Strategic Concealment in Innovation Races*

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Abstract

We investigate a firm's incentives to conceal an intermediate research discovery in order to influence its rival's choice of strategy in an innovation race. To study this, we introduce an innovation game where two firms dynamically allocate their resources between two distinct research and development (R&D) paths towards a final innovation: (i) developing it with the currently available but slower technology; (ii) conducting research to discover a faster new technology for developing it. We fully characterize the equilibrium behavior of the firms in the cases where their research progress is public and private information. Then, we extend the private information setting by allowing firms to conceal or license their intermediate discoveries. We show that when the reward of winning the race is high, firms sometimes conceal their interim discoveries, which inefficiently retards the pace of innovation.

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1 Introduction

In the course of research and development (R&D), firms often discover interim knowledge that brings them closer to ultimate success in producing a final innovation. When multiple firms race towards such innovation, a firm's optimal R&D strategy is likely to be influenced by the information about whether its rivals have made intermediate breakthroughs. Thus, a firm may want to conceal intermediate discoveries in order to hinder its rivals from adjusting their R&D strategies. On the other hand, it may prefer to disclose an intermediate discovery because this can open the opportunity for monetization via licensing the technological breakthrough. In this paper, we introduce and study an innovation race model that captures the tradeoffs between licensing and concealing interim discoveries and characterize firms' equilibrium behavior.

We consider a situation where two firms race towards developing an innovative good, such as a COVID-19 vaccine or a full self-driving (FSD) vehicle. The first firm to develop the product receives a reward (e.g., a transitory flow of monopoly profit) and the other firm does not. At each point in time, the firms allocate their limited resources between two routes for developing the product and incur constant flow costs. One route is to conduct basic *research* to discover a new technology that does not directly deliver the product but makes developing it faster, e.g., messenger RNA (mRNA) or light detection and ranging (LIDAR) technology.[1][2] This route requires two breakthroughs: discovering the new technology and developing the product with it. The other route is to *develop* the product with a currently available but slow technology, namely the incumbent technology. For example, the viral

[1] The mRNA technology was not utilized in practice before the COVID-19 outbreak. Thus, pharmaceutical firms had to first acquire basic knowledge in order to employ this new methodology. The advantage of possessing this intermediate technology is that firms can develop vaccines in a laboratory by using readily available materials. Hence vaccines can be developed faster with the mRNA technology than with older methods. Moderna and Pfizer-BioNTech utilize mRNA technology to develop COVID-19 vaccines. For more information, see the web page of the Centers for Disease Control and Prevention (CDC): https://www.cdc.gov/coronavirus/2019-ncov/vaccines/different-vaccines/mrna.html.

[2] LIDAR is a laser radar that can provide extensive and reliable information surrounding a vehicle including an object's distance, size, position, and velocity if it is moving. Most FSD vehicle developers including Waymo—formerly the Google self-driving car project—use LIDAR combined with cameras. The main drawback of LIDAR is its current high cost. Thus, to develop a commercializable FSD vehicle, firms first need to discover a way to make LIDAR less expensive. Once LIDAR becomes affordable, it will be relatively easy to develop a commercializable FSD vehicle. In this sense, successfully developing an FSD vehicle with the LIDAR technology can be understood as a route requiring two breakthroughs.

vector method for developing a COVID vaccine and the camera-based vision technology for developing an FSD vehicle can be considered incumbent technologies.[3][4] This path requires a single breakthrough but the arrival rate is relatively low. We assume that the path with the new technology is more efficient: the total expected completion time of doing research for the new technology and developing the product with it is shorter than that of developing with the incumbent strategy. Thus, the socially efficient policy is to have both firms allocate all their resources to research, and once one of them discovers the new technology, have it share the breakthrough with the other firm to prevent duplication of research costs.

In the context of this framework, we investigate three different settings. First, we consider the case where it is public information whether a firm has discovered the new technology or not. In this setting, a firm's strategy depends not only on its own technological breakthrough but also on its rival's progress. We show that there exists a unique equilibrium and its form is determined by the relative efficiency of the new technology. The efficiency measure is defined to be inversely proportional to the expected total completion time of the path with the new technology, i.e., doing research is more attractive when efficiency is high. It is shown that when efficiency is extreme (high or low), a firm's equilibrium strategy does not depend on its rival's progress. Specifically, when the new technology is highly efficient, both firms allocate all their resources to research (i.e., perform research only); and when the new technology is not much more efficient, both firms allocate all their resources to development (i.e., develop with the incumbent technology only) regardless of their rival's status. On the contrary, when efficiency is intermediate, the equilibrium strategy of each firm does depend on its rival's progress. In this case, both firms begin by conducting research, but once one firm makes the intermediate technological breakthrough, the other switches to developing with the incumbent technology, namely it pursues a *fall-back* strategy.

[3]The viral vector technology was used during recent disease outbreaks including the 2014-2016 Ebola outbreak in West Africa. Many pharmaceutical firms had access to this methodology when the COVID-19 outbreak began. Indeed, this technology was utilized to develop COVID-19 vaccines by Oxford-AstraZeneca and Janssen (Johnson&Johnson). For more information, see the web page of the CDC: https://www.cdc.gov/coronavirus/2019-ncov/vaccines/different-vaccines/viralvector.html.

[4] Unlike other companies, Tesla's approach towards developing an FSD vehicle is to use only cameras without LIDAR (Templeton, 2019). Since camera technology is already very cheap, no cost-saving breakthrough is needed to implement it. However, the quality of information attained from cameras is inferior to that attained from LIDAR, thus it will take more time to develop an FSD vehicle utilizing only cameras.

Next, we analyze the situation where technological discoveries are private information, i.e., a firm cannot observe its rivals' technological progress. As in the public information setting, when efficiency is high, each firm conducts research until it succeeds or its rival produces the final innovation. Similarly, when efficiency is low, both firms endeavor to develop with the incumbent technology. This invariance occurs because, in the extreme cases of very high and very low efficiency, firms do not use the information about their rival's progress even when it is observable. However, in the case of intermediate efficiency, the firms cannot use the fall-back strategy as in the public information setting since they are no longer able to make their resource allocations contingent on their rivals' state of technology. Instead, their resource allocations must depend on their 'beliefs' about their rivals' progress. We characterize the unique symmetric equilibrium that is Markov with respect to these beliefs. The equilibrium strategy has a cutoff structure: firms conduct research exclusively up to a certain date (belief), then they start allocating their resources between developing with the incumbent technology and researching the new one, namely they employ a *stationary fall-back* strategy. The most intriguing feature of this equilibrium is that beliefs remain constant once the allocation of resources to development begins. This stationarity derives from two conflicting effects in the belief evolution. First, as time passes, it becomes more likely that one's rival has found the new technology (the *duration effect*). On the other hand, the lack of one's rival producing the final innovation (which is publically observable) implies that it is less likely that the new intermediate technology has been discovered (the *still-in-the-race effect*).

Last, we extend the private information setting by allowing firms to protect their discoveries by using either a *patent* or a *trade secret*. First, when a firm treats the new technology as a trade secret, it conceals the discovery, i.e., its rival still cannot observe its progress. However, this does not prohibit the firm's rival from discovering the new technology independently. Second, when a firm files a patent, it discloses the discovery of the new technology. If its rival has not yet made the technological breakthrough, then the exclusive right to use the new technology is bestowed on the patenting firm. In addition, the patenting firm may *license* the new technology, i.e., it may permit its rival to use the new technology for a fee. Once the licensee pays the fee, both firms race for the final innovation employing the new technology. On the other hand, if the rival firm has already discovered the new technology, i.e., it was protected as a trade secret. Then, the patenting firm cannot claim the exclusive right—rather, the new technology is now considered common property—and both firms can use it without making transfers.[5][6]

We first show that if a firm files a patent and the rival firm does not possess the new technology, the patenting firm always licenses, and both firms are able to develop with the new technology, which is the socially efficient outcome. Once a firm files a patent, its rival can only try to develop the product with the old slow technology. Given this, the patenting firm can extract rent from its rival by allowing it to use the new technology for a fee. This is an application of the classical result of Coase (1960) in the sense that the socially efficient outcome can be achieved when the property right of the new technology is given to a firm and trade involves no transaction costs. Therefore, disclosing the new technology implies licensing it.

Finally, we explore whether a firm prefers to disclose or conceal the new technology. We show that this decision crucially depends on the size of the reward of winning the race: when the reward is high, firms may prefer to conceal their discoveries, whereas when the reward is low, they disclose and license them. Intuitively, this is because concealment involves a higher chance of winning the race, which is more attractive when the reward is high. Whereas, disclosure delivers an immediate payment from licensing, which is more appealing when the reward is low. More specifically, when a firm conceals a discovery, its rival does not know whether it possesses the new technology. Thus, per the results from the private information setting, the rival firm continues allocating some of its resources to researching the new technology. This is not desirable for the rival, especially when efficiency is intermediate, because if it knew that the other firm already possessed the new technology, then its best response would be to allocate all its resources to development with the incumbent technology (i.e., to employ the fall-back strategy). In this sense, concealing the new technology hinders the rival firm from strategically responding to its discovery.

[5] When a firm files a patent, the firm with the trade secret can dispute the patent based on 35 U.S. Code §273 - Defense to infringement based on prior commercial use.

[6] For more information about trade secrets and patents, see the web page of the World Intellectual Property Organization: https://www.wipo.int/about-ip/en/. Also see Lobel (2013) for examples.

Concealment is detrimental not only to the rival firm but also to social surplus because it generates duplicate research efforts. This slows down the pace of innovation. On the contrary, the socially efficient outcome could be achieved by disclosing and licensing the new technology. These results on firms' incentives for concealment imply a simple policy intervention. Reducing the reward of winning the race (e.g., weakening the transitory monopoly power in the innovative product market by imposing a tax,) reduces incentives to conceal and promotes licensing, thus speeding up the pace of ultimate innovation.

Related Literature

This paper primarily contributes to the literature on patent vs. secrecy by introducing a novel incentive to conceal a firm's discovery: hindering its rival's strategic response. Previous studies mainly focused on the limited protection power of patents. For example, the seminal article by Horstmann et al. (1985) posits that "patent coverage may not exclude profitable imitation." Thus, in their framework, the main reason why a firm may choose secrecy over a patent is not to be imitated.[7] Another limitation of a patent is that it expires in a finite time. For instance, Denicolò and Franzoni (2004) consider a framework where a patent gives the patenting firm monopoly power only for a certain period of time (and no profit after expiration), whereas secrecy can give indefinite monopoly power to a firm but it can be leaked or duplicated by a rival with some probability. On the contrary, in this paper, we abstract from the restrictions of patents and focus analysis on the potential advantages of concealment.

Another hallmark of this paper is its consideration of 'interim' discoveries. Therefore, it is naturally related to the literature on licensing of interim R&D knowledge, e.g., Bhattacharya et al. (1992); d'Aspremont et al. (2000); Bhattacharya and Guriev (2006); Spiegel (2008). In these papers it is assumed that firms already know which of them has superior knowledge, i.e., the firm that will license the technology is exogenously given. Unlike in those studies, we allow firms to choose when to license (and even allow them not to license), i.e., the licensing decision is endogenous.

[7] Many subsequent papers study the imitation threat and potential patent infringement, e.g., Gallini (1992); Takalo (1998); Anton and Yao (2004); Kultti et al. (2007); Kwon (2012); Zhang (2012).

We also contribute to the innovation literature by introducing a model with two characteristics. First, there are different avenues towards innovation: developing with the incumbent technology and doing research for the new technology. Second, one of the paths involves multiple stages: once a firm discovers the new technology, then the firm develops the innovative product with it.

With respect to the first characteristic, there is a recent branch of the literature that studies races where there are different routes to achieve a final objective. Das and Klein (2020) and Akcigit and Liu (2016) study a patent race where two firms compete for a breakthrough and there are two methods to get the breakthrough: a safe method and a risky method. In Das and Klein (2020) the safe method has a known constant arrival intensity while the risky method has an unknown constant arrival intensity. In Akcigit and Liu (2016), instead, the safe method has a known payoff associated with breakthrough arrival, while there is uncertainty about the payoff if the risky method is used. In this paper, firms face no uncertainty about whether the innovation is feasible. Instead, they are uncertain whether their rival possesses the new and faster technology.

The second characteristic, multi-stage innovation, is also widely studied in the literature, e.g., Scotchmer and Green (1990); Denicolò (2000); Green and Taylor (2016); Song and Zhao (2021). Our paper shares the framework with these in that we use two sequential Poisson discovery processes and ask whether a firm would patent the first discovery or not. A feature setting apart from their works is that there is another path that only requires one but slower breakthrough toward innovation. This feature connects our model to Carnehl and Schneider (2022) and Kim (2022) in the sense that players can choose between a sequential approach—which requires two breakthroughs—and a direct approach, which requires one but risky or slower breakthrough.[8] Our model mainly differs from theirs in that multiple players compete by choosing between these approaches, whereas Carnehl and Schneider (2022) consider a problem by a single decision maker and Kim (2022) studies a contracting setup between a principal and an agent. In their studies, a key factor for a player to choose the direct approach

[8] In Carnehl and Schneider (2022), an agent is uncertain whether the direct approach is feasible or not, i.e., this approach is risky. On the other hand, in Kim (2022), there is no uncertainty on the feasibility of the direct approach, but its completion rate is slower than the ones for the sequential approach. In this sense, our framework is closer to Kim (2022).

is a deadline that is either exogenously given or endogenously determined to reduce moral hazard. In contrast to these, a deadline is not involved in our model. Rather, the race with the rival firm may induce a firm to develop with the incumbent technology, which can be considered as a direct approach.

Last, this paper is related to the recent literature on information disclosure in priority races, e.g., Hopenhayn and Squintani (2016); Bobtcheff et al. (2017). In those papers, once a firm makes a breakthrough, the innovation value grows as time passes until one of the firms files a patent. Thus, firms face a tradeoff between disclosing to claim the priority and delaying in order to grow the innovation value. On the contrary, in this paper, the value of innovation is fixed and the discovery of the new technology only allows the firm to develop the innovative product faster. Therefore, a firm may delay the disclosure purely to confound the rival's R&D decisions.

