$$z = -\log_e\left(\frac{1}{y}-1\right) \qquad e^{-z} = \frac{1}{y}-1 \qquad \cancel{e^{z} = \frac{1-y}{y}} \qquad h_\theta(x) = \frac{1}{1+e^{-\theta^T x}} = \frac{1}{1+e^{-z}}$$

$$e^{-z}+1 = \frac{1}{y} \qquad y = \frac{1}{e^{-z}+1}$$

$$z = \theta^T X$$

$$z = w_0 x_1 + \cdots + w_n x_n$$

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log h_\theta(x^{(i)}) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right] \qquad \frac{e^{-\theta^T x}}{1+e^{-\theta^T x}}$$

$$y^{(i)}\log\left(\frac{1}{1+e^{-\theta^T x}}\right) = -y^{(i)}\log(1+e^{-\theta x})$$

$$(1-y^{(i)})\cdot\left[\log(e^{-\theta^T x}) - \log(1+e^{-\theta^T x})\right] = (1-y^{(i)})\left[-\theta x - \log(1+e^{-\theta x})\right]$$

$$-\frac{1}{m}\sum_{i=1}^{m}\left[\cancel{-y^{(i)}\log(1+e^{-\theta x})} - \theta x - \log(1+e^{-\theta x}) + y^{(i)}\theta x + \cancel{y^{(i)}\log(1+e^{-\theta x})}\right]$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\theta x - (\theta x + \log(1+e^{-\theta x}))\right] \qquad \theta x = \log_e e^{\theta x}$$

$$\log_e e^{\theta x} + \log_e(1+e^{-\theta x})$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\theta x - \log(e^{\theta x}+1)\right] \qquad = \log_e(e^{\theta x}+1)$$

$$\therefore J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\theta x^{(i)} - \ln(e^{\theta x^{(i)}}+1)\right].$$

Cost function 미분 (Derivation)

$$w_0 x_1 + w_1 x_1 + \cdots + w_n x_n$$

$$\frac{\partial}{\partial\theta_j}\cdot(y^{(i)}\cdot\theta x^{(i)}) = y^{(i)}\cdot x_j^{(i)}, \qquad \frac{\partial}{\partial\theta_j}\cdot\left(\ln(e^{\theta x^{(i)}}+1)\right) = \frac{x_j^{(i)} e^{\theta x^{(i)}}}{e^{\theta x^{(i)}}+1}$$

$$= x_j^{(i)}\cdot\left(\frac{1}{1+e^{-\theta x^{(i)}}}\right) = x_j^{(i)}\cdot h_\theta(x^{(i)})$$

$$\therefore \frac{\partial J(\theta)}{\partial\theta_j} = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\cdot x_j^{(i)} - x_j^{(i)}\cdot h_\theta(x^{(i)})\right]$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[x_j^{(i)}\left(h_\theta(x^{(i)}) - y^{(i)}\right)\right]$$

weight update.

$$\theta_j := \theta_j - \alpha \frac{\partial J(\theta)}{\partial \theta_j}$$ 모든 θ 동시에 update.

$$:= \theta_j - \alpha \sum_{i=1}^{m} \left(h_{(\theta)}(x^{(i)}) - y^{(i)} \right) \cdot x_j^{(i)}$$