Complex network approaches to nonlinear time series analysis

Reik V. Donner[a,*], Yong Zou[b,*], Norbert Marwan[a], Jonathan F. Donges[a], Jürgen Kurths[a,c,d,e,f]

[a]*Potsdam Institute for Climate Impact Research, P. O. Box 60 12 03, 14412 Potsdam, Germany*
[b]*Department of Physics, East China Normal University, Shanghai 200062, China*
[c]*Institute for Complex Systems and Mathematical Biology, University of Aberdeen, Aberdeen AB243UE, United Kingdom*
[d]*Department of Physics, Humboldt University Berlin, Newtonstraße 15, 12489 Berlin, Germany*
[e]*Department of Control Theory, Nizhny Novgorod State University, Gagarin Avenue 23, 606950 Nizhny Novgorod, Russia*
[f]*Institute of Applied Physics of the Russian Academy of Sciences, 603950 Nizhny Novgorod, Russia*

Abstract

In the last about 10 years, there has been a growing body of literature aiming at the utilization of complex network methods for the characterization of dynamical systems based on time series. While both nonlinear time series analysis and complex network theory are widely considered to be established fields of complex systems sciences with strong links to Nonlinear Dynamics and Statistical Physics, the thorough combination of both approaches has become an active field of research during the last decade, which has allowed addressing fundamental questions regarding the structural organization of nonlinear dynamics as well as the successful treatment of a variety of applications from a broad range of disciplines.

With its three main concepts, phase space / recurrence networks, visibility graphs and transition / Markov chain networks having made their way from abstract concepts to widely used methodologies, the field of time series networks has become mature. These three concepts, as well as several variants thereof, have been studied in great detail regarding their specific properties, potentials and limitations and provided fundamental new insights into the dynamics of complex systems. In addition, these approaches have already found a wide range of applications from such diverse fields as climatology, neurophysiology and economics, to mention only a few examples, demonstrating the great potentials of time series networks to tackling real-world contemporary scientific problems.

To this end, there exists no thorough overview paper covering all existing approaches of time series networks. Consequently, we believe that the time is ripe to deliver such a review covering the methodological foundations, interpretation and (potential) applications of the existing zoo of methods from this field. We are confident that Physics Reports would be an excellent forum for the first review that integrates the state of research on all corresponding concepts that exist so far.

Keywords:

Contents

*Corresponding author
Email addresses: reik.donner@pik-potsdam.de (Reik V. Donner), yzou@phy.ecnu.edu.cn (Yong Zou), marwan@pik-potsdam.de (Norbert Marwan), donges@pik-potsdam.de (Jonathan F. Donges), kurths@pik-potsdam.de (Jürgen Kurths)

Table 1: Summary of the definitions of vertices and the criteria for the existence of edges in existing complex network approaches to time series analysis.

Method	Vertex	Edge	Directedness
Proximity networks			
Cycle networks	Cycle	Correlation or phase space distance between cycles	undirected
Correlation networks	State vector	Correlation coefficient between state vectors	undirected
Recurrence networks			
k-nearest neighbor networks	State (vector)	Recurrence of states (fixed neighborhood mass)	directed
adaptive nearest neighbor networks	State (vector)	Recurrence of states (fixed number of edges)	undirected
ε-recurrence networks	State (vector)	Recurrence of states (fixed neighborhood volume)	undirected
Visibility graphs	Scalar state	Mutual visibility of states	undirected
Transition networks	Discrete state	Transitions between states	directed

1. Introduction (Reik+Yong+All)

1.1. Nonlinear time series analysis

A bit more on practical motivation - limitations of existing time series analysis methods regarding certain common types of problems like detection of subtle dynamical transitions, time-series reversibility tests, etc.? See comment of Jonathan (added below at Discussion section)!

1.2. Complex networks

1.3. Transformations from time series to network domain

[1, 2]

2. Complex network theory (Reik, in Appendix?)

2.1. Basic concepts

2.2. Network characteristics

2.2.1. Vertex characteristics

2.2.2. Edge characteristics

2.2.3. Global network characteristics

2.3. Stylized facts of complex networks

2.3.1. Random network models

2.3.2. Scale-free networks

2.3.3. Small-world networks

2.3.4. Assortative mixing

2.4. Coupled networks

2.4.1. General foundations

2.4.2. Vertex characteristics

2.4.3. Global characteristics

2.4.4. Relationships with multi-layer and multiplex networks

3. Recurrence networks in phase space (Reik)

3.1. Theoretical background

3.1.1. Phase space and attractor reconstruction

3.1.2. Recurrences and recurrence plots

3.1.3. Related approaches to time series analysis

3.2. Types of recurrence networks

3.2.1. epsilon-recurrence networks

3.2.2. k-nearest neighbor networks

3.2.3. Adaptive neighbor networks

briefly mention the results by Hui Yang on the "embeddability" of different times using force-directed placement algorithms (Chaos paper) - discussion of advantages and disadvantages of different neighborhood definitions

