

### 1. $X \sim POI(\lambda)$

- $f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, \dots$
  - $E(X) = V(X) = \lambda$
  - $P_N(z) = e^{\lambda(z-1)}$
  - $M_N(t) = e^{\lambda(e^t-1)}$
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### 2. $X \sim Bin(m, q)$

- $f(x|q) = \binom{m}{x} q^x (1-q)^{m-x}, x = 0, 1, \dots, m$
  - $E(X) = mq; V(X) = mq(1-q)$
  - $P_N(z) = [1 + q(z-1)]^m$
  - $M_N(t) = [1 + q(e^t - 1)]^m$
  - **Special case:**
    - When  $m = 1, X \sim Bernoulli(q)$
    - $f(x|q) = q^x (1-q)^{1-x}, x = 0, 1$
    - $E(X) = q; V(X) = q(1-q)$
    - $P_N(z) = [1 + q(z-1)]$
    - $M_N(t) = [1 + q(e^t - 1)]$
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### 3. $X \sim NB(r, \beta)$

- $f(x|\beta) = \frac{r(r+1)\dots(r+x-1)\beta^x}{x!(1+\beta)^{r+x}}$   
 $= \binom{r+x-1}{x} \frac{\beta^x}{(1+\beta)^{r+x}}, x = 0, 1, \dots$
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$$= \binom{r+x-1}{x} \left( \frac{1}{1+\beta} \right)^r \left( \frac{\beta}{1+\beta} \right)^x$$

$$\bullet E(X) = r\beta; V(X) = r\beta(1 + \beta)$$

$$\bullet P_N(z) = [1 - \beta(Z - 1)]^{-r}$$

$$\bullet M_N(t) = [1 - \beta(e^t - 1)]^{-r}$$

•Special Case:

–When  $r = 1$ ,  $X \sim \text{Geometric}(\beta)$

$$-f(x|\beta) \equiv \frac{\beta^x}{(1+\beta)^{1+x}}, x = 0, 1, \dots$$

$$-E(X) = \beta; V(X) = \beta(1 + \beta)$$

$$-P_N(z) = [1 - \beta(Z - 1)]$$

$$-M_N(t) = [1 - \beta(e^t - 1)]$$


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4.  $X \sim N(\mu, \sigma)$

$$\bullet f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in R$$

$$\bullet E(X) = \mu; V(X) = \sigma^2$$

$$\bullet F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$


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5.  $X \sim \text{Gamma}(\alpha, \theta)$

$$\bullet f(x|\alpha, \theta) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, x > 0$$


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- $F(x|\alpha, \theta) = 1 - \sum_{j=0}^{\alpha-1} \frac{(\frac{x}{\theta})^j e^{-\frac{x}{\theta}}}{j!}$
  - $E(X) = \alpha\theta; V(X) = \alpha\theta^2$
  - $E(X^k) = \theta^k \alpha(\alpha+1) \cdots (\alpha+k-1)$
  - **Special case: When  $\alpha = 1$ ,  $X \sim EXP(\theta)$** 
    - $f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0$
    - $F(x|\theta) = 1 - e^{-\frac{x}{\theta}}$
    - $E(X) = \theta; V(X) = \theta^2$
    - $E(X^k) = \theta^k$
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## 6. $X \sim InvGamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\frac{\theta}{x}}, x > 0$
  - $E(X^k) = \frac{\theta^k}{(\alpha-1) \cdots (\alpha-k)}$ , if  $k$  is a positive integer
  - **Special case: When  $\alpha = 1$ ,  $X \sim InvExp(\theta)$** 
    - $f(x|\theta) = \theta x^{-2} e^{-\frac{\theta}{x}}, x > 0$
    - $F(x|\theta) = e^{-\frac{\theta}{x}}, x > 0$
    - $E(X^k) = \theta^k \Gamma(1-k), k < 1$
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## 7. $X \sim Pareto(\alpha, \theta)$

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- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, x > 0$
  - $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha$
  - $E(X^k) = \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}$
  - $E(X \wedge x) = \frac{\theta}{\alpha-1} \left[ 1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1} \right]$
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8.  $X \sim \text{SingleParameterPareto}(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
  - $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x}\right)^\alpha$
  - $E(X^k) = \frac{\alpha\theta^k}{\alpha-k}, k < \alpha$
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### 9. $X \sim \text{Beta}(a, b, \theta)$

- $f(x|a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)\theta^{a+b-1}} x^{a-1}(\theta-x)^{b-1}, 0 < x < \theta$
  - $E(X^k) = \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}$  if  $k$  is positive integer.
  - Special case: When  $a = 1, b = 1$ ,  
 $x \sim U(0, \theta)$   
 $f(x) = \frac{1}{\theta}, 0 < x < \theta$   
 $E(X) = \frac{\theta}{2}, V(X) = \frac{\theta^2}{12}$
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### 10. $X \sim \text{LogNormal}(\mu, \sigma)$

- $f(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, x > 0$
  - $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$
  - $E(X^k) = e^{k\mu + \frac{1}{2}k^2\sigma^2}$
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### 11. $X \sim \text{Weibull}(\tau, \theta)$

- $f(x|\tau, \theta) = \frac{\tau}{\theta^\tau} x^{\tau-1} e^{-(x/\theta)^\tau}, x > 0$
  - $F(x) = 1 - e^{-(x/\theta)^\tau}$
  - $E(X^k) = \theta^k \Gamma(1 + k/\tau), k > -\tau$
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12.  $X \sim \text{InverseWeibull}(\tau, \theta)$

- $f(x|\tau, \theta) = \tau\theta^\tau x^{-\tau} e^{-(\theta/x)^\tau}, x > 0$
  - $F(x) = e^{-(\theta/x)^\tau}$
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13.  $X \sim \text{InvGaussian}(\mu, \theta)$

- $E(X) = \mu; V(X) = \frac{\mu^3}{\theta}$
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