

# Chapter 2 Limited Fluctuation

202401

Credibility

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## 2 Limited Fluctuation Credibility

### 2.1 Introduction

Limited fluctuation credibility is also known as classical credibility. Suppose that the random variable (usually claim number for one period or claim amount or aggregate claim amount for one period) is  $W$ , with mean  $E(W) = \mu$  and  $V(W) = \sigma^2$ , and suppose a sample of  $n$  independent observations  $W_1, W_2, \dots, W_n$  is available.

Suppose that there is a manual premium of amount  $M$  for the claim distribution. The manual premium is an amount which has been determined by some past experience and underwriting expertise.  $M$  is not necessarily equal to  $E(W)$ , but is some preconceived estimate, we usually do not know the value of  $E(W)$ .

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Under the full credibility approach, the estimate of  $E(W)$  (also called the premium estimate for  $W$ ) is chosen as one of the following two possible values:

1.  $\overline{W} = \frac{W_1 + \dots + W_n}{n}$ , and we say that full credibility is applied, or
2.  $M$ .

We say that full credibility standard is satisfied if the probability relation

$$P(|\overline{W} - \mu| < k\mu) \geq P \quad (1)$$

is satisfied where  $k$  is the maximum fluctuation you will accept, the size of the confidence interval on each side as a percentage of the mean. Usually  $k = 0.05$ , but other values of  $k$  are possible, such as 0.02, 0.1.  $P$  is the probability parameter. Usually  $P = 0.9$ , but other values of  $P$  are possible, such as 0.95.

Restate equation (1) as

$$P\left(\left|\frac{\overline{W} - \mu}{\sigma/\sqrt{n}}\right| \leq \frac{k\mu\sqrt{n}}{\sigma}\right) \geq P$$

Let

$$y = \inf_z \left\{ P\left(\left|\frac{\overline{W} - \mu}{\sigma/\sqrt{n}}\right| \leq z\right) \geq P \right\}.$$

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If  $\overline{W}$  is continuous random variable, “ $\geq$ ” can be replaced by “ $=$ ” and

$$P \left( \left| \frac{\overline{W} - \mu}{\sigma/\sqrt{n}} \right| \leq y \right) = P.$$

This quantity may be difficult to find for a general random variable  $W$ . However, if  $\overline{W}$  is assumed to be (approximately) normal, then  $y$  is the  $\frac{1+p}{2}$  percentile of the standard normal distribution, then for a random variable  $W$ , full credibility is given to  $\overline{W}$  (meaning that  $n$ , the number of observation of  $W$  is large enough so that  $\overline{W}$  is chosen as the estimated premium) when  $\frac{k\mu\sqrt{n}}{\sigma} \geq y$ , solve for  $n$ , we get

$$n \geq \frac{y^2 \sigma^2}{k^2 \mu^2} = \left( \frac{y}{k} \right)^2 \left( \frac{\sigma}{\mu} \right)^2 = n_0 CV^2$$

where  $n_0 = \left( \frac{y}{k} \right)^2$  and  $CV$  is the coefficient of variation for the random variable  $W$ .

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There are 3 things we can calculate credibility for:

1. Number of claims.
2. Claims sizes.
3. Aggregate losses or pure premium.

**Note:** Some common use of  $P$  and its corresponding value of  $y$  are

$P$	0.90	0.95	0.98	0.99
$y$	1.645	1.96	2.326	2.576

$n_0 = 1082.41$  for  $P = .90$  and  $k = .05$ .

## 2.2 Credibility for Frequency ( $N$ )

This means we want the number of claim to be within  $k$  of the expected  $P$  of the time.

