# ITIS 6260/8260 Quantum Computing

## Lecture 1: A Layman's Guide

Yongge Wang

UNC Charlotte, USA

January 14, 2019

# Outline

# Outline

1. History
2. Qubit
3. Qubit Measurements

# Outline

# What is a quantum computer?

- A quantum computer is a machine that performs calculations based on the laws of quantum mechanics, which is the behavior of particles at the sub-atomic level.

# Why quantum mechanics?

- In early 20th century, the theories of physics at that time were predicting absurdities such as the existence of an 'ultraviolet catastrophe' involving infinite energies (*an ideal black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases*), or electrons spiraling inexorably into the atomic nucleus
- This resulted in the creation of the modern theory of quantum mechanics (1920s)
- What is quantum mechanics? Quantum mechanics is a mathematical framework or set of rules for the construction of physical theories

# Why quantum mechanics?

- In early 20th century, the theories of physics at that time were predicting absurdities such as the existence of an 'ultraviolet catastrophe' involving infinite energies (*an ideal black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases*), or electrons spiraling inexorably into the atomic nucleus
- This resulted in the creation of the modern theory of quantum mechanics (1920s)
- What is quantum mechanics? Quantum mechanics is a mathematical framework or set of rules for the construction of physical theories

# Why quantum mechanics?

- In early 20th century, the theories of physics at that time were predicting absurdities such as the existence of an ‘ultraviolet catastrophe’ involving infinite energies (*an ideal black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases*), or electrons spiraling inexorably into the atomic nucleus
- This resulted in the creation of the modern theory of quantum mechanics (1920s)
- What is quantum mechanics? Quantum mechanics is a mathematical framework or set of rules for the construction of physical theories

# Is cloning possible?

- Is it possible to clone an unknown quantum state, that is, construct a copy of a quantum state?
- If cloning were possible, then it would be possible to signal faster than light using quantum effects.
- This no-cloning theorem, discovered in the early 1980s, is one of the earliest results of quantum computation and quantum information.

# Is cloning possible?

- Is it possible to clone an unknown quantum state, that is, construct a copy of a quantum state?
- If cloning were possible, then it would be possible to signal faster than light using quantum effects.
- This no-cloning theorem, discovered in the early 1980s, is one of the earliest results of quantum computation and quantum information.

# Is cloning possible?

- Is it possible to clone an unknown quantum state, that is, construct a copy of a quantum state?
- If cloning were possible, then it would be possible to signal faster than light using quantum effects.
- This no-cloning theorem, discovered in the early 1980s, is one of the earliest results of quantum computation and quantum information.

# How to obtain complete control over single quantum systems

- Applications of quantum mechanics prior to the 1970s typically involved a gross level of control over a bulk sample containing an enormous number of quantum mechanical systems, none of them directly accessible
- Since 1970s, many techniques for controlling single quantum systems have been developed.
- Methods to trap a single atom in an 'atom trap' (cf. ion trap)
- The scanning tunneling microscope (1986 Nobel Prize) can be used to move single atoms around, creating designer arrays of atoms at will.

# How to obtain complete control over single quantum systems

- Applications of quantum mechanics prior to the 1970s typically involved a gross level of control over a bulk sample containing an enormous number of quantum mechanical systems, none of them directly accessible
- Since 1970s, many techniques for controlling single quantum systems have been developed.
- Methods to trap a single atom in an 'atom trap' (cf. ion trap)
- The scanning tunneling microscope (1986 Nobel Prize) can be used to move single atoms around, creating designer arrays of atoms at will.

# How to obtain complete control over single quantum systems

- Applications of quantum mechanics prior to the 1970s typically involved a gross level of control over a bulk sample containing an enormous number of quantum mechanical systems, none of them directly accessible
- Since 1970s, many techniques for controlling single quantum systems have been developed.
- Methods to trap a single atom in an ‘atom trap’ (cf. ion trap)
- The scanning tunneling microscope (1986 Nobel Prize) can be used to move single atoms around, creating designer arrays of atoms at will.

# How to obtain complete control over single quantum systems

- Applications of quantum mechanics prior to the 1970s typically involved a gross level of control over a bulk sample containing an enormous number of quantum mechanical systems, none of them directly accessible
- Since 1970s, many techniques for controlling single quantum systems have been developed.
- Methods to trap a single atom in an ‘atom trap’ (cf. ion trap)
- The scanning tunneling microscope (1986 Nobel Prize) can be used to move single atoms around, creating designer arrays of atoms at will.

