# Robust Passive Location in Zero-Calibrated Environment Using Smoothed Ordinal Constraints

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Abstract—Passive locating by capturing radio signal strength (RSS) from mobile phone's WiFi probing messages in zerocalibrated environments is a challenging problem, because of 1) the lack of accurate RSS-distance model; 2) measurement noise, and 3) the dynamic environment factors affecting the measurements. This paper investigates the noise features of RSS signals by practical experiments, and investigates whether the feasible region bounded by trustworthy ordinal relationships (TOR) among RSS measurements can help to improve the location accuracy. A constrained non-linear optimization model is proposed to apply the TOR constraints for target localization. A series of methods for smoothed feasible region mergence over time are investigated, including 1) Ordinal constraint fusion in one time-slot (OS); 2) Ordinal constraint intersection of multiple time-slots (IM); 3) Ordinal constraint fusion of multiple time-slots using expansion and kernel (EK). The remarkable accuracy and robustness improvements benefited from using TOR constraints were demonstrated by both simulations and practical experiments compared with state-of-the-art passive locating methods.

#### I. Introduction

Passive indoor position estimation problem which predicts user's indoor position by capturing WiFi-probing messages from mobile phones attracts great attentions. When a mobile phone broadcasts the WiFi probing message automatically for WiFi scanning, the RSS signal of the probing message can be utilized to locate the user in intrusive and uncooperative manner. It has great value in many applications, such as customer behavior and interest analysis [1] [2] [3] [4].

However, noncooperation of users makes the passive locating more difficult than the cooperative cases, mainly because of the highly dynamic application scenarios. During passive locating, the mobile phone maybe in pocket, in bag, or in hand. The inner state of the phone maybe working, sleeping or power saving. Users may take phones of different brands, which have different probing patterns. The locating process is also impacted by the environment changes, i.e., temperature and humidity, which can hardly be awared by the locating system.

Approaches for dealing with environment dynamics in passive location have been investigated in the literature. One of the major approaches is based on learning of radio-map [5]. However, the learning process of radio-map is laborious and less practical in passive location due to large application areas and dynamic application scenarios. The dynamics can hardly be covered even if very high training cost is paid. Therefore,

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passive location in zero-calibrated environment is extremely important for the sense of practice.

In studies of zero-calibrated passive locating, one approach is to learn a suitable RSS-distance model in the specific environment [6] [7], which was addressed by either offline parameter learning or online parameter adaptation by regression models. The RSS-distance model maps RSS to distance between the probe device and the mobile phone. Probe devices refer to devices capturing RSS data of nearby mobile phones. But the learned RSS-distance model is generally simple, characterized by several parameters, which cannot well model the signal noise and environment dynamics. Further, in passive locating, the number of RSS signals captured for locating a mobile phone is generally very limited at a time instance. As a result, the noisy RSS-distance model and the limited number of measurements, generally cause passive location suffer very coarse location accuracy.

An optimization model is generally adopted in localization algorithms to find a position that best matches all observations. Under the condition of small and rough observation set, the optimization is unreliable and less noise-tolerant for passive locating. The noise impacts are hard to exclude, because 1) majority voting [8] may fail for the limited observation number; 2) residue checking [9] may not work well because of the lack of accurate RSS-distance model.

Regarding these practical limitations, this paper exploits the ordinal relationships among the RSS data to improve the location robustness and to exclude the noise impacts. Orders generally depend less on the accurate values, which tends to be robust if noise is bounded and positive correlated. We conduct experiments and they show positive correlation features of RSS because RSS signals are affected by the same environment or the same state changes of phones.

Therefore, system and method are proposed to extract a set of *trustworthy ordinal relationships (TOR)* in each time-slot. An ordinal relationship between two RSS values is trustworthy if their difference of RSS values is larger than a predefined threshold. Each TOR narrows down the target location to a half plane, and multiple TORs define a convex polygon, representing the *feasible region* of the target.

The key to effectively utilize the ordinal constraints is to find a way, in which the feasible region can be smoothly expanded according to the target movement and narrowed down smoothly by the newly obtained TORs. The desired case is to maintain a feasible region around the target, which only exclude the unreasonable noise. Therefore, a series of methods is investigated based on the successively collected TORs. 1) Ordinal constraint fusion in *one time-slot* (OS); 2) Ordinal constraint intersection of multiple time-slots (IM); 3) Ordinal constraint fusion of multiple time-slots with expansion and kernel (EK)A key advantage of ordinal constraint fusion is that the expansion and reduction of feasible region are just simple modifications of the ordinal constraints. We show the third method enables a smoothed feasible region around the target when the target moves slowly. Then, by applying the feasible region as constraints in the location estimation, a constrained nonlinear optimization model is proposed to estimate the target's location.

