1. $\mathbf{X} \sim \mathbf{Bernoulli}(\mathbf{p})$

- $f(x) = p^x q^{1-x}, x = 0, 1$
- $\bullet M_X(t) = pe^t + q$
- \bullet E(X) = p
- $\bullet V(X) = pq$

2. $\mathbf{X} \sim \mathbf{Binomial}(\mathbf{n}, \mathbf{p})$

- $\bullet f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \dots, n$
- $\bullet M_X(t) = (pe^t + q)^n$
- \bullet E(X) = np
- $\bullet V(X) = npq$

3. $\mathbf{X} \sim \mathbf{HYP}(\mathbf{n}, \mathbf{M}, \mathbf{N})$

- \bullet $E(X) = \frac{nM}{N}$
- $V(X) = n\frac{M}{N} \left(1 \frac{M}{N}\right) \frac{N-n}{N-1}$

4. $\mathbf{X} \sim \mathbf{GEO}(\mathbf{p})$

- $f(x) = pq^{x-1}$ x = 1, 2, 3, ...
- $F(x) = 1 q^x$ x = 1, 2, 3, ...
- $\bullet M_X(t) = \frac{pe^t}{1 qe^t}$
- \bullet $E(X) = \frac{1}{p}$
- $V(X) = \frac{q}{p^2}$

5. $\mathbf{X} \sim \mathbf{NegativeBinomial}(\mathbf{r}, \mathbf{p})$

- $f(x) = {x-1 \choose r-1} p^r q^{x-r}, x = r, r+1, \dots$
- $M_X(t) = \left(\frac{pe^t}{1 qe^t}\right)^r$
- $\bullet E(X) = \frac{r}{p}$
- $V(X) = \frac{rq}{p^2}$

6. $\mathbf{X} \sim \mathbf{POI}(\mu)$

- $f(x) = \frac{e^{-\mu}\mu^x}{x!}$ x = 0, 1, 2, ...
- $\bullet M_X(t) = e^{\mu(e^t 1)}$
- \bullet $E(X) = \mu$
- $\bullet V(X) = \mu$

202201

7. $\mathbf{X} \sim \mathbf{DU}(\mathbf{N})$

•
$$f(x) = \frac{1}{N}, X = 1, 2, \dots, N$$

•
$$M_X(t) = \frac{1}{N} \frac{e^t - e^{(N+1)t}}{1 - e^t}$$

$$\bullet F(x) = \frac{x(1+x)}{2N}$$

$$\bullet E(X) = \frac{N+1}{2}$$

$$\bullet V(X) = \frac{N^2 - 1}{12}$$

8. $\mathbf{X} \sim \mathbf{U}(\mathbf{a}, \mathbf{b})$

•
$$f(x) = \frac{1}{b-a}$$
, $a < x < b$ and zero otherwise

•
$$F(x) = \frac{x-a}{b-a}$$
, $a < x < b$

$$\bullet M_X(t) = \frac{e^{tb} - e^{ta}}{b - a}$$

$$\bullet E(X) = \frac{a+b}{2}$$

$$\bullet V(X) = \frac{(b-a)^2}{12}$$

9. $\mathbf{X} \sim \mathbf{Gamma}(\alpha, \theta)$

•
$$f(x) = \frac{1}{\theta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\theta}, x > 0$$

•
$$F(x) = 1 - \sum_{i=0}^{\alpha-1} \frac{(x/\theta)^i}{i!} e^{x/\theta}$$

$$\bullet M_X(t) = (\frac{1}{1-\theta t})^{\alpha}$$

$$\bullet E(X) = \alpha \theta$$

 $\bullet V(X) = \alpha \theta^2$

10. $\mathbf{X} \sim \mathbf{EXP}(\theta)$

• $f(x) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ and zero otherwise.

