

TEST 2 MARKING GUIDE

Name: _____ Student ID: _____ Mark: _____ /100

FACULTY:	FES, UTAR	COURSE CODE:	UECM2453
PROGRAMME/YEAR:	AS /Y2, Y3	COURSE TITLE:	FINANCIAL ECONOMICS II
SESSION:	202301	LECTURER:	DR YONG CHIN KHIAN

1. CO3: Explain the cash flow characteristics of the following exotic options: Asian, barrier, compound, gap, and exchange.

- (a) [Fill in the blank with correct answer] Let $x(t)$ be the value of €1 in terms of US dollars at time t . You are given that:

- The continuously compounded risk-free rate in US is 7.0%.
- Under the risk-neutral measure, the stochastic differential equation of x is

$$dx(t) = 0.022x(t)dt + 0.16d\tilde{Z}(t), \quad x(0) = 0.9$$

where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.

- A call option that gives the option holder the right to pay \$0.02 six months from today to buy a call option that gives the the right to buy €1 using \$0.95 one year from now is costs \$0.0368.

Calculate the price of a put option that gives the option holder the right to sell at \$0.02 six months from today a call that gives the right to buy €1 using \$0.95 one year from now. [0.0134](#) (6 marks)

- (b) [Fill in the blank with correct answer] A British company will receive \$1,000,000 at the end of 6 month. To hedge its currency risk, it buys an option allowing to exchange dollars for pounds at a rate of £0.64/\$. You are given:

- The spot exchange rate is £0.65/\$.
- The continuously compounded risk-free interest rate for dollars is 0.06.
- The continuously compounded risk-free interest rate for pounds is 0.04.
- The volatility of the exchange rate between the two currencies is 0.1.
- The Black-Scholes framework is assumed to apply to the currency rate.

Calculate the cost in pounds of the hedge. [15860.00](#) (7 marks)

- (c) [Fill in the blank with correct answer] Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time- t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively. You are given:

- $S_1(0) = 45$ and $S_2(0) = 90.0$.
- Stock 1's volatility is 0.15.
- Stock 2's volatility is 0.25.
- The correlation between the continuously compounded returns of the two stocks is -0.4 .
- The continuously compounded risk-free interest rate is 5.3%.
- A one-year European option with payoff $\max\{76.0 - \min[2.0S_1(1), S_2(1)], 0\}$ has a current (time-0) price of -0.1391.

Consider a European option that gives its holder the right to buy either 2.0 shares of Stock 1 or one share of Stock 2 at a price of 76.0 one year from now. Calculate the current (time-0) price of this option. [5.63](#) (7 marks)

- (d) [Show your workings. If no workings are shown, ZERO is awarded] Let $S(t)$ denote the price at time t of a stock. Consider a 10-month European gap option. If the stock price after 10-month is less than 28, the payoff is $28.5 - S\left(\frac{10}{12}\right)$; otherwise, the payoff is zero. You are given:

- $S(0) = 30$.
- The stock will pay a dividend of amount 2 after 3-months. This is the only dividend that will be paid before the gap option expires.
- The prepaid forward price of the stock follows a geometric Brownian motion with a volatility of 31%.
- The continuously compounded risk-free rate of interest is 8%.

Calculate the price of the gap option.

(15 marks)

Ans.

$$F_{0,10/12}^P(S) = 30 - 2e^{-0.08(3/12)} = 28.0396$$

$$F_{0,10/12}^P(K_2) = 28e^{-0.08(10/12)} = 26.1942$$

$$d_1 = \frac{\ln(28.0396/26.1942) + (0.31^2/2)(10/12)}{0.31\sqrt{10/12}} = 0.3821$$

$$d_2 = 0.3821 - 0.31\sqrt{10/12} = 0.0991$$

$$N(d_1) = N(0.38) = 0.648; N(d_2) = N(0.0991) = 0.5398$$

The price of the gap put option is

$$\begin{aligned} & F_{0,10/12}^P(K_1)N(-d_2) - F_{0,10/12}^P(S)N(-d_1) \\ &= 28.5e^{-0.08(10/12)}(1 - 0.5398) - 28.0396(1 - 0.648) \\ &= \boxed{2.3999} \end{aligned}$$

- (e) [Show your workings. If no workings are shown, ZERO is awarded] Assume the Black-Scholes framework for a stock whose time- t price is $S(t)$. You are given:

- $S(0) = 80$
- S pays dividends of amount $0.033S(t)dt$ between time- t and time $t + dt$.
- $V[\ln S(t)] = 0.0625t$
- The continuously compounded risk-free interest rate is 0.082.