# Quantum Computing

- Strong Church-Turing thesis states that a probabilistic Turing machine (ie a classical computer that can make fair coin flips) can efficiently simulate any realistic model of computing
- Therefore if we are interested in which problems can be solved efficiently on a realistic model of computation, we can restrict attention to a probabilistic Turing machine (or an equivalent model)

# Quantum Computing

- Strong Church-Turing thesis states that a probabilistic Turing machine (ie a classical computer that can make fair coin flips) can efficiently simulate any realistic model of computing
- Therefore if we are interested in which problems can be solved efficiently on a realistic model of computation, we can restrict attention to a probabilistic Turing machine (or an equivalent model)

# Physics and Computation

- Information is stored in a physical medium and manipulated by physical processes
- Therefore the laws of physics dictate the capabilities and limitations of any information processor
- The “classical” laws of physics are (in macroscopic systems moving relatively slowly) a good approximation to the laws of physics

# Physics and Computation

- Information is stored in a physical medium and manipulated by physical processes
- Therefore the laws of physics dictate the capabilities and limitations of any information processor
- The “classical” laws of physics are (in macroscopic systems moving relatively slowly) a good approximation to the laws of physics

# Physics and Computation

- Information is stored in a physical medium and manipulated by physical processes
- Therefore the laws of physics dictate the capabilities and limitations of any information processor
- The “classical” laws of physics are (in macroscopic systems moving relatively slowly) a good approximation to the laws of physics

# Physics and Computation

- Realisations are getting smaller (and faster) and reaching a point where “classical” physics is no longer a sufficient model for the laws of physics

# Physics and Computation

- However the theory of quantum physics is a much better approximation to the laws of physics
- The probabilistic Turing machine is implicitly a “classical” device and it is not known in general how to use it simulate quantum mechanical systems
- A computer designed to exploit the quantum features of Nature (a quantum computer) seems to violate the Strong Church-Turing thesis

# Physics and Computation

- However the theory of quantum physics is a much better approximation to the laws of physics
- The probabilistic Turing machine is implicitly a “classical” device and it is not known in general how to use it simulate quantum mechanical systems
- A computer designed to exploit the quantum features of Nature (a quantum computer) seems to violate the Strong Church-Turing thesis

# Physics and Computation

- However the theory of quantum physics is a much better approximation to the laws of physics
- The probabilistic Turing machine is implicitly a “classical” device and it is not known in general how to use it simulate quantum mechanical systems
- A computer designed to exploit the quantum features of Nature (a quantum computer) seems to violate the Strong Church-Turing thesis

# Physics and Computation

- If the quantum computers are a reasonable model of computation, and classical devices cannot efficiently simulate them, then the strong Church-Turing thesis needs to be modified to state that a quantum Turing machine can efficiently simulate any realistic model of computation
- Feynman attempted to show that quantum system could be used to do computations (1982)
- Deutsch showed that any physical process could be modeled perfectly by a quantum computer (1985)

# Physics and Computation

- If the quantum computers are a reasonable model of computation, and classical devices cannot efficiently simulate them, then the strong Church-Turing thesis needs to be modified to state that a quantum Turing machine can efficiently simulate any realistic model of computation
- Feynman attempted to show that quantum system could be used to do computations (1982)
- Deutsch showed that any physical process could be modeled perfectly by a quantum computer (1985)

# Physics and Computation

- If the quantum computers are a reasonable model of computation, and classical devices cannot efficiently simulate them, then the strong Church-Turing thesis needs to be modified to state that a quantum Turing machine can efficiently simulate any realistic model of computation
- Feynman attempted to show that quantum system could be used to do computations (1982)
- Deutsch showed that any physical process could be modeled perfectly by a quantum computer (1985)

# Physics and Computation

- No real advantage of quantum computers was showed until Peter Shor
- In 1994, Peter Shor came up with a quantum algorithm to factor very large numbers in polynomial time.
- In 1997, Lov Grover develops a quantum search algorithm with $O(\sqrt{N})$ complexity

# Physics and Computation

- No real advantage of quantum computers was showed until Peter Shor
- In 1994, Peter Shor came up with a quantum algorithm to factor very large numbers in polynomial time.
- In 1997, Lov Grover develops a quantum search algorithm with $O(\sqrt{N})$ complexity

# Physics and Computation

- No real advantage of quantum computers was showed until Peter Shor
- In 1994, Peter Shor came up with a quantum algorithm to factor very large numbers in polynomial time.
- In 1997, Lov Grover develops a quantum search algorithm with $O(\sqrt{N})$ complexity

# Counter-intuitive quantum information theory

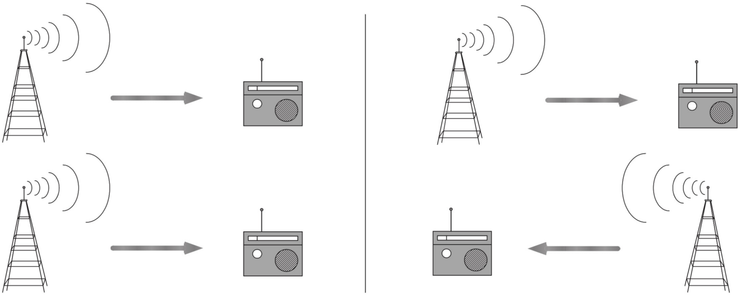


Figure 1.1. Classically, if we have two very noisy channels of zero capacity running side by side, then the combined channel has zero capacity to send information. Not surprisingly, if we reverse the direction of one of the channels, we still have zero capacity to send information. Quantum mechanically, reversing one of the zero capacity channels can actually allow us to send information!