Roadmap

We introduce the model in the next section, then characterize equilibria in the private and the public information settings in Section 3 and 4. In Section 5, we extend the private information setting by allowing firms to disclose their discoveries. We conclude in Section 6. All proofs appear in the appendix.

2 Model

Two risk-neutral firms ($i \in \{A, B\}$) race to develop an *innovative product*. Time is continuous and infinite $t \in [0, \infty)$. Neither firm discounts the future and each owns a unit of resources per unit of time. The innovative good can be developed by either a slower incumbent technology or a faster new technology. At the beginning of the race, both firms have access to the incumbent technology, but not to the new technology. To utilize the new technology, firms need to make a discovery through conducting research.

At each time t, Firm i can use a fraction of its resources either to do 'research' to discover the new technology (σ_t^i) or to 'develop' the innovative product with the incumbent technology ($1-\sigma_t^i$). With the incumbent technology, the firm develops the innovative product stochastically at rate $\lambda_L(1-\sigma_t^i)$. From the research, firms discover the new technology stochastically at rate $\mu\sigma_t^i$. Hereafter, we simply denote that at time t, Firm i *develops* when $\sigma_t^i = 0$ and *does research* when $\sigma_t^i = 1$ unless stated otherwise. The new technology does not generate any direct benefit to a firm, but helps the development faster. That is, once the new technology is obtained by a firm, it will develop the product stochastically at rate $\lambda_H > \lambda_L$.

The firm that develops the innovative product first (either with the incumbent technology or the new one) receives a reward worth Π, i.e., it is a winner-takes-all competition.[9] The other firm does not capture any rents from innovating second. Firms pay a flow cost c at each point of time until making the innovation or quitting the race. Whether a firm makes the ultimate innovation is publicly observable, however, a firm cannot observe how its opponent allocates time between attempting to discover the new technology and attempting to develop the innovative product with the old technology.

We fix λ_L and c throughout the paper. To facilitate, we introduce two relevant parameters measuring the efficiency and the relative intensity of the new technology:

$$\eta \equiv \frac{\mathbb{E}[\text{ completion time with the incumbent technology }]}{\mathbb{E}[\text{ total completion time with the new technology }]} = \frac{\frac{1}{\lambda_L}}{\frac{1}{\mu}+\frac{1}{\lambda_H}},$$

$$\delta \equiv \frac{\mathbb{E}[\text{ research completion time with the new technology }]}{\mathbb{E}[\text{ total completion time with the new technology }]} = \frac{\frac{1}{\mu}}{\frac{1}{\mu}+\frac{1}{\lambda_H}}.$$

Most of the results will be presented in terms of these parameters and Π. Note that the more efficient technology will deliver a shorter expected completion time. Thus, η will increase as the new technology becomes more efficient. We assume that the new technology is more efficient than the incumbent technology: $\eta > 1$. Also note that when η is fixed, a higher δ implies that the new technology is more research-intensive (or less development-intensive). This is because when firms try to achieve innovation via the new technology, they are expected to spend more time in research as δ increases. Last, we assume that the

[9] A winner-takes-all payoff structure has been commonly used in the innovation race literature, e.g., Loury (1979); Lee and Wilde (1980); Denicolò and Franzoni (2010). Following the literature, we can regard Π as the societal value of having the innovative product, and the first firm that introduces the innovative product becomes the monopolist and captures all the social value, e.g., by using the first-degree price discrimination. In this article, we abstract away from the market after the race and focus on the activities during the race.

incumbent technology is profitable: $\Pi \geq c/\lambda_L$ or equivalently $\pi \equiv \lambda_L \Pi / c \geq 1$.

We conclude the section by introducing the planner's problem, namely the first-best case. Consider a social planner who is able to control firms' resource allocations and observe progress. In addition, the planner can force a firm to share its technology to the other firm. The goal of the planner is to maximize the joint expected profit. Since there is no discounting in time and one of firms will receive the reward, the planner's problem is equivalent to the minimization problem of the expected completion time of the product development. Then, the planner's optimal solution is to take the more efficient path:

(i) both firms fully allocate their resources to research and the research is completed at rate 2μ;

(ii) if one of the firms discovers the new technology, and it immediately shares the new technology to the other firm, and then the firms develop the innovative product with the new technology—the development is completed at rate $2\lambda_H$.

Hence, the (ex-ante) expected completion time is given as follows:

$$L_{FB} = \frac{1}{2\mu} + \frac{1}{2\lambda_H} = \frac{1}{2\lambda_L \eta}. \tag{1}$$

3 Public Information Setting

We begin by exploring a setting where firms' research progress is public information, i.e., firms can observe whether their opponents have discovered the new technology. In this case, the list of firms that have obtained the new technology is common knowledge and we can regard it as a state: $\omega \in \Omega \equiv \{\{A, B\}, \{A\}, \{B\}, \emptyset\}$. We assume the firms employ Markov strategies, i.e., Firm i's strategy is defined by $\sigma^i : \Omega \to [0, 1]$. Since a firm possessing the new technology derives no value from research, we can restrict attention to the strategies such that $\sigma^i(\omega) = 0$ when $i \in \omega$. A pair of strategies (σ^A, σ^B) constitutes a Markov perfect equilibrium if, for any state, each firm's strategy is the best response to the opponent's strategy.

Before characterizing the equilibrium of this game, we introduce three benchmark strategies.

Definition 3.1. (a) A *research strategy* is such that a firm fully allocates the resources to research regardless of the opponent's technology level ($\sigma^i(\emptyset) = \sigma^i(\{j\}) = 1$).

(b) A *fall-back strategy* is such that (i) a firm fully allocates the resources to doing research when neither firm has obtained the new technology ($\sigma^i(\emptyset) = 1$); (ii) a firm fully allocates the resources to developing with the incumbent technology if its opponent discovers the new technology ($\sigma^i(\{j\}) = 0$).

(c) An *incumbent strategy* is such that a firm fully allocates the resources to developing with the incumbent technology regardless of the opponent's technology level ($\sigma^i(\emptyset) = \sigma^i(\{j\}) = 0$).

The following proposition shows that it is a Markov perfect equilibrium for both firms to simultaneously use one of the above strategies, depending on parameter values. The proof is in Appendix A.

Proposition 1. *Suppose that firms can observe whether their opponents have made a technological breakthrough. Then, the Markov perfect equilibrium is uniquely characterized as follows.*

(a) If $\eta \geq \overline{\eta}(\delta) \equiv 1 + \delta$*, both firms play the research strategies;*

(b) If $\overline{\eta}(\delta) > \eta > \underline{\eta}(\delta) \equiv \frac{1}{2}\left(1 + \sqrt{1 + 4\delta(1-\delta)}\right)$*, both firms play the fall-back strategies;*

(c) If $\underline{\eta}(\delta) \geq \eta$*, both firms play the incumbent strategies.*

The above proposition provides a clear picture of how the efficiency of the new technology (η) affects the firms' R&D decisions. When the new technology is sufficiently efficient and research is relatively easy ($\eta \geq \overline{\eta}(\delta)$), the firms do research regardless of whether their opponent has discovered the new technology. When the new technology is relatively inefficient ($\eta \leq \underline{\eta}(\delta)$), the firms do not engage in research at all. In the intermediate case

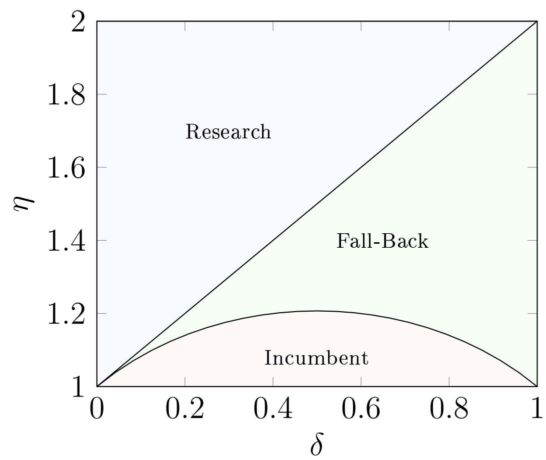


Figure 1: Markov Perfect Equilibrium under the Public Information Setting

($\underline{\eta}(\delta) < \eta < \overline{\eta}(\delta)$), the firms' R&D decisions are affected by the opponent's progress: when neither firm has made the technological breakthrough, both firms do research; but once a firm obtains the new technology, the follower—the firm without the new technology—switches to develop with the incumbent technology.

The proposition also shows that the thresholds depend on δ, the relative intensity of the new technology. Figure 1 illustrates how these thresholds depend on δ. First, to determine the threshold for the equilibrium with the research strategy, we need to consider the case when one firm (the leader) has discovered the new technology and the other (the follower) has not. Say that Firm i is the follower and Firm j is the leader. Firm i needs to determine whether to follow j ($\sigma^i(\{j\}) = 1$) or to switch to the incumbent technology ($\sigma^i(\{j\}) = 0$). When it is difficult to attain the new technology, the follower would choose to follow only if the new technology is efficient enough. Thus, the threshold for the equilibrium with the research strategy is increasing as δ increases.

Second, why is the threshold for the equilibrium with the incumbent strategy hump-shaped? To answer this question, we need to consider the situation where neither firm yet possesses the new technology. The firms allocate resources by taking into account the difficulty and the advantage of becoming a leader. Consider the case with $\delta < 1/2$, i.e., attaining the new technology is relatively easy, but the leader's advantage is relatively weak (due to low λ_H). In this case, the major determinant is the difficulty of becoming a leader. Fix the efficiency (η) and marginally increase δ. Then it becomes more difficult to attain leadership, which makes the incumbent strategy more attractive. Next, consider the case with $\delta > 1/2$, i.e., attaining the new technology is relatively difficult, but it is more advantageous to become a leader (due to high λ_H). In this case, the major determinant is the leader's advantage. If δ decreases, the leader's advantage decreases, which again makes the incumbent strategy more attractive.

We conclude this section by deriving the expected completion time of developing the product in the public information setting. When $\underline{\eta}(\delta) \geq \eta$, the expected completion time is $\frac{1}{2\lambda_L}$ since both firms develop with the incumbent technology. Next, when $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$, the expected time until one of the firms discover new technology is $\frac{1}{2\mu}$. Then, a firm develops with the new technology and the other firm develops with the incumbent technology, thus, the expected completion time from then on is $\frac{1}{\lambda_H+\lambda_L}$. Therefore, the (total) expected completion time is $\frac{1}{2\mu} + \frac{1}{\lambda_H+\lambda_L}$. Last, when $\eta \geq \overline{\eta}(\delta)$, unlike in the previous case, the firm without the new technology keeps doing research. Then, the expected time until either the firm with the new technology develops the product or the firm without the new technology discovers it is $\frac{1}{\lambda_H+\mu}$. With the probability $\frac{\mu}{\lambda_H+\mu}$, the firm without the new technology discovers it earlier than the product development, then it takes an additional expected completion time $\frac{1}{2\lambda_H}$. Therefore, the total expected completion time is

$$\frac{1}{2\mu} + \frac{1}{\lambda_H + \mu} + \frac{\mu}{\lambda_H + \mu} \cdot \frac{1}{2\lambda_H} = \frac{1}{2}\left(\frac{1}{\mu} + \frac{1}{\lambda_H} + \frac{1}{\lambda_H + \mu}\right).$$

The expected completion time is summarized as follows:

$$L_{public} = \begin{cases} \frac{1}{2}\left(\frac{1}{\mu} + \frac{1}{\lambda_H} + \frac{1}{\lambda_H + \mu}\right), & \text{if } \eta \geq \overline{\eta}(\delta), \\ \frac{1}{2\mu} + \frac{1}{\lambda_H + \lambda_L}, & \text{if } \eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta)), \\ \frac{1}{2\lambda_L}, & \text{if } \eta \leq \underline{\eta}(\delta). \end{cases} \tag{2}$$

4 Private Information Setting

Now we assume that firms' research progress is private information, i.e., firms cannot observe whether their opponents have the new technology or not. In this case, the firms can only condition their strategies on their own progress and calendar time t. Again, a firm with the new technology will fully allocate its resources to develop with that technology. Thus, we focus on the dynamic resource allocation problem of a firm that has not discovered the new technology: a strategy for a player can be therefore described by a function $\sigma : \mathbb{R}_+ \to [0, 1]$ that represents the allocation at a given time conditional on the new technology not being discovered. Let $\mathcal{S}$ be the set of such strategies.

4.1 Evolution of Beliefs and Recursive Formulation

Although firms cannot observe their opponents' technology levels, they do form beliefs about whether their opponents have acquired the new technology. Let p_t^i be Firm i's belief that Firm j possesses the new technology given that neither firm has made the innovative product by t and that Firm j follows the resource allocation $\sigma^j : \mathbb{R}_+ \to [0, 1]$.[10] Since it is common knowledge that neither firm possesses the new technology at the beginning of the race, we have $p_0^i = p_0^j = 0$, which will serve as an initial condition for the belief evolution. The following lemma characterizes how the belief p^i evolves conditional on the absence of innovation. The proof is provided in Appendix B.

Lemma 4.1 (Evolution of Beliefs). *Suppose that Firm i does not observe whether Firm j has made a technological breakthrough and believes that Firm j follows the resource allocation σ^j. Then, in the absence of innovation, Firm i's belief, p^i, evolves via the following differential equation:*

$$-\frac{d}{dt}\log(1-p_t^i) = \frac{\dot{p}_t^i}{1-p_t^i} = \mu \cdot \sigma_t^j - \left\{\lambda_H - (1-\sigma_t^j)\lambda_L\right\} \cdot p_t^i. \tag{3}$$

The left hand side of (3) is the opposite of the time derivative for the log belief that Firm j does not possess the new technology. The right hand side of (3) captures two distinct effects

[10] It is technically convenient to assume strategies depend on time rather than beliefs about the other firms breakthrough status because it is cumbersome to analyze all off-path beliefs. As we show below, in any symmetric equilibrium there is a unique mapping from calendar time to beliefs.

in updating the belief. First, given that Firm j has not yet attained the new technology by time t, the research succeeds at rate $\mu \cdot \sigma_t^j$ and it may raise the belief. The first term of (3) represents this effect, which we dub the duration effect (DE). On the other hand, the fact that firm j has not produced the innovative good indicates that it is less likely to have the new technology in hand. The second term of (3) reflects this effect, which we dub the still-in-the-race effect (SRE).[11] Notice that this term is proportional to $\lambda_H - (1-\sigma_t^j)\lambda_L$, which is the rate of successful innovation development given the new technology net of that without the new technology.