Analytical results are presented which verifies the bounded and positive correlated features of RSS noise under different dynamic conditions. Evaluations were conducted using both simulations and experiments, which showed much improvement of locating accuracy than the state-of-the-art methods in challenging observation conditions.

#### II. PROBLEM MODEL AND PRELIMINARIES

#### A. Model and Notations

Suppose the area-of-interest in which a user is moving is a bounded area M. A mobile phone taken by the user broadcasts probing signal periodically. In the area, there are N probe devices, which refer to the devices which capture RSS signal of nearby mobile phones. When the probing signal from the phone is captured by a probe device, the device stores the RSS, MAC address of the phone, and a time stamp. See Figure 1 for illustration of the system and Table I for notations. Denote the RSS captured by probe device i at time t by  $S_i^t$  and the position of probe device i by  $X_i$ . The position of user at time t is denoted by  $Y^t$ . The ultimate task is to estimate  $Y^t$  by  $X_i$  and  $S_i^{\tau}$  by all measurements collected from 1 to t.

$$\widehat{Y}^t = F(X_i, S_i^{\tau} : 1 \le i \le N, 1 \le \tau \le t) \tag{1}$$

where  $\widehat{Y}^t$  is the estimation of  $Y^t$ .

The settings of passive locating problem:

- No offline radio-map training, since training is labor intensive.
- ullet Number of probe devices is limited, i.e., N is small, which is a general case in practice.
- High probability of RSS signal loss (a large proportion of S<sub>i</sub><sup>T</sup> cannot be detected) and loud RSS noise.

TABLE I
LIST OF NOTATIONS AND EXPLANATION

Notations	Explanation					
M	the bounded area where the user is moving					
N	the number of probe devices					
$X_i^t$	the RSS captured by probe device $i$ at time $t$					
$X_i$	the position of probe device $i$					
$Y_i$	the position of user at time $t$					
$R_t$	the feasible region at time $t$					
$R'_t$	the expansion of $R_t$					
$R_t''$	the kernel of $R_t$					

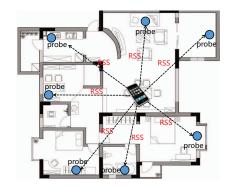


Fig. 1. System of passive indoor location

Without RSS radio-map training, traditional ways of RSS-based locating is to translate RSS into distance The measured RSS value is generally translated into distance for location calculation, which is mainly based on the free-space signal propagation model [10]:

$$RSS = RSS_0 - 10 * n_0 * \log_{10} d + e$$
 (2)

where  $RSS_0$  and  $n_0$  are two parameters related to transmission media [10]. e is the measurement noise.

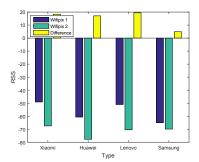
## B. Positive correlation of probing RSS

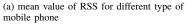
We conducted practical experiments in an office environment to investigate the RSS noise patterns. 3 dynamic factors impacting RSS data are considered, i.e., brands, inner states and positions of phones. 20-minute RSS data is captured from the static phone for each phone condition, e.g., <xiaomi, scan, inbag>. One detailed experiment setting is depicted as follow. In order to investigate the effect of phone brands on RSS noise patterns, we analyze RSS signal from <xiaomi, huawei, samsung, lenove> while keeping other conditions unchanged.

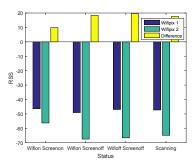
The mean value of RSS for two probe devices and their difference are plotted in Figure 2. Wifipixes refer to probe devices in the figure. The correlated coefficients under all phone conditions are in Table II, where the value in row i and column j is the correlated coefficient between RSS of probe device i and j. It can be concluded RSS difference between 2 probe devices is bounded and measurement noise e is highly positive correlated. This is reasonable since when phone condition changes, RSS of different probe devices tends to have bounded shifting in the same direction. A way to make use of the positive correlation of RSS signals is to consider robust ordinal relations in dynamic environment.

#### C. Non-satisfactory of traditional algorithms

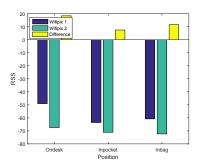
1) Weighted k-nearest neighbor: Weighted k-nearest neighbor (KNN) method [11] is an efficient way to locate user's position by finding the weighted centroid of k probe devices. Although it is less affected by correlated noise of RSS, the number and the topology of probe devices play an important role in impacting the location result of weighted KNN.







