Test 2 Marking Guide

Name: Student ID: Mark: /100

FACULTY:	FES, UTAR	COURSE CODE:	UECM1404
PROGRAMME/YEAR:	AS, FM /Y1	COURSE TITLE:	THEORY OF INTEREST
SESSION:	202301	LECTURER:	DR YONG CHIN KHIAN

CO3: Apply the concept of annuity and compound interest to solve problems in the amortization and sinking fund schedule such as finding the loan amount, interest charged, periodic payments and length of the loan.

1. [Fill in the blank with correct answer] You ara given $\delta_t = 1/(73+t)$ for $0 \le t \le 5$. Calculate $s_{\overline{5|}}$. 5.133356856514751. (7 marks)

2. [Fill in the blank with correct answer] Chass deposits 290 per month beginning one month from now. The monthly deposits increases by 5% every two years. At a nominal interest rate of 9% convertible monthly, calculate the accumulated value of the deposits at the end of 24 years. 353019.0516967079. (7 marks)

3. [Fill in the blank with correct answer] Justin and Maggie each take out a 17-year loan L. Justin repays his loan using the amortization method, at an annual effective interest rate of i. He makes an annual payment of 1100 at the end of each year. Maggie repays her loan using the sinking fund method. She pays interest annually, also at an effective interest rate of i. In addition, Maggie makes level annual deposits at the end of each year for 17 years into a sinking fund. The annual effective rate on the sinking fund is 4.74%, and she pays off the loan after 17 years. Maggie's total payment each year is equal to 9% of the original loan amount. Calculate L. 12364.506100477638. (7 marks)

4. [Fill in the blank with correct answer] You took a mortgage loan of 400,000 on January 1, 2021 which required to pay 25 equal annual payments at 10% interest with the first payment due on January 1, 2022. The bank sold your mortgage to an investor immediately after receiving your 9th payment. The yield to the investor is 6%. Determine the bank's overall return on its investment. 0.11790752. (7 marks)

5. [Fill in the blank with correct answer] A 11-year loan of 5500 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

   (i) Equal annual payments at an annual effective rate of 6.25%.

   (ii) Installments of 500.0 each year plus interest on the unpaid balance at an annual effective rate of i.

   The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine i. 0.06877. (7 marks)

6. [Fill in the blank with correct answer] Two loans for equal amounts are repaid at an effective annual interest rate of 4%. Loan 1 is to be repaid with 30 equal annual payments. Loan 2 is to be repaid by 30 annual payments, each containing equal principal amounts and an interest amount based on the unpaid balance. Payments are made at the end of each year. The annual payment for Loan 1 first exceed the annual payment for Loan 2 with the n-th payment. Determine n. 12.63. (7 marks)

7. [Show your workings. If no workings are shown, ZERO is awarded] Mike takes out a 30-year loan on January 1, 2022 for 80,000 at an annual effective interest rate of 7%. Payments are made at the end of each year. On January 1, 2032, Mike takes out a 20-year loan for 40,000 at an annual effective interest rate of 9%. Payments are also made at the end of each year. Calculate the total amount of principal repaid during year 2002 on both loans.

(14 marks)

Ans.

$80,000 = R_1 a_{\overline{30}|7\%}$
$80,000 = R_1 \left[\frac{1-1.07^{-30}}{0.07}\right]$
$R_1 = 6446.91$

$40,000 = R_2 a_{\overline{20}|9\%}$
$40,000 = R_2 \left[\frac{1-1.09^{-20}}{0.09}\right]$
$R_2 = 4,381.86$

Payments repaid in 2002
$= R_1 v_{7\%}^{30-11+1} + R_2 v_{9\%}^{20-1+1}$
$= 6446.91(1.07^{20}) + 4381.86(1.09^{20})$
$= \boxed{2{,}447.8636}$

8. [Show your workings. If no workings are shown, ZERO is awarded] The proceeds of a 15,000 death benefit are left on deposit with an insurance company for seven years at an annual effective interest rate of 9%. The balance at the end of seven years is paid to the beneficiary in 180 equal monthly payments of X, with the first payment made immediately. During the payout period, interest is credited at an annual effective interest rate of 7%. Calculate X.

(15 marks)

Ans.

```
                   7%
  9%             <--->
<--->            X    X         X
|---|-------|---|---------|
0   1       7  7+1/12   17
15,000
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$15,000(1.09^7) = 12X\ddot{a}^{(12)}_{\overline{15}|7\%}$

$\ddot{a}^{(12)}_{\overline{15}|7\%} = \frac{1-v^n}{d^{(12)}} = \frac{1-1.07^{15}}{12[1-1.07^{-1/12}]} = 9.4497$

$X = \frac{15,000(1.09^7)}{12(9.4497)} = \boxed{241.8118}$

9. [Show your workings. If no workings are shown, ZERO is awarded] Allan buys a house and takes out a 120,000 45-year mortgage. The interest rate is 11% convertible monthly and Allan makes monthly payments of 1,362 for the first 3 years. Determine how large his monthly payment needs to be for the remaining 42 years in order to pay off the mortgage at the end of the 45-year period.

(15 marks)

Ans.

The monthly effective interest rate is $j = \frac{i^{(12)}}{12} = \frac{0.11}{12} = 0.0092$

Let R be the monthly payment for the remaining 42 years

$120,000 = 1,362a_{\overline{36}|j} + Ra_{\overline{504}|j}v_j^{36}$

$120,000 = 1,362\left(\frac{1-1.0092^{-36}}{0.0092}\right) + R\left(\frac{1-1.0092^{-504}}{0.0092}\right)(1.0092^{-36})$

$R = \boxed{1{,}013.27}$

10. [Show your workings. If no workings are shown, ZERO is awarded] If

$$_{\frac{1}{6}|}\ddot{a}^{(3)}_{\overline{n}|} + 5a^{(3)}_{\overline{n}|} = ca^{(3)}_{\overline{n}|}.$$

Determine c.

(14 marks)

Ans.

$_{\frac{1}{6}|}\ddot{a}^{(3)}_{\overline{n}|} + a^{(3)}_{\overline{n}|}$ is

```
      1/3 1/3  1/3  1/3       1/3       1/3   1/3
  |---|----|----|---|----|-----------|-----|-----|
      1/6 2/6  3/6  1      7/6     n-1/6   n-2/6  n
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which is $2a^{(3)}_{\overline{n}|}$.
Thus $_{\frac{1}{6}|}\ddot{a}^{(3)}_{\overline{n}|} + 5a^{(3)}_{\overline{n}|} = {}_{\frac{1}{6}|}\ddot{a}^{(3)}_{\overline{n}|} + a^{(3)}_{\overline{n}|} + 4a^{(3)}_{\overline{n}|} = 2a^{(3)}_{\overline{n}|}. + 4a^{(3)}_{\overline{n}|} = 6a^{(3)}_{\overline{n}|}$.
Thus, $c = 6$