자료구조

L09 Sorting (2)

2022년 1학기

국민대학교 소프트웨어학부

Overview

- **Merge Sort**
- Quicksort

Merge Sort

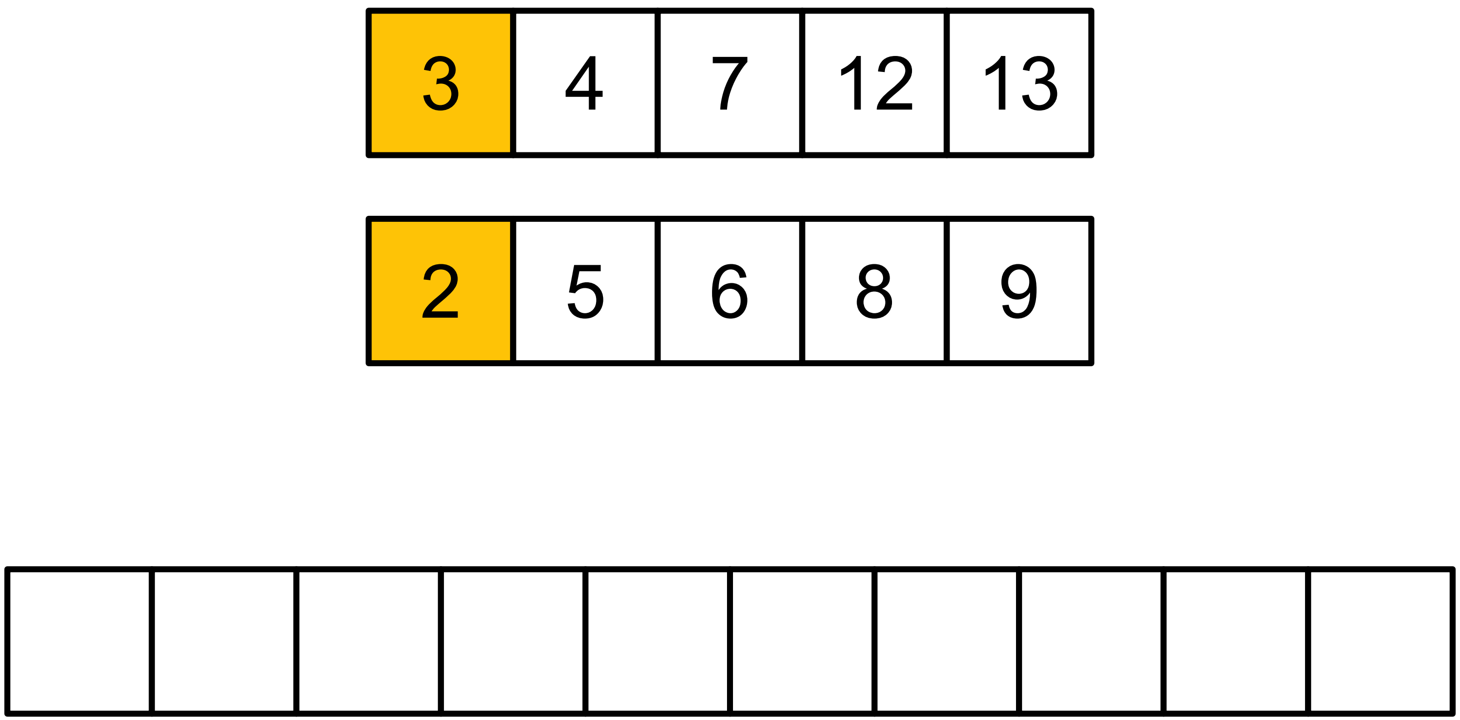
- 입력 배열을 반으로 쪼개서 각각 정렬하고 합친다
- 분할 정복 (Divide and Conquer)의 대표적 예시

Merging two sorted values

3	4	7	12	13

2	5	6	8	9

Merging two sorted values



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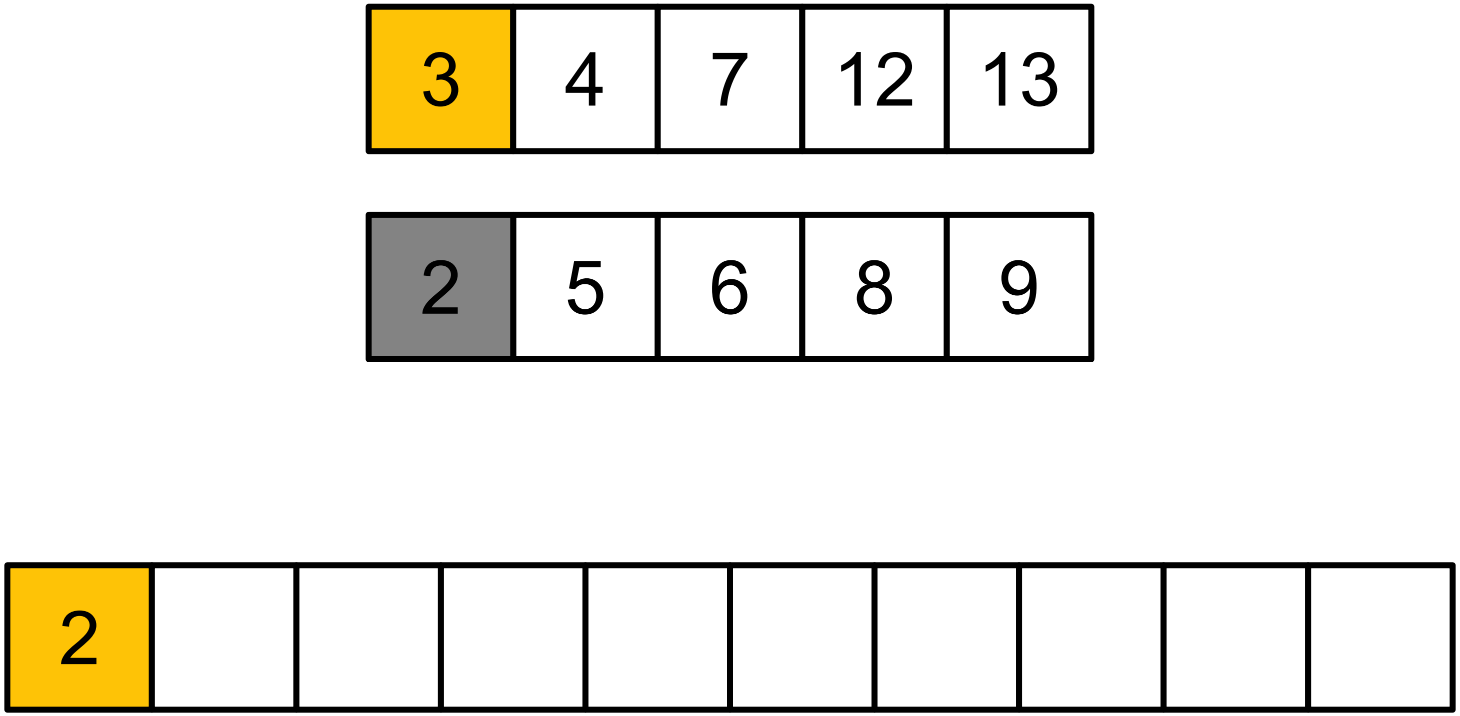

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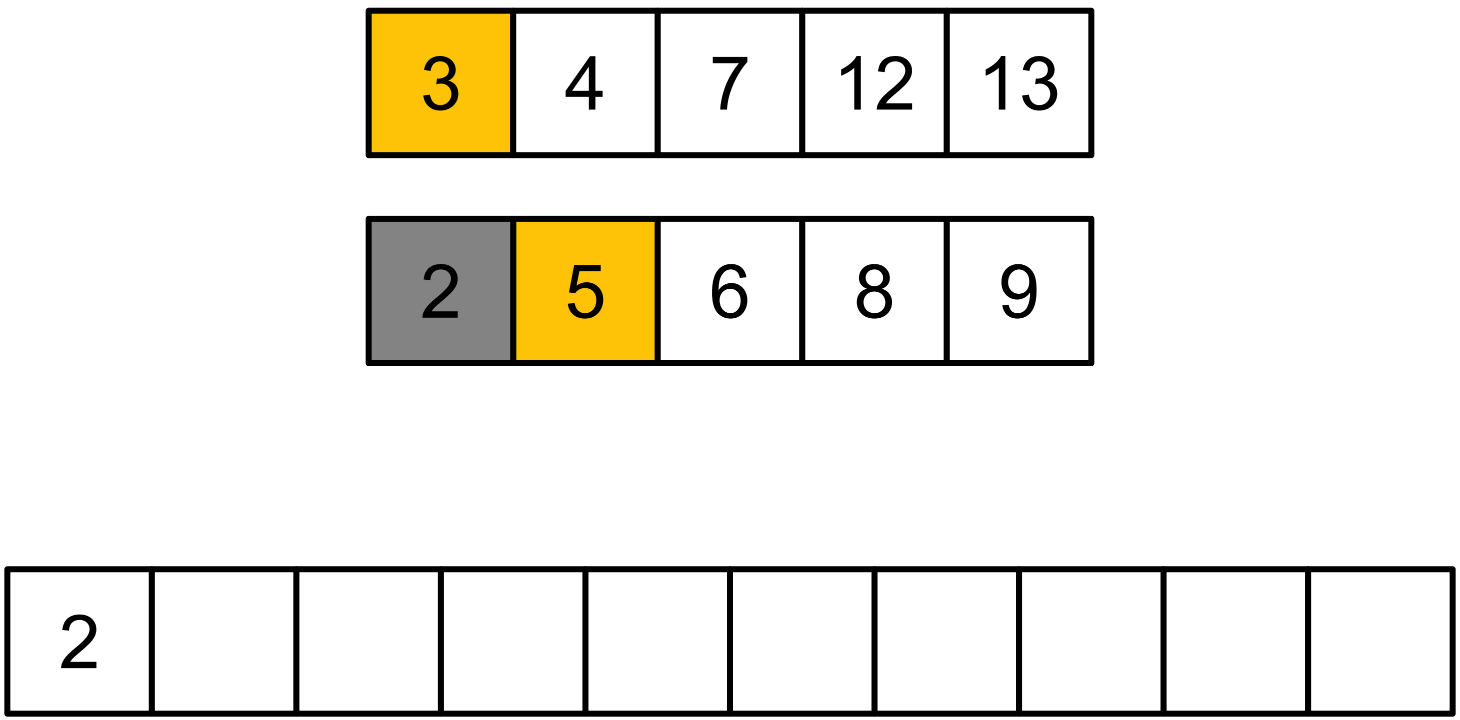


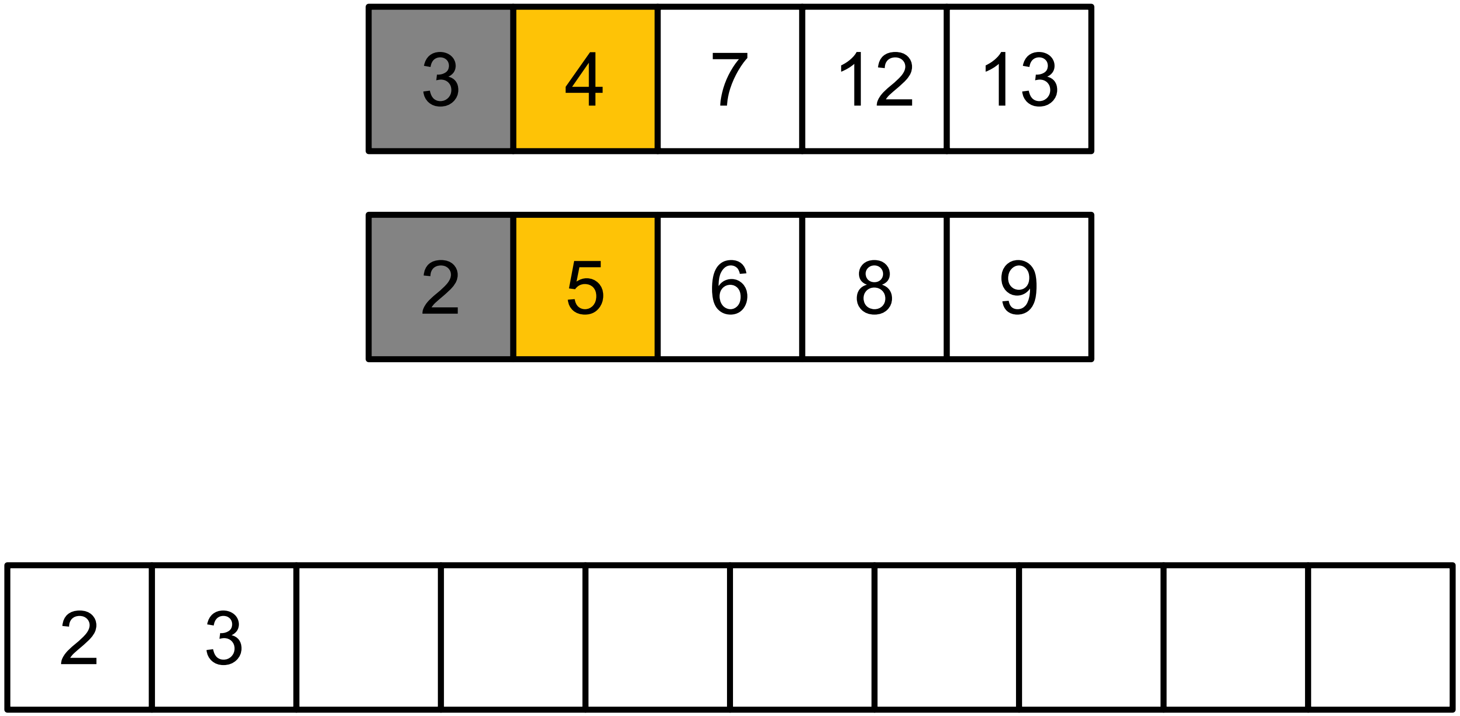
Merging two sorted values

3	4	7	12	13

2	5	6	8	9

2	3								

Merging two sorted values



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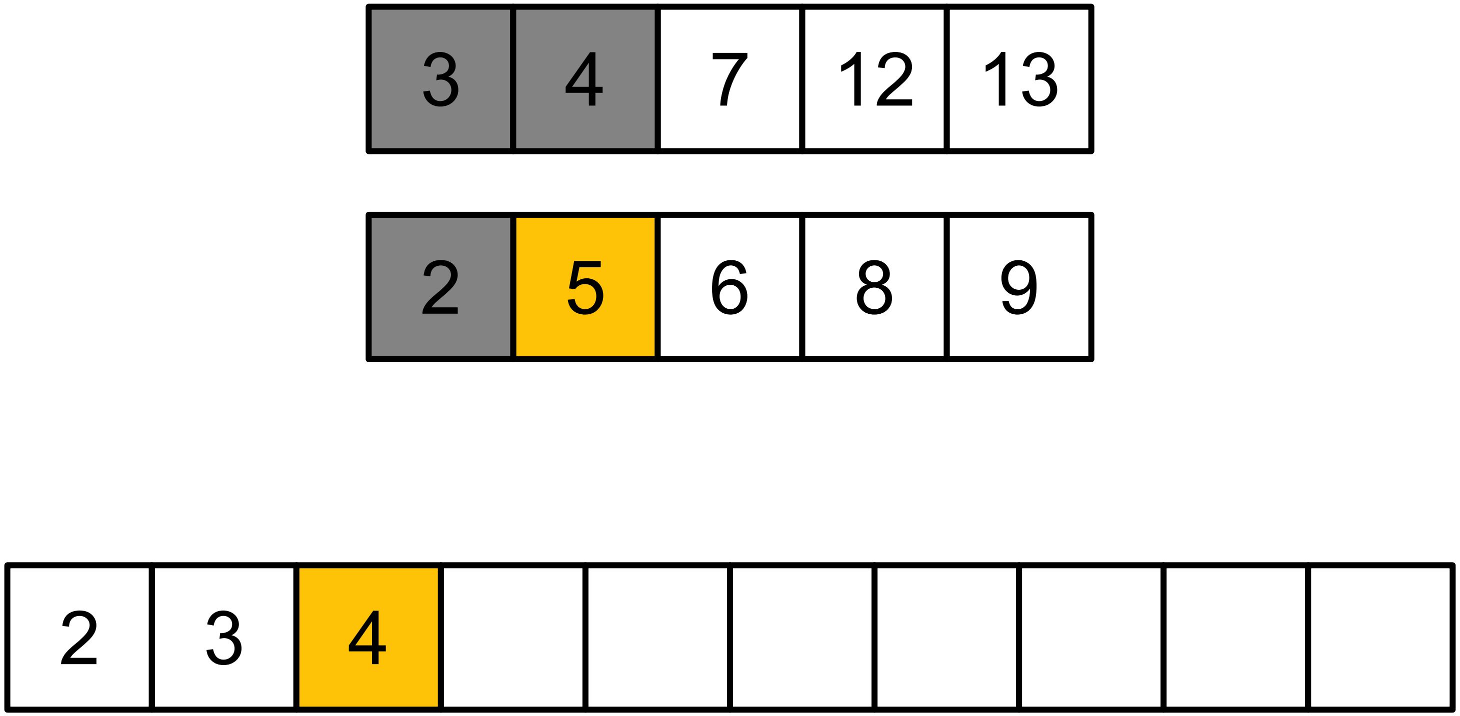

