**MEME16203Linear Models**

**Assignment 4**

**UNIVERSITI TUNKU ABDUL RAHMAN**

| | | | |
|---|---|---|---|
| Faculty: | FES | Unit Code: | MEME16203 |
| Course: | MAC | Unit Title: | Linear Models |
| Year: | 1,2 | Session: | May 2023 |

Due by: 7/8/2023

Q1. Consider the model $Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon ijk,\ i = 1, 2, 3; j = 1, 2; k = 1,$ where $\epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$. Determine which of the following hypotheses are testable. Justify your answer.

(a) $H_0 : \gamma_{11} = 0.$ (4 marks)

(b) $H_0 : \mu + \alpha_1 + \frac{1}{2}(\beta_1 + \beta_2 + \gamma_{11} + \gamma_{12}) = 0.$ (3 marks)

(c) $H_0 : (\alpha_1 - \alpha_2) + \gamma_{11} - \gamma_{21}.$ (3 marks)

Q2. An researcher recorded moisture content for three types of cheese made by two different methods. Three pieces of cheese were measure for each type and each method. The data are shown below.

| Treatment | Moisture Content Measurements | | |
|---|---|---|---|
| Type A made with Method 1 | $y_{11} = 38.02$ | $y_{12} = 39.79$ | $y_{13} = 37.79$ |
| Type B made with Method 1 | $y_{21} = 36.74$ | $y_{22} = 33.41$ | $y_{23} = 38.41$ |
| Type C made with Method 1 | $y_{31} = 38.02$ | $y_{32} = 35.00$ | $y_{33} = 34.00$ |
| Type A made with Method 2 | $y_{41} = 39.96$ | $y_{42} = 39.06$ | $y_{43} = 38.01$ |
| Type B made with Method 2 | $y_{51} = 34.58$ | $y_{52} = 36.52$ | $y_{53} = 35.52$ |
| Type C made with Method 2 | $y_{61} = 34.60$ | $y_{62} = 36.05$ | $y_{63} = 38.0$ |

Consider the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$ , where $\epsilon_{ij} \sim NID(0, \sigma^2)$, $i = 1, 2, 3, 4, 5, 6,$ and $j = 1, 2, 3$. This model can be expressed in matrix form as $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$.

Express the each of the following hypotheses in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$ . Use R to compute the value of the corresponding $SSH_0$, $SSE$ and F-statistic.

(a) After averaging across the two methods of making cheese, the average moisture content is the same for all three types of cheese. (5 marks)

(b) For each type of cheese, the average moisture content is not affected by the method for making cheese. (This hypothesis allows the average moisture content to vary across types of cheese). (5 marks)

Q3. An researcher recorded moisture content for three types of cheese made by two different methods. A $3 \times 2$ factorial experiment with types of cheese made by two different methods was conducted. The data had unequal replications among the six treatment combinations of the two factors, Cheese and Method. The collected data are given below.

| | Cheese | | |
|---|---|---|---|
| Method | 1 | 2 | 3 |
| 1 | 38.49 39.99 | 39.47 39.88 | 39.0 38.79 |
| | | 38.49 | 39.01 39.13 |
| 2 | 38.83 38.83 | 39.75 | 39.4 39.24 |
| | 39.18 | | 39.31 |

Consider the model $y_{ijk} = \mu_{ij} + \epsilon_{ijk}$ , where $\epsilon_{ijk} \sim NID(0, \sigma^2)$, $i = 1, 2$, and $j = 1, 2, 3$ and $k = 1, \ldots, n_{ij}$. This model can be expressed in matrix form as $\mathbf{Y} = \mathbf{D}\boldsymbol{\beta} + \boldsymbol{\epsilon}$. Examine type III sums of squares for these data.

(a) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for Method effects in the form $H_0 : \mathbf{C_1}\boldsymbol{\beta} = \mathbf{0}$. (5 marks)

(b) Present a formula for $SS_{H_{0,1}}$ , corresponding to the null hypothesis in part (a), and state it's distribution when the null hypothesis is true. (5 marks)

(c) Compute $SS_{H_{0,1}}$. (5 marks)

(d) Specify the $\mathbf{C_2}$ matrix needed to write the null hypothesis associated with the F-test for Cheese effects in the form $H_0 : \mathbf{C_2}\boldsymbol{\beta} = \mathbf{0}$. (5 marks)

(e) Present a formula for $SS_{H_{0,2}}$ , corresponding to the null hypothesis in part (d), and state it's distribution when the null hypothesis is true. (5 marks)

(f) Compute $SS_{H_{0,2}}$. (5 marks)

(g) Verify that the hypothesis in part (d) is testable. (5 marks)

(h) Suppose $SSE = 1.097608$, compute the test statistic for testing the null hypothesis associated with the F-test for Cheese effects. (5 marks)

Q4. An researcher recorded moisture content for four types of cheese made by two different methods. Three pieces of cheese were measure for each type and each method. The data are shown below.

| Treatment | Moisture Content Measurements | | |
|---|---|---|---|
| Type A made with Method 1 | $y_{111} = 38.02$ | $y_{112} = 39.79$ | |
| Type A made with Method 2 | $y_{121} = 39.96$ | $y_{122} = 39.06$ | $y_{123} = 38.01$ |
| Type B made with Method 1 | $y_{211} = 36.74$ | $y_{212} = 33.41$ | |
| Type B made with Method 2 | $y_{221} = 34.58$ | $y_{222} = 36.52$ | $y_{223} = 35.52$ |
| Type C made with Method 1 | $y_{311} = 38.02$ | $y_{312} = 35.00$ | $y_{313} = 34.00$ |
| Type C made with Method 2 | $y_{321} = 34.60$ | | |
| Type D made with Method 1 | $y_{411} = 48.02$ | $y_{412} = 45.00$ | $y_{413} = 42.00$ |
| Type D made with Method 2 | $y_{421} = 44.60$ | $y_{422} = 46.05$ | |

Consider the model $y_{ijk} = \mu_{ij} + \epsilon_{ijk}$ , where $\epsilon_{ijk} \sim NID(0, \sigma^2)$, $i = 1, 2, 3, 4$, and $j = 1, 2$ and $k = 1, ..., n_i$. This model can be expressed in matrix form as $\mathbf{Y} = \mathbf{D}\boldsymbol{\beta} + \boldsymbol{\epsilon}$. Examine type III sums of squares for these data.

(a) Use R to complete the ANOVA table below.

| Source | Type III SS | DF | Mean Square | F Value | P-Value |
|---|---|---|---|---|---|
| Type | | | | | |
| Method | | | | | |
| Type×Method | | | | | |

(10 marks)

(b) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for type effects in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. What can you conclude from the results of this test? (10 marks)

(c) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for method effects in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. What can you conclude from the results of this test? (10 marks)

(d) Specify the $\mathbf{C}$ matrix needed to write the null hypothesis associated with the F-test for interaction effects in the form $H_0 : \mathbf{C}\boldsymbol{\beta} = \mathbf{0}$. What can you conclude from the results of this test? (10 marks)