Project 1

Voeurn Yong Ann
y.voeurn@innopolis.university

1. Description of a neuron, numerical simulation of a model neuron

In this report, we present a model of Spiking Neurons (Izhikevich Neuron model). The ordinary differential equations of system [1] forms as following:

$$\frac{dv}{dt} = 0.004v^2 + 5v + 140 - u + I \tag{1}$$

$$\frac{du}{dt} = a(bv - u) \tag{2}$$

with the auxiliary after-spike resetting:

$$\text{if } v \geq 30mV, \text{ then} \begin{cases} v = c \\ u = u + d \end{cases} \tag{3}$$

- The parameter a describes the time scale of the recovery variable u. Smaller values result in slower recovery. A typical value is a =0.02.
- The parameter b describes the sensitivity of the recovery variable u to the subthreshold fluctuations of the membrane potential v. Greater values couple v and u more strongly resulting in possible subthreshold oscillations and low-threshold spiking dynamics. A typical value is b = 0.2.
- The parameter c describes the after-spike reset value of the membrane potential v caused by the fast high-threshold K+ conductance. A typical value is c = -65 mV
- The parameter d describes after-spike reset of the recovery variable u caused by slow high-threshold Na+ and K+ conductances. A typical value is d = 2.

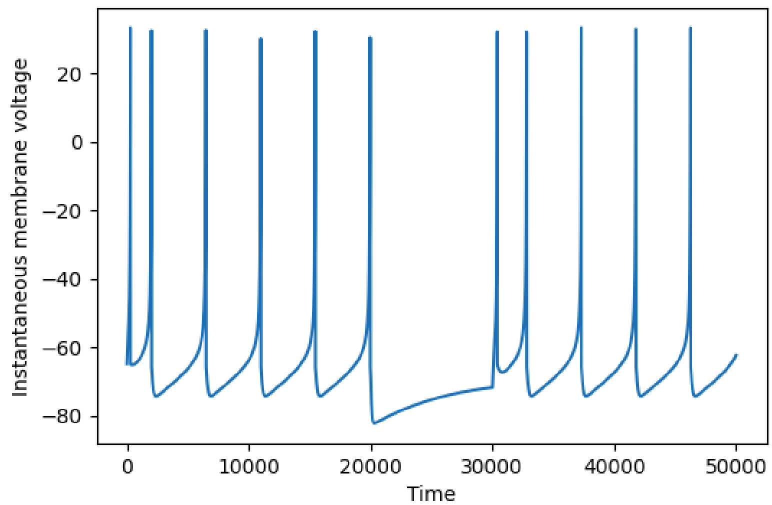


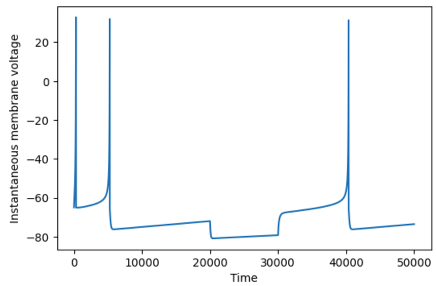
Figure 1: A simple Izhikevich Neuron model

The models illustrates in graphs (i) to (v) in figure are the pattern of neocortical neurons in the mammalian brain. These patterns can be classified into types according to the spiking as following:

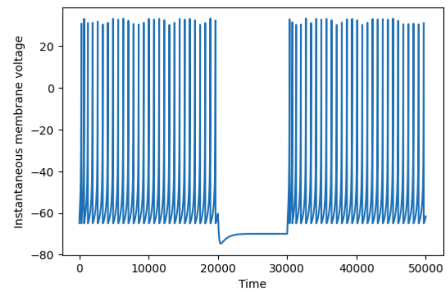
- RS (regular spiking) neurons are the most typical neurons in the cortex as seen in fig (i).
- IB (intrinsically bursting) neurons fire a stereotypical burst of spikes followed by a repetitive single spike as seen in fig (i).
- CH (chattering) neurons can fire stereotypical bursts of closely spaced spikes as seen in fig (iv).
- FS (fast spiking) neurons can fire periodic trains of action potentials with extremely high frequency practically without any adaptation (slowing down) as seen in fig (ii).
- TC (thalamo-cortical) neurons have two firing regimes: When at rest (v is around 60 mV) and then depolarized, they hey exhibit tonic firing as seen in fig (v).

