MEME15203 Statistical Inference

Assignment 4

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Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Q1. Consider a distribution with pdf $f(x) = \theta(1-x)^{\theta-1}$ if $0 < x < 1$ and zero otherwise. Based on a random sample of size $n = 1$, find the most powerful test of $H_0 : \theta = 4$ against $H_1 : \theta = 3$ with $\alpha = 0.08$, then compute the power of the test for the alternative $\theta = 3$.

(10 marks)

Q2. Suppose that a discrete random variable X has pdf $f(x|\theta)$ specified in the table below for some θ.

θ	x				
	1	2	3	4	5
4	5/15	4/15	3/15	2/15	1/15
3	4/30	5/30	6/30	7/30	8/30
2	1/15	2/15	3/15	4/15	5/15
1	0.2	0.2	0.2	0.2	0.2

(a) Identify a test $\phi(x)$ that is MP size $\alpha = .03$ for testing $H_0 : \theta = 1$ vs $H_1 : \theta = 2$ and carefully state why it has this property.

(b) Your test from (a) could be used to test $H_0 : \theta = 1$ vs $H_1 : \theta = 2$ or 3. In this context, is it UMP of its size? Explain carefully.

(c) Either identify a UMP size $\alpha = .4$ test of $H_0 : \theta = 1$ vs $H_1 : \theta = 2$ or 4 or argue very carefully that no such test exists.

(15 marks)

Q3. Assume that X is a discrete random variable. Based on an observed value of X, derive the most powerful test of $H_0 : X = GEO(0.07)$ versus $H_1 : X \sim P0I(0.93)$ with $\alpha = 0.14$. Then find the power of this test under the alternative.

(15 marks)

Q4. Let $X_1, \ldots, X_{35}$ denote a random sample from a Weibull distribution, $X_i \sim WEI(2, \theta)$. Show that a UMP size 0.04 test of $H_0 : \theta \leq 8$ versus $H_1 : \theta > 8$ using Theorem 3 is $\{\sum X_i^2 \leq k\}$, and then determine k.

(10 marks)

Q5. A particular model for random pairs (X, Y) with parameter $\gamma > 0$ has joint pdf

$$f(x, y|\gamma) = \frac{\gamma}{2\pi\sqrt{x^2 + y^2}} e^{-\gamma\sqrt{x^2+y^2}}, (x, y) \in \mathbb{R}^2.$$

(a) For n iid data pairs $(X_1, X_2), \ldots, (X_n, Y_n)$, identify a statistic in which there is monotone likelihood ratio.

(b) For $n = 1$, find the UMP size $\alpha = 0.04$ test of $H_0 : \gamma \geq 6$ vs $H_1 : \gamma < 6$ in as explicit form as possible.

(20 marks)

Q6. Let $X_1, X_2, \ldots, X_m$ denote a random sample from the exponential density with mean θ_1 and let $Y_1, Y_2, \ldots, Y_n$ denote an independent random sample from an exponential density with mean θ_2.

(a) Find the likelihood ratio criterion for testing $H_0 : \theta_1 = \theta_2$ versus $H_1 : \theta_1 \neq \theta_2$

(b) Show that the test in part (a) is equivalent to an exact F test.

(20 marks)

Q7. If $X_i|\lambda \sim POI(\lambda)$ and a Bayesian uses a prior for λ that is Gamma with parameters $\alpha = 7$ and $\theta = \frac{1}{80}$, suppose $x_1, x_2, \ldots, x_5$ have been observed, what is the Bayes test of $H_0 : \lambda \leq 3$ versus $H_1 : \lambda > 3$?

(10 marks)