(b) mean value of RSS for different inner status of mobile phone



(c) mean value of RSS for different placement of mobile phone

Fig. 2. correlation of RSS

TABLE II
CORRELATION COEFFICIENT AMONG RSS OF 6 WIFIPIXES

wifipix	1	2	3	4	5	6
1	1.0000	0.7226	0.6448	0.8134	0.6201	0.5303
2	0.7226	1.0000	0.7741	0.7276	0.6250	0.6807
3	0.6448	0.7741	1.0000	0.6586	0.4972	0.7384
4	0.8134	0.7276	0.6586	1.0000	0.6179	0.5151
5	0.6201	0.6250	0.4972	0.6179	1.0000	0.6576
6	0.5303	0.6807	0.7384	0.5151	0.6576	1.0000

- 2) Least square method: Least square method [12] is an optimization method for passive locating. A set of linear equations about the location of user can be derived from observations and RSS-distance model to calculate an estimation position. Its locating accuracy is greatly impacted by empirical function from RSS to distance and observation topology.
- 3) Max likelihood method: By assuming observation noise is zero-mean, the location of the user can be estimated by maximum likelihood function [13]. Correlation of noise is generally not considered in the function.

So above methods in passive location generally encounter problems due to high and correlated RSS noise. Methods to deal with the RSS noise for robust passive location are critically required.

#### III. ORDER-BASED PASSIVE LOCATING

In this section, the ordinal enhanced passive target localization method is introduced.

## A. System Overview

System structure of ordinal constrained passive localization is shown in Figure 3, which mainly consists of:

- 1) Collaborative filter (CF) to deal with RSS data loss and to smooth the measurement noise
- 2) *Trustworthy Ordinal Relationship (TOR)* extraction to bound the feasible region of the user, and feasible region smoothing method as the target is moving
- A constrained nonlinear optimization model to calculate the location of user by the smoothed ordinal constraints

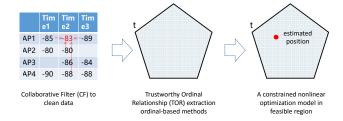


Fig. 3. System structure of ordinal passive localization

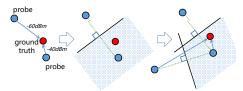
#### B. Collaborative Filter based Data Preprocessing

In order to guarantee the reliability of ordinal relationships, we propose *collaborative filter* (CF) to deal with data loss and noise. For locating one target, the RSS measurements from 1 to t, captured at the N probe devices form a matrix of N rows and t columns. Usually many entries of the matrix are missing and the existing entries are noisy. The goal of data preprocessing is to fill the missing entries and to smooth the existing entries, which consists of two steps. 1) Predict unknown RSS data based on known data; 2) Smooth the noisy RSS data by polynomial fitting over time.

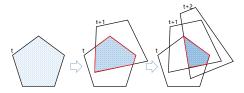
Collaborative filter (CF) [14] was exploited to make predictions for unknown RSS data, which considers both data correlation over time and across probe devices. After filling empty values, RSS data is smoothed by polynomial fitting over time to reduce the impacts of RSS noise [15]. More details can be referred to [16] [17].

## C. Exploit the Robust Ordinal Relationships

Due to bounded and positive correlated characteristics of RSS noise, trustworthy ordinal relationships (TOR) constrained by predefined threshold are reliable most of the time. Detailedly speaking, for two RSS measurements  $S_i^t$  and  $S_j^t$ , without loss of generality, if  $S_i^t - S_j^t \geq T_h$ , we call there is a TOR between  $S_i^t$  and  $S_j^t$ , in which  $T_h$  is a noise threshold. TOR is generally robust to dynamic noise because: 1) The RSS noise is bounded and positive correlated among all probe devices; 2) Due to inaccurate empirical function from RSS to distance, the ordinal relation is more reliable than the values of RSS.



(a) Feasible region generation by TOR in one time-slot



(b) Mergence of feasible region in successive time-slots

Fig. 4. Overview of original constrained passive location

Practically, at time t, an ordinal relation can give a reliable constraint to the location of user. Suppose at time t,  $S_i^t \geq S_j^t$ , this indicates  $(X_i - Y^t)^2 \leq (X_j - Y^t)^2$ . Then

$$2(X_j - X_i)Y^t \le X_j^2 - X_i^2$$

This is a linear constraint of  $Y^t$  representing a half space cut on the feasible region of  $Y^t$ . This kind of cut is named as ordinal cut (OC). Multiple OCs will characterize a polygon, which is the trustworthy feasible region characterized by TORs among the probe devices.

#### D. OC mergence and Constraint Optimization

Therefore, to improve location robustness, the ordinal constraints are exploited. Figure 4 illustrates the basic idea of utilizing the ordinal constraints to enhance the robustness and accuracy of passive locating.

Figure 4a shows how TORs characterize a feasible region in one time-slot (denoted by  $R_t$ ). Each TOR characterizes an OC, and multiple OCs define the feasible region. Figure 4b illustrates how the feasible regions obtained in successive time-slots can be merged to a smaller feasible region to improve the location accuracy. The feasible region is characterized by a set of inequalities.