# Understanding Quantum Mechanics

- In order to understand quantum mechanics principles, we need to discuss
  - states
  - measurements
  - reversible transformations
  - composite systems

# Understanding Quantum Mechanics

- In order to understand quantum mechanics principles, we need to discuss
  - states
  - measurements
  - reversible transformations
  - composite systems

# Understanding Quantum Mechanics

- In order to understand quantum mechanics principles, we need to discuss
  - states
  - measurements
  - reversible transformations
  - composite systems

# Understanding Quantum Mechanics

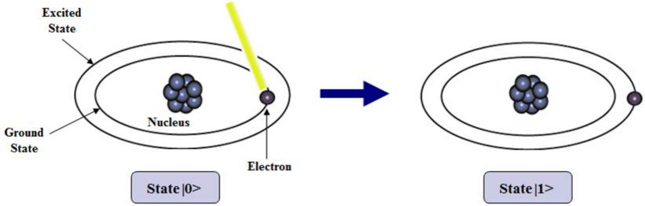
- In order to understand quantum mechanics principles, we need to discuss
  - states
  - measurements
  - reversible transformations
  - composite systems

# Understanding Quantum Mechanics

- In order to understand quantum mechanics principles, we need to discuss
  - states
  - measurements
  - reversible transformations
  - composite systems

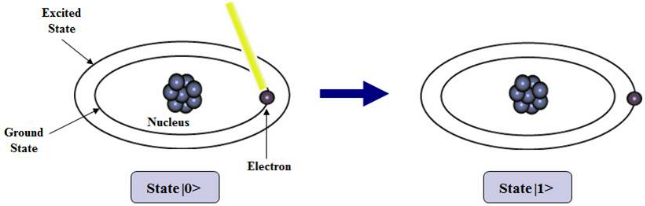
# Qubit

- A bit of data (called *qubit*) is represented by a single atom that is in one of two states denoted by $|0\rangle$ and $|1\rangle$.
- A physical implementation of a qubit could use the two energy levels of an atom. An excited state representing $|1\rangle$ and a ground state representing $|0\rangle$



# Qubit

- A bit of data (called *qubit*) is represented by a single atom that is in one of two states denoted by $|0\rangle$ and $|1\rangle$.
- A physical implementation of a qubit could use the two energy levels of an atom. An excited state representing $|1\rangle$ and a ground state representing $|0\rangle$



# Qubit with Photon Polarization

- Any object (atom, baseball etc.) could be described by quantum mechanics (but too complicated to describe baseball in this way)
- A photon has two essential properties (or two quantum variables): momentum and polarization
- polarization is simpler for discussion
- state: the most complete characterization that one can give of a quantum variable at a particular moment in time
- the state of a quantum variable can change from one moment to the next

# Qubit with Photon Polarization

- Any object (atom, baseball etc.) could be described by quantum mechanics (but too complicated to describe baseball in this way)
- A photon has two essential properties (or two quantum variables): momentum and polarization
- polarization is simpler for discussion
- state: the most complete characterization that one can give of a quantum variable at a particular moment in time
- the state of a quantum variable can change from one moment to the next

# Qubit with Photon Polarization

- Any object (atom, baseball etc.) could be described by quantum mechanics (but too complicated to describe baseball in this way)
- A photon has two essential properties (or two quantum variables): momentum and polarization
- polarization is simpler for discussion
- state: the most complete characterization that one can give of a quantum variable at a particular moment in time
- the state of a quantum variable can change from one moment to the next

# Qubit with Photon Polarization

- Any object (atom, baseball etc.) could be described by quantum mechanics (but too complicated to describe baseball in this way)
- A photon has two essential properties (or two quantum variables): momentum and polarization
- polarization is simpler for discussion
- state: the most complete characterization that one can give of a quantum variable at a particular moment in time
- the state of a quantum variable can change from one moment to the next

# Qubit with Photon Polarization

- Any object (atom, baseball etc.) could be described by quantum mechanics (but too complicated to describe baseball in this way)
- A photon has two essential properties (or two quantum variables): momentum and polarization
- polarization is simpler for discussion
- state: the most complete characterization that one can give of a quantum variable at a particular moment in time
- the state of a quantum variable can change from one moment to the next

# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

# Qubit with Photon Polarization

- what is light?
    - Wave and Particle Duality
    - light particle: photon
- The set of possible polarization states of a photon:
    1. linear
    2. cicular
    3. elliptical
- The linear polarizations are the easiest ones to visualize

# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

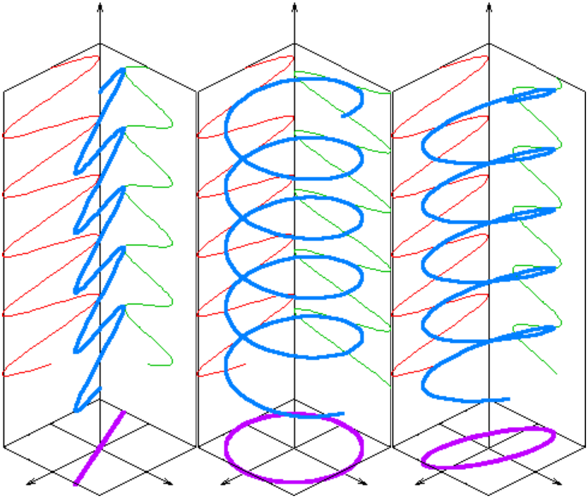
# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

