TEST2 Q2(B)

1. Let $S(t)$ be time-t price of a nondividend-paying stock and $P(S(t), t)$ be the time-t price of a 0.25-year at the money European put option written on the stock, when the time-t stock price is $S(t)$. You are given that

   - $S(0) = 51$.
   - The true stock price process is
   $$dS(t) = 0.13S(t)dt + 0.26S(t)dZ(t)$$
   where $Z(t)$ is a standard Brownian motion under the true measure.
   - The true stochastic process satisfied by the put option is
   $$dP(S(t), t) = a(S(t), t)dt + b(S(t), t)dZ(t)$$
   for some a and b.
   - $r = 0.065$.

   Calculate $a(51, 0)$.

   (1 mark)

   Ans.
   $d_1 = \frac{\ln(51/51)+[0.065+\frac{1}{2}(0.26^2)](0.25)}{0.26\sqrt{0.25}} = 0.19$
   $d_2 = d_1 - 0.26\sqrt{0.25} = 0.06$
   $N(d_1) = N(0.19) = 0.5753$
   $N(d_2) = N(0.06) = 0.5239$
   $V = p[51, 51, 0.25] = 51e^{-0.065(0.25)}(1 - 0.5239) - 51(1 - 0.5753) = 2.23002$
   $\Delta_P = -e^{-0.065(0.25)}(1 - 0.5239) = \text{-0.4247}$
   $\Omega = \frac{S\Delta_P}{V} = \frac{51(-0.4247)}{2.23002} = \text{-9.71278}$
   $m_V = \Omega\alpha + (1 - \Omega)r = -9.71278(0.13) + (1 - -9.71278)(0.065) = \text{-0.56633}$
   Comparing with
   $$\frac{dP(S(t), t)}{P(S(t), t)} = m_V dt + s_V dZ(t),$$
   we have
   $a(51, 0) = m_V(V) = -0.56633(2.23002) = \boxed{\text{-1.2629}}$

2. Let $S(t)$ be time-t price of a nondividend-paying stock and $P(S(t), t)$ be the time-t price of a 0.5-year at the money European put option written on the stock, when the time-t stock price is $S(t)$. You are given that

   - $S(0) = 62$.
   - The true stock price process is
   $$dS(t) = 0.12S(t)dt + 0.24S(t)dZ(t)$$
   where $Z(t)$ is a standard Brownian motion under the true measure.
   - The true stochastic process satisfied by the put option is
   $$dP(S(t), t) = a(S(t), t)dt + b(S(t), t)dZ(t)$$
   for some a and b.

- $r = 0.064$.

Calculate $a(62, 0)$.

(1 mark)

Ans.

$d_1 = \frac{\ln(62/62)+[0.064+\frac{1}{2}(0.24^2)](0.5)}{0.24\sqrt{0.5}} = 0.2734$

$d_2 = d_1 - 0.24\sqrt{0.5} = 0.1037$

$N(d_1) = N(0.27) = 0.6064$

$N(d_2) = N(0.1) = 0.5398$

$V = p[62, 62, 0.5] = 62e^{-0.064(0.5)}(1 - 0.5398) - 62(1 - 0.6064) = 3.23062$

$\Delta_P = -e^{-0.064(0.5)}(1 - 0.5398) =$ -0.3936

$\Omega = \frac{S\Delta_P}{V} = \frac{62(-0.3936)}{3.23062} =$ -7.55372

$m_V = \Omega\alpha + (1 - \Omega)r = -7.55372(0.12) + (1 - -7.55372)(0.064) =$ -0.35901

Comparing with

$$\frac{dP(S(t), t)}{P(S(t), t)} = m_V dt + s_V dZ(t),$$

we have

$a(62, 0) = m_V(V) = -0.35901(3.23062) = \boxed{\text{-1.1598}}$