$W$  = the number of claims ( $N$ )  $\sim$  a distribution with mean  $\mu == E(N) = \mu_f$  and variance  $\sigma^2 = V(N) = \sigma_f^2$ . So

$$n \geq n_0 \left( \frac{\sigma}{\mu} \right)^2 = n_0 \left( \frac{\sigma_f^2}{\mu_f^2} \right) = \left( \frac{n_0}{\mu_f} \right) \left( \frac{\sigma_f^2}{\mu_f} \right)$$

The exposure unit for number of claims is the number of exposures (risks), then

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$$e_F = n_0 \left( \frac{\sigma_f^2}{\mu_f^2} \right) = \left( \frac{n_0}{\mu_f} \right) \left( \frac{\sigma_f^2}{\mu_f} \right)$$

### Notes:

- $\frac{\sigma_f^2}{\mu_f} = \frac{V(N)}{E(N)}$  is easier to calculate than the coefficient of variation.

Distribution	$\mu_f = E(N)$	$\sigma_f^2 = V(N)$	$\frac{\sigma_f^2}{\mu_f}$
$N \sim \text{Poi}(\lambda)$	$\lambda$	$\lambda$	1
$N \sim \text{Bin}(m, q)$	$mq$	$mq(1 - q)$	$1 - q$
$N \sim \text{NB}(r, \beta)$	$r\beta$	$r\beta(1 + \beta)$	$1 + \beta$

- Thus,

– For  $N \sim \text{Poi}(\lambda)$ ,  $\boxed{e_F = \frac{n_0}{\lambda}}$

– For  $N \sim \text{Bin}(m, q)$ ,  $\boxed{e_F = \frac{n_0}{mq}(1 - q)}$

– For  $N \sim \text{NB}(r, \beta)$ ,  $\boxed{e_F = \frac{n_0}{r\beta}(1 + \beta)}$

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### Example 1.

Suppose the number of claims per exposure follows a Poisson distribution with mean 2.2. You want number of claims for a group to be within 10% of expected 90% of the time. How many members ( $e_F$ ) must the group have for full credibility? [123.0](#)

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### Example 2.

Suppose the number of claims per month follows a Binomial distribution with parameters  $m = 1500$  and  $q = 0.069$ . You want number of claims for a period to be within 6% of expected 90% of the time. How many months must the group have for full credibility. [6.7614](#)



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To obtain the **expected number of claims**( $n_F$ ) needed for full credibility, we multiply  $e_F$  by  $\mu_f$ , the expected number of claim per exposure. Thus

$$n_F = \mu_f \times e_F = \mu_f \times n_0 \left( \frac{\sigma_f^2}{\mu_f^2} \right) = n_0 \left( \frac{\sigma_f^2}{\mu_f} \right)$$

Thus,

- For  $N \sim \text{Poi}(\lambda)$ ,  $\boxed{N_F = n_0}$
- For  $N \sim \text{Bin}(m, q)$ ,  $\boxed{N_F = n_0(1 - q)}$
- For  $N \sim \text{NB}(r, \beta)$ ,  $\boxed{N_F = n_0(1 + \beta)}$

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## Example 3.

Suppose the number of claims per exposure follows a Negative Binomial distribution with parameters  $r = 3$  and  $\beta = 4$ . You want number of claims for a group to be within 7% of expected 95% of the time. How many claims( $n_F$ ) must you observe to give full credibility to the experience?

[3920.0](#)

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### Example 4.

Suppose the number of claims per exposure follows a Poisson distribution. You want number of claims for a group to be within 10% of expected 99% of the time. How many claims( $n_F$ ) must you observe to give full credibility to the experience? [663.58](#)

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To obtain the **aggregate losses/total claims** needed for full credibility, we multiply  $n_F$  by  $\mu_X$ , the expected claim size. Thus

$$\text{Aggregate losses, } S_F = n_0 \mu_X \left( \frac{\sigma_f^2}{\mu_f} \right)$$

and hence,

- For  $N \sim \text{Poi}(\lambda)$ ,  $\boxed{S_F = n_0 \mu_X}$
- For  $N \sim \text{Bin}(m, q)$ ,  $\boxed{S_F = n_0 \mu_X (1 - q)}$
- For  $N \sim \text{NB}(r, \beta)$ ,  $\boxed{S_F = n_0 \mu_X (1 + \beta)}$

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### Example 5.

