Test 1 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM1404
Programme/Year:	AS, FM /Y1	Course Title:	Theory of Interest
Session:	202301	Lecturer:	Dr Yong Chin Khian

1. CO1: Use the concepts of derivatives and functions to solve equations in the context of theory of interest.

   (a) [Fill in the blank with correct answer] You are given $\delta_t = \frac{3}{1+t}$. A payment of 320 at the end of 5 years and 640 at the end of 10 years has the same present value as a payment of 170 at the end of 4 years and X at the end of 9 years. Calculus X. 602.32. (6 marks)

   (b) [Fill in the blank with correct answer] Jeff puts 1000 into a fund that pays an effective annual rate of discount of 23% for the first two years and a force of interest of rate $\delta = 2/(12-t), 2 \leq t \leq 4$, for the next two years. At the end of four years, the amount in Jeff's account is the same as what it would have been if he had put 1000 into an account paying interest at the nominal rate of i per annum compounded quarterly for four years. Calculate i. 0.2497. (7 marks)

   (c) [Fill in the blank with correct answer] It is known that

$$1 + \frac{i^{(n)}}{n} = \frac{1 + \frac{i^{(3)}}{3}}{1 + \frac{i^{(4)}}{4}}.$$

   Find n. 12. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] A loan of 12,000 is made at an interest rate of 16% compounded quarterly. The loan is to be repaid with three payments: 4,800 at the end of first year, 9,600 at the end of 5-th year, and the balance at the end of the tenth year. Calculate the amount of final payment.

(15 marks)

Ans.

Let B be the final payment,

$$12,000 = 4,800\left(1+\frac{i^{(4)}}{4}\right)^{-4\times1} + 9,600\left(1+\frac{i^{(4)}}{4}\right)^{-4\times5} + B\left(1+\frac{i^{(4)}}{4}\right)^{-4\times10}$$

$$12,000 = 4,800\,(1.04)^{-4} + 9,600\,(1.04)^{-20} + B\,(1.04)^{-4\times10}$$

$$B = \boxed{16878.5891}$$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Investment A for 280,000 is invested at a nominal rate of interest, j, convertible semiannually. After 8 years, it accumulates to 1,597,923.23. Investment B for 280,000 is invested at a nominal rate of discount, k, convertible quarterly. After 4 years, it accumulates to 1,977,268.74. Investment C for 280,000 is invested at an annual effective rate of interest equal to j in year one and an annual effective rate of discount equal to k in year two. Calculate the value of investment C at the end of two years.

(15 marks)

Ans.

$$280,000(1+j/2)^{2\times8} = 1,597,923.23$$

$$j = \left(\left[\frac{1,597,923.23}{280,000}\right]^{1/(2\times8)} - 1\right)\times 2 = 0.23$$

$$280,000(1-k/4)^{-4\times4} = 1,977,268.74$$

$$k = \left(1 - \left[\frac{1,977,268.74}{280,000}\right]^{-1/(4\times4)}\right)\times 4 = 0.46$$

$$AV_C = 280,000(1+j)(1-k)^{-1} = 280,000(1+0.23)(1-0.46)^{-1} = \boxed{637,777.78}$$

2. CO2: Formulate equations to solve problems involving interest/yield rates.

(a) [Fill in the blank with correct answer] Kenton borrows 240,000 on January 1, 2023 to be repaid in 24 semiannual annual installments at an effective annual rate of interest of 11%. The first payment is due on June 1, 2023. Instead of semiannual payment he decides to make monthly payments equal to one-sixth of the semiannual payment beginning on February 1, 2023. Dertermine how many months will be needed to pay off the loan. 137.86. (6 marks)

(b) [Fill in the blank with correct answer] Annual deposits of 130 are made at the beginning of each year for 13 years. Find the accumulated value at the end of 13 years if the effective rate of interest is 12% for the first 8 years and 11% for the last 5 years. 3916.33. (7 marks)

(c) [Fill in the blank with correct answer] An investment requires an initial payment of 130,000 and annual payments of 13,000 at the end of the first 19 years. Starting at the end of the 20 year, the investment returns 10 equal payments of X. Determine X to yield an annual effective rate of 8% over the 29-year period. 163908.97. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] At time $t = 0$, Edbert deposits Q into a fund crediting interest at an effective annual interest rate of 10%. At the end of each year in years 9 through 25, Edbert withdraws an amount sufficient to purchase an annuity-due of 100 for 12 years at a nominal interest rate of 12% compounded monthly. Immediately after the withdrawal at the end of year 25, the fund value is zero. Calculate Q.

(15 marks)

Ans.

Edbert withdraws $100\ddot{a}_{\overline{144}|1\%} = 100\left[\frac{1-1.01^{-144}}{0.01}\right](1.01) = 7689.85$

$$
\begin{aligned}
Q &= 7689.85 a_{\overline{17}|10\%}(1.1^{-8}) \\
&= 7689.85\left[\frac{1-1.1^{-17}}{0.1}\right](1.1^{-8}) \\
&= \boxed{28{,}776.29}
\end{aligned}
$$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Tom borrows 400 at an annual effective interest rate of 6% and agrees to repay it with 25 annual installments. The amount of each payment in the last 15 years is set at twice that in the first 10 years. At the end of 10 years, Tom has the option to repay the entire loan with a final payment X, in addition to the regular payment. This will yield the lender an annual effective rate of 6.7% over the 10-year period. Calculate X.

(15 marks)

Ans.

```
      6%
     <--->
L = 400 R    R      R   2R          2R
     |---|---|------|---|---------|
     0   1   2      10            25
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```
     6.7%
     <--->
L = 400 R    R      R+X
     |---|---|------|
     0   1   2      10
```

$a_{\overline{25}|6\%} = \frac{1-1.06^{-25}}{0.06} = 12.7834$
$a_{\overline{10}|6\%} = \frac{1-1.06^{-10}}{0.06} = 7.3601$
$400 = 2Ra_{\overline{25}|6\%} - Ra_{\overline{10}|6\%}$
$400 = R(2(12.7834) - 7.3601)$
$R = \frac{400}{2(12.7834)-7.3601} = 21.9699$
$a_{\overline{10}|6.7\%} = \frac{1-1.067^{-10}}{0.067} = 7.122$
$400 = Ra_{\overline{10}|6.7\%} + Xv^{10}_{6.7\%}$
$400 = 21.9699(7.122) + X(1.067)^{-10}$
$X = \frac{400-21.9699(7.122)}{1.067^{-10}} = \boxed{465.80}$