Test 2 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM2453
Programme/Year:	AS /Y2, Y3	Course Title:	Financial Economics II
Session:	202301	Lecturer:	Dr Yong Chin Khian

1. CO3: Explain the cash flow characteristics of the following exotic options: Asian, barrier, compound, gap, and exchange.

   (a) [Fill in the blank with correct answer] Let $x(t)$ be the value of €1 in terms of US dollars at time t. You are given that:

   - The continuously compounded risk-free rate in US is 6.6%.
   - Under the risk-neutral measure, the stochastic differential equation of x is

     $$dx(t) = 0.025x(t)dt + 0.12d\tilde{Z}(t), \quad x(0) = 0.9$$

     where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.
   - A call option that gives the option holder the right to pay \$0.02 six months from today to buy a call option that gives the the right to buy €1 using \$0.95 one year from now is costs \$0.0224.

   Calculate the price of a put option that gives the option holder the right to sell at \$0.02 six months from today a call that gives the right to buy €1 using \$0.95 one year from now. 0.0113 (6 marks)

   (b) [Fill in the blank with correct answer] A British company will receive \$1,000,000 at the end of 6 month. To hedge its currency risk, it buys an option allowing to exchange dollars for pounds at a rate of £0.64/\$. You are given:

   - The spot exchange rate is £0.65/\$.
   - The continuously compounded risk-free interest rate for dollars is 0.06.
   - The continuously compounded risk-free interest rate for pounds is 0.04.
   - The volatility of the exchange rate between the two currencies is 0.1.
   - The Black-Scholes framework is assumed to apply to the currency rate.

   Calculate the cost in pounds of the hedge. 15860.00 (7 marks)

   (c) [Fill in the blank with correct answer] Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively. You are given:

   - $S_1(0) = 50$ and $S_2(0) = 100.0$.
   - Stock 1's volatility is 0.16.
   - Stock 2's volatility is 0.28.
   - The correlation between the continuously compounded returns of the two stocks is -0.4.
   - The continuously compounded risk-free interest rate is 6.5%.
   - A one-year European option with payoff $\max\{85.0 - \min[2.0S_1(1), S_2(1)], 0\}$ has a current (time-0) price of -1.3433.

   Consider a European option that gives its holder the right to buy either 2.0 shares of Stock 1 or one share of Stock 2 at a price of 85.0 one year from now. Calculate the current (time-0) price of this option. 3.95 (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Let $S(t)$ denote the price at time t of a stock. Consider a 10-month European gap option. If the stock price after 10-month is less than 28, the payoff is $28.5 - S\left(\frac{10}{12}\right)$; otherwise, the payoff is zero. You are given:n

- $S(0) = 30$.
- The stock will pay a dividend of amount 3 after 5-months. This is the only dividend that will be paid before the gap option expires.
- The prepaid forward price of the stock follows a geometric Brownian motion with a volatility of 25%.
- The continuously compounded risk-free rate of interest is 9%.

Calculate the price of the gap option.

(15 marks)

Ans.

$F^P_{0,10/12}(S) = 30 - 3e^{-0.09(5/12)} = 27.1104$

$F^P_{0,10/12}(K_2) = 28e^{-0.09(10/12)} = 25.9768$

$d_1 = \frac{\ln(27.1104/25.9768)+(0.25^2/2)(10/12)}{0.25\sqrt{10/12}} = 0.3013$

$d_2 = 0.3013 - 0.25\sqrt{10/12} = 0.0731$

$N(d_1) = N(0.3) = 0.6179$; $N(d_2) = N(0.0731) = 0.5279$

The price of the gap put option is

$F^P_{0,10/12}(K_1)N(-d_2) - F^P_{0,10/12}(S)N(-d_1)$

$= 28.5e^{-0.09(10/12)}(1 - 0.5279) - 27.1104(1 - 0.6179)$

$= \boxed{2.1238}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Assume the Black-Scholes framework for a stock whose time-t price is $S(t)$. You are given:

- $S(0) = 75$
- S pays dividends of amount $0.037S(t)dt$ between time-t and time $t + dt$.
- $V[\ln S(t)] = 0.0961t$
- The continuously compounded risk-free interest rate is 0.099.

Compute the price of $\min(S(0.4), 80)$ that mature at time 0.4.

(15 marks)

Ans.

Note that $\min(S(0.4), 80.0) = S(0.4) - [S(0.4) - 80, 0]_+$, then the price of $\min(S(0.4), 80.0)$ is $F^P_{t,T}(S) - c[75, 80.0]$

$V[\ln S(t)] = \sigma^2 t = 0.0961t \to \sigma = 0.31$

$d_1 = \frac{\ln(75)/(80)+(0.099-0.037+\frac{1}{2}0.31^2)(0.4)}{0.31\sqrt{0.4}} =$ -0.1047

$d_2 = d_1 - \sigma\sqrt{T} =$ -0.3007

$N(d_1) = N(-0.1) = 0.4602$; $N(d_2) = N(-0.3) = 0.3821$;

$c(S(0), 80.0, 0.4) = 75e^{-0.037(0.4)}(0.4602) - 80.0e^{0.099(0.4)}(0.3821) = 4.626778$

The price is $75e^{-0.037(0.4)} + c[75, 80.0] = 75e^{-0.037(0.4)} - 4.626778 = \boxed{69.2714}$

2. CO2: Demonstrate the calculation and the use of option price partial derivatives

(a) [Fill in the blank with correct answer] Let $S(t)$ be the time$-t$ price of a nondividend paying stock. You are given that $S(t)$ follows the stochastic differential equation

$$dS(t) = 0.1S(t)dt + 0.25d\tilde{Z}(t), S(0) = 2,$$

where $\tilde{Z}(t)$ is a standard Brownian motion under the risk-neutral measure.
A market maker has just written a contingent claim that pays the $S^3(3)$ after 3 years. He then immediately delta-hedge his position by trading stocks and cash(W). Calculate W. -51.18 (6 marks)

