The Direct vs. the Sequential Approach in Project Management

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Abstract

I study a dynamic principal-agent problem where an agent can choose either to directly complete a project (a direct approach) or to split the project into two subprojects (a sequential approach). The sequential approach allows the principal to better monitor the agent by checking the completion of the subproject. However, the inflexible nature of this approach may generate inefficiencies. In addition, when the agent is under time pressure, it is harder to achieve multiple goals sequentially than to accomplish a single goal directly. The optimal contract is determined by the interplay of these three economic forces: monitoring, efficiency, and time pressure. I show that the form of the optimal contract depends crucially on the project size. This finding explains why the direct approach is popular for small-sized projects whereas the sequential approach is more common for larger ones.

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You want your software out soon and relatively defect-free, but more than that, you need a way to examine how your team is doing along the way. (*Cockburn, 2006*)

1 Introduction

When an agent is assigned to complete a project, there may be various ways of achieving the goal. One possible approach is to try to attack the project directly (a direct approach). Another option is to divide the project into several subprojects and complete these one by one (a sequential approach). For example, for software development, there are two popular methodologies: *waterfall* and *agile*. According to McCormick (2012), the waterfall methodology requires the developers to follow a rigid development schedule. They can move on to the next stage only after fully accomplishing and documenting the previous stage: a sequential approach. On the contrary, under the agile methodology, the programmers work in more flexible environments and are not required to document every stage. They complete the project by using trial and error, i.e., repeatedly produce output until the product meets the standard: a direct approach.[1]

In the presence of multiple approaches to complete a project, a principal, who delegates the project to an agent, faces a choice problem between these approaches as well as an incentive design problem. In the previous example, a project manager, who hires programmers to develop software, not only determines which methodology to adopt but also devises a scheme to incentivize programmers not to shirk. In such environments, how can the principal optimally design a contract? Under what conditions, would each approach be recommended in the optimal contract?

In this paper, I address these questions by investigating a dynamic principal-agent model with two approaches that reflect the economic situations described above. To complete a project, the direct approach requires a breakthrough which arrives at a low rate. On the other hand, the sequential approach requires two symmetric breakthroughs which arrive at higher rates. Each breakthrough in this approach can be understood as the completion of a subproject. The agent can apply effort to either of the approaches or shirk for private benefit. The principal does not observe the allocation of the agent's effort, which creates a situation of moral hazard.

On top of this basic setup, I impose two key assumptions. First, the first breakthrough of

[1] Another feature of the agile methodology is that programmers learn from early errors. Therefore, their speed of completing the project may increase as errors accumulate. However, to simplify the analysis I abstract from this possibility and focus on the directness aspect of the agile methodology.

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the sequential approach is observable and contractually verifiable. In the software development example, under the waterfall methodology, every stage should be 'documented,' thus the project manager can observe and verify progress. In many R&D projects, the result of each phase is often publicly announced. Thus, it is plausible to assume that the first breakthrough is observable. This assumption implies that the first breakthrough can help the principal monitor the agent. Hence, the sequential approach has an advantage in monitoring the agent compared with the direct approach.

A second assumption is that the direct approach is more efficient than the sequential one. In the previous example, while programmers are flexible in the agile methodology, they need to follow a rigid plan under the waterfall methodology. According to McCormick (2012), it is difficult to make last-minute changes under the waterfall paradigm, whereas programmers can incorporate changes more flexibly under agile project management. Thus, the inflexible nature of the waterfall approach may cause some inefficiency. In addition, waterfall project management requires proper documentation for each stage but agile does not. Documenting each step does not produce any additional value, rather the purpose is to monitor progress. Thus, when there is no moral hazard, documenting each stage is simply a waste of time and resources. In general, the principal may need to sacrifice some efficiency to monitor the agent—so it is natural to focus on the case where the direct approach is technically more efficient than the sequential one.

These assumptions on *monitoring* and *efficiency* imply a tradeoff between the two approaches. In addition to these elements, there is a third important economic force that affects the choice of methodology: *time pressure*. It is based on the observation that it is hard to achieve two breakthroughs in a short period of time, whereas it is relatively easier to make one breakthrough. Thus, time pressure gives another advantage to the direct approach. Time pressure matters because the principal endogenously imposes a deadline to mitigate moral hazard. Without the deadline, the agent could shirk forever without completing the project. The optimal contract is determined by the interplay of these three economic forces.

