**MEME15203 Statistical Inference**

**Assignment 3**

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| Faculty: | FES | Unit Code: | MEME15203 |
| Course: | MAC | Unit Title: | Statistical Inference |
| Year: | 1,2 | Lecturer: | Dr Yong Chin Khian |
| Session: | January 2022 | | |
| Due by: | 17/3/2022 | | |

Q1. Consider a random sample of size $n$ from a two-parameter exponential distribution, $X_i \sim EXP(1, \eta)$.

(a) Show that $T(X) = X_{1:n}$ is complete and sufficient for $\eta$. (5 marks)

(b) Find the UMVUE of $\eta$. (5 marks)

(c) Find the UMVUE of $\eta^2$. (5 marks)

(d) Use Basu's Theorem to show that $X_{1:n}$ and $S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$ are independent. (5 marks)

Q2. Suppose that $X_1, \ldots, X_n$ is a random sample from a Negative Binomial distribution, $X_i \sim \text{NB}(\text{r} = 6,\ \theta)$,

(a) Show that the p.d.f. of $X$ belongs to the regular exponential family. (5 marks)

(b) Based on the answer in (a), find a complete and sufficient statistic for $\theta$. (5 marks)

(c) Find the UMVUE of $\left[\frac{4\theta}{1-4(1-\theta)}\right]^{6n}$. (5 marks)

Q3. Consider a random sample of size $n$ from a gamma distribution $X_i \sim GAM(\alpha, \theta)$ and let $\bar{X} = (1/n)\sum X_i$ and $\tilde{X} = (\prod X_i)^{1/n}$ be the sample mean and geometric mean respectively.

(a) Show that $\bar{X}$ and $\tilde{X}$ are jointly complete and sufficient for $\theta$ and $\alpha$.

(b) Find the UMVUE of $\mu = \alpha\theta$. (5 marks)

(c) Find the UMVUE of $\mu^n$. (5 marks)

(d) Show that the distribution of $T = \frac{\bar{X}}{\tilde{X}}$ does not depend on $\theta$. (5 marks)

(e) Show that $\bar{X}$ and $T$ are stochastically independent random variables. (5 marks)

Q4. Suppose that $X_1, \ldots, X_n$ is a random sample from a Poisson distribution, $X_i \sim \text{POI}(\theta)$. Find the UMVUE of $P(X = 0 \text{ or } 2) = (1 + \frac{1}{2}\theta^2)e^{-\theta}$ using Rao-Blackwell theorem. (15 marks)

Q5. Let $X_1, \ldots, X_n$ be a random sample from a Bernoulli distribution, $X_i \sim Bin(1, p)$; $0 < p < 1$. Find the UMVUE of $p^2$. (10 marks)

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Q6. Show that $X \sim N(0, \theta)$ is not a complete family. (5 marks)

Q7. Let $X_1, X_2, \ldots, X_n$ be random sample of size $n$ from a Gamma distribution with probability density function

$$\frac{1}{\theta^2} x e^{-x/\theta}, x > 0$$

zero otherwise. Find the UMVUE of $\gamma = P(X > t)$ using Rao-Blackwell theorem.

(15 marks)