1. $\mathbf{X} \sim \mathbf{Bernoulli(p)}$

• $f(x) = p^x q^{1-x}$, x = 0.1

 $\bullet M_{\rm Y}(t) = pe^t + q$

 $\bullet E(X) = p$

 $\bullet V(X) = pq$

2. $\mathbf{X} \sim \mathbf{Binomial}(\mathbf{n}, \mathbf{p})$

• $f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$

 $\bullet M_X(t) = (pe^t + q)^n$

 $\bullet E(X) = np$

 $\bullet V(X) = npq$

3. $\mathbf{X} \sim \mathbf{HYP}(\mathbf{n}, \mathbf{M}, \mathbf{N})$

• $f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, $x = 0, 1, \dots, \min(n, M), n - x \le N - M.$

• $E(X) = \frac{nM}{N}$

• $V(X) = n \frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1}$

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4. $\mathbf{X} \sim \mathbf{GEO}(\mathbf{p})$

• $f(x) = pq^{x-1}$ x = 1, 2, 3, ...

• $F(x) = 1 - q^x$ $x = 1, 2, 3, \dots$

 $\bullet M_X(t) = \frac{pe^t}{1-qe^t}$

• $E(X) = \frac{1}{p}$

• $V(X) = \frac{q}{n^2}$

5. $X \sim NegativeBinomial(r, p)$

• $f(x) = {x-1 \choose r-1} p^r q^x, x = r, r+1, \dots$

• $M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$

 $\bullet E(X) = \frac{r}{n}$

 $\bullet V(X) = \frac{rq}{n^2}$

6. $\mathbf{X} \sim \mathbf{POI}(\mu)$

• $f(x) = \frac{e^{-\mu}\mu^x}{x!}$ x = 0, 1, 2, ...

 $\bullet M_{\mathbf{Y}}(t) = e^{\mu(e^t - 1)}$

 \bullet $E(X) = \mu$

 $\bullet V(X) = \mu$

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Common Distributions

7. $\mathbf{X} \sim \mathbf{DU}(\mathbf{N})$

• $f(x) = \frac{1}{N}, X = 1, 2, \dots, N$

 $\bullet M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$

 $F(x) = \frac{x(1+x)}{2N}$ $E(X) = \frac{N+1}{2}$

• $V(X) = \frac{N^2 - 1}{12}$

8. $\mathbf{X} \sim \mathbf{U}(\mathbf{a}, \mathbf{b})$

• $f(x) = \frac{1}{b-a}$, a < x < b and zero otherwise

 \bullet $F(x) = \frac{x-a}{b-a}, a < x < b$

• $M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$

 $\bullet E(X) = \frac{a+b}{2}$

 $\bullet \ V(X) = \frac{(b-a)^2}{10}$

9. $\mathbf{X} \sim \mathbf{Gamma}(\alpha, \theta)$

• $f(x) = \frac{1}{\theta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\theta}, x > 0$

• $F(x) = 1 - \sum_{i=0}^{\alpha - 1} \frac{(x/\theta)^i}{i!} e^{x/\theta}$

 $\bullet M_X(t) = (\frac{1}{1-2t})^{\alpha}$

 $\bullet E(X) = \alpha \theta$

Common Distributions

• $V(X) = \alpha \theta^2$

10. $\mathbf{X} \sim \mathbf{EXP}(\theta)$

• $f(x) = \frac{1}{4}e^{-x/\theta}$, x > 0 and zero otherwise.

