2D Strange Attractors

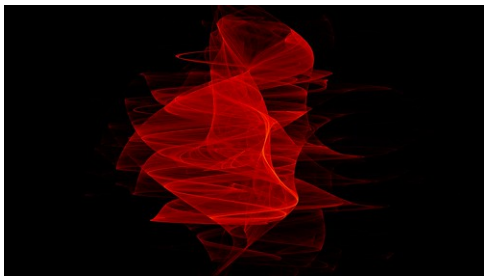
Posted by softologyblog on March 4, 2017
Strange Attractors are plots of relatively simple formulas. They are created by repeating (or iterating) a formula over and over again and using the results at each iteration to plot a point. The result of each iteration is fed back into the equation. After millions of points have been plotted fractal structures appear. The repeated points fall within a basin of attraction (they are attracted to the points that make up these shapes).

I recently revisited my old strange attractor code in Visions of Chaos (https://softology.pro/voc.htm) to add some new variations. This post will show many of the strange attractor formulas and some 4K resolution sample images they create. The images were created using over 1 billion points each. They have also been oversampled at least 3×3 pixels to reduce aliasing artifacts.

Bedhead Attractor

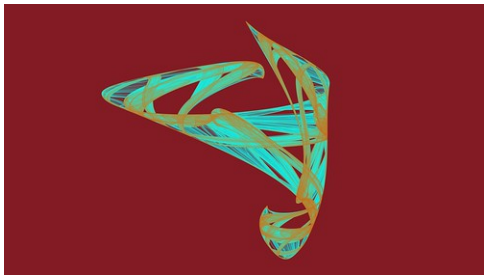
Discovered by Ivan Emrich (http://jaguarfacedman.deviantart.com/).

```
x and y both start at 1.0

xnew=sin(x*y/b)*y+cos(a*x-y)
ynew=x+sin(y)/b

Variables a and b are floating point values bewteen -1 and +1
```



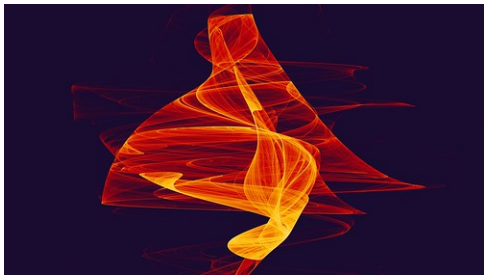
(https://c1.staticflickr.com/5/4311/36236201766_821d757426_o.png)

A=0.65343 B=0.7345345



(https://c1.staticflickr.com/5/4300/36236203746_5b4af3edfa_o.png)

A=-0.81 B=-0.92



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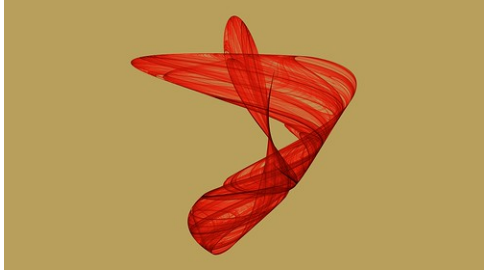
A=-0.64 B=0.76

(https://c1.staticflickr.com/5/4326/35882492370_ef8dc0b8d5_o.png)

A=0.06 B=0.98



(https://c1.staticflickr.com/5/4294/36276169775_c25c96cc22_o.png)

A=-0.67 B=0.83

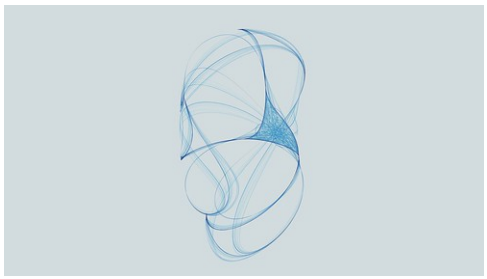
Clifford Attractor

Discovered by Clifford A Pickover (https://en.wikipedia.org/wiki/Clifford_A._Pickover). I found them explained on Paul Bourke (http://paulbourke.net/)'s page here (http://paulbourke.net/fractals/clifford/).

```
x and y both start at 0.1

xnew=sin(a*y)+c*cos(a*x)
ynew=sin(b*x)+d*cos(b*y)

Variables a,b,c and d are floating point values bewteen -3 and +3
```



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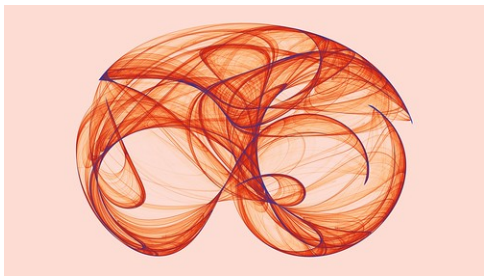
A=-1.7 B=1.3 C=-0.1 D=-1.21



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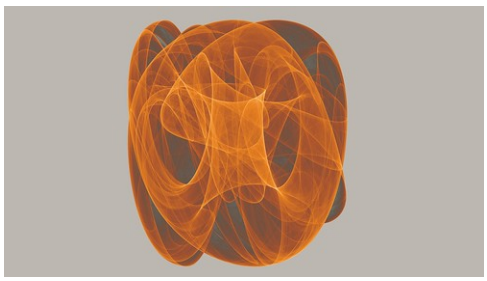
A=-1.7 B=1.8 C=-0.9 D=-0.4



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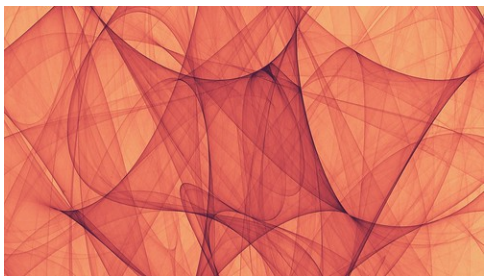
A=1.5 B=-1.8 C=1.6 D=2