Compute the price of $\min(S(0.1), 85)$ that mature at time 0.1.

(15 marks)

Ans.

Note that $\min(S(0.1), 85.0) = S(0.1) - [S(0.1) - 85, 0]_+$, then the price of $\min(S(0.1), 85.0)$ is $F_{t,T}^P(S) - c[80, 85.0]$

$$V[\ln S(t)] = \sigma^2 t = 0.0625t \rightarrow \sigma = 0.25$$

$$d_1 = \frac{\ln(80)/(85) + (0.082 - 0.033 + \frac{1}{2}0.25^2)(0.1)}{0.25\sqrt{0.1}} = -0.6653$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.7444$$

$$N(d_1) = N(-0.67) = 0.2514; N(d_2) = N(-0.74) = 0.2296;$$

$$c(S(0), 85.0, 0.1) = 80e^{-0.033(0.1)}(0.2514) - 85.0e^{0.082(0.1)}(0.2296) = 0.689117$$

$$\text{The price is } 80e^{-0.033(0.1)} + c[80, 85.0] = 80e^{-0.033(0.1)} - 0.689117 = \boxed{79.0473}$$

2. CO2: Demonstrate the calculation and the use of option price partial derivatives

- (a) [Fill in the blank with correct answer] Let $S(t)$ be the time- t price of a nondividend paying stock. You are given that $S(t)$ follows the stochastic differential equation

$$dS(t) = 0.05S(t)dt + 0.27d\tilde{Z}(t), S(0) = 3,$$

where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.

A market maker has just written a contingent claim that pays the $S^3(3)$ after 3 years. He then immediately delta-hedge his position by trading stocks and cash(W). Calculate W. [-140.48](#) (6 marks)

- (b) [Fill in the blank with correct answer] Let $S(t)$ be time- t price of a nondividend-paying stock and $P(S(t), t)$ be the time- t price of a 0.25-year at the money European put option written on the stock, when the time- t stock price is $S(t)$. You are given that

- $S(0) = 51$.
- The true stock price process is

$$dS(t) = 0.13S(t)dt + 0.26S(t)dZ(t)$$

where $Z(t)$ is a standard Brownian motion under the true measure.

- The true stochastic process satisfied by the put option is

$$dP(S(t), t) = a(S(t), t)dt + b(S(t), t)dZ(t)$$

for some a and b .

- $r = 0.065$.

Calculate $a(51, 0)$. [-79.05](#)

(7 marks)

- (c) [Fill in the blank with correct answer] Consider a European call option on a stock following the Black-Scholes framework. The option expires in one year. Using the Black-Scholes formula for this option, you obtain:

$$N(d_1) = 0.6406 \quad N(d_2) = 0.3594.$$

Calculate the volatility of the option. [1.64](#)

(7 marks)

- (d) [Show your workings. If no workings are shown, ZERO is awarded] For a 1-year European call option on a stock:

- The strike price is 68.
- The stock's current price is 74.
- The continuously compounded risk-free interest rate is 0.06.
- The stock pays a dividend of 2 every 3 months, starting immediately after the call option is written. The dividend at the end of one year is paid before the option may be exercised.
- The annual volatility of a prepaid forward on the stock is 0.33.
- The stock follows the Black-Scholes framework.

Calculate the price of the option.