All the spikes in these study were applied the same input of current signal(I) as in fig(vi).

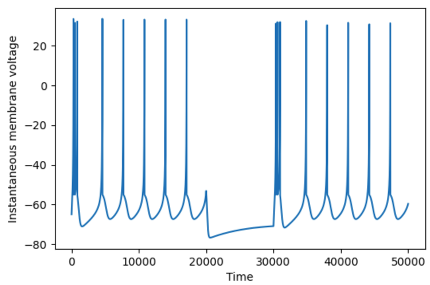
The corresponding parameters value of a,b,c,d of type of spike can be define through the graph in figure 3.



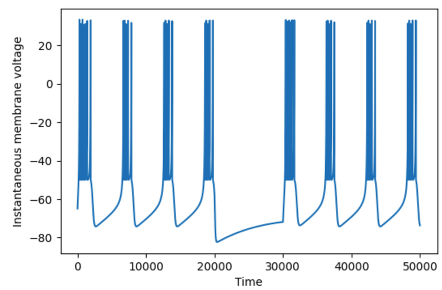
(i) RS (regular spiking)



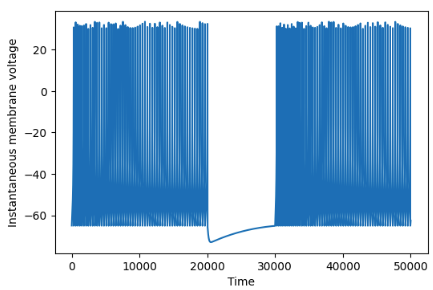
(ii) FS (fast spiking



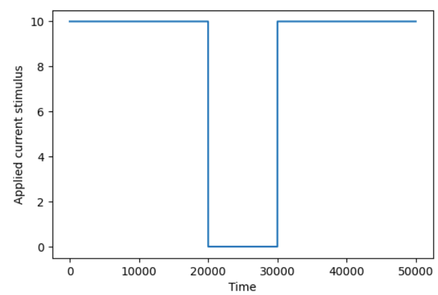
(iii) IB (intrinsically bursting



(iv) CH (chattering)



(v) TC (thalamo-cortic



(vi) Input current

Figure 2: Known types of neurons correspond to different values of the parameters

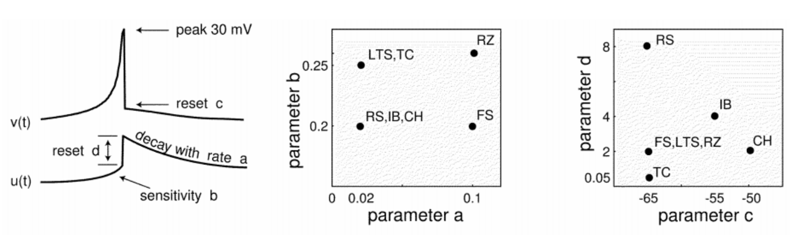


Figure 3: The corresponding parameters a,b,c,d

2. Regime map

This section describes about different regimes of a neuron dynamics, plotting time series and phase portraits of the signal, calculation of a regime map.

The Phase portraits with a = 0.05 is illustrated in fig.4. According to [2], the parameters of the plane can be divided into five regions as following :

- A : a stable limit cycle, where b = 0.1011.
- B : a saddle and a stable limit cycle, b = 0.1008.
- C : a sink, a saddle, a stable limit cycle and an unstable limit cycle, b = 0.10065.
- D : a sink, a saddle and a stable limit cycle, b = 0.1005.
- E : a sink and a saddle, b = 0.1002.

Throughout the graph in fig.5, there are slightly changes between these regions. However, we should notice that there exits a saddle-node bifurcation between region A and B; a saddle separatrix loop occurs between C and D; a big saddle homoclinic orbit bifurcation between D and E. Beside the bifurcation region there is no longer a stable limit cycle.

