

实验二：数字滤波器的设计

1. 根据实验原理，编写代码，得出实验结果，并画出波形图

2. 归纳、总结实验结果

试用双线性 Z 变换法设计：低通数字滤波器，给定技术指标：

$$f_{passband} = 100Hz$$

$$f_{stopband} = 300Hz$$

$$\alpha_p = 3dB$$

$$\alpha_s = 20dB$$

$$F_{sampling} = 1000Hz$$

用所设计的滤波器对实际心电图信号采样序列进行仿真滤波处理，并分别打印出滤波前后的心电图信号波形图，观察总结滤波作用与效果。

心电图信号采样序列 $x(n)$ ：人体心电图信号在测量过程中往往受到工业高频干扰，所以必须经过低通滤波处理后，才能作为判断心脏功能的有用信息。下面给出一实际心电图信号采样序列样本 $x(n)$ ，其中存在高频干扰。在实验中，以 $x(n)$ 作为输入序列，滤出其中的干扰成分。

```
x(n)={-4,   -2,   0,   -4,   -6,   -4,   -2,   -4,   -6,   -6,
      -4,   -4,   -6,   -6,   -2,   6,  12,   8,   0,  -16,
     -38, -60,  -84,  -90,  -66,  -32,  -4,   -2,  -4,   8,
      12,  12,  10,   6,   6,   6,   4,   0,   0,   0,
       0,   0,   -2,  -4,   0,   0,   0,   -2,  -2,   0,
       0,   -2,  -2,  -2,  -2,   0}
```