To facilitate analysis, I begin by characterizing the optimal contract in the case where both approaches are equally efficient. Here, we only need to consider the interaction between monitoring and time pressure. Monitoring gives a generic advantage to the sequential approach while time pressure gives an advantage to the direct approach when the deadline is imminent. This suggests the principal should recommend the sequential approach when the deadline is distant and the direct approach when it is near at hand. However, it is possible that the monitoring advantage is so strong that the principal prefers the sequential approach even at the deadline. On the contrary, it is also possible that the optimal deadline is short enough that the principal would want to recommend the direct approach even at contract

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inception. Therefore, depending on the economic environment, various types of contracts may be optimal even in this simple setting.

The main result of this paper is that the form of the optimal contract depends crucially on the size of the project; i.e., the gross value to the principal from completing the project. To see why, we first need to understand the difference between the incentive schemes implementing each approach. If the principal wants to induce the direct approach, the agent needs to be compensated by an immediate payment upon project success. On the contrary, if the principal wants to induce the sequential approach, the principal extends the deadline after observing the first breakthrough and pays the agent only upon the successful completion of the entire project. Thus, the principal chooses between approaches by comparing the expected payoffs from the immediate payment scheme and the deadline extension scheme. When the principal uses the deadline extension scheme, the agent may work for a longer period of time. This means that under the sequential approach the probability of ultimate success is higher, but the expected running cost is higher as well. At contract inception, the project size must be scaled by the probability of successful completion. It follows that the sequential approach is preferred when the project size is large and the direct approach is preferred when the project size is small. Based on this observation, I show that the optimal contract is derived as follows:

(a) when the project size is small enough, the optimal deadline is short and the principal only recommends the direct approach;

(b) when the project size is large enough, monitoring is highly advantageous and the principal only recommends the sequential approach;

(c) when the size of the project is intermediate, there is a switching point such that the principal first recommends the sequential approach and then switches to recommending the direct approach until the deadline is reached.

Next, I introduce an efficiency loss to the sequential approach from monitoring. When the efficiency loss is small enough, I show that a similar result as in the previous case holds: there are three regions of the project size that characterize the form of the optimal contract. In other words, the characterization of the optimal contract when the approaches are equally efficient is robust to a small efficiency loss. This is mainly because efficiency dominates monitoring only if the deadline is distant. I also consider the case where the efficiency loss is large. In this case, even if the project size is moderately large, the principal prefers to choose the direct approach over the sequential approach to avoid the efficiency cost. Nevertheless, if the project size is very large, the principal prefers to monitor to some degree. In fact, there

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is a cutoff for the project size such that the principal recommends the direct approach when the project size is below the cutoff. Moreover, if the project size is above the threshold, then the principal begins by recommending the direct approach, switches to recommend the sequential approach, then switches back to recommend the direct approach until the deadline.

I additionally explore three extensions of the model. First, I relax the assumption that the two subprojects are symmetric in the sequential approach. I show that a similar result holds if the arrival rate ratio of the second breakthrough to the first breakthrough is below some cutoff. If it is above the cutoff, the principal prefers to recommend the sequential approach even when the project size is small. This is because monitoring becomes more effective as the ratio increases. Second, I introduce value on completion of the subproject, that is, once the first subproject is completed, the outside options for both players increase. I present a numerical example showing that (i) the principal's expected payoffs increases in those outside options; (ii) the optimal deadline increases as the agent's outside option increases; (iii) the optimal deadline decreases as the principal's outside option increases. Last, I explore how discounting distorts the efficiency of each approach. I demonstrate that introducing impatience makes the sequential approach relatively less attractive because this approach requires a long horizon.