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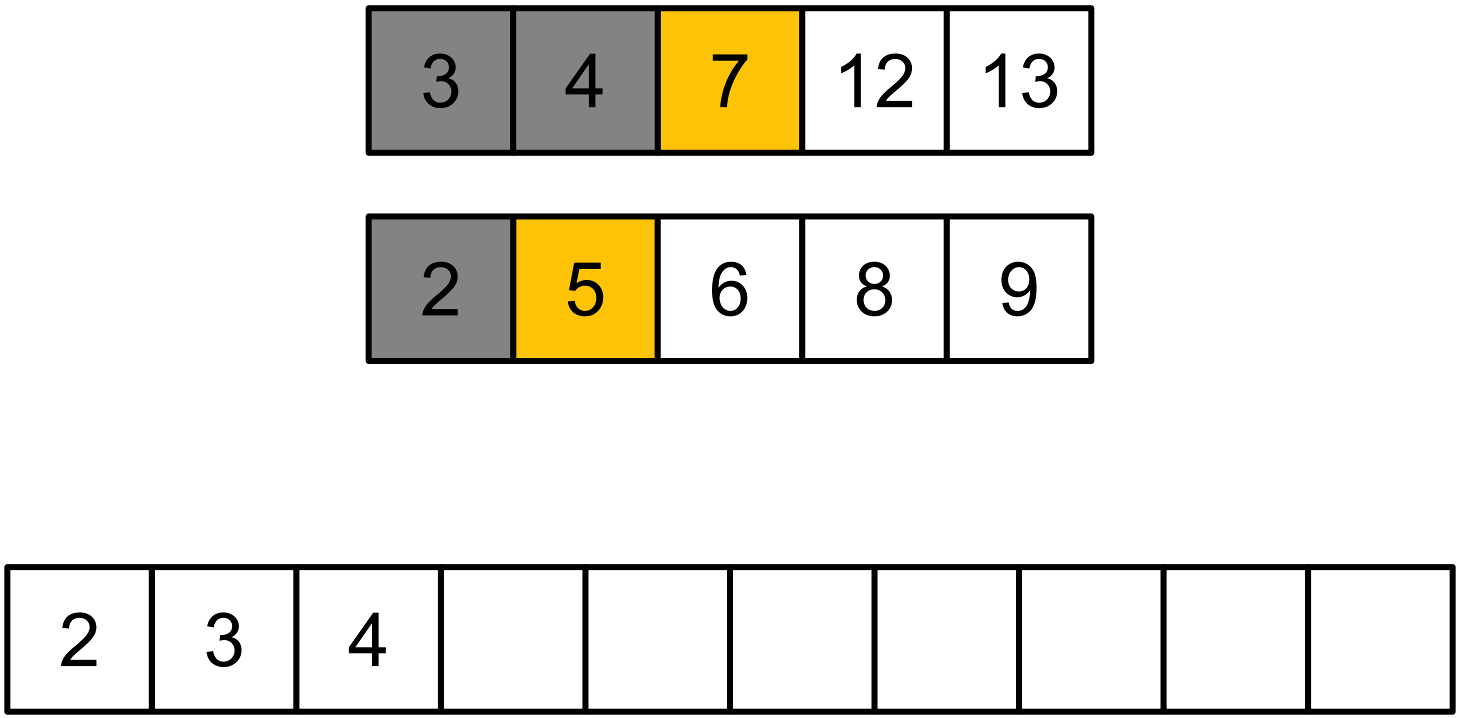

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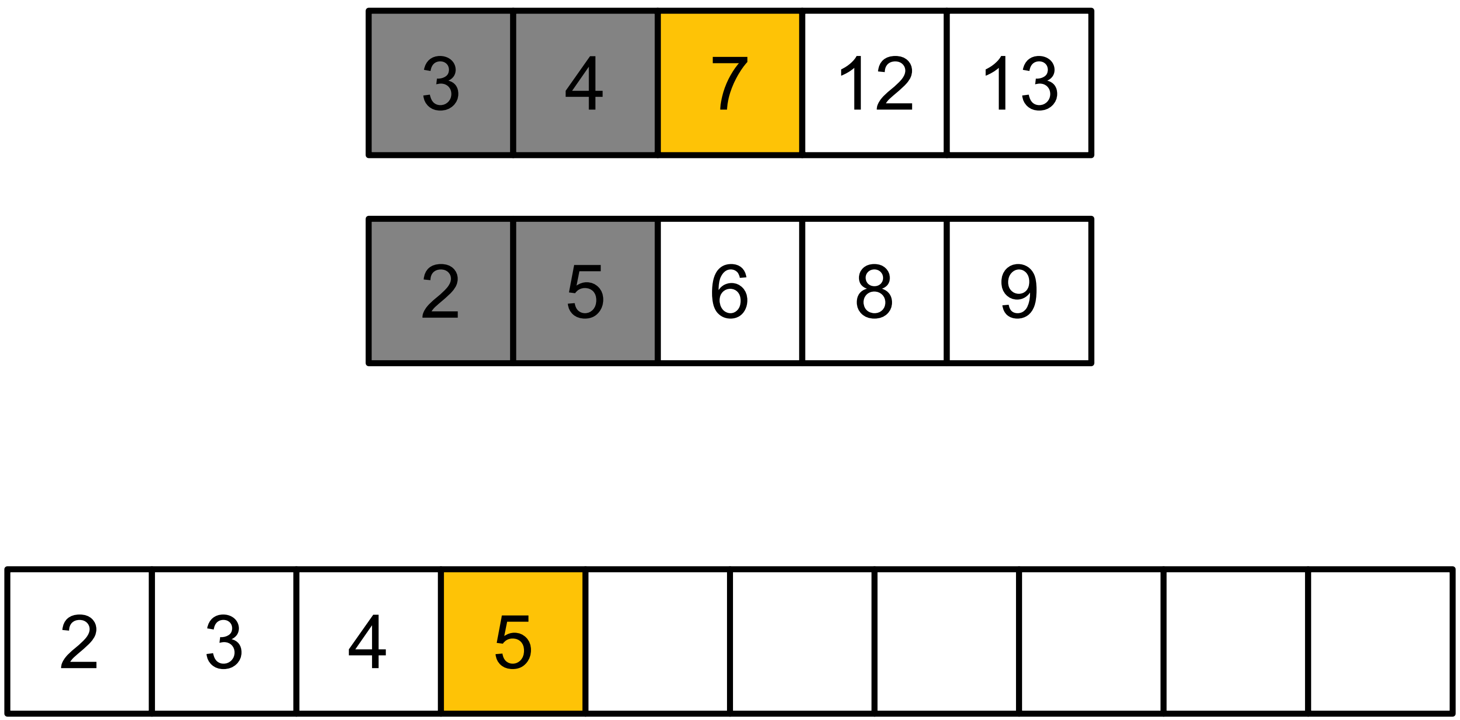

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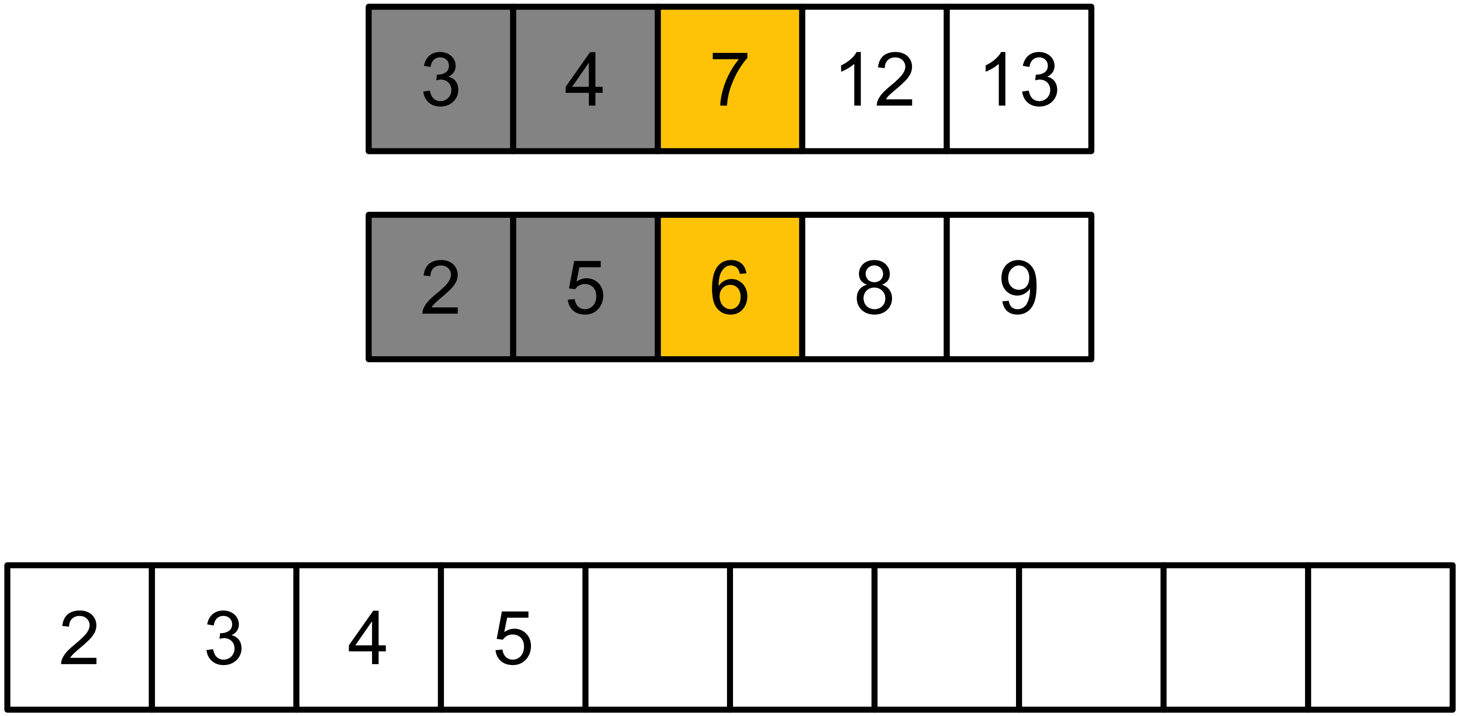

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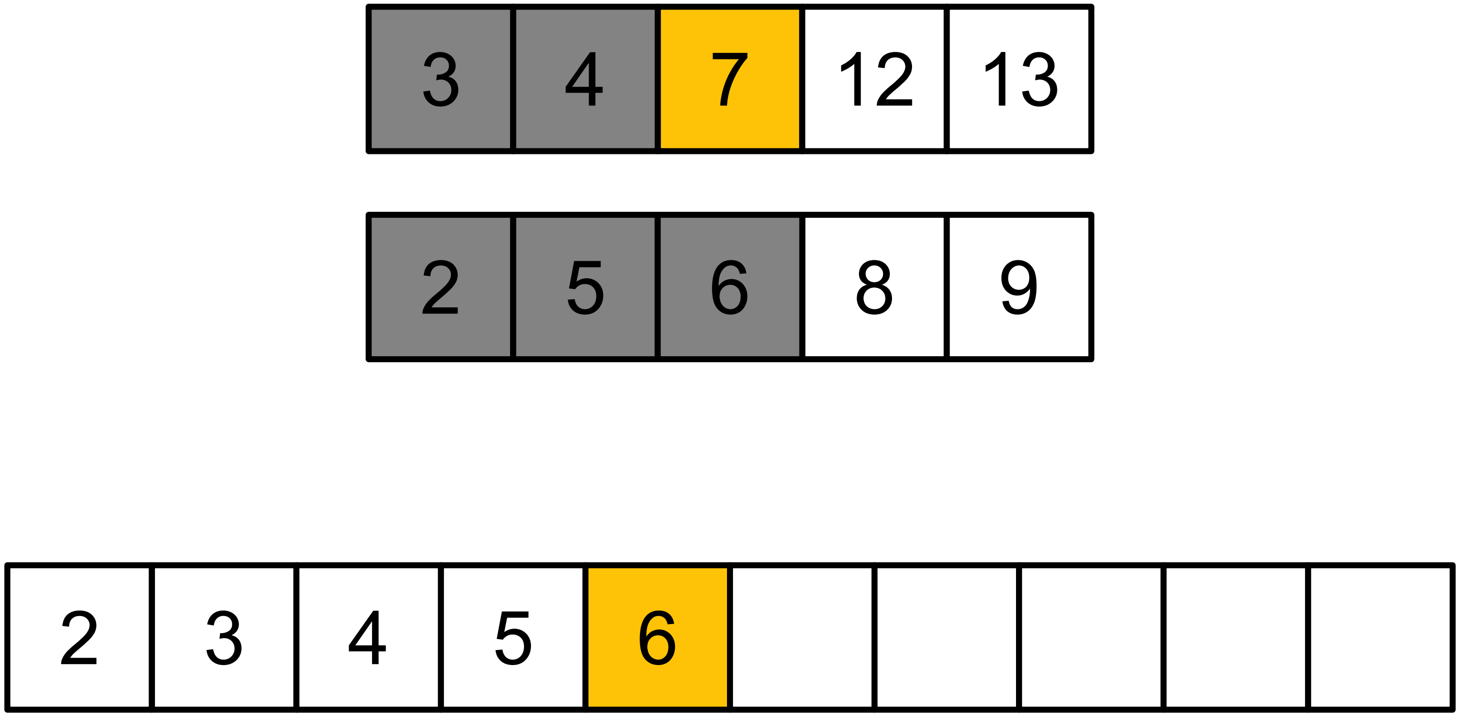

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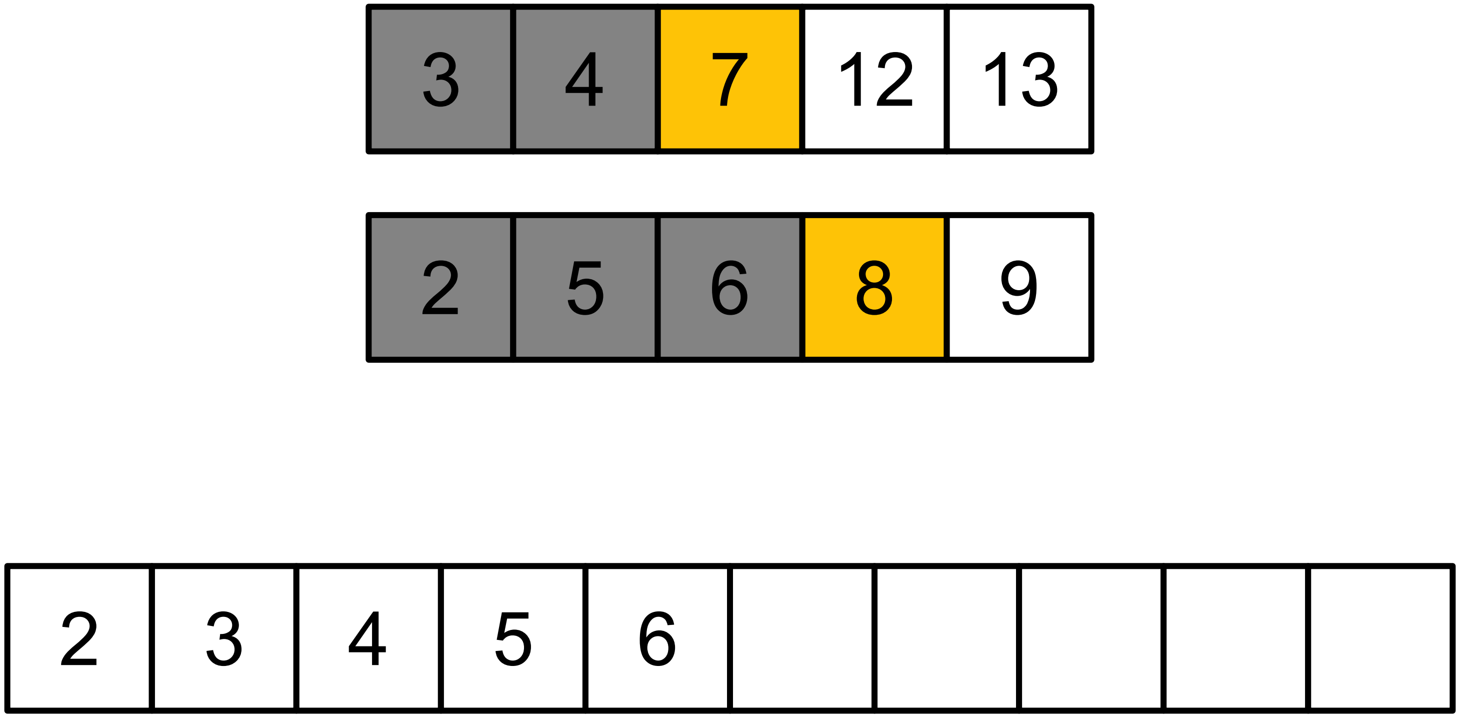