# Qubit with Photon Polarization

- what is light?
  - Wave and Particle Duality
  - light particle: photon
- The set of possible polarization states of a photon:
  1. linear
  2. cicular
  3. elliptical
- The linear polarizations are the easiest ones to visualize

# Wave characterization of photons

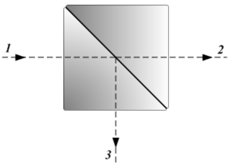


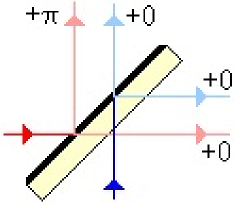
# Circular polarizer

The effects of a polarizer (e.g., polarizing sunglasses) on the sky in a color photograph. The right picture has the polarizer, the left does not. Some photographic polarizers are called circular polarizers. These actually select a linear polarization state from the scene, but then convert it into circularly-polarized light for the camera

# Beam-splitter and phase shift

- A cube, made from two triangular glass prisms which are glued together at their base (polarizing beam splitters, e.g. Wollaston prism, use birefringent materials, splitting light into beams of differing polarization)

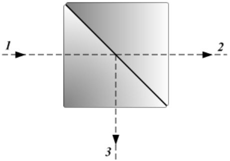


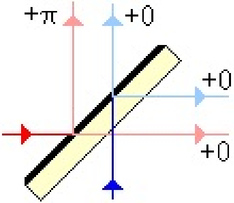




- A phase shift is a beam splitter with a reflective dielectric coating on one side. It gives a phase shift of 0 or $\pi$, depending on the side from which it is incident

# Beam-splitter and phase shift

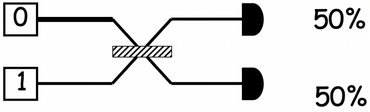
- A cube, made from two triangular glass prisms which are glued together at their base (polarizing beam splitters, e.g. Wollaston prism, use birefringent materials, splitting light into beams of differing polarization)



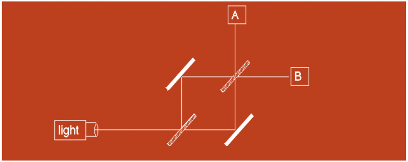

- A phase shift is a beam splitter with a reflective dielectric coating on one side. It gives a phase shift of 0 or $\pi$, depending on the side from which it is incident

# Beam-splitter with a single photon



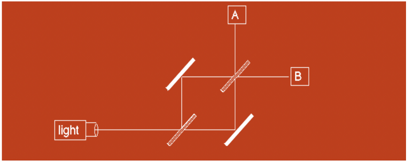

- The simplest explanation is that the beam-splitter acts as a classical coin-flip, randomly sending each photon one way or the other.

# An interferometer



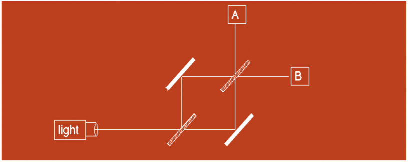

- Equal path lengths, rigid mirrors.
- Only one photon in the apparatus at a time.
- All photons leaving the source arrive at B.
- Why?

# An interferometer



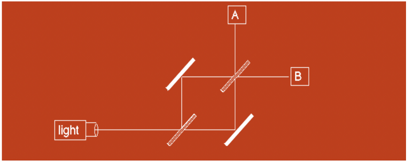

- Equal path lengths, rigid mirrors.
- Only one photon in the apparatus at a time.
- All photons leaving the source arrive at B.
- Why?

# An interferometer



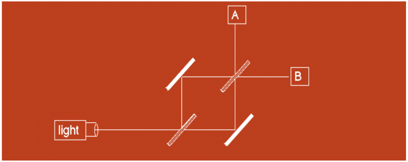

- Equal path lengths, rigid mirrors.
- Only one photon in the apparatus at a time.
- All photons leaving the source arrive at B.
- Why?

# An interferometer



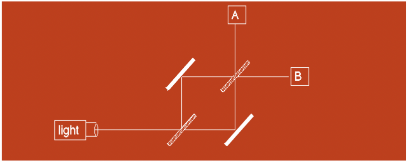

- Equal path lengths, rigid mirrors.
- Only one photon in the apparatus at a time.
- All photons leaving the source arrive at B.
- Why?

# Why?



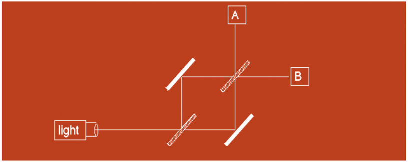

- There is a quantity that we'll call the "amplitude" for each possible path that a photon can take.
- The amplitudes can interfere constructively and destructively, even though each photon takes only one path.
- The amplitudes at detector A interfere destructively; those at detector B interfere constructively.