The findings of this paper have implications for software development and scientific research. In the case of software development, my results imply that the agile methodology is more suitable for small-sized projects whereas the waterfall methodology fits better for large-sized projects. This is consistent with the argument of McCormick (2012):

> *If the projects are smaller projects, then using the agile model is certainly profitable, but if it is a large project, then it becomes difficult to judge the efforts and the time required for the project in the software development life cycle.*

My results also explain why applied scientific research (e.g., development of a new drug) is typically staged. The magnitude of applied research projects is usually large, implying the superiority of the sequential approach. In contrast, the immediate value of basic research (e.g., chemistry) is lower than applied research because "basic research is performed without thought of practical ends" (Bush, 1945).[2] My results imply that the direct approach is preferred for basic research because such projects tend to be smaller. For instance, the

[2] Bush argues that although broad and basic studies seem to be less important than applied ones, they are essential to combat diseases because progress in the treatment "will be made as the result of fundamental discoveries in subjects unrelated to those diseases, and perhaps entirely unexpected by the investigator." However, since this paper does not consider externalities, I abstract from this possibility and focus on project size.

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Research Project designation (R01) grant by the National Institute of Health (NIH) supports "a discrete, specified, circumscribed project" rather than a staged project.[3]

I discuss the related literature below. In Section 3, I introduce the model and derive the first-best contract. In Section 4, I characterize the optimal contract when there is no efficiency loss from monitoring. I introduce an efficiency loss and characterize the optimal contract in Section 5. I conclude by exploring the three possible extensions in Section 6. In the Appendix, I provide the proofs for propositions and theorems in Section 3, 4 and 5. The proofs for the lemmas appearing in the Appendix and for the propositions and theorems appearing in Section 6 are located in the Supplementary Appendix.

2 Related Literature

The current paper utilizes Poisson processes which are widely used to address dynamic moral hazard, e.g., Biais et al. (2010); Green and Taylor (2016a); Bonatti and Hörner (2017); Varas (2017); Sun and Tian (2017). The most closely related study is Green and Taylor (2016a), who study a model in which multiple breakthroughs are needed to complete a project and in which an agent must be incentivized to exert unobservable effort. The sequential approach considered here comprises the baseline model of Green and Taylor (2016a)[4] However, the option to complete the project directly, which is not considered in their setup, allows players to face a choice problem between two approaches.

Another paper that has a similar flavor is Schneider and Wolf (2020). They consider a two-armed bandit problem where an arm requires one breakthrough (the doing arm) but another arm requires two breakthroughs (the thinking arm) to succeed. The arrival rates for the thinking arm are known to the agent whereas the arrival rate for the doing arm is not: the agent needs to infer whether the method is feasible or not by experimenting. The presence of this uncertainty is one key difference between their paper and this one. In addition, their main analysis focuses on a decision problem by a single agent whereas this paper considers a principal-agent contracting model. Despite these differences, we share a common insight in that the chosen approaches may switch up to two times. However, the economic forces that drive choosing the thinking arm (or the sequential approach) are somewhat different. In their paper, as the agent pulls the doing arm and does not achieve any success, the belief that the initial method is feasible goes down. When the belief becomes sufficiently low, the thinking arm would be chosen because it may be more efficient than the doing arm. Thus,

[3] `https://grants.nih.gov/grants/funding/r01.htm`

[4] To be precise, this is true for the tangible first breakthrough case of the working version of the paper (Green and Taylor, 2016b). In the published version of the paper, the authors only consider the case where the principal cannot observe the first breakthrough.

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experimentation and efficiency are key driving forces for choosing the thinking arm. On the contrary, in this paper, the principal recommends the sequential approach to monitor the agent, not because beliefs about the direct approach have deteriorated.[5]

The problem of choosing approaches is naturally related to multitasking in the sense that the agent has multiple options to pursue. In their seminal paper, Holmstrom and Milgrom (1991) consider an economic situation where a production worker faces multiple tasks such as producing output and maintaining quality in a static environment.[6] Several subsequent multitasking problems are also explored in dynamic settings (Manso, 2011; Capponi and Frei, 2015; Varas, 2017; Szydlowski, 2019). A common assumption in these studies is that each task has a different payoff structure.[7] For example, Manso (2011) studies a two-armed bandit problem in a simple agency model with two periods. The main assumption is that if the agent chooses to experiment (pulls the risky arm), the payoff is stochastic, and if the agent chooses to exploit (pulls the safe arm), the payoff is constant. In contrast, the two approaches in this paper have the same ultimate payoff. The difference in the approaches is 'how' the ultimate breakthrough is made–via the direct approach or via the sequential approach.