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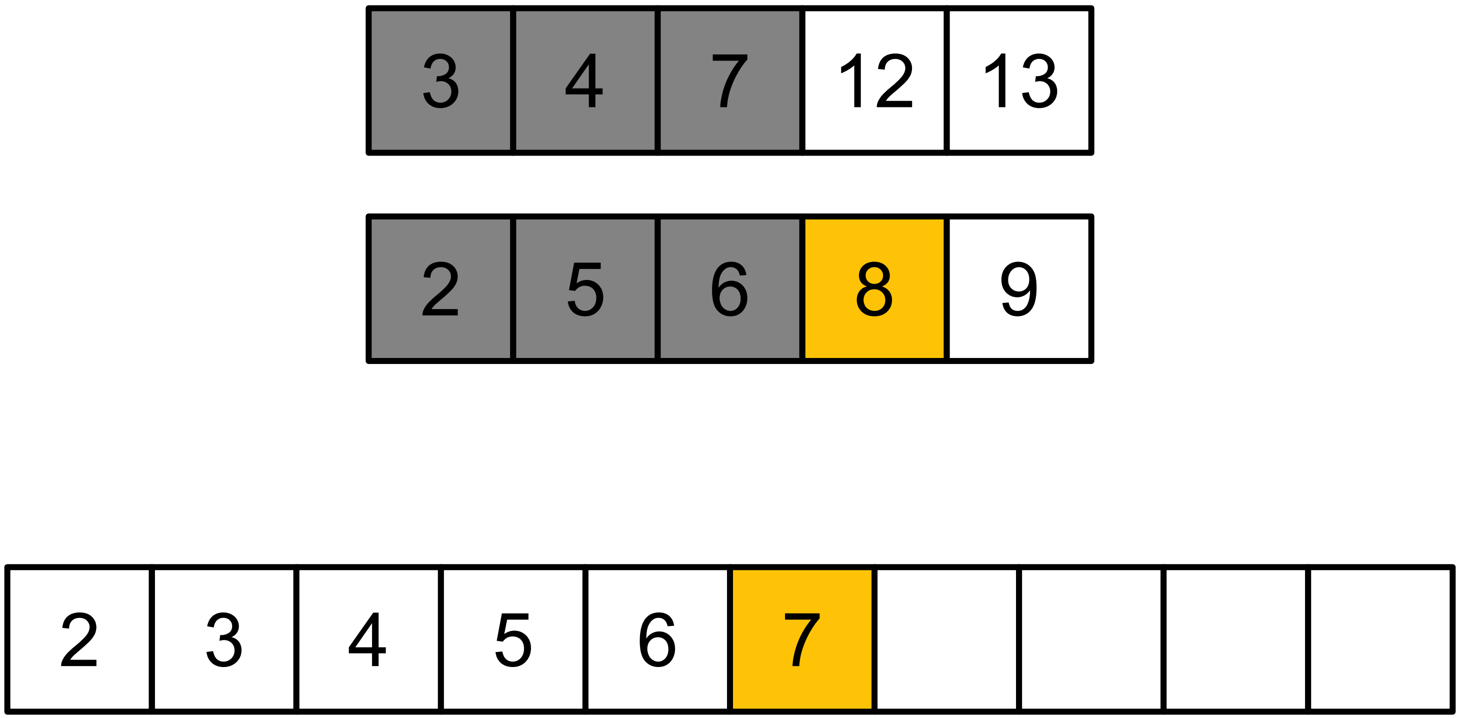

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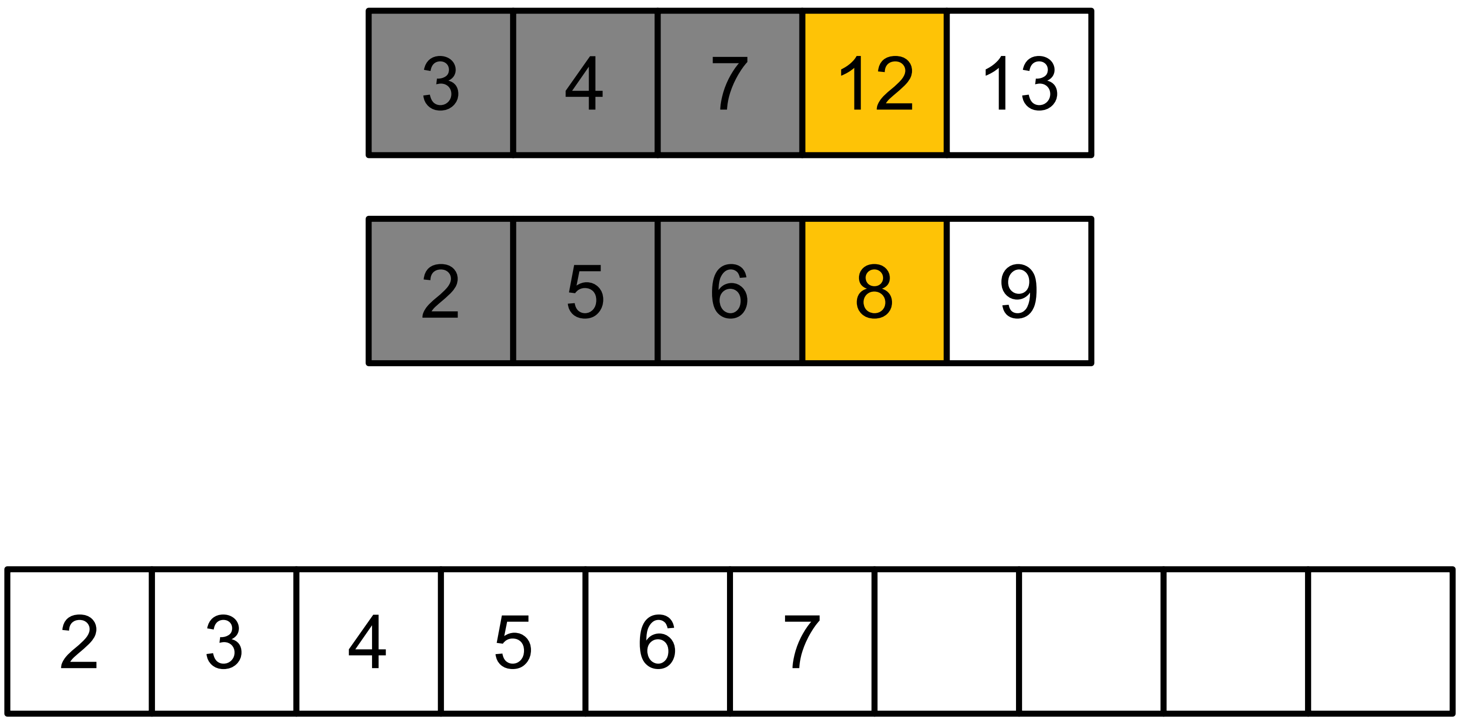

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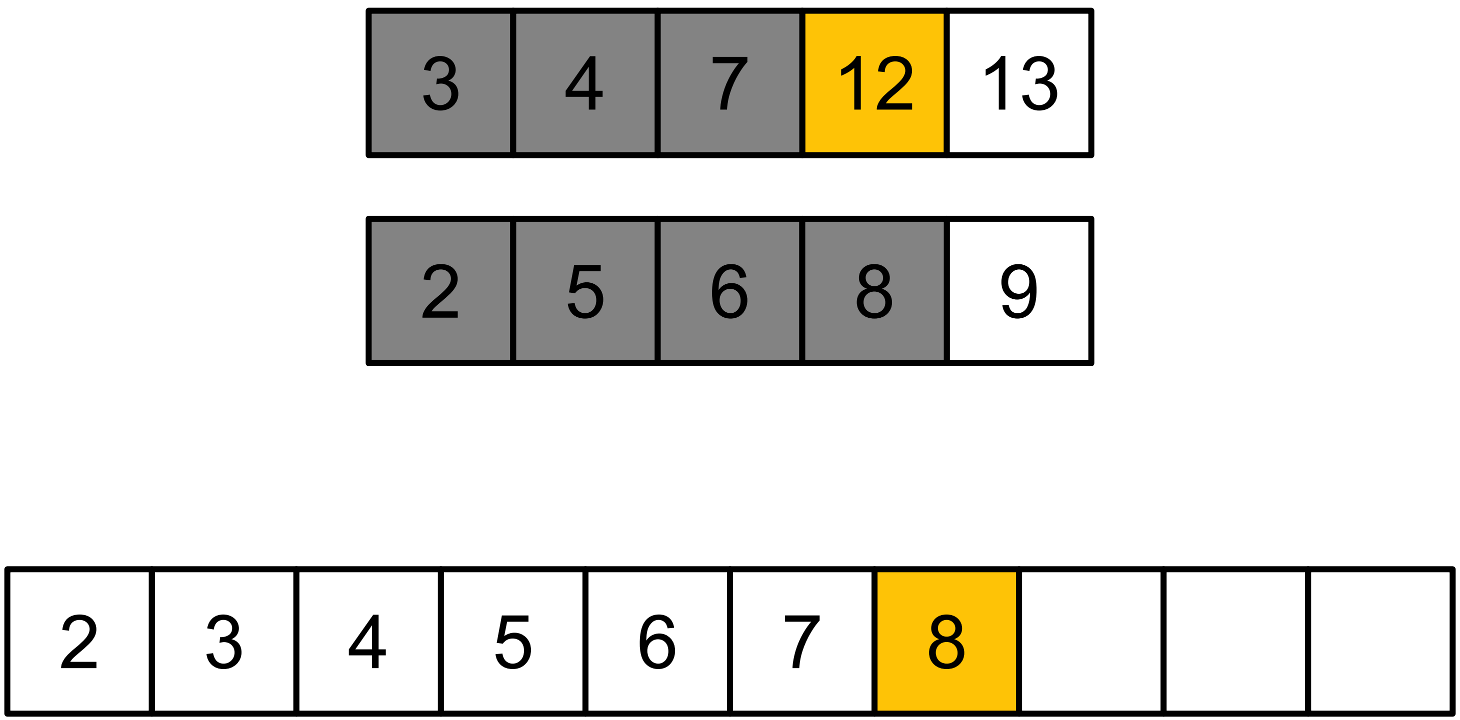

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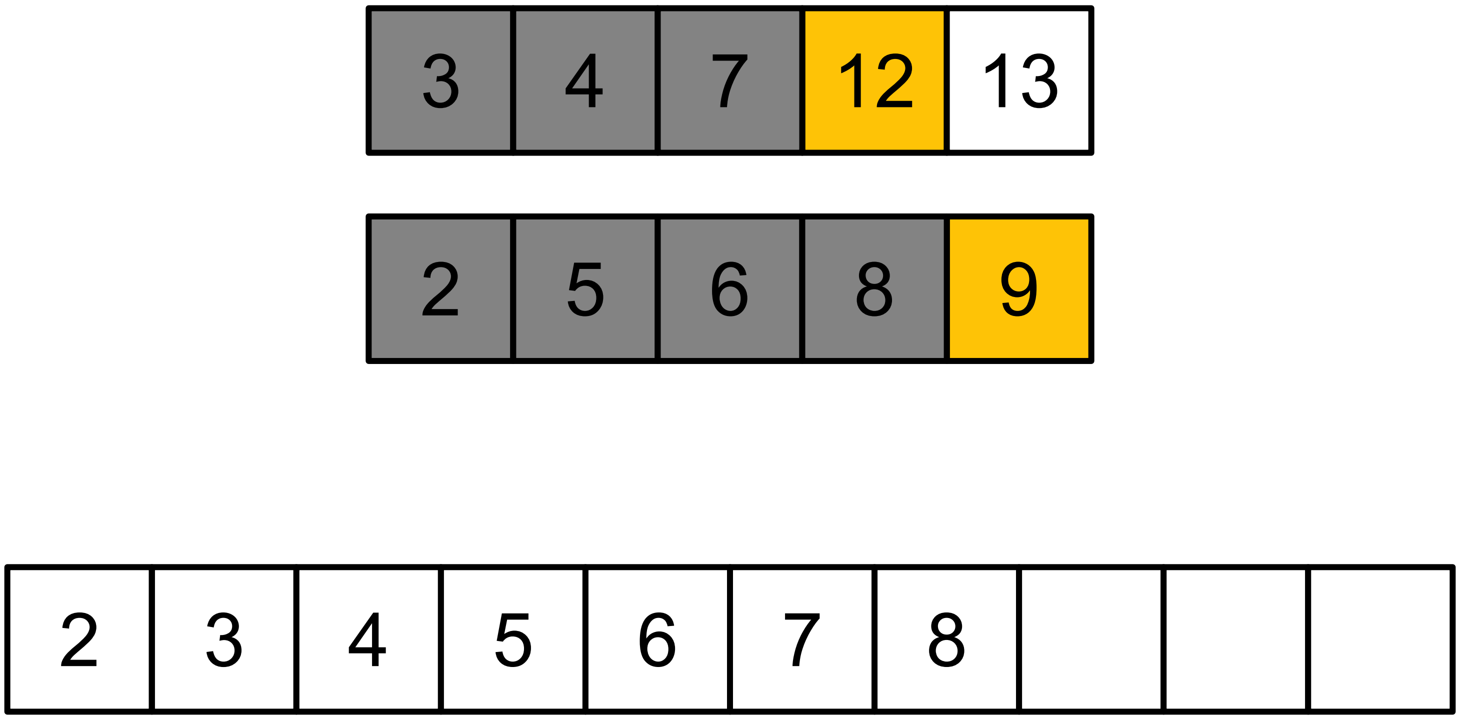

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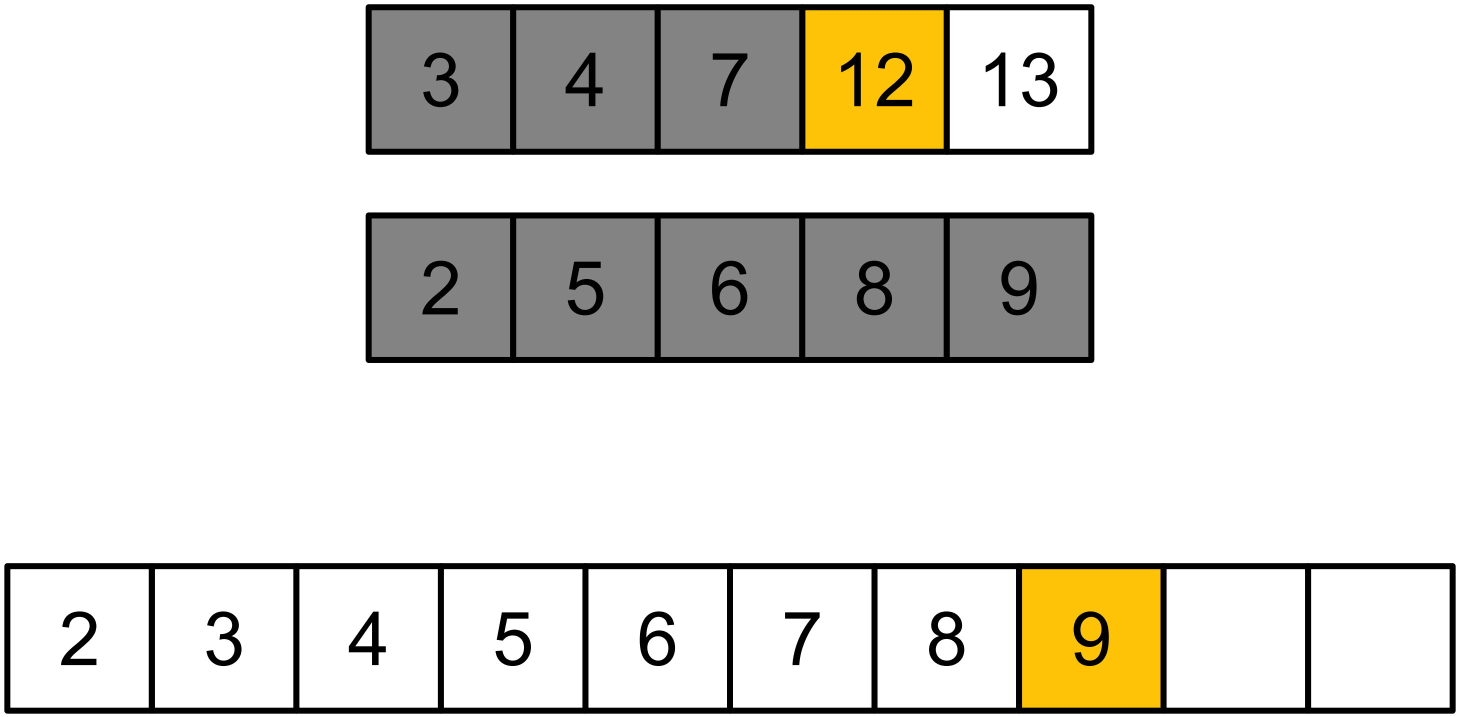