# Why?



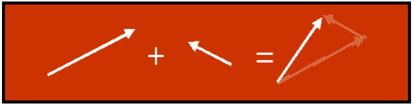

- There is a quantity that we'll call the "amplitude" for each possible path that a photon can take.
- The amplitudes can interfere constructively and destructively, even though each photon takes only one path.
- The amplitudes at detector A interfere destructively; those at detector B interfere constructively.

# Why?



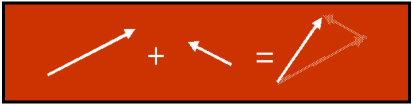

- There is a quantity that we'll call the "amplitude" for each possible path that a photon can take.
- The amplitudes can interfere constructively and destructively, even though each photon takes only one path.
- The amplitudes at detector A interfere destructively; those at detector B interfere constructively.

# Calculating interference



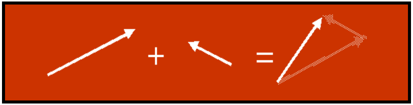

- Arrows for each possibility.
- Arrows flip 180 at mirrors, rotate 90 counter-clockwise when reflected from beam splitters.
- Add arrows and square the length of the result to determine the probability for any possibility.

# Calculating interference



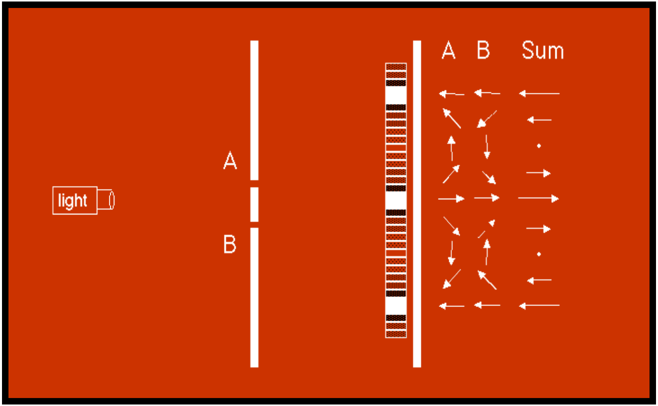

- Arrows for each possibility.
- Arrows flip 180 at mirrors, rotate 90 counter-clockwise when reflected from beam splitters.
- Add arrows and square the length of the result to determine the probability for any possibility.

# Calculating interference

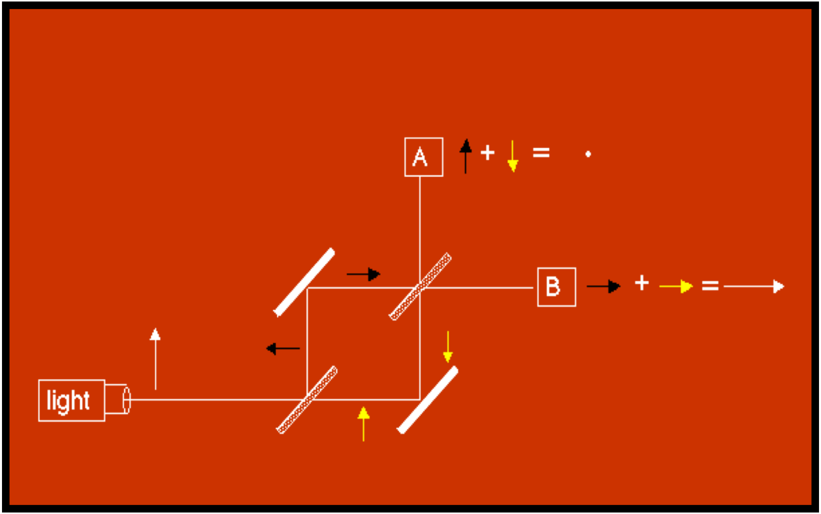



- Arrows for each possibility.
- Arrows flip 180 at mirrors, rotate 90 counter-clockwise when reflected from beam splitters.
- Add arrows and square the length of the result to determine the probability for any possibility.

