Test 1 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM2453
Programme/Year:	AS /Y2	Course Title:	Financial Economics II
Session:	202301	Lecturer:	Dr Yong Chin Khian

CO1: Explain the properties of the lognormal distribution and its applicability to option pricing.

1. [Fill in the blank with correct answer] Suppose that X follows the stochastic differential equation

$$dX(t) = -10dt + 4dZ(t)$$

where $Z(t)$ is a standard Brownian motion.
Let $W(t) = e^{5tX(t)}$. If

$$dW(t) = a[W(t), t]dt + b[W(t), t]dZ(t),$$

find $a(6, 7)$. 56701.54 (7 marks)

2. [Fill in the blank with correct answer] You are given:

   - $S(t)$ is the time-t price of a stock.
   - The stock pays dividend continuously at a constant rate proportional to its price.
   - The true stock price process is given by

$$\frac{dS(t)}{S(t)} = cdt + \sigma dZ(t)$$

   where $Z(t)$ is a standard Brownian motion under the true probability measure, and c and σ are constant.
   - The risk-neutral stock price process is given by

$$\frac{dS(t)}{S(t)} = 0.048dt + 0.19d\tilde{Z}(t)$$

   where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.
   - $Z(5) = \tilde{Z}(5) - 1.55$.

   Find c. 0.11 (7 marks)

3. [Fill in the blank with correct answer] Let $S(t)$ be the time-t price of a nondividend-paying stock, you are given that:

   - The stock price process under the true probability measure is

$$d[\ln S(t)] = 0.05795dt + 0.11dZ(t), S(0) = 1$$

   where $Z(t)$ is a standard Brownian motion under the true probability measure.
   - The sharpe ratio stock price risk is 0.24545.

   Compute the price of a contingent claim that pays $\sqrt[5]{S(3)}$ at time 3. 0.91 (7 marks)

4. [Fill in the blank with correct answer] Suppose that X follows the stochastic differential equation

$$dX(t) = -12dt + 3dZ(t)$$

where $Z(t)$ is a standard Brownian motion.
Let $W(t) = e^{3tX(t)}$. If

$$dW(t) = a[W(t), t]dt + b[W(t), t]dZ(t),$$

find $a(10, 8)$. 23042.88 (7 marks)

5. [Fill in the blank with correct answer] Let $S(t)$ be the time-t price of a nondividend-paying stock, you are given that:

- The stock price process is
$$d[\ln S(t)] = 0.31 dZ(t)$$
where $Z(t)$ is a standard Brownian motion under the true probability measure.
- The continuously compounded risk-free of interest is 0.042

If $F^P_{0,3}(S^4) = e^{-\gamma}E[S^4(3)]$, find γ. 0.20 (7 marks)

6. [Fill in the blank with correct answer] You are given:

- $S(t)$ is the time-t price of a nondividend-paying stock.
- $S(t)$ follows a geometric Brownian motion.
- The current stock price is 30.
- The expected return on the stock is 0.22.
- The stock's volatility is 0.26.

Calculate $E[S(3)I(S(3) > 30)]$. 55.40 (7 marks)

7. [Show your workings. If no workings are shown, ZERO is awarded] Stock prices follow geometric Brownian motion:
$$d \ln S(t) = 0.032dt + 0.26dZ(t)$$
Suppose $S(0) = 47$. Calculate $P[S(2) < 44)$.

(14 marks)

Ans.

$\hat{d}_2 = \frac{\ln(47/44)+(0.032)(2)}{0.26\sqrt{2}} = 0.35$

$P[S(2) < 44) = N(-\hat{d}_2) = N(-0.35) = \boxed{0.3632}$

$S(t) = 47e^{0.032t+0.26Z(t)}$

$P[S(2) < 44)$
$= P\left[\ln S(2) < \ln 44\right]$
$= P\left[\ln 47 + 0.032(2) + 0.26Z(2) < \ln 44\right]$
$= P(Z(2) < -0.4998)$
$= P(Z < \frac{-0.4998}{\sqrt{2}})$
$= N(-0.35)$
$= \boxed{0.3632}$

8. [Show your workings. If no workings are shown, ZERO is awarded] You are given:

   - $S(t) = S(0)e^{0.12t+0.22Z(t)}$
   - $\delta = 0.02$
   - $F_{t,T}$ is a forward on the stock.
   - $r = 0.05$

   $d(\ln F)$ follows the process $\alpha dt + \sigma dZ(t)$. Determine α.