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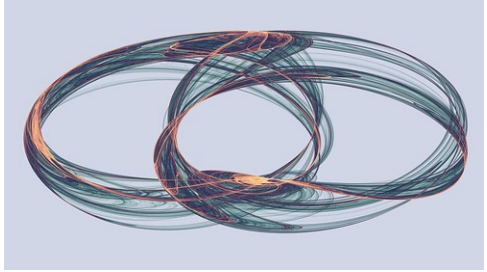
A=-2.239 B=-2.956 C=1.272 D=1.419



(https://c1.staticflickr.com/3/2902/33219989475_b03991785f_o.png)

A=-1.7 B=1.8 C=-1.9 D=-0.4

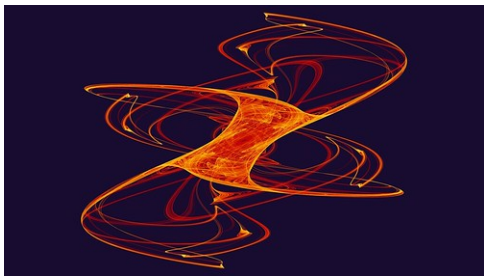
Fractal Dream Attractor

Discovered by Clifford A Pickover (https://en.wikipedia.org/wiki/Clifford_A._Pickover) and discussed in his book “Chaos In Wonderland”.

```
x and y both start at 0.1

xnew=sin(y*b)+c*sin(x*b)
ynew=sin(x*a)+d*sin(y*a)

Variables a and b are floating point values bewteen -3 and +3
Variables c and d are floating point values between -0.5 and +1.5
```



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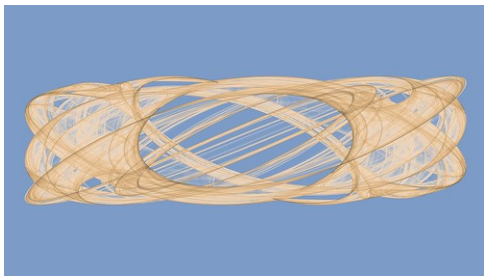
A=-0.966918 B=2.879879 C=0.765145 D=0.744728



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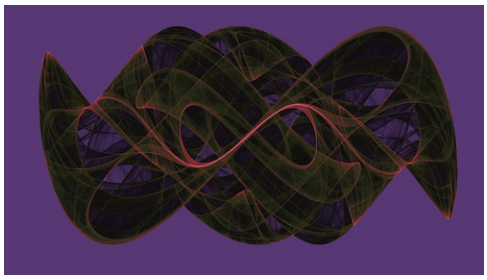
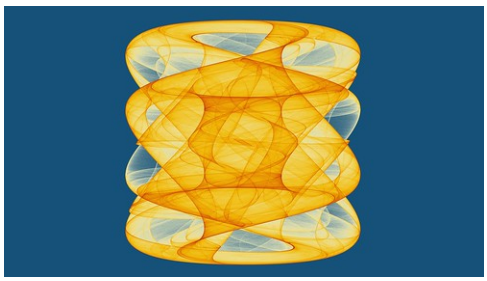
A=-2.9585 B=-2.2965 C=-2.8829 D=-0.1622



(https://c1.staticflickr.com/1/656/32376243764_4755b41b7c_o.png)

A=-2.8276 B=1.2813 C=1.9655 D=0.597



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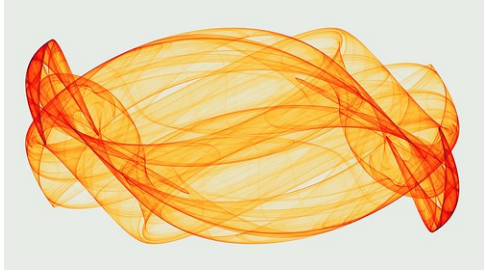
A=-1.1554 B=-2.3419 C=-1.9799 D=2.1828



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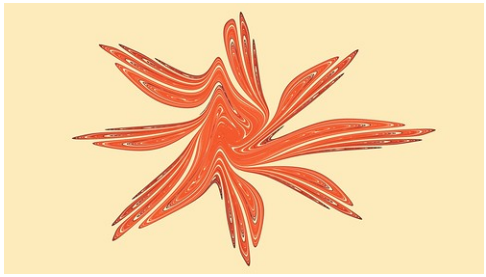
A=-1.9956 B=-1.4528 C=-2.6206 D=0.8517

Gumowski-Mira Attractor

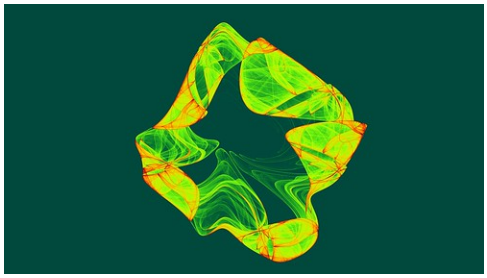
The Gumowski-Mira equation was developed in 1980 at CERN by I. Gumowski and C. Mira to calculate the trajectories of sub-atomic particles. It can also be used to create attractor images.

```
x and y both start at any floating point value between -20 and +20

t=x
xnew=b*y+w
w=a*x+(1-a)*2*x*x/(1+x*x)
ynew=w-t

The a and b parameters can be any floating point value between -1 and +1.
```