3 The Model

3.1 Setup

A principal (she) hires an agent (he) to complete a project. The project is conducted in continuous time and can be potentially operated over an infinite horizon: $t \in [0, \infty)$. The project requires an ultimate breakthrough, which I denote by success or use the term "the (main) project succeeds." When the project succeeds, the principal realizes a payoff $\Pi > 0$ and the game ends. While the project is running, the principal incurs an operating cost of $c > 0$ per unit of time. The principal is assumed to have an infinite amount of resources to fund the project while the agent is protected by limited liability. The principal and the agent are both risk-neutral and patient, i.e., they do not discount the future.[8]

[5]Schneider and Wolf (2020) do provide some analysis of contracting in their supplementary material. However, the setup is not fleshed out enough to identify the effect of monitoring—they do not allow the principal to extend the deadline once the first breakthrough of the thinking arm is made.

[6]Dewatripont et al. (2000) and Laux (2001) also study multitasking problems in static environments.

[7]The only paper that does not have this assumption is Varas (2017). He considers a dynamic model with a Poisson process in which the agent chooses between a good project and a bad project. These projects look identical to the principal and yield the same payoff, but differ in the rate of failure.

[8]I explore the case of a positive discount factor in Section 6.3.

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There are two routes to achieving success. One is to try to complete the project directly and I accordingly call this the direct approach. Another way is to split the project into two subprojects and I call this the sequential approach. At each point in time t, the agent allocates his 1 unit of effort among three alternatives: the direct approach (a_t), the sequential approach (b_t), and shirking (l_t): $a_t + b_t + l_t = 1$ and $a_t,\ b_t,\ l_t \geq 0$. I will also refer to choosing the direct approach as *going for the project* and choosing the sequential approach as *splitting the project*. The allocation of efforts is unobservable to the principal. Then, at time t, the main project succeeds at the rate $\lambda_G a_t$, the subproject succeeds at the rate $\lambda_S b_t$, and the agent receives ϕl_t as a private flow benefit from shirking. I assume that the marginal private benefit ϕ is positive but less than c, thus shirking is not optimal when the agent's effort allocation is observable to the principal. It is easier to achieve the success of the subproject than the success of the main project, i.e., λ_S is greater than λ_G. Completing the subproject does not have any independent value for the principal or the agent.[9] However, the success of the subproject is observable by both players and contractually verifiable by a court. Thus observing the completion of the subproject can be considered as a type of monitoring. Once the subproject succeeds, the agent only needs to complete one more subproject (with the same arrival rate) to make the entire project succeed. Thus, after the first subproject is finished, the agent allocates his 1 unit of effort per period between working (a_t) and shirking (l_t) and the main project succeeds at the rate $\lambda_S a_t$.[10]

3.2 Contract

At the beginning of the game, the principal offers a contract to the agent and fully commits to all contractual terms. If the agent rejects the offer, the principal and the agent receive zero payoffs. To simplify the notation, I denote 'main breakthrough' as the success on the main project and 'intermediate breakthrough' as the success on the subproject. Note that if the agent has not made either the main or the intermediate breakthrough, the calendar time is the only relevant variable summarizing the public history. A contract is denoted by $\Gamma \equiv \left\{T, a, b, R, \hat{\Gamma}\right\}$, where each variable is defined as follows at the calendar time t:[11]

[9] In Section 6.2, I consider the case where the completion of the first subproject raises outside options for the agent and the principal.

[10] I assume that the arrival rates for two breakthrough in the sequential approach is the same, i.e., the two subprojects are symmetric. In Section 6.1, I relax this assumption and analyze the case where the two subprojects are asymmetric.

[11] A mixture of contracts also generates another contract. For example, a contract with a soft deadline—randomly terminating the agent after reaching the soft deadline—as in Green and Taylor (2016a) can be represented by a mixture of two contracts defined here. However, mixing contracts are never optimal in this paper. This is mainly because the value function (that will be defined in Section 4) is always concave. Thus I focus on pure contracts.

