MEME16203Linear Models

Assignment 5

UNIVERSITI TUNKU ABDUL RAHMAN

Faculty:	FES	Unit Code:	MEME16203
Course:	MAC	Unit Title:	Linear Models
Year:	1,2	Session:	May 2023
Due by:			

Q1. A study was conducted on human subjectsto measure the effects of 12 different foods on serum glocuse levels. On each of 6 days, 12 subjectse randomly assigned to the 12 food with one subject for each food. Each subject was given an assigned food, and then blood samples were taken from each subject at 4 time points: 15, 30, 45, and 60 minutes after consumption ofvthe food. The same process was repeated each day with 12 different subjects, so a total of 72 subjects were used in the experiment. The level of the protein of interest was measured in each of the 288 blood samples. For $i = 1, \ldots, 6$, $j = 1, \ldots, 12$, and $k = 1, \ldots, 4$, let y_{ijk} be the serum glocuse measurement on day i for food j at time k. For $i = 1, \ldots, 6$, $j = 1, \ldots, 12$, and $k = 1, \ldots, 4$, consider the model

$$y_{ijk} = \mu_{jk} + d_i + e_{ijk},$$

where μ_{jk} terms are unknown fixed parameters and the other terms are random effects defined as follows. Let $\mathbf{d} = [d_1, \ldots, d_6]^T$. For $i = 1, \ldots, 6$ and $j = 1, \ldots, 12$, let $\mathbf{e}_{ij} = [e_{ij1}, \ldots, e_{ij4}]^T$.
Suppose

$$\mathbf{d} \sim N(\mathbf{0}, \sigma_d^2 \mathbf{I}_{6\times 6}),$$

and

$$\mathbf{e}_{ij} \sim N(\mathbf{0}, \mathbf{\Sigma}_e) \text{ for } i = 1, \ldots, 6 \text{ and } j = 1, \ldots, 12,$$

where σ_d^2 is an unknown positive variance parameter and

$$\mathbf{\Sigma}_e = \sigma_e^2 \begin{bmatrix} 1 & \phi & \phi^2 & \phi^3 \\ \phi & 1 & \phi & \phi^2 \\ \phi^2 & \phi & 1 & \phi \\ \phi^3 & \phi^2 & \phi & 1 \end{bmatrix}$$

for some unknown variance parameter $\sigma_e^2 > 0$ and some unknown correlation parameter ϕ. Finally, suppose that $\mathbf{d}$ and $\mathbf{e}_{11}, \ldots, \mathbf{e}_{6,12}$ are all independent. In terms of model parameters, give a simplified expression for the variance of the generalized least squares estimator of each of the following:

(a) μ_{41}

(b) $\bar{\mu}_{4.}$

(c) $\mu_{14} - \mu_{24}$

(d) $\mu_{11} - \mu_{14}$

Q2. A repeated measures study was conducted to examine the effects of 4 different store displays for a household product on sales in 8 successive time period. 32 stores were randomly selected, and 8 were assigned at random to each display. Consider the model

$$Y_{ijk} = \mu + \alpha_i + S_{ij} + \tau_k + \gamma_{ik} + e_{ijk}$$

where y_{ijk} is the sales amount at the k^{th} time point for the j^{th} store with the i^{th} store display, α_i is the fixed store display effect, τ_k is the fixed time effect and γ_{ik} is the fixed store display $\times$ time effect, $S_{ij} \sim NID(0, \sigma_S^2)$ and is independent of $e_{ijk} \sim NID(0, \sigma_e^2)$.

(a) Find $V(\mathbf{Y_{ij}})$, for this model?

(b) Provide the formulas for the estimator of σ_e^2 and σ_S^2.

(c) What is the correlation between observations taken on the same store?

(d) Find the estimator of $V(\bar{Y}_{ij.})$ and provide it's degrees of freedom.

(e) Find the estimator of $V(\bar{Y}_{i.k})$ and provide it's Satterthwaith degrees of freedom.