## 1. $\mathbf{X} \sim Bernoulli(p)$

• 
$$f(x) = p^x q^{1-x}, x = 0, 1$$

$$\bullet M_X(t) = pe^t + q$$

$$\bullet \ E(X) = p$$

$$ullet V(X) = pq$$

### 2. $\mathbf{X} \sim Binomial(n, p)$

• 
$$f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$$

$$\bullet M_X(t) = (pe^t + q)^n$$

$$ullet E(X) = np$$

$$\bullet \ V(X) = npq$$

3. 
$$\mathbf{X} \sim HYP(n, M, N)$$

$$\bullet f(x) = \frac{\binom{N}{x} \binom{N-M}{n-x}}{\binom{N}{n}},$$

$$x = 0, 1, \dots, \min(n, 1)$$

$$x = 0, 1, ..., \min(n, M), n - x \le N - M.$$
•  $E(X) = \frac{nM}{N}$ 

$$ullet E(X) = rac{nM}{N}$$

$$\bullet \, V(X) = n \frac{M}{N} \left( 1 - \frac{M}{N} \right) \frac{N - n}{N - 1}$$

## Common Distributions

#### 4. $\mathbf{X} \sim GEO(p)$

• 
$$f(x) = pq^{x-1}$$
  $x = 1, 2, 3, ...$ 

• 
$$F(x) = 1 - q^x$$
  $x = 1, 2, 3, ...$   
•  $M_X(t) = \frac{pe^t}{1 - qe^t}$ 

$$\bullet \ M_X(t) = \frac{pe^t}{1 - \alpha e^t}$$

$$ullet E(X) = rac{1}{p} \ ullet V(X) = rac{q}{p^2}$$

$$\bullet \ V(X) = \frac{q}{n^2}$$

# 5. $\mathbf{X} \sim NegativeBinomial(r, p)$

• 
$$f(x) = \binom{x-1}{r-1} p^r q^{x-r}, x = r, r + 1, \dots$$

$$\bullet M_X(t) = \left(\frac{pe^t}{1 - qe^t}\right)^T$$

$$\bullet E(X) = \frac{r}{p}$$

$$\bullet V(X) = \frac{rq}{p^2}$$

$$ullet E(X) = rac{r}{p}$$

$$\bullet \ V(X) = \frac{rq}{n^2}$$

6. 
$$\mathbf{X} \sim POI(\mu)$$

• 
$$f(x) = \frac{e^{-\mu}\mu^x}{x!}$$
  $x = 0, 1, 2, \dots$ 

$$M_X(t) = e^{\mu(e^t - 1)}$$

$$\bullet$$
  $E(X) = \mu$ 

$$V(X) = \mu$$

#### 7. $\mathbf{X} \sim DU(N)$

• 
$$f(x) = \frac{1}{N}, X = 1, 2, \dots, N$$

• 
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$
  
•  $F(x) = \frac{x(1+x)}{2N}$   
•  $E(X) = \frac{N+1}{2}$   
•  $V(X) = \frac{N^2 - 1}{12}$ 

$$\bullet F(x) = \frac{x(1+x)}{2N}$$

$$E(X) = \frac{N+1}{2}$$

#### 8. $\mathbf{X} \sim U(a,b)$

$$F(x) = \frac{x-a}{b-a}, a < x < 0$$

$$\bullet M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$$

$$\bullet E(X) = \frac{a+b}{2}$$

$$\bullet V(X) = \frac{(b-a)^2}{12}$$

#### 9. $\mathbf{X} \sim Gamma(\alpha, \theta)$

$$\bullet \ f(x) = \frac{1}{\theta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\theta}, x>0$$

• 
$$F(x) = 1 - \sum_{i=0}^{\alpha - 1} \frac{(x/\theta)^i}{i!} e^{-x/\theta}$$
  
•  $M_X(t) = (\frac{1}{1-\theta t})^{\alpha}$ 

$$ullet$$
  $M_X(t)=(rac{1}{1- heta t})^{\mathcal{O}}$ 

$$\bullet E(X) = \alpha \theta$$

MEME15203 Statistical Inference©Dr Yong Chin Khian

## Common Distributions

# $f(x) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ and zero otherwise.

• 
$$F(x) = 1 - e^{-x/\theta}, x > 0$$

$$\bullet \ M_X(t) = \left(\frac{1}{1 - \theta t}\right)$$

$$\bullet \ E(X) = \theta,$$

#### 11. $\mathbf{X} \sim WEI(\tau, \theta)$

$$\bullet f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau - 1} e^{-(x/\theta)^{\tau}}, x > 0$$
 and zero otherwise.

