MEME15203 Statistical Inference

Assignment 2

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Q1. Consider a random sample from a Exponential distribution, $X_i \sim Exp(\theta)$. Find the asymtotic normal distribution of $Y_n = \bar{X}_n^4$.

(10 marks)

Q2. Suppose that $X_i \sim N(\mu, \sigma^2), i = 1, \ldots, 13$, $Z_j \sim N(0,1), j = 1, \ldots, 5$, and $W_k \sim \chi^2(v), k = 1, \ldots, 12$ and all random variables are independent. State the distribution of each of the following variables if it is a "named" distribution. [For example $X_1 + X_2 \sim N(2\mu, 2\sigma^2)$]

(a) $\frac{Z_i^2/W_1}{1+Z_1^2/W1}$

(b) $\frac{\frac{\sum_{k=1}^{5} W_k}{\sum_{j=1}^{5}(Z_j-\bar{Z})^2}}{1+\frac{\sum_{k=1}^{5} W_k}{\sum_{j=1}^{5}(Z_j-\bar{Z})^2}}$

(c) $\frac{W_1}{W_1+W_2+W_3+W_4}$

(10 marks)

Q3. Suppose that $X_i \sim N(\mu, \sigma^2), i = 1, \ldots, 11$, $Z_j \sim N(0,1), j = 1, \ldots, 9$, and $W_k \sim \chi^2(v), k = 1, \ldots, 10$ and all random variables are independent. State the distribution of each of the following variables if it is a "named" distribution. [For example $X_1 + X_2 \sim N(2\mu, 2\sigma^2)$]

(a) $\frac{8\sum_{i=1}^{11}(X_i-\bar{X})^2}{10\sigma^2\sum_{j=1}^{9}(Z_j-\bar{Z})^2}$.

(b) $\frac{W_1}{\sum_{k=1}^{9} W_k}$

(c) $\frac{\bar{X}}{\sigma^2} + \frac{\sum_{i=1}^{11} Z_i}{11}$

(10 marks)

Q4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from the density function given by

$$f(x) = \begin{cases} \frac{5}{\theta}x^4 e^{-x^5/\theta}, & \theta > 0, x > 0, \\ 0, & \text{otherwise} \end{cases}$$

(a) find the MME of θ.

(b) Find the MLE of θ.

(c) Find the CRLB of θ.

(15 marks)

Q5. Let $X_1, X_2, \ldots, X_n$ denote a random sample from an exponentially distributed population with mean $\lambda = \frac{1}{\theta}$. Let $\Theta \sim \chi^2(2v)$.

(a) Find the Bayes estimator for $\lambda = \frac{1}{\theta}$ under square error loss.

(b) Show that it is a biased but consistent estimator for $\lambda = \frac{1}{\theta}$.

(20 marks)

Q6. A pmf on the integers with an integer parameter, θ , is

$$f(x|\theta) = \frac{1}{2} I[x = \theta - 5 \text{ or } x = \theta + 5].$$

For X_1 and X_2 iid from this distribution, compare MSE's for the two estimators of θ,

$$\hat{\theta} = \begin{cases} X_1 + 5, & \text{if } X_1 = X_2 \\ \bar{X}, & \text{if } X_1 \neq X_2 \end{cases} \text{ or } \tilde{\theta} = \hat{\theta} - \frac{5}{2}.$$

(15 marks)

Q7. Consider a random sample of size n from a distribution with discrete pdf $f(x : p) = p(1-p)^x; x = 0, 1, \ldots,$ zero otherwise.

(a) Find the MLE of p.

(b) Find the MLE of $\theta = \frac{1-p}{p}$.

(c) Find the CRLB for variance of unbiased estimators of θ.

(d) Is MLE of θ a UMVUE?

(e) Is MLE of θ MSE consistent?

(f) Find the asymptotic distribution of the MLE of θ.

(20 marks)