1. $\mathbf{X} \sim Bernoulli(p)$

 $\bullet f(x) = p^x q^{1-x}, x = 0, 1$

- $\bullet M_X(t) = pe^t + q$
- \bullet E(X) = p
- $\bullet V(X) = pq$

2. $\mathbf{X} \sim Binomial(n, p)$

- $\bullet f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$
- $\bullet M_X(t) = (pe^t + q)^n$
- \bullet E(X) = np
- $\bullet V(X) = npq$

3. $\mathbf{X} \sim HYP(n, M, N)$

- $f(x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}},$ $x = 0, 1, \dots, \min(n, M), n - x \le N - M.$
- \bullet $E(X) = \frac{nM}{N}$
- $V(X) = n\frac{M}{N} \left(1 \frac{M}{N}\right) \frac{N-n}{N-1}$

4. $\mathbf{X} \sim GEO(p)$

• $f(x) = pq^{x-1}$ x = 1, 2, 3, ...

Common Distributions

- $F(x) = 1 q^x$ x = 1, 2, 3, ...
- $\bullet M_X(t) = \frac{pe^t}{1 qe^t}$
- \bullet $E(X) = \frac{1}{p}$
- $V(X) = \frac{q}{p^2}$

5. $\mathbf{X} \sim NegativeBinomial(r, p)$

- $f(x) = {x-1 \choose r-1} p^r q^{x-r}, x = r, r+1, \dots$
- $M_X(t) = \left(\frac{pe^t}{1 qe^t}\right)^r$
- \bullet $E(X) = \frac{r}{p}$
- $\bullet V(X) = \frac{rq}{p^2}$

6. $\mathbf{X} \sim POI(\mu)$

- $f(x) = \frac{e^{-\mu}\mu^x}{x!}$ $x = 0, 1, 2, \dots$
- $\bullet M_X(t) = e^{\mu(e^t 1)}$
- \bullet $E(X) = \mu$
- $\bullet V(X) = \mu$

7. $\mathbf{X} \sim DU(N)$

•
$$f(x) = \frac{1}{N}, X = 1, 2, \dots, N$$

•
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$

$$\bullet F(x) = \frac{x(1+x)}{2N}$$

$$\bullet E(X) = \frac{N+1}{2}$$

$$\bullet V(X) = \frac{N^2 - 1}{12}$$

8. $\mathbf{X} \sim U(a,b)$

•
$$f(x) = \frac{1}{b-a}$$
, $a < x < b$ and zero otherwise

•
$$F(x) = \frac{x-a}{b-a}$$
, $a < x < b$

$$\bullet M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$$

$$\bullet E(X) = \frac{a+b}{2}$$

$$\bullet V(X) = \frac{(b-a)^2}{12}$$

9. $\mathbf{X} \sim Gamma(\alpha, \theta)$

•
$$f(x) = \frac{1}{\theta^{\alpha} \Gamma(\alpha)} x^{\alpha - 1} e^{-x/\theta}, x > 0$$

•
$$F(x) = 1 - \sum_{i=0}^{\alpha-1} \frac{(x/\theta)^i}{i!} e^{x/\theta}$$

$$\bullet M_X(t) = (\frac{1}{1-\theta t})^{\alpha}$$

$$\bullet E(X) = \alpha \theta$$

• $V(X) = \alpha \theta^2$

10. $\mathbf{X} \sim EXP(\theta)$

• $f(x) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ and zero otherwise.

•
$$F(x) = 1 - e^{-x/\theta}, x > 0$$

•
$$M_X(t) = \left(\frac{1}{1-\theta t}\right)$$

$$\bullet E(X) = \theta,$$

•
$$V(X) = \theta^2$$

11. $\mathbf{X} \sim WEI(\tau, \theta)$

• $f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau - 1} e^{-(x/\theta)^{\tau}}, x > 0$ and zero otherwise.

$$\bullet F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

•
$$E(X) = \theta \Gamma \left(1 + \frac{1}{\tau} \right)$$

•
$$E(X^2) = \theta^2 \left[\Gamma \left(1 + \frac{2}{\tau} \right) - \Gamma^2 \left(1 + \frac{1}{\tau} \right) \right]$$

12. $\mathbf{X} \sim PAR(\alpha, \theta)$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$$

•
$$F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$$

•
$$E(X) = \frac{\theta}{\alpha - 1}$$

• $E(X^2) = \frac{2\theta^2}{(\alpha - 1)(\alpha - 2)}$

• $V(X) = \frac{\theta^2}{(\alpha - 1)^2(\alpha - 2)}$

13. $\mathbf{X} \sim Beta(a,b)$

• $f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for 0 < x < 1

 $\bullet E(X) = \frac{a}{a+b}$

 $\bullet V(X) = \frac{ab}{(a+b)^2(a+b+1)}$

14. **X** ~ $N(\mu, \sigma^2)$

• $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, for $x \in \mathbb{R}$, $\mu \in \mathbb{R}$ and $\sigma > 0$.

• $F(x) = \Phi(\frac{x-\mu}{\sigma})$

 $\bullet M_X(t) = e^{\mu t + \sigma^2 t^2/2}$

 \bullet $E(X) = \mu$

• $V(X) = \sigma^2$

15. $\mathbf{X} \sim LN(\mu, \sigma)$

• $f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x - \mu)^2/2\sigma^2}$, for x > 0, $\mu > 0$ and $\sigma > 0$

• $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$

 $\bullet E(X) = e^{\mu + \frac{\sigma^2}{2}}$

 $V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$

16. $\mathbf{X} \sim CAU($ theta, η)

• $f(x) = \frac{1}{\theta \pi \left[1 + \left(\frac{x - \eta}{\theta}\right)^2\right]}$

 $\bullet F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} \left(\frac{x-\eta}{\theta}\right)$

17. $\mathbf{X} \sim EXP(\eta, \theta)$

• $f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$ $x > \eta$

 $\bullet F(x) = 1 - e^{-\frac{x - \eta}{\theta}}$

• $M_X(t) = \frac{e^{\eta t}}{1 - \theta t}$

 $\bullet E(X) = \eta + \theta$

 $\bullet \ V(X) = \theta^2$

18. $\mathbf{X} \sim DE(\eta, \theta)$

- $f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$ $-\infty < x < \infty$ and zero otherwise.
- $F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \le \eta \\ \frac{1}{2}[1 e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$
- $\bullet M_X(t) = \frac{e^{\eta t}}{1 \theta^2 t^2}$
- \bullet $E(X) = \eta$
- $V(X) = 2\theta^2$
- 19. $\mathbf{X} \sim \text{ Single Parameter Pareto } (\alpha, \theta)$
 - $f(x) = \frac{\alpha \theta^{\alpha}}{x^{\alpha+1}}, x > \theta$
 - $F(x) = 1 (\frac{\theta}{x})^{\alpha}$
 - $E(X) = \frac{\alpha \theta}{\alpha 1}$
 - $E(X^2) = \frac{\alpha \theta^2}{\alpha 2}$