Student: Voeurn YongAnn

Institute of Robotics,
Innopolis University

Report

"Dynamics of Non-Linear Robotic Systems"

Homework-05

1. Robot parameter

Type: PRR
$a_1 = a_2 = a_3 = 1$m; $l_{c1} = l_{c2} = l_{c3} = 0.5$m; $m_1 = m_3 = m_2 = 50$kg;
$I_1 = I_2 = I_3 = 10 \text{kg} \cdot \text{m}^2$

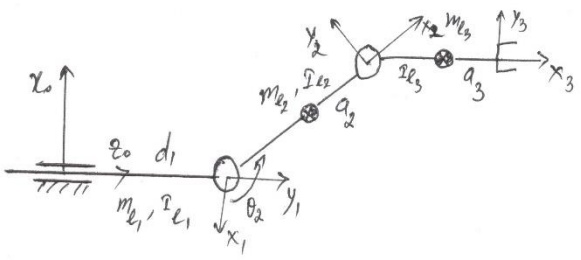


Figure 1Robot PRR

The initial conditions for the velocities and accelerations:

$$\omega_0^0 = \dot{\omega}_0^0 = \mathbf{0},$$

$$\ddot{P}_0^0 - g_0^0 = [0\ g\ 0]^T$$

The current link frame:

$$r_{1,c1}^1 = \begin{bmatrix} 0 \\ l_{c3} \\ 0 \end{bmatrix}, \quad r_0^1 = \begin{bmatrix} 0 \\ d_1 \\ 0 \end{bmatrix}, \quad r_2^2, c_2 = \begin{bmatrix} l_{c2} \\ 0 \\ 0 \end{bmatrix} \quad r_{1,2}^2 = \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix} \; r_3^3, c_3 = \begin{bmatrix} l_{c3} \\ 0 \\ 0 \end{bmatrix} \quad r_{2,3}^3 = \begin{bmatrix} a_3 \\ 0 \\ 0 \end{bmatrix}$$

Vector transformation

$$R_1^0 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad R_2^1 = \begin{bmatrix} C_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$R_3^2 = \begin{bmatrix} C_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Axis rotation

$$\boldsymbol{z}_{m_i}^{i-1} = \boldsymbol{z}_0 = [0 \quad 0 \quad 1]^T$$

2. **Forward recursion:** Link1

ω_1^1=0, $\dot{\omega}_1^1$=0,

$$\ddot{P}_1^1 = \begin{bmatrix} -g \\ \ddot{d}_1 \\ 0 \end{bmatrix}, \quad \ddot{P}_{c1}^1 = \begin{bmatrix} -g \\ \ddot{d}_1 \\ 0 \end{bmatrix}$$

Forward recursion: Link2

$$\omega_2^2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} \quad ; \quad \dot{\omega}_2^2 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$\ddot{P}_2^2 = R_2^{1^T} \ddot{P}_1^1 + \dot{\omega}_2^2 \times r_1^2 + \omega_2^2 \times (\omega_2^2 \times r_1^2) = \begin{bmatrix} C_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -g \\ \ddot{d}_1 \\ 0 \end{bmatrix} + \ddot{\theta}_2 k \times a_2 i + \dot{\theta}_2 k \times (\dot{\theta}_2 k \times a_2 i)$$

$$\ddot{P}_2^2 = \begin{bmatrix} s_2 \ddot{d}_1 - a_2 \dot{\theta}_2^2 - g c_2 \\ 0 c_2 \ddot{d}_1 + a_2 \ddot{\theta}_2^2 - g s_2 \\ 0 \end{bmatrix}$$

$$\ddot{P}_{c_2}^2 = \ddot{P}_2^2 + \dot{\omega}_2^2 \times r_{2,c_2}^2 + \omega_2^2 \times (\omega_2^2 \times r_{2,c_2}^2)$$

$$\ddot{P}_{c_2}^2 = \begin{bmatrix} s_2 \ddot{d}_1 - (l c_2 + a_2) \dot{\theta}_2^2 - g c_2 \\ c_2 \ddot{d}_1 + (l c_2 + a_2) \ddot{\theta}_2 + g s_2 \\ 0 \end{bmatrix}$$

