MEME16203Linear Models

Assignment 4

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Faculty:	FES	Unit Code:	MEME16203
Course:	MAC	Unit Title:	Linear Models
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Q1. Consider the model $Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon ijk$, $i = 1, 2; j = 1, 2; k = 1, 2$, where $\epsilon_{ijk} \sim N(0, \sigma^2)$. Determine which of the following hypotheses are testable. Justify your answer.

(a) $H_0 : \beta_1 - \beta_2$. (4 marks)

(b) $H_0 : \mu + \alpha_1 + \beta_2 + \gamma_{12}$. (4 marks)

(c) $H_0 : (\beta_1 - \beta_2) + \frac{1}{2}(\gamma_{11} - \gamma_{12} + \gamma_{21} - \gamma_{22})$. (4 marks)

(d) $H_0 : \gamma_{11} = 0$.

(e) $H_0 : \mu + \alpha_1 + \frac{1}{2}(\beta_1 + \beta_2 + \gamma_{11} + \gamma_{12}) = 0$. (4 marks)

[Total:20 marks]

Q2. An researcher recorded moisture content for three types of cheese made by two different methods. Two pieces of cheese were measure for each type and each method. The data are shown below.

Treatment	Moisture Content Measurements	
Type A made with Method 1	$y_{11} = 39.02$	$y_{12} = 38.79$
Type B made with Method 1	$y_{21} = 35.74$	$y_{22} = 35.41$
Type C made with Method 1	$y_{31} = 37.02$	$y_{32} = 36.00$
Type A made with Method 2	$y_{41} = 38.96$	$y_{42} = 39.01$
Type B made with Method 2	$y_{51} = 35.58$	$y_{52} = 35.52$
Type C made with Method 2	$y_{61} = 35.70$	$y_{62} = 36.0$

Consider the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, where $\epsilon_{ij} \sim NID(0, \sigma^2)$, $i = 1, 2, 3, 4, 5, 6$, and $j = 1, 2$. This model can be expressed in matrix form as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$.

(a) Suppose the null hypothesis is:
"for each type of cheese, the average moisture content is not affected by the method for making cheese. (This hypothesis allows the average moisture content to vary across types of cheese)."
Express the hypothesis in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. (5 marks)

(b) You are given one of the solution to the normal equation is

$$\mathbf{b} = \begin{bmatrix} 31.68 & 7.28 & 3.95 & 4.89 & 7.36 & 3.92 & 4.24 \end{bmatrix}^T .$$

Compute the SS_{H_0} corresponding to the null hypothesis in part (a), and state it's distribution when the null hypothesis is true. (10 marks)

(c) Suppose $SSE = 0.662$, compute the value of the corresponding $F-$statistic and report the degrees of freedom. (5 marks)

[Total:20 marks]

Q3. In a study to examine the effect of 4 drugs on 3 experimentally induced diseases in dogs, each drug-disease combination was given to six randomly selected dogs. The measurement (y) to be analyzed was the increase in systolic blood pressure (mm Hg) due to treatment. Unfortunately, some dogs were unable to complete the experiment. The data are shown in the following table.

	Disease		
Drug	$j=1$	$j=2$	$j=3$
$i=1$	42, 44, 36,13, 19, 22	33, 26, 33,21	31, -3, 25,25, 24
$i=2$	28, 23, 24,42, 13	34, 33, 31,36	3, 26, 28,32, 3, 16
$i=3$	1, 29, 19	11, 9, 7,1, -6	21, 1, 9,3
$i=4$	24, 9, 22,-2, 15	27, 12, 12,-5, 16, 15	22, 7, 25,5, 12

Consider the model $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$ where $\epsilon_{ijk} \sim NID(0, \sigma^2)$ and y_{ijk} denotes the change in systolic blood pressure (mm Hg) for the $k-$th dog given the $j-$th disease and treated with the $i-$th drug.

(a) Note that the application of the lm() function in R imposes some restrictions to solve the normal equations. What are the restrictions? (5 marks)

(b) Give an interpretation of μ, β_3 and $\mu + \alpha_2 + \beta_3 + \gamma_{23}$ with respect to the restricted model and the mean change in systolic blood pressure. (15 marks)

(c) The effects model under the R baseline restriction has parameter vector for mean responses

$$\boldsymbol{\delta} = (\mu.\alpha_2, \alpha_3, \alpha_4, \beta_2, \beta_3, \gamma_{22}, \gamma_{23}, \gamma_{32}, \gamma_{33}, \gamma_{42}, \gamma_{43})^T$$

(i) Determine a matrix $\mathbf{C}$ so that the testable hypothesis $H_0 : \mathbf{C}\boldsymbol{\delta} = 0$ is the hypothesis $H_0 : \mu_{.j} = \frac{1}{4}\sum_{i=1}^{4} \mu_{ij}$ are equal. (5 marks)

(ii) Determine a matrix $\mathbf{C}$ so that the testable hypothesis $H_0 : \mathbf{C}\boldsymbol{\delta} = 0$ is the hypothesis $H_0 : \mu_{i.} = \frac{1}{3}\sum_{j=1}^{3} \mu_{ij}$ are equal. (5 marks)

[Total:30 marks]

Q4. An researcher recorded moisture content for four types of cheese made by two different methods. Three pieces of cheese were measure for each type and each method. The data are shown below.

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Treatment	Moisture Content Measurements		
Type A made with Method 1	$y_{111} = 38.02$	$y_{112} = 39.79$	
Type A made with Method 2	$y_{121} = 39.96$	$y_{122} = 39.06$	$y_{123} = 38.01$
Type B made with Method 1	$y_{211} = 36.74$	$y_{212} = 33.41$	
Type B made with Method 2	$y_{221} = 34.58$	$y_{222} = 36.52$	$y_{223} = 35.52$
Type C made with Method 1	$y_{311} = 38.02$	$y_{312} = 35.00$	$y_{313} = 34.00$
Type C made with Method 2	$y_{321} = 34.60$		
Type D made with Method 1	$y_{411} = 48.02$	$y_{412} = 45.00$	$y_{413} = 42.00$
Type D made with Method 2	$y_{421} = 44.60$	$y_{422} = 46.05$	

Consider the model $y_{ijk} = \mu_{ij} + \epsilon_{ijk}$, where $\epsilon_{ijk} \sim NID(0, \sigma^2)$, $i = 1, 2, 3, 4$, and $j = 1, 2$ and $k = 1, ..., n_i$. This model can be expressed in matrix form as $\mathbf{Y} = \mathbf{D}\boldsymbol{\beta} + \boldsymbol{\epsilon}$. Examine type III sums of squares for these data.

(a) Use R tp complete the ANOVA table below.

Source	Type III SS	DF	Mean Square	F Value	P-Value
Type					
Method					
Type×Method					

(9 marks)

(b) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for type effects in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. What can you conclude from the results of this test? (7 marks)

(c) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for method effects in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. What can you conclude from the results of this test?

(d) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for interaction effects in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. What can you conclude from the results of this test? (7 marks)

[Total:30 marks]