MEME15203 Statistical Inference

Assignment 3

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Due by:	18/3/2024		

Q1.   Suppose that $X_1, \ldots, X_n$ is a random sample from a Poisson distribution, $X_i \sim$ POI(θ),

(a)Find a complete and sufficient statistic for θ.

(b)Find the UMVUE for θ

(c)Find the UMVUE of $e^{-7\theta}$ using Lehmann Scheffe Theorem.

(d)Find the UMVUE of $e^{-7\theta}$ using Rao Blackwell Theorem.

(20 marks)

Q2.   Let $X_1, X_2, \ldots, X_n$ be random sample of size n from an Exponential distribution with unknown mean θ. Find the UMVUE of $\gamma = e^{-t/\theta}$ using Rao-Blackwell theorem.

(20 marks)

Q3.   Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with pdf

$$f(x;\theta) = 8\theta x^{8\theta-1} I_{(0,1)}(x).$$

Find the UMVUE of θ,

(20 marks)

Q4.   Let $X_1, X_2, \ldots, X_n$ be random sample of size n from $f(x|\theta) = \binom{k}{x}\theta^x(1-\theta)^{k-x}$, $x = 0, 1, \ldots, k$. Find the uniformly minimum variance unbiased estimator (UMVUE) of $g(\theta) = \binom{k}{2}\theta^2(1-\theta)^{k-x}$.

(20 marks)

Q5.   Let $X_1, \ldots, X_{30}$ be a random sample from a distribution with probability density function(p.d.f.)

$$f(x) = \frac{\theta^6}{\Gamma(6)}x^5 e^{-\theta x} I(0, \infty), \theta > 0.$$

(a)Show that the p.d.f. of X belongs to the regular exponential family.

(b)Find a complete and sufficient statistic for θ.

(c)Find the UMVUE for $V(X_1)$.

(d)Find the UMVUE for θ.

(20 marks)

This assessment consists of 5 questions on 1 printed pages