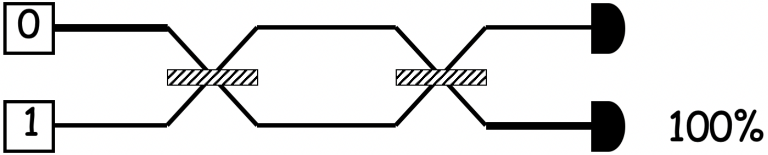
# Double slit interference



# Interference in the interferometer

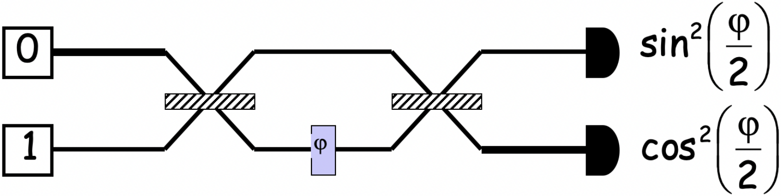


# Quantum Interference

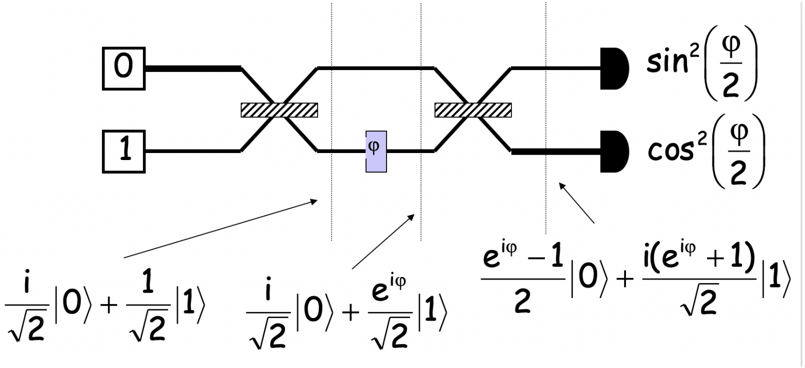



The simplest explanation must be wrong, since it would predict a 50-50 distribution.

# More experimental data



# New Theory




The particle can exist in a linear combination or superposition of the two paths

# Superposition

- A single qubit can be forced into a superposition of the two states denoted by the addition of the state vectors:

$$|\phi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$$

  where $\alpha_1$ and $\alpha_2$ are complex numbers with $|\alpha_1|^2 + |\alpha_2|^2 = 1$
- A qubit in superposition exists partly in all its particular theoretically possible states (i.e., both of states $|0\rangle$ and $|1\rangle$) simultaneously. But when measured, it gives a result corresponding to only one of the possible states (this is further described in interpretation of quantum mechanics).
- For a qubit $|\phi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$, $\alpha_1$ represents the probability of the superposition collapsing to $|0\rangle$.

# Superposition

- A single qubit can be forced into a superposition of the two states denoted by the addition of the state vectors:

$$|\phi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$$

  where $\alpha_1$ and $\alpha_2$ are complex numbers with $|\alpha_1|^2 + |\alpha_2|^2 = 1$
- A qubit in superposition exists partly in all its particular theoretically possible states (i.e., both of states $|0\rangle$ and $|1\rangle$) simultaneously. But when measured, it gives a result corresponding to only one of the possible states (this is further described in interpretation of quantum mechanics).
- For a qubit $|\phi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$, $\alpha_1$ represents the probability of the superposition collapsing to $|0\rangle$.

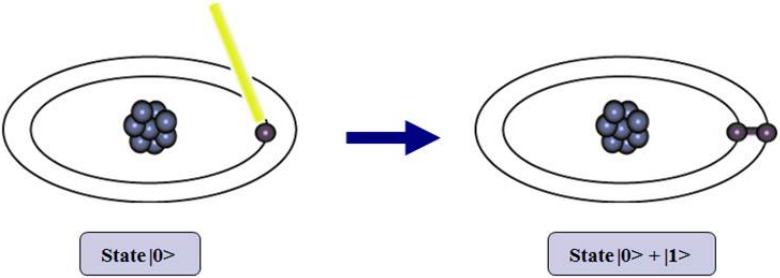
# Superposition

- A single qubit can be forced into a superposition of the two states denoted by the addition of the state vectors:

  $$|\phi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$$

  where $\alpha_1$ and $\alpha_2$ are complex numbers with $|\alpha_1|^2 + |\alpha_2|^2 = 1$
- A qubit in superposition exists partly in all its particular theoretically possible states (i.e., both of states $|0\rangle$ and $|1\rangle$) simultaneously. But when measured, it gives a result corresponding to only one of the possible states (this is further described in interpretation of quantum mechanics).
- For a qubit $|\phi\rangle = \alpha_1|0\rangle + \alpha_2|1\rangle$, $\alpha_1$ represents the probability of the superposition collapsing to $|0\rangle$.

# Superposition



# Quantum Operations

The operations are induced by the apparatus linearly, if

$$|0\rangle \to \frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\text{and } |1\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

Then

$$\begin{aligned}\alpha_0|0\rangle + \alpha_1|1\rangle &\to \alpha_0\left(\tfrac{i}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle\right) + \alpha_1\left(\tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{i}{\sqrt{2}}|1\rangle\right)\\ &= \left(\alpha_0\tfrac{i}{\sqrt{2}} + \alpha_1\tfrac{1}{\sqrt{2}}\right)|0\rangle + \left(\alpha_0\tfrac{1}{\sqrt{2}} + \alpha_1\tfrac{i}{\sqrt{2}}\right)|1\rangle\end{aligned}$$

# Quantum Operations

- A linear operation takes states $\alpha_0|0\rangle + \alpha_1|1\rangle$ satisfying $|\alpha_0|^2 + |\alpha_1|^2 = 1$
- It maps them to states $\alpha'_0|0\rangle + \alpha'_1|1\rangle$ satisfying $|\alpha'_0|^2 + |\alpha'_1|^2 = 1$
- The linear operation must be UNITARY