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1. $T \in \mathbb{R}_+ \cup \{\infty\}$: the deadline date at which the project is terminated conditional on no main or intermediate breakthrough. $T = \infty$ means that no deadline is included in the contract;

2. $a_t \in [0, 1]$ for $0 \leq t \leq T$: the recommended effort to the completion of the main project (direct approach) conditional on no main or intermediate breakthrough;

3. $b_t \in [0, 1]$ for $0 \leq t \leq T$: the recommended effort to the completion of the subproject (sequential approach) conditional on no main or intermediate breakthrough;

4. $R_t \geq 0$ for $0 \leq t \leq T$: the monetary payment from the principal to the agent for the success of the main project conditional on no intermediate breakthrough;[12]

5. $\hat{\Gamma}^t \equiv \{a^t, R^t, T^t\}$: an updated contract when the intermediate breakthrough arrives at time t conditional on no main breakthrough;

   (a) $T^t \in \{\tilde{T} \; : \; \tilde{T} \geq t\} \cup \{\infty\}$: the deadline date at which the project is terminated conditional on the intermediate breakthrough at time t and no main breakthrough. $T^t = \infty$ means that there is no deadline;

   (b) $a^t_s \in [0, 1]$ for $t \leq s \leq T$: the recommended effort to the completion of the main task (working on the remaining subproject) at time $s \geq t$ conditional on the intermediate breakthrough at time s and no main breakthrough;

   (c) $R^t_s \geq 0$ for $t \leq s \leq T$: the monetary payment from the principal to the agent for the main breakthrough at time s conditional on the intermediate breakthrough at time t and no main breakthrough.

Action processes a^t and (a, b) induce probability distributions $\mathbb{P}^{a^t}$ over a date of a main breakthrough τ_m and $\mathbb{P}^{a,b}$ over a pair of dates of main and intermediate breakthroughs (τ_m, τ_s). Let $\mathbb{E}^{a^t}$ and $\mathbb{E}^{a,b}$ denote the corresponding expectation operators. When the subproject is completed at time t and the agent adheres to the recommended action of $\hat{\Gamma}^t$, the principal's expected utility at time t is given by

$$\hat{P}^t(\hat{\Gamma}^t) = \mathbb{E}^{a^t}\left[\left(\Pi - R^t_{\tau_m}\right) \cdot \mathbf{1}_{\{t \leq \tau_m \leq T^t\}} - \int_t^{T^t \wedge \tau_m} c \; ds\right],$$ [13]

[12] Since both the principal and the agent are risk neutral and do not discount the future, without loss of generality, all monetary payments to the agent can be backloaded (see, e.g., Ray (2002)). The nonnegativity of R_t is due to limited liability.

[13] For each x and y, let $x \wedge y$ denote the minimum of x and y, and let $x \vee y$ denote the maximum of x and y.

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where the first term in the expectation is the net profit from the success and the second term is the cumulative operating cost. The agent's expected utility is given by

$$\hat{U}^t(\hat{\Gamma}^t) = \mathbb{E}^{a^t}\left[R^t_{\tau_m} \cdot \mathbf{1}_{\{t \le \tau_m \le T^t\}} + \int_t^{T^t \wedge \tau_m} \phi(1 - a^t_s) ds\right],$$

where the first term is the payoff from the success and the second term is the benefit from shirking.

At time 0, if the agent adheres to the recommended actions of Γ^L, the principal's (ex ante) expected utility is given by

$$P_0(\Gamma) = \mathbb{E}^{a,b}\left[(\Pi - R_{\tau_m}) \cdot \mathbf{1}_{\{\tau_m \le \tau_s \wedge T\}} + \hat{P}_{\tau_s}(\hat{\Gamma}_{\tau_s}) \cdot \mathbf{1}_{\{\tau_s < \tau_m \wedge T\}} - \int_0^{T \wedge \tau_m \wedge \tau_s} c \, dt\right],$$

where the first term is the net profit from the main breakthrough, the second term is the expected payoff from the intermediate breakthrough at time τ_s, and the last term is the cumulative operating cost. The agent's expected utility is given by

$$U_0(\Gamma) = \mathbb{E}^{a,b}\left[R_{\tau_m} \cdot \mathbf{1}_{\{\tau_m \le T\}} + \hat{U}_{\tau_s}(\hat{\Gamma}^{\tau_s}) \cdot \mathbf{1}_{\{\tau_s < \tau_m \wedge T\}} + \int_0^{T \wedge \tau_m \wedge \tau_s} \phi(1 - a_t - b_t) \, dt\right],$$

where the first term is the payoff from the main breakthrough, the second term is the expected payoff from the intermediate breakthrough at time τ_s, and the last term is the benefit from shirking. By using the agent's expected payoffs, I define incentive compatibility (IC) of contracts as follows.

