Test 2 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM2453
Programme/Year:	AS /Y2, Y3	Course Title:	Financial Economics II
Session:	202301	Lecturer:	Dr Yong Chin Khian

1. CO3: Explain the cash flow characteristics of the following exotic options: Asian, barrier, compound, gap, and exchange.

   (a) [Fill in the blank with correct answer] Let $x(t)$ be the value of €1 in terms of US dollars at time t. You are given that:

   - The continuously compounded risk-free rate in US is 7.0%.
   - Under the risk-neutral measure, the stochastic differential equation of x is

   $$dx(t) = 0.022x(t)dt + 0.16d\tilde{Z}(t), \quad x(0) = 0.9$$

     where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.
   - A call option that gives the option holder the right to pay \$0.02 six months from today to buy a call option that gives the the right to buy €1 using \$0.95 one year from now is costs \$0.0368.

   Calculate the price of a put option that gives the option holder the right to sell at \$0.02 six months from today a call that gives the right to buy €1 using \$0.95 one year from now. 0.0134 (6 marks)

   (b) [Fill in the blank with correct answer] A British company will receive \$1,000,000 at the end of 6 month. To hedge its currency risk, it buys an option allowing to exchange dollars for pounds at a rate of £0.64/\$. You are given:

   - The spot exchange rate is £0.65/\$.
   - The continuously compounded risk-free interest rate for dollars is 0.06.
   - The continuously compounded risk-free interest rate for pounds is 0.04.
   - The volatility of the exchange rate between the two currencies is 0.1.
   - The Black-Scholes framework is assumed to apply to the currency rate.

   Calculate the cost in pounds of the hedge. 15860.00 (7 marks)

   (c) [Fill in the blank with correct answer] Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively. You are given:

   - $S_1(0) = 45$ and $S_2(0) = 90.0$.
   - Stock 1's volatility is 0.15.
   - Stock 2's volatility is 0.25.
   - The correlation between the continuously compounded returns of the two stocks is -0.4.
   - The continuously compounded risk-free interest rate is 5.3%.
   - A one-year European option with payoff $\max\{76.0 - \min[2.0S_1(1), S_2(1)], 0\}$ has a current (time-0) price of -0.1391.

   Consider a European option that gives its holder the right to buy either 2.0 shares of Stock 1 or one share of Stock 2 at a price of 76.0 one year from now. Calculate the current (time-0) price of this option. 5.63 (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Let $S(t)$ denote the price at time t of a stock. Consider a 10-month European gap option. If the stock price after 10-month is less than 28, the payoff is $28.5 - S\left(\frac{10}{12}\right)$; otherwise, the payoff is zero. You are given:n

- $S(0) = 30$.
- The stock will pay a dividend of amount 2 after 3-months. This is the only dividend that will be paid before the gap option expires.
- The prepaid forward price of the stock follows a geometric Brownian motion with a volatility of 31%.
- The continuously compounded risk-free rate of interest is 8%.

Calculate the price of the gap option.

(15 marks)

Ans.

$F^P_{0,10/12}(S) = 30 - 2e^{-0.08(3/12)} = 28.0396$
$F^P_{0,10/12}(K_2) = 28e^{-0.08(10/12)} = 26.1942$
$d_1 = \frac{\ln(28.0396/26.1942)+(0.31^2/2)(10/12)}{0.31\sqrt{10/12}} = 0.3821$
$d_2 = 0.3821 - 0.31\sqrt{10/12} = 0.0991$
$N(d_1) = N(0.38) = 0.648$; $N(d_2) = N(0.0991) = 0.5398$
The price of the gap put option is
$F^P_{0,10/12}(K_1)N(-d_2) - F^P_{0,10/12}(S)N(-d_1)$
$= 28.5e^{-0.08(10/12)}(1 - 0.5398) - 28.0396(1 - 0.648)$
$= \boxed{2.3999}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Assume the Black-Scholes framework for a stock whose time-t price is $S(t)$. You are given:

- $S(0) = 80$
- S pays dividends of amount $0.033S(t)dt$ between time-t and time $t + dt$.
- $V[\ln S(t)] = 0.0625t$
- The continuously compounded risk-free interest rate is 0.082.

Compute the price of $\min(S(0.1), 85)$ that mature at time 0.1.

(15 marks)

Ans.

Note that $\min(S(0.1), 85.0) = S(0.1) - [S(0.1) - 85, 0]_+$, then the price of $\min(S(0.1), 85.0)$ is $F^P_{t,T}(S) - c[80, 85.0]$
$V[\ln S(t)] = \sigma^2 t = 0.0625t \to \sigma = 0.25$
$d_1 = \frac{\ln(80)/(85)+(0.082-0.033+\frac{1}{2}0.25^2)(0.1)}{0.25\sqrt{0.1}} =$ -0.6653
$d_2 = d_1 - \sigma\sqrt{T} =$ -0.7444
$N(d_1) = N(-0.67) = 0.2514$; $N(d_2) = N(-0.74) = 0.2296$;
$c(S(0), 85.0, 0.1) = 80e^{-0.033(0.1)}(0.2514) - 85.0e^{0.082(0.1)}(0.2296) = 0.689117$
The price is $80e^{-0.033(0.1)} + c[80, 85.0] = 80e^{-0.033(0.1)} - 0.689117 = \boxed{79.0473}$

2. CO2: Demonstrate the calculation and the use of option price partial derivatives

(a) [Fill in the blank with correct answer] Let $S(t)$ be the time$-t$ price of a nondividend paying stock. You are given that $S(t)$ follows the stochastic differential equation

$$dS(t) = 0.05S(t)dt + 0.27d\tilde{Z}(t), S(0) = 3,$$

where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.
A market maker has just written a contingent claim that pays the $S^3(3)$ after 3 years. He then immediately delta-hedge his position by trading stocks and cash(W). Calculate W. -140.48 (6 marks)

(b) [Fill in the blank with correct answer] Let $S(t)$ be time-t price of a nondividend-paying stock and $P(S(t), t)$ be the time-t price of a 0.25-year at the money European put option written on the stock, when the time-t stock price is $S(t)$. You are given that

- $S(0) = 51$.
- The true stock price process is

$$dS(t) = 0.13S(t)dt + 0.26S(t)dZ(t)$$

where $Z(t)$ is a standard Brownian motion under the true measure.