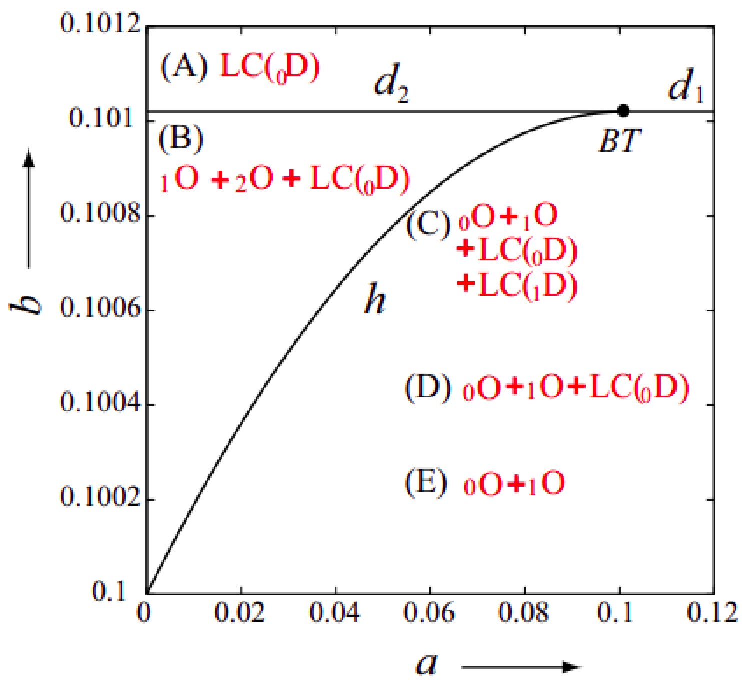
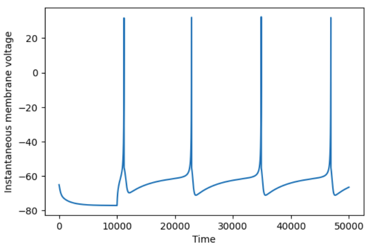
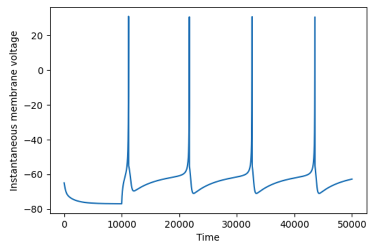


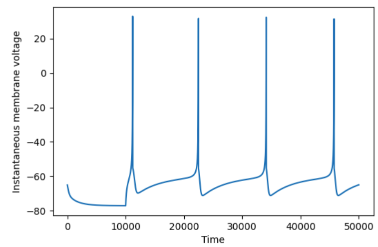
Figure 4: Bifurcation diagram in the a-b plane(c = 55, d = 4, I = 10) [2]