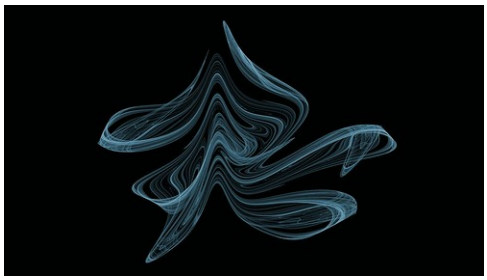
(https://c2.staticflickr.com/4/3733/33064114222_459645840d_o.png)

Initial X=0 Initial Y=0.5 A=0.008 B=-0.7



(https://c1.staticflickr.com/1/763/32837851970_4a8dc5a072_o.png)

Initial X=-0.723135391715914 Initial Y=-0.327585775405169 A=0.79253300698474 B=0.345703079365194



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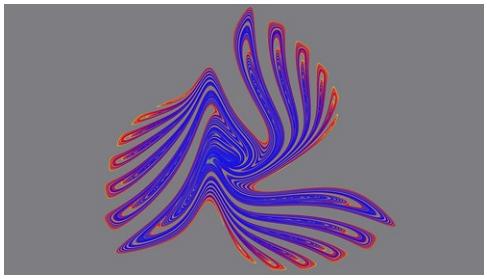
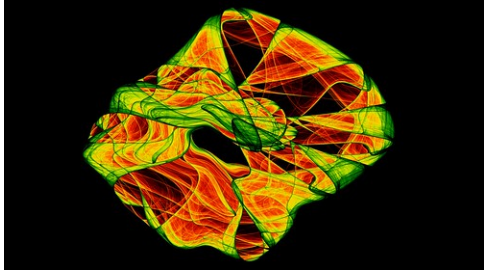
Initial X=-0.312847771216184 Initial Y=-0.710899183526635 A=0.579161538276821 B=-0.820410779677331



(https://c1.staticflickr.com/3/2916/33092660921_ec05c78c21_o.png)

Initial X=-0.325819793157279 Initial Y=0.485735820140 69 A=0.062683217227459 B=-0.436713613104075



(https://c1.staticflickr.com/1/596/32837828190_525f84d874_o.png)

Initial X=0.78662442881614 Initial Y=0.919355855789036 A=0.900278024375439 B=0.661233567167073

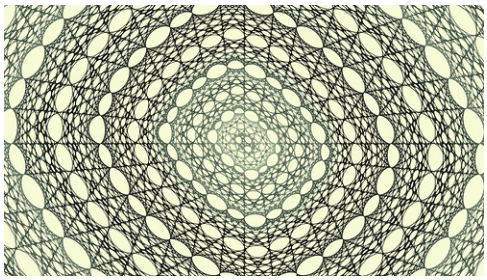
Hopalong Attractor

The Hopalong attractor was discovered by Barry Martin.

```
x and y both start at 0

xnew=y-1-sqrt(abs(b*x-1-c))*sign(x-1)
ynew=a-x-1

The parameters a, b and c can be any floating point value between 0 and +10.
```



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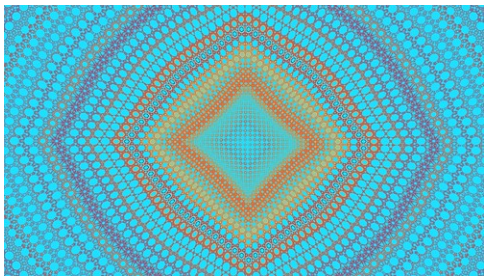
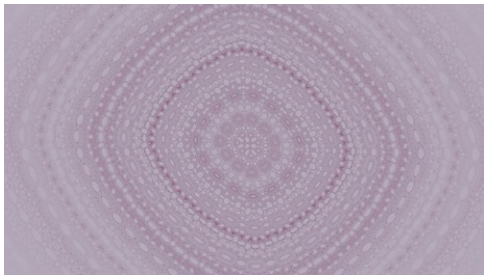
A=7.16878197155893 B=8.43659746693447 C=2.55983412731439



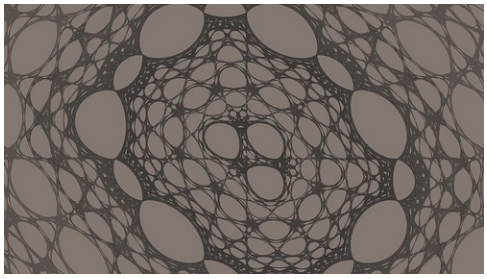
(https://c1.staticflickr.com/3/2918/33221061325_700e94be3f_o.png)

A=7.7867514709942 B=0.132189802825451 C=8.14610984409228



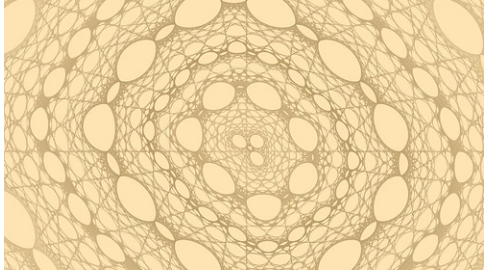
(https://c1.staticflickr.com/3/2822/32376924804_92d9c83a8e_o.png)

A=9.74546888144687 B=1.56320227775723 C=7.86818214459345



(https://c2.staticflickr.com/4/3832/33221002805_072cb5d13d_o.png)

A=9.8724800767377 B=8.66862616268918 C=8.66950439289212



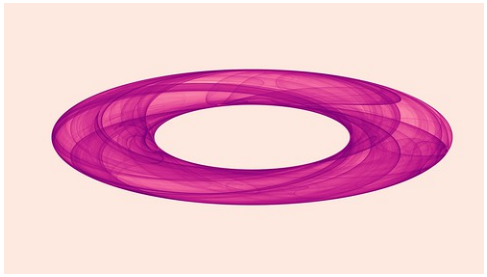
(https://c1.staticflickr.com/1/726/33064359412_cf207d097f_o.png)

