Secret Communication under Plausible Deniability

FSU Theory Reading Group

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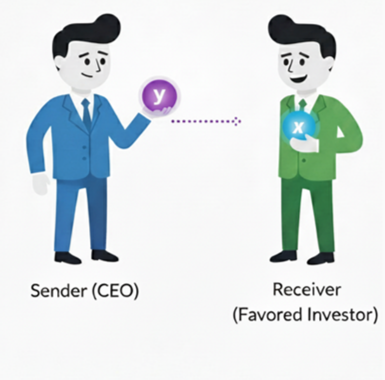
Motivation

Motivation: Insider Trading

- A CEO wants to secretly pass a valuable "tip" to a favored investor, who already has his own private information.
- The investor uses *all* this information—his own & the CEO's tip—to make a decision.
  - The CEO wants to give the most helpful tip possible.
- An outside investigator may be watching.
  - How can CEO give a useful tip that is also "safe" and won't get them into trouble even if they are investigated?



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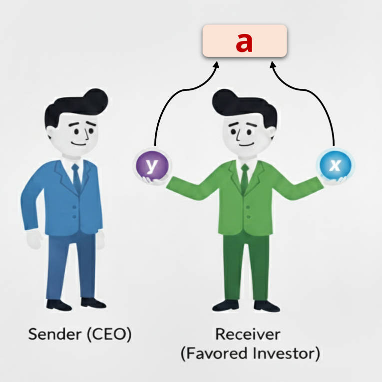

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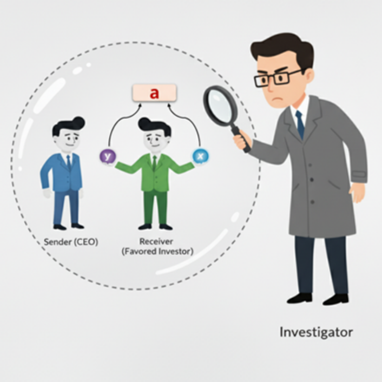

Motivation: What Makes Communication "Safe"?

- To avoid trouble, the Sender's tip must be defensible under investigation.
  1. **Secrecy**: The tip must be meaningless on its own. [Shannon, 1949]
     - An explicit message like "Buy the stock!" is not secret. The message itself is incriminating evidence.
  2. **Plausible Deniability**: The Receiver's final action must be justifiable without revealing the tip.
     - The investor must be able to claim, "Given what I already knew privately, my decision makes perfect sense." This creates a credible cover story.
- **Main Question:** What is the most informative signal the Sender can design under these constraints?

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Setup

Setup: Preliminaries

- Players: Sender, (informed) Receiver and (outside) Investigator
- The state of the world $\omega \in \Omega := \{\omega_1, \cdots, \omega_N\}$ with prior $\mu \in \Delta(\Omega)$
    - Assume that μ has full support.
- Receiver receives a signal x from a fixed information structure $f : \Omega \to \Delta(X)$ where $X = \{x_1, \cdots, x_J\}$
- Sender designs a joint information structure $h : \Omega \to \Delta(X \times Y)$, generating an additional signal $y \in Y$
    - The marginal distribution of X must correspond to f
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Setup: Utility and Optimal Actions

- The set of actions (by Receiver) is an ordered set $A = \{a_1, \cdots, a_L\}$
- A utility function $u : A \times \Omega \to \mathbb{R}$ satisfies the *single-crossing* property if for any actions $a' > a$ and states $\omega' > \omega$:

$$u(a', \omega) \ \geq (>) \ u(a, \omega) \qquad \Longrightarrow \qquad u(a', \omega') \ \geq (>) \ u(a, \omega').$$

  Let $\mathcal{U}_{SC}$ denote the class of single-crossing utilities.

- Optimal actions given $u \in \mathcal{U}_{SC}$:
  - $a_{x,y}(u)$ given (x, y)
  - $a_x(u)$ $(a_y(u))$ given x (y)
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Setup: Observable Information across Players

Receiver

$u \in \mathcal{U}_{SC}$

Sender & Receiver

$h : \Omega \to \Delta(X \times Y)$

Public Information

$\mathcal{U}_{SC}$

$f : \Omega \to \Delta(X)$

$A = \{a_1, \cdots, a_L\}$

$g : \Omega \to \Delta(Y)$

$x, \ y$

$\mu \in \Delta(\Omega)$

$a_{\emptyset}(u), \ a_{x,y}(u)$

Setup: Designing Information

The sender's objective is to design a joint information as informative as possible subject to constraints

1. $h : \Omega \to \Delta(X \times Y)$ satisfies **secrecy** iff for all $u \in \mathcal{U}_{SC}$ and $y \in Y$,

$$a_y(u) = a_\emptyset(u).$$

2. $h : \Omega \to \Delta(X \times Y)$ satisfies **plausible deniability** iff for all $u \in \mathcal{U}_{SC}$ and $(x, y) \in X \times Y$, there exists $\tilde{u} \in \mathcal{U}_{SC}$ such that