3.3. Analytical theory

3.3.1. Foundations: random geometric graphs

3.3.2. Analytical description of epsilon-recurrence networks

3.4. Practical considerations

3.4.1. Dependence on embedding parameters

3.4.2. Choice of recurrence rate or threshold

3.4.3. Application to stochastic systems

maybe switch 3.4 and 3.5 ?

re-organize according to common stylized network facts: scale-free degree distributions, small-world behavior, assortativity

3.5. General properties of recurrence networks

3.5.1. Degree distributions

3.5.2. Transitivity and fractal dimension

3.5.3. Path-based characteristics

3.5.4. Stability and robustness against noise

3.6. Inter-system recurrence networks

3.6.1. Cross-recurrence plots

3.6.2. Coupled networks framework

3.6.3. Analytical description

3.6.4. Geometric signatures of coupling

3.6.5. Relationships with multi-layer / multiplex recurrence networks

3.7. Joint recurrence networks

3.7.1. Joint recurrence plots

3.7.2. Network interpretation

3.7.3. Network properties and synchronization

3.7.4. Algorithmic variants

3.8. Other proximity-based time series networks

3.8.1. Correlation networks

3.8.2. Cycle networks

4. Visibility graphs (Yong)

4.1. Historical roots

In the last about 10 years, there has been a growing body of literature aiming at the utilization of complex network methods for the characterization of dynamical systems based on time series. While both nonlinear time series analysis and complex network theory are widely considered to be established fields of complex systems sciences with strong links to Nonlinear Dynamics and Statistical Physics, the thorough combination of both approaches has become an active field of research during the last decade, which has allowed addressing fundamental questions regarding the structural organization of nonlinear dynamics as well as the successful treatment of a variety of applications from a broad range of disciplines.

With its three main concepts, phase space / recurrence networks, visibility graphs and transition / Markov chain networks having made their way from abstract concepts to widely used methodologies, the field of time series networks has become mature. These three concepts, as well as several variants thereof, have been studied in great detail regarding their specific properties, potentials and limitations and provided fundamental new insights into the dynamics of complex systems. In addition, these approaches have already found a wide range of applications from such diverse fields as climatology, neurophysiology and economics, to mention only a few examples, demonstrating the great potentials of time series networks to tackling real-world contemporary scientific problems.

