Stiffness analyse Robot – Tripteron robot

Link to Code: Link

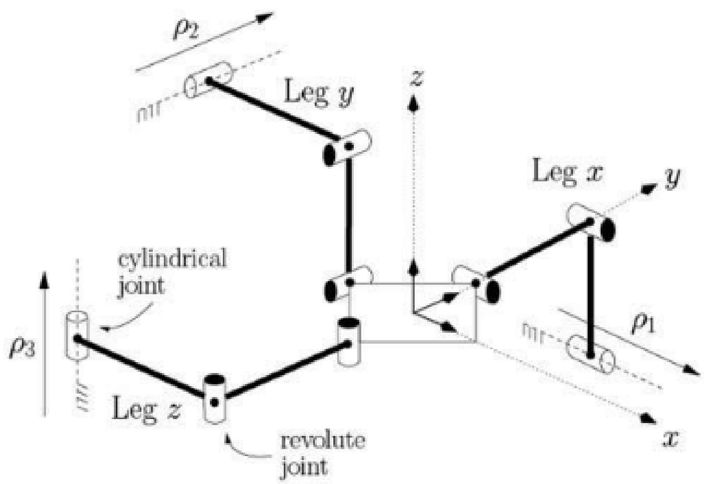

Figure 1: tripteron robot

1 Robot description

The tripteron is a parallel mechanism composed of three legs, each consisting of a 4-DOF serial mechanism whose links are connected by, from base to platform, an actuated prismatic joint fixed at the base, and three revolute joints whose axes are parallel to each other but not orthogonal to the direction of the prismatic joint.

1.1 Inverse kinematics

The three legs can be divided into RRR planar with actuated prismatic joint.

$$\rho_1 = x; \rho_2 = y; \rho_3 = z; \tag{1}$$

The tripteron has orthogonal properties so that it can be written as following:
For the first leg p_z it inverse kinematics solution can be solved by considered the translation along Z and the rotation in XY axis. It follows the same sequence for others legs.

$$c2 = (p_x^2 + p_y^2 - a_1^2 - a_2^2)/(2a_1a_2)$$
$$\mathrm{s2} = \sqrt{(1 - c_2^2)};$$
$$\mathrm{s1} = ((\mathrm{a}_1 + a_2c_2)p_y - a_2s_2p_x)/(p_x^2 + p_y^2);$$

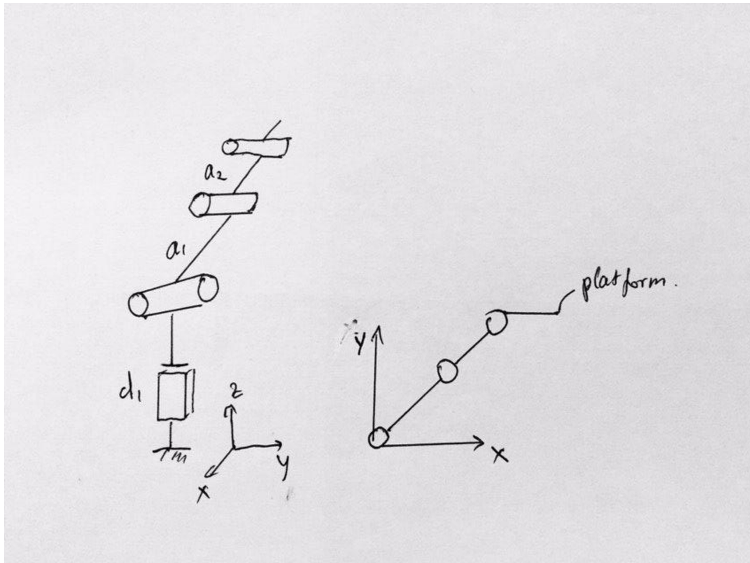

Figure 2: Kinematics model for the first legs of robot

$$c1= ((a_1 + a_2c_2)p_x + a_2s_2p_y)/(p_x^2 + p_y^2);$$
$$d1= pz$$
$$q1 = atan2(s1,c1);$$
$$q2 = atan2(s2,c2);$$
$$q3 = -q1-q2;$$