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2 by 2 Example

2 by 2 Example: Optimal Secret Information (w/o PD)

- $\Omega = \{\omega_L, \omega_H\}$, $X = \{L, H\}$

	L	H
ω_L	.8	.2
ω_H	.2	.8

2 by 2 Example: Optimal Secret Information (w/o PD)

- $\Omega = \{\omega_L, \omega_H\}$, $X = \{L, H\}$, $Y = \{T, F\}$

	LT	HF	LF	HT
ω_L	.8	.2	0	0
ω_H	0	0	.2	.8

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- This joint information structure is most informative and satisfies *secrecy*,

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- This joint information structure is most informative and satisfies *secrecy*, but it violates the *plausible deniability*

2 by 2 Example: Optimal Secret Information w/ PD

- $\Omega = \{\omega_L, \omega_H\}$, $X = \{L, H\}$, $Y = \{T, M(eaningless)\}$

	LT	HM	LM	HT
ω_L	.6	.2	.2	0
ω_H	0	.2	.2	.6

2 by 2 Example: Optimal Secret Information w/ PD

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	T	M
ω_L	.6	.4
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- This joint information structure satisfies secrecy and plausible deniability,

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	LT	HM	LM	HT
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ω_H	0	.2	.2	.6

	T	M
ω_L	.6	.4
ω_H	.6	.4

- This joint information structure satisfies secrecy and plausible deniability, and it is shown to be Blackwell more informative than any joint information structures that satisfies two conditions

2 by 2 Case: Theorem

- $\Omega = \{\omega_L, \omega_H\}$, $X = \{L, H\}$, wlog $f_L \geq f_H$

$$f = \begin{array}{c|cc} & L & H \\ \hline \omega_L & f_L & 1 - f_L \\ \omega_H & f_H & 1 - f_H \end{array}$$

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$$h^* = \begin{array}{c|cccc} & LT & LM & HM & HT \\ \hline \omega_L & f_L - f_H & f_H & 1 - f_L & 0 \\ \omega_H & 0 & f_H & 1 - f_L & f_L - f_H \end{array}$$

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Theorem 1

Suppose that $|\Omega| = |X| = 2$. For any joint information structure h satisfying plausible deniability and secrecy, h^* is more Blackwell informative than h.

Binary State and Multiple Signals

Binary State and Multiple Signals

- $\Omega = \{\omega_L, \omega_H\}$, $X = \{x_1, \cdots, x_J\}$, $f_L^j \equiv f(x_j|\omega_L)$ and $f_H^j \equiv f(x_j|\omega_H)$
- Define

$$\mathcal{L} = \left\{x_j \in X \ : \ f_L^j \geq f_H^j\right\} \qquad \text{and} \qquad \mathcal{H} = \left\{x_j \in X \ : \ f_L^j < f_H^j\right\}$$

- Define $h^*(x_j, y|\omega)$ as follows:

	jT	jM
ω_L	$f_L^j - f_H^j$	f_H^j
ω_H	0	f_H^j

$x_j \in \mathcal{L}$

	jM	jT
ω_L	f_L^j	0
ω_H	f_L^j	$f_H^j - f_L^j$

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ω_H	f_L^j	$f_H^j - f_L^j$

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Binary State and Multiple Signals

Theorem 2

Suppose that $|\Omega| = 2$. For any joint information structure h satisfying plausible deniability and secrecy, h^* is more Blackwell informative than h.

▸ Proof Sketch

Multiple States and Binary Signal

Multiple States and Binary Signal

- $\Omega = \{\omega_1, \cdots, \omega_n\}$, $X = \{L, H\}$, wlog MLRP holds:

$$f_1^L \geq \cdots \geq f_n^L \qquad \text{and} \qquad f_1^H \leq \cdots \leq f_n^H$$

- Construct h^{**} with a set of signals $Y = \{1, \cdots, n-1, M\}$: Details
  - Upon receiving $y = M$, the posterior is the same as the prior
  - $y = k$ serves as a cutoff signal together with x:

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  - Upon receiving $y = M$, the posterior is the same as the prior
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    Prior belief:

ω_1	$\cdots$	ω_k	ω_{k+1}	$\cdots$	ω_n
μ_1	$\cdots$	μ_k	μ_{k+1}	$\cdots$	μ_n

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  - Upon receiving $y = M$, the posterior is the same as the prior
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    Posterior belief with (L, k):

ω_1	$\cdots$	ω_k	ω_{k+1}	$\cdots$	ω_n
$\frac{\mu_1}{\sum_{i=1}^k \mu_i}$	$\cdots$	$\frac{\mu_k}{\sum_{i=1}^k \mu_i}$	0	$\cdots$	0

