Test 1 Marking Guide

Name: Student ID: Mark: /100

FACULTY:	FES, UTAR	COURSE CODE:	UECM2453
PROGRAMME/YEAR:	AS /Y2	COURSE TITLE:	FINANCIAL ECONOMICS II
SESSION:	202301	LECTURER:	DR YONG CHIN KHIAN

CO1: Explain the properties of the lognormal distribution and its applicability to option pricing.

1. [Fill in the blank with correct answer] Suppose that X follows the stochastic differential equation

$$dX(t) = -13dt + 4dZ(t)$$

where $Z(t)$ is a standard Brownian motion.
Let $W(t) = e^{2tX(t)}$. If

$$dW(t) = a[W(t), t]dt + b[W(t), t]dZ(t),$$

find $a(10, 9)$. 23582.56 (7 marks)

2. [Fill in the blank with correct answer] You are given:

- $S(t)$ is the time-t price of a stock.
- The stock pays dividend continuously at a constant rate proportional to its price.
- The true stock price process is given by

$$\frac{dS(t)}{S(t)} = cdt + \sigma dZ(t)$$

where $Z(t)$ is a standard Brownian motion under the true probability measure, and c and σ are constant.
- The risk-neutral stock price process is given by

$$\frac{dS(t)}{S(t)} = 0.06dt + 0.13d\tilde{Z}(t)$$

where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.
- $Z(4) = \tilde{Z}(4) - 1.92$.

Find c. 0.12 (7 marks)

3. [Fill in the blank with correct answer] Let $S(t)$ be the time-t price of a nondividend-paying stock, you are given that:

- The stock price process under the true probability measure is

$$d[\ln S(t)] = 0.04555dt + 0.17dZ(t), S(0) = 1$$

where $Z(t)$ is a standard Brownian motion under the true probability measure.
- The sharpe ratio stock price risk is 0.04118.

Compute the price of a contingent claim that pays $\sqrt[3]{S(6)}$ at time 6. 0.79 (7 marks)

4. [Fill in the blank with correct answer] Suppose that X follows the stochastic differential equation

$$dX(t) = -14dt + 4dZ(t)$$

where $Z(t)$ is a standard Brownian motion.
Let $W(t) = e^{3tX(t)}$. If

$$dW(t) = a[W(t), t]dt + b[W(t), t]dZ(t),$$

find $a(8, 8)$. 34178.08 (7 marks)

5. [Fill in the blank with correct answer] Let $S(t)$ be the time-t price of a nondividend-paying stock, you are given that:

- The stock price process is

$$d[\ln S(t)] = 0.31dZ(t)$$

where $Z(t)$ is a standard Brownian motion under the true probability measure.
- The continuously compounded risk-free of interest is 0.039

If $F_{0,4}^P(S^4) = e^{-\gamma}E[S^4(4)]$, find γ. 0.30 (7 marks)

6. [Fill in the blank with correct answer] You are given:

- $S(t)$ is the time-t price of a nondividend-paying stock.
- $S(t)$ follows a geometric Brownian motion.
- The current stock price is 43.
- The expected return on the stock is 0.1.
- The stock's volatility is 0.32.

Calculate $E[S(7)I(S(7) > 43)]$. 77.45 (7 marks)

7. [Show your workings. If no workings are shown, ZERO is awarded] Stock prices follow geometric Brownian motion:

$$d\ln S(t) = 0.044dt + 0.28dZ(t)$$

Suppose $S(0) = 46$. Calculate $P[S(2) < 44)$.

(14 marks)

Ans.

$\hat{d}_2 = \frac{\ln(46/44)+(0.044)(2)}{0.28\sqrt{2}} = 0.33$

$P[S(2) < 44) = N(-\hat{d}_2) = N(-0.33) = \boxed{0.3707}$

$S(t) = 46e^{0.044t+0.28Z(t)}$

$P[S(2) < 44)$
$= P\left[\ln S(2) < \ln 44\right]$
$= P\left[\ln 46 + 0.044(2) + 0.28Z(2) < \ln 44\right]$
$= P(Z(2) < -0.473)$
$= P(Z < \frac{-0.473}{\sqrt{2}})$
$= N(-0.33)$
$= \boxed{0.3707}$

8. [Show your workings. If no workings are shown, ZERO is awarded] You are given:

   - $S(t) = S(0)e^{0.11t+0.22Z(t)}$
   - $\delta = 0.03$
   - $F_{t,T}$ is a forward on the stock.
   - $r = 0.06$

   $d(\ln F)$ follows the process $\alpha dt + \sigma dZ(t)$. Determine α.