2 Stiffness Analysis

2.1 VJM modeling

The VJM model of each leg of robot can be drawn as follows:

Figure 3: tripteron robot

Forward Kinematics

The global coordinate frame is at the third leg as shown in Figure.1. Then the forward Kinematics equation of can be written as following:
For the first leg

$$T_1 = T_{base}T_x(l)T_y(l)T_x(q_1)T_x(\theta_1)R_x(q_2)T_y(l)T_{3D}(\theta_{2-7})R_x(q_3)T_y(l)T_{3D}(\theta_{8-13})R_x(q_4)T_{tool} \quad (2)$$

For the second leg

$$T_2 = T_{base}T_z(l)T_y(l)T_x(q_5)T_y(l)T_x(\theta_1)R_x(q_6)T_y(l)T_{3D}(\theta_{2-7})R_x(q_7)T_y(l)T_{3D}(\theta_{8-13})R_x(q_8)T_{tool} \tag{3}$$

For the third leg

$$T_3 = T_{base}T_x(q_9)T_y(l)T_x(\theta_1)R_x(q_10)T_y(l)T_{3D}(\theta_{2-7})R_x(q_11)T_y(l)T_{3D}(\theta_{8-13})R_x(q_12)T_{tool} \tag{4}$$

The stiffness matrix can be using the expression below:

$$\mathbf{k}_i = \begin{bmatrix} \frac{E*S}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12E*I_z}{L^3} & 0 & 0 & 0 & \frac{6E*I_y}{L^2} \\ 0 & 0 & \frac{12E*I_y}{L^3} & 0 & \frac{-6E*I_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{G*J}{L} & 0 & 0 \\ 0 & 0 & \frac{-6E*I_y}{L^2} & 0 & \frac{4E*I_y}{L} & 0 \\ 0 & \frac{6E*I_y}{L^2} & 0 & 0 & 0 & \frac{4E*I_z}{L} \end{bmatrix}$$

Where, E is Youngs Modulus = 70×10^9; S is Surface Area; G is Shear Modulus = 25.5×10^9; d is Diameter of link; L is Length of Link.

Jacobian matrix composition

The position part of the Jacobian of the passive joint coordinates can be compute by using derivative of robot transformation matrix:

$$T'_d(q) = T_{base}[T_i(q_i)H'T(q_i)]T_{tool} \tag{5}$$

$$T'_d(\theta) = T_{base}[T_i(q_i)H'T(q_i)]T_{tool} \tag{6}$$

where H' is the differential transformation matrix.
The position part of the Jacobian of each column are:

$$[J(q)]_{1,k} = [T'_d]_{1,4}, [J(q)]_{2,k} = [T'_d]_{2,4}, [J(q)]_{3,k} = [T'_d]_{3,4} \tag{7}$$

It works the same for finding Jacobian of virtual joint coordinates as shown in code (Jq.m, Jt.m).

Stiffness matrix for the loaded mode

By using numerical solution The solution for stiffness matrix

$$K_{c,i} = K_c^0 - K_c^0 \cdot J_q \cdot K_{Cq} \tag{8}$$

where $K_c^0 = (J_\theta \cdot K_\theta^{-1} \cdot J_\theta^T)^{-1}$; $K_{Cq} = (J_q \cdot (K_c^0)^{-1} \cdot J_q)^{-1} \cdot J_q^T \cdot (K_c^0)^{-1}$

2.2 Result

These are result of 6 X 6 stiffness matrices of each link:
$Rank(K_{c1}) = 3$

$$K_{c1} = 1.0 \times 10^{06} \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 5.7269 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 5.7269 \end{bmatrix} \tag{9}$$

$Rank(K_{c1}) = 4$

$$K_{c2} = 1.0 \times 10^{06} \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 4.9235 & 0.00 & -0.6872 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -0.6872 & 0.00 & 4.9235 \end{bmatrix} \tag{10}$$

$Rank(K_{c1}) = 3$

$$K_{c3} = 1.0 \times 10^{08} \begin{bmatrix} 0.3092 & 0.9670 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.9670 & 3.0239 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.0483 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \tag{11}$$

2.3 Deflection Maps

Deflection maps are used to analyze the behavior of end-effector under the influence of external loading. So we measure the maximum deflection for 1N force for different end effector locations. First we need to calculate total robot stiffness matrix deflections for all configurations. Inside the for loops, x,y,z location is passed to inverse kinematics function which returns the joint angle vector. This angle vector is passed to the VJM modeling function to get compliance matrix (kc). Then calculate deflections for all configurations $\delta t = FK$.

