Test 2 Marking Guide

Name: Student ID: Mark: /100

FACULTY:	FES, UTAR	COURSE CODE:	UECM1404
PROGRAMME/YEAR:	AS, FM /Y1	COURSE TITLE:	THEORY OF INTEREST
SESSION:	202306	LECTURER:	DR YONG CHIN KHIAN

CO3: Apply the concept of annuity and compound interest to solve problems in the amortization and sinking fund schedule such as finding the loan amount, interest charged, periodic payments and length of the loan.

1. [Fill in the blank with correct answer] An-annuity-immediate pays 17 at the end of years 1 and 2, 16 at the ends of years 3 and 4, etc.., with payments decreasing by 1 every second year, until nothing is paid. The effective annual rate of interest is 7%. Calculate the present value of this annuity-immediate. 154.14762345437472. (7 marks)

2. [Fill in the blank with correct answer] A loan is being amortized by means of level monthly payments at an annual effective interest rate of 7%. the amount of principal repaid in the 15-th payment is 2,000 and the amount of principal repaid in the t-th payment is 8000. Calculate t. 258.9. (7 marks)

3. [Fill in the blank with correct answer] A 5,000 loan is to be repaid with equal payments at the end of each year for 40 years. The principal portion of the 15th payment is 2.1 times the principal portion of the 9th payment. Calculate the total amount of interest paid on the loan. 21508.800000000003. (7 marks)

4. [Fill in the blank with correct answer] Justin and Maggie each take out a 20-year loan L. Justin repays his loan using the amortization method, at an annual effective interest rate of i. He makes an annual payment of 500 at the end of each year. Maggie repays her loan using the sinking fund method. She pays interest annually, also at an effective interest rate of i. In addition, Maggie makes level annual deposits at the end of each year for 20 years into a sinking fund. The annual effective rate on the sinking fund is 4.66%, and she pays off the loan after 20 years. Maggie's total payment each year is equal to 9% of the original loan amount. Calculate L. 5795.978313810717. (7 marks)

5. [Fill in the blank with correct answer] A loan of 190,000 is repaid with unequal annual payments at the end of each year for 50 years. Each of the first 49 payments is equal to two times the amount of interest then due. The final payment repays the remaining loan balance at that time. Interest is charged at an annual effective rate of 8%. Calculate the amount of the final loan payment. 3449.69. (7 marks)

6. [Fill in the blank with correct answer] If

$$_{\frac{1}{12}|}\ddot{a}^{(6)}_{\overline{n}|} + a^{(6)}_{\overline{n}|} + 26a^{(12)}_{\overline{n}|} = ca^{(12)}_{\overline{n}|}.$$

Determine c. 28. (7 marks)

7. [Show your workings. If no workings are shown, ZERO is awarded] An investor is considering the purchase of 100 ordinary shares in a company. Dividends from the share will be paid annually. The next dividend is due in one year and is expected to be RM0.07 per share. The second dividend is expected to be 8% greater than the first dividend and the third dividend is expected to be 7% greater than the second dividend. Thereafter, dividends are expected to grow at 4% per annum compound in perpetuity. Calculate the present value of this dividend stream at an annual effective rate of interest of 9%.

(15 marks)

Ans.

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     7  7(1.08)  7(1.08)(1.07)  7(1.08)(1.07)(1.04)  7(1.08)(1.07)(1.04^2) .....
  |--|---|---------|-----------------|----------------|-------------------
  0  1   2         3                 4                5
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100 shares provide a first dividend of RM7.

