UECM1703 Introduction to Scientific Computing

TOPIC 5 Practical

UNIVERSITI TUNKU ABDUL RAHMAN

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Q1. Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 28.1 & 16.9 & 17.4 \\ 25.8 & 29.4 & 8.6 \\ 76.8 & 56.5 & 51.4 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 118.4 & 123.6 \\ 146.9 & 125.3 \end{bmatrix}.$$

Use python to compute matrix $\mathbf{C} = \mathbf{A} \otimes \mathbf{B}$. Then obtain the $(2, 2)$ element of $\mathbf{C}$.

Q2. Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 28.1 & 25.8 & 76.8 & 61.0 & 88.8 \\ 16.9 & 29.4 & 56.5 & 70.0 & 73.4 \\ 17.4 & 8.6 & 51.4 & 40.1 & 128.6 \\ 14.6 & 46.4 & 22.0 & 73.9 & 51.6 \\ 24.3 & 12.1 & 45.8 & 51.6 & 82.2 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 118.4 & 123.6 & 121.1 & 115.5 & 107.6 \\ 146.9 & 125.3 & 130.3 & 134.1 & 122.1 \\ 151.4 & 166.4 & 120.8 & 129.2 & 115.6 \\ 160.0 & 154.2 & 137.7 & 159.7 & 152.4 \\ 206.6 & 194.3 & 149.1 & 179.8 & 192.4 \end{bmatrix}.$$

Use python to compute matrix $\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}$. Then obtain the determinat of $\mathbf{C}$.

Q3. Consider the following matrix:

$$\mathbf{C} = \begin{bmatrix} 32.32 & -33.71 & 81.45 & -20.36 & -2.66 \\ -33.71 & 227.97 & -129.15 & 193.79 & -232.12 \\ 81.45 & -129.15 & 390.71 & -72.07 & 91.93 \\ -20.36 & 193.79 & -72.07 & 189.62 & -269.92 \\ -2.66 & -232.12 & 91.93 & -269.92 & 513.0 \end{bmatrix}.$$

Use python to obtain the eigen value and eigen vector of $\mathbf{C}$. Then find the largest values of the eigen values.

Q4. Consider the following linear system:

$$\begin{aligned} 33.1w + 21.9x + 22.4y + 19.6z &= 94.8 \\ 30.8w + 34.4x + 13.6y + 51.4z &= 79.4 \\ 66.8w + 46.5x + 41.4y + 12.0z &= 134.6 \\ 70.0w + 79.0x + 49.1y + 82.9z &= 81.0 \end{aligned}$$

(a) Write the above system in the form $\mathbf{AX} = \mathbf{b}$.

(b) Obtain the solution to the system above using matrix inversion.

(c) Compute $93.8w + 114.6x + 104.4y + 82.0z$.

Q5. You fit the following data to $Y = \beta_0 + \beta_1 X + \epsilon$.

x	y
71	36
34	25
35	25
26	23
58	32
6	17

(a) Write the model above in the form $\mathbf{y} = \mathbf{x}\boldsymbol{\beta} + \boldsymbol{\epsilon}$

(b) Write the Python commands and output using matrix formulation to obtain the estimate of β_0 and β_1.

(c) You are given that $R^2 = \frac{SSR}{SST}$ and adjusted R^2, $R^2_{Adj} = 1 - \frac{SSE/(n-p)}{SST/(n-1)}$, where

- n is the number of observations.
- p is the number of parameters in the model.
- $SSR = \boldsymbol{b}^T\mathbf{x}^{\mathbf{T}}\mathbf{y} - \frac{1}{n}\mathbf{y}^{\mathbf{T}}\mathbf{J}\mathbf{y}$, where $\mathbf{J}$ is an $n \times n$ matrix of one.
- $SSE = \mathbf{y}^{\mathbf{T}}\mathbf{y} - \boldsymbol{b}^T\mathbf{x}^{\mathbf{T}}\mathbf{y}$.
- $SST = \mathbf{y}^{\mathbf{T}}\mathbf{y} - \frac{1}{n}\mathbf{y}^{\mathbf{T}}\mathbf{J}\mathbf{y}$.

Write the Python commands and output to calculated R^2 and adjusted R^2.

Q6. Consider the data below:

y	x_1	x_2	x_3	x_4
275.4	7.9	46.2	15.4	30.2
181.3	4.6	23.9	8.7	16.8
184.7	4.7	24.7	8.9	17.3
162.2	3.9	19.3	7.3	14.1
243.8	6.8	38.7	13.1	25.7
110.2	2.0	7.0	3.6	6.7
280.9	8.1	47.4	15.7	31.0
175.0	4.4	22.4	8.2	15.9
220.7	6.0	33.2	11.5	22.4
196.7	5.1	27.5	9.8	19.0

Determine the predicted value for the mean score of y with $x_1 = 5.9$, $X_2 = 30.8$, $x_3 = 10.2$, and $x_4 = 19.0$.