双线性变换法

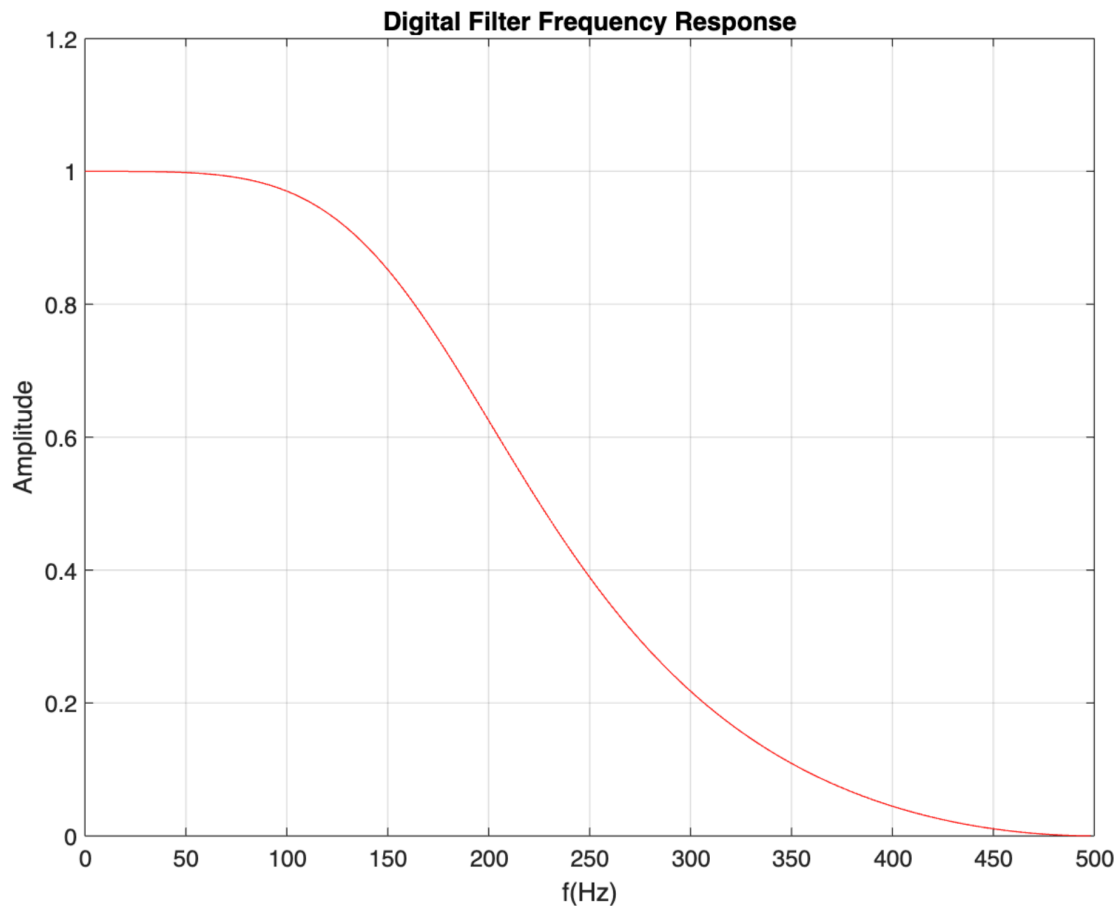
从 s 域映射到正切 \tan ，再从 \tan 映射到 z 域

$$s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$
$$j\Omega = \frac{2}{T} j \tan \frac{\omega}{2} (s = j\Omega)$$
$$z = \frac{1 + sT/2}{1 - sT/2}$$

数字滤波器原理

$$x_0(t) \rightarrow H_1(s) \rightarrow x(t) \rightarrow \text{抽样/量化} \rightarrow$$
$$x(n) \rightarrow H(z) \rightarrow y(n) \rightarrow y_s(t) \rightarrow$$
$$H_2(s) \rightarrow y(t)$$

```
x = [-4 -2 0 -4 -6 -4 -2 -4 -6 -6 -4 -4 -6 -6 -2 6 12 8 0 -16 -38 -
```



```
ECG = [-4 -2 0 -4 -6 -4 -2 -4 -6 -6 -4 -4 -6 -6 -2 6 12 8 0 -16 -38  
lengthOfECG = length(ECG); % lengthOfECG = 56  
fourierFrequency = fft(ECG);  
plot(0: lengthOfECG - 1, fourierFrequency);  
title('Frequency Domain Signal');
```

```
% Digital Filter Specification
```

```
% 定义技术指标
```

```
fpassband=100;
```

```
fstopband=300;
```

```
FSampling = 1000;
```

```
% 求出 Digital 数字角频率
```

```
Wpassband = 2 * pi * fpassband / FSampling;
```

```
Wstopband = 2 * pi * fstopband / FSampling;
```

```
% Analog Domain
```

```
rpPassbandRipple = 3; % 3dB 通带波纹 %通带允许的最大衰减
```

```
rsStopbandAttenuation = 20; % 20dB 阻带衰减 %阻带允许的最小衰减
```

% 对角频率做预畸变

$F_s = 1;$

$\omega_{p1} = 2 * F_s * \tan(W_{passband}/2);$

$\omega_{s1} = 2 * F_s * \tan(W_{stopband}/2);$

% Butterworth filter order and cutoff frequency

% 求出模拟低通滤波器的阶次, 利用函数

$[NorderOfButterworthFilter, WnCutoffFrequencies] = buttord(\omega_{p1}, \omega_{s1}, Rp, Rs);$

% N 代表滤波器阶数, Wn 代表滤波器的截止频率。

% 简单来说就是在 $\Omega_{passband}$ 处, 通带内波纹系数或者说是通带内达到最大衰减为

% 如 (3db), 而在 $\Omega_{stopband}$ 处, 阻带达到最小衰减为 R_s (如 40db)

