Fakerate and Uncertainty Calculation

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Define the ϵ_{f1} and ϵ_{f2} as the fakerates in sample 1 and sample 2 in γ+jets, f_{b1} and f_{b2} are the B jet fractions in these two samples, ϵ_{fb} and ϵ_{fl} are the fakerates we need to calculate. We have

$$\begin{pmatrix} \epsilon_{f1} \\ \epsilon_{f2} \end{pmatrix} = \begin{pmatrix} f_{b1} & (1-f_{b1}) \\ f_{b2} & (1-f_{b2}) \end{pmatrix} \cdot \begin{pmatrix} \epsilon_{fb} \\ \epsilon_{fl} \end{pmatrix} \tag{1}$$

From this, we get

$$\begin{pmatrix} \epsilon_{fb} \\ \epsilon_{fl} \end{pmatrix} = \begin{pmatrix} \frac{1-f_{b2}}{f_{b1}-f_{b2}} & \frac{-(1-f_{b1})}{f_{b1}-f_{b2}} \\ \frac{-f_{b2}}{f_{b1}-f_{b2}} & \frac{f_{b1}}{f_{b1}-f_{b2}} \end{pmatrix} \cdot \begin{pmatrix} \epsilon_{f1} \\ \epsilon_{f2} \end{pmatrix} \tag{2}$$

Assuming there is no correlation between ϵ_{f1}, ϵ_{f2}, f_{b1} and f_{b2}, the uncertainties of the calculated ϵ_{fb} and ϵ_{fl} will be

$$\delta^2_{\epsilon_{fb}} = \frac{1}{(f_{b1}-f_{b2})^2}\left\{(1-f_{b2})^2\left[\delta^2_{\epsilon_1} + \frac{(\epsilon_{f1}-\epsilon_{f2})^2}{(f_{b1}-f_{b2})^2}\cdot\delta^2_{f_{b1}}\right] + (1-f_{b1})^2\left[\delta^2_{\epsilon_2} + \frac{(\epsilon_{f1}-\epsilon_{f2})^2}{(f_{b1}-f_{b2})^2}\cdot\delta^2_{f_{b2}}\right]\right\}$$

$$\delta^2_{\epsilon_{fl}} = \frac{1}{(f_{b1}-f_{b2})^2}\left\{f_{b2}^2\left[\delta^2_{\epsilon_{f1}} + \frac{(\epsilon_{f1}-\epsilon_{f2})^2}{(f_{b1}-f_{b2})^2}\cdot\delta^2_{f_{b1}}\right] + f_{b1}^2\left[\delta^2_{\epsilon_{f2}} + \frac{(\epsilon_{f1}-\epsilon_{f2})^2}{(f_{b1}-f_{b2})^2}\delta^2_{f_{b2}}\right]\right\}$$

When using the calculated ϵ_{fb} and ϵ_{fl} to predict background, what we are actually using is the fakerate averaged by the b jet fraction f_b in QCD(0tag or 1tag), i.e.

$$\begin{aligned} \epsilon_{f,final} &= f_b\epsilon_{fb} + (1-f_b)\epsilon_{fl} \\ &= \frac{(f_b-f_{b2})\epsilon_{f1} + (f_{b1}-f_b)\epsilon_{f2}}{f_{b1}-f_{b2}} \end{aligned} \tag{3}$$

Similarly, the uncertainty will be

$$\delta^2_{\epsilon_{f,final}} = \frac{A}{(f_{b1}-f_{b2})^2} \tag{4}$$

where A is

$$\begin{aligned} A =&(f_b-f_{b2})^2\delta^2_{\epsilon f1} + (f_{b1}-f_b)^2\delta^2_{\epsilon_{f2}} + (\epsilon_{f1}-\epsilon_{f2})^2\delta^2_{f_b} \\ &+ \frac{(f_b-f_{b2})^2(\epsilon_{f2}-\epsilon_{f1})^2}{(f_{b1}-f_{b2})^2} + \frac{(f_b-f_{b1})^2(\epsilon_{f1}-\epsilon_{f2})^2}{(f_{b1}-f_{b2})^2} \end{aligned} \tag{5}$$