The estimation of  $Y^t$  based on  $R_t$  is treated by solving a constrained nonlinear programming problem to minimize the sum of square error, while satisfying the constraints derived by ordinal constraints, i.e.,  $R_t$ .

$$\operatorname{Minimize}_{x} \sum_{i,j} \left( \frac{F(\|Y^{t} - X_{i}\|_{2}) - S_{i}^{t}}{S_{i}^{t}} \right)^{2}$$

Subject to 
$$x \in R_t \cap \mathbf{M}$$

where M represents the bounded area in which a user is moving and F represents the empirical formula to map distance to RSS. This programming can be handled by many standard optimization tools such as interior point method [18] [19] or active set [20] [21].

Some modifications can be made on such optimization. For example  $RSS_0$  and  $n_0$  in empirical function can be taken as optimization variables and weights can be added on the sum of square differences. In addition we can remove the constraints by adding a penalty to objective as follows

$$\operatorname{Minimize}_{x} \sum_{i,j} \left( \frac{F(\|Y^{t} - X_{i}\|_{2}) - S_{i}^{t}}{S_{i}^{t}} \right)^{2} + C \cdot (A_{t}x - b_{t})$$

where C is a vector of penalty weights for each constraint in  $A_t x \leq b_t$ .

Generally, limited number of probe devices and confident  $T_h$  may cause the feasible region  $R_t$  be large, losing the capability for accuracy improvement. Whereas, merged OCs of the successive time-slots may cause  $R_t$  to be separated or be very small, providing wrong restrictions to the target location. Therefore, how to utilize the OCs smoothly to enhance the location accuracy is the key in ordinal constrained location estimation.

## IV. SMOOTHED ORDINAL CONSTRAINTS FOR PASSIVE LOCATION

How to utilize OCs smoothly to improve the location accuracy and robustness is discussed in this section.

#### A. Narrow Down the Feasible Region

TORs should be selected carefully to prevent the ordinal relationship be reversed due to random noise. The cut reversal means  $S_i^t > S_j^t$  but in fact  $(X_i - Y^t)^2 > (X_j - Y^t)^2$  for probe devices i and j, which means the target is predicted to the wrong side of the cut. So the selection of  $T_h$  is the key, but it falls into the dilemma of reliability and accuracy. When  $T_h$  is large, OCs are confident, but the region of  $R_t$  may be too large to effectively assist future optimization. When  $T_h$  is small, the region of  $R_t$  is small, but becomes not reliable, i.e., there is a high risk that the correct location is not in  $R_t$  and the optimization may give incorrect result. In this paper, confident and large  $R_t$  is preferred, because it can further be online narrowed down by OCs of successive time-slots.

## B. Smoothed Mergence of OCs in Successive Time-slots

- 1) Direct Polygon Clipping: Since each reliable region is a polygon, direct mergence of OCs of successive time-slots can be modeled as a polygon clipping problem. However, due to noise, when there are successive time-slots in which the feasible regions don't have overlapped region,  $R_t$  will be separated into multiple components, as shown in Figure 5. In this case  $R_t$  can not be expressed by linear constraints and we can hardly tell which component the target maybe locate in.
- 2) Expand  $R_t$  by Considering Target Movement: To smooth OC mergence in successive time-slots, the continuity of user's movement can be utilized. Let  $\{x \mid Ax \leq b\}$  denote the intersection of several OCs. Each row of A should be normalized to a unit vector and each element of b should be

