Replicating Political Economy Weights in Ossa (2014): An Optimization-based Alternative *

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Abstract

We replicate the political economy weights in Ossa (2014) using a Levenberg-Marquardt optimization framework. The optimization routine yields clearer first-order optimality diagnostics, with Japan showing the most notable differences relative to the heuristic procedure, while most other countries display outcomes that are numerically close to the original implementation. Despite these differences in the underlying weights, the implied optimal tariff profiles remain highly stable across methods.

JEL Codes: C61, C63, F59

Keywords: Political economy weights, Optimal tariffs, Levenberg-Marquardt optimization

*All data used in this paper come directly from the publicly available replication package of Ossa (2014), which is distributed under the Creative Commons Attribution 4.0 License. Ossa (2014)'s replication package is available at https://www.openicpsr.org/openicpsr/project/112717/version/V1/view. This paper's replication package is availabe at https://github.com/yonggeun-jung/replication-ossa2014-peweights. First version: November, 2025.

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1 Introduction

Tariff policy is inherently political. A large body of work has shown that political economy considerations shape both the level and structure of trade protection (e.g., Grossman and Helpman, 1994; Fajgelbaum et al., 2020; Adão et al., 2023; Heins and Jones, 2025; Becko et al., 2025). Yet these forces are difficult to quantify, as political motives are often unobserved or only imperfectly measured. Ossa (2014) offered an elegant solution to this challenge by exploiting the discrepancy between optimal tariffs implied by a structural trade model and the non-cooperative tariffs observed in the data. This gap provides a mapping into a set of political economy weights, λ_{js}, that rationalize countries' tariff choices.

Because the estimation of these weights was not the main objective of the original paper, Ossa (2014) computed them using an efficient but heuristic iterative procedure. While this approach performs well in practice, it does not target a formal optimization criterion, and thus the resulting weights need not satisfy first-order optimality conditions. In this replication study, we revisit the computation of political economy weights by replacing the heuristic algorithm with a Levenberg-Marquardt (LM) optimization method that directly minimizes the associated nonlinear objective[1]. This approach provides a transparent convergence criterion and allows us to assess the extent to which the heuristic output approximates a stationary point of the underlying objective.

Our main findings can be summarized as follows. First, for most countries the LM estimates are numerically close to those produced by the original heuristic, while also satisfying the formal first-order optimality conditions of the underlying objective. Japan is the main case where the LM procedure identifies a different local solution and offers clearer diagnostics, suggesting a more irregular objective landscape. Second, even when the two methods produce noticeable differences in the political economy weights for

[1] See Levenberg (1944); Marquardt (1963); Moré (2006) for foundational references.

a small subset of industries, the implied optimal tariffs remain highly stable across countries and sectors.[2]

Although the two methods generate very similar tariff vectors, examining the underlying political economy weights remains valuable. The weights capture governments' relative preferences across industries, which matter for interpretation and for counterfactual analysis. The optimization-based approach provides transparent diagnostics, reveals when the heuristic is close to a stationary point, and helps clarify how sensitive the equilibrium mapping is to different regions of the parameter space. In this sense, the LM routine complements the original method by offering a structured optimization framework without altering the substantive economic conclusions of Ossa (2014).

The remainder of the paper proceeds as follows. Section 2 reviews the heuristic procedure in Ossa (2014). Section 3 presents the LM formulation and discusses its numerical properties. Section 4 reports the replication results and compares the two approaches. Section 5 concludes. Supplementary tables and figures are provided in the Online Appendix.

2 Ossa (2014)'s Method: Brief Review

Problem. The objective function of government j, denoted G_j, is defined as a weighted sum of welfare across industries W_{js}, where the weights are the political economy parameters λ_{js}. Formally,

$$G_j = \sum_{s=1}^{S} \lambda_{js} W_{js}, \quad s = \{1, \ldots, S\} \tag{1}$$

[2]This stability arises because the mapping from the weights to optimal tariffs is relatively flat in several regions of the parameter space, so distinct configurations of λ_{js} often yield tariff profiles that differ only modestly in levels.

where s denotes industries. Ossa (2014), while not presenting a formal optimization problem, describes a natural approach to identify political economy weights: matching the cross-industry distribution of optimal tariffs to the distribution of factual tariffs, while controlling for the respective means. This matching is implemented by minimizing the residual sum of squares between optimal and noncooperative (or factual) tariffs, subject to the weights being nonnegative and having an average of one. Formally, we interpret this approach as the following optimization problem:

$$\boldsymbol{\lambda}_j = \arg\min_{\boldsymbol{\lambda}_j} \|C(\boldsymbol{\tau}_j^{\text{opt}}(\boldsymbol{\lambda}_j) - \boldsymbol{\tau}_j^{\text{obs}})\|^2 \quad \text{subject to} \quad \lambda_{js} \geq 0, \ \frac{1}{S}\sum_{s=1}^{S} \lambda_{js} = 1 \tag{2}$$

where $C\mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}\mathbf{1}$ is mean-removal operator, $\boldsymbol{\lambda}_j$ is a political economy weight vector $\boldsymbol{\lambda}_j = (\lambda_{j1}, \ldots, \lambda_{jS})$, and $\boldsymbol{\tau}_j$ denotes a vector of tariffs in country j.

Heuristic algorithm. To solve Equation (2), Ossa (2014) employs an iterative heuristic method that adjusts the weight vector $\boldsymbol{\lambda}_j$ to match the pattern of observed tariffs. Industries are partitioned into free-trade and prohibitive sectors[3] using the threshold $\tau_{\text{prohib}} = 2.25$:

$$\mathcal{F} = \{s : \tau_{js}^{\text{obs}} \leq \tau_{\text{prohib}}\}, \quad \mathcal{P} = \{s : \tau_{js}^{\text{obs}} > \tau_{\text{prohib}}\}. \tag{3}$$

For $s \in \mathcal{F}$, the target tariffs are mean-adjusted at each iteration k:[4]

$$\tau_j^{\text{adj}}(\boldsymbol{\lambda}_j^{(k)}) = \tau_j^{\text{obs}} - \bar{\tau}_{\mathcal{F}}^{\text{obs}}\mathbf{1} + \bar{\tau}_{\mathcal{F}}^{\text{opt}}(\boldsymbol{\lambda}_j^{(k)})\mathbf{1} \tag{4}$$

[3] Ossa (2014) imposes an upper bound on prohibitive tariffs to ensure numerical stability and to reflect the economic reality that tariffs above 225% are effectively prohibitive. In Section 3, we apply the same upper-bound restriction in our GN approach.

[4] This transformation removes level differences, focusing the algorithm on matching the relative shape across industries.

where the mean adjustments apply only to free-trade sectors. For $s \in \mathcal{P}$, adjusted targets are set near a fixed upper bound:

$$\tau_{js}^{\text{adj}}(\boldsymbol{\lambda}_j) = UB_s - \varepsilon, \quad UB_s = \max(\tau_{js}^{\text{obs}} + \varepsilon,\ \tau_{\text{prohib}}), \tag{5}$$

with $\varepsilon = 0.03$ in the original codes.