Forward recursion: Link3

$$\omega_3^3 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}; \qquad \dot{\omega}_3^3 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 \end{bmatrix};$$

$$\ddot{P}_3^3 = R_3^{2^T} \ddot{P}_2^2 + \dot{\omega}_3^3 \times r_2^3 + \omega_3^3 \times (\omega_3^3 \times r_2^3)$$

$$= \begin{bmatrix} C_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_2\ddot{d}_1 & a_2\dot{\theta}_2^2 & -gc_2 \\ c_2\ddot{d}_1 & a_2\ddot{\theta}_2^2 & gs_2 \\ 0 & 0 & 0 \end{bmatrix} + (\ddot{\theta}_2 + \ddot{\theta}_3)k \times a_3 i + (\dot{\theta}_2 + \dot{\theta}_3)k \times (\dot{\theta}_2 + \dot{\theta}_3)k \times a_3 i$$

$$\ddot{P}_3^3 = \begin{bmatrix} s_3(a_2\ddot{\theta}_2 + \ddot{d}_2 c_2 + gs_2) - c_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}_1 s_2) - a_3(\dot{\theta}_2 + \dot{\theta}_3)^2 \\ c_3(a_2\ddot{\theta}_2 + \ddot{d}_{c_2} + gs_2) + s_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}s_2 + a_3(\ddot{\theta}_2 + \ddot{\theta}_3) \\ 0 \end{bmatrix}$$

$$\ddot{P}_{c_3}^3 = \ddot{P}_3^3 + \dot{\omega}_3^3 \times r_{3,c_3}^3 + \omega_3^3 \times (\omega_3^3 \times r_{3,c_3}^3)$$

$$= \begin{bmatrix} s_3(a_2\ddot{\theta}_2 + \ddot{d}_2 c_2 + gs_2) - a_3(\dot{\theta}_2 + \dot{\theta}_3)^2 - c_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}_1 s_2) - lc_3(\dot{\theta}_2 + \dot{\theta}_3)^2 \\ c_3(a_2\ddot{\theta}_2 + \ddot{d}_{c_2} + gs_2) + s_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}s_2 + a_3(\ddot{\theta}_2 + \ddot{\theta}_3) + lc_3(\dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$$

3. **Backward recursion:** Link3

$$\boldsymbol{f}_i^i = \boldsymbol{R}_{i+1}^i \boldsymbol{f}_{i+1}^{i+1} + m_i \ddot{\boldsymbol{p}}_{C_i}^i$$

$$f_3^3 = m_3 \ddot{P}_{c_3}$$

$$= m_3 \begin{bmatrix} s_3(a_2\ddot{\theta}_2 + \ddot{d}_2 c_2 + gs_2) - a_3(\dot{\theta}_2 + \dot{\theta}_3)^2 - c_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}_1 s_2) - lc_3(\dot{\theta}_2 + \dot{\theta}_3)^2 \\ c_3(a_2\ddot{\theta}_2 + \ddot{d}_{c_2} + gs_2) + s_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}s_2) + a_3(\ddot{\theta}_2 + \ddot{\theta}_3) + lc_3(\dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$$

$$\tau_3^3 = -f_3^3 \times r_{c_3} + I_3\dot{\omega}_3 + \omega_3 \times I_3\omega_3$$

$$= \begin{bmatrix} 0 \\ 0 \\ I_3(\ddot{\theta}_2 + \ddot{\theta}_3) + m_3 lc_3(a_3(\dot{\theta}_2 + \dot{\theta}_3) + c_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}_1 s_2) + l_{c3}(\ddot{\theta}_2 + \ddot{\theta}_3) + s_3(a_2\ddot{x}1_2^2 + gc_2 - \ddot{d}s_2) \end{bmatrix}$$