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    - $y = k$ serves as a cutoff signal together with x:
      Posterior belief with (H, k):

ω_1	$\cdots$	ω_k	ω_{k+1}	$\cdots$	ω_n
0	$\cdots$	0	$\frac{\mu_{k+1}}{\sum_{i=k+1}^{n} \mu_i}$	$\cdots$	$\frac{\mu_n}{\sum_{i=k+1}^{n} \mu_i}$

Multiple States and Binary Signal

- h^{**} satisfies
  1. **Secrecy**: w/o x, the posterior is the same as the prior
  2. **Plausible Deniability**: when $(x, y) = (L, k)$, the receiver can pretend that her utility does not differ much across actions for higher states greater than k

Theorem 3

Suppose that $|X| = 2$. For any joint information structure h satisfying plausible deniability and secrecy, h^{**} is more Blackwell informative than h.

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  - Introduced **secrecy** and **plausible deniability** as formal constraints for communication.
  - Characterized the *Blackwell-optimal* joint information structures under these constraints for several key cases (2×2, $2 \times N$, $N \times 2$).
- **Future Directions**
  - Characterize the optimal signal for the general $N \times J$ case (multiple states and multiple signals).
  - Explore concrete applications of this framework, e.g., in finance (insider trading) or political economy.

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Appendix

Theorem 1 & 2: Proof Sketch

1. If h satisfies plausible deniability, it satisfies:

$$\begin{cases} h_L^{jk} \leq h_H^{jk}, & \text{if } f_L^j \leq f_H^j, \\ h_L^{jk} \geq h_H^{jk}, & \text{if } f_L^j \geq f_H^j, \end{cases} \quad \forall\, j,\ k, \tag{$\star$}$$

   or equivalently, for all j, k,

$$\begin{cases} \Pr(\omega_H|x_j, y_k) \geq \mu(\omega_H), & \text{if } \Pr(\omega_H|x_j) \geq \mu(\omega_H), \\ \Pr(\omega_H|x_j, y_k) \leq \mu(\omega_H), & \text{if } \Pr(\omega_H|x_j) \leq \mu(\omega_H), \end{cases}$$

2. h^* Blackwell dominates any h satisfying ($\star$)
3. h^* satisfies plausible deniability and secrecy.

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Theorem 1 & 2: Proof Sketch

1. If h satisfies plausible deniability, it satisfies:

$$\begin{cases} h_L^{jk} \leq h_H^{jk}, & \text{if } f_L^j \leq f_H^j, \\ h_L^{jk} \geq h_H^{jk}, & \text{if } f_L^j \geq f_H^j, \end{cases} \quad \forall\, j,\ k, \tag{$\star$}$$

2. h^* Blackwell dominates any h satisfying $(\star)$
   - We show this by constructing a garbling.

3. h^* satisfies plausible deniability and secrecy.

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Theorem 1 & 2: Proof Sketch

1. If h satisfies plausible deniability, it satisfies:

$$\begin{cases} h_L^{jk} \leq h_H^{jk}, & \text{if } f_L^j \leq f_H^j, \\ h_L^{jk} \geq h_H^{jk}, & \text{if } f_L^j \geq f_H^j, \end{cases} \quad \forall\, j,\ k, \tag{$\star$}$$

2. h^* Blackwell dominates any h satisfying ($\star$)

3. h^* satisfies plausible deniability and secrecy.

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Formal Construction of h^{**}

$$
f = \begin{array}{c|cc}
 & L & H \\
\hline
\omega_1 & f_1 & 1 - f_1 \\
\omega_2 & f_2 & 1 - f_2 \\
\vdots & \vdots & \vdots \\
\omega_{n-1} & f_{n-1} & 1 - f_{n-1} \\
\omega_n & f_n & 1 - f_n
\end{array}
$$

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Formal Construction of h^{**}

$$h^{**} = \begin{array}{c|ccccc|ccccc}
 & L_1 & L_2 & \cdots & L_{n-1} & L_M & H_M & H_1 & H_2 & \cdots & H_{n-1} \\
\hline
\omega_1 & f_1 - f_2 & f_2 - f_3 & \cdots & f_{n-1} - f_n & f_n & 1 - f_1 & 0 & 0 & \cdots & 0 \\
\omega_2 & 0 & f_2 - f_3 & \cdots & f_{n-1} - f_n & f_n & 1 - f_1 & f_1 - f_2 & 0 & \cdots & 0 \\
\omega_3 & 0 & 0 & \cdots & f_{n-1} - f_n & f_n & 1 - f_1 & f_1 - f_2 & f_2 - f_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\omega_n & 0 & 0 & \cdots & 0 & f_n & 1 - f_1 & f_1 - f_2 & f_2 - f_3 & \cdots & f_{n-1} - f_n
\end{array}$$

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