Then, the algorithm computes residuals:

$$\boldsymbol{R}_j(\boldsymbol{\lambda}_j^{(k)}) = \boldsymbol{\tau}_j^{\text{opt}}(\boldsymbol{\lambda}_j^{(k)}) - \boldsymbol{\tau}_j^{\text{adj}}(\boldsymbol{\lambda}_j^{(k)}), \tag{6}$$

and updates weights using an adaptive step rule:

$$\boldsymbol{\lambda}_j^{(k+1)} = S \cdot \frac{\max\left(\boldsymbol{\lambda}_j^{(k)} - \eta_k \cdot \text{sign}(\boldsymbol{R}_j) - \overline{\lambda}_j^{(k)} \cdot \mathbf{1} + \mathbf{1},\ \mathbf{0}\right)}{\sum_{s=1}^{S} \max\left(\lambda_{js}^{(k)} - \eta_k\,\text{sign}(R_{js}) - \overline{\lambda}_j^{(k)} + 1,\ 0\right)} \tag{7}$$

where $\overline{\lambda}_j^{(k)} = \frac{1}{S}\sum_{s=1}^{S} \lambda_{js}^{(k)}$. Step sizes η_k follow the multi-scale schedule in the original code.[5] Iterations stop when residuals fall below a tolerance or after the maximum number of iterations. If the tolerance condition is not satisfied, the iterate that minimizes the residual sum of squares is retained. In the original implementation, country $j = 5$ (Japan) is treated slightly differently: after the first few iterations, the algorithm uses the vector of observed tariffs as the warm-start guess instead of the adjusted targets, which improves numerical stability for this specific case. This modification does not affect the structure of the algorithm but helps avoid oscillatory updates in the early iterations.

The heuristic procedure is computationally efficient and converges quickly, but does not allow verification of first-order optimality or provide diagnostic measures of solution quality.[6] However, because it is not derived from a formal optimization problem, the

[5]The implementation cycles through six step sizes ranging from 5×10^{-5} to 0.1.

[6]In our setting, the LM method was approximately twice as slow as Ossa (2014)'s original codes.

procedure does not allow for verification of first-order optimality or provide diagnostic measures of how close the solution is to a stationary point. As with many iterative correction schemes, the procedure may become trapped in local minima, especially in countries with highly nonlinear tariff responses.

3 Levenberg-Marquardt Approach

Optimization framework. We formulate the estimation of political economy weights[7] as a smooth nonlinear least squares problem:

$$F(\boldsymbol{\lambda}_j) = \|\boldsymbol{r}_j(\boldsymbol{\lambda}_j)\|^2, \tag{8}$$

where the residual vector $\boldsymbol{r}_j(\boldsymbol{\lambda}_j) \in \mathbb{R}^S$ mirrors Ossa (2014)'s matching scheme.[8] For $s \in \mathcal{F}$, residuals remove level differences:

$$r_{js}(\boldsymbol{\lambda}_j) = \tau_{js}^{\text{opt}}(\boldsymbol{\lambda}_j) - \left(\tau_{js}^{\text{obs}} - \overline{\tau}_{\mathcal{F}}^{\text{obs}} + \overline{\tau}_{\mathcal{F}}^{\text{opt}}(\boldsymbol{\lambda}_j)\right). \tag{9}$$

For $s \in \mathcal{P}$, we penalize violations of the upper bound UB_s using a smooth hinge:

$$r_{js}(\boldsymbol{\lambda}_j) = g_\alpha\left(\tau_{js}^{\text{opt}}(\boldsymbol{\lambda}_j) - UB_s\right), \quad g_\alpha(x) = \alpha \ln\left(1 + \exp\left(\frac{x}{\alpha}\right)\right) \tag{10}$$

where UB_s follows the same definition as in Equation (5) and $\varepsilon = 0.03$ like the original codes. And $\alpha > 0$ is a small smoothing parameter.[9] Weights satisfy the simplex

[7] We replace Ossa (2014)'s original `mylambdaj.m` file with our `estimate_lambda_lm.m` file for estimating $\boldsymbol{\lambda}_j$, while retaining all other components of the original codes unchanged.

[8] We use $\boldsymbol{R}_j$ to denote residuals in the heuristic algorithm and $\boldsymbol{r}_j$ for the continuous residual vector used in the GN formulation.

[9] As $\alpha \to 0$, $g_\alpha(x) \to \max(x, 0)$. In implementation, α anneals from 5×10^{-2} to 10^{-3} during optimization (1.0 to 10^{-4} for Japan).

constraint:

$$\Omega = \{\boldsymbol{\lambda}_j : \boldsymbol{\lambda}_j \geq \mathbf{0},\ \mathbf{1}'\boldsymbol{\lambda}_j = S\}. \tag{11}$$

Jacobian approximation. Following the structure of the residual definition, the gradient of the objective can be expressed as:

$$\nabla F(\boldsymbol{\lambda}_j) = \left(C\,\mathbf{J}_\tau(\boldsymbol{\lambda}_j)\right)^\top \boldsymbol{r}_j(\boldsymbol{\lambda}_j), \tag{12}$$

where C is the centering matrix introduced in Equation (2) and $\mathbf{J}_\tau(\boldsymbol{\lambda}_j)$ denotes the Jacobian of the optimal tariff vector $\boldsymbol{\tau}^{\text{opt}}(\boldsymbol{\lambda}_j)$ with respect to the weight vector $\boldsymbol{\lambda}_j$. Since $\boldsymbol{\tau}^{\text{opt}}(\boldsymbol{\lambda}_j)$ is obtained from an inner nonlinear optimization routine, analytic derivatives are unavailable. We therefore approximate $\mathbf{J}_\tau$ using central finite differences:

$$\mathbf{J}_{\tau,:,s} \approx \frac{\boldsymbol{\tau}^{\text{opt}}(\boldsymbol{\lambda}_j + \boldsymbol{\delta}_s) - \boldsymbol{\tau}^{\text{opt}}(\boldsymbol{\lambda}_j - \boldsymbol{\delta}_s)}{2h}, \tag{13}$$

where $\boldsymbol{\delta}_s$ is a zero-sum perturbation that preserves feasibility on the simplex constraint Ω of Equation (11), and h is the finite-difference step size.[10]

To reduce computational cost, we adopt a quasi-Newton approach: the Jacobian is recomputed via finite differences every 25 iterations, and updated using Broyden's rank-one formula (Broyden, 1965) in between:

$$\mathbf{J}^{(k+1)} = \mathbf{J}^{(k)} + \frac{(\boldsymbol{y}^{(k)} - \mathbf{J}^{(k)}\boldsymbol{s}^{(k)})(\boldsymbol{s}^{(k)})^\top}{(\boldsymbol{s}^{(k)})^\top \boldsymbol{s}^{(k)}}, \tag{14}$$

where $\boldsymbol{s}^{(k)} = \boldsymbol{\lambda}_j^{(k+1)} - \boldsymbol{\lambda}_j^{(k)}$ and $\boldsymbol{y}^{(k)} = \boldsymbol{\tau}^{\text{opt}}(\boldsymbol{\lambda}_j^{(k+1)}) - \boldsymbol{\tau}^{\text{opt}}(\boldsymbol{\lambda}_j^{(k)})$. This significantly reduces the number of expensive inner optimizations.

[10] In practice we set $h \in [5 \times 10^{-4}, 7 \times 10^{-4}]$.