   (15 marks)

   Ans.

   $\alpha_S - 0.03 - 0.5(0.22^2) = 0.11$
   $\alpha_S = 0.1642$
   $\frac{dF_{t,T}}{F_{t,t}} = (\alpha_S - r)dt + \sigma dZ(t) = (0.1642 - 0.06)dt + 0.22dZ(t)$
   $d(\ln F) = (\alpha_S - r - \frac{1}{2}\sigma^2)dt + \sigma dZ(t) = (0.1642 - 0.06 - .5(0.22^2)dt + 0.22dZ(t)$
   $\alpha = 0.1642 - 0.06 - .5(0.22^2) = \boxed{0.08}$

9. [Show your workings. If no workings are shown, ZERO is awarded] You are given the following information for two nondividend paying stocks X_1 and X_2 with prices $S_1(t)$ and $S_2(t)$ respectively:

$$\frac{dS_1(t)}{S_1(t)} = 0.13dt + 0.31dZ(t); \frac{dS_2(t)}{S_2(t)} = 0.02dt - \sigma dZ(t)$$

$$S_1(0) = 110, S_2(0) = 55.0, r = 0.04$$

   A risk-free portfolio consists of one share of X_1 and c shares of X_2. The cost of this portfolio is borrowed at the risk-free rate so that the net cost outlay is zero. Determine the amount borrowed.

   (14 marks)

   Ans.

   By equality of sharpe ratios,
   $\frac{0.13-0.04}{0.31} = \frac{0.02-0.04}{\sigma}$
   $\sigma = -0.07$
   $c = N(t) = -\frac{\sigma_1 S_1(t)}{\sigma_2 S_2(t)} = -\frac{0.31(110)}{-0.07(55.0)} = 9.0$
   $W = -S_1(t) - cS_2(t) = -1(110) - 9.0(55.0) = -605.0$
   Thus the amount borrowed is $\boxed{605.0}$

10. [Show your workings. If no workings are shown, ZERO is awarded] A forward agreement entered into at time t provides for the exchange of $N(t)$ shares of Fedelity stock for 1 share of Aberdeen stock at time T, $T > t$, with $N(t)$ selected to allow no arbitrage. You are given

- The time-t price of Fedelity stock is $X(t)$, and $X(t)$ satisfies
$$\frac{dX(t)}{X(t)} = 0.14dt + 0.15dZ(t)$$
- The time-t price of Aberdeen stock is $Y(t)$, and $Y(t)$ satisfies
$$\frac{dY(t)}{Y(t)} = 0.25dt + 0.3dZ(t)$$
- Fedelity pays continuous dividends proportional to its price at a rate of 0.014.
- Aberdeen pays continuous dividends proportional to its price at a rate of 0.026.

$N(t)$ satisfies
$$\frac{dN(t)}{N(t)} = \alpha dt + \beta dZ(t).$$

Determine α.

(15 marks)

Ans.

At time-t, there are two ways to acquire 1 share of Aberdeen stock at time T:

(a) Buy $e^{-0.026(T-t))}$ shares of Aberdeen immediately and hold them until time T.

(b) Buy $N(t)e^{-0.014(T-t)}$ shares of Fedelity immediately, and enter into the specified format agreement.

These two ways must have equal cost to avoid arbitrage. So
$N(t)e^{-0.014(T-t)}X(t) = e^{-0.026(T-t))}Y(t)$
$N(t) = \frac{Y(t)e^{-0.012(T-t)}}{X(t)}$
$\ln N(t) = \ln Y(t) - 0.012(T-t) - \ln X(t)$

$$\begin{aligned} d\ln N(t) &= d\ln Y(t) + 0.012dt - d\ln X(t) \\ &= (0.25 - 0.3^2/2)dt + 0.3dZ(t) + 0.012dt - [(0.14 - 0.15^2/2)dt + 0.15dZ(t)] \\ &= 0.0882dt + 0.15dZ(t) \end{aligned}$$

$\frac{dN(t)}{N(t)} = (0.0882 + (0.15)^2/2)dt + 0.15dZ(t)$
$\alpha = (0.0882 + (0.15)^2/2) = \boxed{0.0994}$