A=9.7671244922094 B=4.10973468795419 C=3.78332691499963

Jason Rampe 1

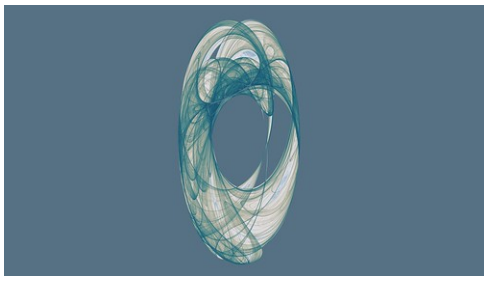
A variation I discovered while trying random formula changes.

```
x and y both start at 0.1

xnew=cos(y*b)+c*sin(x*b)
ynew=cos(x*a)+d*sin(y*a)

Variables a, b, c and d are floating point values between -3 and +3
```



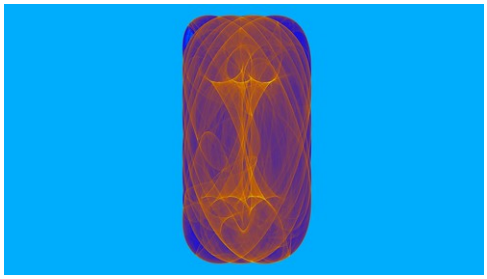
(https://c1.staticflickr.com/3/2935/33179859166_b7703172ca_o.png)

A=2.6 B=-2.5995 C=-2.9007 D=0.3565



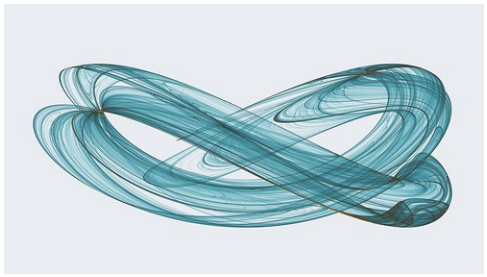
(https://c2.staticflickr.com/4/3780/33064861562_2f9d36e0a7_o.png)

A=1.8285 B=-1.8539 C=0.3816 D=1.9765



(https://c2.staticflickr.com/4/3716/33221552535_c4afc1ccc3_o.png)

A=2.5425 B=2.8358 C=-0.8721 D=2.7044



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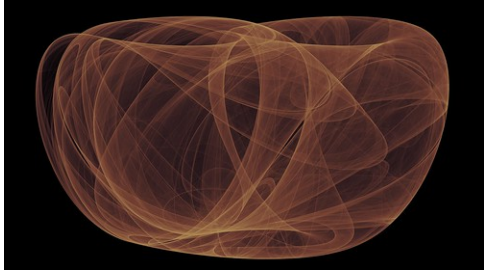
A=-1.8669 B=1.2768 C=-2.9296 D=-0.4121



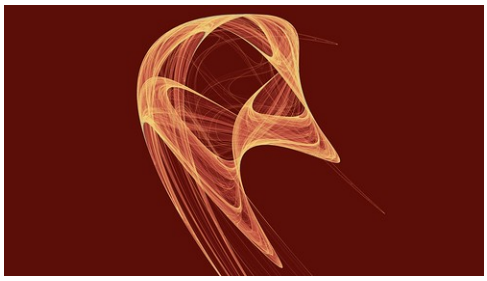
(https://c2.staticflickr.com/4/3738/32406683453_8dea889de9_o.png)

A=-2.7918 B=2.1196 C=1.0284 D=0.1384

Jason Rampe 2

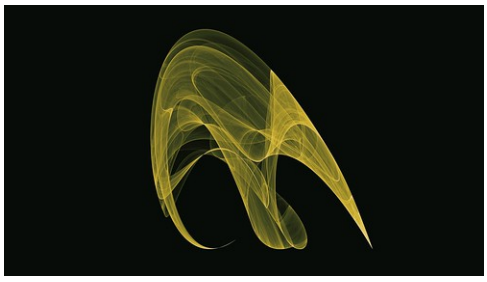
Another variation I discovered while trying random formula changes.

```
x and y both start at 0.1

xnew=cos(y*b)+c*cos(x*b)
ynew=cos(x*a)+d*cos(y*a)

Variables a, b, c and d are floating point values between -3 and +3
```



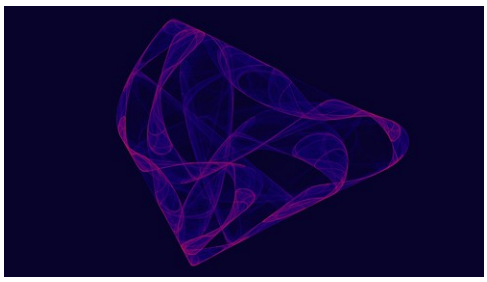
(https://c2.staticflickr.com/4/3894/33221924815_ed72057f1a_o.png)

A=1.546 B=1.929 C=1.09 D=1.41



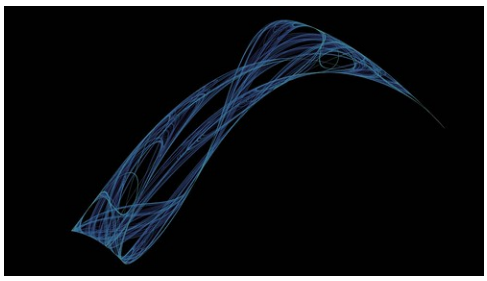
(https://c1.staticflickr.com/3/2830/33180212616_a914ece22c_o.png)

A=2.907 B=-1.9472 C=1.2833 D=1.3206



(https://c2.staticflickr.com/4/3690/33221946405_133dd833be_o.png)