Direction and update. Let $\mathbf{J} = C\,\mathbf{J}_\tau(\boldsymbol{\lambda}_j)$ denote the Jacobian of the residual vector $\boldsymbol{r}_j(\boldsymbol{\lambda}_j)$ with respect to the weights. Search directions are obtained by solving the damped GN system:

$$(\mathbf{J}^\top \mathbf{J} + \gamma I)\boldsymbol{d} = -\mathbf{J}^\top \boldsymbol{r}_j(\boldsymbol{\lambda}_j), \tag{15}$$

with adaptive damping γ. To preserve positivity, updates use the exponential map:

$$\boldsymbol{\lambda}_j^{(k+1)} = S \cdot \frac{\boldsymbol{\lambda}_j^{(k)} \odot \exp(\alpha_k \boldsymbol{d}^{(k)})}{\mathbf{1}'\big(\boldsymbol{\lambda}_j^{(k)} \odot \exp(\alpha_k \boldsymbol{d}^{(k)})\big)}. \tag{16}$$

The step size α_k is selected by a standard Armijo backtracking rule (Nocedal and Wright, 2006).

Convergence and initialization. Iterations terminate when the gradient norm falls below a tolerance $\|\nabla F(\boldsymbol{\lambda}_j)\|_\infty < \text{tol}$ (typically 10^{-4}), or when relative progress becomes negligible. The damping parameter γ adapts during iterations: it decreases by a factor of 10 after each successful step and increases when the line search fails, ensuring robust convergence across different problem structures.

Since $F(\boldsymbol{\lambda}_j)$ is non-convex, we employ a multi-start strategy.[11] The algorithm first attempts optimization from a uniform initial point $\boldsymbol{\lambda}_j^{(0)} = \mathbf{1}$. If the resulting gradient norm exceeds a threshold (e.g., 0.01), additional starting points are tried, including:[12] tariff-proportional weights ($\boldsymbol{\lambda} \propto \boldsymbol{\tau}^{\text{obs}}$), power-transformed weights ($\boldsymbol{\lambda} \propto (\boldsymbol{\tau}^{\text{obs}})^\alpha$ for $\alpha \in \{0.5, 2\}$), inverse weights, entropy-based perturbations, and random draws from uniform and

[11] All initializations are normalized to satisfy $\mathbf{1}^\top \boldsymbol{\lambda}_j = S$. And, the solution with the smallest gradient norm is retained.

[12] To ensure pure replication, we intentionally exclude Ossa (2014)'s estimates from the initial guesses, providing a stricter test of independent convergence. From an optimization standpoint, incorporating the original heuristic solution as an initial guess would, by definition, yield an objective value that is at least as good as the original result, since any well-behaved local search algorithm can only improve upon or reproduce a feasible starting point. Our exclusion therefore provides a stricter test of whether the algorithm can locate comparable optima on its own.

Dirichlet distributions.

Compared to Ossa (2014)'s heuristic, both approaches minimize the same mean-adjusted discrepancy between optimal and observed tariffs. The original method relies on sign-based corrections and ad-hoc step sizes, whereas our LM approach uses gradient information, quasi-Newton updates, and principled line search. Although computationally more expensive due to Jacobian evaluations, this formulation provides a rigorous optimization framework that enables verification of first-order optimality conditions and complements the fast heuristic.

4 Replication Results

This section compares the political economy weights and implied optimal tariffs obtained from the original heuristic procedure of Ossa (2014) with those generated by the LM method. Using the publicly available codes and data provided in the replication package of Ossa (2014), we first verify that our implementation reproduces the results reported in the original paper. We then apply the LM approach to the same dataset in order to evaluate how the two procedures differ in terms of numerical performance and the resulting tariff profiles. To assess numerical optimality on a common basis, we compute the infinity norm of the gradient of the LM objective, $\|\nabla F(\boldsymbol{\lambda})\|_\infty$, for both sets of weights.[13] Detailed numerical results and additional figures are provided in Appendix A.

Table 1 compares the residual sum of squares (RSS), gradient diagnostics, and correlations between the original estimates of Ossa (2014) and those obtained from the LM procedure. For all countries except Japan, the LM algorithm weakly improves or substantially reduces the RSS relative to the original implementation. Japan is the only case in which the LM procedure converges to a different local solution with a noticeably

[13] Since the algorithm in Ossa (2014) does not specify a formal optimization criterion, this gradient evaluation serves as an ex-post diagnostic of how closely its output satisfies the first-order conditions associated with the LM objective.

Table 1: First-Order Optimality and Correlations of Estimates

Country	RSS^{O}	RSS^{LM}	$\lVert\nabla F(\boldsymbol{\lambda}_j^{O})\rVert_\infty$	$\lVert\nabla F(\boldsymbol{\lambda}_j^{LM})\rVert_\infty$	$\rho_{\boldsymbol{\lambda}}$	$\rho_{\boldsymbol{\tau}}$
Brazil	0.000	0.000	0.010	0.000	1.000	0.999
China	0.003	0.000	0.044	0.086	1.000	1.000
EU	0.018	0.017	0.029	0.046	0.999	0.999
India	0.003	0.000	0.069	0.391	0.999	0.999
Japan	0.343	4.360	0.583	0.456	0.372	0.976
ROW	0.000	0.000	0.006	0.002	1.000	1.000
US	0.001	0.000	0.015	0.001	1.000	0.999

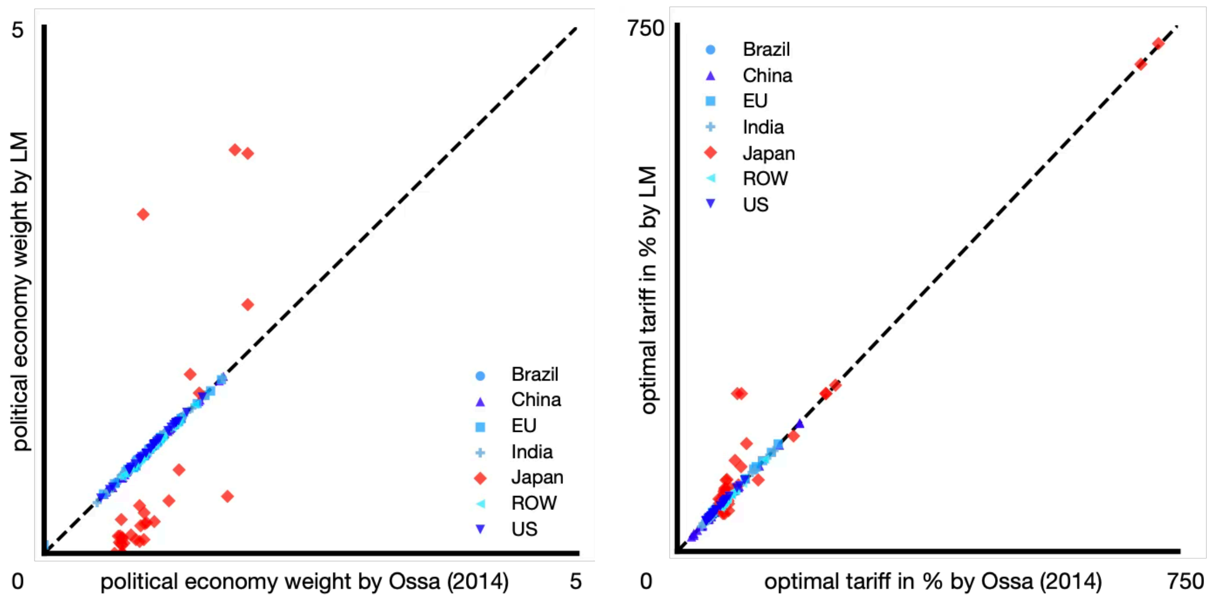
Notes: Data refer to the year 2007 with $S = 33$ industries. $\boldsymbol{\lambda}_j^{O}$ and $\boldsymbol{\lambda}_j^{LM}$ denote the political economy weights for country j obtained from Ossa (2014) and from the proposed Levenberg-Marquardt method, respectively. $\rho_{\boldsymbol{\lambda}}$ and $\rho_{\boldsymbol{\tau}}$ are the correlations between the estimates of weights and optimal tariffs. Gradient norms are evaluated with respect to the Levenberg-Marquardt objective defined in Equation (8), i.e., $\lVert\nabla F(\boldsymbol{\lambda})\rVert_\infty$. ROW denotes the rest of the world.