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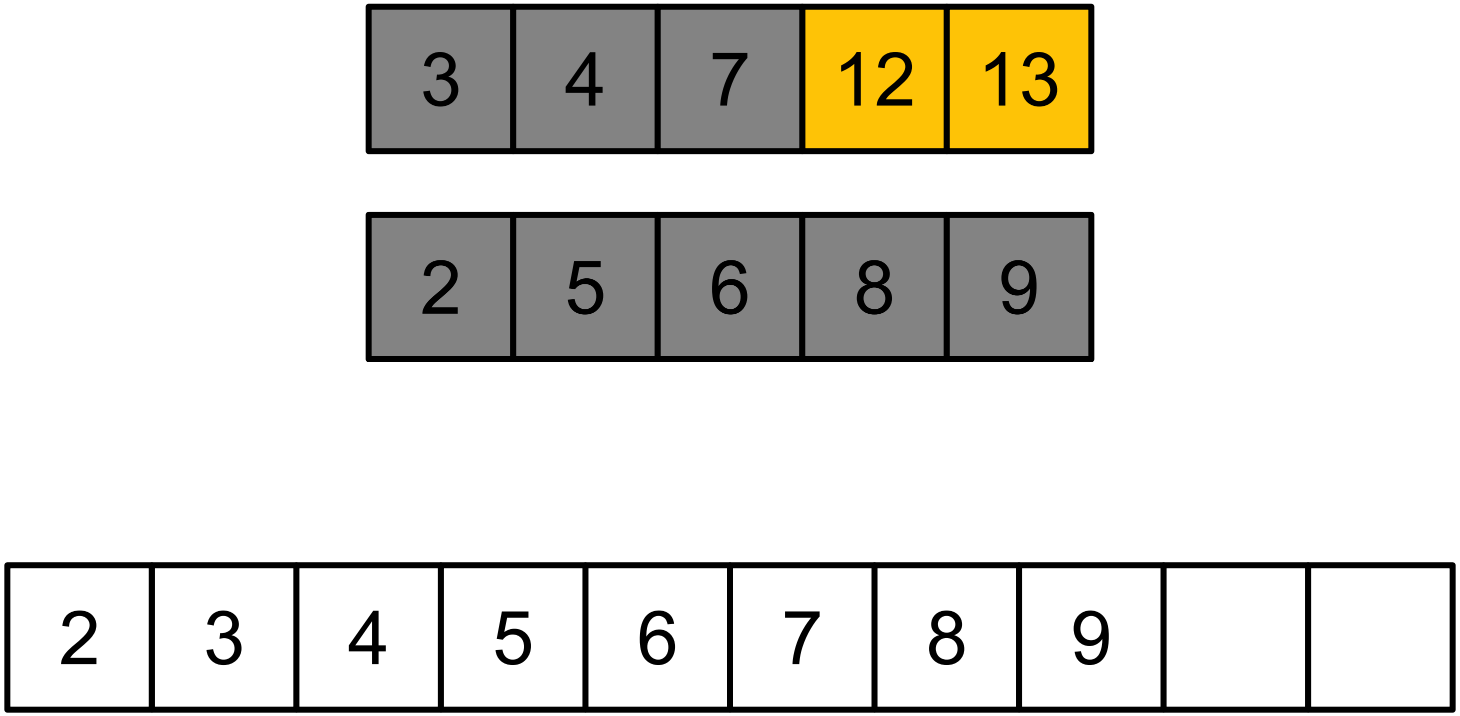


Merging two sorted values

3	4	7	12	13

2	5	6	8	9

2	3	4	5	6	7	8	9	12	13

Merging two sorted values

3	4	7	12	13

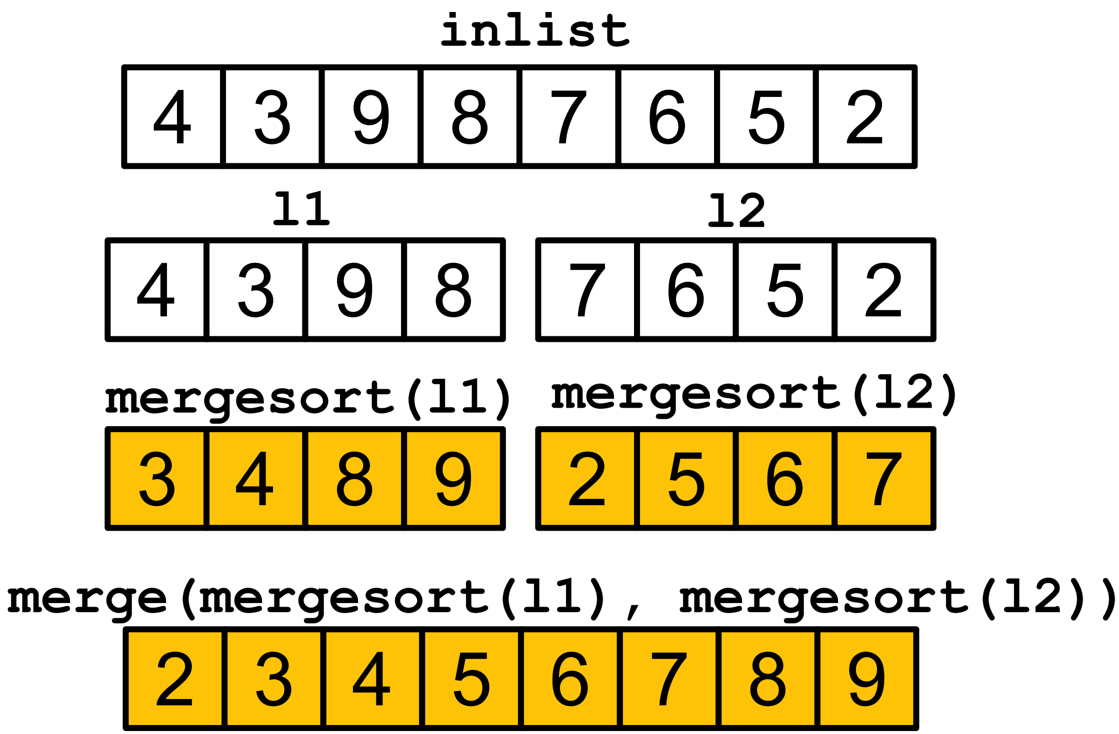
2	5	6	8	9

2	3	4	5	6	7	8	9	12	13

The cost for merging two sorted values: $\Theta(n)$ (n is # values)

Merge Sort - Pseudo Code

```
List mergesort(List inlist) {
  if (inlist.length() <= 1)return inlist;
  List l1 = half of the items from inlist;
  List l2 = other half of items from inlist;
  return merge(mergesort(l1),
               mergesort(l2));
}
```



