Test 2 Marking Guide

Name: Student ID: Mark: /100

Faculty:	LKCFES, UTAR	Unit Code:	UECM3463
Course/Year:	AS /Y2 & Y3	Unit Title:	Loss Models
Session:	202306	Lecturer:	Dr Yong Chin Khian

CO3: Formulate compound random variables including recursion for aggregate deductibles (stop-loss insurance); variances; and probabilities.

1. [Fill in the blank with correct answer] Aggregate claim frequency for an employee dental coverage covering **35** individuals follows a negative binomial distribution with mean 8 and variance 16. Loss size has an exponential distribution with mean **340**. The group expands to 65 individuals and a deductible of 102 is imposed. Calculate the probability of 2 or more claims from the group after these revisions times 1000. 998.060234 (7 marks)

2. [Fill in the blank with correct answer] Losses follow a compound distribution with both frequency and severity having discrete distribution.
   For frequency
   $$P_N(z) = 0.30 + 0.70\left[\frac{(1+0.71(z-1))^8 - (1-0.71)^8}{1-(1-0.71)^8}\right]$$
   For Severity
   $$P_X(z) = 0.42 + 0.39z + 0.08z^2 + 0.06z^3 + 0.05z^4$$
   Calculate the probability that aggregate losses is exactly 3. 0.089900 (7 marks)

3. [Fill in the blank with correct answer] A random variable has an exponential distribution with mean 10. It is to be discretized using the method of rounding with span 60. Determine the mean of the discretized distribution. 2.994647 (7 marks)

4. [Fill in the blank with correct answer] Let the frequency distribution be negative binomial with $r = 4$ and $\beta = 5$. Let the severity distribution has the exponential distribution with mean 39. Determine $F_S(46)$ 0.006000 (7 marks)

5. [Fill in the blank with correct answer] Number of claims follows a zero modified Binomial distribution with $q = 0.85, m = 8$ and $p_0^M = 0.61$. Suppose a deductible is imposed such that the probability of a payment resulting from a loss is now 0.82 rather than 1. Determine the probability that the number of payments exceed 6. 0.097200 (7 marks)

6. [Fill in the blank with correct answer] The number of claims on an insurance coverage follows a zero modified Poisson distribution with mean $\lambda = 5$ and $p_0^M = 0.39$. The size of each claim has the following distribution:

Claim Size, x	0	3	6	9
Probability, $P(X = x)$	0.52	0.2	0.12	0.16

   Calculate the probability of aggregate claims of 9 or more. 0.441400 (7 marks)

7. [Show your workings. If no workings are shown, ZERO is awarded] A company provides insurance to a concert hall for losses due to power failure. You are given:

- The number of power failures in a year has a Binomial distribution with parameters $m = 2$ and $q = 0.53$.
- The distribution of loss amount due to a single power failure follows a gamma distribution $\alpha = 2$ and $\theta = 11$.
- There is an annual deductible of 21.

Calculate the expected amount of claims paid by the insurer in one year.

(15 marks)

Ans.

$X_j \sim Gamma(\alpha = 2, \theta = 11)$
$S = \sum_{j=1}^{n} X_j \sim Gamma(\alpha = \text{2n}, \theta = 11)$
$p_1 = 0.4982$; $p_2 = 0.2809$;

$$\begin{aligned} S_S(x) &= \textstyle\sum_{n=1}^{2} p_n \sum_{j=0}^{2n} (\frac{x}{11})^j e^{-x/11}/j! \\ &= p_1(a_0 + a_1) + p_2(a_0 + a_1 + a_2 + a_3) \\ &= 0.4982e^{-x/11}(1 + \tfrac{x}{11}) + 0.2809e^{-x/11}(1 + \tfrac{x}{11} + (\tfrac{x}{11})^2/2 + (\tfrac{x}{11})^3/6) \\ &= 0.7791e^{-x/11} + 0.070827xe^{-x/11} + 0.001161x^2e^{-x/11} + 3.5e-05x^3e^{-x/11} \end{aligned}$$

$$\begin{aligned} &E[(S - 21)_+] \\ &= \textstyle\int_{21}^{\infty} S_S(x)dx \\ &= \textstyle\int_{21}^{\infty}(0.7791e^{-x/11} + 0.070827xe^{-x/11} + 0.001161x^2e^{-x/11} + 3.5e-05x^3e^{-x/11})dx \\ &= 0.7791(11)e^{-21/11} + 0.070827(11)^2S_2(21) + 2(0.001161)(11)^3S_3(21) + 6(3.5e-05)(11^4S_4(21) \\ &= 0.7791(11)(0.1482) + 0.070827(11)^2(0.4312) + 2(0.001161)(11)^3(0.7013) + 6(3.5e-05)(11^4(0.87314 \\ &= \boxed{9.8173} \end{aligned}$$

Answer2 = 9.817503372187499

8. [Show your workings. If no workings are shown, ZERO is awarded] The number of claims has a Poisson distribution with mean $\lambda = 2.9$. The distribution of the amount of claims(in thousand) is

Amount of claims	1	2	3	4	5	6
Probability	0.16	0.26	0.2	0.11	0.1	0.17

The number of claims and the amount of claims are independent. Determine the expected total amount of claims given that at least 4 thousand have been claimed. .

(15 marks)

Ans.