larger RSS (4.36 versus 0.34). The gradient diagnostics present a more mixed picture. For Brazil, the United States, and the rest of the world, the LM solution delivers very small infinity norms of the gradient ($\lVert\nabla F(\boldsymbol{\lambda}^{LM} j)\rVert_\infty$ between 0 and 2×10^{-3}), indicating that the LM estimates lie close to satisfying the first-order optimality conditions for the LM objective. In contrast, for China, the EU, and India the LM procedure drives the RSS essentially to zero while producing somewhat larger gradient norms, and for Japan it lowers the gradient norm (from 0.583 to 0.456) despite yielding a worse fit.[14]

Despite these numerical differences in the weight vectors, the implied optimal tariffs are highly robust across methods. The correlations between the tariffs obtained from the original procedure and those generated by the LM algorithm exceed 0.99 for all countries except Japan, where the correlation remains high at 0.98. Hence, even when the LM procedure identifies a distinct local optimum in political economy weights, it yields tariff schedules that are nearly indistinguishable from those implied by the original estimates.

A comparison of Tables A1–A2 shows that the numerical discrepancies between the original and LM procedures are highly concentrated in a small subset of industries, and

[14] At boundary solutions, where one or more weights reach the lower bound $\lambda_{js} = 0$, the projected gradient may remain nonzero even when the RSS is minimized, because the unconstrained gradient points outside the feasible simplex.



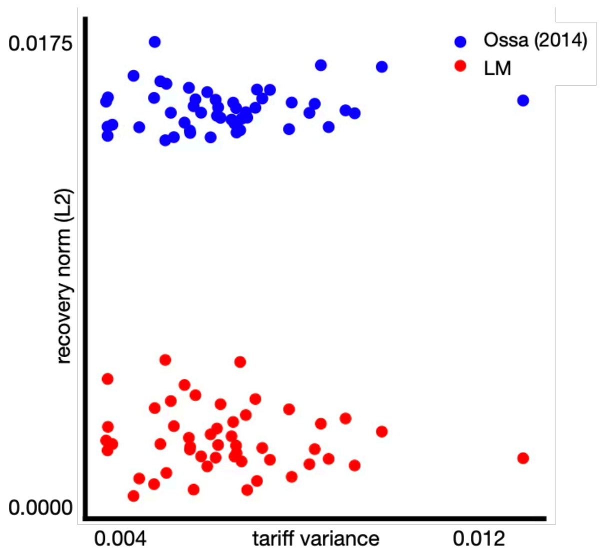
(a) Political Economy Weights (b) Optimal Tariffs

Figure 1: Comparison of Estimates

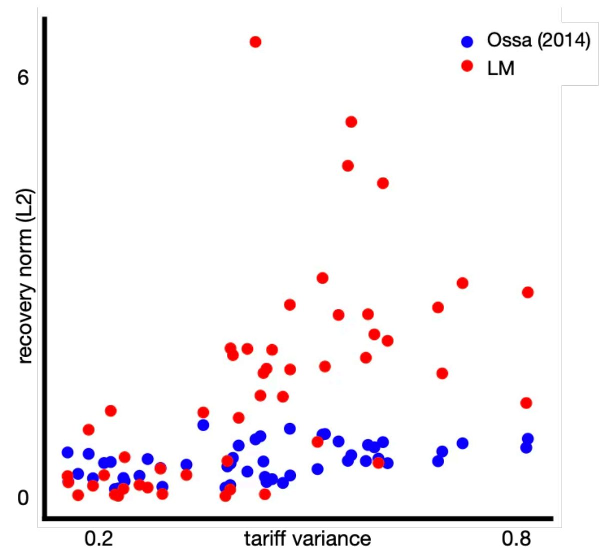
largely concentrated in Japan. For most countries, the political economy weights remain close across methods, with only modest deviations in individual sectors. In contrast, Japan exhibits pronounced differences in several agriculture- and food-related industries, where the LM algorithm either pushes weights toward zero or generates substantially larger values. These discrepancies reflect the presence of multiple local optima and binding constraints in Japan's objective landscape, which amplify small changes in the optimization path.

Despite these substantial differences in the weight vectors, the resulting optimal tariffs reported in Tables A3–A4 are remarkably stable. For all countries expect for India and Japan, the LM and original tariffs align extremely closely across all industries, both in terms of levels and relative magnitudes. Even for Japan and India, the implied tariffs differ only in a limited set of sectors, and the overall tariff profiles remain similar in order of magnitude. These results indicate that the mapping from political-economy weights to optimal tariffs is robust, and that the economic implications of Ossa (2014) are not sensitive to moderate perturbations in the underlying solutions.

Simulations. To assess the performance of both methods under varying degrees of structural deviation from observed data, we conduct two Monte Carlo experiments using Country 1 as a test case. In Scenario 1 (Realistic structure), we generate true λ vectors by adding small Gaussian noise (±5%) to the baseline Ossa (2014)'s estimate, thereby preserving the general shape and variance structure of empirically plausible weights. In Scenario 2 (Unrealistic structure), we generate λ vectors from a uniform distribution over $[0.5, 9.5]$, normalized to unit mean, which produces near-zero correlation with the baseline structure. For each scenario, we run 50 trials: generating a true λ_j^{true}, computing the corresponding optimal tariff via the general equilibrium model, and then recovering λ_j using both methods. We measure recovery accuracy using the L2 norm, $\left\|\hat{\lambda}_j - \lambda_j^{\text{true}}\right\|_2$.