#### Algorithm 1 Trustworthy Ordinal Cut

```
Require: S_i^t: RSS from probe device i at time t, (x_i, y_i): position of probe devicei

Ensure: R_t = \{x \mid A_t x \leq b_t\}: reliable region at time t by slack ordinal cut for (i,j) pair of probe device do if |S_i^t - S_j^t| > T_h then if S_i^t > S_j^t then add 2(x_j - x_i)x + 2(y_j - y_i)y \leq x_j^2 + y_j^2 - x_i^2 - y_i^2 into A_t x \leq b_t else add 2(x_j - x_i)x + 2(y_j - y_i)y \geq x_j^2 + y_j^2 - x_i^2 - y_i^2 into A_t x \leq b_t end if end for R_t = \{x \mid A_t x \leq b_t\}
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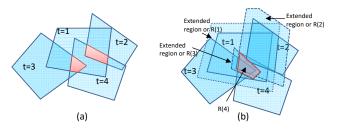


Fig. 5. (a) finding intersections by polygon clipping, which may lead to separated regions. (b) finding intersections by expanding the feasible region to consider the target movement

scaled accordingly for the simplicity of further exposition. We can generate  $R_t'$  from previous  $R_t$  easily by considering user movement. Suppose  $R_t = \{x \mid A_t x \leq b_t\}$  is a reliable region at time t. At time t+1, the user may move, so  $R_t$  should be enlarged to accommodate the user's movements. Take  $\Delta$  to be the upper bound of user's moving distance from time t and t+1, then the *expansion* of  $R_t$  to  $R_t'$  can be modeled as

$$R_t' = \{x \mid A_t x \le b_t + \Delta\} \tag{3}$$

which includes all possible user's position caused by possible movements. But it can be seen from Figure 5b, even if  $R_t$  is expanded by user movement, the noise may still cause the intersection area very small, which makes  $R_t$  over restricted, which is hard to merge with later constraints and have high risks of losing the target.

3) Smoothed Mergence by Considering a Kernel: Therefore, an intuition is that the constraint in time t+1 should not cut down a too large proportion of  $R_t'$ . Otherwise  $R_{t+1}$  will be small and increases the risk of losing the target.

If we think  $R_t$  is reliable, then  $R'_t = \{x \mid A_t x \leq b_t + \Delta\}$  is the expansion of  $R_t$  to consider user movement towards outside of  $R_t$ .  $R_{t+1}$  should always be inside  $R'_t$  as user can not move further than  $\Delta$ . And the *kernel* of  $R_t$ , which is defined as  $R''_t = \{x \mid A_t x \leq b_t - \Delta\}$  should always be inside  $R_{t+1}$  as  $R''_t$  is the overlapping area of all possible translations

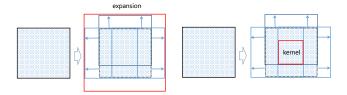


Fig. 6. expansion and kernel

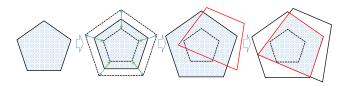


Fig. 7. constraint modification in smoothed mergence

of  $R_t$  by distance less than  $\Delta$ . Extending above observations, we obtain the following rule:

$$R_t'' \subseteq R_{t+1} \subseteq R_t' \tag{4}$$

In this way, we merge the feasible regions of successive time-slots, and can guarantee the feasible region is non-empty and continuously changing over time.

4) Smoothed Mergence by Constriants Modification: Practically, for one OC  $\alpha x \leq \beta$  of time t+1, the deepest possible cut into  $R'_t$  is  $\alpha x \leq \beta^*$  where  $\beta^* = \max\{\alpha x \mid A_t x \leq b_t - \Delta\}$ . Then  $\beta$  should be modified to  $\min\{\beta,\beta^*\}$  to avoid violating  $R_{t+1} \supseteq R''_t$ .  $\beta^*$  can be obtained efficiently by linear programming. The geometric interpretation of this modification is that every cut getting through the kernel should be moved to the border of the kernel with the same direction. In this way the generated region is always non-empty and continuously changing over time. See Figure 6 for demonstration of region expansion and region's kernel.

Suppose the feasible region at t+1 after modification to include the kernal is  $Ax \leq b'$ , then the reliable region  $R_{t+1}$  for t+1 will be:

$$R_{t+1} = \{x \mid R'_t \cap Ax \le b'\} = \{x \mid A_t x \le b_t + \Delta \cap Ax \le b'\}$$
(5)

In this way the OCs in time t+1 are incorporated into previous reliable region and new intersection gives a new reliable region in time t+1. Feasible regions of  $A_t x \leq b_t + \Delta$  and  $Ax \leq b'$  are two polygons geometrically, so  $R_{t+1}$  is the overlap of these two polygons and general polygon clipping algorithm [22] can be used to effectively find  $R_{t+1}$ . The algorithm is given in Algorithm 2. In Figure 7, the second figure shows the kernel and expansion of  $R_t$ ; the third figure shows the modification of OCs; and the last gives  $R_{t+1}$ .

#### C. Algorithm analysis

In this section, the feasibility of our algorithm is investigated. Some lower bounds are given by theoretical analyses of constraint smoothing algorithm.

**Proposition 1.** Suppose at time t, the minimum distance between user and region  $R_t$  satisfies  $d(x, R_t) \leq n\Delta$ , then

Algorithm 2 Region smoothing by expansion and kernel

```
Require: R_t = \{x \mid A_t x \leq b_t\}: reliable region at time t, Ax \leq b: ordinal cut in time t+1

Ensure: R_{t+1} = \{x \mid A_{t+1} x \leq b_{t+1}\}: reliable region at time t+1

for \alpha_t x \leq \beta_t constraint in A_t x \leq b_t do \beta_t' \to \beta_t + \|\alpha_t\|_2 \Delta \beta_t'' \to \beta_t - \|\alpha_t\|_2 \Delta

end for

All \alpha_t x \leq \beta_t' comprise A_t x \leq b_t'
All \alpha_t x \leq \beta_t'' comprise A_t x \leq b_t''
if A_t x \leq b_t'' is feasible then

for \alpha x \leq \beta constraint in Ax \leq b do \beta^* = \max\{\alpha x \mid A_t x \leq b_t''\}
\beta' \to \min\{\beta, \beta^*\}

end for

All \alpha x \leq \beta' comprise Ax \leq b'
R_{t+1} = \{x \mid A_t x \leq b_t' \cap Ax \leq b'\}
else