A=0.8875 B=0.7821 C=-2.3262 D=1.5379



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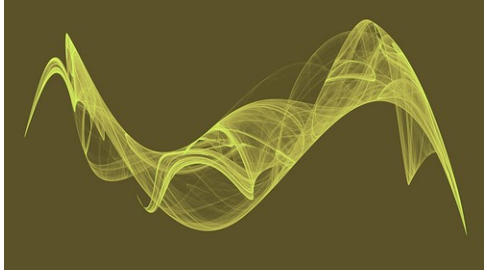
A=-2.4121 B=-1.0028 C=-2.2386 D=0.274



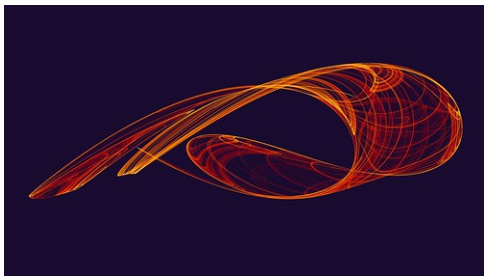
(https://c2.staticflickr.com/4/3819/32377672014_3dd09ce218_o.png)

A=-2.9581 B=0.927 C=2.7842 D=0.6267

Jason Rampe 3

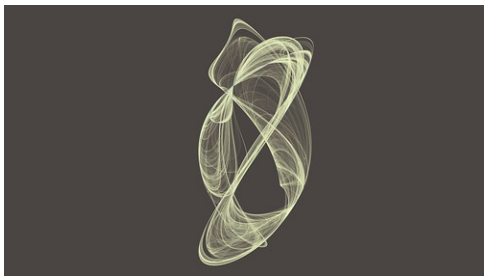
Yet another variation I discovered while trying random formula changes.

```
x and y both start at 0.1

xnew=sin(y*b)+c*cos(x*b)
ynew=cos(x*a)+d*sin(y*a)

Variables a, b, c and d are floating point values between -3 and +3
```



(https://c2.staticflickr.com/4/3732/32407338263_f9b4d8eec1_o.png)

A=2.0246 B=-1.323 C=-2.8151 D=0.2277

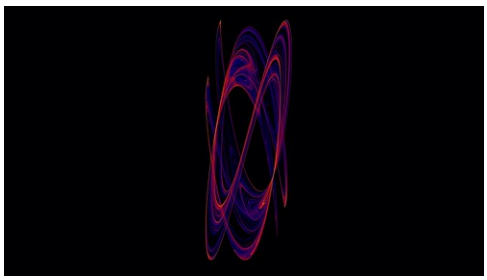


(https://c2.staticflickr.com/4/3700/33065531972_2fa31e569b_o.png)

A=1.4662 B=-2.3632 C=-0.4167 D=2.4162

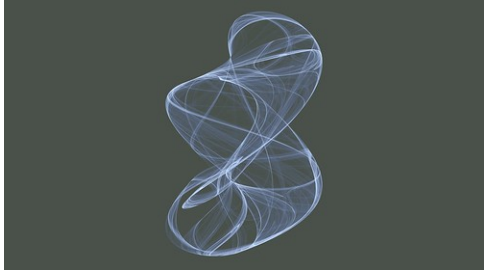
(https://c1.staticflickr.com/3/2822/33094120681_dd2fb81102_o.png)

A=-2.7564 B=-1.8234 C=2.8514 D=-0.8745



(https://c2.staticflickr.com/4/3840/33222284515_723ab9e142_o.png)

A=-2.218 B=1.4318 C=-0.3346 D=2.4993



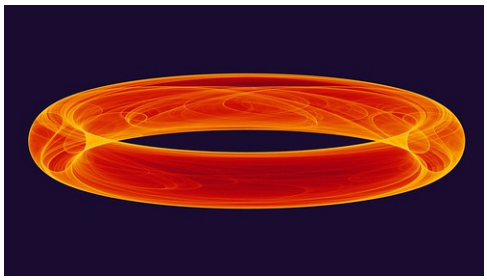
(https://c1.staticflickr.com/3/2929/33222245855_4afa8d5fb3_o.png)

A=1.2418 B=-2.4174 C=-0.7112 D=-1.9802

Johnny Svensson Attractor

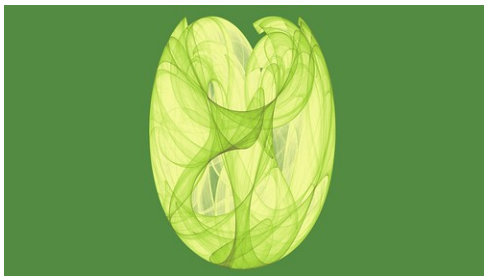
See here (http://paulbourke.net/fractals/peterdejong/).

```
x and y both start at 0.1

xnew=d*sin(x*a)-sin(y*b)
ynew=c*cos(x*a)+cos(y*b)

Variables a, b, c and d are floating point values between -3 and +3
```



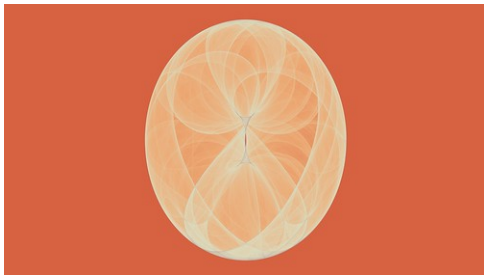
(https://c1.staticflickr.com/3/2887/32413016143_a9bdcfc16c_o.png)