Backward link 2

$$f_2^2 = R_3^2 f_3^3 + m_2 \ddot{P}_{c_2}$$

$$= m_2 \begin{bmatrix} s_2 \ddot{d}_1 - (lc_2 + a_2)\dot{\theta}_2^2 - gc_2 \\ c_2 \ddot{d}_1 + (lc_2 + a_2)\ddot{\theta}_2 + gs_2 \\ 0 \end{bmatrix} +$$

$$m_3 \begin{bmatrix} C_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_3\left(a_2\ddot{\theta}_2 + \ddot{d}_2 c_2 + gs_2\right) - a_3\left(\dot{\theta}_2 + \dot{\theta}_3\right)^2 - c_3\left(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}_1 s_2\right) - lc_3\left(\dot{\theta}_2 + \dot{\theta}_3\right)^2 \\ c_3\left(a_2\ddot{\theta}_2 + \ddot{d}_{c_2} + gs_2\right) + s_3(a_2\ddot{\theta}_2^2 + gc_2 - \ddot{d}s_2 + a_3\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) + lc_3\left(\dot{\theta}_2 + \dot{\theta}_3\right) \\ 0 \end{bmatrix}$$

$$\tau_2^2 = R_3^2 T_3 - f_2^2 \times r_{c_2} - (R_3^2 f_3^3) \times (r_2^1 - r_{2,c_2}^1) + I_2\dot{\omega}_2 + \omega_2 \times I_2\omega_2$$

Backward link 1

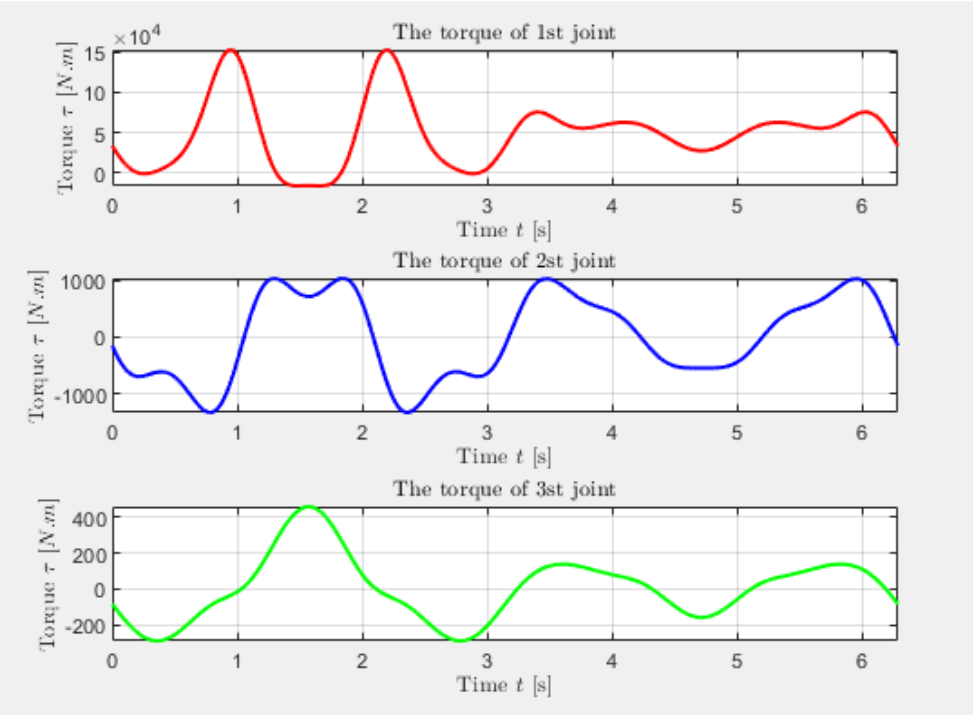
$$f_1^1 = R_2^1 f_2^2 + m_1 \ddot{P}_{c\ 1}^1$$

