ITIS 6260/8260 Quantum Computing

Lecture 7: Quantum Error Correction

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Outline

1. Need for Error correction
   - Need for ECC
   - Classical ECC
2. Quantum ECC
   - Quantum ECC
   - Shor's 9-qubit code
   - Quantum Fault-Tolerance Theorem

Need for ECC

Potential concerns for quantum computation systrem

- Unitary error: U should perform a 45° rotation, but it may perform a 46° rotation
- Decoherence: The accidental measurement of the quantum system by the environment. For example

$$|\phi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

  If the first qubit is measured by the environment. The resulting state lost certain information
- Quantum error correction? Non-cloning theorem: cannot convert $|\phi\rangle|0\rangle$ to $|\phi\rangle|\phi\rangle$

Classical ECC

- To detect one error: Making $\oplus = 0$: 0101011
- To correct one error: repetition code: $0 \to 000$, $1 \to 111$
- von Neumann showed that as long as the physical error probability is small enough, each round of error-correction will make the system better than worse
- modern transistors are much more reliable. For a modern computer with its billions of transistors, we may not see a transitor error in one year.

Errors

- All errors could be considered as a unitary transformation U that operates on $|\phi\rangle|e\rangle$
- Single qubit quantum gates: I, X, Z, XZ. If we can correct I, X, Z, XZ errors, we can correct most one-qubit errors

Correct quantum errors

- Let $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|e\rangle$ be the environment
- Assume that the error E entangles a quantum state with the environment state as $E|\phi\rangle$. It can be described as a unitary transformation
- Since the Pauli operators are a basis for linear transformations on a single qubit, we can write this error transformation as

$$E|\phi\rangle = |e_I\rangle I|\phi\rangle + |e_X\rangle X|\phi\rangle + |e_Z\rangle Z|\phi\rangle + |e_{XZ}\rangle XZ|\phi\rangle$$

- Thus if we can correct against I, X, Z, and XZ, then we can restore the state $U|\phi\rangle$ to just $|\phi\rangle$ for $U \in \{I, X, Z, XZ\}$

Another look from superdense code

- Assume that we have an error on the first qubit of $|\phi_0\rangle = \alpha|0\rangle|v\rangle + \beta|1\rangle|w\rangle$
- A bit-flip results in: $|\phi_1\rangle = \alpha|1\rangle|v\rangle + \beta|0\rangle|w\rangle$
- a phase-flip results in: $|\phi_2\rangle = \alpha|0\rangle|v\rangle - \beta|1\rangle|w\rangle$
- both flips result in: $|\phi_3\rangle = \alpha|1\rangle|v\rangle - \beta|0\rangle|w\rangle$
- these four states consists of an orthogonal basis for the 4-dimensional subspace of all possible states that one can reach from $|\phi_0\rangle$ by transformations on the first qubit

Another look from superdense code

- should not try to measure all the qubits
- should do a measurement that just projects onto one of the four basis vectors
- if the outcome is $|\phi_0\rangle$: no error
- if the outcome is $|\phi_1\rangle$: do a bit flip
- if the outcome is $|\phi_2\rangle$: do a phase flip
- if the outcome is $|\phi_3\rangle$: do both flips
- That is, if can correct bit-flip and phase shift errors, we can correct most one-qubit errors

Shor's 9-qubit code

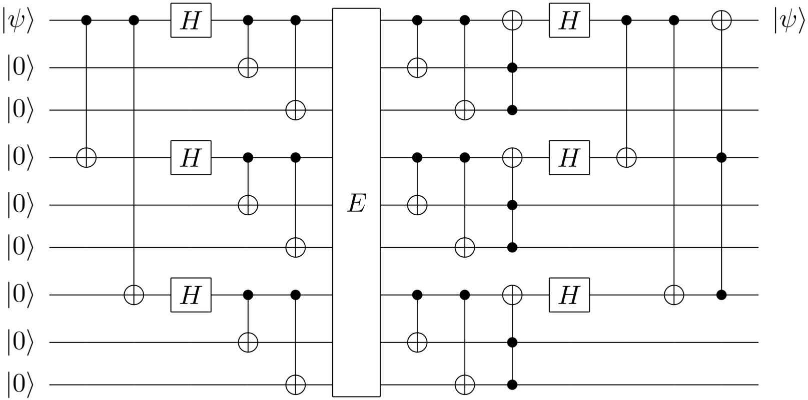
Encode each logical qubit as a 3×3 squre of physical qubits in such a way that each row corresponds to a 3-qubit repetition code to protect against bit-flip errors, and each column corresponds to a 3-qubit repetition code to protect against phase-flip error

$$|0\rangle \rightarrow \left(\frac{|000\rangle + |111\rangle}{\sqrt{2}}\right)^{\otimes 3}$$

$$|1\rangle \rightarrow \left(\frac{|000\rangle - |111\rangle}{\sqrt{2}}\right)^{\otimes 3}$$

Claim: This code detects and corrects a bit flip or a phase flip (and hence, any possible error) on any one of the 9 qubits.

Shor's 9-qubit code



Shor's 9-qubit code

- **correct bit-flip**: Build a quantum circuit to check whether all 3 of the qubits in a given row have the same value. If not, use majority to correct them
- **correct phaser-flip**: Build a quantum circuit that computes the relative phase between $|000\rangle$ and $|111\rangle$ within each column, checks whether all 3 phases have the same value. If not, use majority to correct them

Other improvement

- Andrew Steane achieved the same goal by encoding each logical qubit into 7 physical qubits.
- Raymond Laflamme achieved the same goal by encoding each logical qubit into 5 physical qubits.
- It was proved that 5 qubits are the optimal to achieve 1-qubit error correction (for classical ECC, correcting 1 errors requires three bits)

Quantum Fault-Tolerance Theorem

- (Aharonov, Ben-Or, Zurek et al. 1996) Suppose that, in a quantum computer, each qubit fails at each time step with independent sufficiently small probability ε and we are able to
  1. apply gates to many qubits in parallel,
  2. measure and discard bad qubits,
  3. pump in fresh $|0\rangle$ qubits, and
  4. do extremely fast and reliable classical computation

  Then we can still solve any problem in BQP
- Initial esitmates for ε is 10^{-6}. Improved a lot (e.g. 10^{-3} or 0.5%) in the past. Means thousands of physical qubits for one logical qubit

Q&A

Q&A?