

**MEME15203 Statistical Inference****Assignment 2****UNIVERSITI TUNKU ABDUL RAHMAN**

Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
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Due by:	2/4/2024		

Q1. A random sample of size  $n$  is taken from a distribution with probability density function (pdf)

$$f(x) = \frac{4x^3}{\theta^4}, 0 < x < \theta, \text{ zero otherwise.}$$

- (a) Find the Maximum Likelihood Estimator(MLE) of  $\theta$ . Call it  $\hat{\theta}$ .
- (b) Find the MLE of the median of the distribution.
- (c) Find the Method of Moment Estimator(MME) of  $\theta$ . Call it  $\tilde{\theta}$ .
- (d) Find the constant  $c$  so that  $c\hat{\theta}$  becomes an unbiased estimator of  $\theta$ .
- (e) Find the Mean Square Error(MSE) of  $\hat{\theta}$ .
- (f) Find the MSE of  $\tilde{\theta}$ .

(30 marks)

Q2. Consider a random sample of size  $n$  from a distribution with discrete pdf

$$f(x; p) = p(1 - p)^x; x = 0, 1, \dots, \text{ zero otherwise.}$$

- (a) Find the MLE of  $p$ .
- (b) Find the MLE of  $\theta = \frac{1-p}{p}$ .
- (c) Find the CRLB for variance of unbiased estimators of  $\theta$ .
- (d) Is MLE of  $\theta$  a UMVUE?
- (e) Is MLE of  $\theta$  MSE consistent?
- (f) Find the asymptotic distribution of the MLE of  $\theta$ .

(30 marks)

Q3. Let  $X_1, \dots, X_n$  be a random sample from the uniform distribution on the interval  $(0, \theta)$ , where  $\theta > 0$  is unknown. Let the prior of  $\theta$  be the log-normal distribution with parameter  $(\mu, \sigma)$ , where  $\mu \in R$  and  $\sigma > 0$  are known constants. Find the posterior density of  $\ln(\theta)$ .

(15 marks)

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Q4. Let  $X_1, X_2, \dots, X_n$  denote a random sample from the density function given by

$$f(x) = \begin{cases} \frac{3}{\theta} x^2 e^{-x^3/\theta}, & \theta > 0, x > 0, \\ 0, & \text{otherwise} \end{cases}$$

- (a) find the MME of  $\theta$ .
- (b) Find the MLE of  $\theta$ .
- (c) Find the CRLB of  $\theta$ .

(15 marks)

Q5. Suppose  $X|\theta \sim U(\theta - \frac{1}{2}, \theta + \frac{1}{2})$  and that a prior distribution of  $\theta$  is  $N(0, 1)$ . Find the Bayes estimator of  $\theta$  under squared error loss.

(10 marks)