MEME15203 Statistical Inference

Assignment 2

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Q1. Consider

$$f(x|\theta) = pI(x = 0) + (1 - p)\frac{(\ln\theta)^x}{\theta x!}, x = 0, 1, 2, \ldots.$$

Suppose parameters are $p \in [0, 1]$ and $\theta \geq 0$. Then, for $X_1, X_2, \ldots, X_n$ iid with this marginal distribution, find a method of moments estimator for the parameter vector (p, θ) based on the first two sample moments.

(15 marks)

Q2. Consider a random sample of size n from a distribution with discrete pdf $f(x : p) = p(1 - p)^x; x = 0, 1, \ldots,$ zero otherwise.

(a) Find the MLE of p.

(b) Find the MLE of $\theta = \frac{1-p}{p}$.

(c) Find the CRLB for variance of unbiased estimators of θ.

(d) Is MLE of θ a UMVUE?

(e) Is MLE of θ MSE consistent?

(f) Find the asymptotic distribution of the MLE of θ.

(25 marks)

Q3. Let $Y_1, \ldots, Y_n$ be independent where $Y \sim EXP(\beta x_i)$.

(a) If $y_1, \ldots, y_n$ are observed, derive the MLE $\hat{\beta}$ based on the pairs $(x_1, y_1), \ldots, (x_n, y_n)$.

(b) Determine the distribution of the estimator $\hat{\beta}$.

(c) Determine the mean and varaince of the estimator $\hat{\beta}$.

(d) Find the CRLB of the β.

(20 marks)

Q4. Let $Y_1, \ldots, Y_n$ be independent where $Y \sim POI(\lambda x_i)$.

(a) If $y_1, \ldots, y_n$ are observed, derive the MLE $\hat{\lambda}$ based on the pairs $(x_1, y_1), \ldots, (x_n, y_n)$.

(b) Find a constant c such that $c\hat{\lambda}$ follow the Poisson distrbution. State the corresponding parameter,

(c) Determine the mean and varaince of the estimator $\hat{\lambda}$.

(d) Determine the asymptotic distribution of the estimator $\hat{\lambda}$.

(20 marks)

Q5. Suppose $X|\theta \sim U(\theta - \frac{1}{5}, \theta + \frac{4}{5})$ and that a prior distribution of θ is $N(\mu, 1)$. Find the Bayes estimator of θ under squared error loss.

(20 marks)