Test 1 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Unit Code:	UECM3463
Course/Year:	AS /Y2 & Y3	Unit Title:	Loss Models
Session:	202306	Lecturer:	Dr Yong Chin Khian

1. CO1: Calculate expected values, variances and probabilities for frequency random variables.

   (a) [Fill in the blank with correct answer] For a certain $(a, b, 0)$ distribution,

   - $a = 0.71671$,
   - $b = 1.43342$, and
   - $1000p_0 = 22.734$.

   Calculate the probability of exactly 3 events occurring times 1000, i.e. $1000p_3$. 83.697663 (6 marks)

   (b) [Fill in the blank with correct answer] N^M is a discrete random variable with probability function which is a member of the $(a, b, 1)$ class of distributions. You are given

   $$P(z) = 0.36 + 0.64\left[\frac{e^{5.90(z-1)]} - e^{-5.90}}{1 - e^{-5.90}}\right]$$

   Calculate the variance of the distribution. 11.789200 (7 marks)

   (c) [Fill in the blank with correct answer] A random variable follows a zero-modified Poisson distribution with $\lambda = 0.62$ and $p_0^M = 0.73$. Calculate the third raw moment of the distribution. 1.175427 (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] For a zero-modified Negative Binomial distribution with parameters $r = 2$ and β. You are given $p_0 = 0.44$ and $p_1 = 0.0102$. Find β.

(15 marks)

Ans.

$p_1^M = \frac{(1-p_0^M)}{1-p_0} p_1$ .... [1m]

$0.0102 = (1 - 0.44)\left(\frac{2\beta}{(1+\beta)^3-(1+\beta)}\right)$ .... [1m]

$\frac{1}{(1+\beta)(2+\beta)} = \frac{0.0102}{2(1-0.44)}$ .... [1m]

$2 + 3\beta + \beta^2 = 110$ .... [1m]

$\beta^2 + 3\beta - 108 = 0$ .... [1m]

$\beta = \frac{-3+\sqrt{9+4(108)}}{2} = \boxed{9.00}$ .... [1m]

(e) [Show your workings. If no workings are shown, ZERO is awarded] For a zero-modified Poisson distribution, $p_1 = 0.0038$, $p_2 = 0.0114$, calculate the variance of the distribution.

(15 marks)

Ans.

$N^M \sim ZM - POI(\lambda) \Rightarrow a = 0$ and $\lambda = b$ .... [1m]

$\frac{p_2}{p_1} = \frac{0.0114}{0.0038} = \frac{b}{2} \Rightarrow b = 6.0$ .... [1m]

$p_1^M = cp_1$

$0.0038 = c(6.0)e^{-6.0} \Rightarrow c = \frac{0.0038e^{6.0}}{6.0} = 0.2555$ .... [1m]

$E(N^M) = cE(N) = 0.2555(6.0) = 1.533$ .... [1m]

$E(N^M)^2 = cE(N^2) = 0.2555(6.0 + 6.0^2) = 10.731$ .... [1m]

$V(N^M) = E(N^M)^2 - (E(N^M))^2 = \boxed{8.3809}$ .... [1m]

2. CO2: Calculate expected values, variances, probabilities, and percentiles for severity random variable defined above.

(a) [Fill in the blank with correct answer] Claim severity has the following distribution:

Claim Size	270.0	283.5	297.0	310.5	324.0
Probability	0.34	0.24	0.17	0.13	0.12

Determine the distribution's skewness. 0.548018 (6 marks)

(b) [Fill in the blank with correct answer] X is a random variable representing loss size. You are given that

$$E[X \wedge d] = 393 - \frac{262^3}{2d^2}$$

Loss sizes are affected by 13% inflation. Determine the average payment per loss under a policy with 301 ordinary deductible after inflation. (6 marks) 143.212755 (7 marks)

(c) [Fill in the blank with correct answer] The distribution of X is specified by it's hazard rate function

$$h(x) = \frac{xe^{-0.6x}}{\int_x^\infty se^{-0.6s}ds}, x > 0$$

Calculate $E(X-4)_+$. (6 marks) 0.665300 (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Annual losses follow a Pareto distribution with $\alpha = 5.30$ and $\theta = 1{,}060$. Calculate the difference between $TVaR_{0.96}$ and $VaR_{0.96}$.

(15 marks)

Ans.

$TVaR_p = VaR_p + e(VaR_p)$
$TVaR_p - VaR_p = e(VaR_p)$
.... [1m]
Let $\pi_p = VaR_p$
$X \sim Pareto(\alpha = 5.3, \theta = 1,060)$
$S(\pi_{0.96}) = 1 - 0.96$ .... [1m]
$\left(\frac{1060}{\pi_{0.96}+1060}\right)^{5.3} = 0.04$ .... [1m]
$\pi_p = 885.67$ .... [1m]
$e(\pi_p) = \frac{\theta+\pi_p}{\alpha-1}$ .... [1m]
$e(\pi_{0.96}) = \frac{1060+885.67}{5.3-1} = \boxed{452.48}$ .... [1m]

(e) [Show your workings. If no workings are shown, ZERO is awarded] You are given that the moment generating function of the random variable X is

$$M_X(t) = \exp\left[\frac{\theta}{\mu}\left(1 - \sqrt{1 - \frac{2t\mu^2}{\theta}}\right)\right].$$

Show that the third raw moment of X is $\frac{3\mu^5}{\theta^2} + \frac{3\mu^4}{\theta} + \mu^3$.

(15 marks)

Ans.

$M_X(t) = \exp\left[\frac{\theta}{\mu}\left(1 - \sqrt{1 - \frac{2t\mu^2}{\theta}}\right)\right]$

$k_X(t) = \ln(M_X(t)) = \frac{\theta}{\mu}\left(1 - \left(1 - \frac{2t\mu^2}{\theta}\right)^{1/2}\right)$

$k'_X(t) = \frac{-\theta}{\mu}\left(\frac{1}{2}\right)\left(1 - \frac{2t\mu^2}{\theta}\right)^{-1/2}\left(\frac{-2\mu^2}{\theta}\right) = \mu\left(1 - \frac{2t\mu^2}{\theta}\right)^{-1/2}$

$E(X) = k'_X(0) = \mu$

$k"_X(t) = -\frac{1}{2}\mu\left(1 - \frac{2t\mu^2}{\theta}\right)^{-3/2}\left(\frac{-2\mu^2}{\theta}\right) = \frac{\mu^3}{\theta}\left(1 - \frac{2t\mu^2}{\theta}\right)^{-3/2}$

$V(X) = \frac{\mu^3}{\theta}$

$E(X^2) = \frac{\mu^3}{\theta} + \mu^2$

$k^{(3)}_X(t) = \left(\frac{-3}{2}\right)\left(\frac{\mu^3}{\theta}\right)\left(1 - \frac{2t\mu^2}{\theta}\right)^{-5/2}\left(\frac{-2\mu^2}{\theta}\right) = \frac{3\mu^5}{\theta^2}\left(1 - \frac{2t\mu^2}{\theta}\right)^{-5/2}$

$E(X-\mu)^3 = k^{(3)}_X(0) = \frac{3\mu^5}{\theta^2}$

$E(X^3) - 3E(X^2)E(X) + 2[E(X)]^3 = \frac{3\mu^5}{\theta^2}$

$E(X^3) = \frac{3\mu^5}{\theta^2} + 3\left[\frac{\mu^3}{\theta} + \mu^2\right](\mu) - 2\mu^3 = \frac{3\mu^5}{\theta^2} + \frac{3\mu^4}{\theta} + \mu^3$