Suppose the number of claims per exposure follows a Poisson distribution, and the distribution of claim severity follows a Gamma distribution with a mean of 14 and variance of 36. You want number of claims for a group to be within 6% of expected 90% of the time. How much aggregate losses must be incurred for the group to be given full credibility.

[10523.43](#)

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### Example 6.

Suppose the number of claims per exposure follows a Negative Binomial distribution with parameters  $r = 2$  and  $\beta = 7$ , and the distribution of claim severity follows a Pareto distribution with  $\alpha = 5$  and  $\theta = 7$ . You want number of claims for a group to be within 8% of expected 90% of the time. How much aggregate losses must be incurred for the group to be given full credibility. [5919.43](#)

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### 2.3 Credibility for Severity ( $X$ )

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This means we want the size of each claim to be within  $k$  of the expected  $P$  of the time. In this case  $W$  = the size of the claim( $X$ , Severity)  $\sim$  a distribution with mean  $\mu = \mu_X$  and variance  $\sigma = \sigma_X$ . The exposure unit for claim severity is number of claims, not number of exposures. Let  $n_F$  be the number of claims needed for full credibility, then

$$n_F \geq n_0 \left( \frac{\sigma}{\mu^2} \right) = n_0 \left( \frac{\sigma_X^2}{\mu_X^2} \right) = n_0 CV_X^2$$

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### Example 7.

The claim amount distribution follows an Inverse Gamma distribution with  $\alpha = 6$  and  $\beta = 6$ . You want claim sizes for a group to be within 7% of expected 90% of the time. How many claims( $n_F$ ) must you observe to give full credibility to the experience. [138.06](#)



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To obtain the **number of exposures**,  $e_F$ , we divide  $n_F$  by the expected number of claim per risk,  $\mu_f$ . Thus

$$e_F = \frac{n_0 CV_X^2}{\mu_f} = \frac{n_0 \sigma_X^2}{\mu_X^2 \mu_f}$$

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### Example 8.

Suppose the number of claims per exposure follows a Poisson distribution with mean 2.5, and the claim amount distribution has coefficient of variation equal to 0.75. You want claim sizes for a group to be within 6% of expected 90% of the time. How many members( $e_F$ ) must the group have for full credibility. [169.13](#)

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### Example 9.

Suppose the number of claims per exposure follow a Binomial distribution with parameters  $m = 1360$  and  $q = 0.04$ , and the claim amount distribution follows a single parameter Pareto with  $\alpha = 4$  and  $\theta = 2.4$ . You want claim sizes for a group to be within 4% of expected 90% of the time. How many members ( $e_F$ ) must the group have for full credibility. [3.89](#)

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To obtain the **aggregate losses/total claims** needed for full credibility, we multiply  $n_F$  by  $\mu_X$ , the expected claim size. Thus

$$\text{Aggregate losses, } S_F = n_0 \mu_X CV_X^2$$

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### Example 10.

The claim amount distribution follows a Lognormal distribution with  $\mu = 2.7$  and  $\sigma = 1.2$ . You want claim sizes for a group to be within 5% of expected 90% of the time. How much aggregate losses must be incurred for the group to be given full credibility. [106568.45](#)

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### 2.4 Credibility for Aggregate Claims ( $S = X_1 + \dots + X_N$ )

For aggregate claims, the number of risk is the exposure unit.

Recall the formulas for mean and variance of a compound distribution are:

$$\begin{aligned} E(S) &= E(N)E(X) \\ &= \mu_f \mu_X \end{aligned}$$

$$\begin{aligned} V(S) &= E(N)V(X) + V(N)E^2(X) \\ &= \mu_f \sigma_X^2 + \sigma_f^2 \mu_X^2 \end{aligned}$$

Then,

$$CV_S^2 = \frac{V(S)}{E^2(S)} = \frac{\mu_f \sigma_X^2 + \sigma_f^2 \mu_X^2}{\mu_f^2 \mu_X^2} = \left( \frac{1}{\mu_f} \right) \left( \frac{\sigma_f^2}{\mu_f} + \frac{\sigma_X^2}{\mu_X^2} \right)$$

So exposures needed for full credibility is

$$e_F = n_0 CV_S^2 = \left( \frac{n_0}{\mu_f} \right) \left( \frac{\sigma_f^2}{\mu_f} + CV_X^2 \right)$$

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## Example 11.