(a) Realistic



(b) Unrealistic

Figure 2: Simulation Results by Method

Figure 2 displays the relationship between implied optimal tariff (τ^{opt}) variance and recovery error across both scenarios. In Scenario 1, the LM approach consistently outperforms the heuristic (mean norm: 0.003 for the LM vs. 0.015 for the original), reflecting its ability to refine solutions when the underlying structure resembles observed patterns. However, in Scenario 2, the heuristic proves more robust: while LM occasionally achieves very low errors, it also exhibits large failures (norms exceeding 6.0 in some trials), whereas Ossa (2014)'s original procedure maintains moderate errors across all trials (mean norm: 1.564 for the LM vs. 0.498 for the original). Interestingly, the original heuristic appears better suited to extreme cases, likely because our current LM implementation prioritizes computational efficiency through Broyden (1965)'s updates and a limited number of multistart points, sacrificing robustness in highly irregular landscapes. In practice, practitioners may wish to combine both approaches by using Ossa (2014)'s estimate as an additional initialization point in the multistart procedure, thereby achieving both efficiency and improved accuracy without the computational burden of exhaustive global search.

5 Conclusion

This paper replicates and extends the estimation of political economy weights in Ossa (2014) by recasting the problem as a smooth nonlinear least squares exercise and implementing an LM optimization routine. The original heuristic performs well for most countries, while the LM framework introduces a formal optimality criterion and clearer numerical diagnostics, particularly in nonlinear cases such as Japan. Although the two procedures sometimes generate different industry-level weights, the implied optimal tariffs remain highly stable across methods, since sizable perturbations in weights translate into only modest changes in tariff levels. Simulations further show that the LM approach performs better when the underlying structure resembles empirical patterns, whereas the heuristic remains more robust in highly irregular settings. Taken together, these results suggest that the substantive conclusions of Ossa (2014) are shaped mainly by the structure of the general equilibrium system rather than the numerical algorithm, and that the LM framework serves as a useful complement for understanding identification and robustness.

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A Additional Tables and Figures

Table A1: Political Economy Weights from the Original Codes

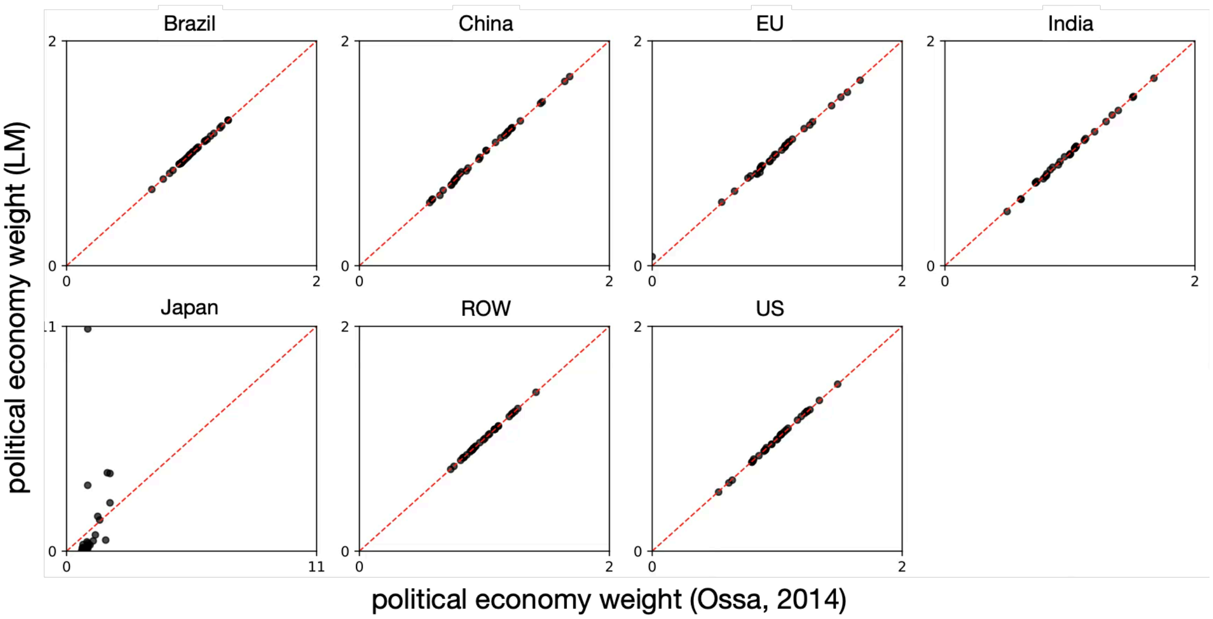
	λ^O_{BRA}	λ^O_{CHN}	λ^O_{EU}	λ^O_{IND}	λ^O_{JPN}	λ^O_{ROW}	λ^O_{US}
Rice	1.23	1.64	1.51	1.38	1.91	1.23	1.19
Wheat	1.29	1.68	1.66	1.67	1.91	1.27	1.24
Other cereal grains	0.98	1.22	1.26	1.04	1.72	1.41	0.91
Vegetables, etc.	0.82	0.96	0.83	1.01	0.90	0.90	0.80
Oil seeds	0.97	1.01	0.87	1.20	1.79	1.20	1.04
Sugar	0.92	1.22	1.10	1.34	1.27	1.00	0.85
Plant-based fibers	0.96	0.58	0.77	0.82	0.73	0.90	0.81
Other crops	0.92	0.64	0.87	1.12	0.74	0.93	0.80
Bovine cattle, etc.	0.92	0.56	1.07	0.92	0.94	0.91	0.81
Other animal prod.	0.68	0.59	0.56	0.50	0.72	0.73	0.53
Dairy	1.24	1.29	1.56	1.29	1.46	1.25	1.34
Wool, etc.	0.91	0.85	0.00	0.86	1.37	0.91	0.64
Forestry	0.77	0.78	0.66	0.61	0.70	0.76	0.61
Bovine meat prod.	1.05	1.16	1.44	1.05	0.95	1.11	1.07
Other meat prod.	1.03	1.09	1.12	1.04	0.91	1.11	0.96
Vegetable oils, etc.	1.13	1.19	1.09	1.50	0.96	1.08	1.03
Other food prod.	0.90	1.01	0.98	1.05	0.93	0.96	0.89
Beverages, tobacco	1.01	1.45	1.28	1.51	0.93	1.09	1.24
Textiles	1.04	1.19	0.96	0.91	0.82	0.99	1.26
Wearing apparel	1.29	1.46	1.22	1.13	1.03	1.22	1.48
Leather prod.	1.18	1.13	1.06	0.97	1.17	1.08	1.22
Wood prod.	0.90	0.77	0.79	0.72	0.71	0.83	0.90
Paper prod.	0.93	0.67	0.88	0.81	0.73	0.86	0.91
Chemical prod.	0.85	0.74	0.84	0.73	0.71	0.81	0.96
Other mineral prod.	0.92	0.87	0.87	0.80	0.72	0.89	1.02
Ferrous metals	1.10	0.73	1.04	1.04	0.90	1.03	1.09
Other metals	1.11	0.80	1.06	1.05	0.94	1.08	1.16
Metal prod.	1.01	0.96	0.94	0.84	0.75	0.91	1.06
Motor vehicles, etc.	0.99	1.18	0.99	0.96	0.73	0.92	0.90
Other transport eq.	0.91	0.76	0.94	0.74	0.73	0.90	1.03
Electronic eq.	0.91	0.82	0.87	0.61	0.66	0.84	1.00
Other machinery	0.94	0.76	0.86	0.79	0.73	0.83	1.00
Other manufactures	1.15	1.22	1.06	1.00	0.86	1.04	1.23
Mean	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Notes: λ^O_j denote the political economy weights computed using the original codes and data from Ossa (2014) for country j. The tariff data correspond to the year 2007. ROW refers to the rest of the world.