R_{t+1} = \{x \mid A_t x \leq b_t'\}
```

at time t+1, the minimum distance between user and region  $R_{t+1}$  satisfies  $d(x, R_{t+1}) \leq (n-1)\Delta$  for large n.

*Proof.* For large n, the minimum distance between  $R_t = \{x \mid A_t x \leq b_t\}$  and x is large. Then for cuts which cut through  $A_t x \leq b_t'$ , with high probability the cut can not be reversed by noise. For cut shifted to the boundary of the kernel, the kernel should be on the same side of the cut as x. This guarantees that  $R_{t+1}$  will approach to x. Since  $A_t \leq b_t'$  enlarge the boundary by  $\Delta$  and further modified cut can not separate x from current region, the minimum distance  $d(x, R_{t+1}) \leq (n-1)\Delta$ .  $\square$ 

This theorem shows  $R_t$  will converge to  $Y^t$  and  $\Delta$  indicates approaching speed when user and  $R_t$  are disjoint.

**Proposition 2.** Suppose at time t, the user is inside the kernel of  $R_t$ , then at time t+1, the minimum distance between  $R_{t+1}$  and x satisfies  $d(x, R_{t+1}) \leq \Delta$ 

*Proof.* If user is inside the kernel of  $R_t$ , by the promise of the algorithm, the kernel must be retained in feasible region of  $R_{t+1}$ . Since the upper bound of user's moving distance is  $\Delta$  in one time unit, the minimum distance  $d(x, R_{t+1}) \leq \Delta$ .  $\square$ 

This theorem shows the  $Y^t$  can not get far away from  $R_t$  when they are close.

**Proposition 3.**  $R_t$  can not grows to infinity at any time t.

*Proof.* When  $R_t$  expands to sufficient large size, any constraints will cut through the kernel of current  $R_t$ . By cut modification in the algorithm, all these cuts should be moved to surround the kernel. For enough probe devices and OCs, the modified cut will reduce the boundary of  $R_t$  by width d which stops  $R_t$  from growing.

This theorem guarantees the validity of  $R_t$ . In general the size of  $R_t$  is stabilized around a certain size related to the number of probe devices and intensity of noise.

#### V. SIMULATION RESULTS

#### A. Experiment Setup

A simulation experiment for passive location is conducted in MATLAB. Since one mobile phone can only be detected by a small number of probe devices, we simulated 8 probe devices to track a mobile phone in a map with the size of  $18.75 \times 10.65 \mathrm{m}^2$ . 20 simulated paths are generated randomly with length 5.25m. Each path is divided into 20 locating points and each RSS signal is calculated by RSS-distance function. We add biased Gaussian noise and random data loss to RSS signal.

We evaluate location robustness and accuracy. For robustness, three factors are analyzed, including 1) signal drifting, 2) standard deviation of unbiased Gaussian noise and 3) the missing ratio of RSS data. For accuracy, we first compare 3 ordinal constraint fusion methods, i.e., ordinal constraint fusion in*one time-slot (OS)* (Section IV-A), ordinal constraint *intersection of multiple time-slots (IM)* (Section IV-B1) and ordinal constraint fusion of multiple time-slots with *expansion and kernel (EK)* (Section IV-B4). Then locating errors of EK will be compared with the three traditional algorithms, 1) Max likelihood, 2) Least square and 3) Weighted KNN.

## B. Robustness VS signal drifting of RSS

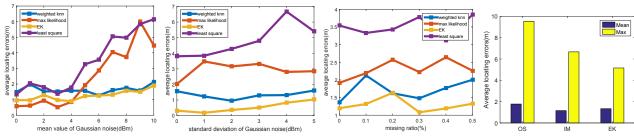
We first investigate the impact of signal drifting on locating performance. The missing ratio and standard deviation of Gaussian noise is set as 0.3 and 5, while signal drifting varies from 1 to 10. Figure 8a shows average locating errors. We find when signal drifting becomes larger, locating errors of EK and weighted KNN increase slightly, while those of max likelihood and least square increase sharply. Max likelihood and least square only work well when signal drifting is very small, but works quite poorly even if signal drifting becomes a little larger. The locating errors of EK are generally smaller than other three locating methods.

## C. Robustness VS mean value of Gaussian noise

Then the effect of standard deviation of Gaussian noise is assessed. The missing ratio is set as 0.3 and RSS drifting is set as 5. The standard deviation of Gaussian noise is set from 1 to 5. The error results are shown in Figure 8b. Locating results of all methods become worse when the standard deviation of Gaussian noise becomes larger. EK shows more accurate locating results.

## D. Robustness VS missing ratio of RSS data

Finally the locating robustness to data missing is evaluated. RSS drifting and standard deviation of Gaussian noise are both set as 5. The missing ratio is chosen from 0.1 to 0.5. The locating errors are plotted in Figure 8c. All locating methods show stable performance even if missing ratio of RSS data is large, which shows the efficiency of CF. The general locating error of EK is smaller than other three methods.