Merge Sort

4	3	9	8	7	6	5	2

Merge Sort

4	3	9	8

7	6	5	2

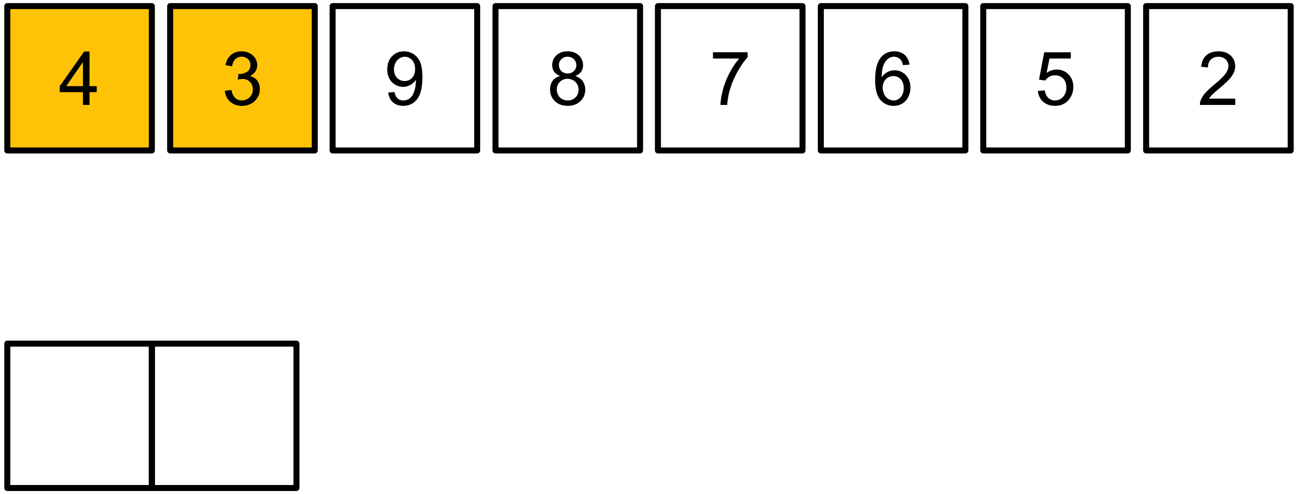
Merge Sort

4	3	9	8	7	6	5	2

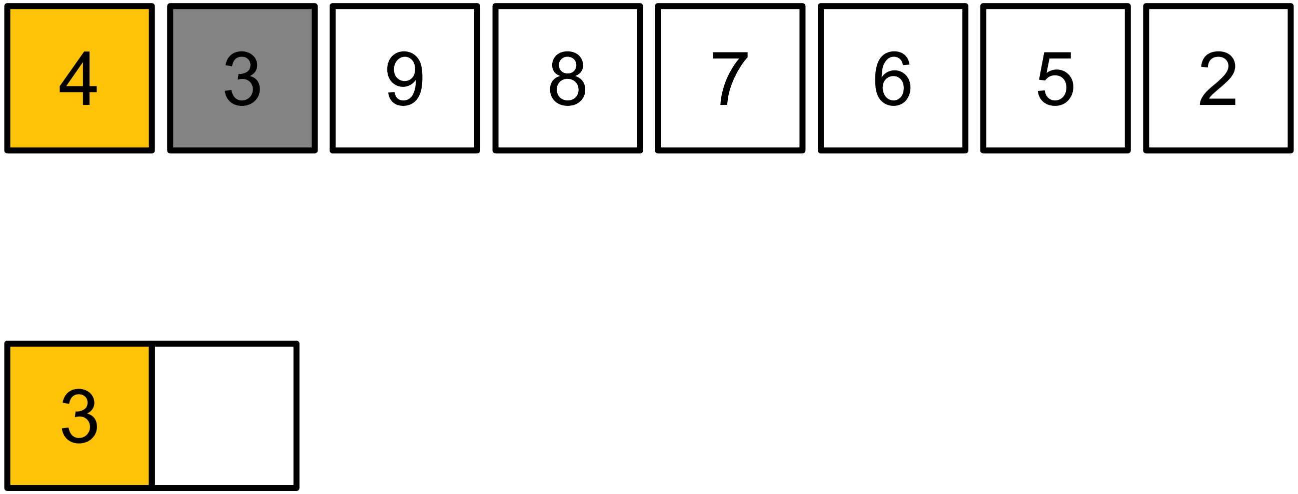
Merge Sort

4	3	9	8	7	6	5	2

Merge Sort



Merge Sort



Merge Sort

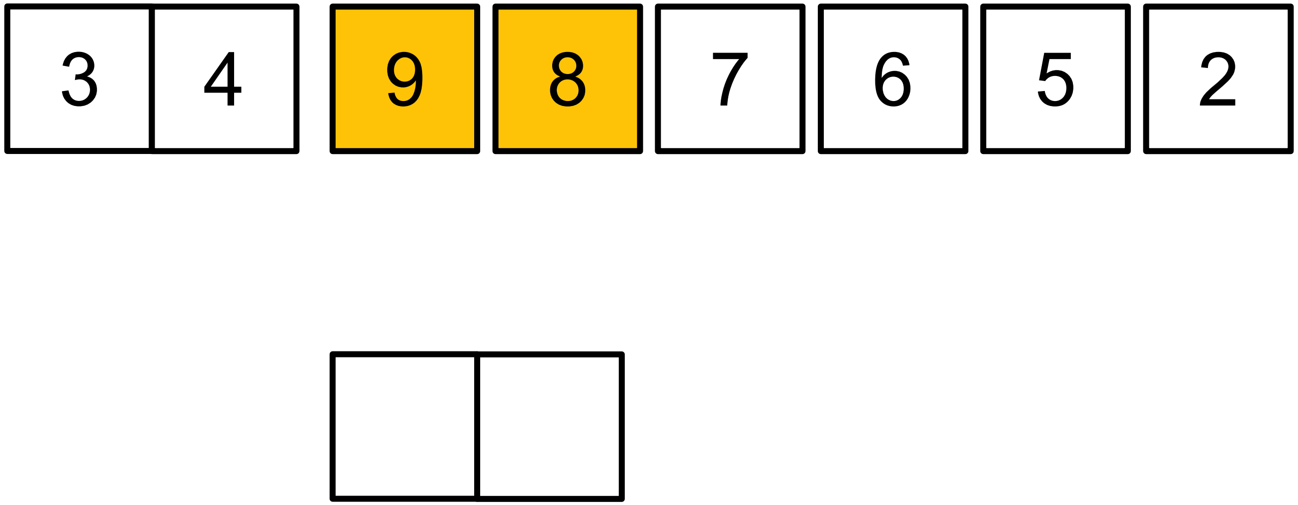
4	3	9	8	7	6	5	2

3	4

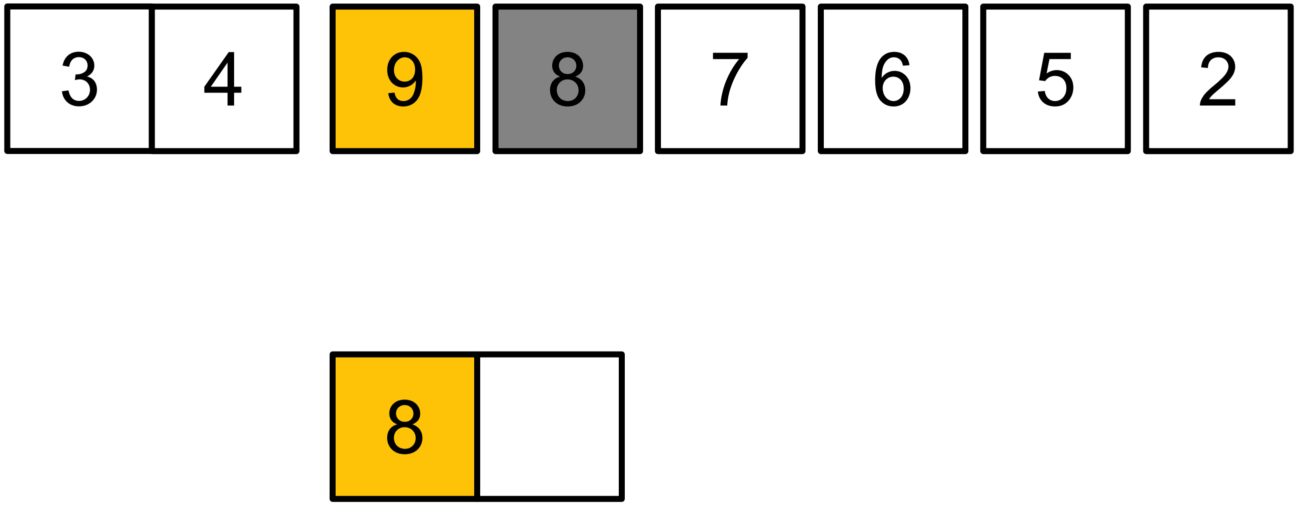
Merge Sort

3	4	9	8	7	6	5	2

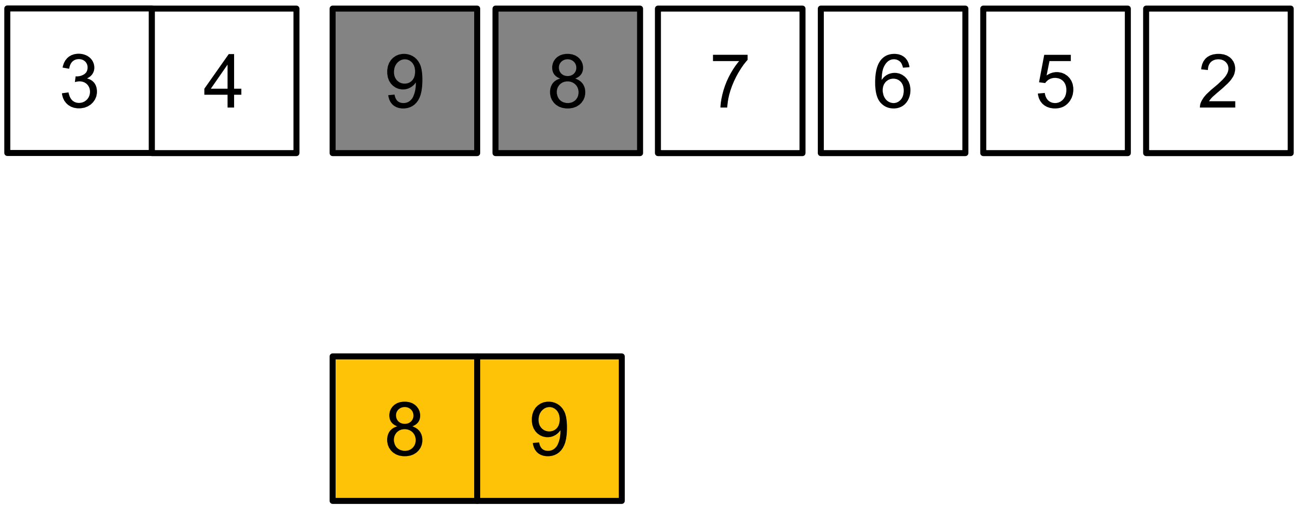
Merge Sort



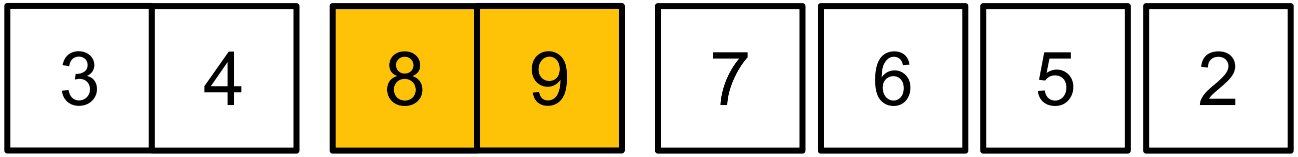
Merge Sort



Merge Sort



Merge Sort



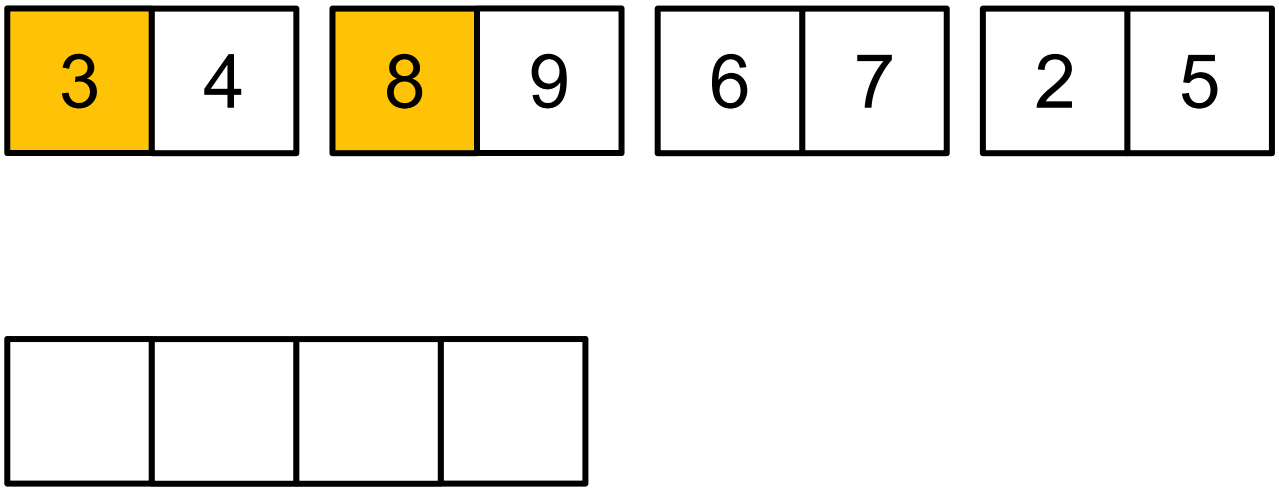
Merge Sort

3	4	8	9	6	7	5	2

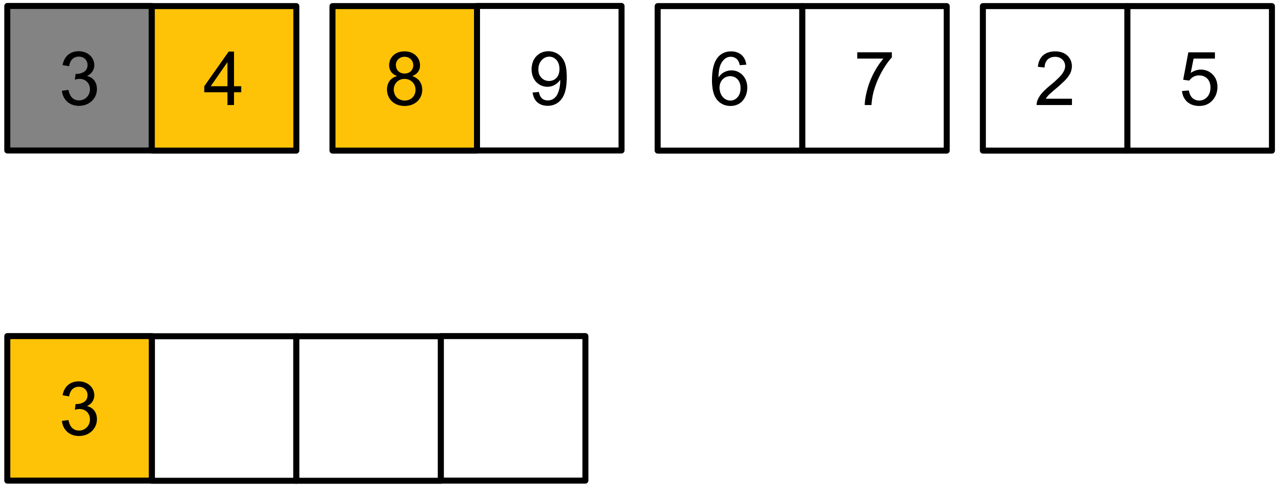
Merge Sort

3	4	8	9	6	7	2	5

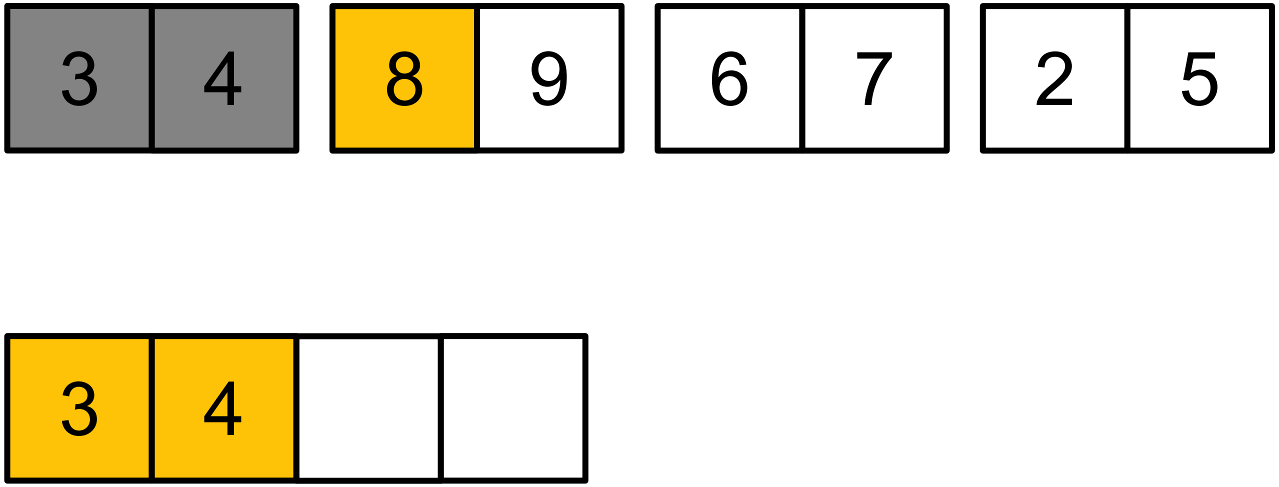
Merge Sort



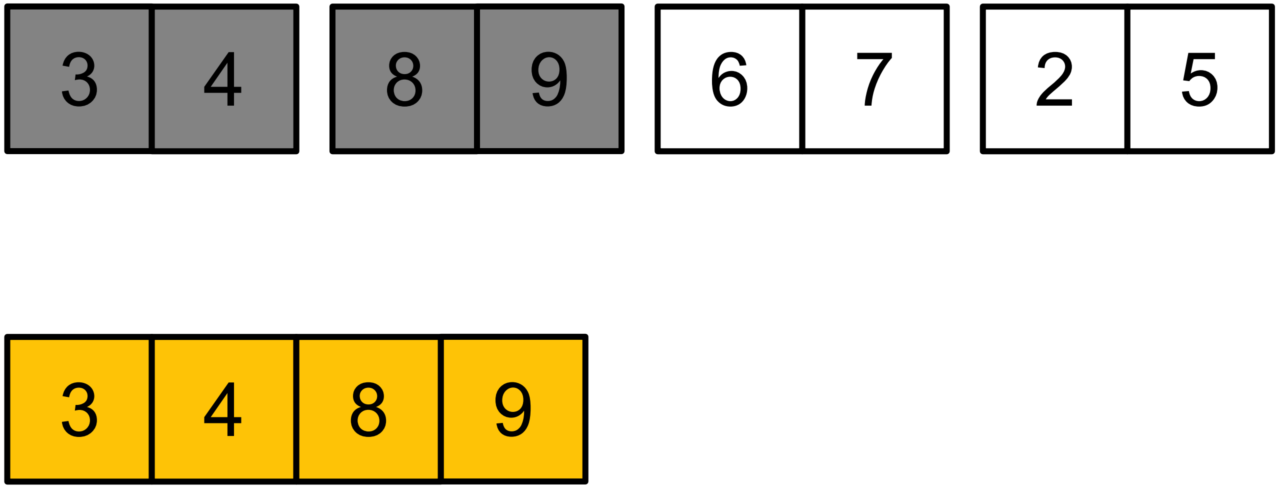
Merge Sort



Merge Sort



Merge Sort



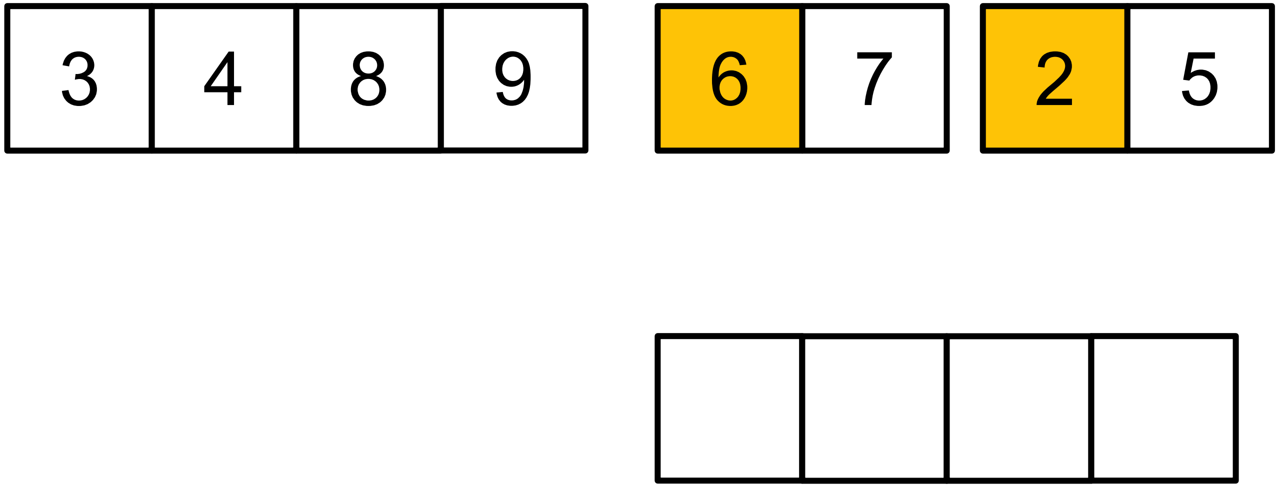
Merge Sort

3	4	8	9

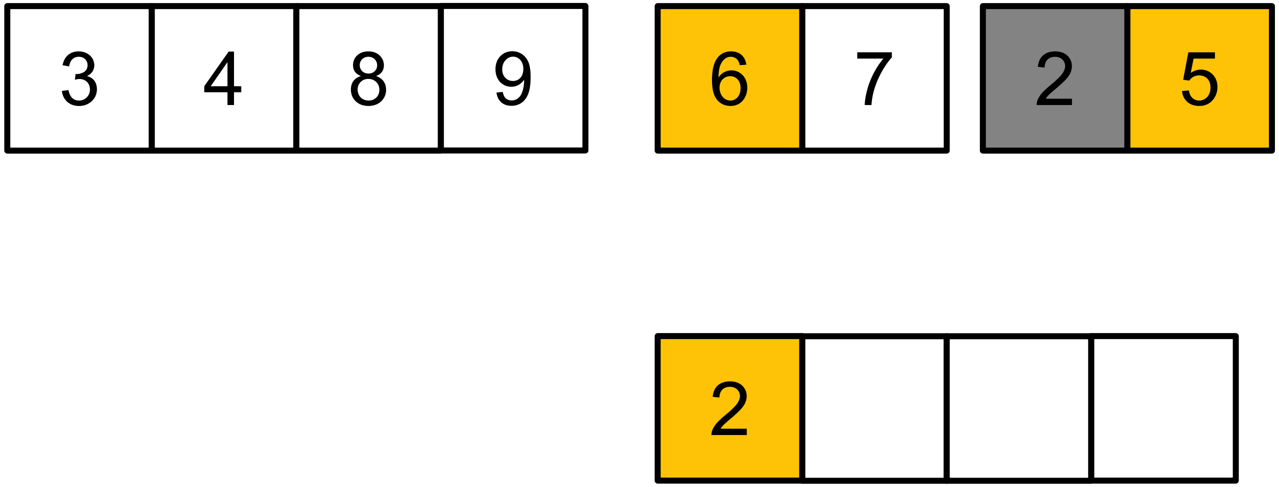
6	7

2	5

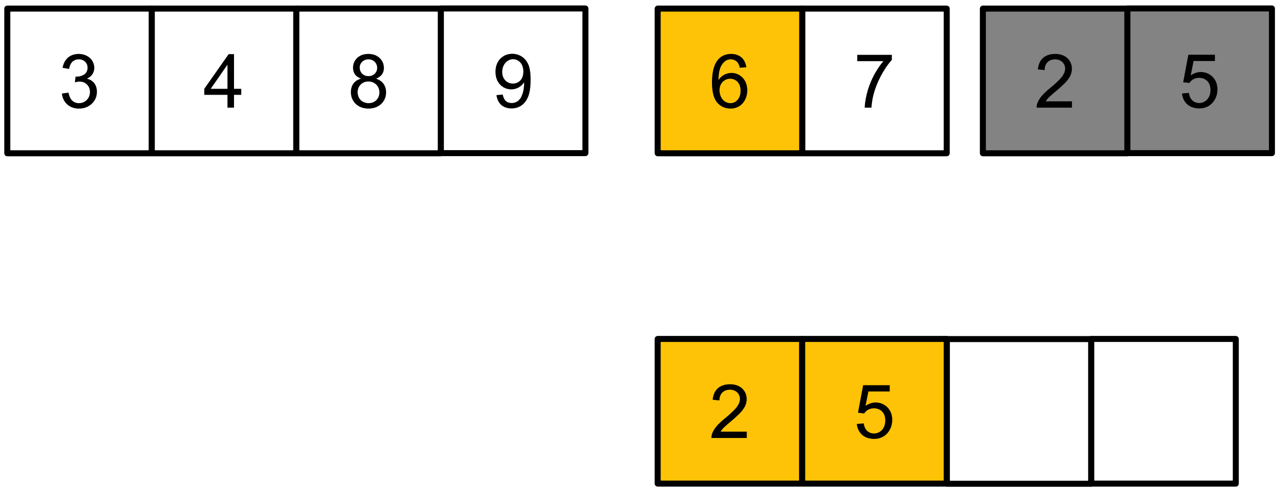
Merge Sort



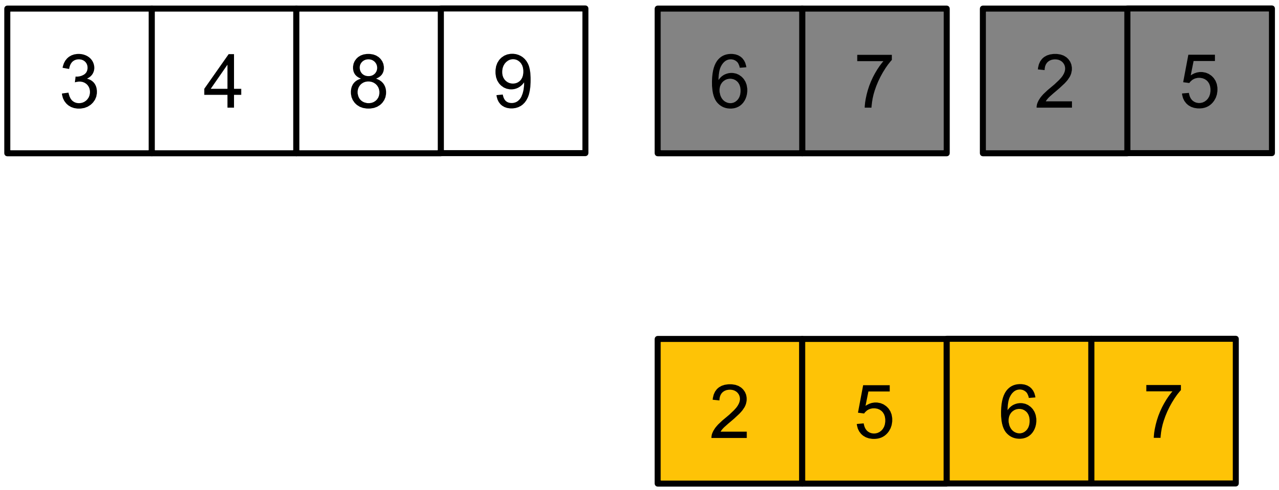
Merge Sort



Merge Sort



Merge Sort

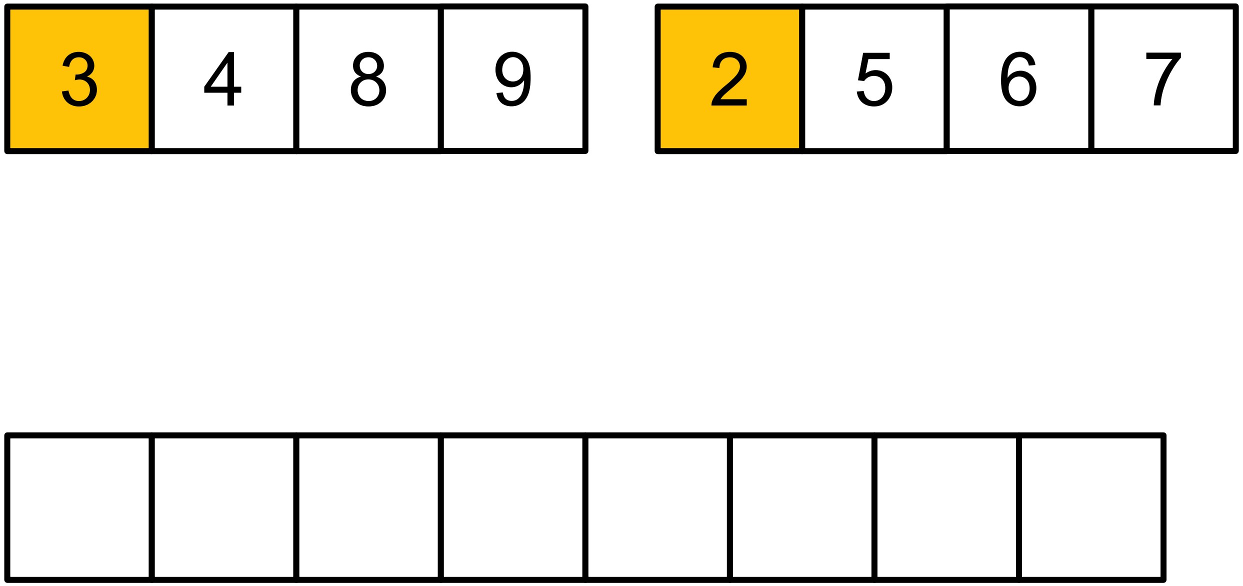


Merge Sort

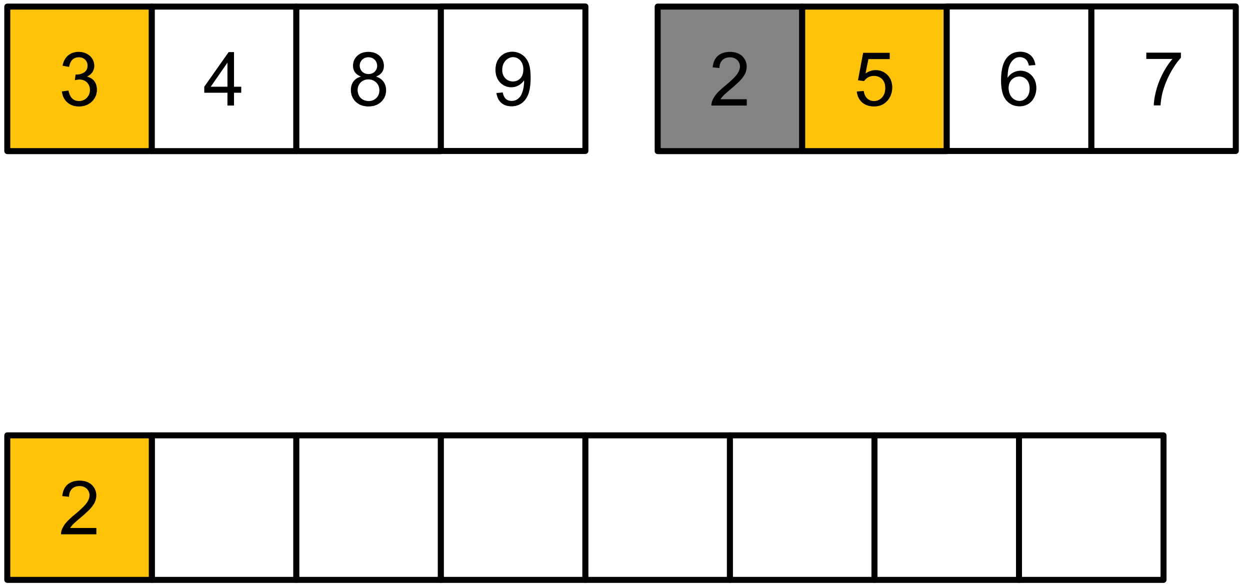
3	4	8	9

2	5	6	7

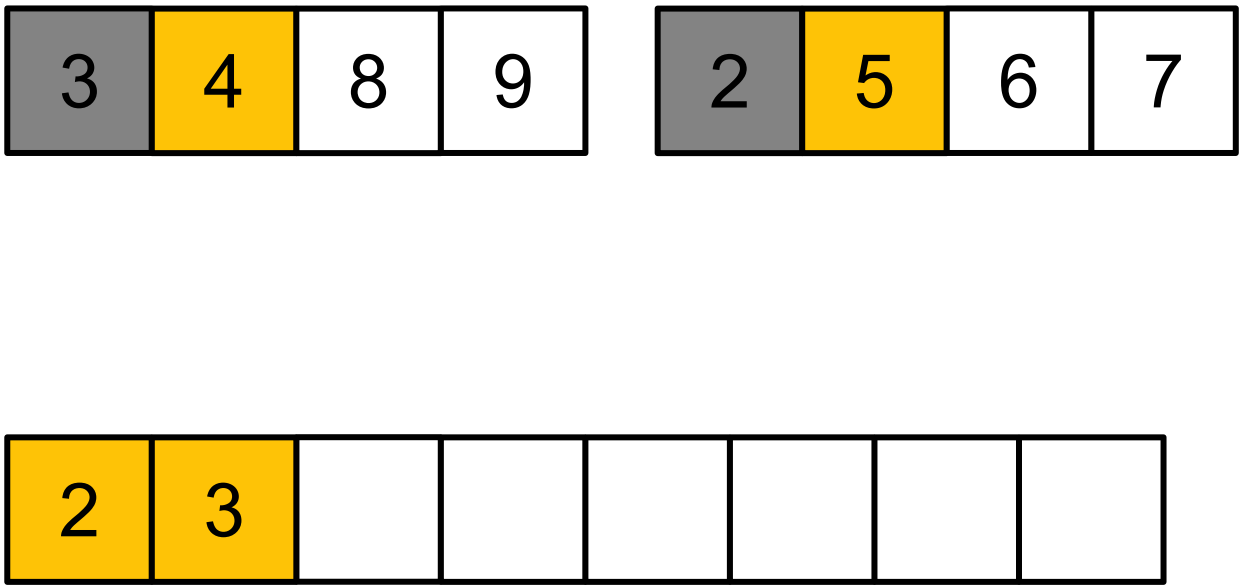
Merge Sort



