MEME15203 Statistical Inference

Assignment 5

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Q1. Let X have probability density function

$$f(x) = \begin{cases} \frac{\Gamma(7)x^4(\theta-x)}{\Gamma(5)\theta^6}, & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

Show that $\frac{X}{\theta}$is a pivotal quantity and use this pivotal quantity to find a 92% lower confidence limit for θ. (20 marks)

Q2. Consider independent random samples from two normal distributions, $X_i \sim N(0, a_1)$ and $Y_j \sim N(0, a_2)$; $i = \ldots, 30$, $j = 1, \ldots, 30$. Derive a $100(1-\alpha)\%$ confidence interval for $\frac{a_1}{a_2}$ based on sufficient statistics. (20 marks)

Q3. Consider independent random samples from two exponential distributions, $X_i \sim EXP(\mu)$ and $Y_j \sim EXP(\lambda)$; $i = 1, \ldots, 30$, $j = 1, \ldots, 30$.

(a) Find the distribution of $(\lambda/\mu)(\bar{X}/\bar{Y})$. (10 marks)

(b) Derive a $100(1-\alpha)\%$ confidence for λ/μ. (10 marks)

Q4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with pdf

$$f(x|\lambda) = \frac{\lambda^5}{\Gamma(5)} x^4 e^{-\lambda x}, x > 0, \text{ zero othewise},$$

the prior density of λ is

$$\pi(\lambda) = \frac{\mu^3}{\Gamma(3)} \lambda^2 e^{-\mu\lambda}, \lambda > 0, \text{ zero othewise},$$

where μ is known. Derive a $100(1-\alpha)\%$ equal probabilty Bayesian confidence interval for λ in terms of χ^2 random variable. (20 marks)

Q5. Losses follow a gamma distribution with $\alpha = 3$ and θ unknown. The prior distribution of θ has density function $\pi(\theta) = \frac{1}{\theta}$, Five losses are observed:

$$[629.6, 178.7, 189.4, 127.0, 665.1].$$

Determine the 95% HPD credible interval for θ. (10 marks)

Q6. You are given:

$$f(x|\theta) = \begin{cases} (\theta+1)x^{\theta} & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\pi(\theta) = \begin{cases} \frac{1}{\theta+1} & \text{for } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that a single observation takes the value $x = 0.33$. Find the upper bound of the 98% HPD credible region for θ. (10 marks)