Suppose the number of claims per exposure follows a Poisson distribution with mean 3.4, and the claim amount distribution follows a  $U[0, 9]$  distribution. You want aggregate claims for a group to be within 3% of expected 98% of the time. How many members ( $e_F$ ) must the group have for full credibility? [2357.36](#)

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### Example 12.

Suppose the claim frequency per exposure follows a distribution with mean = 0.23 and variance = 0.73, and the claim amount distribution follows an Inverse Gaussian distribution with  $\mu = 710$  and  $\theta = 2$ . You want aggregate claims for a group to be within 5% of expected 90% of the time. How many members ( $e_F$ ) must the group have for full credibility? 1,685,613.15



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To obtain the **expected number of claims** needed for full credibility, we multiply  $e_F$  by  $\mu_f$ , the expected number of claim per exposure. Thus

$$\begin{aligned} n_F &= \mu_f \left( \frac{n_0}{\mu_f} \right) \left( \frac{\sigma_f^2}{\mu_f} + \frac{\sigma_X^2}{\mu_X^2} \right) \\ &= n_0 \left( \frac{\sigma_f^2}{\mu_f} + CV_X^2 \right) \end{aligned}$$

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## Example 13.

Suppose the number of claims per exposure follows a Poisson distribution, and the distribution of claim severity has a mean of 5 and variance of 13.9. You want aggregate claims for a group to be within 10% of expected 90% of the time. How many claims( $n_F$ ) must you observe to give full credibility to the experience. [421.06](#)

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### Example 14.

Suppose the claim frequency per exposure follows a distribution with mean = 0.26 and variance = 0.88, and the distribution of claim severity has a mean of 6.9 and variance of 11.7. You want aggregate claims for a group to be within 6% of expected 90% of the time. How many claims( $n_F$ ) must you observe to give full credibility to the experience. 2728.85

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To obtain the **aggregate losses/total claims** needed for full credibility, we multiply  $n_F$  by  $\mu_x$ , the expected claim size. Thus

$$\text{Aggregate losses, } S_F = n_0 \mu_X \left( \frac{\sigma_f^2}{\mu_f} + CV_X^2 \right)$$

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## Example 15.

Suppose the number of claims per exposure follows a Poisson distribution, and the distribution of claim severity follows an Exponential distribution with mean 15. You want aggregate claims for a group to be within 4% of expected 98% of the time. How much aggregate losses must be incurred for the group to be given full credibility. [101442.67](#)

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## Example 16.

Suppose the claim frequency per exposure follows a distribution with mean = 0.39 and variance = 1.48, and the distribution of claim severity has a Gamma distribution with  $\alpha = 4$ , and  $\theta = 2$ . You want aggregate claims for a group to be within 1% of expected 90% of the time. How much aggregate losses must be incurred for the group to be given full credibility. [875641.94](#)

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A summary of the formulas for all possible combinations of experience units used and what the credibility is for is show in table below;

	Credibility for		
Experience expressed in	Number of claims	Claim size (severity)	Aggregate losses/Pure premium
Exposure units, $e_F$	$n_0 CV_f^2$	$\frac{n_0 CV_X^2}{\mu_f}$	$\frac{n_0}{\mu_f} \left( \frac{\sigma_f^2}{\mu_f} + CV_X^2 \right)$
Number of Claims, $n_F$	$n_0 \frac{\sigma_f^2}{\mu_f}$	$n_0 CV_X^2$	$n_0 \left( \frac{\sigma_f^2}{\mu_f} + CV_X^2 \right)$
Aggregate Losses, $s_F$	$n_0 \mu_X \frac{\sigma_f^2}{\mu_f}$	$n_0 \mu_X CV_X^2$	$n_0 \mu_X \left( \frac{\sigma_f^2}{\mu_f} + CV_X^2 \right)$

### Notes:

1.  $(1 + CV_X^2) = \frac{\mu_{2X}}{\mu_X^2} = \frac{E(X^2)}{E^2(X)}$ , where  $\mu_{2X}$  is the second moment of the loss distribution,  $X$ . For distribution like Pareto, it is easier to calculate the second moment than the variance.