(a) mean locating error as a function of mean value of Gaussian noise

(b) mean locating error as a func- (c) mean locating error as a function tion of standard deviation of Gaussian of missing ratio of RSS data noise

(d) mean and max locating error of OC algorithms

Fig. 8. locating error

## E. Accuracy VS OC methods

Locating results of OS, IM and EK are evaluated here. The standard deviation of Gaussian noise and the missing ratio of RSS data is set as 5 and 0.3. RSS drifting varies randomly between 0 to 10. See Figure 8d for average locating errors of 3 OC methods. We can see the average locating errors of 3 OC algorithms are similar. The maximum locating error of EK is smaller than other two OC algorithms.

## F. Accuracy VS zero-calibration algorithms

Then the locating accuracy of EK is compared with other 3 zero-calibration methods. The standard deviation of Gaussian noise is set as 5 and the missing ratio of RSS data is set as 0.3. The RSS is also shifted by a randomly chosen value from 0 to 10. Locating errors of four methods are shown in Figure 9a. We can see the locating error of EK is the smallest. Max likelihood and weighted KNN work the second best, and least square works the worst.

#### VI. EXPERIMENT RESULT

#### A. Experiment Setup

1) Implementation: We choose wifipix [23], produced by Beijing Wifipix Company, as probe device to capture RSS data of nearby mobile phones. Although its scanning period is about 4s, often it can not detect any signal within 1 minute. In addition, sometimes the fluctuation of RSS exceeds 10dbm for the static phone. So RSS signals measured by wifipixes are regarded as weak observations. Buffered data in wifipixes is transmitted to database Postgres by a Node.js program. When we need to locate one mobile phone in specific time span, corresponding RSS data will be extracted by a JAVA program from Postgres. The system structure diagram is shown in Figure 10.

2) Experiment area: The experiment is conducted in DuShi-WangJing 1701, the office environment owning the size of 142 square meters. 6 wifipixes are evenly installed in 6 different areas, such as corridors and offices.

3) Comparing methods: 1) Max likelihood; 2) Least square; 3) Weighted KNN.

4) Categories of experiments: Experiments evaluate 3 categories: 1) RSS noise characteristics, including stability of RSS

difference and data correlation, which is discussed in Section II-B; 2) locating robustness to phone conditions, i.e., brands, inner states and positions of phones; 3) general locating accuracy in 18 different positions.

## B. Robustness VS variable phone conditions

Locating robustness to different conditions is evaluated. We first generate 10 different conditions. One condition consists of different brands, inner states and positions of phones, e.g., <xiaomi, scan, inbag>. Under each condition, 20-minute RSS data of static phone is recorded. The overall locating result is plotted in Figure 9b. Least square works the worst and weighted KNN works the second worst. EK and max likelihood have similar locating results, as targets usually stay in  $R_t$  of EK. But EK works better than max likelihood in some extreme positions.

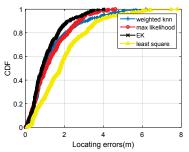
## C. Accuracy VS variable positions

General locating accuracy is evaluated in 18 different positions. 10-minute RSS data is recorded for each position. The average locating results are shown in Figure 9c. It shows that EK gives the most accurate locating result. We also find max likelihood works better than weighted KNN. A possible explanation of this is that the parameters of the empirical formula from RSS to distance are set correctly, which is beneficial to max likelihood.

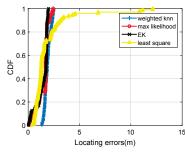
#### VII. CONCLUSION

The paper investigates various usage of TOR in zero-calibrated passive indoor location. TORs are extracted to characterize a feasible region in one time-slot, excluding the impact of positive correlated noise. A series of methods for smoothed feasible region mergence overtime have been investigated, e.g., OS, IM, EK. We find EK can effciently utilize the ordinal constraints overtime. Constrained nonlinear optimizations are proposed to estimate the location under TOR constraints.

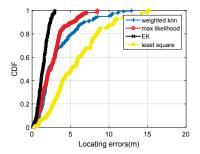
Simulations show that smoothed mergence algorithms can avoid extremely terrible locating results and experiments show that EK reduce almost 1.5m locating errors compared with other zero-calibration methods. Above features prove the validity of TOR in practical passive locating. In future work, we