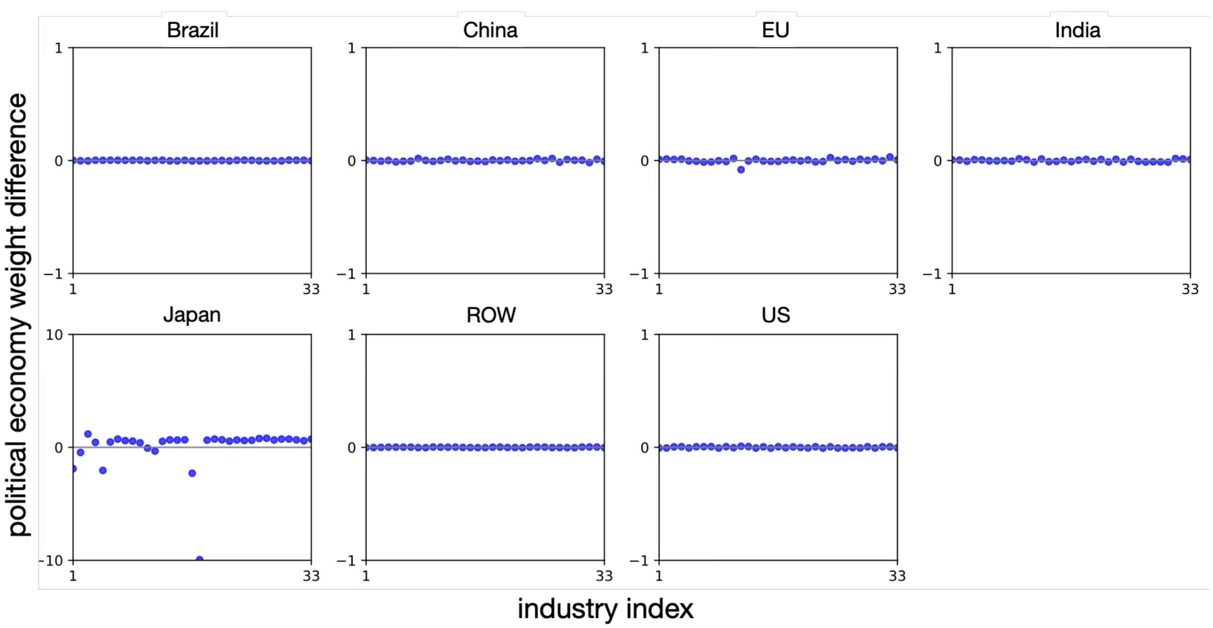
Table A2: Political Economy Weights from the Levenberg-Marquardt Method

	$\lambda^{LM}_{\text{BRA}}$	$\lambda^{LM}_{\text{CHN}}$	λ^{LM}_{EU}	$\lambda^{LM}_{\text{IND}}$	$\lambda^{LM}_{\text{JPN}}$	$\lambda^{LM}_{\text{ROW}}$	λ^{LM}_{US}
Rice	1.23	1.64	1.50	1.38	3.80	1.23	1.20
Wheat	1.29	1.68	1.65	1.67	2.36	1.27	1.24
Other cereal grains	0.98	1.22	1.25	1.04	0.54	1.41	0.90
Vegetables, etc.	0.82	0.96	0.82	1.00	0.45	0.90	0.79
Oil seeds	0.97	1.03	0.87	1.19	3.83	1.20	1.04
Sugar	0.92	1.23	1.10	1.34	0.79	1.00	0.85
Plant-based fibers	0.96	0.58	0.78	0.82	0.00	0.90	0.80
Other crops	0.92	0.63	0.89	1.12	0.14	0.94	0.79
Bovine cattle, etc.	0.92	0.56	1.07	0.93	0.39	0.92	0.82
Other animal prod.	0.68	0.59	0.57	0.48	0.32	0.73	0.53
Dairy	1.24	1.29	1.54	1.28	1.52	1.24	1.34
Wool, etc.	0.91	0.84	0.08	0.88	1.70	0.91	0.63
Forestry	0.77	0.79	0.66	0.59	0.16	0.75	0.61
Bovine meat prod.	1.06	1.16	1.42	1.06	0.28	1.11	1.08
Other meat prod.	1.04	1.10	1.13	1.05	0.26	1.11	0.95
Vegetable oils, etc.	1.12	1.20	1.09	1.50	0.29	1.08	1.04
Other food prod.	0.90	1.02	0.99	1.05	3.22	0.97	0.89
Beverages, tobacco	1.01	1.44	1.28	1.51	10.88	1.09	1.25
Textiles	1.05	1.19	0.96	0.90	0.17	0.99	1.26
Wearing apparel	1.30	1.46	1.22	1.13	0.30	1.22	1.49
Leather prod.	1.18	1.14	1.06	0.99	0.50	1.08	1.23
Wood prod.	0.90	0.77	0.80	0.74	0.17	0.83	0.90
Paper prod.	0.93	0.67	0.89	0.80	0.08	0.86	0.92
Chemical prod.	0.85	0.72	0.81	0.74	0.10	0.81	0.95
Other mineral prod.	0.92	0.87	0.86	0.79	0.09	0.88	1.03
Ferrous metals	1.11	0.72	1.03	1.04	0.11	1.03	1.09
Other metals	1.11	0.82	1.07	1.06	0.13	1.08	1.17
Metal prod.	1.02	0.95	0.93	0.85	0.10	0.92	1.06
Motor vehicles, etc.	0.99	1.18	0.99	0.97	0.00	0.93	0.90
Other transport eq.	0.91	0.76	0.93	0.75	0.00	0.90	1.04
Electronic eq.	0.91	0.83	0.87	0.59	0.00	0.84	0.99
Other machinery	0.94	0.75	0.83	0.77	0.15	0.83	1.00
Other manufactures	1.15	1.22	1.05	0.99	0.13	1.04	1.24
Mean	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Notes: λ^{LM}_j denote the political economy weights computed using the Levenberg-Marquardt method for country j. The tariff data correspond to the year 2007. ROW denotes the rest of the world.



