MEME15203 Statistical Inference

Assignment 2

UNIVERSITI TUNKU ABDUL RAHMAN

Faculty:	FES	Unit Code:	MEME15203
Course:	MAC	Unit Title:	Statistical Inference
Year:	1,2	Lecturer:	Dr Yong Chin Khian
Session:	January 2023		
Due by:	16/3/2023		

Q1. Consider $f(x|\theta) = \begin{cases} p, & x = 0 \\ (1-p)\frac{(\ln\theta)^x}{\theta x!}, & x = 1, 2, 3, \ldots. \\ 0, & \text{otherwise} \end{cases}$

Suppose parameters are $p \in [0,1]$ and $\theta \geq 0$. Then, for $X_1, X_2, \ldots, X_n$ iid with this, find a method of moments estimator for the parameter vector (p, θ) based on the first two sample moments.

(15 marks)

Q2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the probability density function:

$$f(x_i) = \begin{cases} 4\theta x_i^{4\theta-1}, & 0 < x_i < 1, \theta > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the MLE of θ.

(b) Find c such that $c\hat{\theta}$ is an unbiased estimator of θ, where $\hat{\theta}$ is the MLE of θ.

(15 marks)

Q3. Let $X \sim POI(\mu)$. Suppose $\theta = e^{-\mu}$, $\hat{\theta} = e^{-X}$ and $\tilde{\theta} = u(x) = \begin{cases} 1, \text{for } x = 0 \\ 0, \text{for } x = 1, 2, \ldots \end{cases}$.

Compare the MSEs of $\hat{\theta}$ and $\tilde{\theta}$ for estimating θ when $\mu = 5$

(20 marks)

Q4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from the density function given by

$$f(x) = \begin{cases} \frac{4}{\theta}x^3 e^{-x^4/\theta}, & \theta > 0, x > 0, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the MLE of θ.

(b) Find the CRLB of θ.

(c) Find the UMVUE for θ.

(15 marks)

MEME15203 Statistical Inference

Q5. Let $X_1, X_2, \ldots, X_n$ denote a random sample from an exponentially distributed population with mean $\lambda = \frac{1}{\theta}$. Let $\Theta \sim \chi^2(2v)$.

(a) Find the Bayes estimator for $\lambda = \frac{1}{\theta}$ under square error loss.

(b) Show that it is a biased but consistent estimator for $\lambda = \frac{1}{\theta}$.

(20 marks)

Q6. Suppose $X|\theta \sim U(\theta - \frac{1}{6}, \theta + \frac{5}{6})$ and that a prior distribution of θ is $N(\mu, 1)$. Find the Bayes estimator of θ under squared error loss.

(15 marks)