(15 marks)

Ans.

$$F_{t,T}^P(S) = 74 - 2(1 + e^{-0.25(0.06)} + e^{-0.5(0.06)} + e^{-0.75(0.06)} + e^{-0.06}) = 64.2934$$

$$F_{t,T}^P(K) = Ke^{-rt} = 68e^{-0.06(1.0)} = 64.04$$

$$d_1 = \frac{\ln[F_{t,T}^P(S)/F_{t,T}^P(K)] + \sigma^2/2(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln[64.2934/64.04] + 0.33^2/2(1.0)}{0.33\sqrt{1.0}} = 0.18$$

$$d_2 = \frac{\ln[F_{t,T}^P(S)/F_{t,T}^P(K)] - \sigma^2/2(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln[64.2934/64.04] - 0.33^2/2(1.0)}{0.33\sqrt{1.0}} = -0.15$$

$$N(d_1) = N(0.18) = 0.5714$$

$$N(d_2) = N(-0.15) = 0.4404$$

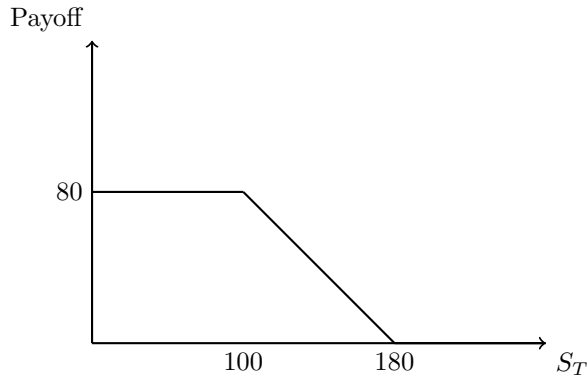
$$c(S(t), K, t) = F_{t,T}^P(S)N(d_1) - F_{t,T}^P(K)N(d_2) -$$

$$\begin{aligned}
c(S(1.0), 68, 1.0) &= 64.2934N(0.18) - 64.04N(-0.15) \\
&= 64.2934(0.5714) - 64.04(0.4404) \\
&= \boxed{8.534}
\end{aligned}$$

- (e) [Show your workings. If no workings are shown, ZERO is awarded] For a stock whose time- t price is $S(t)$, you are given:

$$S(0) = 100 \quad \delta = 3.0\% \quad \sigma = 37\% \quad r = 8.7\%$$

Consider a contingent claim that has the following payoff function at time 3:



Calculate the time-0 price of the contingent claim.

(15 marks)

Ans.

The payoff of the contingent claim is equivalent to longing 1.0-180-strike put and shorting 1.0-100-strike put.

$$d_1(100) = \frac{\ln(100/100) + [0.087 - 0.03 + \frac{1}{2}(0.37^2)](3)}{0.37\sqrt{3}} = 0.59; N(-d_1(100)) = N(-0.59) = 1 - 0.7224 = 0.2776$$

$$d_2(100) = d_1(100) - \sigma\sqrt{t} = 0.59 - 0.37\sqrt{3} = -0.05; N(-d_2(100)) = N(-(-0.05)) = 1 - 0.4801 = 0.5199$$

$$d_1(180) = \frac{\ln(100/180) + [0.087 - 0.03 + \frac{1}{2}(0.37^2)](3)}{0.37\sqrt{3}} = -0.33; N(-d_1(180)) = N(-(-0.33)) = 1 - 0.3707 = 0.6293$$

$$d_2(180) = d_1(180) - \sigma\sqrt{t} = -0.33 - 0.37\sqrt{3} = -0.97; N(-d_2(180)) = N(-(-0.97)) = 1 - 0.166 = 0.834$$

$$p(100, 100) = 100e^{-0.087(3)}N(-d_2(100)) - 100e^{-0.03(3)}N(-d_1(100)) = 14.6762$$

$$p(100, 180) = 180e^{-0.087(3)}N(-d_2(180)) - 100e^{-0.03(3)}N(-d_1(180)) = 58.1209$$

$$\text{The time-0 price of the contingent claim} = 1.0(58.1209) - 1.0(14.6762) = \boxed{43.4447}$$