A natural benchmark is a firm's belief when the opponent fully allocates its resources to research. We characterize this belief in the following lemma. The proof is also in Appendix B.

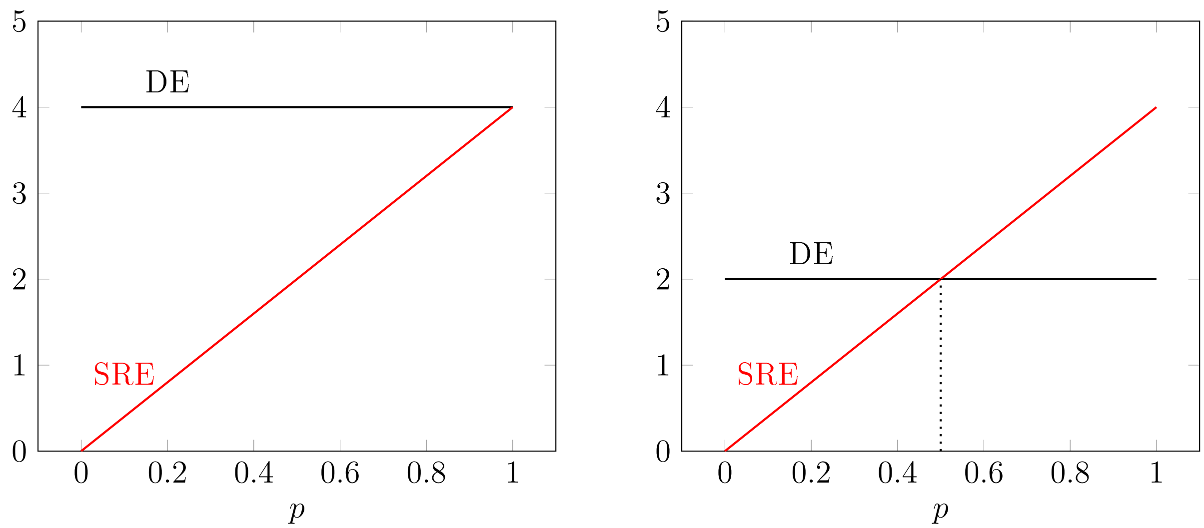
Lemma 4.2. *Suppose that Firm i does not observe whether Firm j has made a technological breakthrough and believes that Firm j fully allocates the resources to research up to time T ($\sigma_t^j = 1$ for all $0 \leq t \leq T$). Then, $p_T^i = q_T$ where*

$$q_T \equiv \frac{\frac{1}{\lambda_H}\left(e^{-\mu T} - e^{-\lambda_H T}\right)}{\frac{1}{\mu}e^{-\mu T} - \frac{1}{\lambda_H}e^{-\lambda_H T}}. \tag{4}$$

In addition, q_T is increasing in T, $\lim_{T\to\infty} q_T = 1$ if $\mu > \lambda_H$ (or equivalently $\delta < 1/2$), and $\lim_{T\to\infty} q_T = \mu/\lambda_H = (1-\delta)/\delta$ if $\mu < \lambda_H$ (or equivalently $\delta > 1/2$).

This result is easily understood as a tradeoff between the duration effect and the still-in-the-race effect. In Figure 2, we illustrate these effects when Firm j fully allocates its resources to research ($\sigma_t^j = 1$ for all $t \geq 0$). Specifically, we provide the graphs of the terms of each effect divided by $(1-p)$: μ (DE), $\lambda_H p$ (SRE). In Figure 2a, we depict the case where $\mu = \lambda_H$, i.e., $\delta = 1/2$. Observe that the duration effect is always larger than the still-in-the-race effect here. If we fix λ_H and increase μ, we can easily see that the duration effect will still dominate the still-in-the-race effect. Hence, when $\delta = \lambda_H/(\lambda_H + \mu) < 1/2$, we can see that the belief keeps increasing up to 1 ($\lim_{T\to\infty} q_T = 1$). On the other hand,

[11] Similar types of the belief updating can be found in the strategic experimentation literature, e.g., Keller et al. (2005); Bonatti and Hörner (2011). The main difference is that the agents form beliefs about whether the project is good or bad in those papers, whereas the firms form beliefs about whether the rival possesses the new technology or not in our model.



(a) $\sigma = 1$, $\mu = 4$, $\lambda_H = 4$ and $\delta = 1/2$. (b) $\sigma = 1$, $\mu = 2$, $\lambda_H = 4$ and $\delta = 2/3$.

Figure 2: Duration Effect (Black) and Still-in-the-Race Effect (Red)

in Figure 2b, we illustrate the case where $\mu < \lambda_H$, i.e., $\delta < 1/2$. Observe that the duration effect is greater than the still-in-the-race effect only when $p < \mu/\lambda_H$. This induces the belief to converge to $(1-\delta)/\delta$ ($\lim_{T\to\infty} q_T = \mu/\lambda_H = (1-\delta)/\delta$).

We now explore the dynamics of the firms' expected payoffs. Suppose that Firm i possesses the new technology and Firm j's strategy is σ^j. Then, the expected payoff of Firm i at time T, denoted by $V_T^{1,i}$, can be written as follows.

$$
\begin{aligned}
V_T^{1,i} = & - cdt + \Pi \cdot \lambda_H dt + 0 \cdot \left(\lambda_H p_T^i + \lambda_L (1-p_T^i)(1-\sigma_T^j)\right) dt \\
& + (1 - \lambda_H dt - \lambda_H p_T^i dt - \lambda_L (1-p_T^i)(1-\sigma_T^j) dt)(V_T^{1,i} + \dot{V}_T^{1,i} dt).
\end{aligned}
$$

Thus, we can derive the Hamilton-Jacobi-Bellman (HJB) equation:

$$
0 = \dot{V}_T^{1,i} + \lambda_H(\Pi - V_T^{1,i}) - \left\{\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)\right\} V_T^{1,i} - c. \tag{HJB$_1$}
$$

This HJB equation gives a clear interpretation on the evolution of $V_T^{1,i}$: at an instant T, (i) Firm i wins the race at rate λ_H and gets the rent Π but loses the continuation payoff $V_T^{1,i}$; (ii) Firm j wins the race at rate $\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)$ and Firm i loses the continuation payoff; (iii) the flow cost c is charged.

Next, let $V_T^{0,i}$ denote the expected payoff of Firm i at time T given that it has not succeeded in research. In this case, the firm needs to choose between doing research and developing with the incumbent technology:

$$
\begin{aligned}
V_T^{0,i} = \max_{\sigma_T^i \in [0,1]} & - cdt + \Pi \cdot \lambda_L(1-\sigma_T^i)dt + V_T^{1,i} \cdot \mu\sigma_T^i dt + 0 \cdot \left(\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)\right) dt \\
& + \left(1 - \lambda_L(1-\sigma_T^i)dt - \mu\sigma_T^i dt - \lambda_H p_T^i dt - \lambda_L(1-p_T^i)(1-\sigma_T^j)dt\right)(V_T^{0,i} + \dot{V}_T^{0,i} dt).
\end{aligned}
$$

By using the linearity of σ_T^i, the corresponding HJB equation can be derived as follows:

$$
\begin{aligned}
0 = {} & \dot{V}_T^{0,i} - \left\{\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)\right\} V_T^{0,i} - c \\
& + \max_{\sigma_T^i \in [0,1]} \left[\sigma_T^i \cdot \mu(V_T^{1,i} - V_T^{0,i}) + (1-\sigma_T^i) \cdot \lambda_L(\Pi - V_T^{0,i})\right].
\end{aligned} \tag{HJB$_0$}
$$

This HJB equation determines whether Firm i allocates the resources to research or development. If $\mu(V_T^{1,i} - V_T^{0,i}) > \lambda_L(\Pi - V_T^{0,i})$, Firm i allocates all the resources to doing research: $\sigma_T^i = 1$. If $\lambda_L(\Pi - V_T^{0,i}) > \mu(V_T^{1,i} - V_T^{0,i})$, Firm i allocates all the resources to developing with the incumbent technology: $\sigma_T^i = 0$. If $\mu(V_T^{1,i} - V_T^{0,i}) = \lambda_L(\Pi - V_T^{0,i})$, Firm i is indifferent between the research and the development: $\sigma_T^i \in [0,1]$.

4.2 Symmetric Markov Equilibrium

To characterize the equilibrium in the private information setting, we focus on strategy profiles that are symmetric and Markov with respect to the belief on the opponent's technology level. Formally, we call $\sigma : \mathbb{R}_+ \to [0,1]$ a *symmetric Markov strategy* if $\sigma^A = \sigma^B = \sigma$, and for $i \in \{A, B\}$, $p_t^i = p_{t'}^i$ implies $\sigma_t^i = \sigma_{t'}^i$ where p^i is the belief process derived from $p_0^i = 0$ and (3). A strategy profile σ constitutes a *symmetric Markov equilibrium* if σ is a symmetric Markov strategy and $\sigma^A = \sigma^B = \sigma$ solves (HJB$_0$) for all $T \geq 0$. The following proposition characterizes the symmetric Markov equilibria of the game.

Proposition 2. *Suppose that the firms cannot observe their opponent's technological level. The following statements hold.*

(a) ***(Cutoff Structure)*** *Any symmetric Markov equilibrium can be characterized by a*

cutoff time $T^* \in \mathbb{R} \cup \{\infty\}$ and a stationary strategy $\sigma^* \in [0,1)$ where both firms fully conduct research up to time T^* ($\sigma_t = 1$ for all $t < T^*$), and choose σ^* from then on ($\sigma_t = \sigma^*$ for all $t > T^*$).

(b) ***(Equilibrium Characterization)*** *The unique symmetric Markov equilibrium is characterized as follows.*

(i) *If $\eta \geq \tilde{\eta}(\delta) \equiv \min\{\overline{\eta}(\delta), 2-\delta\}$, both firms play* ***research strategy*** *($T^* = \infty$). In equilibrium, a firm's expected payoffs with and without the new technology are $V_0^t = \overline{V}_0(q_t)$ and $V_1^t = \overline{V}_1(q_t)$ where q is the belief defined in (4) and*

$$\overline{V}_1(q) \equiv \frac{1}{2}\left(\Pi - \frac{c}{\lambda_H}\right)(1+\delta(1-q)), \tag{5}$$

$$\overline{V}_0(q) \equiv \frac{1}{2}\left(\Pi - \frac{c}{\mu} - \frac{c}{\lambda_H}\right)(1-\delta q) - \frac{c}{2(\lambda_H+\mu)}. \tag{6}$$

(ii) *If $\eta \leq \underline{\eta}(\delta)$, both firms play* ***incumbent strategy*** *($T^* = 0$, $\sigma^* = 0$). In equilibrium, the expected payoff of each firm is $V_t^0 = \frac{\lambda_L \Pi - c}{2\lambda_L}$ for all $t \geq 0$.*[12]

(iii) *If $\tilde{\eta}(\delta) > \eta > \underline{\eta}(\delta)$, both firms play* ***stationary fall-back strategy*** *($T^* \in (0,\infty)$, $\sigma^* \in (0,1)$). Moreover, for all $t \geq T^*$, $\sigma_t = \sigma^*$, $p_t = p^*$, $V_t^1 = V_1^*$ and $V_t^0 = V_0^*$ where*

$$\sigma^* = \frac{\eta}{1-\delta} - \frac{\eta+\delta}{\eta-\delta}, \qquad p^* = \frac{1}{2}\left\{\frac{\eta}{\delta} - \frac{1-\delta}{\eta-1}\right\},$$
$$V_1^* = \frac{\lambda_L}{\mu} \cdot \frac{\lambda_H \Pi - c}{\lambda_H - \lambda_L}, \qquad V_0^* = \frac{\lambda_L}{\mu - \lambda_L} \cdot \frac{\lambda_L \Pi - c}{\lambda_H - \lambda_L}.\text{[13]} \tag{7}$$

The formal proof of the proposition is relegated to Appendix C, but we provide a sketch of the proof here. We begin by showing that the belief derived from a symmetric Markov strategy is nondecreasing in time, i.e., $\dot{p}_t \geq 0$ (Lemma C.1). This is because if $\dot{p}_t < 0$ for some $t > 0$, by the Markov property, the belief cannot go above p_t which contradicts $\dot{p}_t < 0$. Note that $\dot{p}_t < 0$ if $p_t > 0$ and $\sigma_t = 0$. This allows us to focus on the following two cases: (i)

[12] When a firm possesses the new technology—though it is off the equilibrium path—the expected payoff is $V_1^t = \frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$.

[13] See Remark 1 for the expected payoffs for $t \in [0, T^*]$

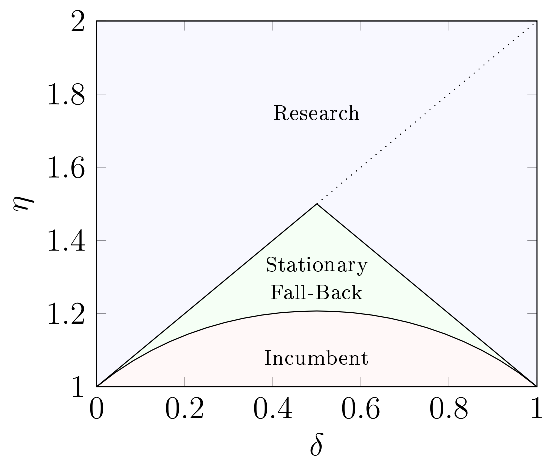


Figure 3: Symmetric Markov Equilibrium under the Private Information Setting

$\sigma_t = 0$ for all $t \geq 0$; or (ii) $\sigma_t > 0$ for all $t \geq 0$ (Lemma C.2). The first case corresponds to incumbent strategy in the observable-breakthrough benchmark ($T^* = 0$ and $\sigma^* = 0$). Even though the strategy space is different from the benchmark, it is qualitatively equivalent since the firm always develops with the incumbent technology. Similarly, a special case of the second case—$\sigma_t = 1$ for all $t \geq 0$—corresponds to research strategy ($T^* = \infty$). In the remaining case, on the equilibrium path, $\sigma_S \in (0, 1)$ for some $S \geq 0$, i.e., firms are indifferent between researching to find the new technology and developing the innovation with the old technology at time S. In this case, we show that from then on ($t \geq S$), the firms continue to be indifferent between research and development (Lemma C.3). In addition, we show that to make firms indifferent for all $t \geq S$, the firms' strategies and beliefs should be stationary: $\sigma_t = \sigma^*$ and $p_t = p^*$ for all $t \geq S$ (Lemma C.4). By identifying the earliest time T at which firms are indifferent, we can show that it corresponds to the stationary fall-back strategy: $\sigma_t = 1$ for all $t < T$ and $\sigma_t = \sigma^*$ for all $t > T$. Thus, we have three types of symmetric Markov equilibria: both firms play (i) the research strategy; (ii) the incumbent strategy; or (iii) the stationary fall-back strategy.

