

**MEME15203 Statistical Inference****Assignment 5****UNIVERSITI TUNKU ABDUL RAHMAN**

Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
Session:	January 2023		
Due by:	07/04/2023		

Q1. Let  $X$  be a single observation from the  $Beta(1, 7\theta)$  pdf. Find the shortest 93% pivotal interval.

(20 marks)

Q2. Let  $X_1, X_2, \dots, X_n$  be a random sample from a Weibull distribution,  $X \sim WEI(\theta, 4)$ .

(a) Show that  $Q = 2 \sum_{i=1}^n X_i^4 / \theta^4 \sim \chi^2(2n)$ .

(b) Use  $Q$  to derive an equal tailed 100 $\gamma\%$  confidence interval for  $\theta$ .

(20 marks)

Q3. Let  $X$  have probability density function

$$f(x) = \begin{cases} \frac{\Gamma(5)x^2(\theta-x)}{\Gamma(3)\theta^4}, & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

Show that  $\frac{X}{\theta}$  is a pivotal quantity and use this pivotal quantity to find a 97% upper confidence limit for  $\theta$ .

(20 marks)

Q4. Let  $X_1, \dots, X_n$ , be a random sample from a gamma distribution with parameters  $\alpha = 3$  and unknown  $\theta$ .

(a) Find a pivotal quantity for the parameter  $\theta$  based on the sufficient statistic.

(b) Derive an equal tail 98% confidence interval for  $\theta$  based on the pivotal quantity from part (a).

(20 marks)

Q5. You are given the following:

$$f(x|\theta) = \frac{8x^7}{\theta^8}, 0 < x < \theta.$$

$$\pi(\theta) = \frac{4}{\theta^5}, \theta > 1.$$

Three observations were observed: 100, 800, 1200. Find a 93% "HPD" credible set for  $\theta$ .

(20 marks)