

1. $\mathbf{X} \sim \text{Bernoulli}(p)$

- $f(x) = p^x q^{1-x}, x = 0, 1$
- $M_X(t) = pe^t + q$
- $E(X) = p$
- $V(X) = pq$

2. $\mathbf{X} \sim \text{Binomial}(n, p)$

- $f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$
- $M_X(t) = (pe^t + q)^n$
- $E(X) = np$
- $V(X) = npq$

3. $\mathbf{X} \sim \text{HYP}(n, M, N)$

- $f(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}},$
 $x = 0, 1, \dots, \min(n, M), n - x \leq N - M.$
- $E(X) = \frac{nM}{N}$
- $V(X) = n \frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1}$

4. $\mathbf{X} \sim \text{GEO}(p)$

- $f(x) = pq^{x-1} \quad x = 1, 2, 3, \dots$
- $F(x) = 1 - q^x \quad x = 1, 2, 3, \dots$
- $M_X(t) = \frac{pe^t}{1-qe^t}$
- $E(X) = \frac{1}{p}$
- $V(X) = \frac{q}{p^2}$

5. $\mathbf{X} \sim \text{NegativeBinomial}(r, p)$

- $f(x) = \binom{x-1}{r-1} p^r q^{x-r}, x = r, r+1, \dots$
- $M_X(t) = \left(\frac{pe^t}{1-qe^t}\right)^r$
- $E(X) = \frac{r}{p}$
- $V(X) = \frac{rq}{p^2}$

6. $\mathbf{X} \sim \text{POI}(\mu)$

- $f(x) = \frac{e^{-\mu} \mu^x}{x!} \quad x = 0, 1, 2, \dots$
- $M_X(t) = e^{\mu(e^t-1)}$
- $E(X) = \mu$
- $V(X) = \mu$

7. $\mathbf{X} \sim DU(N)$

- $f(x) = \frac{1}{N}, X = 1, 2, \dots, N$
- $M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$
- $F(x) = \frac{x(1+x)}{2N}$
- $E(X) = \frac{N+1}{2}$
- $V(X) = \frac{N^2-1}{12}$

8. $\mathbf{X} \sim U(a, b)$

- $f(x) = \frac{1}{b-a}, a < x < b$ and zero otherwise
- $F(x) = \frac{x-a}{b-a}, a < x < b$
- $M_X(t) = \frac{e^{tb} - e^{ta}}{b-a}$
- $E(X) = \frac{a+b}{2}$
- $V(X) = \frac{(b-a)^2}{12}$

9. $\mathbf{X} \sim \text{Gamma}(\alpha, \theta)$

- $f(x) = \frac{1}{\theta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\theta}, x > 0$
- $F(x) = 1 - \sum_{i=0}^{\alpha-1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$
- $M_X(t) = (\frac{1}{1-\theta t})^\alpha$
- $E(X) = \alpha\theta$

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- $V(X) = \alpha\theta^2$

10. $\mathbf{X} \sim EXP(\theta)$

- $f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ and zero otherwise.
- $F(x) = 1 - e^{-x/\theta}, x > 0$
- $M_X(t) = \left(\frac{1}{1-\theta t} \right)$
- $E(X) = \theta,$
- $V(X) = \theta^2$

11. $\mathbf{X} \sim WEI(\tau, \theta)$

- $f(x) = \frac{\tau}{\theta^\tau} x^{\tau-1} e^{-(x/\theta)^\tau}, x > 0$ and zero otherwise.
- $F(x) = 1 - e^{-(x/\theta)^\tau}$
- $E(X) = \theta \Gamma \left(1 + \frac{1}{\tau} \right)$
- $E(X^2) = \theta^2 \left[\Gamma \left(1 + \frac{2}{\tau} \right) - \Gamma^2 \left(1 + \frac{1}{\tau} \right) \right]$

12. $\mathbf{X} \sim PAR(\alpha, \theta)$

- $f(x) = \frac{\alpha \theta^\alpha}{(x+\theta)^{\alpha+1}}, x > 0$
- $F(x) = 1 - \left(\frac{\theta}{x+\theta} \right)^\alpha$
- $E(X) = \frac{\theta}{\alpha-1}$

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- $E(X^2) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)}$
 - $V(X) = \frac{\theta^2}{(\alpha-1)^2(\alpha-2)}$
13. $\mathbf{X} \sim \text{Beta}(a, b)$
- $f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}, 0 < x < 1$
 - $E(X) = \frac{a}{a+b}$
 - $V(X) = \frac{ab}{(a+b)^2(a+b+1)}$
14. $\mathbf{X} \sim N(\mu, \sigma^2)$
- $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, x \in \mathbb{R}, \mu \in \mathbb{R}$
and $\sigma > 0$.
 - $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
 - $M_X(t) = e^{\mu t + \sigma^2 t^2/2}$
 - $E(X) = \mu$
 - $V(X) = \sigma^2$

15. $\mathbf{X} \sim LN(\mu, \sigma)$
- $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x - \mu)^2/2\sigma^2}, x > 0, \mu > 0$ and $\sigma > 0$
 - $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$
 - $E(X) = e^{\mu + \frac{\sigma^2}{2}}$
 - $V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$
16. $\mathbf{X} \sim CAU(\theta, \eta)$
- $f(x) = \frac{1}{\theta\pi \left[1 + \left(\frac{x-\eta}{\theta}\right)^2\right]}$
 - $F(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x-\eta}{\theta}\right)$
17. $\mathbf{X} \sim EXP(\eta, \theta)$
- $f(x) = \frac{1}{\theta} e^{-\frac{x-\eta}{\theta}} \quad x > \eta$
 - $F(x) = 1 - e^{-\frac{x-\eta}{\theta}}$
 - $M_X(t) = \frac{e^{\eta t}}{1 - \theta t}$
 - $E(X) = \eta + \theta$
 - $V(X) = \theta^2$

18. $\mathbf{X} \sim DE(\eta, \theta)$

- $f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta} \quad -\infty < x < \infty$ and zero otherwise.

- $F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \leq \eta \\ \frac{1}{2}[1 - e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$

- $M_X(t) = \frac{e^{\eta t}}{1 - \theta^2 t^2}$

- $E(X) = \eta$

- $V(X) = 2\theta^2$

19. $\mathbf{X} \sim$ Single Parameter Pareto (α, θ)

- $f(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$

- $F(x) = 1 - (\frac{\theta}{x})^\alpha$

- $E(X) = \frac{\alpha\theta}{\alpha-1}$

- $E(X^2) = \frac{\alpha\theta^2}{\alpha-2}$