Next, we need to identify which regions of the parameter space give rise to each of the three types of equilibrium. First, consider parameter values under which the fall-back policy is not the second-best policy ($\eta \geq \overline{\eta}(\delta)$ or $\eta \leq \underline{\eta}(\delta)$). In this case, firms do not change their resource allocations even if they can observe their opponent's technological breakthroughs. Thus, the same strategy profiles (fully conducting research or fully developing with the incumbent technology) will constitute an equilibrium in the private information setting.

Now consider the remaining case ($\overline{\eta}(\delta) > \eta > \underline{\eta}(\delta)$). In this case, the new technology is efficient enough ($\eta > \underline{\eta}(\delta)$) for both firms to begin by doing only research. However, They need to determine whether to keep fully conducting research indefinitely ($T^* = \infty$) or to hedge their bets by switching to the stationary fall-back strategy at some point ($T^* < \infty$ and $\sigma^* \in (0,1)$). The answer crucially depends on the relative intensity δ. When the new technology is more development-intensive ($\delta < 1/2$), if firms keep fully conducting research indefinitely, then the beliefs that their opponent has made a breakthrough converge to 1 by Lemma 4.2. Since they are in the parameter region where firms switch to development with the incumbent technology if they know that their opponent possesses the new technology (or equivalently $p = 1$), they will find it better to choose the stationary fall-back strategy when the belief is sufficiently close to 1. Next, consider the case where the new technology is more research-intensive ($\delta > 1/2$). In contrast to the previous case, there exists a region where both firms play the research strategy indefinitely in equilibrium ($2 - \delta < \eta < \overline{\eta}(\delta) = 1 + \delta$). This result is also easily understood by considering the belief about technological status. By Lemma 4.2, this belief cannot exceed $(1-\delta)/\delta$ in this region of the parameter space. Since the belief is bounded strictly below 1, firms may find it preferable to keep fully conducting research in the private information setting, even though they would switch to development with the incumbent technology if they were able to observe a breakthrough by their opponent. As δ increases, the upper bound of the belief $(1-\delta)/\delta$ decreases, so the firms will keep fully conducting the research even with the relatively low η. Thus, the threshold between the equilibria with the stationary fall-back strategy and the research strategy decreases in δ.

Remark 1. In Lemma C.6, we characterize the generic forms of the expected payoffs when both firms use $\sigma_t = 1$ for all $t \in [0, T]$. It allows us to derive the expected payoffs at each point in time under the equilibrium with the stationary fall-back policy. By using $V^1_{T^*} = V^*_1$ and $V^0_{T^*} = V^*_0$ in (7) as terminal conditions at time T^*, the constants C_0 and C_1 in (26) and (27) can be determined. Moreover, C_1 is negative (see the proof of Lemma C.8). Thus, the expected payoffs V^1_t and V^0_t for any $t \in [0, T^*]$ can be derived.

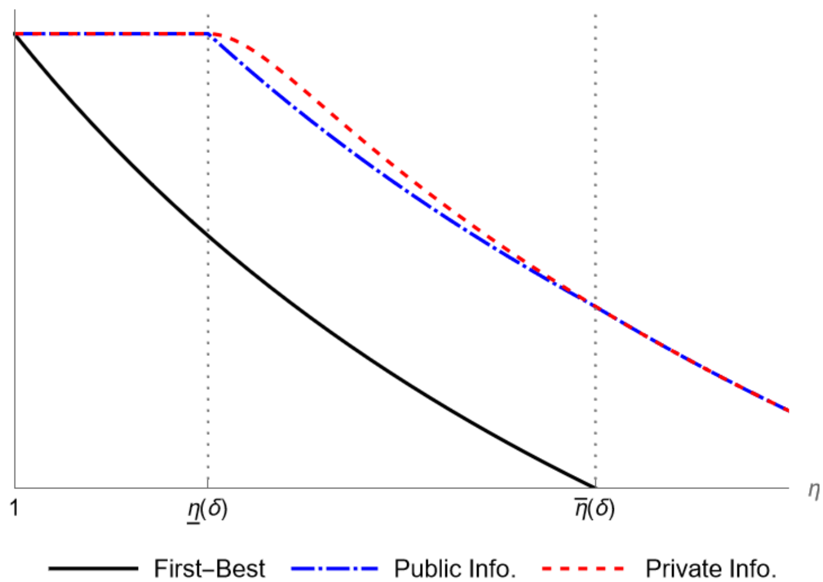


Figure 4: Expected completion times for the first-best case, private and public information settings

4.3 Comparison of Expected Completion Times

Now that we characterized the firms' equilibrium behavior in the private information settings, we compare it with the benchmarks. Recall that the (ex ante) expected completed times for the first-best case and the public information setting are characterized in equations (1) and (2). In Figure 4, we fix δ and display the curves of the expected completion times for the first-best case (black) and the second-best case (blue) with respect to the efficiency measure η. The gap between these two expected completion times is generated since the new technology is not shared.

Observe that when $\eta \geq \overline{\eta}(\delta)$ or $\eta \leq \underline{\eta}(\delta)$, the equilibrium strategy in the private information setting is consistent with the public information setting, i.e., the expected completion times for these settings are same. When $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$, in the private information setting, the firms cannot use the fall-back strategy because they are not able to observe the rivals' technology levels. Rather, they use stationary fall-back strategies ($\eta \in (\underline{\eta}(\delta), \tilde{\eta}(\delta))$) or research strategies ($\eta \in [\tilde{\eta}(\delta), \overline{\eta}(\delta))$), which are suboptimal compared to the public information case. Therefore, the expected completion time would be longer, i.e., a lack of information transmission about the research status retards the pace of innovation. Figure 4 also illustrates these results: there is a gap between the expected completion times of the public and private information settings only if $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$.

5 Disclosure vs. Concealment

We now extend the private information setting by adding options to protect interim discoveries. Once a firm discovers the new technology, it can choose to conceal its discovery and treat it as a trade secret or disclose it to file a patent. In addition, if a firm files a patent, it can decide whether to license it or not. To facilitate the analysis, we assume that $\eta > \underline{\eta}(\delta)$ to rule out the case where firms do not engage in research at all.

5.1 The Game after Patenting

We begin by describing the game that takes place after a firm files a patent. First, consider the case where the rival already had the new technology and protected it as a trade secret. Then, the trade secret right allows the rival firm to dispute the patent. Thus, both firms have the right to use the new technology and the expected payoffs of them are $V_C = \frac{\lambda_H \Pi - c}{2\lambda_H}$.

Next, suppose that the rival firm does not possess the new technology. Then, the patenting firm has the exclusive right to use the new technology and the rival firm has to develop with the incumbent technology. Then, the expected payoffs of the patenting firm and the rival firm are $V_P = \frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$ and $V_R = \frac{\lambda_L \Pi - c}{\lambda_H + \lambda_L}$.

Now we explore whether the patenting firm has an incentive to license the technology. Assume that the patenting firm can make a take-it-or-leave-it (TIOLI) offer $x \in \mathbb{R}_+$ to the rival firm for the right to use the new technology.[14] After licensing, both firms can use the new technology. Thus, the expected payoffs of the patenting firm and the rival firm after licensing are $V_C + x$ and $V_C - x$. Then, the optimal offer x^* should satisfy $V_C - x^* = V_R$. With simple algebra, we can derive that

$$x^* = \frac{(\lambda_H - \lambda_L)(\lambda_H \Pi + c)}{2\lambda_H(\lambda_H + \lambda_L)} = \frac{\lambda_H - \lambda_L}{\lambda_H + \lambda_L}\left(V_C + \frac{c}{\lambda_H}\right) > 0. \tag{8}$$

[14]This assumption implies that the patenting firm has all the bargaining power. If the firm conceals the discovery even in this case, then it will conceal it in the less bargaining power cases.

Then, the expected payoff for the patenting firm after licensing is

$$V_L \equiv V_C + x^* = \left(1 + \frac{(\lambda_H - \lambda_L)c}{\lambda_H(\lambda_H \Pi - c)}\right) V_P > V_P. \tag{9}$$

This implies that the patenting firm can always be better off by licensing the new technology.

5.2 Immediate-Disclosure Equilibrium

We first explore whether the first-best outcome can be achieved by allowing the firms to disclose the new technology and license it. Recall that in the first-best case, both firms do research, and a firm's new technology is immediately spilled over to the rival. Thus, we consider a strategy profile such that a firm with the new technology employs the *immediate-disclosure strategy*—a firm discloses (and licenses) the new technology as soon as it discovers—and a firm without the new technology employs the research strategy ($\sigma_t = 1$ for all $t \geq 0$). Then, we ask whether both firms playing this strategy can be sustained as an equilibrium.

Suppose that a firm (say Firm A) just discovered the new technology and Firm B has not disclosed it yet. Given that Firm B sticks to the immediate disclosure and research strategy, Firm A's belief that Firm B has the new technology is zero. Then, by disclosing the new technology, Firm A expects to license it, i.e., the expected payoff for Firm A after disclosure is V_L. Now consider Firm A's deviation to delay the disclosure by time dt. With the probability $\lambda_H dt$, Firm A wins the race and receives Π. But with the probability μdt, Firm B will discover the new technology and files a patent, but it will be disputed by Firm A's trade secret right. Thus, both firms will race with the new technology from then on and the expected payoff is V_C. With the probability $(1 - \lambda_H dt - \mu dt)$, neither of the events happens and Firm A licenses, then the expected payoff is V_L. Last, the flow cost cdt will be paid. To sum up, Firm A's expected payoff from delaying the disclosure is

$$\begin{aligned}
\Pi \cdot \lambda_H dt + V_C \cdot \mu dt + (1 - \lambda_H dt - \mu dt) \cdot V_L - cdt &= V_L + [(\mu + 2\lambda_H)V_C - (\mu + \lambda_H)V_L]\, dt \\
&= V_L + [\lambda_H V_C - (\mu + \lambda_H)x^*]\, dt.
\end{aligned}$$

Then, from $\delta = \lambda_H/(\mu+\lambda_H)$, the immediate-disclosure and research strategy can be sustained as an equilibrium if and only if $\delta V_C \leq x^*$. By using (8) and some algebra, this inequality is equivalent to:

$$\frac{(1-\delta)(\overline{\eta}(\delta)-\eta)}{2(\eta-1+\delta)} \leq \frac{c}{\lambda_H \Pi - c}. \tag{10}$$

From the assumption that $\lambda_L \Pi \geq c$, observe that (10) always holds if $\eta \geq \overline{\eta}(\delta)$. Recall that firms do research regardless of the rival's progress. It implies that there does not exist any incentive for a firm to conceal its progress. Therefore, the firms would monetize the new technology by licensing it as soon as it discovers, and the first-best outcome would be achieved.

Next, suppose that $\underline{\eta}(\delta) < \eta < \overline{\eta}(\delta)$. Then, (10) is equivalent to:

$$\pi = \frac{\lambda_L \Pi}{c} \leq 1 + \frac{\eta^2 - (1-\delta)^2}{\eta(\overline{\eta}(\delta)-\eta)} \equiv \underline{\pi}(\eta, \delta). \tag{11}$$

Also note that $\tilde{\pi}(\eta, \delta) > 1$ since $\eta > 1 - \delta > 0$. Therefore, when the reward of winning the race is sufficiently low ($1 \leq \pi \leq \underline{\pi}(\eta, \delta)$), the firms would license the new technology as soon as it discovers.

The following proposition formally summarizes the above results.

Proposition 3. *Suppose that one of the following conditions holds: (i) $\eta \geq \overline{\eta}(\delta)$; or (ii) $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$ and $1 \leq \pi \leq \underline{\pi}(\eta, \delta)$. Then, there exists an equilibrium such that the firms fully allocate their resources to research and license as soon as they find the new technology.*

5.3 No-Disclosure Equilibrium

We now explore whether the worst-case scenario for the planner can be realized, i.e., firms never disclose their discoveries and the expected completion time corresponds to the private information setting case.

First, we consider the case where $\tilde{\eta}(\delta) = \min\{2-\delta, \overline{\eta}(\delta)\} \leq \eta < \overline{\eta}(\delta) = 1 + \delta$. By Proposition 2, in the equilibrium under the private information setting, firms do research until it succeeds ($T^* = \infty$ and $\sigma_t = 1$ for all $t \geq 0$). Suppose that both firms stick to this resource allocation strategy and never disclose their discoveries. When Firm A discovers the new technology at time t and never discloses it, the expected payoff of Firm A is $V_1^t = \{1+\delta(1-q_t)\} \cdot V_C$ by Proposition 2 (b-i). If Firm A discloses the discovery at time t, Firm B has the new technology with the probability q_t and does not with the probability $1-q_t$. Thus, the expected payoff from the disclosure is $V_C \cdot q_t + V_L \cdot (1-q_t) = V_C + (1-q_t)x^*$. Therefore, the firm will not disclose if $x^* < \delta V_C$. We can also consider the case where Firm A discovers at time t but conceals until t' and decides to disclose or not at time t'. Even in this case, Firm A faces the same problem as before and will not disclose if $x^* < \delta V_C$. Recall that $x^* < \delta V_C$ is equivalent to $\pi > \underline{\pi}(\eta, \delta)$. Therefore, if $\pi > \underline{\pi}(\eta, \delta)$, there exists an equilibrium such that firms never disclose their discoveries and do research until it succeeds.

