MEME15203 Statistical Inference

Assignment 1

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Wong Wai Kuan
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Due by:			

Q1. Let X_1 and X_2 be a random sample of size $n = 2$ from a continuous distribution with pdf of the form $f(x) = 5x^4$ if $0 < x < 1$ and zero otherwise.

(a) Find the joint pdf of $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$.

(b) Find the pdf of the sample range $R = Y_2 - Y_1$.

(10 marks)

Q2. Let X_1 and X_2 be independent random variables with $X_1 \sim GAM(\alpha_1 = a, \theta = 2)$ and $X_2 \sim GAM(\alpha_2 = b, \theta = 2)$, show that $U = \frac{X_1}{X_1+X_2}$ follow a Beta distribution. Suppose $Y_i \sim GAM(\alpha = 10, \theta = 2)$, using the result above, find the distribution of $V = \frac{Y_1}{\sum_{i=1}^{16} Y_i}$.

(10 marks)

Q3. The waiting time X until delivery of a new component for an industrial operation is uniformly distributed over the interval from 1 to 9 days. The cost of this delay is given by $U = 5X^2 + 8$. Find the probability density function for U using distribution method.

(10 marks)

Q4. In each of the following, random variable X and Y are independent. Use moment generating function methodto find the probability density functions (pdfs)for $V = X + Y$.

(a) $f(x) = \binom{15}{x} 0.8^x 0.2^{15-x}, x = 0, 1, \ldots, 15$
$f(y) = \binom{13}{y} 0.8^y 0.2^{13-y}, y = 0, 1, \ldots, 13$

(b) $X \sim POI(\lambda = 1.6)$
$Y \sim POI(\lambda = 2.7)$

(c) $X \sim N(\mu = 15, \sigma^2 = 5^2)$
$Y \sim N(\mu = 6, \sigma^2 = 7^2)$

(d) $f(x) = \frac{1}{\Gamma(3)6^3} x^2 e^{-\frac{x}{6}}, x > 0$
$f(y) = \frac{1}{\Gamma(3)6^3} y^2 e^{-\frac{y}{6}}, y > 0$

(12 marks)

Q5. Let Y_n denote the maximum of a random sample of size n from a distribution of the continuous type that has cdf $F(x)$ and pdf $f(x) = F'(x)$. Find the limiting distribution of $W_n = n[1 - F(Y_n)]$.

(10 marks)

Q6. Consider a random sample from a Exponential distribution, $X_i \sim Exp(\theta)$. Find the asymtotic normal distribution of $Y_n = [\ln(\bar{X}_n)]^9$.

(8 marks)

Q7. Consider a random sample from a Geometric distribution, $X_i \sim GEO(p)$. Let $W_i = e^{X_i}$ and $V_n = W_1 \times W_2 \times \cdots W_n$. $V_n^{\frac{1}{n}}$ converges in probability to a constant, identify the constant.

(5 marks)

Q8. Let $X_2, X_3, X_4, \ldots$ be a sequence of random variable such that

$$f_{X_n}(x) = \begin{cases} -n \ln\left(1 - \frac{1}{4n}\right)\left(1 - \frac{1}{4n}\right)^{nx} & x > 0 \\ 0 & \text{otherwise} \end{cases}.$$

Find the limiting distribution of X_n.

(5 marks)

Q9. Let $Y_n \sim GAM(\alpha n, \theta)$. Find the limiting distribution of $Z_n = \frac{Y_n - \alpha n\theta}{\sqrt{\alpha n}\theta}$ as $n \to \infty$, using moment generating function.

(10 marks)

Q10. Suppose that $X_i \sim N(\mu, \sigma^2), i = 1, \ldots, 14$, $Z_j \sim N(0, 1), j = 1, \ldots, 9$, and $W_k \sim \chi^2(50), k = 1, \ldots, 13$ and all random variables are independent. State the distribution of each of the following variables if it is a "named" distribution. [For example $X_1 + X_2 \sim N(2\mu, 2\sigma^2)$]

(a) $\frac{8\sum_{i=1}^{14}(X_i - \bar{X})^2}{13\sigma^2\sum_{j=1}^{9}(Z_j - \bar{Z})^2}$.

(b) $\frac{8\sum_{k=1}^{9} W_k}{450\sum_{j=1}^{9}(Z_j - \bar{Z})^2}$

(c) $\frac{\sqrt{700}(\bar{X} - \mu)}{\sigma\sqrt{W_1}}$

(d) $\frac{W_1}{W_1 + W_2 + W_3 + W_4}$

(e) $\frac{Z_i^2/W_1}{1 + Z_1^2/W1}$

(f) $\frac{\frac{\sum_{k=1}^{9} W_k}{\sum_{j=1}^{9}(Z_j - \bar{Z})^2}}{1 + \frac{\sum_{k=1}^{9} W_k}{\sum_{j=1}^{9}(Z_j - \bar{Z})^2}}$

(20 marks)