• $F(x) = 1 - e^{-x/\theta}, x > 0$

• $M_X(t) = \left(\frac{1}{1-\theta t}\right)$

 $\bullet E(X) = \theta.$

 $\bullet V(X) = \theta^2$

11. $\mathbf{X} \sim \mathbf{WEI}(\tau, \theta)$

• $f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau - 1} e^{-(x/\theta)^{\tau}}, x > 0$ and zero otherwise

 $\bullet F(x) = 1 - e^{-(x/\theta)^{\tau}}$

• $E(X) = \theta \Gamma \left(1 + \frac{1}{\tau} \right)$

• $E(X^2) = \theta^2 \left[\Gamma \left(1 + \frac{2}{\tau} \right) - \Gamma^2 \left(1 + \frac{1}{\tau} \right) \right]$

12. $\mathbf{X} \sim \mathbf{PAR}(\alpha, \theta)$

• $f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$

 $\bullet F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$

 $\bullet E(X) = \frac{\theta}{\alpha - 1}$

•
$$E(X^2) = \frac{2\theta^2}{(\alpha - 1)(\alpha - 2)}$$

•
$$V(X) = \frac{\theta^2}{(\alpha - 1)^2(\alpha - 2)}$$

13. $\mathbf{X} \sim \mathbf{Beta}(\mathbf{a}, \mathbf{b})$

$$\bullet \ f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}, \text{for } 0 < x < 1$$

$$\bullet \ E(X) = \frac{a}{a+b}$$

$$\bullet V(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

14. $\mathbf{X} \sim \mathbf{N}(\mu, \sigma^2)$

•
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$$
, for $x \in R$, $\mu \in R$ and $\sigma > 0$.

•
$$F(x) = \Phi(\frac{x-\mu}{\sigma})$$

$$\bullet M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$\bullet$$
 $E(X) = \mu$

•
$$V(X) = \sigma^2$$

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Common Distributions

18. $\mathbf{X} \sim \mathbf{DE}(\eta, \theta)$

• $f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$ $-\infty < x < \infty$ and

zero otherwise.

•
$$F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \leq \eta \\ \frac{1}{2}[1 - e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$$

$$\bullet M_X(t) = \frac{e^{\eta t}}{1 - \theta^2 t^2}$$

$$\bullet E(X) = \eta$$

•
$$V(X) = 2\theta^2$$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{x^{\alpha+1}}, x > \theta$$

$$\bullet F(x) = 1 - (\frac{\theta}{\theta})^{\alpha}$$

•
$$E(X) = \frac{\alpha \theta}{\alpha - 1}$$

•
$$E(X^2) = \frac{\alpha \theta^2}{\alpha - 2}$$

15.
$$\mathbf{X} \sim \mathbf{LN}(\mu, \sigma)$$

•
$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x - \mu)^2/2\sigma^2}$$
, for $x > 0$, $\mu \in R$ and $\sigma > 0$

•
$$F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

$$\bullet \ E(X) = e^{\mu + \frac{\sigma^2}{2}}$$

$$\bullet V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

16. $\mathbf{X} \sim \mathbf{CAU}(\mathbf{theta}, \eta)$

•
$$f(x) = \frac{1}{\theta \pi \left[1 + \left(\frac{x-\eta}{\theta}\right)^2\right]}$$

$$\bullet F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} \left(\frac{x-\eta}{\theta}\right)$$

17. $\mathbf{X} \sim \mathbf{EXP}(\eta, \theta)$

•
$$f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$$
 $x > \eta$

•
$$F(x) = 1 - e^{-\frac{x-\eta}{\theta}}$$

•
$$M_X(t) = \frac{e^{\eta t}}{1-\theta t}$$

$$\bullet E(X) = \eta + \theta$$

$$V(X) = \theta^2$$

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$$\bullet\; E(X) = \eta$$

$$V(X) = 2\theta^2$$

19.
$$\mathbf{X} \sim \text{ Single Parameter Pareto } (\alpha, \theta)$$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{x^{\alpha+1}}, x > \theta$$

•
$$F(x) = 1 - (\frac{\theta}{x})^{\alpha}$$

$$\bullet\; E(X) = \tfrac{\alpha\theta}{\alpha-1}$$

$$E(X^2) = \frac{\alpha \theta^2}{\alpha - 2}$$