Next, we consider the case where $\underline{\eta}(\delta) < \eta < \tilde{\eta}(\delta)$. By Proposition 2 (b-iii), in the equilibrium under the private information setting, firms employ the stationary fall-back strategy (for some $T^* \in (0,\infty)$ and $\sigma^* \in (0,1)$, $\sigma_t = 1$ for all $0 \leq t < T^*$ and $\sigma_t = \sigma^*$ for all $t \geq T^*$). Suppose that Firm A discovers the new technology at $t \geq T^*$. If Firm A keeps the discovery secret, the expected payoff of Firm A is $V_1^* = \dfrac{2\delta}{\eta - 1 + \delta} V_C$. In addition, $V_1^* < \overline{V}_1(p^*) = \{1+\delta(1-p^*)\} V_C$ (see Lemma C.8). On the other hand, if Firm A discloses the discovery, the expected payoff from the disclosure is $V_C + (1-p^*)x^*$. Then, Firm A does not disclose under the condition stronger than $x^* < \delta V_C$. In this case, there exists $\overline{\pi}(\eta, \delta) > \underline{\pi}(\eta, \delta)$ such that Firm A does not disclose when $\pi > \overline{\pi}(\eta, \delta)$. The following proposition formally states this result.

Proposition 4. *Suppose that $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$. Then, there exists $\overline{\pi}(\eta, \delta) > \underline{\pi}(\eta, \delta)$ such that for all $\lambda_L \Pi / c > \overline{\pi}(\eta, \delta)$, the following strategy can be sustained as an equilibrium: (i) firms never disclose their discoveries; (ii) and employs the equilibrium resource allocations in the private setting: $\sigma_t = 1$ for all $t < T^*$ and $\sigma_t = \sigma^*$ for all $t > T^*$ for some $T^* \in (0, \infty]$ and $\sigma^* \in (0,1)$.*

6 Conclusion

In this article, we study the long-lasting question of patent vs. secrecy from a different angle from the literature: a firm's concealing motive to hinder the rival's strategic response. We introduce an innovation race model with multiple paths and show that firms' disclosing decisions depend on the reward of winning the race. Based on this result, we can argue that reducing the reward for the winner may promote licensing which would speed up the pace of innovation.

There are many avenues open for further research. For example, we assume that there are exogenously given two paths towards innovation, and one of the paths requires two breakthroughs. However, in practice, there are numerous ways to make an innovation, and it often requires more than two breakthroughs. We also assume that a firm's R&D resources are fixed over time, but we could also allow firms to endogenously choose how much effort to put in each point in time. Finally, we assume the contest structure is given by the winner-takes-all competition, but we might consider a contest designing problem. We leave these intriguing questions and others for future work.

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Appendix

A Proofs for Public Information Setting

A.1 Transformation

First, we transform the conditions in terms of μ.

Lemma A.1. *Define* $\overline{\mu} \equiv \frac{2\lambda_L\lambda_H}{\lambda_H-\lambda_L}$ *and* $\underline{\mu} \equiv \frac{\lambda_L(\lambda_H+\lambda_L)}{\lambda_H-\lambda_L}$. *Then,* $\eta \geq \overline{\eta}(\delta)$ *is equivalent to* $\mu \geq \overline{\mu}$ *and* $\eta \leq \underline{\eta}(\delta)$ *is equivalent to* $\mu \leq \underline{\mu}$.

Proof of Lemma A.1. Observe that

$$\begin{aligned}\eta - \overline{\eta}(\delta) = \eta - (1+\delta) =& \frac{\mu\lambda_H}{\lambda_L(\lambda_H+\mu)} - \frac{\lambda_H}{\lambda_H+\mu} - 1\\ =& \frac{\mu(\lambda_H-\lambda_L) - 2\lambda_L\lambda_H}{\lambda_L(\lambda_H+\mu)} = \frac{\lambda_H-\lambda_L}{\lambda_L(\lambda_H+\mu)}(\mu - \overline{\mu}).\end{aligned}$$

Therefore, $\eta \geq \overline{\eta}(\delta)$ is equivalent to $\mu \geq \overline{\mu}$.

Also observe that

$$\begin{aligned}\left(\underline{\eta}(\delta) - \eta\right)\left(\eta - \frac{1-\sqrt{1+4\delta(1-\delta)}}{2}\right) =& -\eta^2 + \eta + (1-\delta)\delta\\ =& -\left(\frac{\mu\lambda_H}{\lambda_L(\lambda_H+\mu)}\right)^2 + \frac{\mu\lambda_H}{\lambda_L(\lambda_H+\mu)} + \frac{\mu\lambda_H}{(\lambda_H+\mu)^2}\\ =& \frac{\mu\lambda_H(\lambda_H-\lambda_L)}{\lambda_L^2(\lambda_H+\mu)^2}\left(\underline{\mu} - \mu\right).\end{aligned}$$

Note that $\eta - \left\{1 - \sqrt{1+4\delta(1-\delta)}\right\}/2 > 0$ from $\eta > 1$. Thus, $\eta \leq \underline{\eta}(\delta)$ is equivalent to $\mu \leq \underline{\mu}$. $\square$

A.2 Proof of Proposition 1

A.2.1 Best Responses

Assume that a firm possesses the new technology. The following lemma characterizes the opponent firm's optimal strategy.

Lemma A.2. *Suppose that Firm i possesses the new technology and the opponent Firm j does not. Firm j prefers to develop with the incumbent technology if and only if $\eta < \overline{\eta}(\delta)$.*

Proof of Lemma A.2. Let U^j_ω denote Firm j's expected profit at the state ω. Observe that $U^A_{\{A,B\}} = U^B_{\{A,B\}} = \frac{1}{2}\left(\Pi - \frac{c}{\lambda_H}\right) = V_C$.

Now consider the case with $\omega = \{i\}$, i.e., Firm i possesses the new technology. Then, Firm j chooses between doing research and developing with the incumbent technology:

$$U^j_{\{i\}} = \max\left\{\frac{\mu V_C - c}{\mu + \lambda_H}, \ \frac{\lambda_L \Pi - c}{\lambda_L + \lambda_H}\right\}.$$

Observe that

$$\begin{aligned}
(\lambda_L + \lambda_H)(\mu + \lambda_H)\left\{\frac{\mu V_C - c}{\mu + \lambda_H} - \frac{\lambda_L \Pi - c}{\lambda_L + \lambda_H}\right\} =&(\lambda_L + \lambda_H)(\mu V_C - c) - (\mu + \lambda_H)(\lambda_L \Pi - c)\\
=&\mu \cdot \{(\lambda_L + \lambda_H)V_C - (\lambda_L \Pi - c)\} - \lambda_L(\lambda_H \Pi + c)\\
=&\mu \cdot \left\{\left(\frac{\lambda_H - \lambda_L}{2}\right)\Pi + \frac{\lambda_H - \lambda_L}{2\lambda_H} c\right\} - \lambda_L(\lambda_H \Pi + c)\\
=&(\lambda_H \Pi + c)\left[\left(\frac{\lambda_H - \lambda_L}{2\lambda_H}\right)\mu - \lambda_L\right]\\
=&(\lambda_H \Pi + c)\frac{\lambda_H - \lambda_L}{2\lambda_H}\{\mu - \overline{\mu}\}
\end{aligned}$$

Then, $U^j_{\{i\}} = \dfrac{\mu V_C - c}{\mu + \lambda_H}$ if and only if $\mu \geq \overline{\mu}$, or equivalently, $\eta \geq \overline{\eta}(\delta)$ by Lemma A.1. Thus, when Firm j possesses the new technology, Firm i conducts research if $\eta \geq \overline{\eta}(\delta)$ and develops with the incumbent technology if $\eta < \overline{\eta}(\delta)$. $\square$

A.2.2 MPE Characterization

We can divide the proof of Proposition 1 into following three lemmas.

Lemma A.3. *Assume $\eta < \underline{\eta}(\delta)$. Then it is the unique equilibrium for both firms to play the incumbent strategy.*

Proof of Lemma A.3. By Lemma A.2, once a firm finds the new technology, i.e., $\omega = \{A\}$ or $\{B\}$, the other one switches to developing with the incumbent technology since $\eta < \underline{\eta}(\delta) \leq \overline{\eta}(\delta)$. In these cases, The expected payoffs of the firms are given as follows:

$$U^A_{\{A\}} = U^B_{\{B\}} = \frac{\lambda_H \Pi - c}{\lambda_L + \lambda_H} \qquad \text{and} \qquad U^B_{\{A\}} = U^A_{\{B\}} = \frac{\lambda_L \Pi - c}{\lambda_L + \lambda_H}. \tag{12}$$

It implies that Firm i's optimal strategy under the state j is $\sigma^i_{\{j\}} = 0$.

Now consider the state $\emptyset$. Let $\pi_i(\sigma^i_\emptyset, \sigma^j_\emptyset)$ be the expected payoff of Firm i when it plays strategy $\sigma^i_\emptyset$ when neither firm possesses the new technology and plays the optimal continuation strategy thereafter.

Under $\sigma^i_\emptyset = 1$ and $\sigma^j_\emptyset = 0$, Firm i's expected profit is

$$\pi_i(1,0) = \frac{\mu U^i_{\{i\}} - c}{\mu + \lambda_L} = \frac{-c}{\mu + \lambda_L} + \frac{\mu}{\mu + \lambda_L} \frac{\lambda_H \Pi - c}{\lambda_L + \lambda_H}$$

Firm i's expected profit when both firms play the incumbent strategy is $\pi(0,0) = \frac{\lambda_L \Pi - c}{2\lambda_L}$. Then,

$$\pi_i(0,0) - \pi_i(1,0) = \frac{(\lambda_L \Pi + c)(\lambda_H - \lambda_L)(\underline{\mu} - \mu)}{2\lambda_L(\mu + \lambda_L)(\lambda_H + \lambda_L)}. \tag{13}$$

When $\eta < \underline{\eta}(\delta)$, $\mu < \underline{\mu}$ by Lemma A.1. Given $\sigma^j_\emptyset = 0$, $\sigma^i_\emptyset = 0$ is the best response, i.e., both firms playing the incumbent strategies constitutes an equilibrium.

To ensure the uniqueness, we can similarly look for the best response for $\sigma^j_\emptyset = 1$. Notice that:

$$\pi_i(1,1) = \frac{\mu \cdot U^i_{\{i\}} + \mu \cdot U^i_{\{j\}} - c}{2\mu} \qquad \text{and} \qquad \pi_i(0,1) = \frac{\lambda_L \Pi + \mu U^i_{\{j\}} - c}{\lambda_L + \mu}.$$

Then, we also have

$$\pi_i(0,1) - \pi_i(1,1) = \frac{(\mu \Pi + c)(\lambda_H - \lambda_L)(\underline{\mu} - \mu)}{2\mu(\mu + \lambda_L)(\lambda_H + \lambda_L)}. \tag{14}$$

Thus, under the assumption that $\eta < \underline{\eta}(\delta)$ or equivalently $\mu < \underline{\mu}$, $\sigma^i_\emptyset = 0$ is the dominant strategy at the state $\emptyset$, and both firms playing the incumbent strategy is the unique equilibrium. $\square$

Lemma A.4. *Assume $\underline{\eta}(\delta) < \eta < \overline{\eta}(\delta)$. Then, it is the unique equilibrium for both firms to play fall-back strategies.*

Proof of Lemma A.4. By the assumption $\eta < \overline{\eta}(\delta)$ and Lemma A.2, the equations (12) in Lemma A.3 hold, that is, Firm i's optimal strategy under the state j is $\sigma^i_{\{j\}} = 0$.

Next, consider the state $\omega = \emptyset$. By the equations (13) and (14) in Lemma A.3, we have $\pi_i(1,0) \geq \pi_i(0,0)$ and $\pi_i(1,1) \geq \pi_i(0,1)$ under the assumption that $\eta \geq \underline{\eta}(\delta)$. It implies that $\sigma^i_\emptyset = 1$ is the dominant strategy at the state $\emptyset$.

Thus, the strategy profile that both firms play fall-back strategies constitutes the unique equilibrium. $\square$

Lemma A.5. *Assume $\eta > \overline{\eta}(\delta)$. Then it is the unique equilibrium for both firms to play the research strategy.*

Proof of Lemma A.5. Recall that $\eta > \overline{\eta}(\delta)$ is equivalent to $\mu > \overline{\mu} \equiv \frac{2\lambda_L\lambda_H}{\lambda_H - \lambda_L}$.

By Lemma A.2, it is efficient for a firm to do research once the opponent obtains the new technology, so:

$$U^i_{\{i\}} = \frac{\lambda_H \Pi + \mu V_C - c}{\mu + \lambda_H} \qquad \text{and} \qquad U^i_{\{j\}} = \frac{\mu V_C - c}{\mu + \lambda_H}.$$

At the state $\emptyset$, if Firm j does research, doing research gives Firm i:

$$\pi_i(1,1) = \frac{\mu U^i_{\{i\}} + \mu U^i_{\{j\}} - c}{\mu + \mu} = \frac{1}{2}\Pi - \frac{c}{2\mu} - \frac{c}{\lambda_H + \mu} - \frac{\mu}{\lambda_H + \mu}\frac{c}{2\lambda_H}.$$

Developing with the incumbent technology, instead, gives the firm:

$$\pi_i(0,1) = \frac{\lambda_L \Pi + \mu U^i_{\{j\}} - c}{\lambda_L + \mu}$$

We get that:

$$\pi_i(1,1) - \pi_i(0,1) = \frac{(\lambda_H - \lambda_L)(\lambda_H \Pi + c)\mu^2 - \lambda_H(\lambda_H \lambda_L \Pi + 3\lambda_L c - \lambda_H c)\mu - \lambda_H^2 \lambda_L c}{2\mu\lambda_H(\mu + \lambda_H)(\mu + \lambda_L)}.$$

We want to show that the numerator, $K_1(\mu)$, is positive. To do so notice that:

$$K_1(\bar{\mu}) = \lambda_H^2 \lambda_L \left(\frac{2\lambda_H \lambda_L \Pi}{\lambda_H - \lambda_L} + c \right) > 0.$$

Moreover,

$$\left. \frac{\partial K_1}{\partial \mu} \right|_{\mu = \bar{\mu}} = \lambda_H \left(3\lambda_L \lambda_H \Pi + (\lambda_L + \lambda_H) c \right) > 0,$$

and

$$\frac{\partial^2 K}{\partial \mu^2} = 2(\lambda_H \Pi + c)(\lambda_H - \lambda_L) > 0.$$

Thus, it must be that $K_1(\mu)$ is positive for all $\mu \geq \bar{\mu}$.