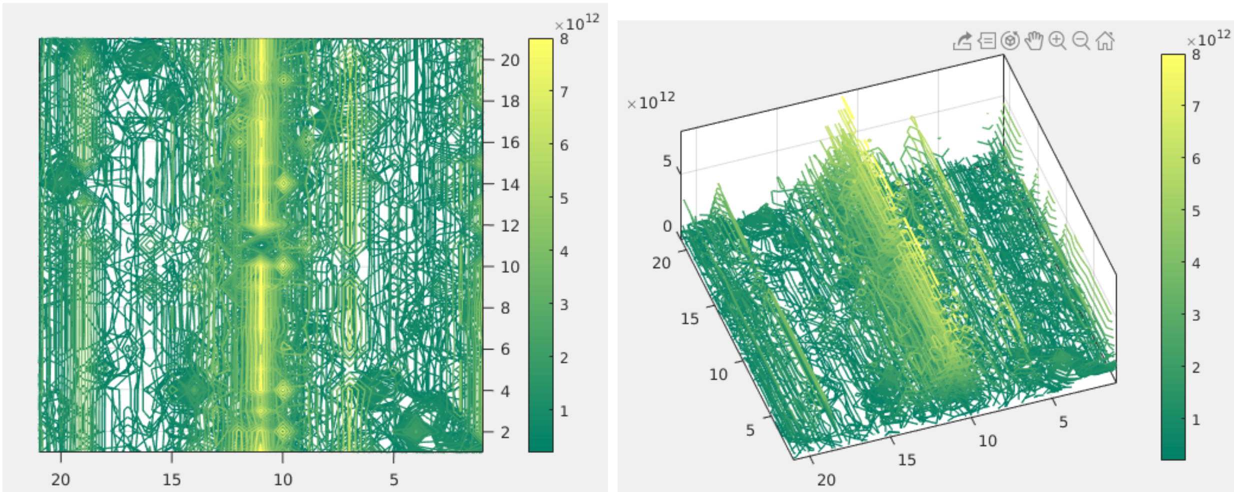

Figure 4: Deflection Maps top view

3 MSA

The first step in MSA modeling is deriving the MSA model of the robot. The robot is split into four parts: the robot platform and three legs. This allows taking into account a complex platform elasticity of the robot if needed. The MSA model of each leg of robot can be drawn as follows:

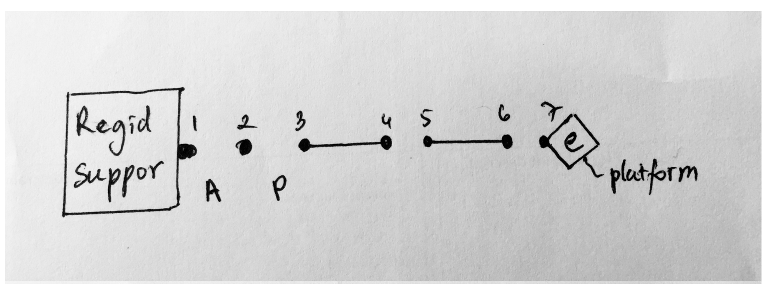

Figure 5: MSA model

The total robot stiffness matrix was obtained in two steps: calculating stiffness for each leg and aggregating legs in total robot structure.