% 而我们默认求得的 Wn 是在(-3db)时的频率

% Butterworth filter prototype

% 设计模拟低通原型滤波器 $G(p)$, 其调用格式是

$[Zeros, Poles, KGains] = buttap(NorderOfButterworthFilter);$

% $G(p)$ 的 Zeros 零点, Poles 极点, KGains 增益用于计算 N 阶归一化

% (3dB 截止频率 $\Omega_c=1$) 模拟低通原型滤波器系统函数

% 的零、极点和增益因子。

% Convert zero-pole-gain filter parameters to transfer function for

% 求模拟低通原型滤波器 $G(p)$ 的分子分母系数

$[b_{numerator}, a_{denominator}] = zp2tf(Zeros, Poles, KGains);$

% $G(p)$ 的分子、分母系数从零、极点模型得到

% 系统函数的分子、分母多项式系数向量 ba 、 aa

% Change cutoff frequency for lowpass analog filter

% 求出 $G(p)$ 的分子、分母系数。

% bt Transformed numerator and denominator coefficients, returned

% at Transformed numerator and denominator coefficients, returned

$[bt, at] = lp2lp(b_{numerator}, a_{denominator}, WnCutoffFrequencies);$

% $H(s)$ 的分子、分母系数改变低通模拟滤波器的截止频率,

% 原滤波器是以多项式系数 Bap, Aap 给出的, 改后的滤波器是带截止频率 Wn 的

% $H(z)$ 的分子、分母系数

$[bz_{numeratorcoefficients}, az_{denominatorcoefficients}] = bilinear(bt, at, Ts);$

% H 频率响应 rad / seconds

% Frequency response of digital filter

$[H_{frequencyResponse}, W_{angularFrequencies}] = freqz(bz_{numeratorcoefficients}, az_{denominatorcoefficients}, W_{angularFrequencies});$