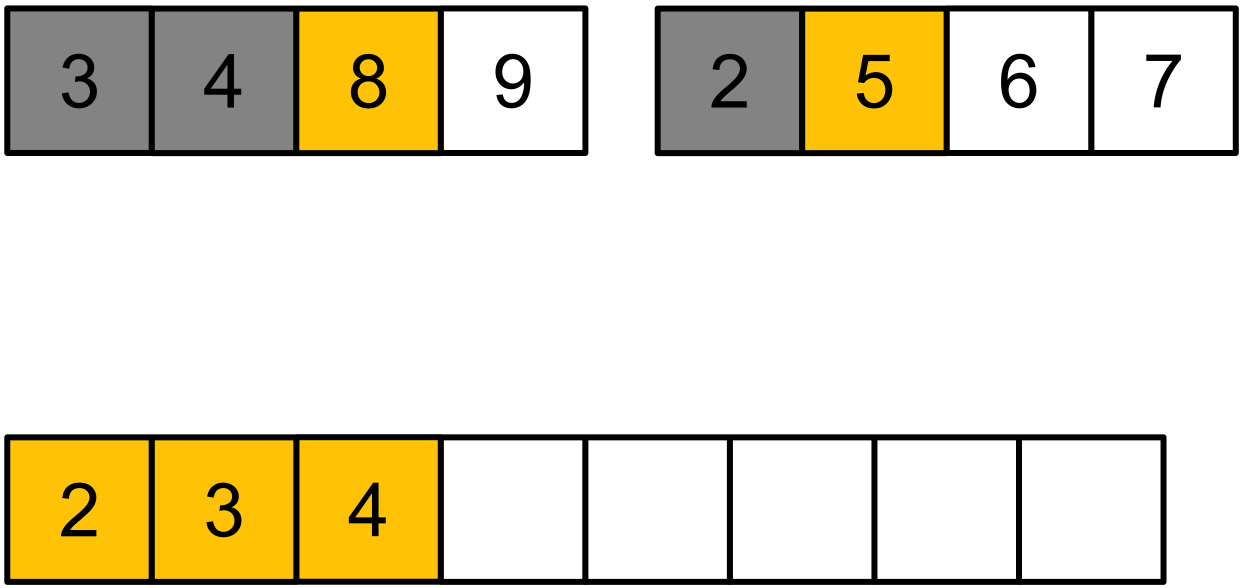
Merge Sort



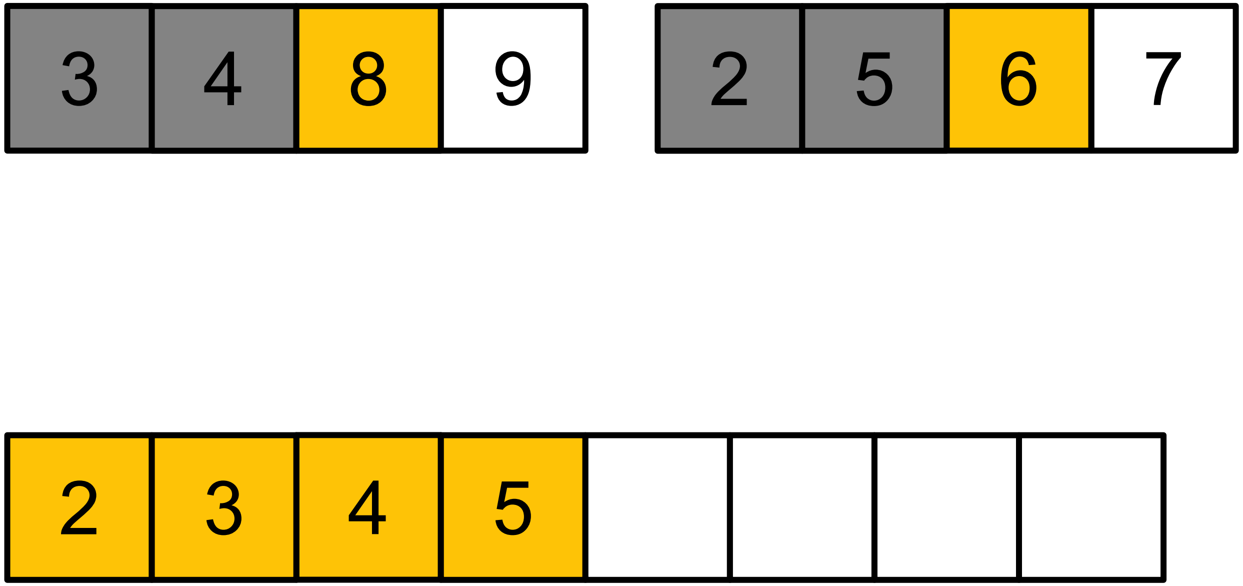
Merge Sort



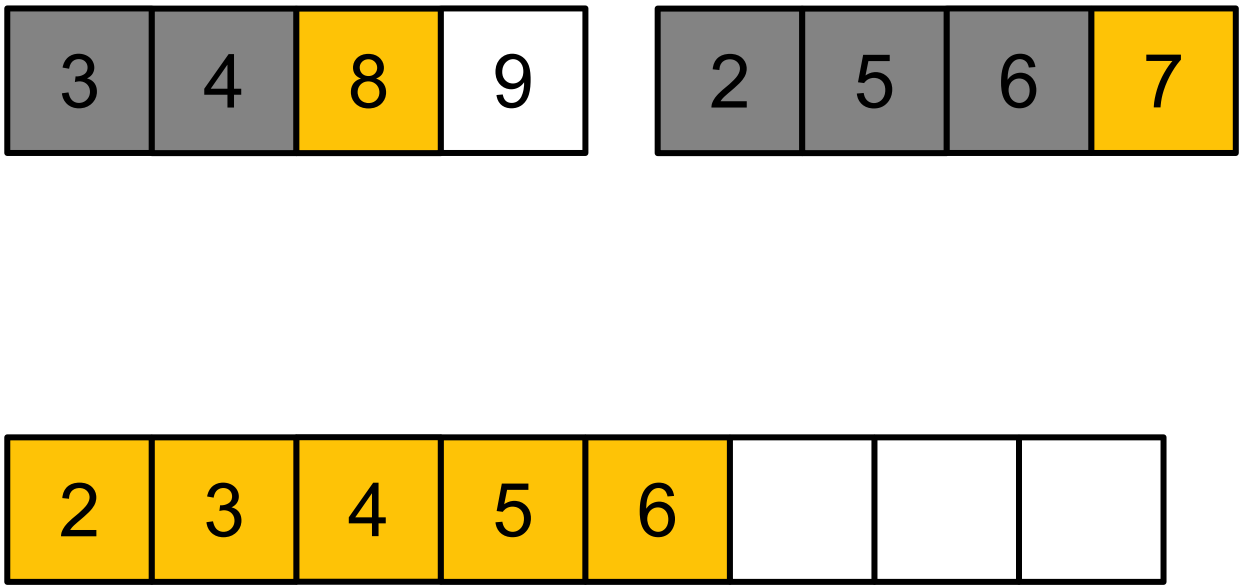
Merge Sort



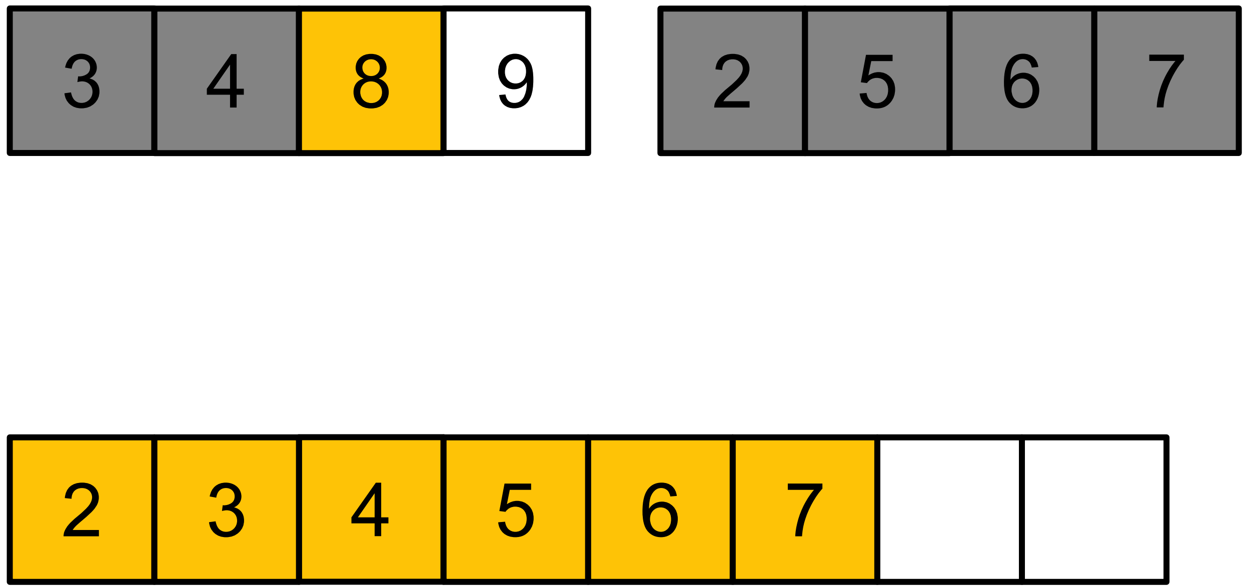
Merge Sort



Merge Sort



Merge Sort



Merge Sort

3	4	8	9		2	5	6	7

2	3	4	5	6	7	8	9

Merge Sort

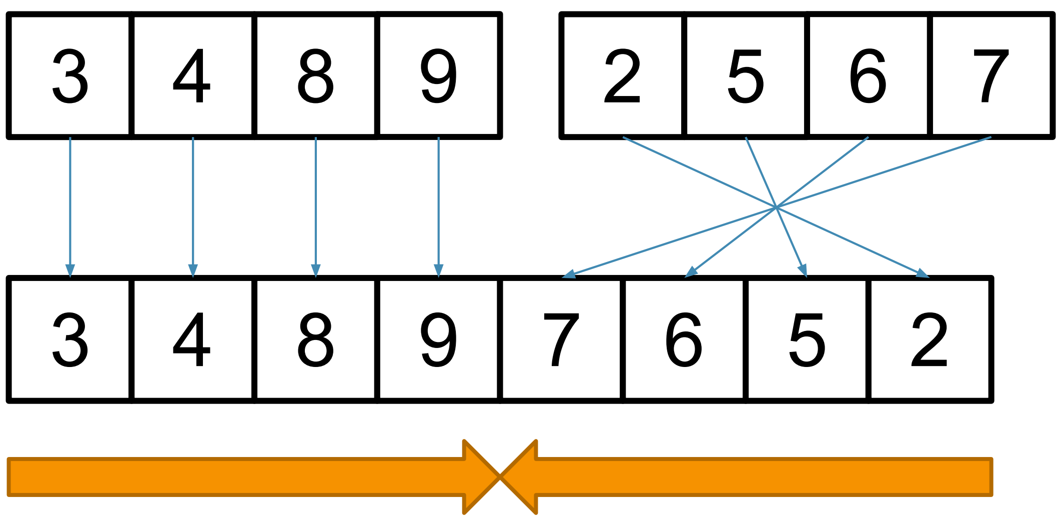
2	3	4	5	6	7	8	9

Merge Sort Implementation

```
static <E extends Comparable<? super E>>
void mergesort(E[] A, E[] temp, int l, int r) {
    int mid = (l + r) / 2; // Select midpoint
    if (l == r) return; // List has one element
    mergesort(A, temp, l, mid); // Mergesort first half
    mergesort(A, temp, mid + 1, r); // Mergesort second half
    for (int i = l; i <= r; i++) // Copy subarray to temp
        temp[i] = A[i];
    // Do the merge operation back to A
    int i1 = l; int i2 = mid + 1;
    for (int curr = l; curr <= r; curr++) {
        if (i1 == mid + 1) // Left sublist exhausted
            A[curr] = temp[i2++];
        else if (i2 > r) // Right sublist exhausted
            A[curr] = temp[i1++];
        else if (temp[i1].compareTo(temp[i2]) < 0) // Get smaller
            A[curr] = temp[i1++];
        else A[curr] = temp[i2++];
    }
}
```

