

**MEME15203 Statistical Inference****Assignment 4****UNIVERSITI TUNKU ABDUL RAHMAN**

Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
Session:	January 2022		
Due by:	7/4/2022		

- Q1. Let  $X \sim NB(r, 0.49)$ . Derive the most powerful test of size  $\alpha = 0.134$  of  $H_0 : r = 1$  against  $H_1 : r = 3$  based on an observed value of  $X$ . Compute the power of this test for the alternative  $r = 3$ . (20 marks)
- Q2. Consider a random sample of size  $n$  from a uniform distribution,  $X_i \sim U(0, \theta)$ . Find the UMP test of size  $\alpha$  of  $H_0 : \theta \leq \theta_0$  versus  $H_1 : \theta > \theta_0$  by first deriving a most powerful test of simple hypotheses and then extending it to composite hypotheses. (20 marks)
- Q3. Let  $X_1, X_2, \dots, X_n$  denote a random sample from a normal distribution with mean  $\mu$ (unknown) and variance  $\sigma^2$ . For testing  $H_0 : \sigma^2 = \sigma_0^2$  against  $H_1 : \sigma^2 < \sigma_0^2$ , show that the likelihood ratio test is equivalent to the  $\chi^2$  test. (20 marks)
- Q4. Let  $X_1, \dots, X_{20}$  denote a random sample from a Weibull distribution,  $X_i \sim WEI(2, \theta)$ . Show that a UMP size 0.03 test of  $H_0 : \theta \geq 2$  versus  $H_1 : \theta < 2$  using Theorem 3 is  $\{\sum X_i^2 \leq k\}$ , and then determine  $k$ . (20 marks)
- Q5. Consider a random sample of size  $n$  from a Bernoulli distribution,  $X_i \sim BIN(10, p)$ . Derive a UMP test of  $H_0 : p \geq p_0$  versus  $H_1 : p < p_0$  using monotone likelihood ratio property. (10 marks)
- Q6. If  $X_i | \lambda \sim POI(\lambda)$  and a Bayesian uses a prior for  $\lambda$  that is Gamma with parameters  $\alpha = 7$  and  $\theta = \frac{1}{100}$ , suppose  $x_1, x_2, \dots, x_n$  have been observed, what is the Bayes test of  $H_0 : \lambda \leq 5$  versus  $H_1 : \lambda > 5$ ? (10 marks)