3.1 Link description

The node 1 is connected to the rigid base and described by the following constraint equation:

$$\begin{bmatrix} 0_{6\times6} & I_{6\times6} \end{bmatrix} \begin{bmatrix} W_1 \\ \Delta t_1 \end{bmatrix} = 0 \tag{12}$$

Flexible links 3-4, 5-6 constraints on deflection and loading could be described as:

$$\begin{bmatrix} -I_{6\times6} & 0_{6\times6} & 0_{6\times6} & 0_{6\times6} & QK_{3,4}^{11}Q^T & QK_{3,4}^{12}Q^T & 0_{6\times6} & 0_{6\times6} \\ 0_{6\times6} & -I_{6\times6} & 0_{6\times6} & 0_{6\times6} & QK_{3,4}^{12}Q^T & QK_{3,4}^{23}Q^T & 0_{6\times6} & 0_{6\times6} \\ 0_{6\times6} & 0_{6\times6} & -I_{6\times6} & 0_{6\times6} & 0_{6\times6} & 0_{6\times6} & QK_{5,6}^{11}Q^T & QK_{5,6}^{12}Q^T \\ 0_{6\times6} & 0_{6\times6} & 0_{6\times6} & -I_{6\times6} & 0_{6\times6} & 0_{6\times6} & QK_{5,6}^{12}Q^T & QK_{5,6}^{22}Q^T \end{bmatrix} \begin{bmatrix} W_3 \\ W_4 \\ W_5 \\ W_6 \\ \Delta t_3 \\ \Delta t_4 \\ \Delta t_5 \\ \Delta t_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

Note that the first leg the axis rotation is x where $Q_i = \begin{bmatrix} R_{xi} & 0_{3\times3} \\ 0_{3\times3} & R_{xi} \end{bmatrix}$. For second and third leg, the rotation axis y,z respectively and Q_i is follow the same sequence above.

Rigid platform presented as a rigid link 7-e:

$$\begin{bmatrix} 0_{6\times6} & 0_{6\times6} & D_{7,e} & I_{6\times6} \\ I_{6\times6} & D_{7,e}^T & 0_{6\times6} & 0_{6\times6} \end{bmatrix} \begin{bmatrix} W_7 \\ W_e \\ \Delta t_7 \\ \Delta t_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{14}$$

where

$$D_{7,e} = \begin{bmatrix} I_{3\times3} & [D_{7,e}]_\times^T \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{15}$$

$[D_{7,e}]_\times$ is denotes the 3x3 skew-symmetric matrix derived from the vector $D_{7,e}$ describes the link geometry and is directed from the 7th to the e-th node.

3.2 Joint descriptions

Active elastic joint 1-2 is described by the following equation:

$$\begin{bmatrix} 0_{5\times6} & 0_{5\times6} & \Lambda_{1,2}^r & -\Lambda_{1,2}^r \\ I_{6\times6} & I_{6\times6} & 0_{6\times6} & 0_{6\times6} \\ \Lambda_{1,2}^e & 0_{6\times6} & K_{act}\Lambda_{1,2}^e & -K_{act}\Lambda_{1,2}^e \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ \Delta t_1 \\ \Delta t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

The passive joints 2-3, 4-5, 6-7:

$$\begin{bmatrix}
0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & \Lambda^r_{2,3} & -\Lambda^r_{2,3} & 0_{5\times6} & 0_{5\times6} \\
0_{5\times6} & 0_{5\times6} \\
\Lambda^r_{2,3} & \Lambda^r_{2,3} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} \\
0_{5\times6} & 0_{5\times6} \\
\Lambda^p_{2,3} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & \Lambda^p_{2,3} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} \\
0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & \Lambda^r_{4,5} & -\Lambda^r_{4,5} \\
0_{5\times6} & 0_{5\times6} \\
0_{5\times6} & 0_{5\times6} & \Lambda^r_{4,5} & \Lambda^r_{4,5} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} \\
0_{5\times6} & 0_{5\times6} \\
0_{1\times6} & 0_{1\times6} & \Lambda^p_{4,5} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} & 0_{1\times6} & \Lambda^p_{4,5} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} \\
0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} \\
\Lambda^r_{6,7} & -\Lambda^r_{6,7} \\
0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & \Lambda^r_{6,7} & \Lambda^r_{6,7} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} & 0_{5\times6} \\
0_{5\times6} & 0_{5\times6} \\
0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & \Lambda^p_{6,7} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} \\
0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & \Lambda^p_{6,7}0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6} & 0_{1\times6}
\end{bmatrix}
\begin{bmatrix} W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ \Delta t_2 \\ \Delta t_3 \\ \Delta t_4 \\ \Delta t_5 \\ \Delta t_6 \\ \Delta t_7 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

where For the first leg the joint 4-5 and 6-7 represent rotation and elasticity around x-axis. So,

$$\Lambda^r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} ; \Lambda^e = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{18}$$

R_{ij} is ijth element of rotation matrix in joint. Λ^p calculated the same way as Λ^e.

