

**MEME15203 Statistical Inference****Assignment 3****UNIVERSITI TUNKU ABDUL RAHMAN**

Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
Session:	January 2022		
Due by:	24/3/2022		

- Q1. Consider a random sample of size  $n$  from a single-parameter pareto distribution,  $X_i \sim SP(\alpha = 2, \theta)$ . Find the UMVUE of the  $p^{th}$  percentile. (10 marks)
- Q2. Suppose that  $X_1, \dots, X_n$  is a random sample from a Negative Binomial distribution,  $X_i \sim NB(r = 6, \theta)$ ,
- Show that the p.d.f. of  $X$  belongs to the regular exponential family. (5 marks)
  - Based on the answer in (a), find a complete and sufficient statistic for  $\theta$ . (5 marks)
  - Find the UMVUE of  $\left[ \frac{4\theta}{1-4(1-\theta)} \right]^{6n}$ . (5 marks)
- Q3. Consider a random sample of size  $n$  from a gamma distribution  $X_i \sim GAM(\alpha, \theta)$  and let  $\bar{X} = (1/n) \sum X_i$  and  $\tilde{X} = (\prod X_i)^{1/n}$  be the sample mean and geometric mean respectively.
- Show that  $\bar{X}$  and  $\tilde{X}$  are jointly complete and sufficient for  $\theta$  and  $\alpha$ . (5 marks)
  - Find the UMVUE of  $\mu = \alpha\theta$ . (5 marks)
  - Find the UMVUE of  $\mu^n$ . (5 marks)
  - Show that  $\bar{X}$  and  $T$  are stochastically independent random variables. (5 marks)
- Q4. Suppose that  $X_1, \dots, X_n$  is a random sample from a Poisson distribution,  $X_i \sim POI(\lambda)$ . Find the UMVUE of  $P(X_1 + X_2 = 0 \text{ or } 1) = (1 + 2\lambda)e^{-2\lambda}$  using Rao-Blackwell theorem. (10 marks)
- Q5. Consider a random sample of size  $n$  from a distribution with pdf
- $$f(x; \theta) = \frac{(\ln \theta)^x}{\theta x!}, x = 1, 1, \dots; \theta > 1, \text{ zero, otherwise.}$$
- Find a complete sufficient statistic for  $\theta$ . (5 marks)
  - Find the UMVUE of  $\ln \theta$ . (5 marks)
  - Find the UMVUE of  $(\ln \theta)^2$ . (5 marks)
- Q6. Show that  $X \sim N(0, \theta)$  is not a complete family. (10 marks)

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- Q7. Let  $X_1, X_2, \dots, X_n$  be random sample of size  $n$  from a Gamma distribution with probability density function

$$\frac{1}{\theta^2} x e^{-x/\theta}, x > 0$$

zero otherwise. Find the UMVUE of  $\gamma = P(X > t)$  using Rao-Blackwell theorem.

(20 marks)