

TEST 1 MARKING GUIDE

Name: _____ Student ID: _____ Mark: _____ /100

FACULTY:	FES, UTAR	UNIT CODE:	UECM3463
COURSE/YEAR:	AS /Y2 & Y3	UNIT TITLE:	LOSS MODELS
SESSION:	202306	LECTURER:	DR YONG CHIN KHIAN

1. CO1: Calculate expected values, variances and probabilities for frequency random variables.

(a) [Fill in the blank with correct answer] For a certain $(a, b, 0)$ distribution,

- $a = 0.5$,
- $b = 2.0$, and
- $1000p_0 = 31.25$.

Calculate the probability of exactly 3 events occurring times 1000, i.e. $1000p_3$.
[136.718750](#) (6 marks)

(b) [Fill in the blank with correct answer] N^M is a discrete random variable with probability function which is a member of the $(a, b, 1)$ class of distributions. You are given

$$P(z) = 0.21 + 0.79 \left[\frac{e^{6.20(z-1)} - e^{-6.20}}{1 - e^{-6.20}} \right]$$

Calculate the variance of the distribution. [11.249000](#) (7 marks)

(c) [Fill in the blank with correct answer] A random variable follows a zero-modified Poisson distribution with $\lambda = 0.7000000000000001$ and $p_0^M = 0.64$. Calculate the third raw moment of the distribution. [1.797087](#) (7 marks)

- (d) [Show your workings. If no workings are shown, ZERO is awarded] For a zero-modified Negative Binomial distribution with parameters $r = 2$ and β . You are given $p_0 = 0.22$ and $p_1 = 0.0142$. Find β .

(15 marks)

Ans.

$$p_1^M = \frac{(1-p_0^M)}{1-p_0} p_1 \dots\dots\dots [1m]$$

$$0.0142 = (1 - 0.22) \left(\frac{2\beta}{(1+\beta)^3 - (1+\beta)} \right) \dots\dots\dots [1m]$$

$$\frac{1}{(1+\beta)(2+\beta)} = \frac{0.0142}{2(1-0.22)} \dots\dots\dots [1m]$$

$$2 + 3\beta + \beta^2 = 110 \dots\dots\dots [1m]$$

$$\beta^2 + 3\beta - 108 = 0 \dots\dots\dots [1m]$$

$$\beta = \frac{-3 + \sqrt{9 + 4(108)}}{2} = \boxed{9.00} \dots\dots\dots [1m]$$

- (e) [Show your workings. If no workings are shown, ZERO is awarded] For a zero-modified Poisson distribution, $p_1 = 0.0017$, $p_2 = 0.005$, calculate the variance of the distribution.

(15 marks)

Ans.

$$N^M \sim ZM - POI(\lambda) \Rightarrow a = 0 \text{ and } \lambda = b \dots\dots\dots [1m]$$

$$\frac{p_2}{p_1} = \frac{0.005}{0.0017} = \frac{b}{2} \Rightarrow b = 5.8824 \dots\dots\dots [1m]$$

$$p_1^M = cp_1$$

$$0.0017 = c(5.8824)e^{-5.8824} \Rightarrow c = \frac{0.0017e^{5.8824}}{5.8824} = 0.1037 \dots\dots\dots [1m]$$

$$E(N^M) = cE(N) = 0.1037(5.8824) = 0.61 \dots\dots\dots [1m]$$

$$E(N^M)^2 = cE(N^2) = 0.1037(5.8824 + 5.8824^2) = 4.1983 \dots\dots\dots [1m]$$

$$V(N^M) = E(N^M)^2 - (E(N^M))^2 = \boxed{3.8262} \dots\dots\dots [1m]$$

2. CO2: Calculate expected values, variances, probabilities, and percentiles for severity random variable defined above.

(a) [Fill in the blank with correct answer] Claim severity has the following distribution:

Claim Size	150.0	157.5	165.0	172.5	180.0
Probability	0.29	0.27	0.27	0.08	0.09

Determine the distribution's skewness. [0.590756](#) (6 marks)

(b) [Fill in the blank with correct answer] X is a random variable representing loss size. You are given that

$$E[X \wedge d] = 408 - \frac{272^3}{2d^2}$$

Loss sizes are affected by 10% inflation. Determine the average payment per loss under a policy with 313 ordinary deductible after inflation. (6 marks) [136.694872](#)
(7 marks)

