1.  $\mathbf{X} \sim Bernoulli(p)$ 

• 
$$f(x) = p^x q^{1-x}, x = 0, 1$$

$$\bullet \ M_X(t) = pe^t + q$$

$$\bullet\; E(X)=p$$

$$\bullet V(X) = pq$$

2.  $\mathbf{X} \sim Binomial(n, p)$ 

• 
$$f(x) = \binom{n}{r} p^x q^{n-x}, x = 0, 1, \dots, n$$

$$\bullet M_X(t) = (pe^t + q)^n$$

$$\bullet$$
  $E(X) = np$ 

$$\bullet V(X) = npq$$

3.  $\mathbf{X} \sim HYP(n, M, N)$ 

• 
$$E(X) = \frac{nM}{N}$$

• 
$$V(X) = n\frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1}$$

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4.  $\mathbf{X} \sim GEO(p)$ 

• 
$$f(x) = pq^{x-1}$$
  $x = 1, 2, 3, \dots$ 

• 
$$F(x) = 1 - q^x$$
  $x = 1, 2, 3, \dots$ 

• 
$$M_X(t) = \frac{pe^t}{1 - qe^t}$$

$$\bullet\; E(X) = \tfrac{1}{p}$$

$$\bullet V(X) = \frac{q}{p^2}$$

5.  $\mathbf{X} \sim NegativeBinomial(r, p)$ 

• 
$$f(x) = {x-1 \choose r-1} p^r q^{x-r}, x = r, r+1, \dots$$

• 
$$M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$$

$$\bullet E(X) = \frac{r}{p}$$

• 
$$V(X) = \frac{rq}{p^2}$$

6.  $\mathbf{X} \sim POI(\mu)$ 

• 
$$f(x) = \frac{e^{-\mu}\mu^x}{x!}$$
  $x = 0, 1, 2, \dots$ 

$$\bullet \ M_X(t) = e^{\mu(e^t - 1)}$$

$$\bullet\; E(X) = \mu$$

$$\bullet \ V(X) = \mu$$

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## **Common Distributions**

7.  $\mathbf{X} \sim DU(N)$ 

• 
$$f(x) = \frac{1}{N}, X = 1, 2, \dots, N$$

• 
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$

$$F(x) = \frac{x(1+x)}{2N}$$

$$E(X) = \frac{N+1}{2}$$

$$\bullet E(X) = \frac{N+1}{2}$$

• 
$$V(X) = \frac{N^2 - 1}{12}$$

8.  $\mathbf{X} \sim U(a,b)$ 

• 
$$f(x) = \frac{1}{b-a}$$
,  $a < x < b$  and zero otherwise

$$\bullet$$
  $F(x) = \frac{x-a}{b-a}, a < x < b$ 

• 
$$M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$$

$$\bullet E(X) = \frac{a+b}{2}$$

$$\bullet \ V(X) = \frac{(b-a)^2}{12}$$

9.  $\mathbf{X} \sim Gamma(\alpha, \theta)$ 

• 
$$f(x) = \frac{1}{\theta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\theta}, x > 0$$

• 
$$F(x) = 1 - \sum_{i=0}^{\alpha - 1} \frac{(x/\theta)^i}{i!} e^{x/\theta}$$

$$\bullet M_X(t) = (\frac{1}{1-\theta t})^{\alpha}$$

$$\bullet E(X) = \alpha \theta$$

## Common Distributions

•  $V(X) = \alpha \theta^2$ 

10.  $\mathbf{X} \sim EXP(\theta)$ 

• 
$$f(x) = \frac{1}{\theta}e^{-x/\theta}, x > 0$$
 and zero otherwise.

• 
$$F(x) = 1 - e^{-x/\theta}, x > 0$$

• 
$$M_X(t) = \left(\frac{1}{1-\theta t}\right)$$

$$\bullet E(X) = \theta$$

$$\bullet \ V(X) = \theta^2$$

11.  $\mathbf{X} \sim WEI(\tau, \theta)$ 

• 
$$f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau - 1} e^{-(x/\theta)^{\tau}}, x > 0$$
 and zero otherwise.

$$\bullet F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

• 
$$E(X) = \theta \Gamma \left( 1 + \frac{1}{\tau} \right)$$

• 
$$E(X^2) = \theta^2 \left[ \Gamma \left( 1 + \frac{2}{\tau} \right) - \Gamma^2 \left( 1 + \frac{1}{\tau} \right) \right]$$

12.  $\mathbf{X} \sim PAR(\alpha, \theta)$ 

• 
$$f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$$

• 
$$F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$$

• 
$$E(X) = \frac{\theta}{\alpha - 1}$$

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• 
$$E(X^2) = \frac{2\theta^2}{(\alpha - 1)(\alpha - 2)}$$

• 
$$V(X) = \frac{\theta^2}{(\alpha - 1)^2(\alpha - 2)}$$

13.  $\mathbf{X} \sim Beta(a, b)$ 

$$\bullet \ f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}, \text{for } 0 < x < 1$$

$$\bullet \ E(X) = \frac{a}{a+b}$$

$$\bullet \ V(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

14. **X** ~  $N(\mu, \sigma^2)$ 

• 
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$$
, for  $x \in \mathbb{R}$ ,  $\mu \in \mathbb{R}$  and  $\sigma > 0$ .

• 
$$F(x) = \Phi(\frac{x-\mu}{\sigma})$$

$$\bullet M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$\bullet$$
  $E(X) = \mu$ 

• 
$$V(X) = \sigma^2$$

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18.  $\mathbf{X} \sim DE(\eta, \theta)$ 

•  $f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$   $-\infty < x < \infty$  and zero otherwise.

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zero otherwise.

• 
$$F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \leq \eta \\ \frac{1}{2}[1 - e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$$

$$\bullet M_X(t) = \frac{e^{\eta t}}{1 - \theta^2 t^2}$$

$$\bullet \ E(X) = \eta$$

$$\bullet V(X) = 2\theta^2$$

19.  $\mathbf{X} \sim \text{Single Parameter Pareto } (\alpha, \theta)$ 

• 
$$f(x) = \frac{\alpha \theta^{\alpha}}{x^{\alpha+1}}, x > \theta$$

• 
$$F(x) = 1 - (\frac{\theta}{x})^{\alpha}$$

$$\bullet E(X) = \frac{\alpha \theta}{\alpha - 1}$$

• 
$$E(X^2) = \frac{\alpha \theta^2}{\alpha - 2}$$

15. 
$$\mathbf{X} \sim LN(\mu, \sigma)$$

• 
$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x - \mu)^2/2\sigma^2}$$
, for  $x > 0$ ,  $\mu > 0$  and  $\sigma > 0$ 

• 
$$F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

$$\bullet E(X) = e^{\mu + \frac{\sigma^2}{2}}$$

$$\bullet V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

16. 
$$\mathbf{X} \sim CAU(theta, \eta)$$

• 
$$F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} (\frac{x-\eta}{\theta})$$

17.  $\mathbf{X} \sim EXP(\eta, \theta)$ 

• 
$$f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$$
  $x > \eta$ 

$$\bullet \ F(x) = 1 - e^{-\frac{x - \eta}{\theta}}$$

• 
$$M_X(t) = \frac{e^{\eta t}}{1-\theta t}$$

$$\bullet E(X) = \eta + \theta$$

• 
$$V(X) = \theta^2$$

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