

**MEME15203 Statistical Inference****Assignment 5****UNIVERSITI TUNKU ABDUL RAHMAN**

Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Due by:	2/4/2024		

Q1. Let  $X_1, X_2, \dots, X_{150}$  be a random sample from a Weibull distribution,  $X \sim WEI(\theta, 5)$ . Derive an equal tailed  $100\gamma\%$  confidence interval for  $\theta$  using a pivotal quantity.

(30 marks)

Q2. Consider a random sample of size 22 from a uniform distribution,  $X_i \sim U(0, \theta)$ ,  $\theta > 0$ , and let  $X_{n:n}$  be the largest order statistic. Find the constant  $c$  such that  $(x_{n:n}, cx_{n:n})$  is a 99% confidence interval for  $\theta$ .

(30 marks)

Q3. Let  $X_1, \dots, X_n$ , be a random sample from a gamma distribution with parameters  $\alpha = 4$  and unknown  $\theta$ .

(a) Find a pivotal quantity for the parameter  $\theta$  based on the sufficient statistic.

(b) Derive an equal tail 98% confidence interval for  $\theta$  based on the pivotal quantity from part (a).

(15 marks)

Q4. You are given the following:

$$f(x|\theta) = \frac{5x^4}{\theta^5}, 0 < x < \theta.$$

$$\pi(\theta) = \frac{6}{\theta^7}, \theta > 1.$$

Three observations were observed: 500, 700, 1000. Find a 96% "HPD" credible set for  $\theta$ .

(15 marks)

Q5. Let  $X_1, \dots, X_n$ , be a random sample from  $N(\theta, 50)$ . Assume that the prior distribution of  $\Theta$  is  $N(60, 16)$ .

(a) Derive the posterior distribution of  $\Theta$ .

(b) Find a Bayesian interval of  $\theta$  with confidence coefficient  $1 - \alpha$ .

(c) Find the corresponding non-Bayesian confidence interval of  $\theta$  using pivotal quantity method.

(10 marks)