# Quantum Operations

- A linear operation takes states $\alpha_0|0\rangle + \alpha_1|1\rangle$ satisfying $|\alpha_0|^2 + |\alpha_1|^2 = 1$
- It maps them to states $\alpha_0'|0\rangle + \alpha_1'|1\rangle$ satisfying $|\alpha_0'|^2 + |\alpha_1'|^2 = 1$
- The linear operation must be UNITARY

# Quantum Operations

- A linear operation takes states $\alpha_0|0\rangle + \alpha_1|1\rangle$ satisfying $|\alpha_0|^2 + |\alpha_1|^2 = 1$
- It maps them to states $\alpha_0'|0\rangle + \alpha_1'|1\rangle$ satisfying $|\alpha_0'|^2 + |\alpha_1'|^2 = 1$
- The linear operation must be UNITARY

# UNITARY matrices

$$U = \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix}$$

if unitary if and only if

$$UU^{+} = \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix} \begin{bmatrix} u_{00}^{*} & u_{10}^{*} \\ u_{01}^{*} & u_{11}^{*} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Linear algebra

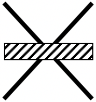
Let

$$\begin{aligned} |0\rangle &= |\leftrightarrow\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ |1\rangle &= |\updownarrow\rangle &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$

Then

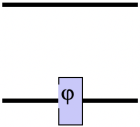
$$\alpha_0|0\rangle + \alpha_1|1\rangle = \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} = |\alpha\rangle$$

# Linear algebra



corresponds to

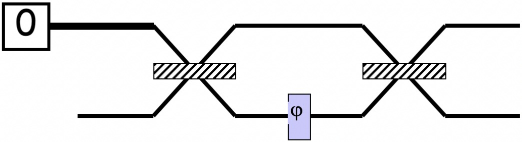
$$\begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$$




corresponds to

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

# Linear algebra




corresponds to

$$\begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

# Abstraction

Except when addressing a particular physical implementation, we will simply talk about "basis" states $|0\rangle$ and $|1\rangle$

and **unitary operations** like

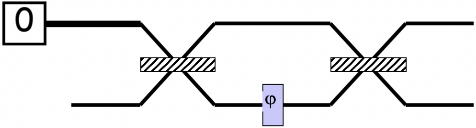
$-\boxed{H}-$ and $-\boxed{\varphi}-$

where $-\boxed{H}-$ corresponds to $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$

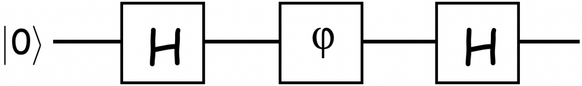
and $-\boxed{\varphi}-$ corresponds to $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$

# Abstraction

An arrangement like




is represented with a network *like*



# Qubit Measurements

- We want to design a box to measure the polarization state of photons
- The input for the box is a photon, the output is the value of $\theta$ or the values of $x$ and $y$.
- According to quantum mechanics, no such device could ever be built
- Are we hopeless?

# Qubit Measurements

- We want to design a box to measure the polarization state of photons
- The input for the box is a photon, the output is the value of $\theta$ or the values of $x$ and $y$.
- According to quantum mechanics, no such device could ever be built
- Are we hopeless?

# Qubit Measurements

- We want to design a box to measure the polarization state of photons
- The input for the box is a photon, the output is the value of $\theta$ or the values of $x$ and $y$.
- According to quantum mechanics, no such device could ever be built
- Are we hopeless?

# Qubit Measurements

- We want to design a box to measure the polarization state of photons
- The input for the box is a photon, the output is the value of $\theta$ or the values of $x$ and $y$.
- According to quantum mechanics, no such device could ever be built
- Are we hopeless?

# Qubit Measurements

- Quantum mechanics told us that we could only ask binary questions about photon polarization states, giving the photon a choice between two orthogonal states.
- That is, we can ask: which of the following two polarizations do you have: vertical or horizontal?
- The problem is that the photon may not have either of the polarizations
- in these situations, the measurement results is probabilistic

# Qubit Measurements

- Quantum mechanics told us that we could only ask binary questions about photon polarization states, giving the photon a choice between two orthogonal states.
- That is, we can ask: which of the following two polarizations do you have: vertical or horizontal?
- The problem is that the photon may not have either of the polarizations
- in these situations, the measurement results is probabilistic

# Qubit Measurements

- Quantum mechanics told us that we could only ask binary questions about photon polarization states, giving the photon a choice between two orthogonal states.
- That is, we can ask: which of the following two polarizations do you have: vertical or horizontal?
- The problem is that the photon may not have either of the polarizations
- in these situations, the measurement results is probabilistic

# Qubit Measurements

- Quantum mechanics told us that we could only ask binary questions about photon polarization states, giving the photon a choice between two orthogonal states.
- That is, we can ask: which of the following two polarizations do you have: vertical or horizontal?
- The problem is that the photon may not have either of the polarizations
- in these situations, the measurement results is probabilistic

# Qubit Linear polarization measurement

- A device to measure linear polarization is denoted by a pair of orthogonal vectors $|x\rangle$ and $|y\rangle$
- For two vectors $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the inner product of them is $\langle a|b\rangle = a_1 b_1 + a_2 b_2$
- Then the result of measuring a photon in polarization state $|a\rangle$ is that:
  - with probability $\langle a|x\rangle$, it is measured as $|x\rangle$
  - with probability $\langle a|y\rangle$, it is measured as $|y\rangle$
- It is easy to show that: with probability 1, $|x\rangle$ is measured as $|x\rangle$, and with probability 1, $|y\rangle$ is measured as $|y\rangle$.

