**MEME16203Linear Models**

**Assignment 5**

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| Faculty: | FES | Unit Code: | MEME16203 |
| Course: | MAC | Unit Title: | Linear Models |
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| Due by: | | | |

Q1. A study was conducted on human subjects to measure the effects of 12 different foods on serum glocuse levels. On each of 6 days, 12 subjects were randomly assigned to the 12 food with one subject for each food. Each subject was given an assigned food, and then blood samples were taken from each subject at 4 time points: 15, 30, 45, and 60 minutes after consumption ofvthe food. The same process was repeated each day with 12 different subjects, so a total of 72 subjects were used in the experiment. The level of the protein of interest was measured in each of the 288 blood samples. For $i = 1, \ldots, 6$, $j = 1, \ldots, 12$, and $k = 1, \ldots, 4$, let $y_{ijk}$ be the serum glocuse measurement on day $i$ for food $j$ at time $k$. For $i = 1, \ldots, 6$, $j = 1, \ldots, 12$, and $k = 1, \ldots, 4$, consider the model

$$y_{ijk} = \mu_{jk} + d_i + e_{ijk},$$

where $\mu_{jk}$ terms are unknown fixed parameters and the other terms are random effects defined as follows. Let $\mathbf{d} = [d_1, \ldots, d_6]^T$. For $i = 1, \ldots, 6$ and $j = 1, \ldots, 12$, let $\mathbf{e}_{ij} = [e_{ij1}, \ldots, e_{ij4}]^T$.
Suppose

$$\mathbf{d} \sim N(\mathbf{0}, \sigma_d^2 \mathbf{I}_{6\times 6}),$$

and

$$\mathbf{e}_{ij} \sim N(\mathbf{0}, \mathbf{\Sigma}_e) \text{ for } i = 1, \ldots, 6 \text{ and } j = 1, \ldots, 12,$$

where $\sigma_d^2$ is an unknown positive variance parameter and

$$\mathbf{\Sigma}_e = \sigma_e^2 \begin{bmatrix} 1 & \phi & \phi^2 & \phi^3 \\ \phi & 1 & \phi & \phi^2 \\ \phi^2 & \phi & 1 & \phi \\ \phi^3 & \phi^2 & \phi & 1 \end{bmatrix}$$

for some unknown variance parameter $\sigma_e^2 > 0$ and some unknown correlation parameter $\phi$. Finally, suppose that $\mathbf{d}$ and $\mathbf{e}_{11}, \ldots, \mathbf{e}_{6,12}$ are all independent. In terms of model parameters, give a simplified expression for the variance of the generalized least squares estimator of each of the following:

(a) $\mu_{41}$ (10 marks)

(b) $\bar{\mu}_{4.}$ (10 marks)

(c) $\mu_{14} - \mu_{24}$ (10 marks)

(d) $\mu_{11} - \mu_{14}$ (10 marks)

(e) $\mu_{12} - \mu_{34}$ (10 marks)

Q2. A repeated measures study was conducted to examine the effects of 4 different store displays for a household product on sales in 8 successive time period. 32 stores were randomly selected, and 8 were assigned at random to each display. Consider the model

$$Y_{ijk} = \mu + \alpha_i + S_{ij} + \tau_k + \gamma_{ik} + e_{ijk}$$

where $y_{ijk}$ is the sales amount at the $k^{th}$ time point for the $j^{th}$ store with the $i^{th}$ store display, $\alpha_i$ is the fixed store display effect, $\tau_k$ is the fixed time effect and $\gamma_{ik}$ is the fixed store display $\times$ time effect, $S_{ij} \sim NID(0, \sigma_S^2)$ and is independent of $e_{ijk} \sim NID(0, \sigma_e^2)$.

(a) Find $V(\mathbf{Y_{ij}})$, for this model? (10 marks)

(b) Provide the formulas for the estimator of $\sigma_e^2$ and $\sigma_S^2$. (10 marks)

(c) What is the correlation between observations taken on the same store? (10 marks)

(d) Find the estimator of $V(\bar{Y}_{ij.})$ and provide it's degrees of freedom. (10 marks)

(e) Find the estimator of $V(\bar{Y}_{i.k})$ and provide it's Satterthwaith degrees of freedom. (10 marks)