1. $\mathbf{X} \sim Bernoulli(p)$
   - $f(x) = p^x q^{1-x}, x = 0, 1$
   - $M_X(t) = pe^t + q$
   - $E(X) = p$
   - $V(X) = pq$

2. $\mathbf{X} \sim Binomial(n, p)$
   - $f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \ldots, n$
   - $M_X(t) = (pe^t + q)^n$
   - $E(X) = np$
   - $V(X) = npq$

3. $\mathbf{X} \sim HYP(n, M, N)$
   - $f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$,
     $x = 0, 1, \ldots, \min(n, M), n - x \leq N - M.$
   - $E(X) = \frac{nM}{N}$
   - $V(X) = n\frac{M}{N}\left(1 - \frac{M}{N}\right)\frac{N-n}{N-1}$

4. $\mathbf{X} \sim GEO(p)$
   - $f(x) = pq^{x-1} \quad x = 1, 2, 3, \ldots$
   - $F(x) = 1 - q^x \quad x = 1, 2, 3, \ldots$
   - $M_X(t) = \frac{pe^t}{1-qe^t}$
   - $E(X) = \frac{1}{p}$
   - $V(X) = \frac{q}{p^2}$

5. $\mathbf{X} \sim NegativeBinomial(r, p)$
   - $f(x) = \binom{x-1}{r-1} p^r q^{x-r}, x = r, r+1, \ldots$
   - $M_X(t) = \left(\frac{pe^t}{1-qe^t}\right)^r$
   - $E(X) = \frac{r}{p}$
   - $V(X) = \frac{rq}{p^2}$

6. $\mathbf{X} \sim POI(\mu)$
   - $f(x) = \frac{e^{-\mu}\mu^x}{x!} \quad x = 0, 1, 2, \ldots$
   - $M_X(t) = e^{\mu(e^t-1)}$
   - $E(X) = \mu$
   - $V(X) = \mu$

MEME15203 Statistical Inference©Dr Yong Chin Khian

7. $\mathbf{X} \sim DU(N)$

- $f(x) = \frac{1}{N}, X = 1, 2, \ldots, N$
- $M_X(t) = \frac{1}{N}\frac{e^t - e^{(N+1)t}}{1-e^t}$
- $F(x) = \frac{x(1+x)}{2N}$
- $E(X) = \frac{N+1}{2}$
- $V(X) = \frac{N^2-1}{12}$

8. $\mathbf{X} \sim U(a, b)$

- $f(x) = \frac{1}{b-a}, a < x < b$ and zero otherwise
- $F(x) = \frac{x-a}{b-a}, a < x < b$
- $M_X(t) = \frac{e^{tb} - e^{ta}}{b-a}$
- $E(X) = \frac{a+b}{2}$
- $V(X) = \frac{(b-a)^2}{12}$

9. $\mathbf{X} \sim Gamma(\alpha, \theta)$

- $f(x) = \frac{1}{\theta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\theta}, x > 0$
- $F(x) = 1 - \sum_{i=0}^{\alpha-1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$
- $M_X(t) = (\frac{1}{1-\theta t})^\alpha$
- $E(X) = \alpha\theta$

- $V(X) = \alpha\theta^2$

10. $\mathbf{X} \sim EXP(\theta)$

- $f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ and zero otherwise.
- $F(x) = 1 - e^{-x/\theta}, x > 0$
- $M_X(t) = \left(\frac{1}{1-\theta t}\right)$
- $E(X) = \theta,$
- $V(X) = \theta^2$

11. $\mathbf{X} \sim WEI(\tau, \theta)$

- $f(x) = \frac{\tau}{\theta^\tau} x^{\tau-1} e^{-(x/\theta)^\tau}, x > 0$ and zero otherwise.
- $F(x) = 1 - e^{-(x/\theta)^\tau}$
- $E(X) = \theta\Gamma\left(1 + \frac{1}{\tau}\right)$
- $E(X^2) = \theta^2\left[\Gamma\left(1 + \frac{2}{\tau}\right) - \Gamma^2\left(1 + \frac{1}{\tau}\right)\right]$

12. $\mathbf{X} \sim PAR(\alpha, \theta)$

- $f(x) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, x > 0$
- $F(x) = 1 - (\frac{\theta}{x+\theta})^\alpha$
- $E(X) = \frac{\theta}{\alpha-1}$

- $E(X^2) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)}$
- $V(X) = \frac{\theta^2}{(\alpha-1)^2(\alpha-2)}$

13. $\mathbf{X} \sim Beta(a, b)$
    - $f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$, $0 < x < 1$
    - $E(X) = \frac{a}{a+b}$
    - $V(X) = \frac{ab}{(a+b)^2(a+b+1)}$

14. $\mathbf{X} \sim N(\mu, \sigma^2)$
    - $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, $x \in \mathbb{R}$, $\mu \in \mathbb{R}$ and $\sigma > 0$.
    - $F(x) = \Phi(\frac{x-\mu}{\sigma})$
    - $M_X(t) = e^{\mu t + \sigma^2 t^2/2}$
    - $E(X) = \mu$
    - $V(X) = \sigma^2$

15. $\mathbf{X} \sim LN(\mu, \sigma)$
    - $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x - \mu)^2/2\sigma^2}$, $x > 0$, $\mu > 0$ and $\sigma > 0$
    - $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$
    - $E(X) = e^{\mu + \frac{\sigma^2}{2}}$
    - $V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$

16. $\mathbf{X} \sim CAU(\theta, \eta)$
    - $f(x) = \frac{1}{\theta\pi\left[1 + \left(\frac{x-\eta}{\theta}\right)^2\right]}$
    - $F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1}(\frac{x-\eta}{\theta})$

17. $\mathbf{X} \sim EXP(\eta, \theta)$
    - $f(x) = \frac{1}{\theta} e^{-\frac{x-\eta}{\theta}} \quad x > \eta$
    - $F(x) = 1 - e^{-\frac{x-\eta}{\theta}}$
    - $M_X(t) = \frac{e^{\eta t}}{1-\theta t}$
    - $E(X) = \eta + \theta$
    - $V(X) = \theta^2$

18. $\mathbf{X} \sim DE(\eta, \theta)$

- $f(x) = \frac{1}{2\theta} e^{-|x-\eta|/\theta} \qquad -\infty < x < \infty$ and zero otherwise.
- $F_X(x) = \begin{cases} \frac{1}{2} e^{(x+\eta)/\theta}, & x \leq \eta \\ \frac{1}{2}[1 - e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$
- $M_X(t) = \frac{e^{\eta t}}{1-\theta^2 t^2}$
- $E(X) = \eta$
- $V(X) = 2\theta^2$

19. $\mathbf{X} \sim$ Single Parameter Pareto (α, θ)

- $f(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
- $F(x) = 1 - (\frac{\theta}{x})^\alpha$
- $E(X) = \frac{\alpha\theta}{\alpha-1}$
- $E(X^2) = \frac{\alpha\theta^2}{\alpha-2}$