**UECM3463 Practice**

**PRACTICE**

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Q1. You are given the following:

- Losses follow a Weibull distribution with parameters $\theta = 22$ and $\tau = 2$.
- The insurance coverage has an ordinary deductible of 13.

If the insurer makes a payment, what is the probability that an insurer's payment is less than or equal to 33.

Q2. The losses experienced by an insurance company have the following probability distribution:

| Loss size | Probability |
|---|---|
| 0 | 0.60 |
| 130 | 0.25 |
| 230 | 0.10 |
| 1,260 | 0.05 |

Calculate the $CTE_{0.74}$(or $TVaR_{0.74}$).

Q3. For an insurance coverage, lossess (before application of any deductible) follow a Pareto distribution with parameters $\alpha = 4$ and $\theta = 10000$. The coverage is subject to a deductible of 1000.0. Calculate the deductible needed to double the loss elimination ratio.

Q4. The number of claims in a period has a Binomial distribution with parameters $m = 8$ and $q = 0.21$. The amount of each claim $X$ follows $P(X = x) = 0.25$, $x = 1, 2, 3, 4$. The number of claims and claim amounts are independent. $S$ is the aggregate claim amount in the period. Calculate $F_S(4)$.

Q5. Claim sizes follow an exponential distribution with $\theta$ =14.00. Claim counts are independent of claim sizes, and have the following distribution:

| $n$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P_n$ | 0.40 | 0.36 | 0.21 | 0.03 |

Calculate $F_S(6)$.

Q6. For a certain insurance, individual losses in 2020 were Pareto distributed with parameters $\alpha = 5$ and $\theta = 1200$. A deductible of 120.0 is applied to each loss. In 2021, individual losses have increased 7%. A deductible of 120.0 is still applied to each loss. Determine the standard deviation of amount paid per loss.

Q7. For a discrete probability distribution, you are given the recursion relation

$$p_k = (3.48/k + 0.87)p_{(k-1)}, k = 1, 2, \ldots$$

Determine $p_3$.

Q8. $N^M$ is a discrete random variable with probability function which is a member of the $(a, b, 1)$ class of distributions. You are given

$$P(z) = 0.32 + 0.68\left[\frac{e^{2.10(z-1)]} - e^{-2.10}}{1 - e^{-2.10}}\right]$$

Calculate the variance of the distribution.

Q9. The number of losses follows a Binomial distribution with $m = 45$ and $q = 0.24$. Loss sizes follow an inverse exponential distribution with $\theta = 150$. Let $N$ be the number of losses for amount less than 300. Determine the standard deviation of $N$.

Q10. A random sample of 8 claims $x_1, \ldots, x_8$ is taken from the probability density function

$$f(x_i) = \frac{\alpha\theta^\alpha}{(x_i)^{\alpha+1}}, \alpha, \theta > 0, x_i > \theta.$$

In ascending order the observations are: 1,330, 1,374,1,402, 1,425, 1,500, 1,532, 1,592, 2,034

Suppose the parameters are $\alpha = 3$ and $\theta = 1330$. Commonly used critical values for this test are

| $\alpha$ | 0.10 | 0.05 | 0.025 | 0.01 |
|---|---|---|---|---|
| Critical Value | $\frac{1.22}{\sqrt{n}}$ | $\frac{1.36}{\sqrt{n}}$ | $\frac{1.48}{\sqrt{n}}$ | $\frac{1.63}{\sqrt{n}}$ |

Determine the result of the test at 0.1 significant level.

Q11. You are given the following:

- 121 observed losses have been recorded an are grouped as follows:

| Interval | Number of Losses |
|---|---|
| [0,1) | 18 |
| [1,5) | 40 |
| [5,10) | 21 |
| [10,15) | 22 |
| [15, $\infty$) | 20 |

- The random variable $X$ underlying the observed losses, is believed to follow the exponential disribution with mean 5.

Determine the value of Pearson's goodness-of-fit statistic.

Q12. You are given a sample of 10 observations from the following distribution:

$$f(X) = \frac{1}{2\theta^3}x^2e^{-x/\theta}, x > 0$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|---|---|---|---|---|---|---|---|---|---|
| 137.04 | 172.84 | 45.58 | 101.64 | 86.24 | 96.67 | 183.52 | 82.26 | 176.85 | 341.03 |

Determine the value of the Akaike Information Criterion (AIC).

Q13. A random sample of 10 claims $x_1, \ldots, x_{10}$ is taken from the probability density function

$$f(x_i) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x_i^{\alpha-1} e^{-\frac{x_i}{\theta}}, x_i > 0.$$

In ascending order the observations are: 91.87, 147.5, 153.31, 168.4, 175.52, 225.24, 274.12, 279.52, 288.5, 493.43

Suppose the parameters are $\alpha = 4$ and $\theta = 50$. Commonly used critical values for this test are

| $\alpha$ | 0.10 | 0.05 | 0.025 | 0.01 |
|---|---|---|---|---|
| Critical Value | $\frac{1.22}{\sqrt{n}}$ | $\frac{1.36}{\sqrt{n}}$ | $\frac{1.48}{\sqrt{n}}$ | $\frac{1.63}{\sqrt{n}}$ |

Determine the result of the test at 0.1 significant level.

Q14. You are given a sample of 5 observations from $Pareto(\alpha, \theta = 1830)$ distribution:

$$2,191.62 \qquad 2,799.33 \qquad 1,831.07 \qquad 2,064.33 \qquad 1,930.42.$$

Determine the value of the Bayesian Information Criterion (BIC).

Q15. Suppose that $X_1, \ldots, X_n$ denotes a random sample from the probability density function given by

$$f(x|\theta_1, \theta_2) = \begin{cases} \left(\frac{1}{\theta_1}\right) e^{-(x-\theta_2)/\theta_1}, & x > \theta_2 \\ 0, & \text{otherwise.} \end{cases}$$

The following random sample of 8 has been observed:

$$63, \quad 124, \quad 19, \quad 48, \quad 32, \quad 27, \quad 36, \quad 54$$

Determne the likelihood test statistc for testing $H_0 : \theta_1 = 82.8$ versus $H_1 : \theta_1 > 82.8$ with $\theta_2$ unknown.