Test 1 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM1404
Programme/Year:	AS, FM /Y1	Course Title:	Theory of Interest
Session:	202306	Lecturer:	Dr Yong Chin Khian

1. CO1: Use the concepts of derivatives and functions to solve equations in the context of theory of interest.

   (a) [Fill in the blank with correct answer] A deposit of 680 is made into a fund which pays an annual effective interest rate of 5% for 11 years. At the same time, 340 is deposited into another fund which pays an annual effective rate of discount of d for 11 years. The amounts of interest earned over the 11 years are equal for both funds. Calculate d. 0.07722329475104872. (6 marks)

   (b) [Fill in the blank with correct answer] Find the nominal rate of interest convertible semiannually which is equivalent to a nominal rate of discount of 20% per annum convertible quarterly. 0.21606648199446. (7 marks)

   (c) [Fill in the blank with correct answer] The risk-free force of interest δ_t at time t is given by:

   $$\delta_t = \begin{cases} 0.07, & 0 < t \leq 15 \\ 0.1 + 0.003t, & t > 15. \end{cases}$$

   Calculate the equivalent constant force of interest from time $t = 10$ to time $t = 20$. 0.11125. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] You are given a loan on which interest is charged over 4-year period, as follows:

- an effective rate of discount of 0.075 for the first year;
- a nominal rate of discount of 0.054 compounded every 2 years for the second year;
- a nominal rate of interest of 0.050 compounded semiannually for the third year; and
- a force of interest of 0.057 for the forth year.

Calculate the annual effective rate of interest over the 4-year period.

(15 marks)

Ans.

$$\begin{aligned}\text{AV} &= (1-d)^{-1}(1-\tfrac{d^{(1/2)}}{1/2})^{-1/2}(1+\tfrac{i^{(2)}}{2})^2 e^{\int_3^4 \delta dt} \\ &= (1-0.075)^{-1}(1-\tfrac{0.054}{1/2})^{-1/2}(1+\tfrac{0.05}{2})^2 e^{\int_3^4 0.057 dt} \\ &= 1.2731\end{aligned}$$

$(1+i)^4 = 1.2731$
$i = 1.2731^{1/4} - 1 = \boxed{0.0622}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Fund P grows at an annual interest rate of $i > 0$ for n years, an at annual interest rate of $j > 0$ for the next n years. Fund Q grows at an annual interest rate of $k > 0$ for $2n$ years. Fund P equals 1.5 times fund Q after n years. the amount in the two funds are equal after $2n$ years. Show that $j < k < i$.

(15 marks)

Ans.

$(1+i)^n(1+j)^n = (1+k)^{2n}$ ..... (1)
$(1+i)^n = 1.5(1+k)^n$ ..... (2)
$\frac{(1)}{(2)}, (1+j)^n = \frac{2}{3}(1+k)^n$ ..... (3)
From (2), $(1+i)^n = 1.5(1+k)^n, \Rightarrow i > k$
From (3), $(1+j)^n = \frac{2}{3}(1+k)^n, \Rightarrow j < k$

2. CO2: Formulate equations to solve problems involving interest/yield rates.

(a) [Fill in the blank with correct answer] You took a loan of 400,000 which required to pay 50 equal annual payments at 12% interest. The payments are due at the end of each year. The bank sold your loan to an investor immediately after receiving your 9th payment. With yield to the investor of 8%, the price the investor pay was 576,422. Determine the bank's overall return on its investment. 0.147. (6 marks)

(b) [Fill in the blank with correct answer] A loan of amount 13000 with annual effective interest rate of 6%, made at time $t = 0$, is to be repaid by 19 annual payments of R, beginning at time $t = 1$ and ending at time $t = 19$. At time $t = 14$, the borrower has financial troubles and can only pay $(R - 821.33)$. If he then returns to his original payment schedule of 1 at times $t = 15$ through $t = 18$, determine how much will his payment at time $t = 19$ need to be in order to pay the loan off in full.
2264.19. (7 marks)

(c) [Fill in the blank with correct answer] It is known that $w = \frac{1+\frac{i^{(3)}}{3}}{1+\frac{i^{(4)}}{4}}$. Express $s_{\overline{n}|}$ in terms of w. $s_{\overline{n}|} = \frac{w^{12n}-1}{w^{12}-1}$. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Kelvin wish to accumulate 80,000 in a fund at the end of 25 years. He plans to deposit 67 into the fund at the end of each of the first 120 months. He then plans to deposit $67 + k$ into the fund at the end of each of the last 180 months. Assume the fund earns interest at an annual effective rate 3.9%. Determine k.

(15 marks)

Ans.

Let j be the monthly effective interest rate.
$(1 + j)^{12} = (1 + 0.039) \Rightarrow j = (1.039)^{1/12} - 1 = 0.0032$
$67 s_{\overline{300}|j} + k s_{\overline{180}|j} = 80,000$
$67 \left[\frac{1.0032^{300}-1}{0.0032}\right] + k \left[\frac{1.0032^{180}-1}{0.0032}\right] = 80,000$
$67(502.41) + k(242.9) = 80,000$
$k = \boxed{190.77}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] An investment requires an initial payment of 140,000 and annual payments of 14,000 at the end of the first 15 years. Starting at the end of the 16 year, the investment returns 11 equal payments of X. Determine X to yield an annual effective rate of 7% over the 26-year period.

(15 marks)

Ans.

$a_{\overline{11}|} = \frac{1-(1+0.07)^{-11}}{0.07} = 7.4987$

$s_{\overline{15}|} = \frac{(1+0.07)^{15}-1}{0.07} = 25.1290$

$Xa_{\overline{11}|} = 140,000(1+0.07)^{15} + 14,000s_{\overline{15}|}$

$X = \frac{140,000(1+0.07)^{15}+14,000s_{\overline{15}|}}{a_{\overline{11}|}}$

$X = \frac{140,000(1+0.07)^{15}+14,000(25.1290)}{7.4987} = \boxed{98{,}426.8}$