A=1.40 B=1.56 C=1.40 D=-6.56



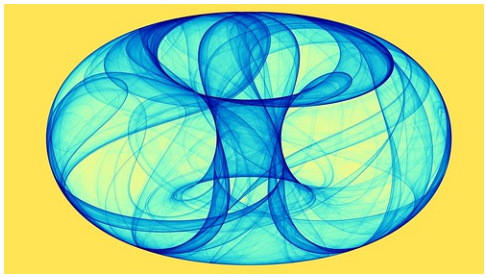
(https://c2.staticflickr.com/4/3832/32844844480_061684c3aa_o.png)

A=-2.538 B=1.362 C=1.315 D=0.513



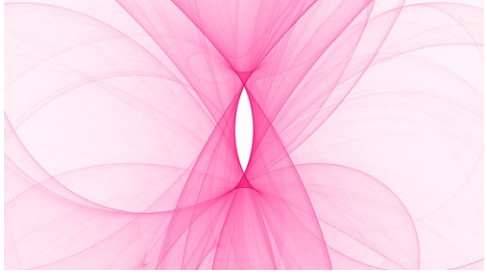
(https://c1.staticflickr.com/1/776/33071346732_ae74291d9d_o.png)

A=1.913 B=2.796 C=1.468 D=1.01



(https://c1.staticflickr.com/1/612/32383801024_cd911b6d76_o.png)

A=-2.337 B=-2.337 C=0.533 D=1.378



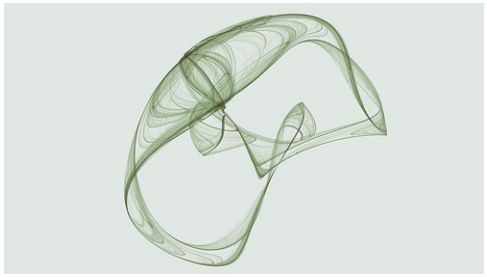
(https://c2.staticflickr.com/4/3670/33227897245_cdb1770cec_o.png)

A=-2.722 B=2.574 C=1.284 D=1.043

Peter DeJong Attractor

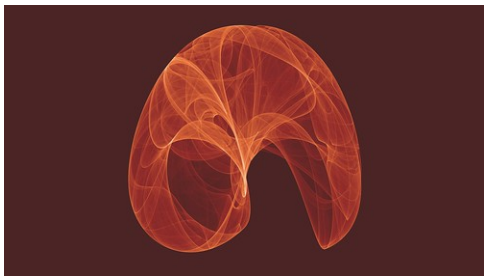
See here (http://paulbourke.net/fractals/peterdejong/).

```
x and y both start at 0.1

xnew=sin(y*a)-cos(x*b)
ynew=sin(x*c)-cos(y*d)

Variables a, b, c and d are floating point values between -3 and +3
```



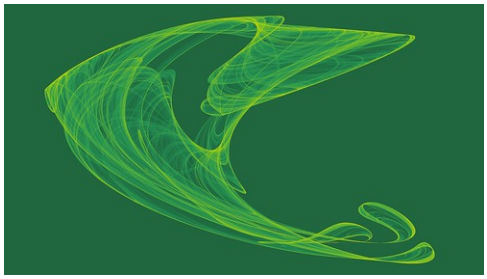
(https://c2.staticflickr.com/4/3679/32413704063_5b0b776998_o.png)

A=0.970 B=-1.899 C=1.381 D=-1.506



(https://c2.staticflickr.com/4/3864/32413714853_4d36b5313d_o.png)

A=1.4 B=-2.3 C=2.4 D=-2.1



(https://c1.staticflickr.com/1/672/33187029376_5293da36de_o.png)

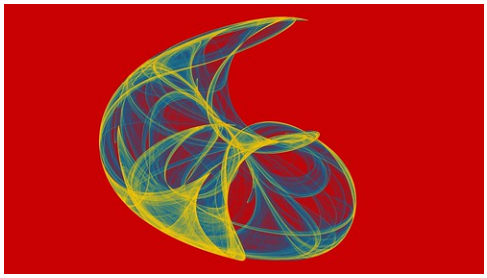
A=2.01 B=-2.53 C=1.61 D=-0.33

(https://c2.staticflickr.com/4/3885/33228649655_75db0bb1b8_o.png)

A=-2.7 B=-0.09 C=-0.86 D=-2.2

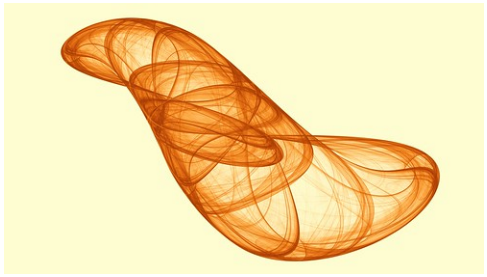
(https://c1.staticflickr.com/1/768/33228658045_c7f5027e27_o.png)

A=-0.827 B=-1.637 C=1.659 D=-0.943



(https://c1.staticflickr.com/3/2912/33072153912_2ed77e168b_o.png)

A=-2 B=-2 C=-1.2 D=2



(https://c1.staticflickr.com/3/2942/33072089382_b82c6e58c9_o.png)

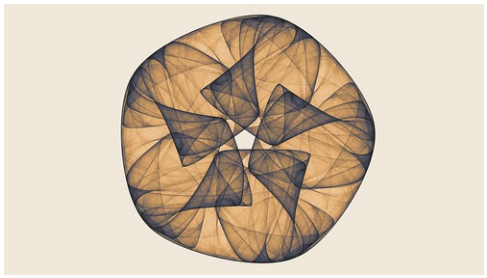
A=-0.709 B=1.638 C=0.452 D=1.740

Symmetric Icon Attractor

These attractors came from the book “Symmetry in Chaos” by Michael Field and Martin Golubitsky. They give symmetric results to the attractors formed.

