MEME15203 Statistical Inference

Assignment 3

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
Session:	January 2023		
Due by:	23/03/2023		

Q1. Suppose $X \overset{iid}{\sim} POI(\mu)$ and $\gamma = P(X > 0) = 1 - P(X = 0) = 1 - e^{-\mu}$. Find the UMVUE for γ.

(10 marks)

Q2. Let $X_1, X_2, \ldots, X_n$ be a random sample from $X_i \sim Beta(1, 7\theta)$. Find the UMVUE of θ,

(10 marks)

Q3. Consider a random sample of size n from a Poisson distribution, $X_i \sim POI(\theta)$. Find the UMVUE of θ^2

(10 marks)

Q4. Suppose that $X_1, \ldots, X_{39}$ is a random sample from a Poisson distribution, $X_i \sim$ POI(θ). Find the UMVUE of $e^{-9\theta}$ using Rao-Blackwell theorem.

(10 marks)

Q5. Let $X_1, X_2, \ldots, X_n$ be random sample of size n from a Gamma distribution with probability density function

$$\frac{1}{\theta^2} x e^{-x/\theta}, x > 0$$

zero otherwise. Find the UMVUE of $\gamma = P(X > t)$ using Rao-Blackwell theorem.

(10 marks)

Q6. Let $X_1, \ldots, X_n$ be a sample form a population with density $f(x, \theta)$ given by

$$f(x, \theta) = \begin{cases} \frac{1}{\sigma} \exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right], & \text{if } x \geq \mu \\ 0 & \text{otherwise} \end{cases}$$

(a) Identify a two-dimensional sufficient statistic for the parameter vector $\boldsymbol{\theta} = (\mu, \sigma)$ with $-\infty < \mu < \infty$, $\sigma > 0$, and carefully argue that it is sufficient.

(b) Suppose $\sigma = 1$, find a complete sufficient statistic for μ.

(c) Find the UMVUE for μ.

(d) Use Basu's Theorem to show that $X_{1:n}$ and $W = \frac{(X_i - \bar{X})^2}{n}$ are independent.

(20 marks)

Q7. Suppose that $X_1, \ldots, X_{30}$ is a random sample from a Gamma distribution, $X_i \sim \text{GAM}(\alpha = 6, \theta)$,

(a) Show that the p.d.f. of X belongs to the regular exponential family.

(b) Find a complete and sufficient statistic for θ.

(c) Find the UMVUE for $\frac{1}{1-9\theta}$.

(15 marks)

Q8. Suppose $X_1, \ldots, X_n$ is a random sample from a normal distribution, $X_i \sim N(\mu, 16)$. Use the Rao-Blackwell theorem to find the UMVUE of $\nu = P[X \leq c]$.

(15 marks)