On the other hand, if Firm j develops with the incumbent technology at state $\emptyset$, doing research gives Firm i:

$$\pi_i(1,0) = \frac{\mu U^i_{\{i\}} - c}{\mu + \lambda_L}.$$

If instead Firm i develops with the incumbent technology,

$$\pi_i(0,0) = \frac{\lambda_L \Pi - c}{2\lambda_L}.$$

Subtracting we get:

$$\pi_i(1,0) - \pi_i(0,0) = \frac{c(\lambda_H - \lambda_L)\mu^2 + \lambda_H(\lambda_L(\lambda_H - \lambda_L)\Pi + (\lambda_H - 3\lambda_L)c)\mu - \lambda_H^2 \lambda_L(\lambda_L \Pi + c)}{2\lambda_H \lambda_L(\mu + \lambda_H)(\mu + \lambda_L)}.$$

Again, we want to show that the numerator, $K_0(\mu)$, is positive. To do so notice that:

$$K_0(\bar{\mu}) = \lambda_H^2 \lambda_L(\lambda_L \Pi + c) > 0$$

Moreover,

$$\left. \frac{\partial K_0}{\partial \mu} \right|_{\mu = \bar{\mu}} = \lambda_H(\lambda_L(\lambda_H - \lambda_L)\Pi + (\lambda_H + \lambda_L)c) > 0,$$

and

$$\frac{\partial^2 K_0}{(\partial\mu)^2} = 2c(\lambda_H - \lambda_L) > 0$$

Thus, it must be that $K_0(\mu)$ is positive for all $\mu \geq \bar{\mu}$. $\square$

B Private Information: Evolution of Beliefs

Proof of Lemma 4.1. Suppose that Firm j allocates σ_t^j attention to research conditional on not having found the good project so far. Finally, let $\Sigma_t = \int_0^t \sigma_s^j \, ds$.

Let p_t^i be the belief of Firm i that Firm j obtained the new technology by time t.

$$\begin{aligned}\frac{1-p_t}{p_t} &= \frac{\Pr(\text{no success in research and development})}{\Pr(\text{success in research but no success in development})} \\ &= \frac{e^{-\mu\Sigma_t}\cdot e^{-\lambda_L(t-\Sigma_t)}}{\int_0^t \mu\cdot\sigma_s\cdot e^{-\mu\Sigma_s}\cdot e^{-\lambda_L(s-\Sigma_s)}\cdot e^{-\lambda_H(t-s)}\, ds}\end{aligned} \tag{15}$$

Let U_t and R_t be the numerator and the denominator of the right hand side of (15). Note that

$$\begin{aligned}\frac{\partial U_t}{\partial t} &= -U_t\cdot\big((\mu-\lambda_L)\sigma_t^j + \lambda_L\big) \\ \frac{\partial R_t}{\partial t} &= \mu\cdot\sigma_t^j\cdot U_t - \lambda_H\cdot R_t\end{aligned}$$

By Differentiating (15) and multiplying R_t/U_t,

$$\begin{aligned}-\frac{\dot{p}_t^i}{(1-p_t^i)p_t^i} =& \frac{-U_t\cdot\big((\mu-\lambda_L)\sigma_t^j+\lambda_L\big)\cdot R_t - \mu\cdot\sigma_t^j\cdot U_t^2 + \lambda_H\cdot R_t\cdot U_t}{R_t^2}\cdot\frac{R_t}{U_t} \\ =& -\big((\mu-\lambda_L)\sigma_t^j+\lambda_L\big) + \lambda_H - \mu\cdot\sigma_t^j(1-p_t^i)/p_t^i \\ =& \big\{\lambda_H - \lambda_L(1-\sigma_t^j)\big\} - \mu\cdot\sigma_t^j/p_t^i.\end{aligned}$$

By multiplying $-(1-p_t^i)p_t^i$, we have (3). $\square$

Proof of Lemma 4.2. By plugging $\sigma_t = 1$ to (3), we have $\dot{p}_t = (\mu - \lambda_H p_t)(1 - p_t)$. By rearranging the differential equation, we can derive that

$$\lambda_H - \mu = (\lambda_H - \mu)\frac{\lambda_H \dot{p}_t}{(\mu - \lambda_H p_t)(\lambda_H - \lambda_H p_t)} = \frac{d}{dt}\log\left(\frac{\lambda_H - \lambda_H p_t}{\mu - \lambda_H p_t}\right).$$

Then, from $p_0 = 0$, we can derive that

$$\frac{\lambda_H(1 - p_T)}{\mu - \lambda_H p_T} = \frac{\lambda_H}{\mu}e^{(\lambda_H - \mu)T}.$$

By rearranging the above equation, we have (4).

Observe that

$$\dot{q}_T = \frac{\mu(\lambda_H - \mu)^2 e^{(\lambda_H + \mu)T}}{(\lambda_H e^{\lambda_H T} - \mu e^{\mu T})^2} > 0.$$

Thus, q is increasing in T.

When $\mu > \lambda_H$,

$$\lim_{T\to\infty} q_T = \lim_{T\to\infty} \frac{\frac{1}{\lambda_H}\left(e^{(\lambda_H - \mu)T} - 1\right)}{\frac{1}{\mu}e^{(\lambda_H - \mu)T} - \frac{1}{\lambda_H}} = 1.$$

When $\mu < \lambda_H$,

$$\lim_{T\to\infty} q_T = \lim_{T\to\infty} \frac{\frac{1}{\lambda_H}\left(1 - e^{(\mu - \lambda_H)T}\right)}{\frac{1}{\mu} - \frac{1}{\lambda_H}e^{(\mu - \lambda_H)T}} = \frac{\mu}{\lambda_H}.$$

□

C Private Information: Equilibria

C.1 Cutoff Structure

C.1.1 Lemmas

Lemma C.1. *If the belief process $\{p_t\}$ is derived from a symmetric Markov strategy σ, then $\dot{p}_t \geq 0$ for all $t \geq 0$.*

Proof of Lemma C.1. Suppose that $\dot{p}_t < 0$ for some $t \geq 0$. Note that the belief is nonnegative. Then, $p_{t-\eta} > p_t \geq 0$ for a small $\eta > 0$. Also note that $p_0 = 0$ and $\dot{p}_0 = \mu \cdot \sigma_0 \geq 0$ by (3).

By the continuity of p, there exists $t' < t$ such that $p_{t'} = p_t$ and $\dot{p}_{t'} \geq 0$. However, since the strategy is Markov, $\sigma_{t'} = \sigma_t$, which gives $\dot{p}_{t'} = \dot{p}_t$ and contradicts $\dot{p}_{t'} \geq 0 > \dot{p}_t$. Therefore, $\dot{p}_t \geq 0$ for all $t \geq 0$. $\square$

Lemma C.2. *If σ constitutes a symmetric Markov equilibrium, $\sigma_t = 0$ for all $t \geq 0$ or $\sigma_t > 0$ for all $t \geq 0$.*

Proof of Lemma C.2. Consider the case with $p_t > 0$. If $\sigma_t = 0$, by (3), $\dot{p}_t = -(\lambda_H - \lambda_L)p_t(1-p_t)$. Since p_t cannot be equal to 1, $\dot{p}_t < 0$, which contradicts the previous result. Therefore, $\sigma_t > 0$ whenever $p_t > 0$. If $\sigma_0 = 0$, then $\dot{p}_0 = 0$ and the belief stays at 0. By the Markov property, $\sigma_t = 0$ for all $t \geq 0$. If $\sigma_0 > 0$, then $\dot{p}_0 > 0$ and $p_t > 0$ for a small enough $t > 0$. Then, $\sigma_t = 0$ will never be chosen, i.e., $\sigma_t > 0$ for all $t \geq 0$. $\square$

Lemma C.3. *If σ constitutes a symmetric Markov equilibrium and $\sigma_S \in (0, 1)$ for some $S \geq 0$, then $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ for all $t \geq S$.*

Proof of Lemma C.3. By Lemma C.2, if $\sigma_S > 0$ for some $S \geq 0$, $\sigma_t > 0$ for all $t > 0$, thus, $\mu(V_t^1 - V_t^0) \geq \lambda_L(\Pi - V_t^0)$ for all $t \geq 0$.

Assume the contrary. Then, we can properly define $T \equiv \inf\{t > S \mid \mu(V_t^1 - V_t^0) > \lambda_L(\Pi - V_t^0)\}$. Then, $\mu(V_s^1 - V_s^0) = \lambda_L(\Pi - V_s^0)$ holds for $S \leq s \leq T$, and for some $\delta > 0$, $\mu(V_s^1 - V_s^0) > \lambda_L(\Pi - V_s^0)$ for all $T < s < T + \delta$.

When $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$, we show that $\mu(\dot{V}_t^1 - \dot{V}_t^0) \leq -\lambda_L \dot{V}_t^0$ if and only if

$$\lambda_H p_t + \lambda_L(1 - p_t)(1 - \sigma_t) \leq \frac{\mu(\lambda_H - \lambda_L)(\Pi - V_t^1) - \lambda_L c}{\lambda_L \Pi}.$$

First, by (HJB$_1$) and (HJB$_0$), we have

$$\mu \dot{V}_t^1 = \mu(\lambda_H + X_t)V_t^1 - \mu(\lambda_H \Pi - c), \tag{16}$$

$$(\mu - \lambda_L)\dot{V}_t^0 = (\mu - \lambda_L)X_t V_t^0 - (\mu - \lambda_L)(\mu(V_t^1 - V_t^0) - c), \tag{17}$$

where $X_t = \lambda_H p_t + \lambda_L(1-p_t)(1-\sigma_t)$. Also note that $(\mu-\lambda_L)(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^1)$. Then, $\mu(\dot{V}_t^1 - \dot{V}_t^0) \leq -\lambda_L \dot{V}_t^0$ is equivalent to:

$$
\begin{aligned}
\lambda_L \Pi \cdot X_t =& \left\{\mu V_t^1 - (\mu-\lambda_L)V_t^0\right\} X_t \\
\leq& \mu(\lambda_H \Pi - c) - \mu\lambda_H V_t^1 + (\mu-\lambda_L)c - \mu(\mu-\lambda_L)(V_t^1 - V_t^0) \\
=& \mu(\lambda_H - \lambda_L)(\Pi - V_t^1) - \lambda_L c.
\end{aligned}
$$

Let $\sigma_{T^-} := \lim_{t\to T^-} \sigma_t$ and $\sigma_{T^+} := \lim_{t\to T^+} \sigma_t$. Note that $\sigma_{T^+} = 1$. By the continuity of p and V^1, we have $p_{T^-} = p_{T^+} = p_T$ and $V_{T^-}^1 = V_{T^+}^1 = V_T^1$.

First, consider the case with $\sigma_{T^-} < 1$.[15] In this case, we have

$$
\begin{aligned}
X_{T^+} =& \lambda_H p_T + \lambda_L(1-p_T)(1-\sigma_{T^+}) \\
<& \lambda_H p_T + \lambda_L(1-p_T)(1-\sigma_{T^-}) = \frac{\mu(\lambda_H - \lambda_L)(\Pi - V_T^1) - \lambda_L c}{\lambda_L \Pi}.
\end{aligned}
$$

Then, $\mu(\dot{V}_{T^+}^1 - \dot{V}_{T^+}^0) < -\lambda_L \dot{V}_{T^+}^0$. Since $\mu(V_T^1 - V_T^0) = \lambda_L(\Pi - V_T^0)$, for small enough $\eta > 0$, $\lambda_L(\Pi - V_{T+\eta}^0) > \mu(V_{T+\eta}^1 - V_{T+\eta}^0)$ which contradicts $\sigma_{T^+} = 1$.

Next, consider the case with $\sigma_{T^-} = 1$. Note that $\mu(V_T^1 - V_T^0) = \lambda_L(\Pi - V_T^0)$ and $\mu(\dot{V}_T^1 - \dot{V}_T^0) = -\lambda_L \dot{V}_T^0$. If we show that $\mu(\ddot{V}_{T^+}^1 - \ddot{V}_{T^+}^0) < -\lambda_L \ddot{V}_{T^+}^0$, it contradicts $\sigma_{T^+} = 1$. Observe that $\sigma_{T^+} = 1$ and $\dot{\sigma}_{T^+} = 0$, thus, $\dot{X}_{T^+} = \lambda_H \dot{p}_T$. By taking derivatives for (16) and (17), we can derive that

$$
\begin{aligned}
\mu \ddot{V}_{T^+}^1 =& \mu\lambda_H \left[(1+p_T)\dot{V}_T^1 + \dot{p}_T V_T^1\right], \\
(\mu - \lambda_L)\ddot{V}_{T^+}^1 =& (\mu-\lambda_L)\left[\lambda_H p_T \dot{V}_T^0 + \lambda_H \dot{p}_T V_T^0 - \mu(\dot{V}_T^1 - \dot{V}_T^0)\right].
\end{aligned}
$$

[15] It is possible that σ_{T^-} is not properly defined. Even in this case, the following argument still holds by considering a converging subsequence of $\{\sigma_t\}$.

