

Subjects: Rigid motion; Rotation & Translation; Kinematics; python programming

(1) A and B are 2x2 matrices given as:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Calculate the following two matrix multiplications:

(1.1)  $C = A B$

(1.2)  $D = B A$

(2) You are given a 2D point  $P = (2, 0)$ . What is the coordinate vector  $Q$  if you apply rotation by 30 degrees? Compute it by using the equation:  $Q = R(\theta=30) P$

(3) Describe a python function to make the set of vertices of a regular N-gon by sampling N points around the circle of unit radius.

**def makeRegularNgon(N):**

**vertices = []**

**# provide your code here**

**return vertices** # vertices is a numpy array of shape (N, 2)

(4) Describe a python function to obtain a list of every pixel coordinate connecting two end points  $(x_0, y_0)$  and  $(x_1, y_1)$ :

**def getLineCoordinates(x0, y0, x1, y1):**

**coordinates = []**

**# your code here**

**return coordinates**

Here, the variable **coordinates** is a numpy array of shape (N, 2) where N is the number of points. This process is called 'rasterization'.

(5) Following is a part of the `main()` function to draw a star shape in the canvas image:

```
V = makeRegularNgon(5)
```

```
drawStar(canvas, V)
```

Describe the python function **drawStar()** to draw a star shape using vertices of a regular pentagon. You may assume that **drawLine(canvas, p, q)** has already been defined, where p & q are two end points of the line segment connecting p and q.

\*You are given vertices of a rectangle defined by: `P = np.array([ [0,0], [6, 0], [6, 2], [0, 2] ])`. Use this for the problems from (6) to (10).

(6) What is the 3x3 matrix for rotation of 60 degrees?

(7) Compute the 3x3 matrix H that (1) rotate P by 45 degrees and (2) translate by (3, 1). Sketch the result of this transformation; it doesn't have to be 100% exact but should be good enough.

(8) Now (1) translate P by (3,1) and (2) rotate by 30 degrees. What is the transformation matrix H? Provide a sketch of the result of this transformation.

(9) Provide a python code to draw a windmill of 4 wings, rotating 5 degrees per frames.

(10) Provide a python code to animate a clock. Because we have only one rectangle, the two needles indicating hour and minute have the same length. Use red color for minute-needle and green color for hour-needle. The animation must be such that when the minute-needle rotates 360 degrees, the hour-needle rotates 15 degrees. The clock starts from 12 o'clock. Describe the **main()** function only.