   (15 marks)

   Ans.

   $\alpha_S - 0.02 - 0.5(0.22^2) = 0.12$
   $\alpha_S = 0.1642$
   $\frac{dF_{t,T}}{F_{t,t}} = (\alpha_S - r)dt + \sigma dZ(t) = (0.1642 - 0.05)dt + 0.22dZ(t)$
   $d(\ln F) = (\alpha_S - r - \frac{1}{2}\sigma^2)dt + \sigma dZ(t) = (0.1642 - 0.05 - .5(0.22^2)dt + 0.22dZ(t)$
   $\alpha = 0.1642 - 0.05 - .5(0.22^2) = \boxed{0.09}$

9. [Show your workings. If no workings are shown, ZERO is awarded] You are given the following information for two nondovidend paying stocks X_1 and X_2 with prices $S_1(t)$ and $S_2(t)$ respectively:

   $$\frac{dS_1(t)}{S_1(t)} = 0.12dt + 0.3dZ(t); \frac{dS_2(t)}{S_2(t)} = 0.021dt - \sigma dZ(t)$$

   $$S_1(0) = 180, S_2(0) = 90.0, r = 0.04$$

   A risk-free portfolio consists of one share of X_1 and c shares of X_2. The cost of this portfolio is borrowed at the risk-free rate so that the net cost outlay is zero. Determine the amount borrowed.

   (14 marks)

   Ans.

   By equality of sharpe ratios,
   $\frac{0.12-0.04}{0.3} = \frac{0.021-0.04}{\sigma}$
   $\sigma = -0.07$
   $c = N(t) = -\frac{\sigma_1 S_1(t)}{\sigma_2 S_2(t)} = -\frac{0.3(180)}{-0.07(90.0)} = 8.42$
   $W = -S_1(t) - cS_2(t) = -1(180) - 8.42(90.0) = -937.8$
   Thus the amount borrowed is $\boxed{937.8}$

10. [Show your workings. If no workings are shown, ZERO is awarded] A forward agreement entered into at time t provides for the exchange of $N(t)$ shares of Fedelity stock for 1 share of Aberdeen stock at time T, $T > t$, with $N(t)$ selected to allow no arbitrage. You are given

- The time-t price of Fedelity stock is $X(t)$, and $X(t)$ satisfies

$$\frac{dX(t)}{X(t)} = 0.11dt + 0.15dZ(t)$$

- The time-t price of Aberdeen stock is $Y(t)$, and $Y(t)$ satisfies

$$\frac{dY(t)}{Y(t)} = 0.3dt + 0.26dZ(t)$$

- Fedelity pays continuous dividends proportional to its price at a rate of 0.015.
- Aberdeen pays continuous dividends proportional to its price at a rate of 0.028.

$N(t)$ satisfies

$$\frac{dN(t)}{N(t)} = \alpha dt + \beta dZ(t).$$

Determine α.

(15 marks)

Ans.

At time-t, there are two ways to acquire 1 share of Aberdeen stock at time T:

(a) Buy $e^{-0.028(T-t))}$ shares of Aberdeen immediately and hold them until time T.

(b) Buy $N(t)e^{-0.015(T-t)}$ shares of Fedelity immediately, and enter into the specified format agreement.

These two ways must have equal cost to avoid arbitrage. So
$N(t)e^{-0.015(T-t)}X(t) = e^{-0.028(T-t))}Y(t)$
$N(t) = \frac{Y(t)e^{-0.013(T-t)}}{X(t)}$
$\ln N(t) = \ln Y(t) - 0.013(T-t) - \ln X(t)$

$$\begin{aligned} d\ln N(t) &= d\ln Y(t) + 0.013dt - d\ln X(t) \\ &= (0.3 - 0.26^2/2)dt + 0.26dZ(t) + 0.013dt - [(0.11 - 0.15^2/2)dt + 0.15dZ(t)] \\ &= 0.1805dt + 0.11dZ(t) \end{aligned}$$

$\frac{dN(t)}{N(t)} = (0.1805 + (0.11)^2/2)dt + 0.11dZ(t)$
$\alpha = (0.1805 + (0.11)^2/2) = \boxed{0.1865}$