(a) Political Economy Weight Comparison



(b) Political Economy Weight ($\lambda_j^O - \lambda_j^{LM}$) by Industry

Figure A1: Comparison of Political Economy Weight Across Methods

Table A3: Optimal Tariffs in % from the Original Codes

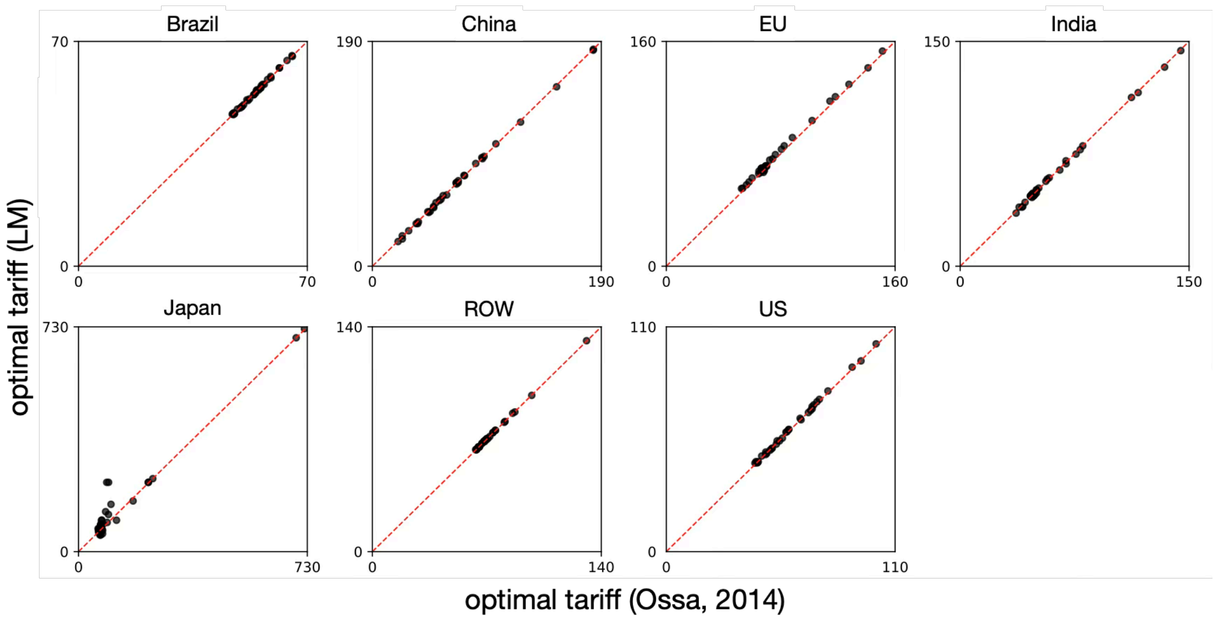
	$\tau^{\text{opt},O}_{\text{BRA}}$	$\tau^{\text{opt},O}_{\text{CHN}}$	$\tau^{\text{opt},O}_{\text{EU}}$	$\tau^{\text{opt},O}_{\text{IND}}$	$\tau^{\text{opt},O}_{\text{JPN}}$	$\tau^{\text{opt},O}_{\text{ROW}}$	$\tau^{\text{opt},O}_{\text{US}}$
Rice	47.39	183.39	118.20	75.91	694.45	69.53	45.85
Wheat	47.52	183.47	151.14	134.00	237.00	68.38	43.10
Other cereal grains	47.02	90.83	101.96	56.12	103.70	131.01	44.23
Vegetables, etc.	49.50	69.89	66.13	69.41	95.55	71.82	52.89
Oil seeds	47.74	58.69	53.49	78.66	720.73	97.48	58.70
Sugar	52.31	102.50	88.15	112.26	174.11	81.17	50.59
Plant-based fibers	53.59	30.12	52.56	42.53	64.66	63.49	43.89
Other crops	55.07	37.66	66.60	80.35	69.99	73.77	50.97
Bovine cattle, etc.	47.65	21.31	80.48	50.24	91.51	64.74	42.61
Other animal prod.	51.83	45.97	57.82	41.03	86.51	69.90	47.77
Dairy	54.38	76.57	141.04	65.29	222.49	80.67	70.07
Wool, etc.	50.01	51.00	66.32	47.61	222.97	63.93	48.26
Forestry	50.51	55.71	56.05	40.53	74.39	63.78	43.69
Bovine meat prod.	47.25	71.28	127.71	49.84	76.12	75.30	53.27
Other meat prod.	54.60	69.74	82.43	58.35	79.01	85.82	49.88
Vegetable oils, etc.	54.19	76.00	66.32	116.65	76.72	68.79	47.78
Other food prod.	51.50	71.33	76.24	69.31	96.00	75.15	55.81
Beverages, tobacco	56.32	152.96	114.41	144.60	89.91	87.16	89.33
Textiles	61.46	92.72	69.98	49.98	73.10	70.76	93.59
Wearing apparel	65.32	123.05	72.29	46.30	76.76	73.69	100.84
Leather prod.	65.43	69.51	68.85	47.05	121.21	70.09	73.57
Wood prod.	58.80	50.74	64.76	46.03	73.15	66.98	64.77
Paper prod.	55.61	36.37	66.76	47.71	68.17	65.26	58.93
Chemical prod.	53.69	48.06	68.59	46.01	71.17	65.59	68.29
Other mineral prod.	56.82	57.00	68.21	48.49	69.03	68.67	72.42
Ferrous metals	55.91	24.88	64.63	51.51	70.84	64.91	57.51
Other metals	48.62	24.78	60.01	46.87	68.35	63.19	57.97
Metal prod.	61.44	61.70	70.18	47.12	67.29	67.05	70.93
Motor vehicles, etc.	57.83	91.45	74.32	57.01	63.93	68.26	54.44
Other transport eq.	49.97	38.19	67.58	38.61	63.55	65.16	64.32
Electronic eq.	55.82	52.75	69.34	36.63	62.43	63.78	69.35
Other machinery	58.89	46.76	67.92	48.24	71.58	65.54	70.13
Other manufactures	63.80	85.83	69.51	49.08	67.41	68.12	77.70
Mean	54.18	71.28	78.48	61.80	132.24	72.70	60.41

Notes: $\tau^{\text{opt},O}_{j}$ denote the optimal tariffs computed using the original codes and data from Ossa (2014) for country j. The tariff data correspond to the year 2007. ROW refers to the rest of the world.

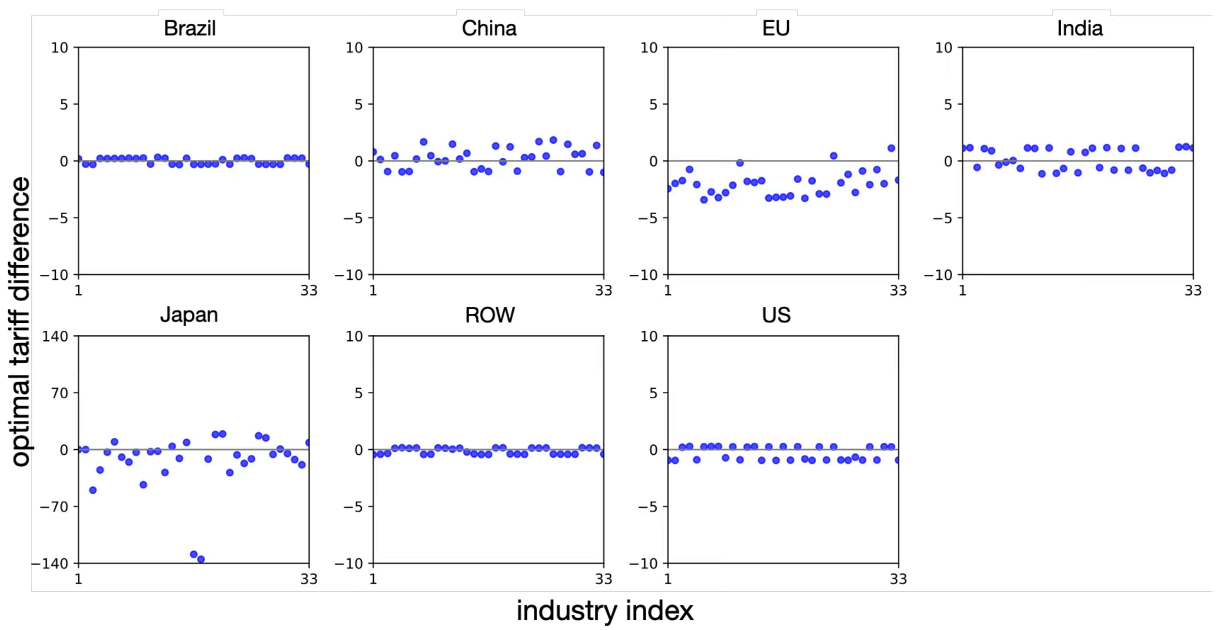
Table A4: Optimal Tariffs in % from the Levenberg-Marquardt Method