# Qubit Linear polarization measurement

- A device to measure linear polarization is denoted by a pair of orthogonal vectors $|x\rangle$ and $|y\rangle$
- For two vectors $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the inner product of them is $\langle a|b\rangle = a_1 b_1 + a_2 b_2$
- Then the result of measuring a photon in polarization state $|a\rangle$ is that:
  - with probability $\langle a|x\rangle$, it is measured as $|x\rangle$
  - with probability $\langle a|y\rangle$, it is measured as $|y\rangle$
- It is easy to show that: with probability 1, $|x\rangle$ is measured as $|x\rangle$, and with probability 1, $|y\rangle$ is measured as $|y\rangle$.

# Qubit Linear polarization measurement

- A device to measure linear polarization is denoted by a pair of orthogonal vectors $|x\rangle$ and $|y\rangle$
- For two vectors $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the inner product of them is $\langle a|b\rangle = a_1 b_1 + a_2 b_2$
- Then the result of measuring a photon in polarization state $|a\rangle$ is that:
    - with probability $\langle a|x\rangle$, it is measured as $|x\rangle$
    - with probability $\langle a|y\rangle$, it is measured as $|y\rangle$
- It is easy to show that: with probability 1, $|x\rangle$ is measured as $|x\rangle$, and with probability 1, $|y\rangle$ is measured as $|y\rangle$.

# Qubit Linear polarization measurement

- A device to measure linear polarization is denoted by a pair of orthogonal vectors $|x\rangle$ and $|y\rangle$
- For two vectors $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the inner product of them is $\langle a|b\rangle = a_1 b_1 + a_2 b_2$
- Then the result of measuring a photon in polarization state $|a\rangle$ is that:
  - with probability $\langle a|x\rangle$, it is measured as $|x\rangle$
  - with probability $\langle a|y\rangle$, it is measured as $|y\rangle$
- It is easy to show that: with probability 1, $|x\rangle$ is measured as $|x\rangle$, and with probability 1, $|y\rangle$ is measured as $|y\rangle$.

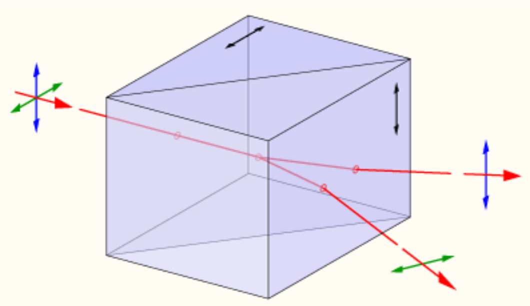
# Qubit Linear polarization measurement

- A device to measure linear polarization is denoted by a pair of orthogonal vectors $|x\rangle$ and $|y\rangle$
- For two vectors $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the inner product of them is $\langle a|b\rangle = a_1 b_1 + a_2 b_2$
- Then the result of measuring a photon in polarization state $|a\rangle$ is that:
  - with probability $\langle a|x\rangle$, it is measured as $|x\rangle$
  - with probability $\langle a|y\rangle$, it is measured as $|y\rangle$
- It is easy to show that: with probability 1, $|x\rangle$ is measured as $|x\rangle$, and with probability 1, $|y\rangle$ is measured as $|y\rangle$.

# Qubit Linear polarization measurement

- A device to measure linear polarization is denoted by a pair of orthogonal vectors $|x\rangle$ and $|y\rangle$
- For two vectors $|a\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, the inner product of them is $\langle a|b\rangle = a_1 b_1 + a_2 b_2$
- Then the result of measuring a photon in polarization state $|a\rangle$ is that:
  - with probability $\langle a|x\rangle$, it is measured as $|x\rangle$
  - with probability $\langle a|y\rangle$, it is measured as $|y\rangle$
- It is easy to show that: with probability 1, $|x\rangle$ is measured as $|x\rangle$, and with probability 1, $|y\rangle$ is measured as $|y\rangle$.

# Wollaston prism



A Wollaston prism is an optical device that manipulates polarized light. It separates randomly polarized or unpolarized light into two orthogonal, linearly polarized outgoing beams.

# Polarizing filters

On the other hand, polarizing filters simple absorb photons with one polarizations and transmits photons having the orthogonal polarizations. For example, polarizing sunglasses will absorb horizontally polarized light (glare) and transmits vertically polarized light.

# Q&A

Q&A?