MEME15203 Statistical Inference

Assignment 1

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Wong Wai Kuan
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Q1. Suppose that X_1 and X_2 denote a random sample of size 2 from a gamma distribution $X_i \sim GAM(0.5, 8)$. Find the pdf of $\frac{X_1}{X_2}$.

(10 marks)

Q2. Let X_1 and X_2 be independent random variables with $X_1 \sim GAM(\alpha_1 = a, \theta = 2)$ and $X_2 \sim GAM(\alpha_2 = b, \theta = 2)$, show that $U = \frac{X_1}{X_1+X_2}$ follow a Beta distribution. Suppose $Y_i \sim GAM(\alpha = 5, \theta = 2)$, using the result above, find the distribution of $V = \frac{Y_1}{\sum_{i=1}^{11} Y_i}$.

(10 marks)

Q3. Let Y_3 denote the third smallest item of a random sample of size n from a distribution of the continuous type that has cdf $F_X(x)$ and pdf $f_X(x) = F'_X(x)$. Find the probability density function (p.d.f.) of $W_n = nF_{Y_3}(y)$.

(10 marks)

Q4. In each of the following, random variable X and Y are independent. Write the full expression for the probability density functions (pdfs)for X, Y and $V = X + Y$. No proofs need to be given. Also simply giving the name of the pdf is not enough.

(a)$X \sim Bin(n = 11, p = 0.5)$
$Y \sim Bin(n = 9, p = 0.5)$

(b)$X \sim POI(\lambda = 1.3)$
$Y \sim POI(\lambda = 2.6)$

(c)$X \sim N(\mu = 12, \sigma^2 = 4^2)$
$Y \sim N(\mu = 8, \sigma^2 = 6^2)$

(d)$X \sim GAM(\alpha = 3, \theta = 10)$
$Y \sim GAM(\alpha = 3, \theta = 10)$

(12 marks)

Q5. Let Y_1 denote the minimum of a random sample of size n from a distribution that has pdf $f(x) = e^{-(x-\theta)}, \theta < x < \infty$, zero otherwise. Let $Z_n == n(Y_1 - \theta)$. Investigate the limiting distribution of Z_n.

(10 marks)

Q6. Consider a random sample from a Exponential distribution, $X_i \sim Exp(\theta)$. Find the asymtotic normal distribution of $Y_n = \bar{X}_n^5$.

(8 marks)

Q7. Let the random variable Y_n have a distribution that is $Bin(n,p)$. Prove that $\left(\frac{Y_n}{n}\right)\left(1-\frac{Y_n}{n}\right)$ converges in probability to a constant, identify the constant.

(5 marks)

Q8. Let $X_2, X_3, X_4, \ldots$ be a sequence of random variable such that

$$F_{X_n}(x) = \begin{cases} 1-\left(1-\frac{1}{9n}\right)^{nx} & x>0 \\ 0 & \text{otherwise} \end{cases}.$$

Find the limiting distribution of X_n.

(5 marks)

Q9. Let $Y_n \sim GAM(n,\theta)$. Find the limiting distribution of $Z_n = \frac{Y_n - n\theta}{\sqrt{n}\theta}$ as $n \to \infty$, using moment generating function.

(10 marks)

Q10. Suppose that $X_i \sim N(\mu,\sigma^2), i = 1,\ldots,15$ and $Z_i \sim N(0,1), i = 1,\ldots,20$ and all variables are independent. State the distribution of each of the following variables if it is a "named" distribution or otherwise state "unknown."

(a) $\dfrac{\sqrt{15}(\bar{X}-\mu)}{\sigma S_Z}$

(b) $\dfrac{\sum_{i=1}^{15}(X_i-\mu)^2}{\sigma^2} + \sum_{i=1}^{20}(Z_i-\bar{Z})^2$

(c) $\dfrac{\bar{X}}{\sigma^2} + \dfrac{\bar{Z}}{\sigma}$

(d) Let $W = \dfrac{\sqrt{15}(\bar{X}-\mu)}{\sigma}$, find $\bar{W}^2$

(e) $\frac{(19)\sum_{i=1}^{15}(X_i-\bar{X})^2}{(14)\sigma^2\sum_{i=1}^{20}(Z_i-\bar{Z})^2}$

(20 marks)