(b) [Fill in the blank with correct answer] Let $S(t)$ be time-t price of a nondividend-paying stock and $P(S(t), t)$ be the time-t price of a 0.5-year at the money European put option written on the stock, when the time-t stock price is $S(t)$. You are given that

- $S(0) = 62$.
- The true stock price process is

$$dS(t) = 0.12S(t)dt + 0.24S(t)dZ(t)$$

  where $Z(t)$ is a standard Brownian motion under the true measure.
- The true stochastic process satisfied by the put option is

$$dP(S(t), t) = a(S(t), t)dt + b(S(t), t)dZ(t)$$

  for some a and b.
- $r = 0.064$.

Calculate $a(62, 0)$. -95.74 (7 marks)

(c) [Fill in the blank with correct answer] Consider a European call option on a stock following the Black-Scholes framework. The option expires in one year. Using the Black-Scholes formula for this option, you obtain:

$$N(d_1) = 0.6628 \quad N(d_2) = 0.3372.$$

Calculate the volatility of the option. 1.71 (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] For a 1-year European call option on a stock:

- The strike price is 73.
- The stock's current price is 79.
- The continuously compounded risk-free interest rate is 0.08.
- The stock pays a dividend of 5 every 3 months, starting immediately after the call option is written. The dividend at the end of one year is paid before the option may be exercised.
- The annual volatility of a prepaid forward on the stock is 0.35.
- The stock follows the Black-Scholes framework.

Calculate the price of the option.

(15 marks)

Ans.

$F^P_{t,T}(S) = 79 - 5(1 + e^{-0.25(0.08)} + e^{-0.5(0.08)} + e^{-0.75(0.08)} + e^{-0.08}) = 54.9707$

$F^P_{t,T}(K) = Ke^{-rt} = 73e^{-0.08(1.0)} = 67.3875$

$d_1 = \frac{\ln[F^P_{t,T}(S)/F^P_{t,T}(K)] + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln[54.9707/67.3875] + 0.35^2/2)(1.0)}{0.35\sqrt{1.0}} =$ -0.41

$d_2 = \frac{\ln[F^P_{t,T}(S)/F^P_{t,T}(K)] - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln[54.9707/67.3875] - 0.35^2/2)(1.0)}{0.35\sqrt{1.0}} =$ -0.76

$N(d_1) = N(-0.41) = 0.3409$

$N(d_2) = N(-0.76) = 0.2236$

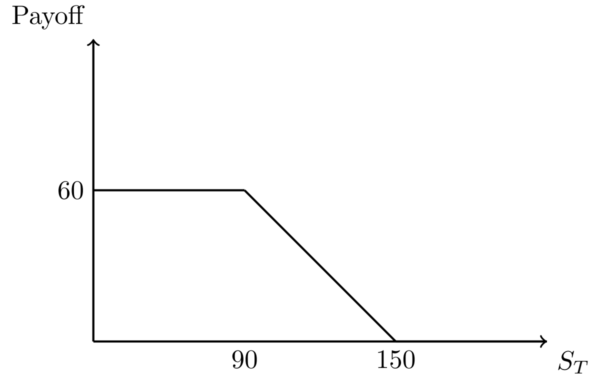
$c(S(t), K, t) = F^P_{t,T}(S)N(d_1) - F^P_{t,t}(K)N(d_2)-$

$$
\begin{aligned}
c(S(1.0), 73, 1.0) &= 54.9707N(-0.41) - 67.3875N(-0.76) \\
&= 54.9707(0.3409) - 67.3875(0.2236) \\
&= \boxed{3.6717}
\end{aligned}
$$

(e) [Show your workings. If no workings are shown, ZERO is awarded] For a stock whose time$-t$ price is $S(t)$, you are given:

$$S(0) = 90 \qquad \delta = 2.1\% \qquad \sigma = 30\% \qquad r = 8.4\%$$

Consider a contingent claim that has the following payoff function at time 4:



Calculate the time-0 price of the contingent claim.

(15 marks)

Ans.

The payoff of the contingent claim is equivalent to longing 1.0-150-strike put and shorting 1.0-90-strike put.

$d_1(90) = \frac{\ln(90/90)+[0.084-0.021+\frac{1}{2}(0.3^2)](4)}{0.3\sqrt{4}} = 0.72$; $N(-d_1(90)) = N(-0.72) = 1 - 0.7642 = 0.2358$

$d_2(90) = d_1(90) - \sigma\sqrt{t} = 0.72 - 0.3\sqrt{4} = 0.12$; $N(-d_2(90)) = N(-0.12) = 1 - 0.5478 = 0.4522$

$d_1(150) = \frac{\ln(90/150)+[0.084-0.021+\frac{1}{2}(0.3^2)](4)}{0.3\sqrt{4}} = -0.13$; $N(-d_1(150)) = N(- -0.13) = 1 - 0.4483 = 0.5517$

$d_2(150) = d_1(150) - \sigma\sqrt{t} = -0.13 - 0.3\sqrt{4} = -0.73$; $N(-d_2(150)) = N(- -0.73) = 1 - 0.2327 = 0.7673$

$p(90, 90) = 90e^{-0.084(4)}N(-d_2(90)) - 90e^{-0.021(4)}N(-d_1(90)) = 9.5716$

$p(90, 150) = 150e^{-0.084(4)}N(-d_2(150)) - 90e^{-0.021(4)}N(-d_1(150)) = 36.597$

The time-0 price of the contingent claim $= 1.0(36.597) - 1.0(9.5716) = \boxed{27.0254}$