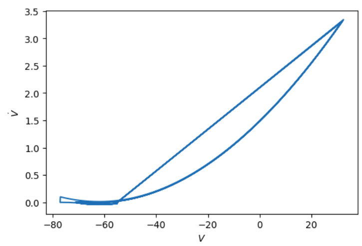
(i) Region A



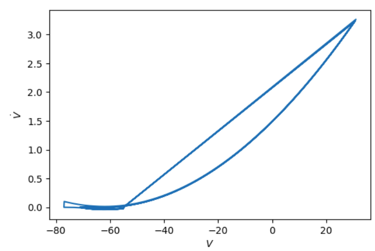
(ii) Region B



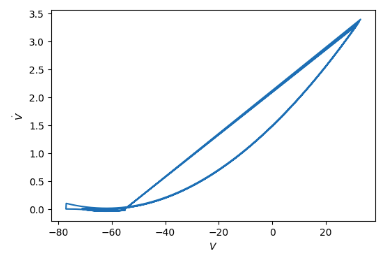
(iii) Region C



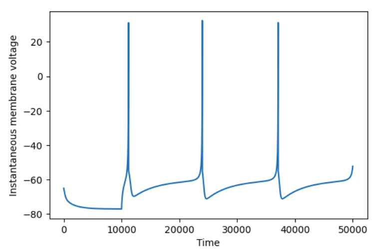
(iv) Phase plane A



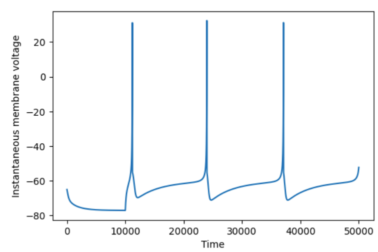
(v) Phase plane B



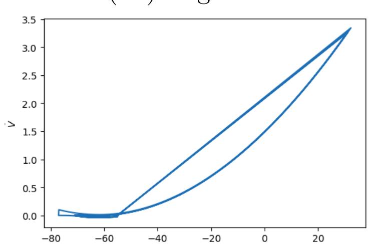
(vi) Phase plane c



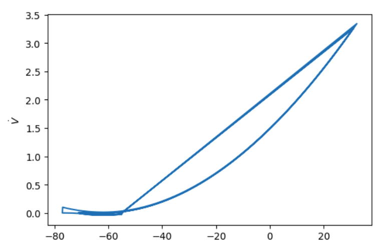
(vii) Region D



(viii) Region E



(ix) Phase plane D



(x) Phase plane E

Figure 5: Phase portraits of bifurcation

3. Reference to a model with noise

The task is about adding noise to the system, solving the system of differential equations with noise, analyzing the influence of noise amplitude on the system dynamics.

The electrical activity of each neuron of this ensemble is described by a set of two ordinary nonlinear coupled differential equations [1]:

$$\frac{dv_i}{dt} = 0.004v_i^2 + 5v_i + 140 - u + I_i^{DC} + I_i^{syn} \tag{4}$$

$$\frac{du_i}{dt} = a(bv_i - u_i) \tag{5}$$

with the auxiliary after-spike resetting:

$$\text{if } v_i \geq 30mV, \text{ then } \begin{cases} v = c \\ u_i = u_i + d \end{cases} \tag{6}$$

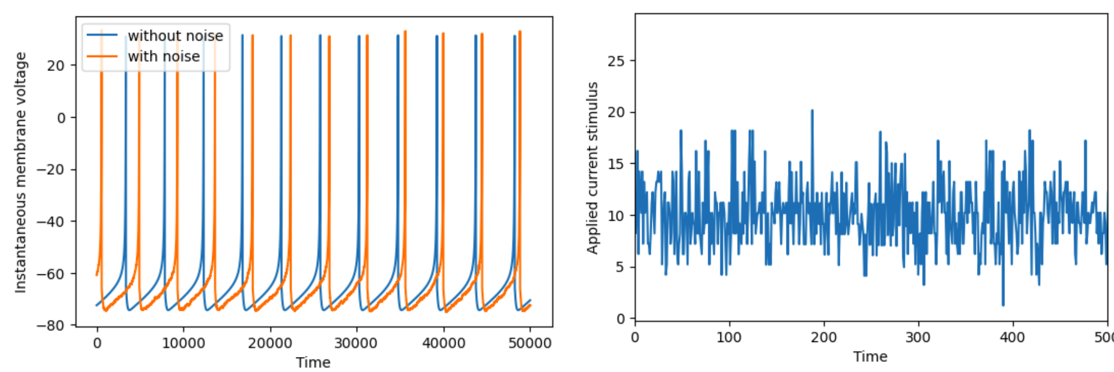
In section we set a = 0.02, b = 0.2, c = 65, and d = 8, which corresponds to regular spiking pattern [1].

The term I_i in Equation (7) denotes synaptic current received by post-synaptic neuron i [3] and was defined as following:

$$I_i^{syn} = \frac{1}{D_i} \sum_j g_{ji} \frac{e^{-\frac{t-t_j}{\tau_s}} - e^{-\frac{t-t_j}{\tau_f}}}{\tau_s - \tau_f} (V_0 - v_i) \tag{7}$$

Poisson noise is added into the system and the influence of amplitude of noise is observed on the membrane potential as shown below.

First, We choose values of I_i^{DC} randomly from a Poisson distribution with mean value 10 according to [3]. The result is shown in fig 6 and fig 7.

