ITIS 6260/8260 Quantum Computing

Lecture 6: non-cloning and quantum teleportation

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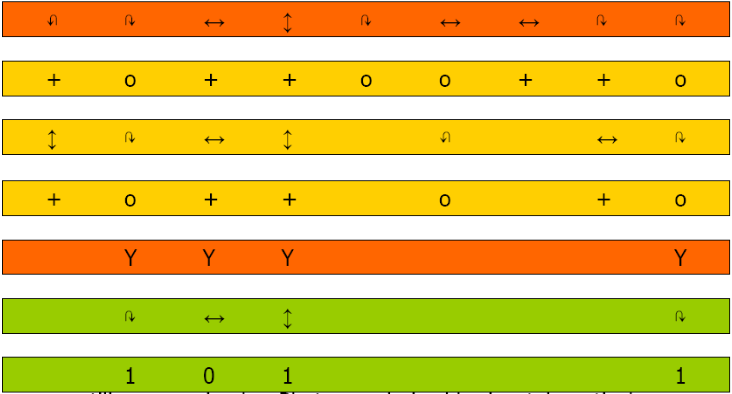
Outline

Quantum Key Distribution

- Alice sends random sequence of 4 types of polarized photons over the quantum channel: horizontal $|0\rangle$, vertical $|1\rangle$, right-circular $(|0\rangle - |1\rangle)/\sqrt{2}$, left-circular $(|0\rangle + |1\rangle)/\sqrt{2}$
- Bob randomly choose one of the following basis to measure each photon: $(|0\rangle, |1\rangle)$ or $((|0\rangle - |1\rangle)/\sqrt{2}, (|0\rangle + |1\rangle)/\sqrt{2})$
- Bob tells Alice the bases he used over the public channel
- Alice informs Bob which bases were correct
- Alice and Bob discard the data from incorrectly measured photons
- The polarization data is converted to a bit string

Quantum Key Distribution



+: rectilinear, o: circular. Photons polarized horizontal, vertical, right-circular, and left-circular

Non-cloning theorem

- Assume that a unitary transformation U can copy two pure states $|\phi\rangle$ and $|\psi\rangle$. Then

$$U(|\phi\rangle \otimes |0\rangle) = |\phi\rangle \otimes |\phi\rangle$$
$$U(|\psi\rangle \otimes |0\rangle) = |\psi\rangle \otimes |\psi\rangle$$

- Taking the inner product of the two equations gives

$$\langle\psi|\phi\rangle\langle 0|0\rangle = (\langle\psi|\phi\rangle)^2$$

- That is, $\langle\psi|\phi\rangle = 0$ or $\langle\psi|\phi\rangle = 1$
- Either $|\phi\rangle = |\psi\rangle$ or $|\phi\rangle$ and $|\psi\rangle$ are orthogonal
- Conclusion: unitary transmission can only clone orthogonal pure states

Quantum teleportation (1991)

- By the limitations of relativity, we can not send information faster than the speed of light
- Quantum teleportation shows that It is possible for Alice, with a pre-shared entanglement qubit with Bob, to transmit a qubit to Bob over a classical communication channel
- Indeed, it could be show that it is optimal to transmit a qubit using a preshared entangled qubit and two classicial bits

Quantum teleportation (high level protocol)

- Alice shares a Bell pair with Bob
- Alice has a qubit $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$
- Alice applies some transformation to $|\phi\rangle$ that entangles it with her half of the Bell pair
- Alice measures $|\phi\rangle$ and her half of the Bell pair and tells Bob (over phone) the measurement outcome
- Bob applies some transformation to his entangled qubit
- By non-cloning theorem, only Bob has the qubit $|\phi\rangle$ at the end

Quantum teleportation (detailed protocol)

