Common Distributions

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1. $\mathbf{X} \sim Bernoulli(p)$

•
$$f(x) = p^x q^{1-x}, x = 0, 1$$

$$\bullet \ M_X(t) = pe^t + q$$

$$\bullet$$
 $E(X) = p$

$$\bullet V(X) = pq$$

2. $\mathbf{X} \sim Binomial(n, p)$

•
$$f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$$

$$\bullet M_X(t) = (pe^t + q)^n$$

$$\bullet$$
 $E(X) = np$

$$\bullet V(X) = npq$$

3. $\mathbf{X} \sim HYP(n, M, N)$

•
$$f(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}},$$

 $x = 0, 1, \dots, \min(n, M), n - x \le N - M.$

$$\bullet E(X) = \frac{nM}{N}$$

•
$$V(X) = n \frac{M}{N} \left(1 - \frac{M}{N} \right) \frac{N-n}{N-1}$$

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4. $\mathbf{X} \sim GEO(p)$

•
$$f(x) = pq^{x-1}$$
 $x = 1, 2, 3, ...$

•
$$F(x) = 1 - q^x$$
 $x = 1, 2, 3, ...$

$$\bullet M_X(t) = \frac{pe^t}{1 - qe^t}$$

$$\bullet E(X) = \frac{1}{p}$$

$$\bullet V(X) = \frac{q}{p^2}$$

5. $\mathbf{X} \sim NegativeBinomial(r, p)$

•
$$f(x) = {x-1 \choose r-1} p^r q^{x-r}, x = r, r+1, \dots$$

•
$$M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$$

$$\bullet$$
 $E(X) = \frac{r}{p}$

$$\bullet \ V(X) = \frac{rq}{p^2}$$

6. $\mathbf{X} \sim POI(\mu)$

•
$$f(x) = \frac{e^{-\mu}\mu^x}{x!}$$
 $x = 0, 1, 2, ...$

$$\bullet M_X(t) = e^{\mu(e^t - 1)}$$

$$\bullet$$
 $E(X) = \mu$

$$\bullet V(X) = \mu$$

7. $\mathbf{X} \sim DU(N)$

•
$$f(x) = \frac{1}{N}, X = 1, 2, \dots, N$$

•
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$

$$\bullet F(x) = \frac{x(1+x)}{2N}$$

$$\bullet E(X) = \frac{N+1}{2}$$

$$\bullet V(X) = \frac{N^2 - 1}{12}$$

8. **X** ~ U(a, b)

•
$$f(x) = \frac{1}{b-a}$$
, $a < x < b$ and zero otherwise

•
$$F(x) = \frac{x-a}{b-a}$$
, $a < x < b$

$$\bullet \ M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$$

•
$$E(X) = \frac{a+b}{2}$$

$$V(X) = \frac{(b-a)^2}{12}$$

9. $\mathbf{X} \sim Gamma(\alpha, \theta)$

•
$$f(x) = \frac{1}{\theta^{\alpha} \Gamma(\alpha)} x^{\alpha - 1} e^{-x/\theta}, x > 0$$

•
$$F(x) = 1 - \sum_{i=0}^{\alpha - 1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$$

•
$$M_X(t) = (\frac{1}{1-\theta t})^{\alpha}$$

$$\bullet$$
 $E(X) = \alpha \theta$

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•
$$V(X) = \alpha \theta^2$$

10. $\mathbf{X} \sim EXP(\theta)$

• $f(x) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ and zero otherwise.

•
$$F(x) = 1 - e^{-x/\theta}, x > 0$$

•
$$M_X(t) = \left(\frac{1}{1-\theta t}\right)$$

$$\bullet$$
 $E(X) = \theta,$

•
$$V(X) = \theta^2$$

11. $\mathbf{X} \sim WEI(\tau, \theta)$

• $f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau - 1} e^{-(x/\theta)^{\tau}}, x > 0$ and zer otherwise.

$$\bullet F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

•
$$E(X) = \theta \Gamma \left(1 + \frac{1}{\tau} \right)$$

•
$$E(X^2) = \theta^2 \left[\Gamma \left(1 + \frac{2}{\tau} \right) - \Gamma^2 \left(1 + \frac{1}{\tau} \right) \right]$$

12. $\mathbf{X} \sim PAR(\alpha, \theta)$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$$

•
$$F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$$

•
$$E(X) = \frac{\theta}{\alpha - 1}$$

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•
$$E(X^2) = \frac{2\theta^2}{(\alpha - 1)(\alpha - 2)}$$

•
$$V(X) = \frac{\theta^2}{(\alpha - 1)^2(\alpha - 2)}$$

13. $\mathbf{X} \sim Beta(a,b)$

•
$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}, \ 0 < x < 1$$

$$\bullet$$
 $E(X) = \frac{a}{a+b}$

$$\bullet V(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

14. **X** ~ $N(\mu, \sigma^2)$

•
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, x \in \mathbb{R}, \mu \in \mathbb{R}$$

and $\sigma > 0$.

$$\bullet F(x) = \Phi(\frac{x-\mu}{\sigma})$$

•
$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$\bullet$$
 $E(X) = \mu$

•
$$V(X) = \sigma^2$$

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15. $\mathbf{X} \sim LN(\mu, \sigma)$

•
$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x - \mu)^2/2\sigma^2}, x > 0, \mu = 0$$
 and $\sigma > 0$

•
$$F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

$$\bullet E(X) = e^{\mu + \frac{\sigma^2}{2}}$$

$$V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

16. $\mathbf{X} \sim CAU(\theta, \eta)$

•
$$f(x) = \frac{1}{\theta \pi \left[1 + \left(\frac{x - \eta}{\theta}\right)^2\right]}$$

•
$$F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} (\frac{x-\eta}{\theta})$$

17. $\mathbf{X} \sim EXP(\eta, \theta)$

•
$$f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$$
 $x > \eta$

$$\bullet F(x) = 1 - e^{-\frac{x - \eta}{\theta}}$$

•
$$M_X(t) = \frac{e^{\eta t}}{1-\theta t}$$

$$\bullet$$
 $E(X) = \eta + \theta$

•
$$V(X) = \theta^2$$

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18. $\mathbf{X} \sim DE(\eta, \theta)$

- $f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$ $-\infty < x < \infty$ and zero otherwise.
- $F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \leq \eta \\ \frac{1}{2}[1 e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$
- $\bullet M_X(t) = \frac{e^{\eta t}}{1 \theta^2 t^2}$
- \bullet $E(X) = \eta$
- $V(X) = 2\theta^2$
- 19. $\mathbf{X} \sim \text{Single Parameter Pareto}(\alpha, \theta)$
 - $f(x) = \frac{\alpha \theta^{\alpha}}{r^{\alpha+1}}, x > \theta$
 - $F(x) = 1 (\frac{\theta}{x})^{\alpha}$
 - $E(X) = \frac{\alpha \theta}{\alpha 1}$
 - $E(X^2) = \frac{\alpha \theta^2}{\alpha 2}$