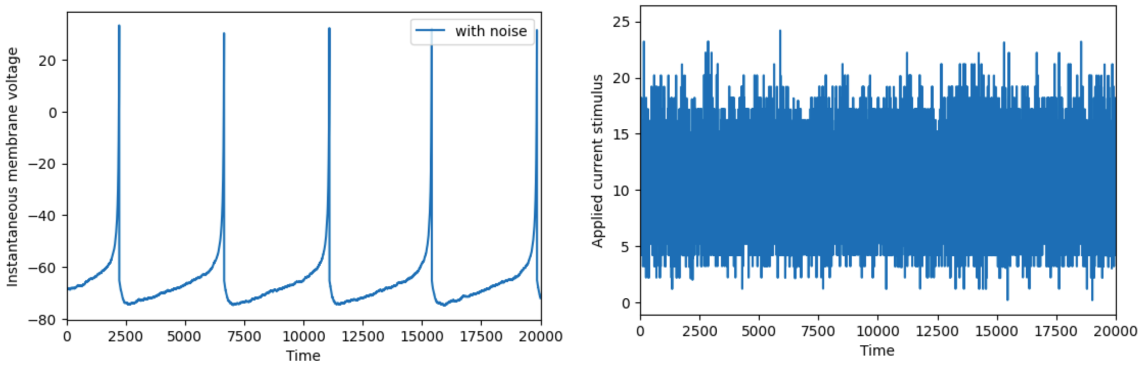


(i) The membrane potential

(ii) The input current I_i^{DC} $+I_i^{syn}$

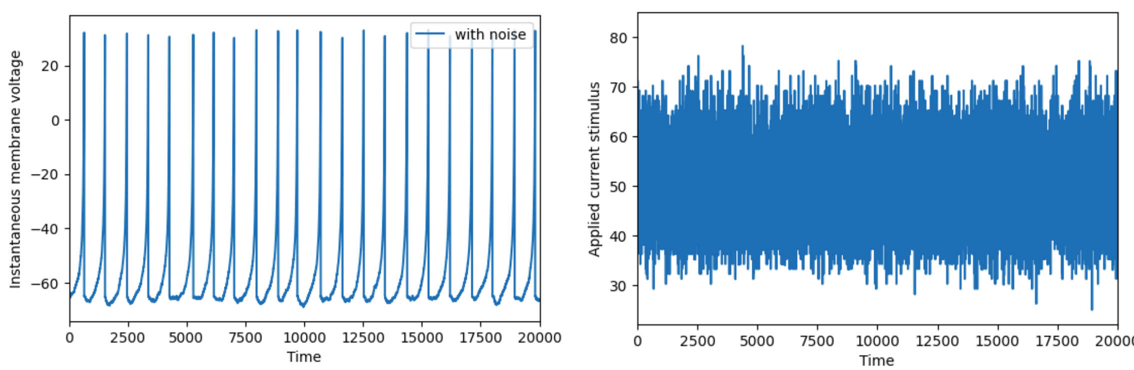
Figure 6: A simple Izhikevich model with/without noise

In the graph below (fig.8), we applied the same values of I_i^{DC} randomly from a Poisson distribution without I_i^{syn}. In fig.9 the magnitude of Poisson noise for I_i^{DC} was increased to 50 and was applied together with I_i^{syn} .



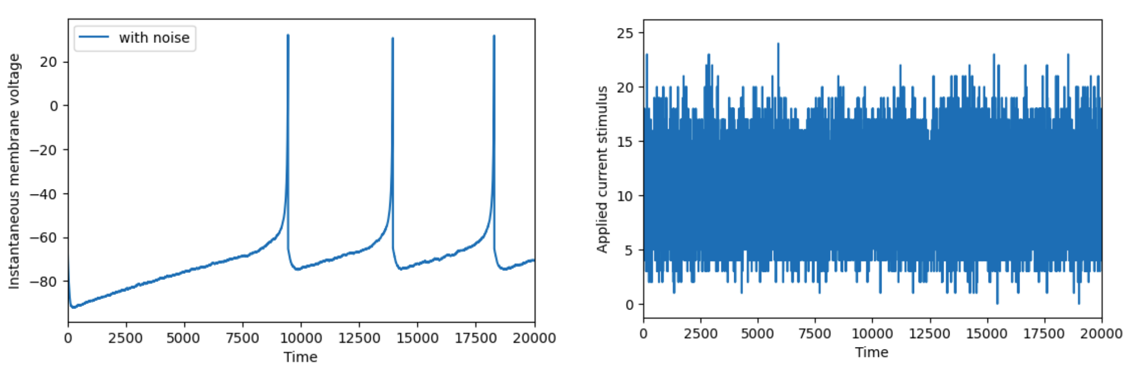
(i) The membrane potential (ii) The input current I_i^{DC}