•
$$F(x) = 1 - e^{-x/\theta}, x > 0$$

•
$$M_X(t) = \left(\frac{1}{1-\theta t}\right)$$

$$\bullet$$
 $E(X) = \theta$,

•
$$V(X) = \theta^2$$

11. $\mathbf{X} \sim \mathbf{WEI}(\tau, \theta)$

• $f(x) = \frac{\tau}{\theta^{\tau}} x^{\tau - 1} e^{-(x/\theta)^{\tau}}, x > 0$ and zero otherwise.

$$\bullet F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

•
$$E(X) = \theta \Gamma \left(1 + \frac{1}{\tau} \right)$$

•
$$E(X^2) = \theta^2 \left[\Gamma \left(1 + \frac{2}{\tau} \right) - \Gamma^2 \left(1 + \frac{1}{\tau} \right) \right]$$

12. $\mathbf{X} \sim \mathbf{PAR}(\alpha, \theta)$

•
$$f(x) = \frac{\alpha \theta^{\alpha}}{(x+\theta)^{\alpha+1}}, x > 0$$

•
$$F(x) = 1 - (\frac{\theta}{x+\theta})^{\alpha}$$

•
$$E(X) = \frac{\theta}{\alpha - 1}$$

202201

- $E(X^2) = \frac{2\theta^2}{(\alpha 1)(\alpha 2)}$
- $V(X) = \frac{\theta^2}{(\alpha 1)^2(\alpha 2)}$
- 13. $\mathbf{X} \sim \mathbf{Beta}(\mathbf{a}, \mathbf{b})$
 - $f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for 0 < x < 1
 - \bullet $E(X) = \frac{a}{a+b}$
 - $\bullet V(X) = \frac{ab}{(a+b)^2(a+b+1)}$
- 14. $\mathbf{X} \sim \mathbf{N}(\mu, \sigma^2)$
 - $f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$, for $x \in R$, $\mu \in R$ and $\sigma > 0$.
 - $F(x) = \Phi(\frac{x-\mu}{\sigma})$
 - $\bullet M_X(t) = e^{\mu t + \sigma^2 t^2/2}$
 - \bullet $E(X) = \mu$
 - $V(X) = \sigma^2$

15. $\mathbf{X} \sim \mathbf{LN}(\mu, \sigma)$

- $f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-(\ln x \mu)^2/2\sigma^2}$, for x > 0, $\mu \in R$ and $\sigma > 0$
- $F(x) = \Phi\left(\frac{\ln x \mu}{\sigma}\right)$
- $\bullet E(X) = e^{\mu + \frac{\sigma^2}{2}}$
- $V(X) = e^{2\mu + \sigma^2} (e^{\sigma^2} 1)$
- 16. $\mathbf{X} \sim \mathbf{CAU}(\mathbf{theta}, \eta)$

•
$$f(x) = \frac{1}{\theta \pi \left[1 + \left(\frac{x - \eta}{\theta}\right)^2\right]}$$

- $\bullet F(x) = \frac{1}{2} + \frac{1}{\pi} tan^{-1} \left(\frac{x-\eta}{\theta}\right)$
- 17. $\mathbf{X} \sim \mathbf{EXP}(\eta, \theta)$
 - $f(x) = \frac{1}{\theta}e^{-\frac{x-\eta}{\theta}}$ $x > \eta$
 - $\bullet F(x) = 1 e^{-\frac{x \eta}{\theta}}$
 - $M_X(t) = \frac{e^{\eta t}}{1 \theta t}$
 - \bullet $E(X) = \eta + \theta$
 - $V(X) = \theta^2$

Common Distributions

7

18. $\mathbf{X} \sim \mathbf{DE}(\eta, \theta)$

- $f(x) = \frac{1}{2\theta}e^{-|x-\eta|/\theta}$ $-\infty < x < \infty$ and zero otherwise.
- $F_X(x) = \begin{cases} \frac{1}{2}e^{(x+\eta)/\theta}, & x \le \eta \\ \frac{1}{2}[1 e^{-(x+\eta)/\theta}], & x > \eta \end{cases}$
- $\bullet M_X(t) = \frac{e^{\eta t}}{1 \theta^2 t^2}$
- \bullet $E(X) = \eta$
- $V(X) = 2\theta^2$
- 19. $\mathbf{X} \sim \text{ Single Parameter Pareto } (\alpha, \theta)$
 - $f(x) = \frac{\alpha \theta^{\alpha}}{x^{\alpha+1}}, x > \theta$
 - $F(x) = 1 (\frac{\theta}{x})^{\alpha}$
 - $E(X) = \frac{\alpha \theta}{\alpha 1}$
 - $E(X^2) = \frac{\alpha \theta^2}{\alpha 2}$