• 
$$F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

• 
$$E(X) = \theta \Gamma \left( 1 + \frac{1}{\tau} \right)$$

• 
$$E(X^2) = \theta^2 \left[ \Gamma \left( 1 + \frac{2}{\tau} \right) - \Gamma^2 \left( 1 + \frac{1}{\tau} \right) \right]$$

#### 12. $\mathbf{X} \sim PAR(\alpha, \theta)$

• 
$$f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$$
  
•  $F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$   
•  $E(X) = \frac{\theta}{\alpha-1}$ 

$$ullet F(x) = 1 - (rac{ heta}{x+ heta})^{\epsilon}$$

$$\bullet$$
  $E(X) = \frac{\theta}{\alpha - 1}$ 

## Common Distributions

•  $E(X^2) = \frac{2\theta^2}{(\alpha - 1)(\alpha - 2)}$ •  $V(X) = \frac{\theta^2}{(\alpha - 1)^2(\alpha - 2)}$ 

13.  $\mathbf{X} \sim Beta(a,b)$ 

•  $f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}, 0 < x < 1$ •  $E(X) = \frac{a}{a+b}$ •  $V(X) = \frac{ab}{(a+b)^2(a+b+1)}$ 

14.  $\mathbf{X} \sim N(\mu, \sigma^2)$ 

•  $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, x \in \mathbb{R}, \mu \in \mathbb{R}$ and  $\sigma > 0$ .

•  $F(x) = \Phi(\frac{x-\mu}{\sigma})$ •  $M_X(t) = e^{\mu t + \sigma^2 t^2/2}$ 

 $E(X) = \mu$ 

 $V(X) = \sigma^2$ 

## Common Distributions

15.  $\mathbf{X} \sim LN(\mu, \sigma)$ 

•  $f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x - \mu)^2/2\sigma^2}, x > 0, \mu > 0$  and  $\sigma > 0$ 

•  $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$ •  $E(X) = e^{\mu + \frac{\sigma^2}{2}}$ 

 $\bullet \ V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$ 

16.  $\mathbf{X} \sim CAU(\theta, \eta)$ •  $f(x) = \frac{1}{\theta \pi \left[1 + \left(\frac{x - \eta}{\theta}\right)^2\right]}$ 

•  $F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} (\frac{x-\eta}{\theta})$ 

17.  $\mathbf{X} \sim EXP(\eta, \theta)$ 

•  $f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$   $x > \eta$ 

•  $F(x) = 1 - e^{-\frac{x-\eta}{\theta}}$ •  $M_X(t) = \frac{e^{\eta t}}{1-\theta t}$ •  $E(X) = \eta + \theta$ 

 $V(X) = \theta^2$ 

18.  $\mathbf{X} \sim DE(\eta, \theta)$ 

• 
$$f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$$
  $-\infty < x < \infty$  and

• 
$$f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta} - \infty < x < \infty$$
 and zero otherwise.  
•  $F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \leq \eta \\ \frac{1}{2}[1 - e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$   
•  $M_X(t) = \frac{e^{\eta t}}{1 - \theta^2 t^2}$   
•  $E(X) = \eta$   
•  $V(X) = 2\theta^2$ 

$$M_X(t) = \frac{e^{\eta t}}{1 - \theta^2 t^2}$$

$$E(X) = n$$

$$V(X) = 2\theta$$

19. **X** ~ Single Parameter Pareto (α, θ)
 • f(x) = αθα/xα+1, x > θ
 • F(x) = 1 - (θ/x)α
 • E(X) = αθ/α-1
 • E(X<sup>2</sup>) = αθ/α-2

$$f(x) = \frac{\alpha \theta^{\alpha}}{\alpha^{\alpha+1}}, x > \theta$$

$$\bullet \ F(x) = 1 - \left(\frac{\theta}{x}\right)$$

$$\bullet E(X^2) = \frac{\alpha \theta^2}{\alpha - 9}$$