Test 1 Marking Guide

Name: Student ID: Mark: /100

Faculty:	FES, UTAR	Course Code:	UECM1404
Programme/Year:	AS /Y1	Course Title:	Theory of Interest
Session:	202206	Lecturer:	Dr Yong Chin Khian

1. CO1: Use the concepts of derivatives and functions to solve equations in the context of theory of interest.

   (a) [Fill in the blank with correct answer] You are given:

   (i) $\delta_t = \frac{1}{4+t}$; and

   (ii) the total interest earned during the first n years on an investment of 1 at time $t = 0$ is 1.9.

   Determine n. 7.60000000. (6 marks)

   (b) [Fill in the blank with correct answer] You are given a loan on which interest is charged over 4-year period, as follows:

   - an effective rate of discount of 0.068 for the first year;
   - a nominal rate of discount of 0.060 compounded every 2 years for the second year;
   - a nominal rate of interest of 0.054 compounded semiannually for the third year; and
   - a force of interest of 0.052 for the forth year.

   Calculate the annual effective rate of interest over the 4-year period.
   0.061743. (7 marks)

   (c) [Fill in the blank with correct answer] At a certain interest rate the present value of the following two payment patterns are equal:

   - 289 at the end of 8 years plus 563 at the end of 16 years.
   - 639.58 at the end of 8 years.

   At the same interest rate, 144.0 invested now plus 173.0 invested at the end of 8 years will accumulate to P at the end of 16 years. Calculate P. 649.190919. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Jeff deposits 18 into a fund today and 36 21-year later. Interest for the first 9 years is credited at a nominal discount rate of d compounded quarterly, and thereafter at a nominal interest rate of 7% compounded semiannually. The accumulated balance in the fund at the end of 32 years is 203. Calculate d.

(15 marks)

Ans.

Equation of value at end of 32 years:

$18\left(1-\frac{d}{4}\right)^{-4\times 9}\left(1+\frac{0.07}{2}\right)^{2\times 23}+36\left(1+\frac{0.07}{2}\right)^{2\times 11}=203$

$18\left(1-\frac{d}{4}\right)^{-36}(1.035)^{46}+36\,(1.035)^{22}=203$

$18\left(1-\frac{d}{4}\right)^{-36}=\frac{203-36(1.035)^{22}}{1.035^{46}}=25.9435$

$1-\frac{d}{4}=\left(\frac{25.9435}{18}\right)^{-1/36}=0.9899$

$d=4(1-0.9899)=\boxed{0.0404}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] If $r=\frac{i^{(3)}}{d^{(3)}}$, express v in terms of r.

(15 marks)

Ans.

$(1+\frac{i^{(3)}}{3})(1-\frac{d^{(3)}}{3})=1$

$1+\frac{i^{(3)}}{3}-\frac{d^{(3)}}{3}-\frac{i^{(3)}d^{(3)}}{9}=1$

$\frac{i^{(3)}-d^{(3)}}{3}=\frac{i^{(3)}d^{(3)}}{9}$

$i^{(3)}-d^{(3)}=\frac{i^{(3)}d^{(3)}}{3}$

$r-1=\frac{i^{(3)}-d^{(3)}}{d^{(3)}}=\frac{i^{(3)}d^{(3)}}{3d^{(3)}}=\frac{i^{(3)}}{3}$

$r=1+\frac{i^{(3)}}{3}$

$v=(1+\frac{i^{(3)}}{3})^{-3}=r^{-3}$

$\boxed{1.0000}$

2. CO2: Formulate equations to solve problems involving interest/yield rates.

(a) [Fill in the blank with correct answer] At a nominal rate of interest i, convertible semiannually, the present value of a series of payment of 1 at the end of every 5 years, forever, is 5.06. Calculate i. 0.036395864466696397. (6 marks)

(b) [Fill in the blank with correct answer] The death benefit on a life insurance policy can be paid in any of the following ways, each of which has the same present value as the death benefit:

- a perpetuity of 170 at the end of each month;
- 242.728624 at the end of each month for n years; and
- a payment of 107050.500000 at the end of n years.

Calculate the amount of the death benefit. 32075.471698113208. (7 marks)

(c) [Fill in the blank with correct answer] You took a loan of 300,000 which required to pay 40 equal annual payments at 10% interest. The payments are due at the end of each year. The bank sold your loan to an investor immediately after receiving your 9th payment. With yield to the investor of 8%, the price the investor pay was 348,187. Determine the bank's overall return on its investment.
0.11343632. (7 marks)

(d) [Show your workings. If no workings are shown, ZERO is awarded] Lee borrows X for 45 years at an annual effective interest rate of 9%. If he pays the principal and accumulated interest in one lump sum at the end of 45 years, he would pay 23421.61 more in interest than if he repaid the loan with 45 level payments at the end of each year. calculate X.

(15 marks)

Ans.

$I_1 = X(1.09^{45} - 1)$

$I_2 = \frac{45X}{a_{\overline{45}|9\%}} - X$

$a_{\overline{45|9\%}|} = \frac{1-1.09^{-45}}{0.09} = 10.8812$

Thus,

$X(1.09^{45} - 1) = \frac{45X}{a_{\overline{45}|9\%}} - X + 23421.61$

$X(1.09^{45} - \frac{45}{a_{\overline{45}|9\%}}) = 23421.61$

$X(1.09^{45} - \frac{45}{10.8812}) = 23421.61$

$X = \boxed{530.00}$

(e) [Show your workings. If no workings are shown, ZERO is awarded] Steven have a 30-year 130,000 mortgage with an 8% interest rate convertibele monthly. Payments are made at the end of the month. Immediate after the 120th payment, he refinance the mortgage. The iterest rate is reduced to 6.5%, convertibele monthly, and the term is reduced to 20 years (so there are 10 years of payments remainning). He also make an additional payment of 17,333 at the time of refinancing. Calculate his new monthly payment.

(15 marks)

Ans.

```
        0.6667%
          <--->
         130000     R   R      R         R
1/2 years |---|---|---...|---...---|
          0   1         120       360

        0.5417%
          <--->
B120+17,333   X   X      X
1/2 years    |---|---|...---|
             120 121 122     240
```

$130000 = Ra_{\overline{360}|0.6667\%}$
$R = \frac{130000}{a_{\overline{360}|0.6667\%}} = 130000\left(\frac{0.0067}{1-1.0067^{-360}}\right) = 957.52$
$B_{120} = Ra_{\overline{240}|0.6667\%} = 957.52\left[\frac{1-1.0067^{-240}}{0.0067}\right] = 114135.7$
$B_{120} + 17,333 = Xa_{\overline{120}|0.5417\%}$
$X = \frac{B_{120}-17,333}{a_{\overline{120}|0.5417\%}} = (114135.7 - 17,333)\left[\frac{0.0054}{1-1.0054^{-120}}\right] = \boxed{1,098.19}$