Q1. Suppose that $X_1, \ldots, X_n$ denotes a random sample from the probability density function given by

$$f(x|\theta_1, \theta_2) = \begin{cases} \left(\frac{1}{\theta_1}\right) e^{-(x-\theta_2)/\theta_1}, & x > \theta_2 \\ 0, & \text{otherwise.} \end{cases}$$

The following random sample of 8 has been observed:

$$61, \quad 122, \quad 16, \quad 46, \quad 29, \quad 24, \quad 33, \quad 51$$

Determne the likelihood test statistc for testing $H_0 : \theta_1 = 82.8$ versus $H_1 : \theta_1 > 82.8$ with θ_2 unknown.

Q2. A random sample of 8 claims $x_1, \ldots, x_8$ is taken from the probability density function

$$f(x_i) = \frac{\alpha\theta^\alpha}{(x_i)^{\alpha+1}}, \alpha, \theta > 0, x_i > \theta.$$

In ascending order the observations are: 1,700, 1,742,1,769, 1,790, 1,860, 1,890, 1,946, 2,338

Suppose the parameters are $\alpha = 4$ and $\theta = 1700$.

Commonly used critical values for this test are

α	0.10	0.05	0.025	0.01
Critical Value	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.48}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Determine the result of the test at 0.1 significant level.

Q3. You are given the following:

- 124 observed losses have been recorded an are grouped as follows:

Interval	Number of Losses
[0,1)	20
[1,5)	45
[5,10)	21
[10,15)	24
[15, ∞)	14

- The random variable X underlying the observed losses, is believed to follow the exponential disribution with mean 5.

Determine the value of Pearson's goodness-of-fit statistic.

Q4. You are given a sample of 10 observations from the following distribution:

$$f(X) = \frac{1}{2\theta^3} x^2 e^{-x/\theta}, x > 0$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
142.15	179.28	47.28	105.43	89.45	100.27	190.36	85.33	183.4

Determine the value of the Akaike Information Criterion (AIC).

Q5. A random sample of 10 claims $x_1, \ldots, x_{10}$ is taken from the probability density function

$$f(x_i) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x_i^{\alpha-1} e^{-\frac{x_i}{\theta}}, x_i > 0.$$

In ascending order the observations are: 111.59, 167.12, 172.79, 187.42, 194.29, 241.61, 287.32, 292.33, 300.64, 486.2

Suppose the parameters are $\alpha = 3$ and $\theta = 41$. Commonly used critical values for this test are

α	0.10	0.05	0.025	0.01
Critical Value	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.48}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Determine the result of the test at 0.1 significant level.

Q6. You are given a sample of 5 observations from $Pareto(\alpha, \theta = 1460)$ distribution:

$$1,748.71 \qquad 2,233.55 \qquad 1,461.06 \qquad 1,647.15 \qquad 1,540.32.$$

Determine the value of the Bayesian Information Criterion (BIC).

Q7. You are given the following:

- Losses follow a Weibull distribution with parameters $\theta = 24$ and $\tau = 3$.
- The insurance coverage has an ordinary deductible of 12.

If the insurer makes a payment, what is the probability that an insurer's payment is less than or equal to 35.

Q8. The losses experienced by an insurance company have the following probability distribution:

Loss size	Probability
0	0.60
160	0.25
260	0.10
1,320	0.05

Calculate the $CTE_{0.73}$(or $TVaR_{0.73}$).

Q9. For an insurance coverage, lossess (before application of any deductible) follow a Pareto distribution with parameters $\alpha = 4$ and $\theta = 5000$. The coverage is subject to a deductible of 500.0. Calculate the deductible needed to double the loss elimination ratio.

Q10. The number of claims in a period has a Binomial distribution with parameters $m = 8$ and $q = 0.55$. The amount of each claim X follows $P(X = x) = 0.25$, $x = 1, 2, 3, 4$. The number of claims and claim amounts are independent. S is the aggregate claim amount in the period. Calculate $F_S(4)$.

Q11. Claim sizes follow an exponential distribution with θ =12.50. Claim counts are independent of claim sizes, and have the following distribution:

n	0	1	2	3
P_n	0.41	0.25	0.19	0.15

Calculate $F_S(5)$.

Q12. For a certain insurance, individual losses in 2020 were Pareto distributed with parameters $\alpha = 6$ and $\theta = 1000$. A deductible of 100.0 is applied to each loss. In 2021, individual losses have increased 6%. A deductible of 100.0 is still applied to each loss. Determine the standard deviation of amount paid per loss.

Q13. For a discrete probability distribution, you are given the recursion relation

$$p_k = (3.04/k + 0.76)p_{(k-1)}, k = 1, 2, \ldots$$

Determine p_3.

Q14. N^M is a discrete random variable with probability function which is a member of the $(a, b, 1)$ class of distributions. You are given

$$P(z) = 0.41 + 0.59 \left[\frac{e^{2.40(z-1)]} - e^{-2.40}}{1 - e^{-2.40}} \right]$$

Calculate the variance of the distribution.

Q15. The number of losses follows a Binomial distribution with $m = 44$ and $q = 0.34$. Loss sizes follow an inverse exponential distribution with $\theta = 200$. Let N be the number of losses for amount less than 400. Determine the standard deviation of N.