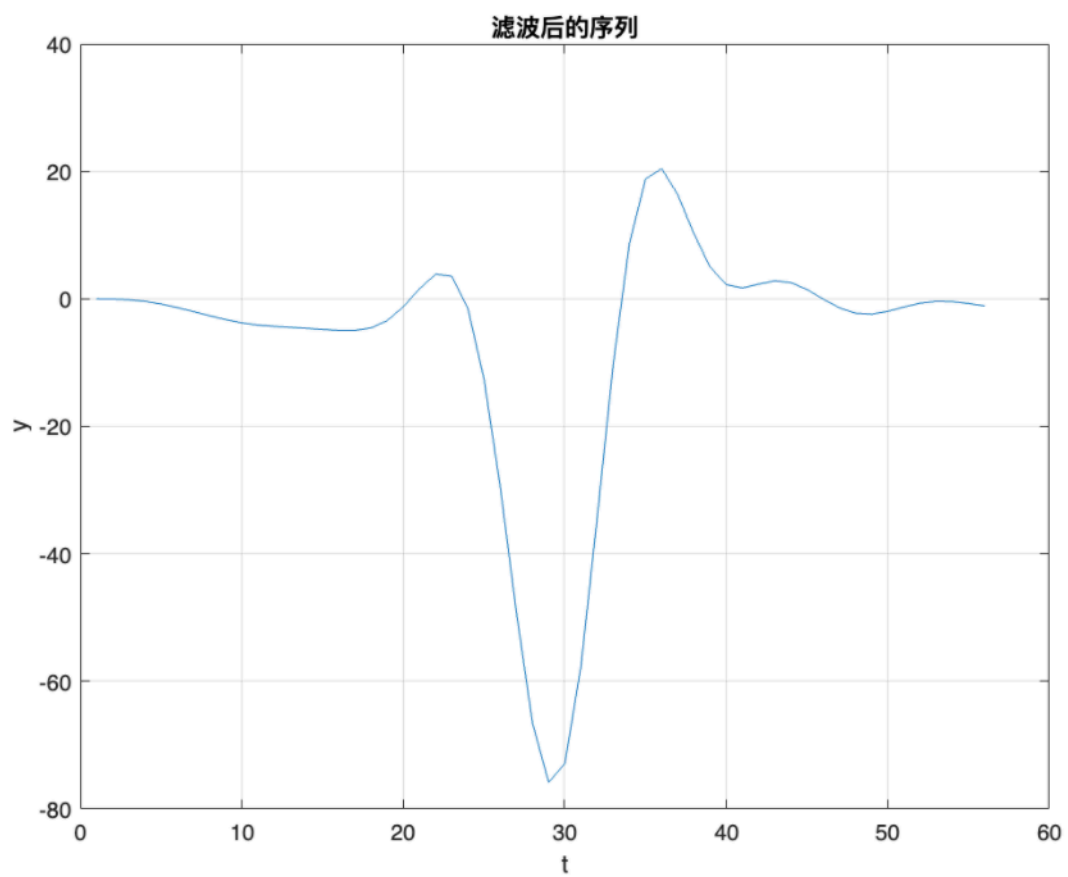
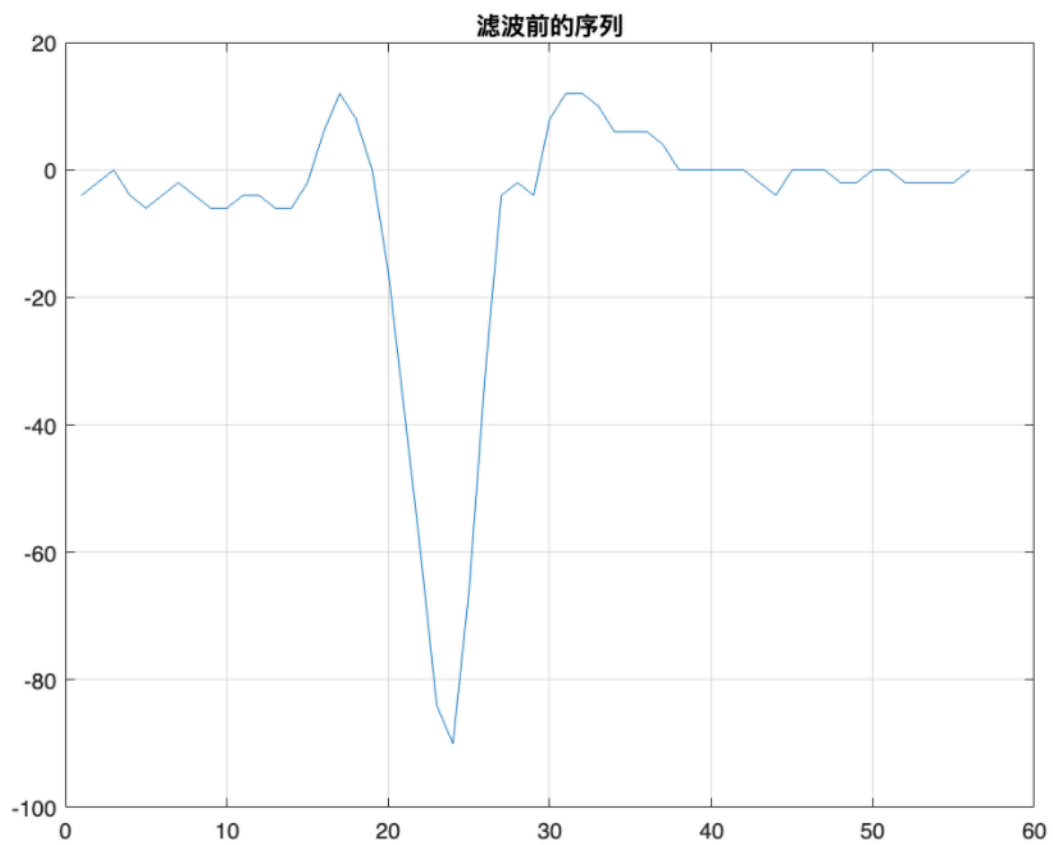
$\text{disp}(bz_{numeratorcoefficients});$