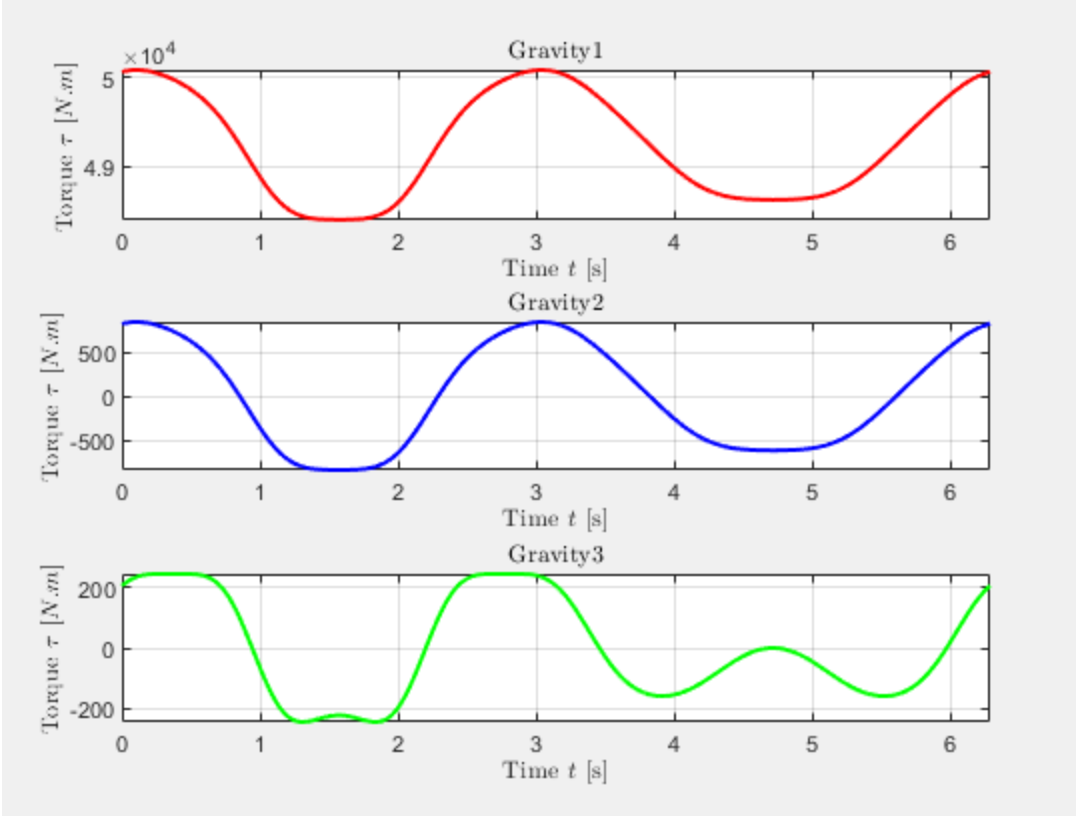
$$= m_1 \begin{bmatrix} -g \\ \ddot{d}_1^1 \\ 0 \end{bmatrix}$$

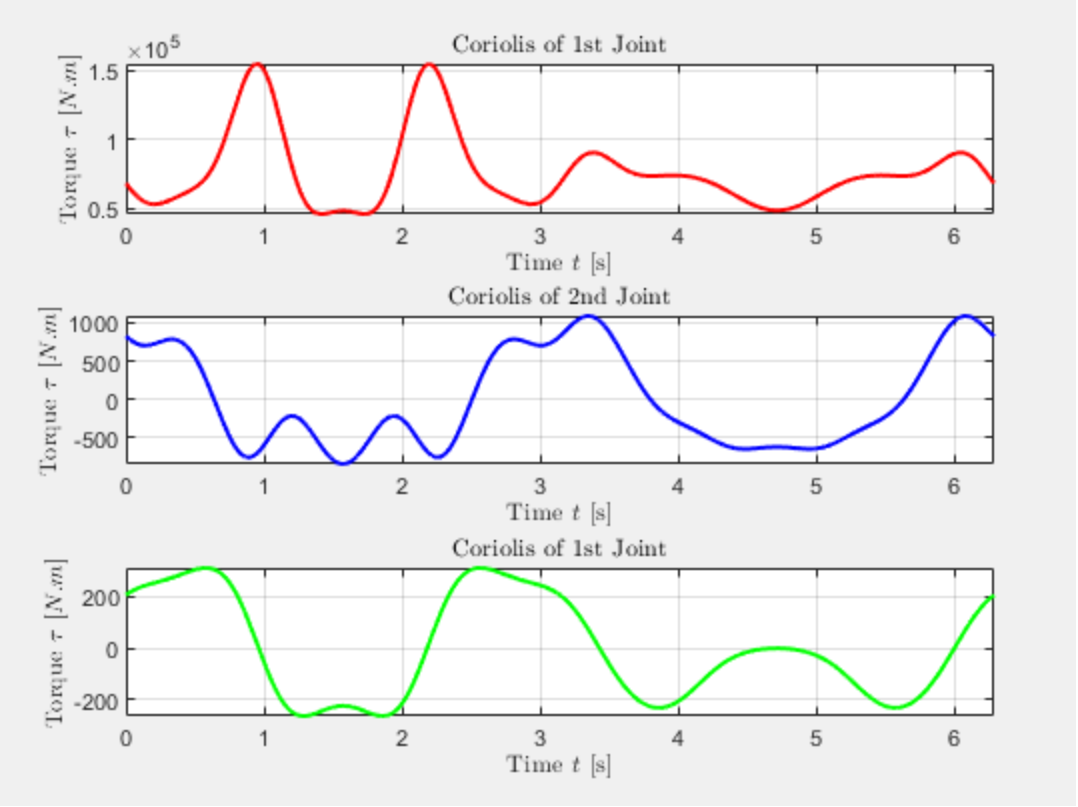
$$\tau_1^1 = R_2^1 T_2 - f_1^1 \times r_{c_1}^1 - R_2^1 f_2^2) \times (r_0^1 - r_{1,c_2}^1) + I_1\dot{\omega}_1 + \omega_1 \times (I_1 \times \omega_1)$$

