$\bullet f(x) = p^x q^{1-x}, x = 0, 1$

$$\bullet \ M_X(t) = pe^t + q$$

$$\bullet\; E(X)=p$$

$$\bullet V(X) = pq$$

2. $\mathbf{X} \sim Binomial(n, p)$

•
$$f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$$

$$\bullet M_X(t) = (pe^t + q)^n$$

$$\bullet$$
 $E(X) = np$

$$\bullet \ V(X) = npq$$

3. $\mathbf{X} \sim HYP(n, M, N)$

•
$$f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}},$$

 $x = 0, 1, \dots, \min(n, M), n - x \le N - M.$

$$\bullet\; E(X) = \tfrac{nM}{N}$$

$$\bullet \ V(X) = n \frac{M}{N} \left(1 - \frac{M}{N} \right) \frac{N - n}{N - 1}$$

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$$\bullet f(x) = pq^{x-1}$$
 $x = 1, 2, 3, \dots$

•
$$F(x) = 1 - q^x$$
 $x = 1, 2, 3, \dots$

•
$$M_X(t) = \frac{pe^t}{1 - qe^t}$$

$$\bullet E(X) = \frac{1}{n}$$

$$\bullet V(X) = \frac{q}{p^2}$$

5. $\mathbf{X} \sim NegativeBinomial(r, p)$

•
$$f(x) = {x-1 \choose r-1} p^r q^{x-r}, x = r, r+1, \dots$$

•
$$M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^r$$

$$\bullet E(X) = \frac{r}{p}$$

$$\bullet V(X) = \frac{rq}{p^2}$$

6. $\mathbf{X} \sim POI(\mu)$

•
$$f(x) = \frac{e^{-\mu}\mu^x}{x!}$$
 $x = 0, 1, 2, \dots$

$$\bullet \ M_X(t) = e^{\mu(e^t - 1)}$$

$$\bullet\; E(X) = \mu$$

$$\bullet \ V(X) = \mu$$

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Common Distributions

7. $\mathbf{X} \sim DU(N)$

•
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$

$$\bullet F(x) = \frac{x(1+x)}{2N}$$

$$\bullet E(X) = \frac{\tilde{N}+1}{2}$$

$$\bullet V(X) = \frac{N^2 - 1}{12}$$

8. $\mathbf{X} \sim U(a,b)$

•
$$F(x) = \frac{x-a}{b-a}$$
, $a < x < b$

$$M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$$

•
$$E(X) = \frac{a+b}{2}$$

$$V(X) = \frac{(b-a)^2}{12}$$

9. $\mathbf{X} \sim Gamma(\alpha, \theta)$

•
$$F(x) = 1 - \sum_{i=0}^{\alpha - 1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$$

•
$$M_X(t) = (\frac{1}{1-\theta t})^{\alpha}$$

$$\bullet \ E(X) = \alpha \theta$$

• $f(x) = \frac{1}{N}, X = 1, 2, \dots, N$

•
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$

$$\bullet \ F(x) = \frac{x(1+x)}{2N}$$

$$\bullet V(X) = \frac{N^2 - 1}{12}$$

• $f(x) = \frac{1}{b-a}$, a < x < b and zero otherwise

$$\bullet \ F(x) = \frac{x-a}{b-a}, a < x < b$$

$$\bullet \ M_X(t) = \frac{e^{tb} - e^{tc}}{b - a}$$

•
$$E(X) = \frac{1}{2}$$

• $V(Y) = \frac{(b-a)^2}{2}$

•
$$f(x) = \frac{1}{\theta^{\alpha} \Gamma(\alpha)} x^{\alpha - 1} e^{-x/\theta}, x > 0$$

•
$$F(x) = 1 - \sum_{i=0}^{\alpha - 1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$$

$$F(Y) = \alpha \theta$$

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Common Distributions

$$V(X) = \alpha \theta^2$$

10. $\mathbf{X} \sim EXP(\theta)$

• $f(x) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ and zero otherwise.

•
$$F(x) = 1 - e^{-x/\theta}, x > 0$$

•
$$M_X(t) = \left(\frac{1}{1-\theta t}\right)$$

$$\bullet \ E(X) = \theta$$

•
$$V(X) = \theta^2$$

11. $\mathbf{X} \sim WEI(\tau, \theta)$

• $f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau-1} e^{-(x/\theta)^{\tau}}, x > 0$ and zero otherwise.

$$\bullet \ F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

•
$$E(X) = \theta \Gamma \left(1 + \frac{1}{\tau} \right)$$

•
$$E(X^2) = \theta^2 \left[\Gamma \left(1 + \frac{2}{\tau} \right) - \Gamma^2 \left(1 + \frac{1}{\tau} \right) \right]$$

12. $\mathbf{X} \sim PAR(\alpha, \theta)$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$$

•
$$F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$$

•
$$E(X) = \frac{\theta}{\alpha - 1}$$

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•
$$V(X) = \frac{\theta^2}{(\alpha - 1)^2(\alpha - 2)}$$

13. $\mathbf{X} \sim Beta(a,b)$

•
$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}, \ 0 < x < 1$$

$$\bullet E(X) = \frac{a}{a+b}$$

$$\bullet V(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

14. **X** ~ $N(\mu, \sigma^2)$

•
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}, x \in \mathbb{R}, \mu \in \mathbb{R}$$
 and $\sigma > 0$.

•
$$F(x) = \Phi(\frac{x-\mu}{\sigma})$$

$$\bullet \ M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$\bullet \; E(X) {=} \; \mu$$

$$\bullet \ V(X) = \sigma^2$$

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Common Distributions

 $18. \mathbf{X} \sim DE(\eta, \theta)$

•
$$f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$$
 $-\infty < x < \infty$ and zero otherwise.

•
$$F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \le \eta \\ \frac{1}{2}[1 - e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$$

$$\bullet M_X(t) = \frac{e^{\eta t}}{1 - \theta^2 t^2}$$

$$\bullet E(X) = \eta$$

•
$$V(X) = 2\theta^2$$

19. $\mathbf{X} \sim \text{Single Parameter Pareto } (\alpha, \theta)$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{x^{\alpha+1}}, x > \theta$$

•
$$F(x) = 1 - (\frac{\theta}{x})^{\alpha}$$

$$\bullet\; E(X) = \tfrac{\alpha\theta}{\alpha-1}$$

$$\bullet\; E(X^2) = \tfrac{\alpha\theta^2}{\alpha-2}$$

•
$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x - \mu)^2/2\sigma^2}, x > 0, \mu > 0$$

0 and $\sigma > 0$

•
$$F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

$$\bullet E(X) = e^{\mu + \frac{\sigma^2}{2}}$$

$$\bullet V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$

16.
$$\mathbf{X} \sim CAU(\theta, \eta)$$

•
$$f(x) = \frac{1}{\theta \pi \left[1 + \left(\frac{x - \eta}{\theta}\right)^2\right]}$$

•
$$F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} (\frac{x-\eta}{\theta})$$

17. $\mathbf{X} \sim EXP(\eta, \theta)$

•
$$f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$$
 $x > \eta$

$$\bullet F(x) = 1 - e^{-\frac{x - \eta}{\theta}}$$

•
$$M_X(t) = \frac{e^{\eta t}}{1 - \theta t}$$

$$\bullet E(X) = \eta + \theta$$

$$\bullet V(X) = \theta^2$$

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