Q7. Consider the data shown below:

y	x	y	x
19	12	19	13
19	13	18	9
18	9	19	14
19	14	19	12
18	11	19	13
17	8	19	13
18	10	18	9
19	14	19	13

You fit the above data to $y = \beta_0 + \beta_1 x + \epsilon$. You are given that:

- n is the number of observations.
- p is the number of parameters in the model.
- $\boldsymbol{b} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$.
- $SSE = \mathbf{y^T y} - \boldsymbol{b}^T \mathbf{X^T y}$.
- $MSE = \frac{SSE}{n-p}$.

Compute MSE.

Q8. Consider the data shown below:

y	x	z	y	x	z
28	17	31	25	12	25
26	13	27	25	11	24
25	12	26	23	8	22
23	7	20	26	14	28
25	11	25	26	14	28
25	11	24	28	17	31
25	12	25	25	12	26
24	10	24	25	11	24

You fit the above data to $y = \beta_0 + \beta_1 x + \beta_2 z + \epsilon$. You are given that:

- n is the number of observations.
- p is the number of parameters in the model.
- $\boldsymbol{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$.
- $SSR = \boldsymbol{b}^T \mathbf{x^T y} - \frac{1}{n}\mathbf{y^T J y}$, where $\mathbf{J}$ is an $n \times n$ matrix of one.

Compute SSR.

Q9. The world 10 largest companies yield the following data (in billion):

Company	x_1 = sales	x_2 = profits	x_3 = assets
1	244.4	25.8	1135.8
2	128.1	13.0	554.4
3	132.4	13.5	576.2
4	104.1	10.4	434.5
5	204.9	21.4	938.5
6	40.2	3.4	115.1
7	250.6	26.4	1167.2
8	120.3	12.2	515.6
9	176.5	18.3	796.5
10	146.9	15.1	648.7

Derive the sample covariance matrix using the NumPy package, then provide $cov(x_2, x_2)$.

Q10. Consider the data shown below:

y	x	z	y	x	z
27	16	30	26	13	26
24	10	24	25	11	25
24	11	24	27	16	30
23	9	22	22	6	19
26	14	28	25	11	25
22	6	19	24	11	24
28	17	31	27	15	29
24	10	23	23	9	22

You fit the above data to $y = \beta_0 + \beta_1 x + \beta_2 z + \epsilon$. You are given that:

- n is the number of observations.
- p is the number of parameters in the model.
- $\boldsymbol{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$.
- $SSE = \mathbf{y^T y} - \boldsymbol{b}^T \mathbf{x^T y}$.
- $MSE = \frac{SSE}{n-p}$
- $SE(\hat{\beta}_j) = \sqrt{MSE \times C_{jj}}$, where C_{jj} is the diagonal element of the $(\mathbf{X^T X})^{-1}$ corresponding to $\hat{\beta}_j$.

Compute $SE(\hat{\beta}_1)$.

Q11. A researcher believes that the number of days the ozone levels exceeded 0.2ppm (y) depends on the seasonal meteorological index (x). The following table gives the data.

Index	16.3	16.8	18.2	17.4	17.0	17.4	14.1	16.8	17.6	17.1
Days	86	114	113	116	79	82	67	80	72	58

You fit the above data to $y = \beta_0 + \beta_1 x + \epsilon$, where y is the number of days the ozone levels exceeded 0.2ppm, and x is the seasonal meteorological index.

- n is the number of observations.
- p is the number of parameters in the model.
- $SSR = \boldsymbol{b}^T \mathbf{x}^\mathbf{T} \mathbf{y} - \frac{1}{n} \mathbf{y}^\mathbf{T} \mathbf{J} \mathbf{y}$, where $\mathbf{J}$ is an $n \times n$ matrix of one.
- $SSE = \mathbf{y}^\mathbf{T} \mathbf{y} - \boldsymbol{b}^T \mathbf{x}^\mathbf{T} \mathbf{y}$.
- $MSE = \frac{SSE}{n-p}$.
- $MSR = \frac{SSR}{p-1}$.

Use Python to calculated F.

Q12. You are given the following data:

No.	x_1	x_2	x_3
1	13.7	1.3	60.7
2	15.6	1.5	70.0
3	4.8	0.3	16.0
4	24.4	2.4	114.1
5	6.6	0.5	24.9
6	11.5	1.0	49.6
7	18.5	1.8	84.6
8	9.4	0.8	39.1
9	10.4	0.9	44.0
10	11.2	1.0	47.9

Derive the sample covariance matrix($\mathbf{S_n}$) using the NumPy package, then determine the determinant of $\mathbf{S_n}$.