Then, we have

$$
\begin{aligned}
\mu\ddot{V}^1_{T^+} - (\mu-\lambda_L)\ddot{V}^0_{T^+} =& \lambda_H p_T \left\{ \mu \dot{V}^1_T - (\mu-\lambda_L)\dot{V}^0_T \right\} + \mu\lambda_H \dot{V}^1_T \\
& + \lambda_H \dot{p}_T \left\{ \mu V^1_T - (\mu-\lambda_L) V^0_T \right\} + \mu(\mu-\lambda_L)(\dot{V}^1_T - \dot{V}^0_T) \\
=& \mu(\lambda_H - \lambda_L)\dot{V}^1_T.
\end{aligned} \tag{18}
$$

Note that $(\lambda_H \Pi - c)/(\lambda_H + \lambda_H p_T)$ is the expected payoff of the firm (with the new technology) at time T if the opponent shuts down when it does not possess the new technology at time T. Then, in equilibrium, the expected payoff cannot exceed this value: $V^1_T < (\lambda_H \Pi - c)/(\lambda_H + \lambda_H p_T)$. From (16), we have $\dot{V}^1_T < 0$. By (18), $\mu(\ddot{V}^1_{T^+} - \ddot{V}^0_{T^+}) < -\lambda_L \ddot{V}^0_{T^+}$, which contradicts $\sigma_{T^+} = 1$. □

Lemma C.4. *Suppose that there exists $0 \le T < \infty$ such that σ is a symmetric Markov equilibrium with $\sigma_t = 1$ for all $t < T$ and $\mu(V^1_t - V^0_t) = \lambda_L(\Pi - V^0_t)$ for all $t \ge T$. Then, for all $t \ge T$, $\sigma_t = \sigma^*$, $p_t = p^*$, $V^1_t = V^*_1$, and $V^0_t = V^*_0$, i.e., σ is a stationary fall-back equilibrium. In addition, $T = \frac{1}{\lambda_H - \mu} \log\left(\frac{\mu(1-p^*)}{\mu - \lambda_H p^*}\right)$.*

Proof of Lemma C.4. To have $0 < \sigma_t < 1$ for all $t \ge T$,

$$
\mu(V^1_t - V^0_t) = \lambda_L(\Pi - V^0_t). \tag{19}
$$

By taking a derivative, we also have

$$
\mu \dot{V}^1_t = (\mu - \lambda_L)\dot{V}^0_t. \tag{20}
$$

Define $X(p_t, \sigma_t) \equiv \lambda_H p_t + \lambda_L(1-p_t)(1-\sigma_t)$. By ($\text{HJB}_1$), we have

$$
\mu \dot{V}^1_t = X(p_t, \sigma_t)\mu \cdot V^1_t - \mu\lambda_H(\Pi - V^1_t) + \mu c. \tag{21}
$$

By (HJB_0) and (19), we also have

$$\begin{aligned}(\mu-\lambda_L)\dot{V}_t^0 =& X(p_t,\sigma_t)(\mu-\lambda_L)V_t^0-(\mu-\lambda_L)\mu(V_t^1-V_t^0)+(\mu-\lambda_L)c \\ =& X(p_t,\sigma_t)(\mu V_t^1-\lambda_L\Pi)-\mu\lambda_L(\Pi-V_t^1)+(\mu-\lambda_L)c.\end{aligned} \tag{22}$$

By using (20), (21) and (22), we have

$$X(p_t,\sigma_t)=\frac{\mu(\lambda_H-\lambda_L)(\Pi-V_t^1)-\lambda_L c}{\lambda_L\Pi}. \tag{23}$$

By plugging (23) into (HJB_1), we can derive that

$$\begin{aligned}0 =& \dot{V}_t^1-\frac{1}{\lambda_L\Pi}\left\{\mu(\lambda_H-\lambda_L)(\Pi-V_t^1)-\lambda_L c\right\}V_t^1+\lambda_H(\Pi-V_t^1)-c \\ =& \dot{V}_t^1-\frac{\mu(\lambda_H-\lambda_L)}{\lambda_L\Pi}\left\{V_t^1-\frac{\lambda_L(\lambda_H\Pi-c)}{\mu(\lambda_H-\lambda_L)}\right\}(\Pi-V_t^1) \\ =& \dot{V}_t^1-\frac{\alpha}{\Pi-\beta}(V_t^1-\beta)(\Pi-V_t^1)\end{aligned} \tag{24}$$

where $\alpha\equiv\frac{\mu(\lambda_H-\lambda_L)(\Pi-\beta)}{\lambda_L\Pi}$ and $\beta\equiv\frac{\lambda_L(\lambda_H\Pi-c)}{\mu(\lambda_H-\lambda_L)}$. Note that

$$\Pi-\beta=\frac{\lambda_L\lambda_H}{\lambda_H-\lambda_L}\left[\left(\frac{1}{\lambda_L}-\frac{1}{\lambda_H}-\frac{1}{\mu}\right)\Pi+\frac{c}{\mu}\right]>0.$$

Thus, α and β are strictly positive.

Also note that $\Pi-c/\lambda_H>V_t^1$ for all $t\geq 0$ since the firm's expected profit under the competition cannot exceed that without the competition. If $V_T^1>\beta$, $V_t^1>\beta$ for all $t\geq T$. If not, there exists $t>T$ such that $V_t>\beta$ and $\dot{V}_t=0$, which contradict (24). Likewise, if $V_T^1<\beta$, $V_t^1<\beta$ for all $t\geq T$. Now suppose that $V_T^1\neq\beta$. By (24), we have

$$\alpha=\frac{(\Pi-\beta)}{(V_t^1-\beta)(\Pi-V_t^1)}\dot{V}_t^1=\frac{d}{dt}\log\left(\frac{|\beta-V_t^1|}{\Pi-V_t^1}\right)$$

By integrating the above equation side-by-side from T to t, we have

$$\begin{aligned} \alpha(t-T) &= \log\left(\frac{|\beta - V_t^1|}{\Pi - V_t^1}\right) - \log\left(\frac{|\beta - V_T^1|}{\Pi - V_T^1}\right) \\ \iff \qquad \frac{|\beta - V_T^1|}{\Pi - V_T^1} e^{\alpha(t-T)} &= \frac{|\beta - V_t^1|}{\Pi - V_t^1}. \end{aligned}$$

Notice that the right-hand-side is bounded above and below since $V_t^1 < \Pi - c/\lambda_H$. The left-hand-side diverges to positive or negative infinite. Thus, it must be that $V_T^1 = \beta$.

In addition, by solving (24) with the initial condition $V_T^1 = \beta$, we have $V_t^1 = \beta = V_1^*$ for all $t \geq T$. By plugging $V_t^1 = \beta$ into (19), we also have

$$V_t^0 = \frac{\mu\beta - \lambda_L \Pi}{\mu - \lambda_L} = \frac{\lambda_L}{\mu - \lambda_L}\left(-\Pi + \frac{\lambda_H \Pi - c}{\lambda_H - \lambda_L}\right) = \frac{\lambda_L}{\mu - \lambda_L} \cdot \frac{\lambda_L \Pi - c}{\lambda_H - \lambda_L} = V_0^*$$

Next, by plugging $V_t^1 = \beta$ into (23), we have

$$\lambda_H p_t + \lambda_L (1 - p_t)(1 - \sigma_t) = \frac{\mu(\lambda_H - \lambda_L)(\Pi - \beta) - \lambda_L c}{\lambda_L \Pi} = \frac{\lambda_H - \lambda_L}{\lambda_L}\mu - \lambda_H,$$

or equivalently,

$$1 - \sigma_t = \frac{\left(\frac{\lambda_H - \lambda_L}{\lambda_L}\right)\mu - \lambda_H(1 + p_t)}{\lambda_L(1 - p_t)}. \tag{25}$$

From (3), we have

$$\begin{aligned} \dot{p}_t =& (1 - p_t)\left[\mu - \lambda_H p_t - (1 - \sigma_t)(\mu - \lambda_L p_t)\right] \\ =& (1 - p_t)(\mu - \lambda_H p_t) - \left\{\left(\frac{\lambda_H - \lambda_L}{\lambda_L} - \lambda_H(1 + p_t)\right)\mu\right\}\left(\frac{\mu}{\lambda_L} - p_t\right) \\ =& \frac{\mu(\lambda_H - \lambda_L)}{\lambda_L^2}(\underline{\mu} - \mu) + \left(\frac{\lambda_H - \lambda_L}{\lambda_L}\right)(2\mu - \overline{\mu})p_t \\ =& \frac{(\lambda_H - \lambda_L)(2\mu - \overline{\mu})}{\lambda_L}(p_t - p^*). \end{aligned}$$

If $p_T \neq p^*$, then the solution of the above differential equation diverges and contradicts $0 \leq p_t \leq 1$ for all $t \geq T$. Therefore, $p_T = p^*$ and it also gives $p_t = p^*$ for all $t \geq T$. Also note that

$$1 - p^* = \frac{(\overline{\mu} - \mu)(\mu - \lambda_L)}{\lambda_L(2\mu - \overline{\mu})}.$$

By plugging this into (25), for all $t \geq T$, we have

$$\begin{aligned}
\sigma_t =& 1 - \frac{\left(\frac{\lambda_H - \lambda_L}{\lambda_L}\right)\mu - 2\lambda_H + \lambda_H(1-p^*)}{\lambda_L(1-p^*)} \\
=& 1 - \frac{\frac{\lambda_H - \lambda_L}{\lambda_L}(\mu - \overline{\mu}) + \lambda_H \frac{(\overline{\mu}-\mu)(\mu-\lambda_L)}{\lambda_L(2\mu-\overline{\mu})}}{\frac{(\overline{\mu}-\mu)(\mu-\lambda_L)}{(2\mu-\overline{\mu})}} \\
=& 1 - \frac{-(\lambda_H - \lambda_L)(2\mu - \overline{\mu}) + \lambda_H(\mu - \lambda_L)}{\lambda_L(\mu - \lambda_L)} \\
=& 1 - \frac{-(\lambda_H - \lambda_L)(\mu - \lambda_L) + \lambda_L(\mu + \lambda_L)}{\lambda_L(\mu - \lambda_L)} = \sigma^*.
\end{aligned}$$

Last, since $\sigma_t = 1$ for all $0 \leq t \leq T$, the belief that the opponent has the new technology at time t is $p^* = p_T = q_T$ where q is defined as in (4). By the definition of q, we can derive that $e^{(\lambda_H - \mu)T} = \frac{\mu(1-p^*)}{\mu - \lambda_H p^*}$, or equivalently, $T = \frac{1}{\lambda_H - \mu} \log\left(\frac{\mu(1-p^*)}{\mu - \lambda_H p^*}\right)$. □

C.1.2 Proof of Proposition 2 (a)

Proof of Proposition 2 (a). By Lemma C.1, σ satisfies $\sigma_t = 0$ for all $t \geq 0$ or $\sigma_t > 0$ for all $t \geq 0$. In the former case, we can set $T^* = 0$ and $\sigma^* = 0$, then we have $\sigma_t = \sigma^*$ for all $t > T^* = 0$. Now consider the latter case. If $\{t \geq 0 | \sigma_t \in (0,1)\}$ is empty, set $T^* = \infty$. Then, $\sigma_t = 1$ for all $t < T^*$. If $\{t \geq 0 | \sigma_t \in (0,1)\}$ is nonempty, we can properly define $T^* \equiv \inf\{t \geq 0 | \sigma_t \in (0,1)\} < \infty$. Then, $\sigma_t = 1$ for all $t < T^*$. In addition, by Lemma C.3, $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ for all $t \geq T^*$. By Lemma C.4, $\sigma_t = \sigma^*$ for all $t \geq T^*$. □

C.2 Equilibrium Characterization

C.2.1 The Equilibrium with Incumbent Strategies

Lemma C.5. *Suppose that σ is the incumbent strategy, i.e., $\sigma_t = 0$ for all $t \geq 0$. Then, $\sigma^A = \sigma^B = \sigma$ constitutes a symmetric Markov equilibrium if and only if $\mu \leq \underline{\mu}$.*

Proof of Lemma C.5. Suppose that the incumbent strategy constitutes an equilibrium. Since neither firm conducts research, the belief that the other firm possesses the new technology is 0, i.e., $p_t = 0$ for all $t \geq 0$. Observe that $V_t^0 = \frac{\lambda_L \Pi - c}{2\lambda_L}$ since both firms develop with the incumbent technology. If a firm happens to have the new technology and the other firm sticks with the strategy, the expected payoff is $\frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$, i.e., $V_t^1 = \frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$ for all $t \geq 0$. To support this equilibrium, from (HJB$_0$), $\mu(V_t^1 - V_t^0) \leq \lambda_L(\Pi - V_t^0)$ needs to hold. By plugging V_t^1 and V_t^0 in, we have

$$\begin{aligned}
& \mu\left(\frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L} - \frac{\lambda_L \Pi - c}{2\lambda_L}\right) \leq \lambda_L\left(\Pi - \frac{\lambda_L \Pi - c}{2\lambda_L}\right) \\
\iff \quad & \frac{\mu(\lambda_L \Pi + c)(\lambda_H - \lambda_L)}{2\lambda_L(\lambda_H + \lambda_L)} \leq \frac{\lambda_L \Pi + c}{2} \\
\iff \quad & \mu \leq \frac{\lambda_L(\lambda_H + \lambda_L)}{\mu(\lambda_H - \lambda_L)} = \underline{\mu}.
\end{aligned}$$

Now suppose that $\mu \leq \underline{\mu}$. By the above inequality, the strategy profile with $\sigma_t = 0$ for all $t \geq 0$ constitutes an equilibrium, i.e., the incumbent equilibrium exists. $\square$

C.2.2 The Equilibrium with Research Strategies

Lemma C.6. *Suppose that for some T, both firms play $\sigma_t = 1$ for all $0 \leq t \leq T$. Then, there exist $C_0, C_1 \in \mathbb{R}$ such that the expected payoffs of the firm with and without the new technology at time $t \in [0, T]$ is given as follows:*

$$V_t^1 = \overline{V}_1(q_t) + C_1 \cdot (1 - q_t) \cdot \left(\frac{\mu - \lambda_H q_t}{1 - q_t}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}, \tag{26}$$

$$V_t^0 = \overline{V}_0(q_t) + \left(C_0\left(\frac{\mu}{\lambda_H} - q_t\right) - C_1 \frac{\mu}{\lambda_H}\right) \cdot \left(\frac{\mu - \lambda_H q_t}{1 - q_t}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}.[16] \tag{27}$$

Moreover, if both firms play the research strategy ($T = \infty$), $C_1 = C_0 = 0$, i.e., $V_t^1 = \overline{V}_1(q_t)$ and $V_t^0 = \overline{V}_0(q_t)$.

[16] If $\mu = \lambda_H$, we need to replace $\left(\frac{\mu - \lambda_H q_t}{1 - q_t}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}$ to $e^{\frac{2}{1 - q_t}}$.

