Test 1 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM1404
Programme/Year:	AS /Y1	Course Title:	Theory of Interest
Session:	202206	Lecturer:	Dr Yong Chin Khian

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1. CO1: Use the concepts of derivatives and functions to solve equations in the context of theory of interest.

   (a) [Fill in the blank with correct answer] You are given:

   (i) $\delta_t = \frac{1}{3+t}$; and

   (ii) the total interest earned during the first n years on an investment of 1 at time $t = 0$ is 2.4.

   Determine n. 7.20000000. (6 marks)

   (b) [Fill in the blank with correct answer] You are given a loan on which interest is charged over 4-year period, as follows:

   - an effective rate of discount of 0.051 for the first year;
   - a nominal rate of discount of 0.051 compounded every 2 years for the second year;
   - a nominal rate of interest of 0.055 compounded semiannually for the third year; and
   - a force of interest of 0.080 for the forth year.

   Calculate the annual effective rate of interest over the 4-year period. 0.061952. (7 marks)

   (c) [Fill in the blank with correct answer] At a certain interest rate the present value of the following two payment patterns are equal:

   - 270 at the end of 7 years plus 538 at the end of 14 years.
   - 731.98 at the end of 7 years.

   At the same interest rate, 135.0 invested now plus 162.0 invested at the end of 7 years will accumulate to P at the end of 14 years. Calculate P. 371.742183. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Jeff deposits 13 into a fund today and 26 25-year later. Interest for the first 7 years is credited at a nominal discount rate of d compounded quarterly, and thereafter at a nominal interest rate of 7% compounded semiannually. The accumulated balance in the fund at the end of 34 years is 176. Calculate d.

(15 marks)

Ans.

Equation of value at end of 34 years:

$13\left(1-\frac{d}{4}\right)^{-4\times 7}\left(1+\frac{0.07}{2}\right)^{2\times 27}+26\left(1+\frac{0.07}{2}\right)^{2\times 9}=176$

$13\left(1-\frac{d}{4}\right)^{-28}(1.035)^{54}+26\,(1.035)^{18}=176$

$13\left(1-\frac{d}{4}\right)^{-28}=\frac{176-26(1.035)^{18}}{1.035^{54}}=19.9265$

$1-\frac{d}{4}=\left(\frac{19.9265}{13}\right)^{-1/28}=0.9849$

$d=4(1-0.9849)=\boxed{0.0604}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] If $r=\frac{i^{(5)}}{d^{(5)}}$, express v in terms of r.

(15 marks)

Ans.

$(1+\frac{i^{(5)}}{5})(1-\frac{d^{(5)}}{5})=1$

$1+\frac{i^{(5)}}{5}-\frac{d^{(5)}}{5}-\frac{i^{(5)}d^{(5)}}{25}=1$

$\frac{i^{(5)}-d^{(5)}}{5}=\frac{i^{(5)}d^{(5)}}{25}$

$i^{(5)}-d^{(5)}=\frac{i^{(5)}d^{(5)}}{5}$

$r-1=\frac{i^{(5)}-d^{(5)}}{d^{(5)}}=\frac{i^{(5)}d^{(5)}}{5d^{(5)}}=\frac{i^{(5)}}{5}$

$r=1+\frac{i^{(5)}}{5}$

$v=(1+\frac{i^{(5)}}{5})^{-5}=r^{-5}$

$\boxed{1.0000}$

2. CO2: Formulate equations to solve problems involving interest/yield rates.

(a) [Fill in the blank with correct answer] At a nominal rate of interest i, convertible semiannually, the present value of a series of payment of 1 at the end of every 2 years, forever, is 6.78. Calculate i. 0.06998630149361107. (6 marks)

(b) [Fill in the blank with correct answer] The death benefit on a life insurance policy can be paid in any of the following ways, each of which has the same present value as the death benefit:

- a perpetuity of 170 at the end of each month;
- 302.708172 at the end of each month for n years; and
- a payment of 73164.400000 at the end of n years.

Calculate the amount of the death benefit. 32075.471698113208. (7 marks)

(c) [Fill in the blank with correct answer] You took a loan of 400,000 which required to pay 45 equal annual payments at 12% interest. The payments are due at the end of each year. The bank sold your loan to an investor immediately after receiving your 7th payment. With yield to the investor of 8%, the price the investor pay was 571,269. Determine the bank's overall return on its investment.
0.15843181. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Lee borrows X for 25 years at an annual effective interest rate of 6%. If he pays the principal and accumulated interest in one lump sum at the end of 25 years, he would pay 2312.85 more in interest than if he repaid the loan with 25 level payments at the end of each year. calculate X.

(15 marks)

Ans.

$I_1 = X(1.06^{25} - 1)$

$I_2 = \frac{25X}{a_{\overline{25}|6\%}} - X$

$a_{\overline{25|6\%}|} = \frac{1-1.06^{-25}}{0.06} = 12.7834$

Thus,

$X(1.06^{25} - 1) = \frac{25X}{a_{\overline{25}|6\%}} - X + 2312.85$

$X(1.06^{25} - \frac{25}{a_{\overline{25}|6\%}}) = 2312.85$

$X(1.06^{25} - \frac{25}{12.7834}) = 2312.85$

$X = \boxed{990.00}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Steven have a 30-year 140,000 mortgage with an 7% interest rate convertibele monthly. Payments are made at the end of the month. Immediate after the 120th payment, he refinance the mortgage. The iterest rate is reduced to 5.5%, convertibele monthly, and the term is reduced to 20 years (so there are 10 years of payments remainning). He also make an additional payment of 18,667 at the time of refinancing. Calculate his new monthly payment.

(15 marks)

Ans.

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        0.5833%
          <--->
         140000      R    R       R          R
1/2 years |---|---|---...|---...---|
          0    1         120        360

        0.4583%
          <--->
B120+18,667   X    X       X
1/2 years    |---|---|...---|
            120  121  122      240
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$140000 = Ra_{\overline{360}|0.5833\%}$
$R = \frac{140000}{a_{\overline{360}|0.5833\%}} = 140000\left(\frac{0.0058}{1-1.0058^{-360}}\right) = 927.67$
$B_{120} = Ra_{\overline{240}|0.5833\%} = 927.67\left[\frac{1-1.0058^{-240}}{0.0058}\right] = 120024.62$
$B_{120} + 18,667 = Xa_{\overline{120}|0.4583\%}$
$X = \frac{B_{120}-18,667}{a_{\overline{120}|0.4583\%}} = (120024.62 - 18,667)\left[\frac{0.0046}{1-1.0046^{-120}}\right] = \boxed{1,101.00}$