2.

Severity Distribution	$(1 + CV_X^2)$
Gamma $(\alpha, \theta)$	$1 + \alpha^{-1}$
Inverse Gamma $(\alpha, \theta)$	$\frac{\alpha-1}{\alpha-2}$
Lognormal $(\mu, \sigma)$	$e^{\sigma^2}$
Two-parameter Pareto $(\alpha, \theta)$	$\frac{2(\alpha-1)}{\alpha-2}$
Single-parameter Pareto $(\alpha, \theta)$	$\frac{(\alpha-1)^2}{\alpha(\alpha-2)}$
Uniform $(0, \theta)$	$1 + \frac{1}{3}$

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### Example 17 (T2Q1).

You are given:

- The number of claims has a Poisson distribution.
- Claims sizes have a Pareto distribution with  $\alpha = 5.0$ ,  $\theta = 0.5$ .
- The number of claims and claim sizes are independent.
- The observed pure premium should be within 10% of the expected pure premium 90% of the time.

Determine the expected number of claims needed for full credibility.



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### Example 18 (T2Q2).

You are given:

- The number of claims has a Poisson distribution with mean 0.03.
- Claims sizes have a log normal distribution distribution with  $\sigma = 0.6$ .
- The number of claims and claim sizes are independent.
- The observed pure premium should be within 7% of the expected pure premium 99% of the time.

Determine the expected number of exposures needed for full credibility.

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### Example 19 (T2Q3).

You are given:

- The number of claims follows a negative binomial distribution with parameters  $r$  and  $\beta = 6$ .
- Claim severity has the following distribution:

Claim Size	Probability
1	0.38
10	0.33
100	0.29

- The number of claims is independent of the severity of claims.

Determine the expected number of claims needed for aggregate losses to be within 6% of expected aggregate losses with 95% probability.

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### Example 20 (T2Q4).

For a group dental plan, each individual's number of claims follow Poisson distribution with parameter  $\lambda$ .  $\lambda$  varies by individual in accordance with the following distribution:

$\lambda$	Probability
1	0.44
2	0.34
5	0.22

Claim sizes follow log normal distribution with parameter  $\mu$  and  $\sigma = 1.10$ . Classical credibility techniques are used. The standard for full credibility of aggregate loss experience is set so that the probability of observed claims being within 6.20% of expected claims is 99%. Determine the number of claims required for full credibility.

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**Example 21** (T2Q5).

The full credibility standard for a company is set according to the methods of classical credibility so that the total number of claims is to be within 3% of the true value with probability  $P$ . This full credibility standard is calculated to 852 claims. The standard is altered so that the total cost of claims is to be within 6% of the true value with probability  $P$ . The claim frequency has a Poisson distribution and the claim severity had the distribution

$$f(x) = \frac{190 - x}{18,050}, \text{ for } 0 < x < 190$$

What is the expected number of claims necessary to obtain full credibility under the new standard? 319.5

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**Example 22** (T2Q6).

Aggregate claims follows a Pareto distribution with parameters  $\alpha = 5$  and  $\theta = 6$ . The full credibility standard is set according to the methods of classical credibility so that actual aggregate claims are within 7% of expected aggregate claims 95% of the time. Determine the amount of expected aggregate claims needed for full credibility.

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### Example 23 (T2Q7).

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You are given the following:

- 100,000 exposures are needed for full credibility.
- The 100,000 exposures standard was selected using a normal approximation so that the actual total cost of claims is within 7.0% of the expected total 95% of the time.
- The number of claims per exposure follows a Poisson distribution with mean  $m$ .
- $m$  was estimated from the following observed data using the maximum likelihood:

Year	Exposures	Number of Claims
1	19,929	1,125
2	25,128	1,547
3	21,343	1,317

If mean claim severity is 1,533, determine the standard deviation of the claim severity distribution.

