UCCM3473 CREDIBILITY THEORY

$1. X \sim POI(\lambda)$

- $f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, \ldots$
- $E(X) = V(X) = \lambda$

$2. X \sim Bin(m, q)$

- $f(x|q) = \binom{m}{x} q^x (1-q)^{m-x}, x = 0, 1, \ldots, m$
- $E(X) = mq; V(X) = mq(1-q)$

$3. X \sim NB(r, \beta)$

- $f(x|\beta) = \frac{r(r+1)\cdots(r+x-1)\beta^x}{x!(1+\beta)^{r+x}}$

  $= \binom{r+x-1}{x} \frac{\beta^x}{(1+\beta)^{r+x}}, x = 0, 1, \ldots$
- $E(X) = r\beta; V(X) = r\beta(1+\beta)$

$4. X \sim N(\mu, \sigma)$

- $f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, x \in R$
- $E(X) = \mu; V(X) = \sigma^2$

$5. X \sim Gamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, x > 0$
- $F(x|\alpha, \theta) = 1 - \sum_{j=0}^{\alpha-1} \frac{(\frac{x}{\theta})^j e^{-\frac{x}{\theta}}}{j!}$

- $E(X) = \alpha\theta; V(X) = \alpha\theta^2$
- $E(X^k) = \theta^k \alpha(\alpha+1)\cdots(\alpha+k-1)$

6.$X \sim InvGamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\frac{\theta}{x}}, x > 0$
- $E(X^k) = \frac{\theta^k}{(\alpha-1)\cdots(\alpha-k)}$, if k is a positive integer

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7.$X \sim Pareto(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, x > 0$
- $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha$
- $E(X^k) = \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}$

8.$X \sim SingleParameterPareto(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
- $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x}\right)^\alpha$
- $E(X^k) = \frac{\alpha\theta^k}{\alpha-k}, k < \alpha$

$9. X \sim Beta(a, b, \theta)$

- $f(x|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)\theta^{a+b-1}} x^{a-1}(\theta - x)^{b-1}, 0 < x < \theta$
- $E(X^k) = \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}$ if k is positve integer.

$10. X \sim LogNormal(\mu, \sigma)$

- $f(x|\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, x > 0$
- $F(x) = \Phi(\frac{x-\mu}{\sigma})$
- $E(X^k) = e^{k\mu + \frac{1}{2}k^2\sigma^2}$

$11. X \sim Weibull(\tau, \theta)$

- $f(x|\tau,\theta) = \frac{\tau}{\theta^\tau} x^{\tau-1} e^{-(x/\theta)^\tau}, x > 0$
- $F(x) = 1 - e^{-(x/\theta)^\tau}$
- $E(X^k) = \theta^k \Gamma(1 + k/\tau), k > -\tau$

$12. X \sim InvGaussian(\mu, \theta)$

- $E(X) = \mu; V(X) = \frac{\mu^3}{\theta}$