Figure 7: A simple Izhikevich model with noise without I_i^{DC} and I_i^{syn}



(i) The membrane potential (ii) The input current $I_i^{DC} + I_i^{syn}$

Figure 9: A simple Izhikevich model with noise, here noise of I_i^{DC} is increased with mean value 50



(i) The membrane potential (ii) The input current I_i^{DC}

Figure 8: A simple Izhikevich model with noise without I_i^{DC} and without I_i^{syn}

4. The coupling between 2 neurons

The task is about adding the coupling between 2 neurons, analyzing the synchronization between neurons for different values of the coupling strength.

The coupled neurons of the gap-junction can be represented as follows:

$$\frac{dv_i}{dt} = 0.004v_i^2 + 5v_i + 140 - u + I_i + \sum_{j=1, i \neq j} w_{ji}(v_j - v_i) \tag{8}$$

$$\frac{du_i}{dt} = a(bv_i - u_i) \tag{9}$$

with auxiliary after-spike resetting:

$$\text{if } v_i \geq 30mV, \text{ then} \begin{cases} v = c \\ u_i = u_i + d \end{cases} \tag{10}$$

where,w_{ij} represents the coupling coefficients. In this paper, the number of coupled neurons is two, then N = 2[4]. The coupling coefficients ,w_{ij} and ,w_{ji} are the same value and we coupled two neurons which have the same parameter values, namely a,b,c,d are constants.

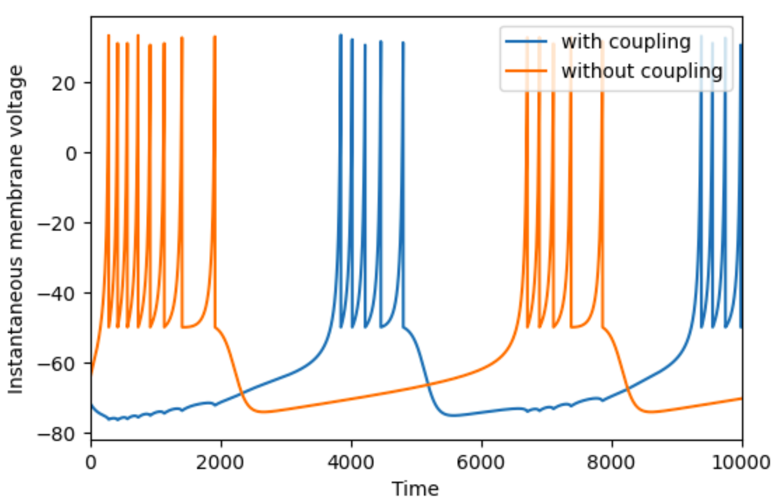
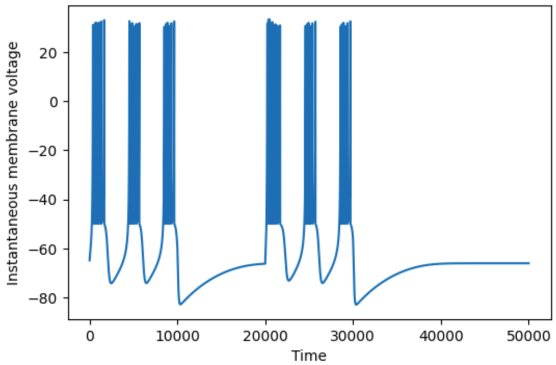
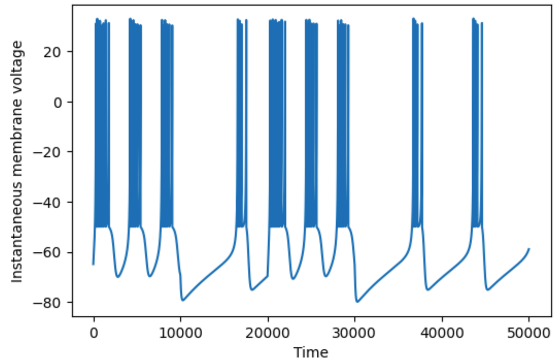


Figure 10: Time series of the membrane potential v with and without coupling.

