MEME16203Linear Models

Assignment 1

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Faculty:	FES	Unit Code:	MEME16203
Course:	MAC	Unit Title:	Linear Models
Year:	1,2	Lecturer:	Dr Yong Chin Khian
Session:	May 2024		
Due by:	12/6/2024		

Q1. Suppose $\mathbf{Z} = \underset{4\times 1}{\mathbf{1}}$, $\mathbf{G} = 4$, $\mathbf{R} = 25 \underset{4\times 4}{\mathbf{I}}$. If $\mathbf{\Sigma} = \mathbf{ZGZ^T} + \mathbf{R}$, find $\mathbf{\Sigma^{-1}}$.

(20 marks)

Q2. Let the 3×1 random vector $\mathbf{y}$ follows a multivariate normal distribution with mean vector $\boldsymbol{\mu} = [7 \quad 13 \quad 5]^T$ and covariance matrix $\mathbf{\Sigma}$ where

$$\mathbf{\Sigma} = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 6 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

Consider the vector $\mathbf{w}$ where

$$\mathbf{w} = \begin{bmatrix} 3y_1 - 3y_2 + 3y_3 - 24 \\ 3y_1 + 3y_2 - 5y_3 - 16 \end{bmatrix}.$$

Find the mean vector and covariance matrix of $\mathbf{W}$.

(20 marks)

Q3. Let $\mathbf{A}$ be an $n \times n$ symmetric matrix with rank $(\mathbf{A}) = r$. Here r may be smaller than n. Let

$$\mathbf{A} = \mathbf{L} \begin{bmatrix} \mathbf{\Delta}_r & 0 \\ 0 & 0 \end{bmatrix} \mathbf{L^T}$$

represent the spectral decomposition of A. Then, $\mathbf{\Delta}_r$ is an $r \times r$ diagonal matrix containing the positive eigenvalues of $\mathbf{A}$, and $\mathbf{L}$ is an $n \times n$ orthogonal matrix where the columns are eignenvectors of $\mathbf{A}$. Show that

$$\mathbf{G} = \mathbf{L} \begin{bmatrix} \mathbf{\Delta}_r^{-1} & 0 \\ 0 & 0 \end{bmatrix} \mathbf{L^T}$$

satisfies the definition of the Moore-Penrose inverse of $\mathbf{A}$.

(20 marks)

Q4. Suppose $\mathbf{X}$ and $\mathbf{W}$ are any two matrices with n rows for which $C(\mathbf{X}) = C(\mathbf{W})$. Show that $\mathbf{P_X} = \mathbf{P_W}$.

(20 marks)

Q5. Suppose $\mathbf{X}$ is an 30×4 matrix. Prove that $\mathcal{C}(\mathbf{X}) = \mathcal{C}(\mathbf{X}(\mathbf{X^T X})^{-}\mathbf{X^T})$.

(20 marks)