UCCM3463 LOSS MODELS

1. $X \sim N(\mu, \sigma)$

- $f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, x \in R$
- $E(X) = \mu; V(X) = \sigma^2$
- $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

2. $X \sim Gamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-\frac{x}{\theta}}, x > 0$
- $F(x) = 1 - \sum_{j=0}^{\alpha-1} \frac{\left(\frac{x}{\theta}\right)^j e^{-\frac{x}{\theta}}}{j!}$
- $E(X) = \alpha\theta; V(X) = \alpha\theta^2$
- $E(X^k) = \theta^k \alpha(\alpha+1)\cdots(\alpha+k-1)$
- $M_X(t) = \frac{1}{(1-\theta t)^\alpha}$
- When $\alpha = 1$, $X \sim Exp(\theta)$
  - $f(x) = \frac{1}{\theta} e^{-x/\theta}$
  - $F(x) = 1 - e^{-x/\theta}$

3.$X \sim InvGamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\frac{\theta}{x}}, x > 0$
- $E(X^k) = \frac{\theta^k}{(\alpha-1)\cdots(\alpha-k)}$, if k is a positive integer
- When $\alpha = 1$, $X \sim InvExp(\theta)$
  - $f(x) = \theta x^{-2} e^{-\frac{\theta}{x}}$
  - $F(x) = e^{-\frac{\theta}{x}}$

4.$X \sim Pareto(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, x > 0$
- $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha$
- $E(X^k) = \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}$
- $E(X \wedge x) = \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right]$

5. $X \sim SingleParameterPareto(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
- $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x}\right)^\alpha$
- $E(X^k) = \frac{\alpha\theta^k}{\alpha-k}, k < \alpha$

6. $X \sim Beta(a, b, \theta)$

- $f(x|a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)\theta^{a+b-1}} x^{a-1}(\theta - x)^{b-1}, 0 < x < \theta$
- $E(X^k) = \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}$ if k is positve integer.

7. $X \sim LogNormal(\mu, \sigma)$

- $f(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, x > 0$
- $F(x) = \Phi(\frac{\ln x-\mu}{\sigma})$
- $E(X^k) = e^{k\mu+\frac{1}{2}k^2\sigma^2}$

$\bullet E(X \wedge x)^k$
$= e^{k\mu+\frac{1}{2}k^2\sigma^2}\Phi(\frac{\ln x-\mu-k\sigma^2}{\sigma})+x^kS(x)$

$8. X \sim Weibull(\tau, \theta)$

$\bullet f(x|\tau,\theta) = \frac{\tau}{\theta^\tau}x^{\tau-1}e^{-(x/\theta)^\tau}, x > 0$

$\bullet F(x) = 1 - e^{-(x/\theta)^\tau}$

$\bullet E(X^k) = \theta^k\Gamma(1 + k/\tau), k > -\tau$

$9. X \sim InvGaussian(\mu, \theta)$

$\bullet f(x) = \left(\frac{\theta}{2\pi x^3}\right)^{\frac{1}{2}} \exp\left(-\frac{\theta z^2}{2x}\right), z = \frac{x-\mu}{\mu}$

$\bullet E(X) = \mu; V(X) = \frac{\mu^3}{\theta}$

$\bullet E(X \wedge x) = x - \mu z\Phi\left[z\left(\frac{\theta}{x}\right)^{\frac{1}{2}}\right] - \mu y \exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{\frac{1}{2}}\right], \quad y =$

$\frac{x+\mu}{\mu}$

10.$X \sim POI(\lambda)$

- $f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, \ldots$
- $E(X) = V(X) = \lambda$
- $P_N(z) = e^{\lambda(z-1)}$
- $M_N(t) = e^{\lambda(e^t-1)}$

11.$X \sim Bin(m, q)$

- $f(x|q) = \binom{m}{x} q^x (1-q)^{m-x}, x = 0, 1, \ldots, m$
- $E(X) = mq; V(X) = mq(1-q)$
- $P_N(z) = [1 + q(z-1)]^m$
- $M_N(t) = [1 + q(e^t - 1)]^m$

12.$X \sim NB(r, \beta)$

- $f(x|\beta) = \frac{r(r+1)\cdots(r+x-1)\beta^x}{x!(1+\beta)^{r+x}}$

$$= \binom{r+x-1}{x}\frac{\beta^x}{(1+\beta)^{r+x}}, x = 0, 1, \ldots$$

$$= \binom{r+x-1}{x}\left(\frac{1}{1+\beta}\right)^r\left(\frac{\beta}{1+\beta}\right)^x$$

- $E(X) = r\beta; V(X) = r\beta(1+\beta)$
- $P_N(z) = [1-\beta(Z-1)]^{-r}$
- $M_N(t) = [1-\beta(e^t-1)]^{-r}$