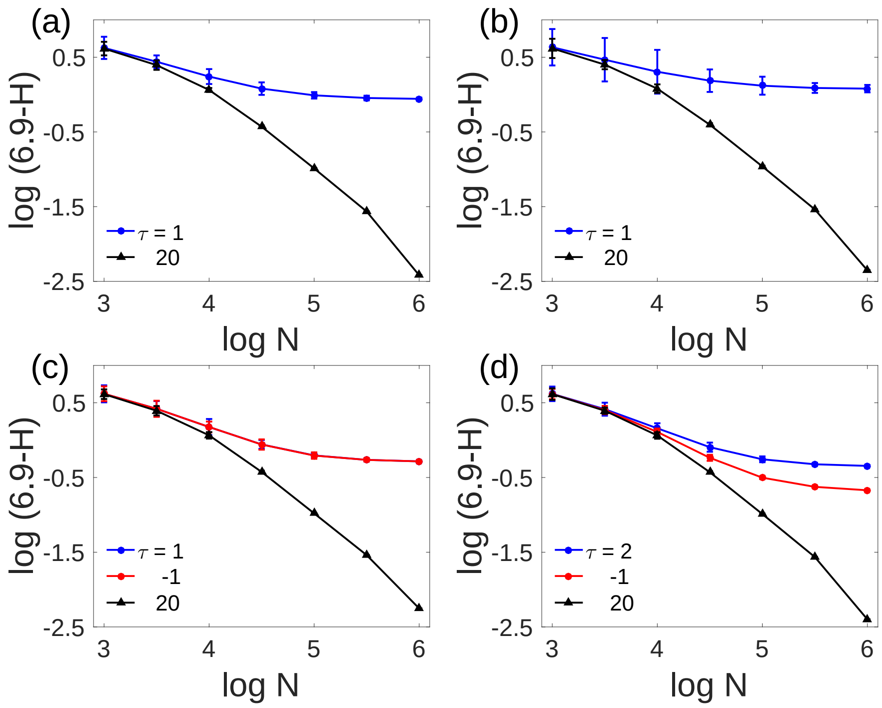

FIG. S3: (Color online) The caption is the same as Fig. S1, but for the co-occurrence entropy $H_{X \to Y}(\tau)$.

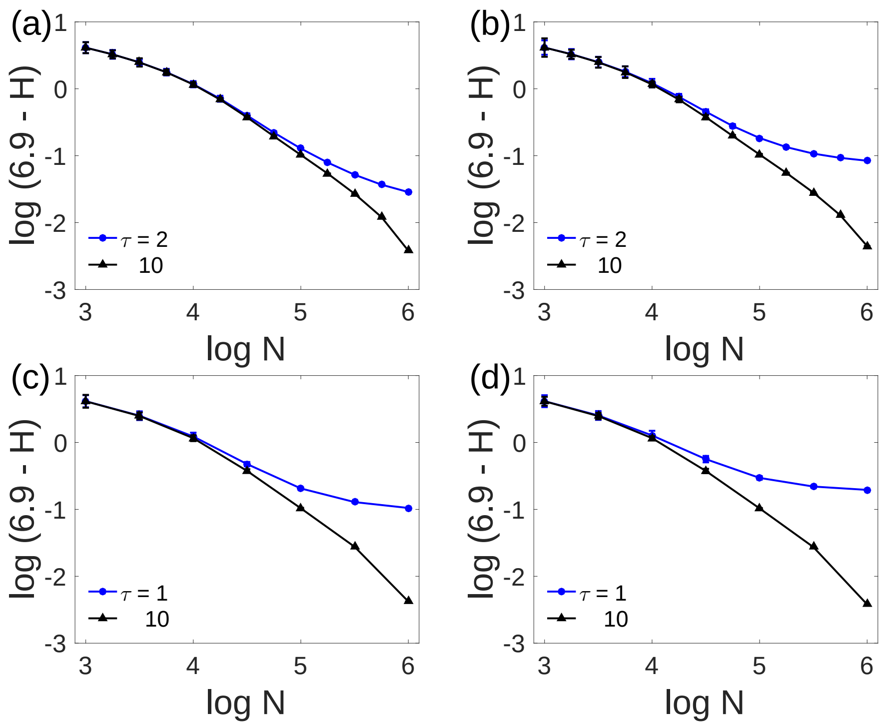

FIG. S4: (Color online) The caption is the same as Fig. S2, but for the co-occurrence entropy $H_{X \to Y}(\tau)$.

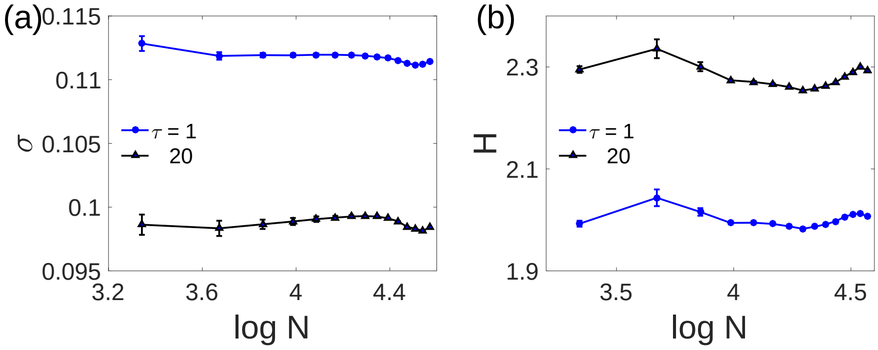

FIG. S5: (Color online) (a) Dependence of σ and, respectively, (b) H on the sample length N at the optimal delay ($\tau = 1$ day, blue) and some arbitrarily chosen large delay ($\tau = 20$ days, black).