MEME16203Linear Models

Assignment 3

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Faculty:	FES	Unit Code:	MEME16203
Course:	MAC	Unit Title:	Linear Models
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Q1. Suppose that $\mathbf{y}$ is $\mathrm{MVN}_n(\boldsymbol{\mu}, \sigma^2\mathbf{I})$ and that $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{R}$ are symmetric $n \times n$ matrices with $\mathbf{PQ} = \mathbf{0}$, $\mathbf{PR} = \mathbf{0}$, and $\mathbf{QR} = \mathbf{0}$. Argue carefully that the three random variables $\mathbf{y^T P y}$, $\mathbf{y^T Q y}$ and $\mathbf{y^T R y}$ are jointly independent. (15 marks)

Q2. Suppose $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon} \overset{iid}{\sim} N(\mathbf{0}, \sigma^2\mathbf{I}$ for some unknown $\sigma^2 > 0$.

(a) Determine the distribution of $\begin{bmatrix} \widehat{\mathbf{Y}} \\ \mathbf{Y} - \widehat{\mathbf{Y}} \end{bmatrix}$. (10 marks)

(b) Determine the distribution of $\widehat{\mathbf{Y}}^{\mathbf{T}}\widehat{\mathbf{Y}}$. (15 marks)

Q3. Consider the model

$$Y_{ij} = \gamma_0 + \gamma_1 X_i + \gamma_2 X_i^2 + \alpha_i + \epsilon_{ij}, \quad i = 1, 2, \ldots, 11; \quad j = 1, \ldots, 5$$

where $\epsilon \sim NID(0, \tau^2)$. This model can be expresses in matrix notation as $\mathbf{Y} = \mathbf{W}\boldsymbol{\gamma} + \boldsymbol{\epsilon}$. Let the matrix $\mathbf{Z}$ be the first 3 columns of the matrix $\mathbf{W}$, define $\mathbf{P_Z} = \mathbf{Z}(\mathbf{Z^T Z})^{-1}\mathbf{Z^T}$ and $\mathbf{P_W} = \mathbf{W}(\mathbf{W^T W})^{-1}\mathbf{W^T}$.

(a) Use Cochran's theorem to derive the distribution of $F = \dfrac{c\mathbf{Y^T}(\mathbf{P_W} - \mathbf{P_Z})\mathbf{Y}}{\mathbf{Y^T}(\mathbf{I} - \mathbf{P_W})\mathbf{Y}}$. Report c, degrees of freedom and a formula for the noncentrality parameter. (20 marks)

(b) Show that the noncentrality parameter is zero if $\alpha_1\mathbf{w_4} + \alpha_2\mathbf{w_5} + \cdots + \alpha_{11}\mathbf{w_{11}} = \mathbf{Zc}$ for some vector $\mathbf{c}$, where $\mathbf{w_j}$ is the j^{th} column of $\mathbf{W}$. (10 marks)

Q4. Consider the model $Y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, where $i = 1, 2, 3$, $j = 1, 2, 3$, and μ, α_1, α_2, α_3, are unknown parameters. Let $\epsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2)$, where σ^2 is unknown.

(a) Determine the distribution of $\frac{\hat{\tau}^2}{35\sigma^2}$ when $\tau = 0$, where $\hat{\tau}$ is the BLUE of $\tau = 2\alpha_1 - 8\alpha_2 + 6\alpha_3$. (10 marks)

(b) Determine the distribution of $S^2 = \sum_{i=1}^{3}\sum_{j=1}^{3}(Y_{ij} - \bar{Y}_{i.})^2$. (10 marks)

(c) Show that $F = \frac{c\hat{\tau}^2}{S^2}$, where c is a constant, has central $F-$distribution when $\tau = 0$. Report c. (10 marks)