Optimized Merge Sort

- 두 가지 최적화 방법
  - 입력 배열 크기가 작을 때 insertion sort 사용하기.
    - Insertion sort: 재귀 없음. 배열 값 복사 안해도 됨.
  - 리스트의 끝을 확인 안하기
    - 두 번째 리스트를 반대 순서로 temp 배열에 넣기



Optimized Merge Sort

```
static <E extends Comparable<? super E>>
void mergesort(E[] A, E[] temp, int l, int r) {
   int i, j, k, mid = (l + r) / 2; // Select the midpoint
   if (l == r) return; // List has one element
   if ((mid - l) >= THRESHOLD) mergesort(A, temp, l, mid);
   else inssort(A, l, mid - l + 1);
   if ((r - mid) > THRESHOLD) mergesort(A, temp, mid + 1, r);
   else inssort(A, mid + 1, r - mid);
   // Do the merge operation. First, copy 2 halves to temp.
   for (i = l; i <= mid; i++) temp[i] = A[i];
   for (j = 1; j <= r - mid; j++) temp[r - j + 1] = A[j + mid];
   // Merge sublists back to array
   for (i = l, j = r, k = l; k <= r; k++)
      if (temp[i].compareTo(temp[j]) < 0) A[k] = temp[i++];
      else A[k] = temp[j--];
}
```

Merge Sort Cost

- Merge sort cost:
  - $\Theta(n \log n)$ in the best, average and worst cases
- Merge sort is also good for sorting linked lists.
  - Because merging requires only sequential access
- Merge sort requires twice the space.

Overview

- Merge Sort
- **Quicksort**

Quicksort

- 또 다른 분할 정복 알고리즘

- 입력 배열이 주어졌을 때
  - 배열의 값 하나를 선택해서 'pivot'으로 지정
  - pivot보다 작은 값이 pivot보다 큰 값 보다 항상 왼쪽에 오도록 배열을 재 정렬 = 'Partition 연산'
    - pivot과 같은 값은 어느 쪽에 있어도 상관 없음

```
static <E extends Comparable<? super E>>
int findpivot(E[] A, int i, int j){
    return (i+j)/2;
}
```

Quicksort

10	1	9	3	4	7	2	5	8	6

```java
static <E extends Comparable<? super E>>
void qsort(E[] A, int i, int j) {
    int pivotindex = findpivot(A, i, j);
    swap(A, pivotindex, j);
    int k = partition(A, i - 1, j, A[j]);
    swap(A, k, j);
    if (i < k - 1) qsort(A, i, k - 1);
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

10	1	9	3	4	7	2	5	8	6
i				pivot					j

```java
static <E extends Comparable<? super E>>
void qsort(E[] A, int i, int j) {
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    swap(A, k, j);
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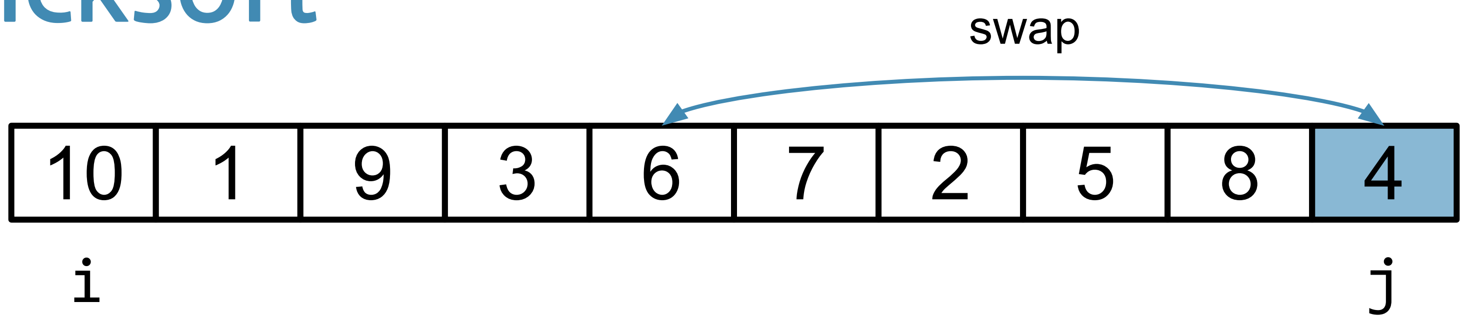
Quicksort

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    swap(A, k, j);
    if (i < k - 1) qsort(A, i, k - 1);
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

10	1	9	3	6	7	2	5	8	4
i									j

```java
static <E extends Comparable<? super E>>
void qsort(E[] A, int i, int j) {
    int pivotindex = findpivot(A, i, j);
    swap(A, pivotindex, j);
    int k = partition(A, i - 1, j, A[j]);
    swap(A, k, j);
    if (i < k - 1) qsort(A, i, k - 1);
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

2	1	3	9	6	7	10	5	8	4
i			k						j

```
static <E extends Comparable<? super E>>
void qsort(E[] A, int i, int j) {
    int pivotindex = findpivot(A, i, j);
    swap(A, pivotindex, j);
    int k = partition(A, i - 1, j, A[j]);
    swap(A, k, j);
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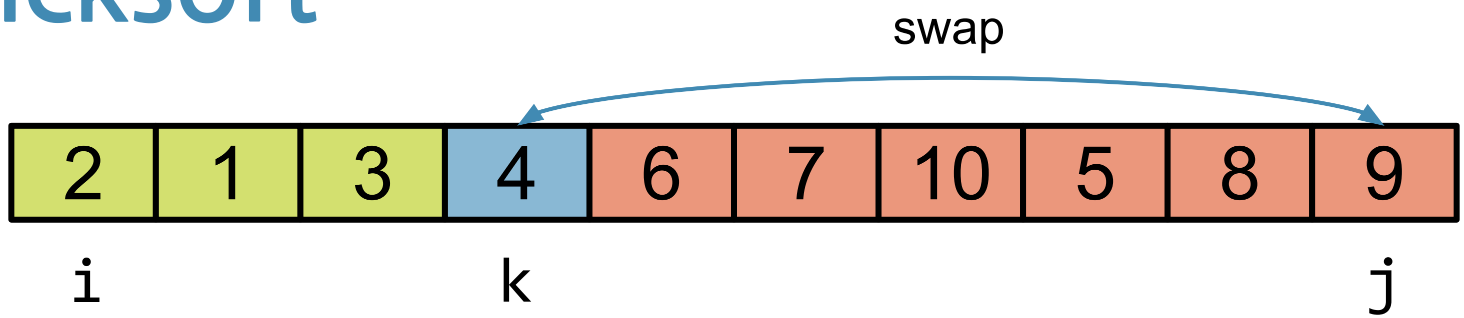
Quicksort

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void qsort(E[] A, int i, int j) {
    int pivotindex = findpivot(A, i, j);
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    swap(A, k, j);
    if (i < k - 1) qsort(A, i, k - 1);
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

2	1	3	4	6	7	10	5	8	9
i			k						j

```
static <E extends Comparable<? super E>>
void qsort(E[] A, int i, int j) {
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    int k = partition(A, i - 1, j, A[j]);
    swap(A, k, j);
    if (i < k - 1) qsort(A, i, k - 1);
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

2	1	3	4	6	7	10	5	8	9
i		j							

```
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void qsort(E[] A, int i, int j) {
    int pivotindex = findpivot(A, i, j);
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    swap(A, k, j);
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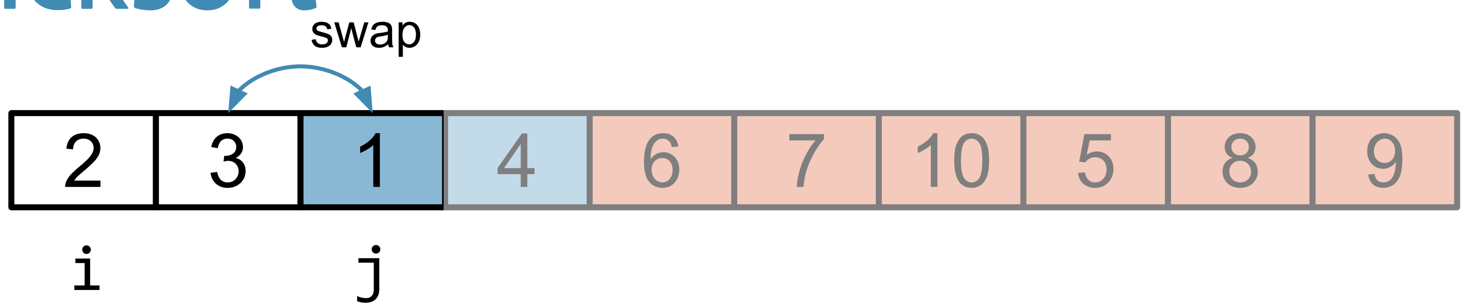
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

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    int pivotindex = findpivot(A, i, j);
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    int k = partition(A, i - 1, j, A[j]);
    swap(A, k, j);
    if (i < k - 1) qsort(A, i, k - 1);
    if (k + 1 < j) qsort(A, k + 1, j);
}
```

Quicksort

2	3	1	4	6	7	10	5	8	9
i,k		j							