Present Value
$= 7v + 7(1.08)v^2 + 7(1.08)(1.07)[1 + 1.04v + 1.04v^2 + \cdots]v^2$
$= 7(1.09^{-1}) + 7(1.08)(1.09^{-1})^2 + 7(1.08)(1.07)[\frac{1}{0.09-0.04}](1.09^{-1})^2$
$= \boxed{148.9555}$

8. [Show your workings. If no workings are shown, ZERO is awarded] A loan is repayable by a decreasing annuity payable annually in arrears for 30 years. The repayment at the end of the first year is 5000 and subsequent repayments reduce by 400 each year. The repayments were calculated using an annual effective rate of interest of 9%. Construct the schedule of armortization for years eight and nine, showing the outstanding balance at the beginning of the year (B_{t-1}), the repayments (R_t), the interest element (I_t), the principal repayment (P_t) and the outstanding balance at the end of the year (B_t) for $t = 8$ and $t = 9$.

(15 marks)

Ans.

$B_7 = PVD = Pa_{\overline{23}|} - Q\left[\frac{a_{\overline{23}|} - 30v^{23}}{i}\right]$

$P = 2200$; $Q = 400$;

$a_{\overline{23}|} = \frac{1-1.09^{-23}}{0.09} = 9.5802$

$B_7 = 2200(9.5802) - 400\left[\frac{9.5802-23(1.09^{-23})}{0.09}\right] = \boxed{\text{-7,417.91}}$

Hence

Year,t	B_{t-1}	R_t	I_t	P_t	B_t
8	-7417.91	2200	-667.61	2867.61	-10285.52
9	-10285.52	1800	-925.7	2725.7	-13011.22

9. [Show your workings. If no workings are shown, ZERO is awarded] Calculate $\ddot{s}^{(12)}_{\overline{10.5}|}$ at an annual effective rate of interest of 13%. Explain what it represents.

(14 marks)

Ans.

$\ddot{s}^{(12)}_{\overline{10.5}|} = \frac{(1+i)^{10.5}-1}{d^{(12)}} = \frac{1.13^{10.5}-1}{12[1-1.13^{-1/12}]} = \boxed{21.4517}$

$\ddot{s}^{(12)}_{\overline{10.5}|}$ represents the accumulated value of an annuity of $\frac{1}{12}$ payable at the beginning of the month for 10.5 years at an annual effective interest rate of 13%.

10. [Show your workings. If no workings are shown, ZERO is awarded] In order to pay off a 6,000 loan, payments of R are made at the end of each quarter. Interest on the first 1,500 of the unpaid balance is at rate $i^{(4)} = 18\%$, while interest on the excess is at $i^{(4)} = 16\%$. If the outstanding loan balance is 5,607 at the end of the first year, find R.

(14 marks)

Ans.

$B_0 = 6000$
$I_1 = 1500.0 * (0.045) + 4500.0(0.04) = 247.5$
$P_1 = R - I_1 = R - 247.5$
$B_1 = B_0 - P_1 = 6000 - [R - 247.5] = 6247.5 - R$
$I_2 = 1500.0 * (0.045) + (6247.5 - R - 1500.0) * 0.04 = 257.4 - 0.04R$
$P_2 = R - I2 = R - [257.4 - 0.04R] = 1.04R - 257.4$
$B_2 = B_1 - P_2 = 6247.5 - R - [1.04R - 257.4] = 6504.9 - 2.04R$
$I_3 = 1500.0 * (0.045) + (6504.9 - 2.04R - 1500.0) * (0.04) = 267.6960 - 0.0816R$
$P_3 = R - I_3 = R - [267.6960 - 0.0816R] = 1.0816R - 267.6960$
$B_3 = B_2 - P_3 = 6504.9000 - 2.0400R - 1.0816R + 267.6960 = 6772.5960 - 3.1216R$
$I_4 = 1500.0 * (0.045) + (6772.5960 - 3.1216R - 1500.0) * (0.04) = 278.4038 - 0.1249R$
$P_4 = R - I_4 = R - [278.4038 - 0.1249R] = 1.1249R - 278.4038$
$B_4 = B_3 - P_4 = 6772.5960 - 3.1216R - 1.1249R + 278.4038 = 7050.9998 - 4.2465R$
$7050.9998 - 4.2465R = 5607.0$
$R = \boxed{340}$