Fig.10 shows the time series of the membrane potential v of a single neuron without coupling and coupled two neurons with the coupling coefficient w = 0.05. Parameter values are set to a = 0.03, b = 0.3, c = 50, d = 2 and I = 10, with these parameters, the neuron exhibits chaotic response.



(i) coupled two neurons with the coupling coefficient w = -0.05.



(ii) coupled two neurons with the coupling coefficient w = -0.01.

Figure 11: the membrane potential v with two different values of coupling strength.

The time series of the membrane potential v of coupled two neurons with the coupling coefficient w = 0.05 and w = -0.01 are shown in Fig.11. From these results (Fig.10 and Fig.11), we can see that the responses of two neurons are changed by the coupling coefficient.

5. Correlation time

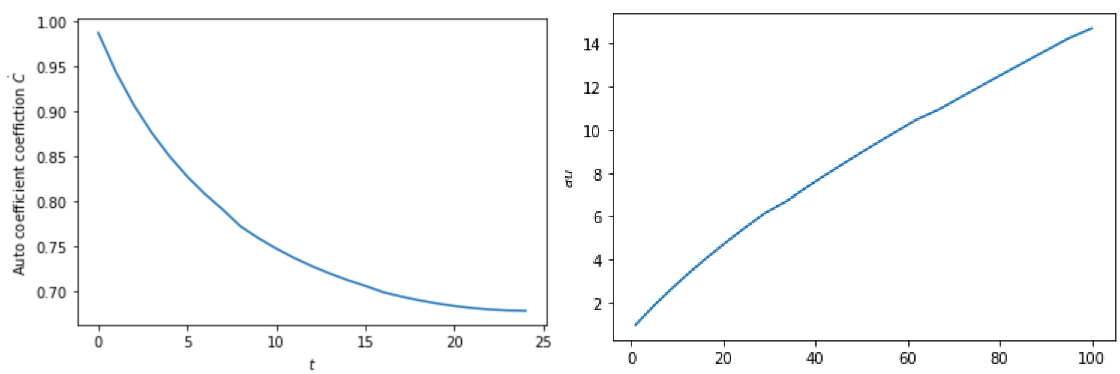
The task is about simulation of a neural network with global topology, analyzing the influence of external stimulus and noise amplitude by calculating characteristic correlation time.

The the normalized autocorrelation function [5] can be represented as follows:

$$C(\tau) = \frac{\prec \tilde{y}(t)\tilde{y}(t+\tau) \succ}{\tilde{y}^2}, \quad \tilde{y} = y - \prec y \succ \tag{11}$$

we calculate the characteristic correlation time as follows:

$$\tau_c = \int_0^{\infty} C^2(t)\, dt \tag{12}$$



(i) The normalized autocorrelation coefficient $C(\tau)$

(ii) The correlation time τ

Figure 12: The correlation

From the result, as shown in fig 12, the normalized autocorrelation coefficient decreases over time, while the correlation time increases when the noise amplitude is increased.

Full implementation of this paper can be found within this Colab's Link

References

[1] E. M. Izhikevich, "Simple model of spiking neurons," *IEEE Transactions on neural networks*, vol. 14, no. 6, pp. 1569–1572, 2003.

[2] A. Tamura, T. Ueta, and S. Tsuji, "Bifurcation analysis of izhikevich model," in *Proc. Int. Symp. Nonlinear Theory Appl.*, pp. 424–427, 2008.

[3] M. Khoshkhou and A. Montakhab, "Beta-rhythm oscillations and synchronization transition in network models of izhikevich neurons: effect of topology and synaptic type," *Frontiers in computational neuroscience*, vol. 12, p. 59, 2018.

[4] K. Naiki, Y. Shimada, K. Fujiwara, and T. Ikeguchi, "Synchronization in a coupled izhikevich neuron model,"

[5] A. S. Pikovsky and J. Kurths, "Coherence resonance in a noise-driven excitable system," *Physical Review Letters*, vol. 78, no. 5, p. 775, 1997.