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### Example 24 (T2Q8).

For a group dental plan, each individual's number of claims follows Poisson distribution with parameter  $\lambda$ .  $\lambda$  varies by individual in accordance with gamma distribution with parameter  $\alpha = 3$ ,  $\theta_1 = 7$ . Claim sizes follow and inverse Gaussian distribution with parameter  $\mu = 1800$ ,  $\theta_2 = 6.3$ . Classical credibility techniques are used. The standard for full credibility of aggregate loss experience is set so that the probability of observed claims being within 10% of expected claims is 90%. Determine the number of claims required for full credibility. 79,479.82

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### Example 25 (T2Q9).

For an insurance coverage you are given:

- Claim counts follow a Poisson distribution.
- Claim sizes follow an exponential distribution with mean  $\mu$ .
- $\mu$  varies by insured according to a gamma distribution with parameters  $\alpha = 4$  and  $\theta = 210$ .

The methods of limited fluctuation credibility are used. 2200 expected claims are required for full credibility.

The full credibility standard is that actual claims should be within 7% of expected claims with probability  $p$ .

Determine  $p$ .



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### Example 26 (T2Q10).

You are given:

- Claim frequency has a Poisson distribution.
- Claims size has a Gamma distribution with  $\alpha = 2.5$ ,  $\theta$  unknown.
- Using the methods of classical credibility, a full credibility standard of 1960 expected claims has been established so that actual aggregate claim costs will be within 6% of expected aggregate claim costs  $P\%$  of the time.

Determine  $P$ .

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### Example 27 (T2Q11).

An insurance company is determining limited fluctuation credibility standards for its automobile losses. You are given the following information:

- The company selects all of its credibility standards to be the number of claims at which there is a 99% probability that the observed amount is within 7% of the mean.
- The standard for full credibility for aggregate loss is 8,911 claims.
- claim frequency follows a Poisson distribution.
- Claim frequency and claim severity are independent.

Calculate the limited-fluctuation credibility standard for claim severity.

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### Example 28 (T2Q12).

Aggregate claims per period have a compound Poisson distribution. You have determined that the number of claims for full credibility is 4,500 claims. It is then discovered that an incorrect value of the coefficient of variation for the severity distribution was used to determine the full credibility standard. The original coefficient of variation used was 0.5133, but the corrected coefficient of variation is 0.8164. Find the corrected number of claims for full credibility.

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### 2.5 Partial Credibility

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When there is inadequate experience for full credibility, we must determine  $Z$ , the credibility factor. This will be used to determine the credibility to determine the credibility premium  $P_C$ :

$$P_C = Z\bar{W} + (1 - Z)M = M + Z(\bar{W} - M)$$

Where  $M$  is the manual premium, the premium initially assumed if there is no credibility.

$Z \in [0, 1]$  is called the credibility factor. The general way to formulate  $Z$  is

$$\begin{aligned} Z &= \sqrt{\frac{\text{Information Available}}{\text{Information Needed for Full Credibility}}} \\ &= \sqrt{\frac{e}{e_F}} = \sqrt{\frac{n}{n_F}} \end{aligned}$$

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If  $n < n_F$ , then

$$P(|Z\bar{W} - Z\mu| < k\mu) = P$$

$$P\left(\left|\frac{Z\bar{W} - Z\mu}{Z(\sigma/\sqrt{n})}\right| < \frac{k\mu\sqrt{n}}{Z\sigma}\right) = P$$

$$\frac{k\mu\sqrt{n}}{Z\sigma} = y$$

$$k\mu\sqrt{n} = Z\sigma y$$

$$Z = \frac{k\mu\sqrt{n}}{\sigma y} = \left(\frac{k}{y}\right) \left(\frac{\mu}{\sigma}\right) \sqrt{n} = \sqrt{\frac{n}{n_F}}$$

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### Example 29 (T2Q13).

An insurance coverage involves credibility based on number of claims only. A full credibility standard is determined so that the number of claims is within 1% of the expected 98% of the times. For a particular group, 785 claims have been observed. Determine an appropriate credibility factor, assuming that the number of claims is Poisson distributed.