$\text{disp}(az_{denominatorcoefficients});$

*% rad / s. $\omega = 2 * \pi / T$*

```
plot(WAngularFrequencies, abs(HFrequencyResponse));
grid on;
xlabel('t');
ylabel('Amplitude');
title('Digital Filter Frequency Response');
t = 1:56
x = [-4,-2,0,-4,-6,-4,-2,-4,-6,-6,-4,-4,-6,-6,-2,6,12,8,0,-16,-38,-
plot(t, x);
title('滤波前的序列');

grid on;
y = filter( ,az,x);
figure(2);
plot(t,y);
grid on;
xlabel('t');ylabel('y');
title('滤波后的序列');
```



%原心电信号

x1=[-4,-2,0,-4,-6,-4,-2,-4,-6,-6,-4,-4,-6,-6,-2,6,12,8,0,...

```

-16,-38,-60,-84,-90,-66,-32,-4,-2,-4,8,12,12,10,6,6,6,4,0,0,...
0,0,0,-2,-4,0,0,0,-2,-2,0,0,-2,-2,-2,-2,0];
m=size(x1,2);
nx1=0:m-1;

X1=fft(x1,1024);%计算xn的1024点fft
n=0:1023;
n1=2*n/1024;%计算1024点DFT对应的采样点频率

% 滤波后信号
y=filter(bz,az,x1);
% plot(nx1,y)
% title('滤波后')
%
%滤波后
X2=fft(y,1024);%计算xn的1024点fft
n=0:1023;
n2=2*n/1024;%计算1024点DFT对应的采样点频率

%%绘图
figure;
subplot(221),plot(nx1,x1);title('滤波前信号')
subplot(222),plot(nx1,y);title('滤波后信号')
subplot(223),plot(n1,abs(X1));xlabel('\pi'),title('滤波前幅频特性曲线')
subplot(224),plot(n2,abs(X2));xlabel('\pi'),title('滤波后幅频特性曲线')

Ts = 1;
figure;
subplot(211)
xx1=n1/Ts/2;
plot(xx1(1:end/2),abs(X1(1:end/2)));xlabel('Hz'),title('滤波前幅频特性')
subplot(212)
xx2=n2/Ts/2;
plot(xx2(1:end/2),abs(X2(1:end/2)));xlabel('Hz'),title('滤波后幅频特性')

%
% n = 0:39;
% y = cos(2*pi*2*n/40);
% stem(n,y);
% stem(n, abs(fft(y)))

clear;
% 读取原始数据, 这里是 n * 1 的数据
Signal = [-4 -2 0 -4 -6 -4 -2 -4 -6 -6 -4 -4 -6 -6 -2 6 12 8 0 -16

```

%指标

```
wp = 2 * 100 / 1000 * pi; ws = 2 * 300 / 1000 * pi; rp = 3; rs = 20; Fs =
```

%计算

```
wp1=Fs*tan(wp/2);
```

```
ws1=Fs*tan(ws/2);
```

```
[N,Wn] = buttord(wp1,ws1,rp,rs,'s');
```

```
[Z,P,K] = buttap(N);
```

```
[Bap,Aap] = zp2tf(Z,P,K);
```

```
[b,a] = lp2lp(Bap,Aap,Wn);
```

```
[bz,az] = bilinear(b,a,Fs);
```