- Alice and Bob share a Bell pair: $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$
- Alice wants to send a qubit $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$. The total state starts as $(\alpha|0\rangle + \beta|1\rangle) \otimes \frac{|00\rangle+|11\rangle}{\sqrt{2}}$
- Alice applies a CNOT gate (with $|\phi\rangle$ as the control bit and her half of the Bell pair as target) to the total state to get

$$\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$

- Alice Hadamards her $|\phi\rangle$ to get

$$\frac{1}{2}\left(\begin{array}{l} \alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle \\ \quad +\beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle \end{array}\right)$$

Quantum teleportation (detailed protocol continued)

$$\frac{1}{2}\left(\begin{array}{c} \alpha|000\rangle+\alpha|100\rangle+\alpha|011\rangle+\alpha|111\rangle \\ +\beta|010\rangle-\beta|110\rangle+\beta|001\rangle-\beta|101\rangle \end{array}\right)$$

- This leads to the four possible scenarios (Alice sees top row, Bob's qubit in second row)

00	01	10	11
$\alpha\|0\rangle+\beta\|1\rangle$	$\alpha\|1\rangle+\beta\|0\rangle$	$\alpha\|0\rangle-\beta\|1\rangle$	$\alpha\|1\rangle-\beta\|0\rangle$

- The impact on Bob's qubit is instant. But before Bob learns Alice's measurement output, Bob only just has the maximally mixed state (by non-communication theorem)

Quantum teleportation (detailed protocol)

00	01	10	11
$\alpha\|0\rangle+\beta\|1\rangle$	$\alpha\|1\rangle+\beta\|0\rangle$	$\alpha\|0\rangle-\beta\|1\rangle$	$\alpha\|1\rangle-\beta\|0\rangle$

- Alice tells Bob her measurement output
- If the first bit sent by Alice is 1, then Bob applies $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- If the second bit sent by Alice is 1, then Bob applies $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- These transformation will bring Bob's qubit to the state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$

Superdense coding

- By contrast of tepeportation, we show that if Alice and Bob share a Bell pair: $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$, Alice can send two classical bits via one qubit
- First we note that Alice can get three different states by applying gates to her qubit
  - applies $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to get $\frac{|01\rangle+|10\rangle}{\sqrt{2}}$
  - applies $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ to get $\frac{|00\rangle-|11\rangle}{\sqrt{2}}$
  - applies $iY = ZX = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ to get $\frac{|01\rangle-|10\rangle}{\sqrt{2}}$
  - together with original Bell pair, these four states form an orthogonal basis

Superdense coding

- Suppose Alice wants to send $b_0 b_1$ to Bob. Then Alice applies the following gates and send her qubit to Bob
  - If $b_0 = 1$, applies NOT gate X
  - If $b_1 = 1$, applies phase gate Z
- Alice's transformation is equivalent to the unitary transformation

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

Superdense coding

After Alice applies the trandformation, we can distinguish the following four cases:

Alice classical bits	Alice qubits
00	$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}}$
01	$\frac{\|00\rangle-\|11\rangle}{\sqrt{2}}$
10	$\frac{\|01\rangle+\|10\rangle}{\sqrt{2}}$
11	$\frac{\|01\rangle-\|10\rangle}{\sqrt{2}}$

Superdense coding

- In order for Bob to decode, he needs the inverse transformation

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

- This is equivalent to CNOT gate (Bob qubit control Alice qubit) and then Hadamard gate on Bob's qubit

Superdense coding

We can distinguish the following four cases:

Bob receives	After CNOT	After Hadamard
$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}}$	$\|0\rangle \otimes \|+\rangle$	$\|0\rangle \otimes \|0\rangle$
$\frac{\|01\rangle+\|10\rangle}{\sqrt{2}}$	$\|1\rangle \otimes \|+\rangle$	$\|1\rangle \otimes \|0\rangle$
$\frac{\|00\rangle-\|11\rangle}{\sqrt{2}}$	$\|0\rangle \otimes \|-\rangle$	$\|0\rangle \otimes \|1\rangle$
$\frac{\|01\rangle-\|10\rangle}{\sqrt{2}}$	$\|1\rangle \otimes \|-\rangle$	$\|1\rangle \otimes \|1\rangle$

Q&A

Q&A?