(c) [Fill in the blank with correct answer] The distribution of X is specified by it's hazard rate function

$$h(x) = \frac{xe^{-0.7x}}{\int_x^\infty se^{-0.7s}ds}, x > 0$$

Calculate $E(X - 3)_+$. (6 marks) [0.717200](#) (7 marks)

- (d) [Show your workings. If no workings are shown, ZERO is awarded] Annual losses follow a Pareto distribution with $\alpha = 4.90$ and $\theta = 1,050$. Calculate the difference between $TVaR_{0.92}$ and $VaR_{0.92}$.

(15 marks)

Ans.

$$TVaR_p = VaR_p + e(VaR_p)$$

$$TVaR_p - VaR_p = e(VaR_p)$$

..... [1m]

$$\text{Let } \pi_p = VaR_p$$

$$X \sim \text{Pareto}(\alpha = 4.9, \theta = 1,050)$$

$$S(\pi_{0.92}) = 1 - 0.92 \dots \dots \dots [1m]$$

$$\left(\frac{1050}{\pi_{0.92} + 1050} \right)^{4.9} = 0.08 \dots \dots \dots [1m]$$

$$\pi_p = 708.12 \dots \dots \dots [1m]$$

$$e(\pi_p) = \frac{\theta + \pi_p}{\alpha - 1} \dots \dots \dots [1m]$$

$$e(\pi_{0.92}) = \frac{1050 + 708.12}{4.9 - 1} = \boxed{450.80} \dots \dots \dots [1m]$$

- (e) [Show your workings. If no workings are shown, ZERO is awarded] You are given that the moment generating function of the random variable X is

$$M_X(t) = \exp \left[\frac{\theta}{\mu} \left(1 - \sqrt{1 - \frac{2t\mu^2}{\theta}} \right) \right].$$

Show that the third raw moment of X is $\frac{3\mu^5}{\theta^2} + \frac{3\mu^4}{\theta} + \mu^3$.

(15 marks)

Ans.

$$M_X(t) = \exp \left[\frac{\theta}{\mu} \left(1 - \sqrt{1 - \frac{2t\mu^2}{\theta}} \right) \right]$$

$$k_X(t) = \ln(M_X(t)) = \frac{\theta}{\mu} \left(1 - \left(1 - \frac{2t\mu^2}{\theta} \right)^{1/2} \right)$$

$$k'_X(t) = \frac{-\theta}{\mu} \left(\frac{1}{2} \right) \left(1 - \frac{2t\mu^2}{\theta} \right)^{-1/2} \left(\frac{-2\mu^2}{\theta} \right) = \mu \left(1 - \frac{2t\mu^2}{\theta} \right)^{-1/2}$$

$$E(X) = k'_X(0) = \mu$$

$$k''_X(t) = -\frac{1}{2}\mu \left(1 - \frac{2t\mu^2}{\theta} \right)^{-3/2} \left(\frac{-2\mu^2}{\theta} \right) = \frac{\mu^3}{\theta} \left(1 - \frac{2t\mu^2}{\theta} \right)^{-3/2}$$

$$V(X) = \frac{\mu^3}{\theta}$$

$$E(X^2) = \frac{\mu^3}{\theta} + \mu^2$$

$$k^{(3)}_X(t) = \left(\frac{-3}{2} \right) \left(\frac{\mu^3}{\theta} \right) \left(1 - \frac{2t\mu^2}{\theta} \right)^{-5/2} \left(\frac{-2\mu^2}{\theta} \right) = \frac{3\mu^5}{\theta^2} \left(1 - \frac{2t\mu^2}{\theta} \right)^{-5/2}$$

$$E(X - \mu)^3 = k^{(3)}_X(0) = \frac{3\mu^5}{\theta^2}$$

$$E(X^3) - 3E(X^2)E(X) + 2[E(X)]^3 = \frac{3\mu^5}{\theta^2}$$

$$E(X^3) = \frac{3\mu^5}{\theta^2} + 3 \left[\frac{\mu^3}{\theta} + \mu^2 \right] (\mu) - 2\mu^3 = \frac{3\mu^5}{\theta^2} + \frac{3\mu^4}{\theta} + \mu^3$$