$p_0 = e^{-2.9} = 0.055$; $p_1 = 2.9e^{-2.9} = 0.1596$, $p_2 = 2.9^2e^{-2.9}/2 = 0.2314$, $p_3 = 2.9^3e^{-2.9}/6 = 0.2237$

$E(N) = \lambda = 2.9$; $E(X) = 3.24$; $E(S) = E(N)E(X) = 2.9(3.24) = 9.396$

$g_0 = p_0 = 0.055$

$g_1 = p_1 f_1 = 0.1596(0.16) = 0.0255$

$g_2 = p_1 f_2 + p_2 f_1^2 = 0.1596(0.26) + 0.2314(0.16^2) = 0.0474$

$g_3 = p_1 f_3 + 2p_2 f_1 f_2 + p_3 f_1^3 = 0.1596(0.2) + 2(0.2314)(0.16)(0.26) + 0.2237(0.16^3) = 0.0521$

$P(S \geq 4) = 1 - g_0 - g_1 - g_2 - g_3 = 1 - 0.055 - 0.0255 - 0.0474 - 0.0521 = 0.82$

$\sum_{k=1}^{3} kg_k = g_1 + 2g_2 + 3g_3 = 0.0255 + 2(0.0474) + 3(0.0521) = 0.2766$

$E(S|S \geq 4) = \frac{E(S) - \sum_{k=1}^{3} kg_k}{P(S \geq 4)} = \frac{9.396 - 0.2766}{0.82} = \boxed{11.1212}$

9. [Show your workings. If no workings are shown, ZERO is awarded] Consider the compound logarithmic distribution with exponential severity distribution. The parameter for logarithmic distribution is $\beta = 4$ and the parameter for exponential distribution is $\theta = 400$. Show that the density of aggregate losses may be expresses as

$$f_S(x) = \frac{e^{-\frac{x}{2000}} - e^{-\frac{x}{400}}}{x \ln(5)}.$$

(14 marks)

Ans.

$f_S(x) = \sum_{n=1}^{\infty} p_n f^{*n}(x)$

$p_n = \frac{\beta^n}{n(1+\beta)^n \ln(1+\beta)} = \frac{4^n}{n(5)^n \ln(5)}$

$f^{*n} = P[\sum X_j = x] = \frac{1}{\Gamma(n)\theta^n} x^{n-1} e^{-x/\theta} = \frac{1}{(n-1)!400^n} x^{n-1} e^{-x/400}$

$$f_S(x) = \sum_{n=1}^{\infty} \left[\frac{4^n}{n(5)^n \ln(5)}\right] \left[\frac{1}{(n-1)!400^n} x^{n-1} e^{-x/400}\right]$$

$$= \frac{1}{\ln(5)} \sum_{n=1}^{\infty} \frac{1}{n!} \left[\frac{4}{2000}\right]^n x^{n-1} e^{-x/400}$$

$$= \frac{e^{-x/400}}{x \ln(5)} \sum_{n=1}^{\infty} \left[\frac{4x}{2000}\right]^n \frac{1}{n!}$$

$$= \frac{e^{-x/400}}{x \ln(5} [e^{4x/2000} - 1]$$

$$= \frac{e^{-x/2000} - e^{-x/400}}{x \ln(5)}$$

10. [Show your workings. If no workings are shown, ZERO is awarded] Show that when in the zero-truncated negative binomial distribution, $r \to 0$ the pf is

$$p_k = \left(\frac{\beta}{1+\beta}\right)^k \frac{1}{k\ln(1+\beta)}.$$

(14 marks)

Ans.

$$\begin{aligned} p_k &= p_{k-1}\left[\tfrac{\beta}{1+\beta} + \tfrac{r-1}{k}\tfrac{\beta}{1+\beta}\right] \\ &= p_{k-1}\tfrac{\beta}{1+\beta}\tfrac{k+r-1}{k} \\ &= p_{k-2}\left(\tfrac{\beta}{1+\beta}\right)^2 \tfrac{k+r-1}{k}\tfrac{k+r-2}{k-1} \\ &= p_1\left(\tfrac{\beta}{1+\beta}\right)^{k-1} \tfrac{k+r-1}{k}\tfrac{k+r-2}{k-1}\cdots\tfrac{r+1}{2} \end{aligned}$$

when $r = 0$

$$\begin{aligned} 1 = \textstyle\sum_{k=1}^{\infty} p_k &= \textstyle\sum_{k=1}^{\infty} p_1\left(\tfrac{\beta}{1+\beta}\right)^{k-1} \tfrac{k-1}{k}\tfrac{k-2}{k-1}\cdots\tfrac{1}{2} \\ &= \textstyle\sum_{k=1}^{\infty} p_1\left(\tfrac{\beta}{1+\beta}\right)^{k-1} \tfrac{1}{k} \\ &= p_1\tfrac{1+\beta}{\beta}\left[-\ln\left(1-\tfrac{\beta}{1+\beta}\right)\right] \end{aligned}$$

using Taylor series expansion for $\ln(1-x)$. Thus

$$p_1 = \left(\frac{\beta}{1+\beta}\right)\frac{1}{\ln(1+\beta)}.$$

and

$$p_k = p_1\left(\frac{\beta}{1+\beta}\right)^{k-1}\frac{1}{k} = \left(\frac{\beta}{1+\beta}\right)\frac{1}{\ln(1+\beta)}\left(\frac{\beta}{1+\beta}\right)^{k-1}\frac{1}{k} = \left(\frac{\beta}{1+\beta}\right)^k\frac{1}{k\ln(1+\beta)}.$$