% 滤波

```
Signal_Filter = filter(bz, az, Signal);
```

```
subplot(2, 1, 1);
```

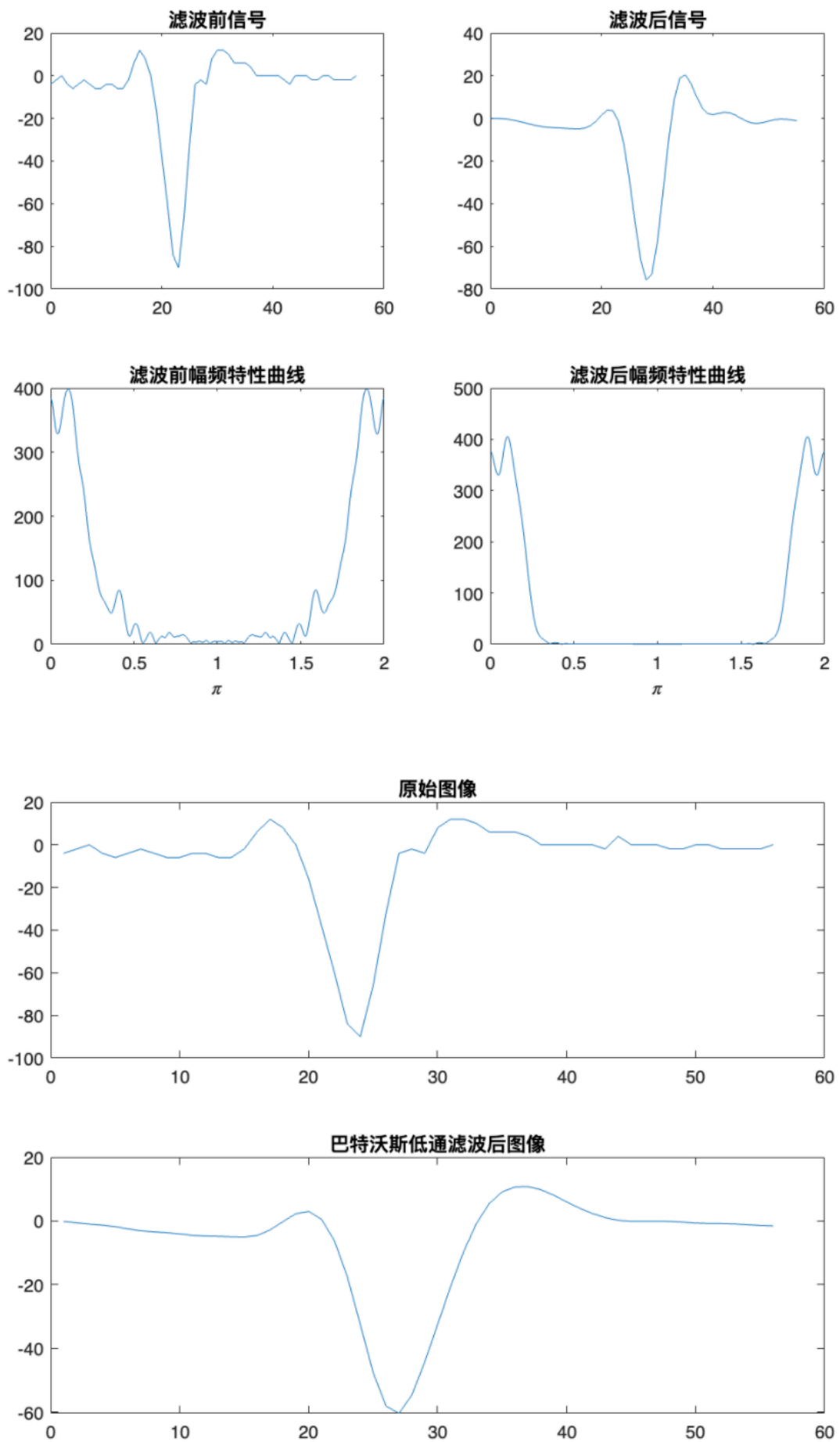
```
plot(Signal);
```

```
title('原始图像');
```

```
subplot(2,1,2);
```

```
plot(Signal_Filter);
```

```
title('巴特沃斯低通滤波后图像');
```



3. 心得体会及其他

Reference

- 猿儿飘飘. 数字信号处理_巴特沃斯低通滤波器实验[EB/OL]. 2022[2022-04-16]. https://blog.csdn.net/weixin_48023487/article/details/124217932.
- Niel de Beaudrap. Writing multiplication dots[EB/OL]. 2013[2022-11-24]. <https://tex.stackexchange.com/questions/113686/writing-multiplication-dots>.
- 设计数字低通滤波器(用matlab实现).doc <https://www.taodocs.com/p-249592419.html>