[0.1205](#)

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### Example 30 (T2Q14).

The average claim size for a group of insureds is 1,700 with standard deviation 8,200. Claim count follows a Poisson distribution. The standard for full credibility is that the total loss should be within 2% of the expected total loss with probability 99%. We observe 6,000 claims and a total loss of 1,630,000 for a group of insureds. If our prior estimate of the total loss is 1,830,000, determine the limited fluctuation credibility estimate of the total loss for the group of insureds. 1,805,580

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### Example 31 (T2Q15).

You are given:

- Claim counts follow a Negative Binomial distribution with parameters  $r = 5$  and  $\beta = 0.41$ .
- Claim sizes follow a lognormal distribution with coefficient variation 3.4.
- Claim sizes and claim counts are independent.
- The number of claims in the first year was 1,490.
- The aggregate loss in the first year was 8,670,000.
- The manual premium for the first year was 4,260,000.
- The exposure in the second year is identical to the exposure in the first year.
- The full credibility standard is to be within 8.70% of the expected aggregate loss 95% of the time.

Determine the limited fluctuation credibility net premium for the second year. 6,358,278



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**Example 32** (T2Q16).

Claim frequency follows a Poisson distribution. The coefficient of variation for claim severity is 3.2. The methods of limited fluctuation credibility are used, with a standard of aggregate losses being within 5% of expected losses 95% of the time. Determine the number of expected claims needed for 9.1% credibility. [143.0](#)

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### Example 33 (T2Q17).

You are given:

- Number of claims follows a Binomial distribution with parameters  $m$  and  $q = 0.15$ .
- The standard for full credibility is set so that the actual aggregate are within 7.80% of expected losses 95% of the time.
- 4190 expected claims are required for 58% credibility.

Determine the coefficient of variation for the claim size distribution.

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### Example 34 (T2Q18).

You are given:

- The losses  $W_j$ ,  $j = 1, \dots, 75$ , are available for a particular policyholder.
- It is reasonable to assume that the  $W_j$ 's are independent and compound Negative Binomial distributed with parameters  $r = 10$  and  $\beta$ .
- $\beta$  varies and follows a gamma distribution with parameters  $\alpha = 7$  and  $\theta = 4$ .
- Claim sizes( $X$ ) follow a distribution with probability density function(p.d.f.)

$$f(x|\mu) = \frac{1}{\mu} e^{-x/\mu}.$$

- $\mu$  varies and follows a distribution with p.d.f

$$f(\mu) = \frac{1200^3}{\Gamma(3)} \mu^{-4} e^{-1200/\mu}.$$

- Claim sizes and claim frequency are independent.
- The full credibility standard is to be within 10% of the expected aggregate losses 95% of the time.

Determine the credibility factor.

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### Example 35 (T2Q19).

For an insurance portfolio, you are given the following:

- The number of claims for each insured follows follows a Poisson distribution.
- The mean claim count for each insured varies. The distribution of mean claim counts is a gamma distribution with  $\alpha_1 = 0.7$  and  $\theta_1 = 5$ .
- The size of claims for each insured follows an Exponential distribution.
- The mean of the size of claims varies. The distribution of the mean of the size of the claim is an inverse gamma with parameters  $\alpha_2 = 3$  and  $\theta_2 = 8000$ .
- The credibility standard is that the aggregate claims must be with 9% of the expected  $P\%$  of the time.
- 919 claims were observed and 50% credibility was assigned to this experience.

Determine  $P$ .

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### Example 36 (T2Q20).

An insurance portfolio has two types of risk, A and B. 50% of the insureds are of type A and 50% are of type B. You are given:

	Number of claims		Size of claims	
Type	Mean	Standard deviation	Mean	Standard deviation
A	0.12	0.12	4	4
B	0.24	0.15	6	6

Given the type of risk, number of claims and size of claims are independent. The methods of limited fluctuation credibility are used, with a standard for full credibility of expected aggregate claims being within 6% of actual aggregate claims 95% of the time. Calculate the credibility given to 1,216 claims.