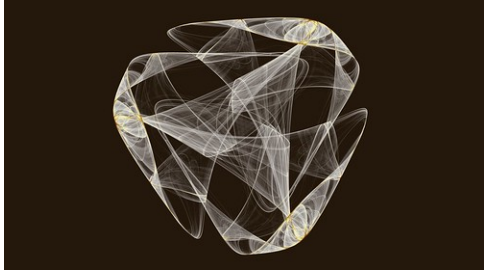
```
x and y both start at 0.01

zzbar=sqr(x)+sqr(y)
p=alpha*zzbar+lambda
zreal=x
zimag=y
for i=1 to degree-2 do
begin
     za=zreal*x-zimag*y
     zb=zimag*x+zreal*y
     zreal=za
     zimag=zb
end
zn=x*zreal-y*zimag
p=p+beta*zn
xnew=p*x+gamma*zreal-omega*y
ynew=p*y-gamma*zimag+omega*x
x=xnew
y=ynew

The Lambda, Alpha, Beta, Gamma, Omega and Degree parameters can be changed to create new plot shapes.
```

These sample images all come from paramters in the “Symmetry in Chaos” book.



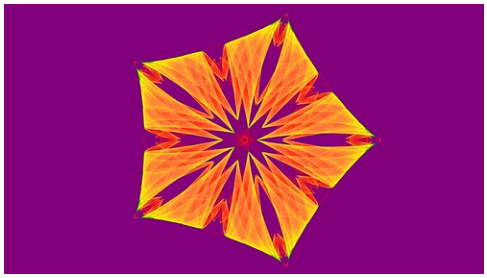
(https://c1.staticflickr.com/1/597/33072555022_c0ff1b26fd_o.png)

L=-2.5 A=5 B=-1.9 G=1 O=0.188 D=5



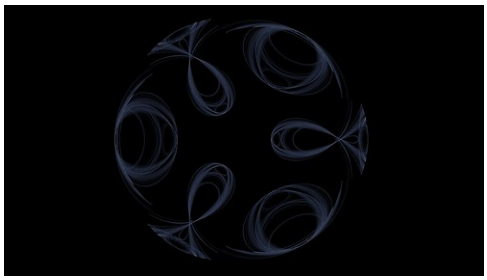
(https://c1.staticflickr.com/1/718/32845967770_4c3fd36554_o.png)

L=1.56 A=-1 B=0.1 G=-0.82 O=0.12 D=3



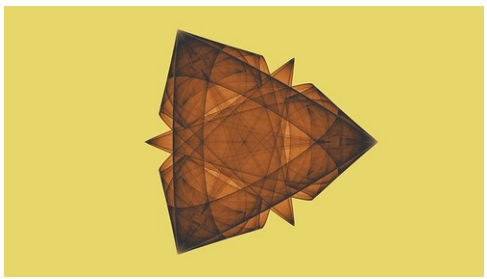
(https://c1.staticflickr.com/1/685/32384964574_e57d6740b1_o.png)

L=-1.806 A=1.806 B=0 G=1 O=0 D=5



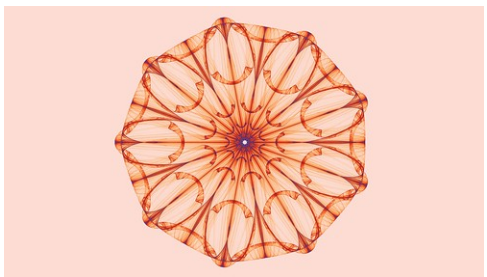
(https://c1.staticflickr.com/1/729/33100892901_139d705688_o.png)

L=-2.195 A=10 B=-12 G=1 O=0 D=3



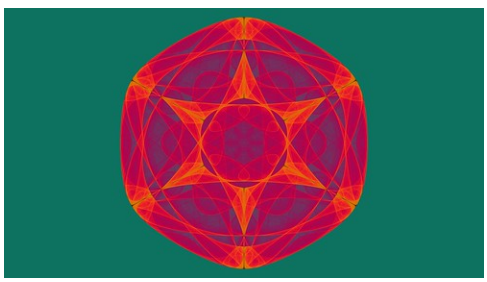
(https://c2.staticflickr.com/4/3903/33072652592_3940971042_o.png)

L=2.5 A=-2.5 B=0 G=0.9 O=0 D=3



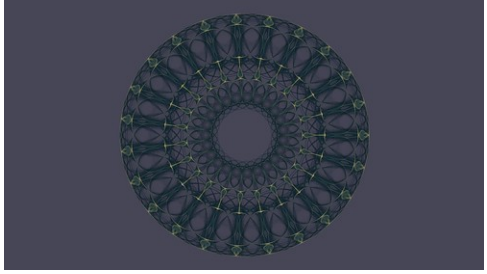
(https://c2.staticflickr.com/4/3950/32385133224_d720c3896e_o.png)

L=-2.05 A=3 B=-16.79 G=1 O=0 D=9



(https://c1.staticflickr.com/1/754/33229512885_1783842f9d_o.png)

L=-2.7 A=5 B=1.5 G=1.0 O=0 D=6

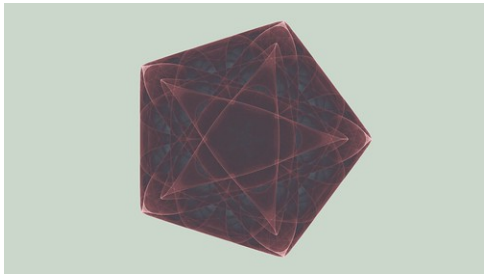


(https://c2.staticflickr.com/4/3774/32414657043_cb324e4a76_o.png)