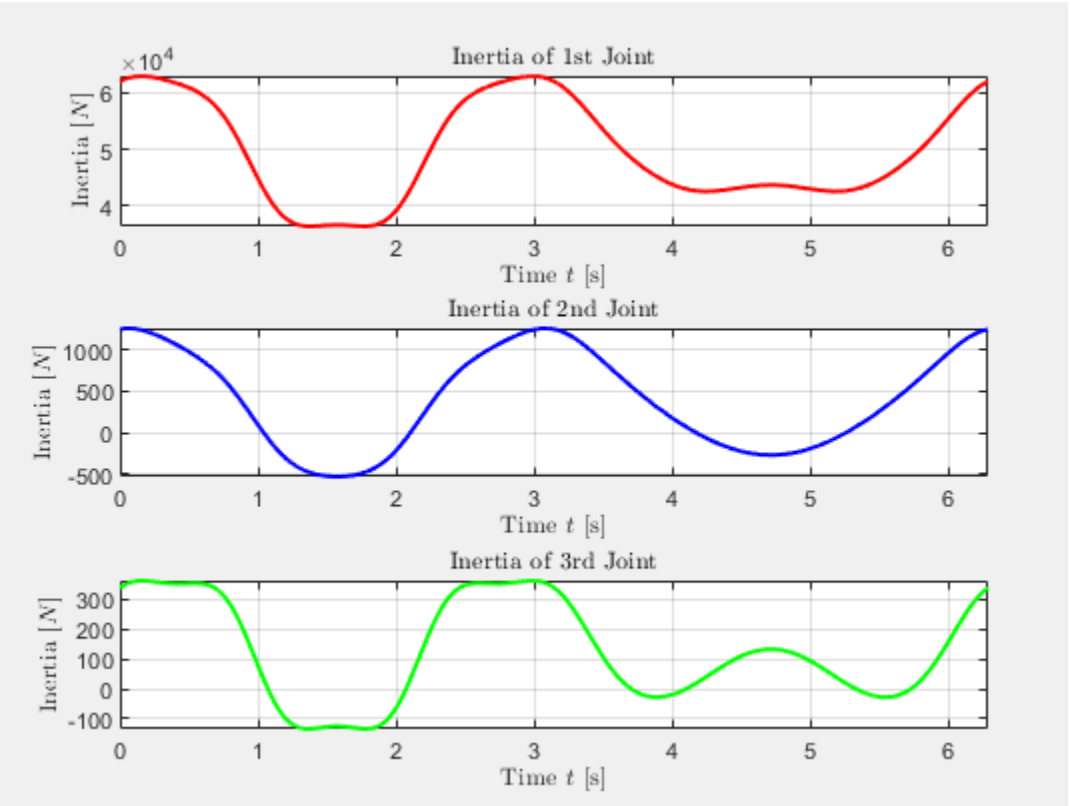
4. The graph of Torque, gravity term, centrifugal and Coriolis term, inertia term is given as:

$$q_1(t) = A_1 \sin(t);\ q_2(t) = A_2 \cos(2t);\ q_3(t) = A_3 \sin(3t);\ A1=A2=A3=1$$




