MEME16203Linear Models

Assignment 1

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Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Due by:			

Q1. Suppose that X and Y have joint probability density function (pdf)

$$f(x,y) = \begin{cases} \frac{2}{3^3}(x+y), & 0 \le x \le y \le 3 \\ 0, \text{otherwise} \end{cases}$$

Find $P[Y < 4X]$. (5 marks)

Q2. The random variable X_1 has an exponential distribution with mean 2. The random variable X_2 is related to X_1 in such a way that $E(X_2|x_1) = 2x_1$ and $V(X_2|x_1) = 3x_1^2$. Find $V(5X_1 + 3X_2)$. (5 marks)

Q3. Let X_1, X_2 be two random variables with joint pdf $f(x_1, x_2) = x_1 e^{-x_2}$, for $0 < x_1 < x_2 < \infty$, zero otherwise. Determine the joint mgf of X_1, X_2. Does $M(t_1, t_2) = M(t_1, 0)M(0, t_2)$? (10 marks)

Q4. Suppose $P[\mu = 1] = 0.3$ and $P[\mu = 2] = 0.7$, and that conditional on μ, $X|\mu \sim POI(\mu)$. Find $V(4X - 4\mu)$.

(5 marks)

Q5. Let X and Y have joint pdf $f(x,y) = cy^2e^{-6y}, 0 < x < y < \infty$ and zero otherwise.

(a)Find the joint pdf of $S = X + Y$ and $T = X$. (5 marks)

(b)Find the marginal pdf of T. (5 marks)

(c)Find the marginal pdf of S. (5 marks)

Q6. Let X be a random variable with a density function given by

$$f(x) = \begin{cases} \frac{3}{2}x^2, & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the density function of $U_1 = 7X$ using distribution method.(2 marks)

(b) Find the density function of $U_2 = 7 - X$ using one to one transformation. (3 marks)

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Q7. A member of the power family of distributions has a distribution function given by

$$F(x) = \begin{cases} 0, & x < 0 \\ (\frac{x}{\theta})^{\alpha}, & 0 \le x \le \theta \\ 1, & x > \theta \end{cases}$$

where $\alpha, \theta > 0$.

(a) For fixed values of α and θ, find a transformation $G(U)$ so that $G(U)$ has a distribution function of F when U possesses a uniform $(0,1)$ distribution. (2 marks)

(b) Given that a random sample of size 5 from a uniform distribution on the interval $(0,1)$ yielded the values $u_1 = 0.027$, $u_2 = 0.06901$, $u_3 = 0.01413$, $u_4 = 0.01523$, and $u_5 = 0.03609$, use the transformation derived in the above result to give values associated with a random variable with a power family distribution with $\alpha = 2$, $\theta = 4$. (3 marks)

Q8. Let X_1 and X_2 be independent random variables with $X_1 \sim GAM(\alpha_1 = a, \theta = 2)$ and $X_2 \sim GAM(\alpha_2 = b, \theta = 2)$, show that $U = \frac{X_1}{X_1+X_2}$ follow a Beta distribution. Suppose $Y_i \sim GAM(\alpha = 7, \theta = 2)$, using the result above, find the distribution of $V = \frac{Y_1}{\sum_{i=1}^{20} Y_i}$. (10 marks)

Q9. Consider a random sample of size n from an exponential distribution, $X_i \sim EXP(1)$. Derive the pdf of the sample range, $R = Y_n - Y_1$, where $Y_1 = \min(X_1, \ldots, X_n)$ and $Y_n = \max(X_1, \ldots, X_n)$. (5 marks)

Q10. Suppose that $X \sim \chi^2(23)$, $S = X + Y \sim \chi^2(62)$, and X and Y are independent. Use MGFs to find the distrbution of $S - X$. (5 marks)

Q11. Suppose that $X_i \sim N(\mu, \sigma^2), i = 1, \ldots, 14$, $Z_j \sim N(0,1), j = 1, \ldots, 8$, and $W_k \sim \chi^2(v), k = 1, \ldots, 13$ and all random variables are independent. State the distribution of each of the following variables if it is a "named" distribution. [For example $X_1 + X_2 \sim N(2\mu, 2\sigma^2)$]

(a) $\frac{7\sum_{i=1}^{14}(X_i - \bar{X})^2}{13\sigma^2 \sum_{j=1}^{8}(Z_j - \bar{Z})^2}$. (4 marks)

(b) $\frac{W_1}{\sum_{k=1}^{8} W_k}$ (3 marks)

(c) $\frac{\bar{X}}{\sigma^2} + \frac{\sum_{i=1}^{14} Z_i}{14}$ (3 marks)

Q12. Suppose that $X_1, \ldots, X_n$, is a random sample from a Pareto distribution, $X \sim PAR(\alpha = 1, \theta = 25)$. Let $Y_n = 1/nX_{n:n}$, find the limiting distribution of Y_n, $F(y)$, state the distribution and it's parameter, then find $F(28.6)$. (5 marks)

Q13. Consider a random sample from a Exponential distribution, $X_i \sim Exp(\theta)$. Find the asymtotic normal distribution of $Y_n = \bar{X}_n^3$.

(5 marks)

Q14. Let the random variable Y_n have a distribution that is $Bin(n, p)$. Prove that Yn/n converges in probability to a constant, identify the constant. (5 marks)

Q15. Consider a random sample from a Geometric distribution, $X_i \sim GEO(p)$. Let $W_i = e^{X_i}$ and $V_n = W_1 \times W_2 \times \cdots W_n$. $V_n^{\frac{1}{n}}$ converges in probability to a constant, identify the constant. (5 marks)