L=2.409 A=-2.5 B=0 G=0.9 O=0 D=23

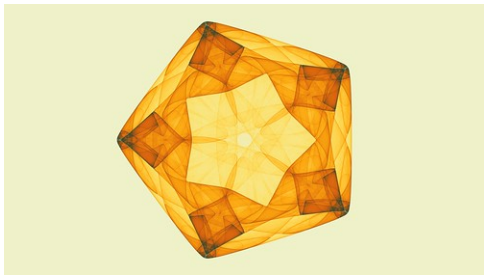
(https://c1.staticflickr.com/1/564/33073271752_c4e81c6a94_o.png)

L=-2.08 A=1 B=-0.1 G=0.167 O=0 D=7



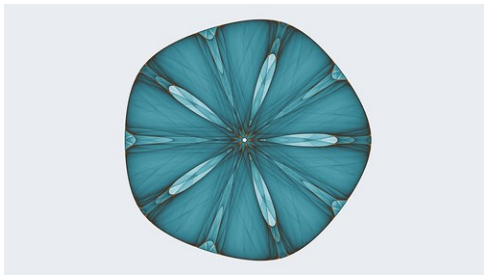
(https://c2.staticflickr.com/4/3817/32385682684_22525d529f_o.png)

L=-2.32 A=2.32 B=0 G=0.75 O=0 D=5



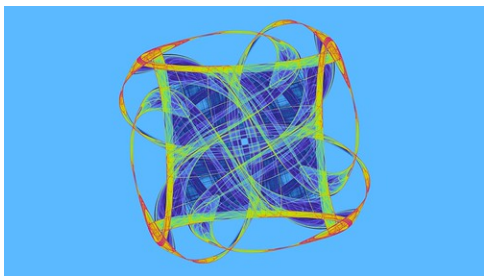
(https://c2.staticflickr.com/4/3914/32846848450_c2eed9370c_o.png)

L=2.6 A=-2 B=0 G=-0.5 O=0 D=5



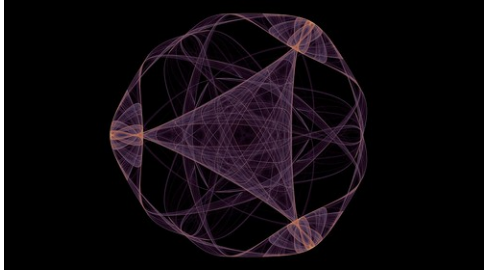
(https://c2.staticflickr.com/4/3893/33073533472_2a784309dd_o.png)

L=-2.34 A=2 B=0.2 G=0.1 O=0 D=5



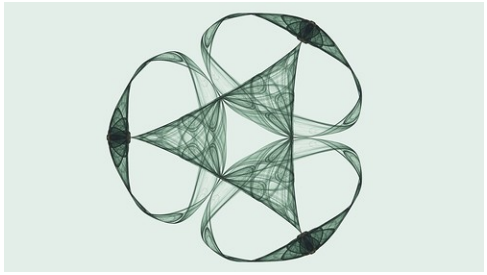
(https://c1.staticflickr.com/1/626/32415017763_2484a56900_o.png)

L=-1.86 A=2 B=0 G=1 O=0.1 D=4



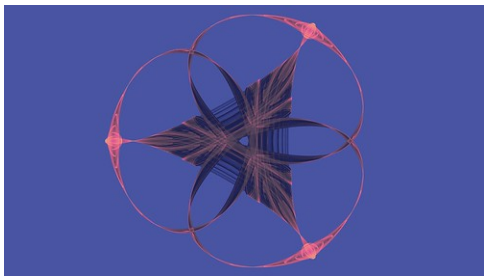
(https://c2.staticflickr.com/4/3778/33101821791_524eb597c8_o.png)

L=1.56 A=-1 B=0.1 G=-0.82 O=0 D=3



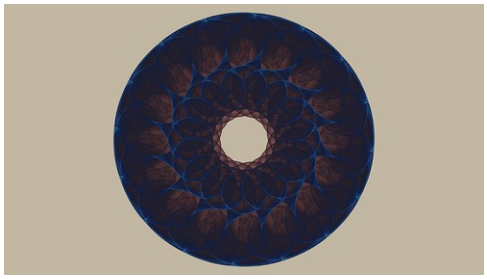
(https://c1.staticflickr.com/1/703/32386005504_f9516be551_o.png)

L=1.5 A=-1 B=0.1 G=-0.805 O=0 D=3



(https://c1.staticflickr.com/3/2813/32847157210_ba8a1e0295_o.png)

L=1.455 A=-1 B=0.03 G=-0.8 O=0 D=3



(https://c1.staticflickr.com/1/768/33188647746_28b35ef625_o.png)

L=2.39 A=-2.5 B=-0.1 G=0.9 O=-0.15 D=16

3D Alternatives

Strange Attractors can also be extended into three dimensions. See here (https://softologyblog.wordpress.com/2009/10/19/3d-strange-attractors/) and here (https://softologyblog.wordpress.com/2011/02/20/further-adventures-with-3d-strange-attractors/) for my previous experiments with 3D Strange Attractors.

All of the images in this post were created using Visions of Chaos (https://softology.pro/voc.htm).

Jason.

This entry was posted in Strange Attractors. Bookmark the permalink.

3 responses to "*2D Strange Attractors*"

cellocgw says:
May 11, 2021 at 8:22 pm
Hi, there's a typo in your Gumowski-mira formulas. The final term should be 1/(1+x^2)^2 . The artwork, however, is 'correct' and beautiful.

Reply

softologyblog says:
May 12, 2021 at 5:32 am
Hi, I used the formulas from http://www.scipress.org/journals/forma/pdf/1502/15020121.pdf when creating these Gumowski-mira images.

Reply

cellocgw says:
May 12, 2021 at 6:04 am
Hi, and apologies for not doing more research — it seems many folks use your version and many others use the extra-squared version (Wolfram, for one).