	$\tau_{\text{BRA}}^{\text{opt},LM}$	$\tau_{\text{CHN}}^{\text{opt},LM}$	$\tau_{\text{EU}}^{\text{opt},LM}$	$\tau_{\text{IND}}^{\text{opt},LM}$	$\tau_{\text{JPN}}^{\text{opt},LM}$	$\tau_{\text{ROW}}^{\text{opt},LM}$	$\tau_{\text{US}}^{\text{opt},LM}$
Rice	47.20	182.59	120.64	74.79	694.45	69.98	46.79
Wheat	47.80	183.34	153.12	132.84	237.00	68.79	44.06
Other cereal grains	47.32	91.77	103.69	56.68	153.69	131.35	44.03
Vegetables, etc.	49.29	69.43	66.88	68.33	120.74	71.71	52.61
Oil seeds	47.53	59.65	55.56	77.77	723.90	97.32	59.60
Sugar	52.10	103.44	91.58	112.61	164.57	81.06	50.34
Plant-based fibers	53.38	29.96	55.28	42.64	74.03	63.35	43.62
Other crops	54.83	35.99	69.82	80.31	85.38	74.20	50.69
Bovine cattle, etc.	47.44	20.85	83.27	50.89	94.84	65.15	43.33
Other animal prod.	51.58	46.04	59.96	39.88	129.82	69.75	47.52
Dairy	54.66	76.57	141.21	64.18	225.00	80.54	70.98
Wool, etc.	49.69	49.53	68.12	48.75	225.00	63.90	48.04
Forestry	50.28	55.54	57.94	39.39	102.62	63.66	43.42
Bovine meat prod.	47.54	70.60	129.46	50.93	72.14	75.51	54.22
Other meat prod.	54.93	70.69	85.70	59.01	89.91	86.20	49.63
Vegetable oils, etc.	53.96	76.71	69.53	115.85	67.93	69.22	48.74
Other food prod.	51.81	72.26	79.43	70.35	225.00	75.58	55.55
Beverages, tobacco	56.62	151.65	117.49	143.86	225.00	87.01	90.26
Textiles	61.75	92.79	71.58	48.86	84.76	70.60	93.33
Wearing apparel	65.58	121.81	75.58	46.90	58.15	74.07	101.67
Leather prod.	65.32	70.41	70.61	45.88	101.97	70.49	74.51
Wood prod.	59.09	50.45	67.66	46.84	101.51	67.40	64.53
Paper prod.	55.38	36.02	69.67	46.61	74.71	65.12	59.84
Chemical prod.	53.42	46.35	68.14	46.83	88.14	65.46	68.05
Other mineral prod.	56.62	56.57	70.13	47.36	80.43	68.53	73.33
Ferrous metals	56.19	23.04	65.81	52.14	53.86	65.31	58.45
Other metals	48.92	25.72	62.79	47.92	53.94	63.60	58.64
Metal prod.	61.74	60.24	71.07	47.96	73.21	67.47	71.85
Motor vehicles, etc.	58.14	90.87	76.40	58.11	63.27	68.67	54.20
Other transport eq.	49.70	37.56	68.35	39.41	68.44	65.00	65.24
Electronic eq.	55.57	53.71	71.35	35.42	74.77	63.63	69.09
Other machinery	58.65	45.40	66.80	46.98	90.32	65.41	69.89
Other manufactures	64.07	86.84	71.20	47.95	58.86	68.52	78.63
Mean	54.18	71.04	80.48	61.64	146.59	72.84	60.75

Notes: $\tau_j^{\text{opt},LM}$ denote the optimal tariffs computed using the Levenberg-Marquardt method for country j. The tariff data correspond to the year 2007. ROW refers to the rest of the world.

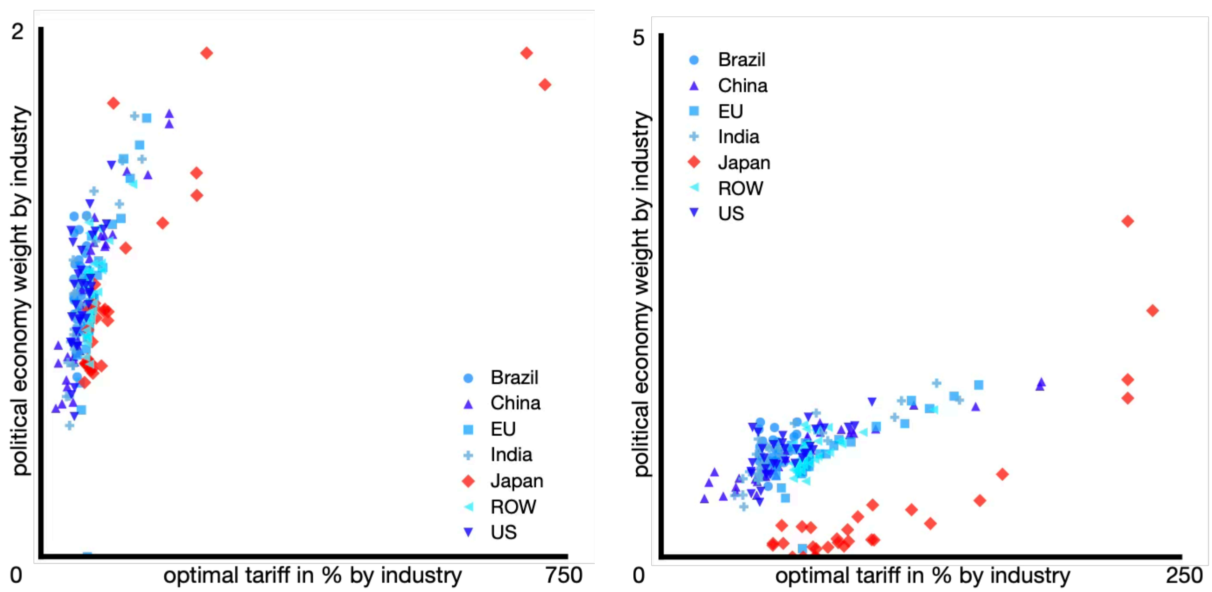


(a) Tariff Comparison



(b) Optimal Tariffs Differences ($\tau_j^{\text{opt},O} - \tau_j^{\text{opt},LM}$) by Industry

Figure A2: Comparison of Optimal Tariff Across Methods



(a) Original Method

(b) LM Method

Figure A3: Political Economy Weights and Optimal Tariffs by Method