AIE6002 인공지능확률통계 기말시험

이름, 학번:

- 반드시 설명을 적으세요.
- 10 points each.
- (For problems 1 & 2) Given a Linear Gaussian System:

$$p(z) = \mathcal{N}(z|\mu_z, \Sigma_z) \tag{1}$$
$$p(y|z) = \mathcal{N}(y|Wz + b, \Sigma_y) \tag{2}$$

1. Derive the mean μ and covariance Σ of $p(z, y)$.
2. Find the marginal distribution $p(y)$ based on the result of Problem 2.
3. $Y \sim Bernoulli(\theta)$ and θ is a discrete random varaible with its PMF $P(\theta = 0.6) = 0.5$ and $P(\theta = 0.3) = 0.5$. Through independent random experiments you obtained observations $1, 1, 0$ of Y.
    - Compute the posterior PMF $P[\theta|Y = \{1, 1, 0\}]$.
4. You are given the observation $N_0 = 3$ and $N_1 = 7$ from 10 times independent experiments of Bernoulli distribution $Bernoulli(\theta)$ where θ denotes $P[Y = 1|\theta]$ and the prior distribution is $Beta(a = 11, b = 9)$.
    1. Find the MAP estimate of the posterior
    2. and Laplace approximation of the posterior distribution $p(\theta|N_0 = 3, N_1 = 7)$.
5. You have an observation $y = 3$ from a Gaussian distribution $\mathcal{N}(\mu, \sigma^2 = 1)$; you know $\sigma^2 = 4$ but you do not know the value of μ. From our bureau of information, you are told that the prior for μ is $\mathcal{N}(0, 4)$.
    - What is the posterior distribution $p_9(\mu|y)$ of μ given the data y?
6. You are going to apply ELBO maximization to obtain an approximation $q(\mu|\phi, \lambda^{-1})$ of the exact posterior $p_9(\mu|y)$; here ϕ is the mean of $q()$ and λ is the precision of $q()$.
    - Find ELBO which is a function of ϕ and λ.
7. Find the optimal values for ϕ and λ in Problem 10.
8. When $X \sim \mathcal{N}(0, 4)$, what is $p(y)$ when $Y = 2X + 3$?
9. Provided a 2D random vector $X = [X_1, X_2]^T$ with a 2D Gaussian distribution of mean μ and covariance Σ, find $P[Y > 2]$ where $Y = X_1 + X_2$:

$$\mu = [2, 0]^T$$

$$\Sigma = \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}$$

10. Let $X \sim U(-1, 1)$ and $Y = X^2$. Clearly Y is dependent on X. However, show that the correlation coefficient $\rho(X, Y) = 0$. The general definition of ρ is given as follows:

$$\rho(X, Y) = \frac{\mathrm{Cov}[X, Y]}{\sqrt{Var[X]Var[Y]}}$$

- Good luck!