Gravity1
×10^4
5
4.9
Torque τ [N.m]
0
1
2
3
4
5
6
Time t [s]
Gravity2
500
0
-500
Torque τ [N.m]
Time t [s]
Gravity3
200
0
-200
Torque τ [N.m]
Time t [s]


Coriolis of 1st Joint
×10^5
Torque τ [N.m]
Time t [s]
Coriolis of 2nd Joint
Torque τ [N.m]
Time t [s]
Coriolis of 1st Joint
Torque τ [N.m]
Time t [s]


Inertia of 1st Joint
×10^4
Inertia [N]
Time t [s]
Inertia of 2nd Joint
Inertia [N]
Time t [s]
Inertia of 3rd Joint
Inertia [N]
Time t [s]

GitHub link: https://github.com/yongan007/hw5_DNLRS.git

reference:
B. Sicilliano -Robotics. Modelling, Planning and Control

5. Result:

T Tau_3 =

I3*(q2__2dot + q3__2dot) + lc3*m3*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))

Tau__2 =

I2*q2__2dot - (m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) - m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2))))*(a2 - lc2) + I3*(q2__2dot + q3__2dot) + lc2*(m2*(a2*q2__2dot + lc2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) - m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))) + lc3*m3*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))

Tau__1 =

I2*q2__2dot - (m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) - m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2))))*(a2 - lc2) + (d1 - lc1)*(cos(q2)*(m2*(a2*q2__dot^2 + g*cos(q2) + lc2*q2__dot^2 - d__2dot*sin(q2)) + m3*cos(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) + m3*sin(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot +

q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))) + sin(q2)*(m2*(a2*q2__2dot + lc2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) - m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2))))) + lc1*(cos(q2)*(m2*(a2*q2__dot^2 + g*cos(q2) + lc2*q2__dot^2 - d__2dot*sin(q2)) + m3*cos(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) + m3*sin(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))) + g*m1 + sin(q2)*(m2*(a2*q2__2dot + lc2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) - m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2))))) + I3*(q2__2dot + q3__2dot) + lc2*(m2*(a2*q2__2dot + lc2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) - m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 - sin(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + cos(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)) + lc3*(q2__dot + q3__dot)^2) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))) + lc3*m3*(a3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + d__2dot*cos(q2) + g*sin(q2)) + lc3*(q2__2dot + q3__2dot) + sin(q3)*(a2*q2__dot^2 + g*cos(q2) - d__2dot*sin(q2)))

G3(q) =

lc3*m3*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2))

G2(q2) =

lc2*(g*m2*sin(q2) - m3*sin(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3)) + m3*cos(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2))) - (m3*sin(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3)) - m3*cos(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2)))*(a2 - lc2) + lc3*m3*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2))

G1(q) =

lc2*(g*m2*sin(q2) - m3*sin(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3)) + m3*cos(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2))) - (m3*sin(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3)) - m3*cos(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2)))*(a2 - lc2) + (d1 - lc1)*(cos(q2)*(g*m2*cos(q2) + m3*sin(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2)) + m3*cos(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3))) + sin(q2)*(g*m2*sin(q2) - m3*sin(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3)) + m3*cos(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2)))) + lc1*(g*m1 + cos(q2)*(g*m2*cos(q2) + m3*sin(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2)) + m3*cos(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3))) + sin(q2)*(g*m2*sin(q2) - m3*sin(q3)*(g*cos(q2)*cos(q3) - g*sin(q2)*sin(q3)) + m3*cos(q3)*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2)))) + lc3*m3*(g*cos(q2)*sin(q3) + g*cos(q3)*sin(q2))

C3(q, q__dot) =

lc3*m3*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))

C2(q, q__dot) =

lc2*(g*m2*sin(q2) - m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) - g*sin(q2)*sin(q3)) + m3*cos(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))) - (a2 - lc2)*(m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) - g*sin(q2)*sin(q3)) - m3*cos(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))) + lc3*m3*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))

