MEME15203 Statistical Inference

Assignment 4

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Due by:	26/3/2024		

Q1. Suppose $X \sim Beta(a = 2\theta, b = 1)$. Based on a random sample of size $n = 1$, find the most powerful test of $H_0 : \theta = 4$ against $H_1 : \theta = 3$ with $\alpha = 0.06$, then compute the power of the test for the alternative $\theta = 3$.

(20 marks)

Q2. Let $X \sim NB(r, 0.53)$. Derive the most powerful test of size $\alpha = 0.136$ of $H_0 : r = 1$ against $H_1 : r = 3$ based on an observed value of X. Compute the power of this test for the alternative $r = 3$.

(20 marks)

Q3. Let $X_1, \ldots, X_n$ denote a random sample from a gamma distribution with probability density function(p.d.f.)

$$f(x) = \begin{cases} \frac{1}{\Gamma(6)\theta^6} x^{6-1} e^{-x/\theta}, & x > 0 \\ 0, & \text{oterwise} \end{cases}$$

(a) show that the uniformly most powerful crtical region of size α for testing $H_0 : \theta \leq 4$ versus $H_1 : \theta > 4$ using monotone likelihood ratio(MLR) property is given by $\sum_{i=0}^{n} X_i \geq c$, where c is a constant.

(b) Determine the value of c for $\alpha = 0.1$ and $n = 16$. [Note; qchisq(0.9,192) = 217.5024]

(c) For the test in (b), find the value of the power for $\theta = 7.7615$. [Note: $pchisq(112.093, 192) = 0.0$]

(25 marks)

Q4. Let $X_1, X_2, \ldots, X_v$ denote a random sample from a gamma distribution $X_i \sim GAM(\alpha_1 = 1, \theta_1)$ and let $Y_1, Y_2, \ldots, Y_w$ denote an independent random sample from a gamma distribution $Y_i \sim GAM(\alpha_2 = 1, \theta_2)$.

(a) Find the likelihood ratio criterion for testing $H_0 : \theta_1 = \theta_2$ versus $H_1 : \theta_1 \neq \theta_2$

(b) Show that the test in part (a) can be based on the statistic

$$T = \frac{v\bar{X}}{v\bar{X} + w\bar{Y}}.$$

This assessment consists of 5 questions on 5 printed pages

    (c)   Find the distribution of T when H_0 is true.

(20 marks)

Q5.   If $X_i|\lambda \sim POI(\lambda)$ and a Bayesian uses a prior for λ that is Gamma with parameters $\alpha = 6$ and $\theta = \frac{1}{90}$, suppose $x_1, x_2, \ldots, x_{10}$ have been observed, what is the Bayes test of $H_0 : \lambda \leq 4$ versus $H_1 : \lambda > 4$?

(15 marks)