For the second leg the joint 4-5 and 6-7 represent rotation and elasticity around y-axis. So,

$$\Lambda^r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} ; \Lambda^e = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{19}$$

For the third leg the joint 4-5 and 6-7 represent rotation and elasticity around z-axis. So,

$$\Lambda^r = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \Lambda^e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

The external loading denotes by the following equation:

$$\begin{bmatrix} I_{6\times6} & 0_{6\times6} \end{bmatrix} \begin{bmatrix} W_e \\ \Delta t_e \end{bmatrix} = W_{ext} \tag{21}$$

Aggregation

Now we can aggregate the equations (11) until (18) to the following type:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} W_{ag} \\ \Delta t_{ag} \\ \Delta t_e \end{bmatrix} = \begin{bmatrix} 0 \\ W_{ext} \end{bmatrix} \tag{22}$$

The overall size of matrix is 96x96 so A is 90x90, B is 90x6, C is 6x90 and D is 6x6. where

$$B = \begin{bmatrix} 0_{42\times6} \\ -I_{6\times6} \\ 0_{42\times6} \end{bmatrix}; C = \begin{bmatrix} 0_{42\times6} & -I_{6\times6} & 0_{42\times6} \end{bmatrix}; D = \begin{bmatrix} 0_{6\times6} \end{bmatrix}$$

So, the Cartesian stiffness matrix K_{Ci} is found according to the following equation:

$$K_{Ci} = D - CA^{-1}B \tag{23}$$

Then the resulting stiffness matrix K_C of the whole manipulator could be found as:

$$K_{Ci} = \sum_{t=1}^{3} K_{Ci} \tag{24}$$

3.3 Result

```
Kc1 =

   1.0e-06 *

    0.0012   -0.0000   -0.0027    0.0022   -0.0000   -0.0173
   -0.0155   -0.1857    0.0693    0.4890    0.0321   -0.8389
    0.0069    0.1761   -0.0564   -0.4937   -0.0320    0.8576
    0.0046    0.0696   -0.0247   -0.1852   -0.0127    0.3226
    0.0014   -0.0306    0.0073    0.0943    0.0066   -0.1730
   -0.0061   -0.0389    0.0174    0.0910    0.0062   -0.1496

Kc2 =

   1.0e-05 *

    0.0128    0.0013   -0.0129    0.1620   -0.0000    0.2185
   -0.0009   -0.0021   -0.0012   -0.0052    0.0000   -0.0023
   -0.0120    0.0005    0.0143   -0.1596   -0.0000   -0.2178
   -0.0024    0.0001    0.0026   -0.0291   -0.0000   -0.0408
    0.0047    0.0001   -0.0053    0.0607    0.0000    0.0823
   -0.0023   -0.0003    0.0027   -0.0316    0.0000   -0.0415

Kc3 =

   1.0e-05 *

    0.0214    0.0406   -0.0787   -0.5289   -0.1201   -0.0000
   -0.0215   -0.0410    0.0789    0.5286    0.1226   -0.0000
   -0.0001   -0.0001   -0.0006   -0.0031   -0.0035    0.0000
    0.0040    0.0077   -0.0149   -0.0984   -0.0240   -0.0000
    0.0040    0.0076   -0.0148   -0.1017   -0.0221    0.0000
   -0.0080   -0.0152    0.0297    0.2001    0.0461   -0.0000
```

Conclusion : These two methods are computational light compare to FEA. It also gives intuition of the deflection(displacement) of robot with the deflection map.