UCCM3463 LOSS MODELS-Distributions

1. $X \sim POI(\lambda)$

- $f(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, \ldots$
- $E(X) = V(X) = \lambda$
- $P_N(z) = e^{\lambda(z-1)}$
- $M_N(t) = e^{\lambda(e^t-1)}$

---

2. $X \sim Bin(m, q)$

- $f(x|q) = \binom{m}{x} q^x (1-q)^{m-x}, x = 0, 1, \ldots, m$
- $E(X) = mq; V(X) = mq(1-q)$
- $P_N(z) = [1 + q(z-1)]^m$
- $M_N(t) = [1 + q(e^t - 1)]^m$
- Special case:
  - When $m = 1$, $X \sim Bernoulli(q)$
  - $f(x|q) = q^x(1-q)^{1-x}, x = 0, 1$
  - $E(X) = q; V(X) = q(1-q)$
  - $P_N(z) = [1 + q(z-1)]$
  - $M_N(t) = [1 + q(e^t - 1)]$

---

3. $X \sim NB(r, \beta)$

- $f(x|\beta) = \frac{r(r+1)\cdots(r+x-1)\beta^x}{x!(1+\beta)^{r+x}}$

  $= \binom{r+x-1}{x} \frac{\beta^x}{(1+\beta)^{r+x}}, x = 0, 1, \ldots$

- $E(X) = r\beta; V(X) = r\beta(1+\beta)$
- $P_N(z) = [1-\beta(Z-1)]^{-r}$
- $M_N(t) = [1-\beta(e^t-1)]^{-r}$
- Special Case:
  - When $r = 1$, $X \sim Geometric(\beta)$
  - $f(x|\beta) = \frac{\beta^x}{(1+\beta)^{1+x}}, x = 0, 1, \ldots$
  - $E(X) = \beta; V(X) = \beta(1+\beta)$
  - $P_N(z) = [1-\beta(Z-1)]$
  - $M_N(t) = [1-\beta(e^t-1)]$

---

4. $X \sim N(\mu, \sigma)$

- $f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, x \in R$
- $E(X) = \mu; V(X) = \sigma^2$
- $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$

---

5. $X \sim Gamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, x > 0$
- $F(x|\alpha, \theta) = 1 - \sum_{j=0}^{\alpha-1} \frac{(\frac{x}{\theta})^j e^{-\frac{x}{\theta}}}{j!}$

- $E(X) = \alpha\theta; V(X) = \alpha\theta^2$
- $E(X^k) = \theta^k \alpha(\alpha+1)\cdots(\alpha+k-1)$
- Special case: When$\alpha = 1$, $X \sim EXP(\theta)$
  - $f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0$
  - $F(x|\theta) = 1 - e^{-\frac{x}{\theta}}$
  - $E(X) = \theta; V(X) = \theta^2$
  - $E(X^k) = \theta^k$

6.$X \sim InvGamma(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\frac{\theta}{x}}, x > 0$
- $E(X^k) = \frac{\theta^k}{(\alpha-1)\cdots(\alpha-k)}$, if k is a positive integer
- Special case: When $\alpha = 1$, $X \sim InvExp(\theta)$
  - $f(x|\theta) = \theta x^{-2} e^{-\frac{\theta}{x}}, x > 0$
  - $F(x|\theta) = e^{-\frac{\theta}{x}}, x > 0$
  - $E(X^k) = \theta^k \Gamma(1-k), k < 1$

7.$X \sim Pareto(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}, x > 0$
- $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha$

- $E(X^k) = \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}$
- $E(X \wedge x) = \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right]$

$8. X \sim Single Parameter Pareto(\alpha, \theta)$

- $f(x|\alpha, \theta) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta$
- $F(x|\alpha, \theta) = 1 - \left(\frac{\theta}{x}\right)^\alpha$
- $E(X^k) = \frac{\alpha\theta^k}{\alpha-k}, k < \alpha$

---

9. $X \sim Beta(a, b, \theta)$

- $f(x|a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)\theta^{a+b-1}} x^{a-1}(\theta-x)^{b-1}, 0 < x < \theta$
- $E(X^k) = \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}$ if k is positve integer.
- Sepecial case: When $a = 1, b = 1$, $x \sim U(0, \theta)$
  - $f(x) = \frac{1}{\theta}, 0 < x < \theta$
  - $E(X) = \frac{\theta}{2}, V(X) = \frac{\theta^2}{12}$

---

10. $X \sim LogNormal(\mu, \sigma)$

- $f(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, x > 0$
- $F(x) = \Phi(\frac{\ln x - \mu}{\sigma})$
- $E(X^k) = e^{k\mu + \frac{1}{2}k^2\sigma^2}$

---

11. $X \sim Weibull(\tau, \theta)$

- $f(x|\tau, \theta) = \frac{\tau}{\theta^\tau} x^{\tau-1} e^{-(x/\theta)^\tau}, x > 0$
- $F(x) = 1 - e^{-(x/\theta)^\tau}$
- $E(X^k) = \theta^k \Gamma(1 + k/\tau), k > -\tau$

$$12. X \sim InverseWeibull(\tau, \theta)$$

- $f(x|\tau,\theta) = \tau\theta^{\tau}x^{-\tau}e^{-(\theta/x)^{\tau}}, x > 0$
- $F(x) = e^{-(\theta/x)^{\tau}}$

$$13. X \sim InvGaussian(\mu, \theta)$$

- $E(X) = \mu; V(X) = \frac{\mu^3}{\theta}$