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4.2. Algorithmic variants

4.2.1. Standard visibility graphs

4.2.2. Horizontal visibility graphs

4.3. Visibility graph properties

4.3.1. Degree distributions

relationship with Hurst exponent in fractal / multifractal time series

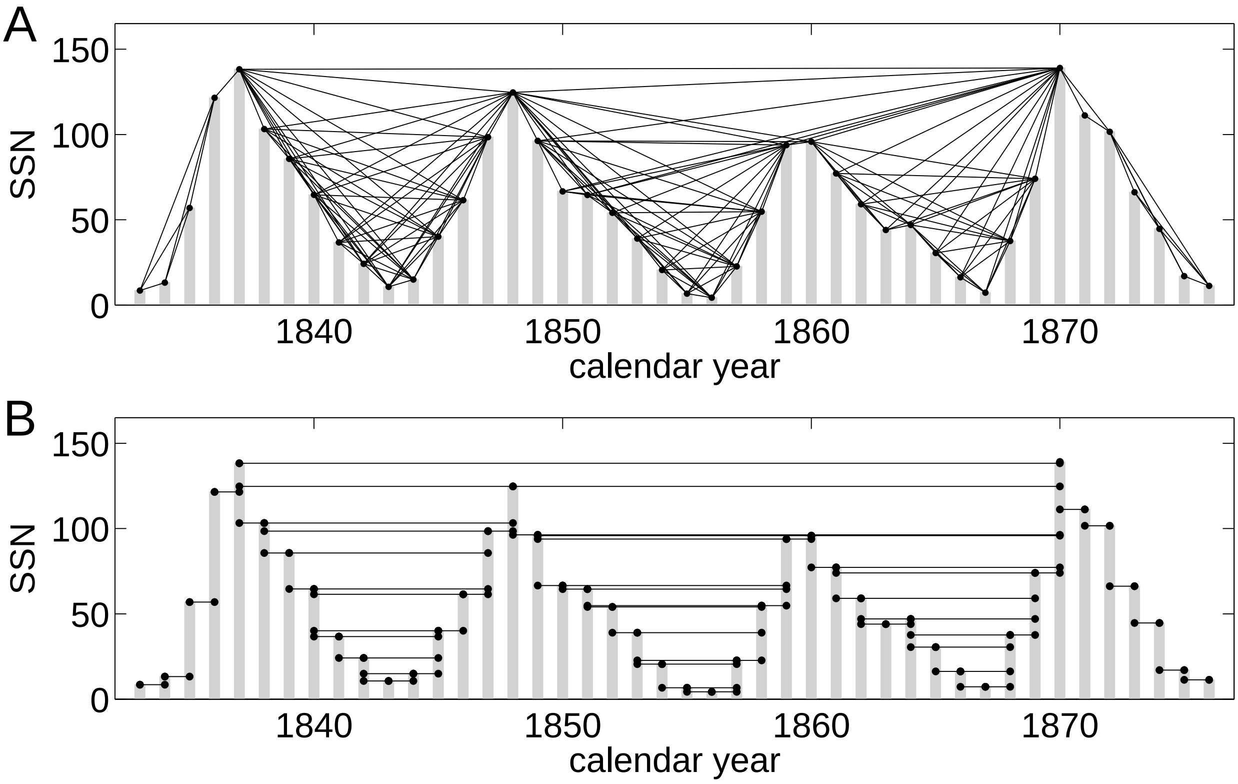

Figure 1: Algorithm of constructing visibility graphs.

4.3.2. Stochastic vs. deterministic dynamics

4.3.3. Local network properties

4.3.4. Global network properties

4.3.5. Properties related to edge lengths

4.3.6. Finite-sample effects

influence of noise, missing values, time-scale uncertainty

also discuss here VGs for (marked) point processes like application to earthquakes (Telesca et al.), draw link to natural time analysis method here???

Maybe that's more like a separate subsection on "practical considerations", together with noise effects, etc.

General question: What information can be gained from VGs? Which networks properties are (when) useful to study?

4.4. Bivariate visibility graph methods

4.4.1. Visibility graph similarity

4.4.2. Joint and excess degrees

4.5. Decomposition of visibility graphs

4.5.1. Time-directed visibility graphs

4.5.2. Tests for time series irreversibility

4.6. Identifying transitions by visibility graph method? Any references?

5. Transition networks (Reik)

5.1. Markov chains

5.2. Symbolic encoding of time series

5.2.1. Threshold-based coarse-graining

5.2.2. Order pattern-based coarse graining

5.2.3. Other approaches

5.3. Network interpretation of Markov chains

6. Other approaches (Reik+Yong+All)

7. Real-world applications (All)

7.1. Recurrence networks

7.1.1. Example I: palaeoclimate data

7.1.2. Example II: soil water

7.2. Visibility graphs

7.2.1. Example I: sunspot numbers

7.2.2. Example II: Asymmetry of sunspots

7.3. Transition networks

7.4. Other approaches

7.5. Please suggest any examples you may find appropriately here. Later, we will agree upon 1 or 2 examples which will discussed in detail for each method and mention briefly about other examples. We will include those examples that have been fully done to avoid futher serious calculations.

8. Existing software (Jonathan+Norbert)

8.1. pyunicorn

8.2. Any others?

9. Discussion (All)

Comments by Jonathan:

- What is the added value of network methods compared to standard methods?

- Why do network methods typically seem to work better with small data sets than standard methods? Does this have to do with N^2 of information units (although not independent) being created from N data points through building an adjacency matrix, leading to more robust statistics?

- What are common applications of network methods? Such as identifying regime shifts / transitions / tipping points in time series, distinguishing different dynamical regimes, analysis of dynamical system structure in phase space, prediction of some types... What more?

We might probably start with taking these points already in the introduction (as motivating questions beyond the methodological interest) and take them up for more detailed discussion at the end of the review.

10. Summary (All)

Acknowledgements

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[2] N. Marwan, M. C. Romano, M. Thiel, J. Kurths, Recurrence plots for the analysis of complex systems, Phys. Rep. 438 (5–6) (2007) 237–329. doi:10.1016/j.physrep.2006.11.001.