(a) CDF of locating error in simulation



(b) CDF of locating error when phone condition changesFig. 9. cumulative distribution function



(c) CDF of locating error when phone position changes





Fig. 10. wifipix and system structure

will explore other constrained algorithms based on TOR and fuse them with particle filter and digital floor map information. We will also investigate whether the system works well in locating multiple devices at the same time, which is a key requirement in large-scale passive locating application.

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#### REFERENCES

- W. W. Smith, "Passive location of mobile cellular telephone terminals," in 1991 IEEE International Carnahan Conference on Security Technology, 1991. Proceedings, 1991, pp. 221–225.
- [2] Z. Li, T. Braun, and D. Dimitrova, "A passive wifi source localization system based on fine-grained power-based trilateration," in WOWMOM, 2015, pp. 1–9.
- [3] Y. Wang, L. Song, Z. Gu, and D. Li, "Intenct: Efficient multi-target counting and tracking by binary proximity sensors," in 2016 13th Annual IEEE International Conference on Sensing, Communication, and Networking (SECON), June 2016, pp. 1–9.
- [4] L. Song and Y. Wang, "Multiple target counting and tracking using binary proximity sensors: Bounds, coloring, and filter," in *Proceedings of* the 15th ACM International Symposium on Mobile Ad Hoc Networking and Computing, ser. MobiHoc '14. New York, NY, USA: ACM, 2014, pp. 397–406.
- [5] A. Kushki, K. N. Plataniotis, A. N. Venetsanopoulos, and C. S. Regazzoni, "Radio map fusion for indoor positioning in wireless local area networks," in *International Conference on Information Fusion*, 2005.
- [6] W. Yang, P. Xing, Y. Liu, E. Department, and H. P. Institute, "A positioning method of wsn based on self-adapted rssi distance model," *Chinese Journal of Sensors & Actuators*, vol. 28, no. 1, pp. 137–141, 2015.

- [7] S. Mahfouz, F. Mourad-Chehade, P. Honeine, and J. Farah, "Non-parametric and semi-parametric rssi/distance modeling for target tracking in wireless sensor networks," *IEEE Sensors Journal*, vol. 16, no. 7, pp. 2115–2126, 2016.
- [8] D. Ruta and B. Gabrys, "Classifier selection for majority voting," Information Fusion, vol. 6, no. 1, pp. 63–81, 2005.
- M. M. Schaffer, "Residue checking apparatus for detecting errors in add, subtract, multiply, divide and square root operations," 1990.
- [10] T. Qiu, Y. Zhou, F. Xia, N. Jin, and L. Feng, "A localization strategy based on n -times trilateral centroid withweight," *International Journal* of Communication Systems, vol. 25, no. 9, p. 11601177, 2013.
- [11] H. Yigit, "Abc-based distance-weighted knn algorithm," Journal of Experimental & Theoretical Artificial Intelligence, vol. 27, no. 2, pp. 1–10, 2014.
- [12] U. Helmke and M. A. Shayman, "Critical points of matrix least squares distance functions," *Linear Algebra & Its Applications*, vol. 215, no. 2, pp. 1–19, 1995.
- [13] G. U. Xiaojie, X. Wang, and L. I. Wenchao, "Detector position optimization in tdoa passive location," *Chinese Journal of Sensors & Actuators*, vol. 24, no. 1, pp. 93–99, 2011.
- [14] J. B. Schafer, F. Dan, J. Herlocker, and S. Sen, "Collaborative filtering recommender systems," in *The Adaptive Web, Methods and Strategies* of Web Personalization, 2007, pp. 46–45.
- [15] A. Lanza, F. Tombari, and L. D. Stefano, "Robust and efficient back-ground subtraction by quadratic polynomial fitting," pp. 1537–1540, 2010.
- [16] X. Ye, Y. Wang, W. Hu, L. Song, Z. Gu, and D. Li, "Warpmap: Accurate and efficient indoor location by dynamic warping in sequence-type radiomap," in SECON, 2016, pp. 1–9.
- [17] X. Su and T. M. Khoshgoftaar, "A survey of collaborative filtering techniques," Advances in Artificial Intelligence, vol. 2009, no. 12, 2009.
- [18] R. H. Byrd, J. C. Gilbert, and J. Nocedal, "A trust region method based on interior point techniques for nonlinear programming," *Mathematical Programming*, vol. 89, no. 1, pp. 149–185, 2000.
- [19] R. A. Waltz, J. L. Morales, J. Nocedal, and D. Orban, "An interior algorithm for nonlinear optimization that combines line search and trust region steps," *Mathematical Programming*, vol. 107, no. 3, pp. 391–408, 2006.
- [20] R. Fletcher and M. J. D. Powell, "A rapidly convergent descent method for minimization," *Computer Journal*, vol. 6, no. 6, pp. 163–168, 1963.
- [21] D. Goldfarb and D. Goldfarb, "A family of variable metric updates derived by variational means," *Mathematics of Computing*, no. 24, pp. 23–26, 1970.
- [22] B. R. Vatti, "A generic solution to polygon clipping," *Communications of the Acm*, vol. 35, no. 7, pp. 56–63, 1992.
- [23] WIFIPIX. http://www.wifipix.com/, 2014.