Proof of Lemma C.6. Consider V_t^n as a value function with respect to the belief process q_t defined as in (4): $V_t^n = V_n(q_t)$. Note that $\dot{V}_t^n = V_n'(q_t)\dot{q}_t = V_n'(q_t)(\mu - \lambda_H q_t)(1-q_t)$. By plugging this into (HJB$_1$), we have

$$0 = V_1'(q)(\mu - \lambda_H q)(1-q) - \lambda_H(1+q)V_1(q) + \lambda_H \Pi - c. \tag{28}$$

By multiplying $(\mu - \lambda_H q)^{-\frac{2\mu}{\mu-\lambda_H}}(1-q)^{\frac{3\lambda_H-\mu}{\mu-\lambda_H}}$ and rearranging the equation, for all $0 = q_0 \leq q \leq q_T$, we can derive that

$$0 = \frac{d}{dq}\left[\frac{(1-q)^{\frac{2\lambda_H}{\mu-\lambda_H}}}{(\mu-\lambda_H q)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}}\left\{V_1(q) - \overline{V}_1(q)\right\}\right]. \tag{29}$$

Therefore, for all $0 = q_0 \leq q \leq q_T$, we have

$$V_1(q) = \overline{V}_1(q) + C_1 \cdot (1-q) \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} \tag{30}$$

for some $C_1 \in \mathbb{R}$. By $V_t^1 = V_1(q_t)$ for all $0 \leq t \leq T$, (26) holds.

Next, plug $\dot{V}_t^0 = V_0'(q_t)(\mu - \lambda_H q_t)(1-q_t)$ into (HJB$_0$):

$$\begin{aligned}
0 =& V_0'(q)(\mu - \lambda_H q)(1-q) - \lambda_H q V_0(q) - c + \mu(V_1(q) - V_0(q)) \\
=& V_0'(q)(\mu - \lambda_H q)(1-q) - V_0(q)(\lambda_H q + \mu) - c \\
&+ \mu\left(\Pi - \frac{c}{\lambda_H}\right)\left(\frac{1}{2} + \frac{\lambda_H(1-q)}{2(\lambda_H+\mu)}\right) + \mu C_1 \cdot (1-q) \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}.
\end{aligned} \tag{31}$$

By multiplying $(1-q)^{\frac{2\lambda_H}{\mu-\lambda_H}}(\mu - \lambda_H q)^{-\frac{3\mu-\lambda_H}{\mu-\lambda_H}}$ and rearranging the equation, $0 \leq q \leq q_T$, we have

$$0 = \frac{d}{dq}\left[\frac{(1-q)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}}{(\mu-\lambda_H q)^{\frac{2\mu}{\mu-\lambda_H}}}\left\{V_0(q) - \overline{V}_0(q) + C_1 \cdot \frac{\mu}{\lambda_H} \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}\right\}\right].$$

Therefore, we have

$$V_0(q) = \overline{V}_0(q) + \left(C_0\left(\frac{\mu}{\lambda_H} - q\right) - C_1\frac{\mu}{\lambda_H}\right) \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}. \tag{32}$$

for some $C_0 \in \mathbb{R}$. By $V_t^0 = V_0(q_t)$ for all $0 \le t \le T$, (27) holds.

Now suppose that both firms play research-first strategy. Then, (26) and (27) hold for all $t \ge 0$. When $\mu > \lambda_H$, by Lemma 4.2, $\lim_{t\to\infty} q_t = 1$. Since $\lim_{t\to\infty}(1-q_t)\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$ and $\lim_{t\to\infty}\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$, to make the value functions converge, $C_1 = C_0 = 0$. When $\mu < \lambda_H$, by Lemma 4.2, $\lim_{t\to\infty} q_t = \mu/\lambda_H$, which also implies $\lim_{t\to\infty}(1-q_t)\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$ and $\lim_{t\to\infty}\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$. Likewise, we also have $C_1 = C_0 = 0$ in this case to make the value functions converge. □

Lemma C.7. *Suppose that σ is the research strategy, i.e., $\sigma_t = 1$ for all $t \ge 0$. Then, $\sigma^A = \sigma^B = \sigma$ constitutes a symmetric Markov equilibrium if and only if $\mu \ge \min\{\overline{\mu}, \hat{\mu}\}$.*

Proof of Lemma C.7. Suppose that both firms play the research-first strategy. By Lemma C.6, the expected payoffs at time t with and without the new technology are $V_t^1 = \overline{V}_1(q_t)$ and $V_t^0 = \overline{V}_0(q_t)$.

Both firms playing the research strategy constitutes an equilibrium if and only if $\mu(\overline{V}_1(q_t) - \overline{V}_0(q_t)) \ge \lambda_L(\Pi - \overline{V}_0(q_t))$ for all $t \ge 0$. Note that

$$\frac{d}{dq}\left[\mu(\overline{V}_1(q) - \overline{V}_0(q)) - \lambda_L(\Pi - \overline{V}_0(q))\right] = -\frac{\left(\Pi - \frac{c}{\mu} - \frac{c}{\lambda_H}\right) + \frac{c}{\lambda_L}}{2\left(\frac{\lambda_H+\mu}{\lambda_H\lambda_L}\right)} < 0.$$

Therefore, it is enough to check whether the following inequality holds:

$$\lim_{t\to\infty}\left[\mu(\overline{V}_1(q_t) - \overline{V}_0(q_t)) - \lambda_L(\Pi - \overline{V}_0(q_t))\right] \ge 0. \tag{33}$$

When $\mu \ge \lambda_H$, by $\lim_{t\to\infty} q_t = 1$, (33) is equivalent to

$$\mu(\overline{V}_1(1) - \overline{V}_0(1)) - \lambda_L(\Pi - \overline{V}_0(1)) = \frac{(\lambda_H\Pi + c)\mu\lambda_L(\mu - \overline{\mu})}{2(\lambda_H + \mu)} \ge 0. \tag{34}$$

When $\lambda_H > \mu$, by $\lim_{t\to\infty} q_t = \mu/\lambda_H$, (33) is equivalent to

$$\begin{aligned}&\mu\left(\overline{V}_1(\mu/\lambda_H) - \overline{V}_0(\mu/\lambda_H)\right) - \lambda_L\left(\Pi - \overline{V}_0(\mu/\lambda_H)\right)\\ &= \frac{(\mu\Pi + c)\lambda_L\lambda_H\left((\lambda_H - 2\lambda_L)\mu - \lambda_L\lambda_H\right)}{2\mu(\lambda_H+\mu)} \geq 0.\end{aligned} \tag{35}$$

Observe that when $\lambda_H < 3\lambda_L$, $\lambda_H < \overline{\mu} < \hat{\mu}$. In this case, by (34), (33) holds iff $\mu \geq \overline{\mu} = \min\{\mu, \hat{\mu}\}$. When $\lambda_H \geq 3\lambda_L$, note that $\lambda_H \geq \overline{\mu} \geq \hat{\mu}$. If $\mu \geq \lambda_H \geq \overline{\mu}$, (33) holds by (34). If $\lambda_H > \mu \geq \hat{\mu}$, (33) holds by (35). Therefore, (33) holds iff $\mu \geq \hat{\mu} = \min\{\mu, \hat{\mu}\}$. □

C.2.3 The Equilibrium with Stationary Fall-back Strategies

Lemma C.8. *Suppose that σ is a stationary fall-back strategy, i.e., for some $T \geq 0$ and $\sigma^* \in [0,1)$, $\sigma_t = 1$ for all $t < T$ and $\sigma_t = \sigma^*$ for all $t > T$. If $\sigma^A = \sigma^B = \sigma$ constitutes a symmetric Markov equilibrium, then $\underline{\eta}(\delta) < \eta < \min\{1+\delta, 2-\delta\}$. Conversely, if $\underline{\eta}(\delta) < \eta < \min\{1+\delta, 2-\delta\}$, there exists a unique symmetric Markov equilibrium and it is a stationary fall-back strategy.*

Proof of Lemma C.8. Suppose that $\sigma^A = \sigma^B = \sigma$ constitutes an equilibrium. By Lemma 4.2 and C.4, $p^* = p_T = q_T > 0$. Since $2\mu > \overline{\mu}$, to have $p^* > 0$, $\mu > \underline{\mu}$ has to hold.

Next, we show that $\mu < \min\{\hat{\mu}, \overline{\mu}\}$. When $\lambda_H < \mu$,

$$\lim_{\tilde{T}\to\infty} q_{\tilde{T}} = 1 > q_T = p^*.$$

By using the definition of p^*, $\overline{\mu} = \underline{\mu} + \lambda_L$, $1 > p^*$ is equivalent to $(\overline{\mu} - \mu)(\mu - \lambda_L) > 0$, thus, $\overline{\mu} > \mu$. Then, $\overline{\mu} > \mu > \lambda_H$ is equivalent to $3\lambda_L > \lambda_H$, which implies $\hat{\mu} > \overline{\mu}$. Therefore, $\min\{\hat{\mu}, \overline{\mu}\} > \mu$ in this case. Consider the case with $\lambda_H > \mu$ and $3\lambda_L \geq \lambda_H$. In this case, $\hat{\mu} > \overline{\mu} \geq \lambda_H > \mu$, thus, $\min\{\hat{\mu}, \overline{\mu}\} > \mu$. Last, consider the case with $\lambda_H > \mu$ and $3\lambda_L < \lambda_H$. Then, we have $\overline{\mu} > \hat{\mu}$ and

$$\lim_{\tilde{T}\to\infty} q_{\tilde{T}} = \frac{\mu}{\lambda_H} > q_T = p^*.$$

By rearranging the inequality, we have $\lambda_L\lambda_H = \lambda_H\underline{\mu} - \lambda_L\overline{\mu} > (\lambda_H - 2\lambda_L)\mu$, which is equivalent to $\min\{\hat{\mu}, \overline{\mu}\} = \hat{\mu} > \mu$.

Now we assume that $\underline{\mu} < \mu < \min\{\overline{\mu}, \hat{\mu}\}$ and show that the stationary fall-back strategy defined in Lemma C.4 constitutes an equilibrium. By the construction of the strategy, for all $t \geq T$, $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$, which supports $\sigma_t \in (0,1)$. Next, we need to show that $\mu(V_t^1 - V_t^0) \geq \lambda_L(\Pi - V_t^0)$ for all $0 \leq t < T$ to support $\sigma_t = 1$. Assume the contrary: $\mu(V_s^1 - V_s^0) < \lambda_L(\Pi - V_s^0)$ for some $0 \leq s < T$. Since $\mu(V_T^1 - V_T^0) = \lambda_L(\Pi - V_T^0)$, there exists $s < t \leq T$ such that $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ and $\mu(\dot{V}_t^1 - \dot{V}_t^0) > -\lambda_L \dot{V}_t^0$, or equivalently,

$$\lambda_L \dot{V}_t^1 > (\mu - \lambda_L)(\dot{V}_t^0 - \dot{V}_t^1). \tag{36}$$

As a first step, we show that there exists $C_1 < 0$ such that V_t^1 is given as (26) in Lemma C.6 for all $0 \leq t < T$. By $V_T^1 = V_1^*$ and $q_T = p^*$, we have

$$C_1 = \frac{1}{(1-p^*)} \left(\frac{1-p^*}{\mu - \lambda_H p^*} \right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} \left(V_1^* - \overline{V}_1(p^*) \right)$$

where $\overline{V}_1$ is defined as in (5). With some algebra and $\min\{\overline{\mu}, \hat{\mu}\} > \mu$, we can derive that

$$\overline{V}_1(p^*) - V_1^* = \left(\Pi - \frac{c}{\lambda_H} \right) \cdot \frac{(\overline{\mu} - \mu)^2 \left(\lambda_L \lambda_H - (\lambda_H - 2\lambda_L)\mu\right)}{2\lambda_L \lambda_H (\lambda_H + \lambda_L)(2\mu - \overline{\mu})} > 0.$$

Therefore, $C_1 < 0$. Then, for all $0 \leq t < T$, we have

$$\dot{V}_t^1 = \dot{q}_t \left[-\left(\Pi - \frac{c}{\lambda_H} \right) \frac{\lambda_H}{2(\lambda_H + \mu)} + C_1 \cdot \frac{\lambda_H (1+q_t)}{1-q_t} \left(\frac{1-q_t}{\mu - \lambda_H q_t} \right)^{\frac{2\lambda_H}{\lambda_H - \mu}} \right] < 0. \tag{37}$$

By (HJB$_1$) and (HJB$_0$), we have

$$\begin{aligned} \dot{V}_t^1 =& \lambda_H (1+q_t) V_t^1 + c - \lambda_H \Pi \\ \dot{V}_t^0 =& \lambda_H q_t V_t^0 + c - \mu(V_t^1 - V_t^0). \end{aligned}$$

By using $\lambda_L(\Pi - V_t^0) = \mu(V_t^1 - V_t^0)$, we can derive that

$$\begin{aligned}\dot{V}_t^0 - \dot{V}_t^1 =& \lambda_H(1+q_t)(V_t^0 - V_t^1) + \mu(V_t^0 - V_t^1) + \lambda_H(\Pi - V_t^0) \\ =& \left[(\lambda_H - \lambda_L)\mu - \lambda_H\lambda_L(1+q_t)\right]\left(\frac{\Pi - V_t^0}{\mu}\right).\end{aligned}$$

Note that $\Pi > V_t^0$ since the expected payoff cannot exceed the rent Π. By using $\Pi > V_t^0$, $p^* \geq q_t$ and $\min\{\overline{\mu}, \hat{\mu}\} > \mu$, we can derive that

$$\begin{aligned}\dot{V}_t^0 - \dot{V}_t^1 \geq& \left[(\lambda_H - \lambda_L)\mu - \lambda_H\lambda_L(1+p^*)\right]\left(\frac{\Pi - V_t^0}{\mu}\right) \\ =& \frac{(\overline{\mu} - \mu)\left(\lambda_L\lambda_H - (\lambda_H - 2\lambda_L)\mu\right)}{2\mu - \overline{\mu}}\left(\frac{\Pi - V_t^0}{\mu}\right) > 0.\end{aligned} \tag{38}$$

Then, (37) and (38) contradict (36). Therefore, $\mu(V_t^1 - V_t^0) \geq \lambda_L(\Pi - V_t^0)$ for all $0 \leq t < T$, and the stationary fall-back strategy constitutes an equilibrium. □