```
static <E extends Comparable<? super E>>
void qsort(E[] A, int i, int j) {
    int pivotindex = findpivot(A, i, j);
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    int k = partition(A, i - 1, j, A[j]);
    swap(A, k, j);
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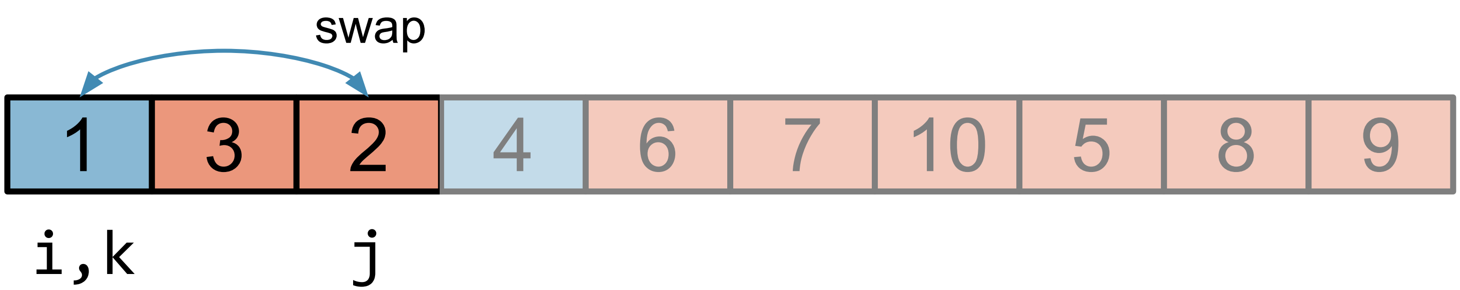
}
```

Quicksort

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}
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Quicksort

1	3	2	4	6	7	10	5	8	9
i,k		j							

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void qsort(E[] A, int i, int j) {
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Quicksort

1	3	2	4	6	7	10	5	8	9
	i	j							

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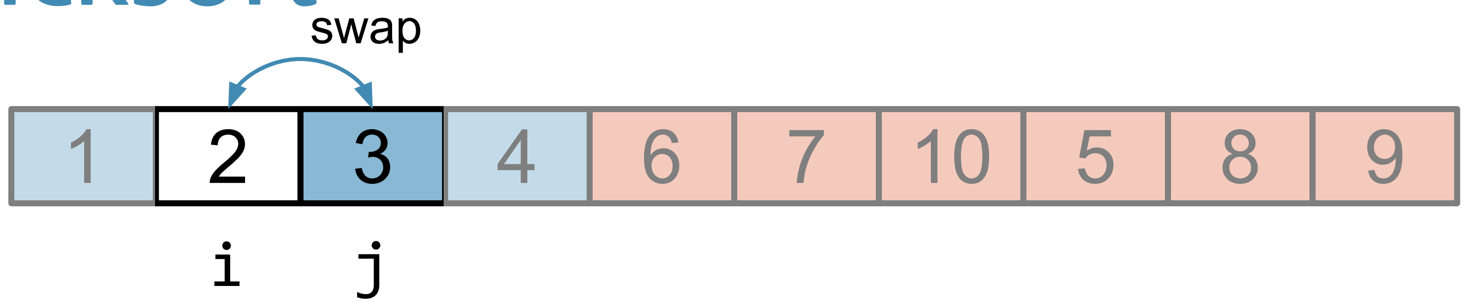
Quicksort

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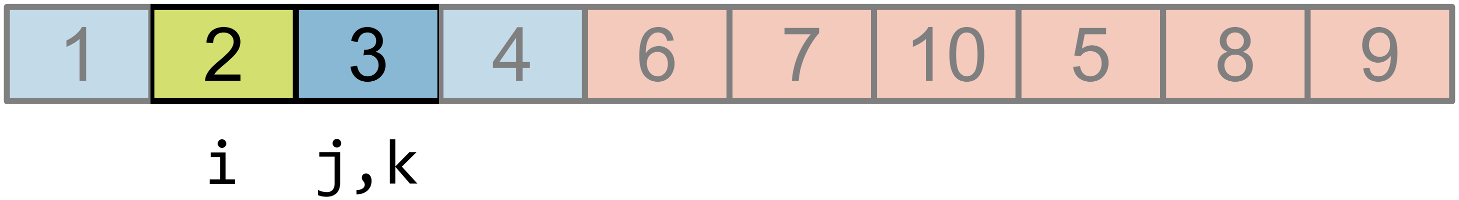
Quicksort

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Quicksort

1	2	3	4	6	7	10	5	8	9
i			k						j

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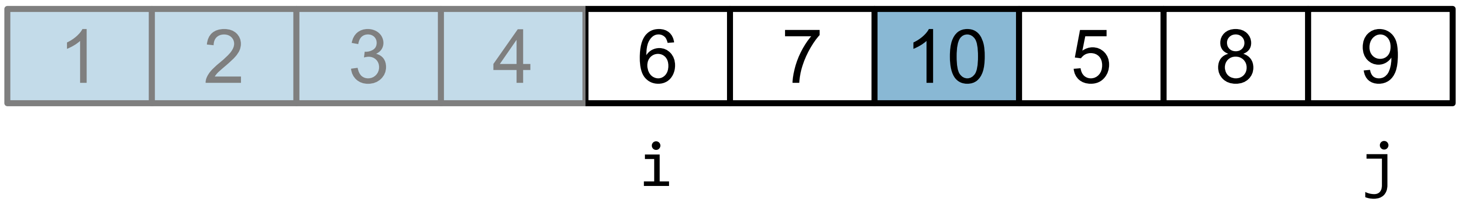
Quicksort

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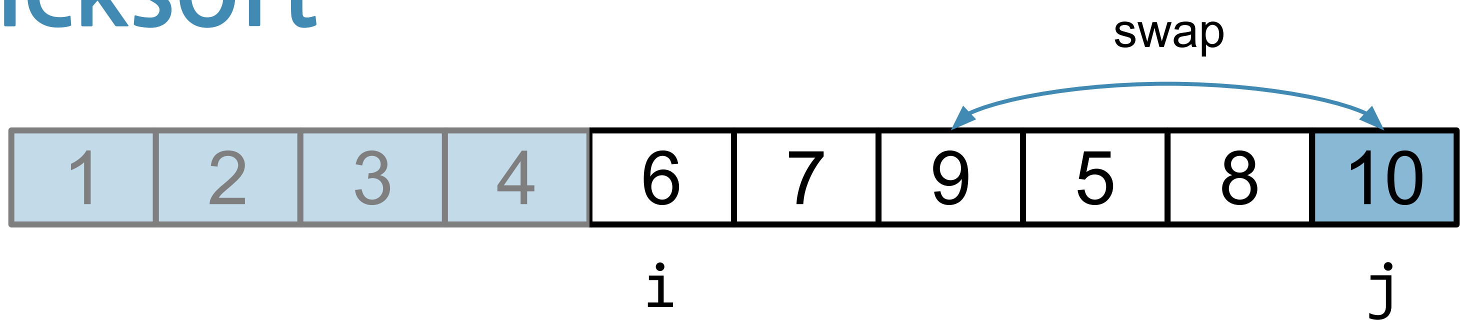
Quicksort

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Quicksort

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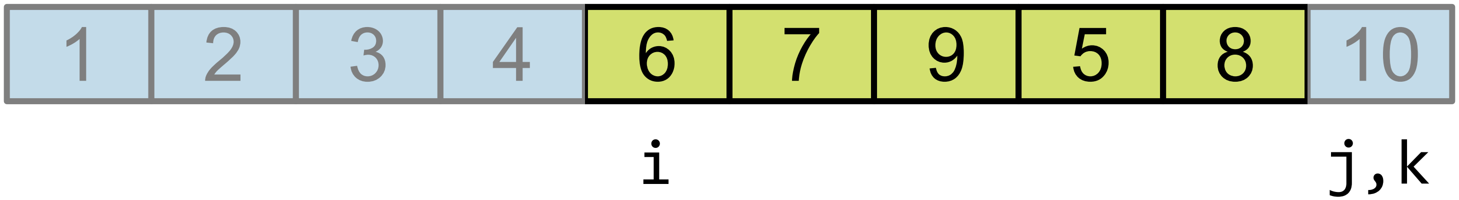
Quicksort

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Quicksort

1	2	3	4	6	7	9	5	8	10
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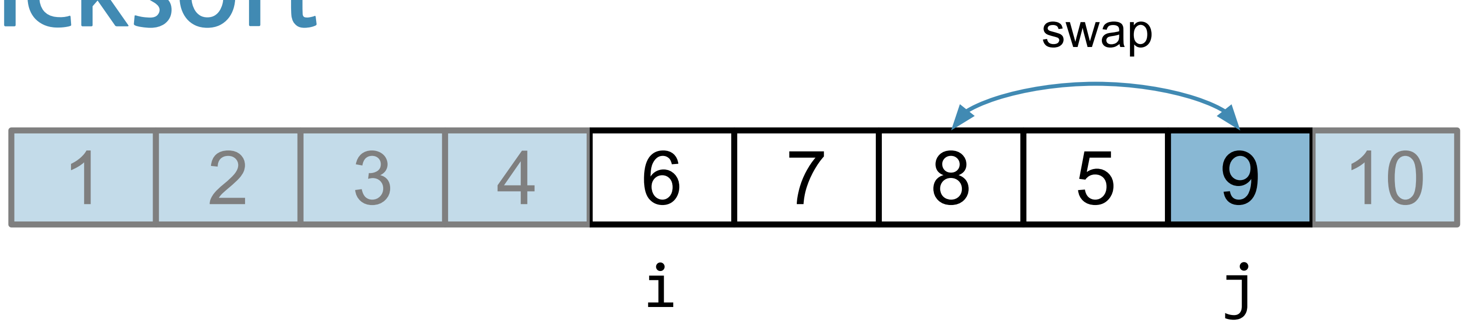
Quicksort

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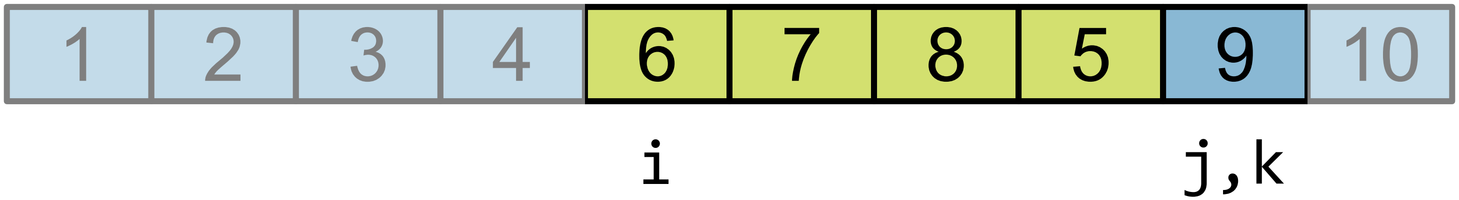
Quicksort

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Quicksort

1	2	3	4	6	7	8	5	9	10
				i				j,k	

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Quicksort

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				i			j		

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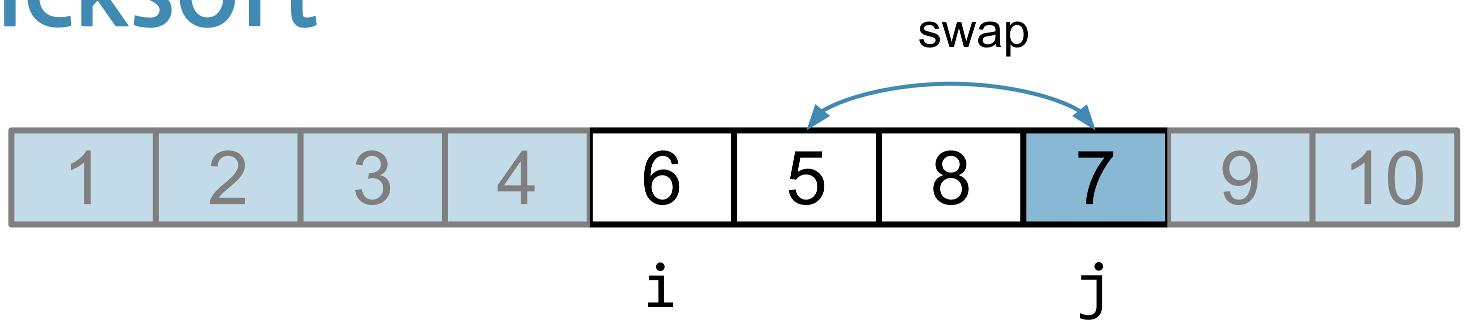
Quicksort

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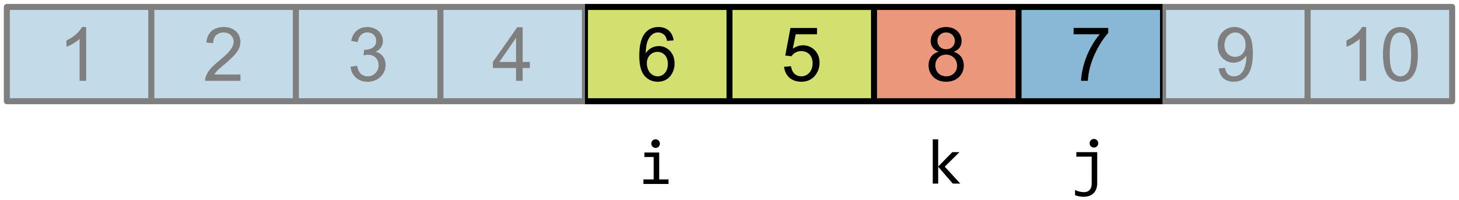
Quicksort

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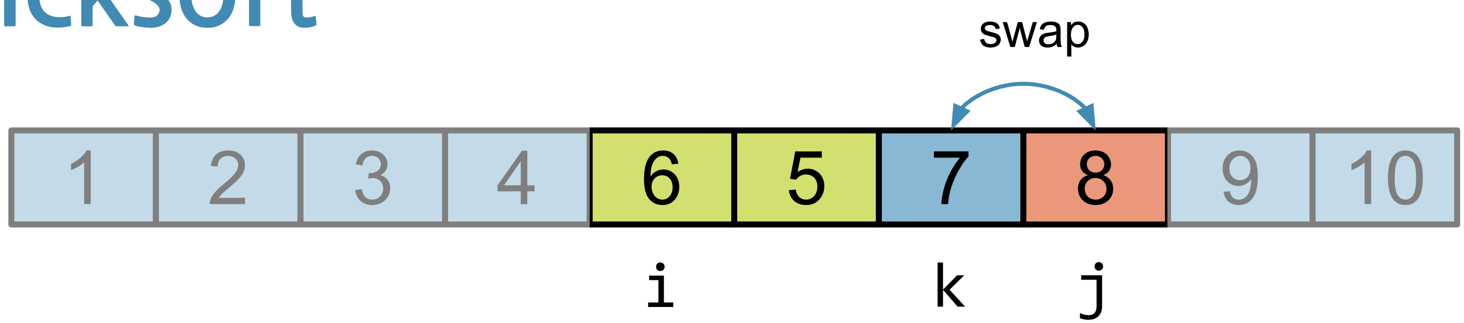
Quicksort

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Quicksort

1	2	3	4	6	5	7	8	9	10
				i		k	j		

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Quicksort

1	2	3	4	6	5	7	8	9	10
				i	j				

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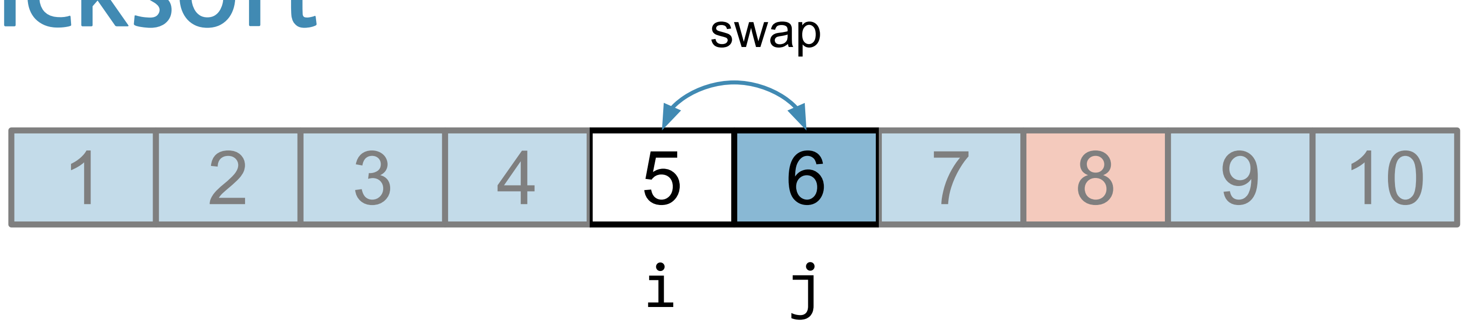
Quicksort

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Quicksort

1	2	3	4	5	6	7	8	9	10
				i	j,k				

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Quicksort

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Quicksort Partition

10	1	9	3	6	7	2	5	8	4

l r

```
static <E extends Comparable<? super E>>
int partition(E[] A, int l, int r, E pivot) {
    do {
        while (A[++l].compareTo(pivot) < 0);
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        swap(A, l, r);
    } while (l < r);
    swap(A, l, r);
    return l;
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```

Quicksort Partition

10	1	9	3	6	7	2	5	8	4
l									r

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Quicksort Partition

10	1	9	3	6	7	2	5	8	4
l						r			

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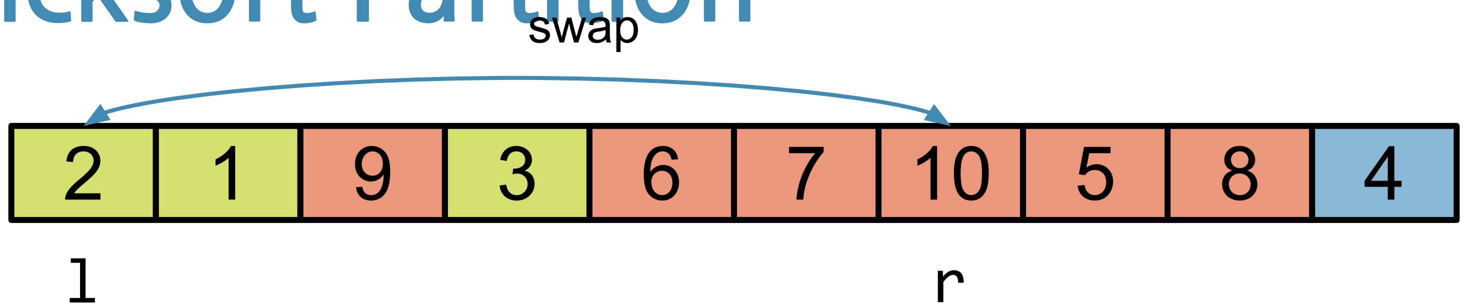
Quicksort Partition

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Quicksort Partition

2	1	9	3	6	7	10	5	8	4
		l				r			

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Quicksort Partition

2	1	9	3	6	7	10	5	8	4
		l	r						

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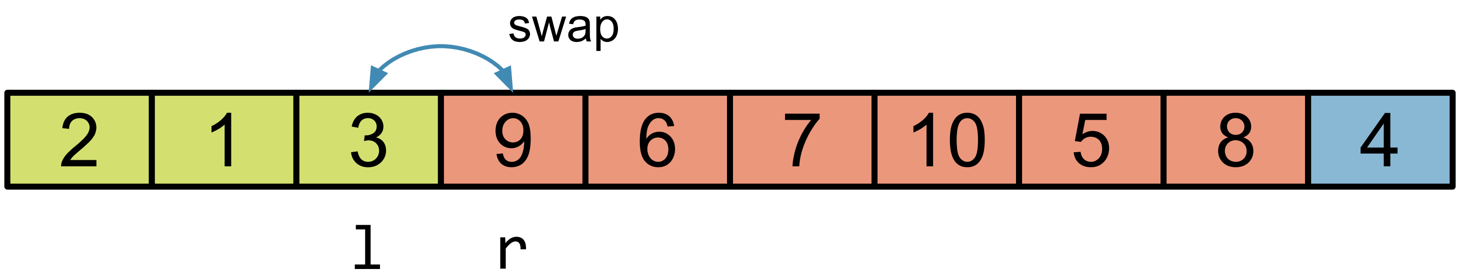


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Quicksort Partition

2	1	3	9	6	7	10	5	8	4
			l,r						

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Quicksort Partition

2	1	3	9	6	7	10	5	8	4
		r	l						

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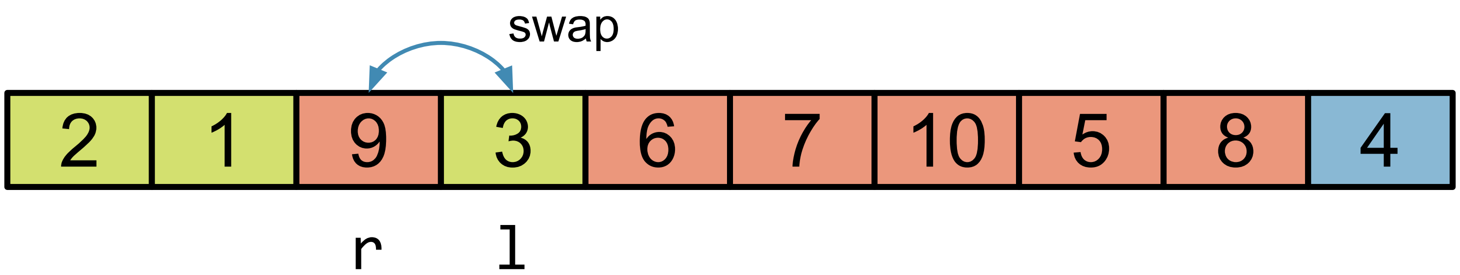
Quicksort Partition

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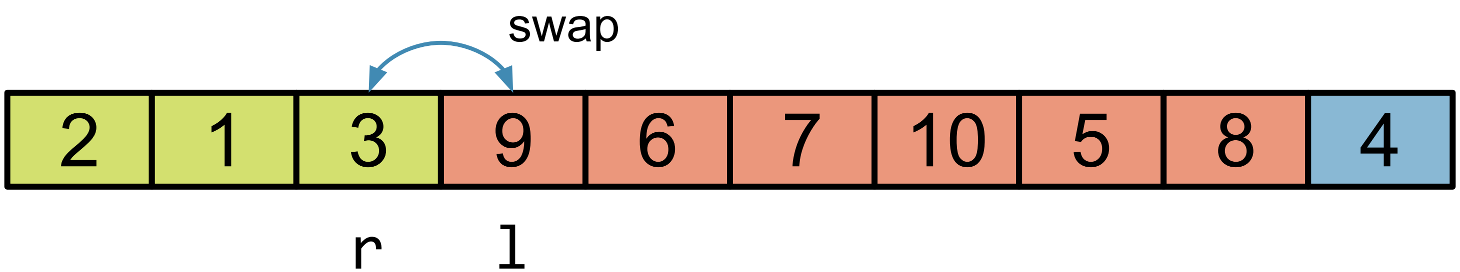
Quicksort Partition

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Quicksort Partition

- Return value: pivot보다 큰 값들의 첫 번째 index
- 비용: $\Theta(n)$

Cost of Quicksort

- Best case: 항상 절반씩 분할되는 경우
- Worst case: pivot보다 모든 값이 큰 경우 (혹은 작은 경우)
- Average case:

$$T(n) = cn + \frac{1}{n}\sum_{k=0}^{n-1}(T(k) + T(n-1-k))$$

$$T(0) = T(1) = c$$

  - 위의 재귀 관계를 풀어보면? (증명?)

$$T(n) = \Theta(n \log n)$$

Optimizations for Quicksort

- Optimizations for Quicksort:
  - Pivot을 더 잘 선택하기 (how?)
  - sublist의 크기가 작을 때 다른 알고리즘 사용하기
    - Quicksort는 n이 작을 때 느린 편 (why?)
    - n이 작은 sublist에서는 insertion sort나 selection sort 사용하기
  - 재귀 호출 없애기: e.g., use stack

What you need to know

- Merge sort
  - Main idea: ‘divide and conquer’
  - Cost analysis
  - Advantage of optimized merge sort
- Quicksort
  - Main idea: ‘divide and conquer’
  - Cost analysis

Questions?