- The true stochastic process satisfied by the put option is

$$dP(S(t), t) = a(S(t), t)dt + b(S(t), t)dZ(t)$$

for some a and b.

- $r = 0.065$.

Calculate $a(51, 0)$. -1.2629 (7 marks)

(c) [Fill in the blank with correct answer] Consider a European call option on a stock following the Black-Scholes framework. The option expires in one year. Using the Black-Scholes formula for this option, you obtain:

$$N(d_1) = 0.6406 \quad N(d_2) = 0.3594.$$

Calculate the volatility of the option. 1.64 (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] For a 1-year European call option on a stock:

- The strike price is 68.
- The stock's current price is 74.
- The continuously compounded risk-free interest rate is 0.06.
- The stock pays a dividend of 2 every 3 months, starting immediately after the call option is written. The dividend at the end of one year is paid before the option may be exercised.
- The annual volatility of a prepaid forward on the stock is 0.33.
- The stock follows the Black-Scholes framework.

Calculate the price of the option.

(15 marks)

Ans.

$F^P_{t,T}(S) = 74 - 2(1 + e^{-0.25(0.06)} + e^{-0.5(0.06)} + e^{-0.75(0.06)} + e^{-0.06}) = 64.2934$

$F^P_{t,T}(K) = Ke^{-rt} = 68e^{-0.06(1.0)} = 64.04$

$d_1 = \frac{\ln[F^P_{t,T}(S)/F^P_{t,T}(K)] + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln[64.2934/64.04] + 0.33^2/2)(1.0)}{0.33\sqrt{1.0}} = 0.18$

$d_2 = \frac{\ln[F^P_{t,T}(S)/F^P_{t,T}(K)] - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln[64.2934/64.04] - 0.33^2/2)(1.0)}{0.33\sqrt{1.0}} = -0.15$

$N(d_1) = N(0.18) = 0.5714$

$N(d_2) = N(-0.15) = 0.4404$

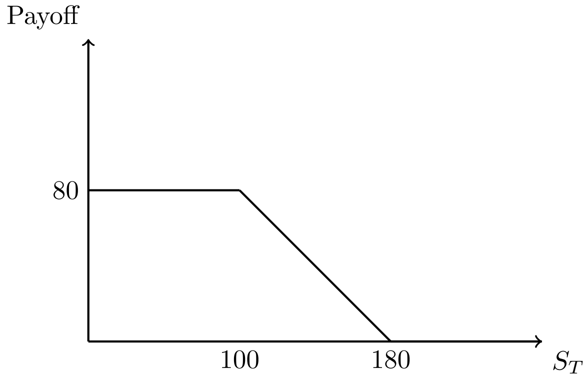
$c(S(t), K, t) = F^P_{t,T}(S)N(d_1) - F^P_{t,t}(K)N(d_2) -$

$$
\begin{aligned}
c(S(1.0), 68, 1.0) &= 64.2934N(0.18) - 64.04N(-0.15) \\
&= 64.2934(0.5714) - 64.04(0.4404) \\
&= \boxed{8.534}
\end{aligned}
$$

(e) [Show your workings. If no workings are shown, ZERO is awarded] For a stock whose time$-t$ price is $S(t)$, you are given:

$$S(0) = 100 \qquad \delta = 3.0\% \qquad \sigma = 37\% \qquad r = 8.7\%$$

Consider a contingent claim that has the following payoff function at time 3:



Calculate the time-0 price of the contingent claim.

(15 marks)

Ans.

The payoff of the contingent claim is equivalent to longing 1.0-180-strike put and shorting 1.0-100-strike put.

$d_1(100) = \frac{\ln(100/100)+[0.087-0.03+\frac{1}{2}(0.37^2)](3)}{0.37\sqrt{3}} = 0.59$; $N(-d_1(100)) = N(-0.59) = 1 - 0.7224 = 0.2776$

$d_2(100) = d_1(100) - \sigma\sqrt{t} = 0.59 - 0.37\sqrt{3} = -0.05$; $N(-d_2(100)) = N(--0.05) = 1 - 0.4801 = 0.5199$

$d_1(180) = \frac{\ln(100/180)+[0.087-0.03+\frac{1}{2}(0.37^2)](3)}{0.37\sqrt{3}} = -0.33$; $N(-d_1(180)) = N(--0.33) = 1 - 0.3707 = 0.6293$

$d_2(180) = d_1(180) - \sigma\sqrt{t} = -0.33 - 0.37\sqrt{3} = -0.97$; $N(-d_2(180)) = N(--0.97) = 1 - 0.166 = 0.834$

$p(100, 100) = 100e^{-0.087(3)}N(-d_2(100)) - 100e^{-0.03(3)}N(-d_1(100)) = 14.6762$

$p(100, 180) = 180e^{-0.087(3)}N(-d_2(180)) - 100e^{-0.03(3)}N(-d_1(180)) = 58.1209$

The time-0 price of the contingent claim $= 1.0(58.1209) - 1.0(14.6762) = \boxed{43.4447}$