C1(q, q__dot) =

(d1 - lc1)*(cos(q2)*(m2*(a2*q2__dot^2 + g*cos(q2) + lc2*q2__dot^2) + m3*cos(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) - g*sin(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))) + sin(q2)*(g*m2*sin(q2) - m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) -

g*sin(q2)*sin(q3)) + m3*cos(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2)))) +
lc1*(cos(q2)*(m2*(a2*q2__dot^2 + g*cos(q2) + lc2*q2__dot^2) + m3*cos(q3)*(a3*(q2__dot + q3__dot)^2 +
lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) - g*sin(q2)*sin(q3)) +
m3*sin(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))) + g*m1 + sin(q2)*(g*m2*sin(q2) -
m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) -
g*sin(q2)*sin(q3)) + m3*cos(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2)))) + lc2*(g*m2*sin(q2) -
m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) -
g*sin(q2)*sin(q3)) + m3*cos(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))) - (a2 -
lc2)*(m3*sin(q3)*(a3*(q2__dot + q3__dot)^2 + lc3*(q2__dot + q3__dot)^2 + cos(q3)*(a2*q2__dot^2 + g*cos(q2)) -
g*sin(q2)*sin(q3)) - m3*cos(q3)*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))) +
lc3*m3*(sin(q3)*(a2*q2__dot^2 + g*cos(q2)) + g*cos(q3)*sin(q2))

B3(q)*q__dot =

I3*(q2__2dot + q3__2dot) + lc3*m3*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) +
cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3))

B2(q)*q__dot =

I2*q2__2dot + (m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot
+ g*sin(q2)) + g*cos(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3)))*(a2 - lc2) +
lc2*(m2*(a2*q2__2dot + lc2*q2__2dot + g*sin(q2)) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot +
q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2))
- g*cos(q2)*cos(q3))) + I3*(q2__2dot + q3__2dot) + lc3*m3*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) +
cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3))

B1(q)*q__dot =

I2*q2__2dot + (m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3)))*(a2 - lc2) + lc2*(m2*(a2*q2__2dot + lc2*q2__2dot + g*sin(q2)) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3))) + (d1 - lc1)*(cos(q2)*(g*m2*cos(q2) + m3*sin(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) - m3*cos(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3))) + sin(q2)*(m2*(a2*q2__2dot + lc2*q2__2dot + g*sin(q2)) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3)))) + lc1*(g*m1 + cos(q2)*(g*m2*cos(q2) + m3*sin(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) - m3*cos(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3))) + sin(q2)*(m2*(a2*q2__2dot + lc2*q2__2dot + g*sin(q2)) + m3*cos(q3)*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3)) + m3*sin(q3)*(sin(q3)*(a2*q2__2dot + g*sin(q2)) - g*cos(q2)*cos(q3)))) + I3*(q2__2dot + q3__2dot) + lc3*m3*(a3*(q2__2dot + q3__2dot) + lc3*(q2__2dot + q3__2dot) + cos(q3)*(a2*q2__2dot + g*sin(q2)) + g*cos(q2)*sin(q3))

b1=

2*I3 + lc3*m3*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2)))

b2=

I2 + 2*I3 + (m3*cos(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*sin(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2))))*(a2 - lc2) + lc2*(m2*(a2 + lc2 + cos(q2) + g*sin(q2)) + m3*cos(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*sin(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2)))) + lc3*m3*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2)))

b3=

I2 + 2*I3 + (m3*cos(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*sin(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2))))*(a2 - lc2) - (cos(q2)*(m2*(sin(q2) - g*cos(q2)) - m3*sin(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*cos(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2)))) - sin(q2)*(m2*(a2 + lc2 + cos(q2) + g*sin(q2)) + m3*cos(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*sin(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2)))))*(d1 - lc1) + lc1*(g*m1 - cos(q2)*(m2*(sin(q2) - g*cos(q2)) - m3*sin(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*cos(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2)))) + sin(q2)*(m2*(a2 + lc2 + cos(q2) + g*sin(q2)) + m3*cos(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*sin(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2))))) + lc2*(m2*(a2 + lc2 + cos(q2) + g*sin(q2)) + m3*cos(q3)*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2))) + m3*sin(q3)*(cos(q3)*(sin(q2) - g*cos(q2)) + sin(q3)*(a2 + cos(q2) + g*sin(q2)))) + lc3*m3*(2*a3 + 2*lc3 - sin(q3)*(sin(q2) - g*cos(q2)) + cos(q3)*(a2 + cos(q2) + g*sin(q2)))