Definition 3.1. A contract $\Gamma = \left\{T, a, b, R, \hat{\Gamma}\right\}$ is *incentive compatible* if

1. for all $t \le T$, the recommended action profile a^t maximizes the agent's expected utility at time t, i.e.,

$$\hat{U}^t(\hat{\Gamma}^t) = \max_{\tilde{a} \in \hat{\mathcal{A}}^t} \mathbb{E}^{\tilde{a}}\left[R^t_{\tau_m} \cdot \mathbf{1}_{\{\tau_m \le T^t\}} + \int_t^{T^t \wedge \tau_m} \phi(1 - \tilde{a}_s) ds\right]$$

   where $\hat{\mathcal{A}}^t \equiv \{\{a_s\}_{t \le s \le T^t} \; : \; a_s \in [0,1]\}$;

2. the recommended action profile (a, b) maximizes the agent's expected utility at time 0, i.e.,

$$U_0(\Gamma) = \max_{(\tilde{a},\tilde{b}) \in \mathcal{A}} \mathbb{E}^{\tilde{a},\tilde{b}}\left[R_{\tau_m} \cdot \mathbf{1}_{\{\tau_m \le T\}} + \hat{U}^{\tau_s}(\hat{\Gamma}^{\tau_s}) \cdot \mathbf{1}_{\{\tau_s < \tau_m \wedge T\}} + \int_0^{T \wedge \tau_m \wedge \tau_s} \phi(1 - \tilde{a}_t - \tilde{b}_t) \, dt\right]$$

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where $\mathcal{A} \equiv \{\{a_t, b_t\}_{0\leq t\leq T} \ : \ (a_t, b_t, a_t + b_t) \in [0,1]^3\}$.

The objective of the principal is to find a contract Γ that maximizes her ex ante expected utility $P_0(\Gamma)$ subject to the incentive compatibility constraint and the individual rationality constraint, i.e., $U_0(\Gamma) \geq 0$. Designate such a contract as an *optimal contract*.

3.3 The First-Best Contract

In this subsection, I assume that the agent's allocation of effort is observable to the principal and I characterize the first-best contract. I introduce two benchmark contracts and show that, depending on parameter values, one or the other of them is first-best optimal. The two contracts are defined as follows:

1. **The direct-only contract without a deadline** (Γ_G): (i) $T = \infty$; (ii) $(a_t, b_t, R_t) = (1, 0, 0)$ for all $t \geq 0$; (iii) terminate the contract if the agent deviates from the recommended action.

2. **The sequential-only contract without a deadline** (Γ_S): (i) $T = \infty$; (ii) $(a_t, b_t, R_t) = (0, 1, 0)$ for all $t \geq 0$; (iii) $\hat{\Gamma}^t$ is defined as follows: (a) $(a^t_s, R^t_s) = (1, 0)$ for all $s \geq t$; (b) $T^t = \infty$; (iv) terminate the contract if the agent deviates from the recommended action.[14]

The direct-only contract without a deadline means that the agent indefinitely tries to complete the project directly, i.e., chooses only the direct approach until the main project succeeds. On the other hand, the sequential-only contract without a deadline means that the agent indefinitely tries to achieve the success of the subproject, i.e., chooses only the sequential approach until the subproject succeeds, then works until the main project succeeds. Note that the agent does not switch approaches over time for either contract. In these contracts, I do not impose any deadline because the agent's effort is observable. In the following sections, the agent's effort is no longer observable thus a deadline will be introduced to mitigate moral hazard.

The probability distribution of τ_m for the direct-only contract without a deadline is given by $\lambda_G e^{-\lambda_G \tau_m}$. On the other hand, for the sequential-only contract, the probability distribution of τ_m conditional on the success of subproject at τ_s is $\lambda_S e^{-\lambda_S(\tau_m - \tau_s)}$ for $\tau_m > \tau_s$ and 0 for $\tau_m \leq \tau_s$. The marginal probability distribution of τ_s for the sequential-only contract

[14] Since the agent is protected by limited liability, he is indifferent between following the recommendation and being terminated. We can resolve this problem by setting $R_t = \epsilon > 0$ in the direct-only contract, setting $R^t_s = \epsilon$ in the sequential-only contract, and then sending ϵ to zero.

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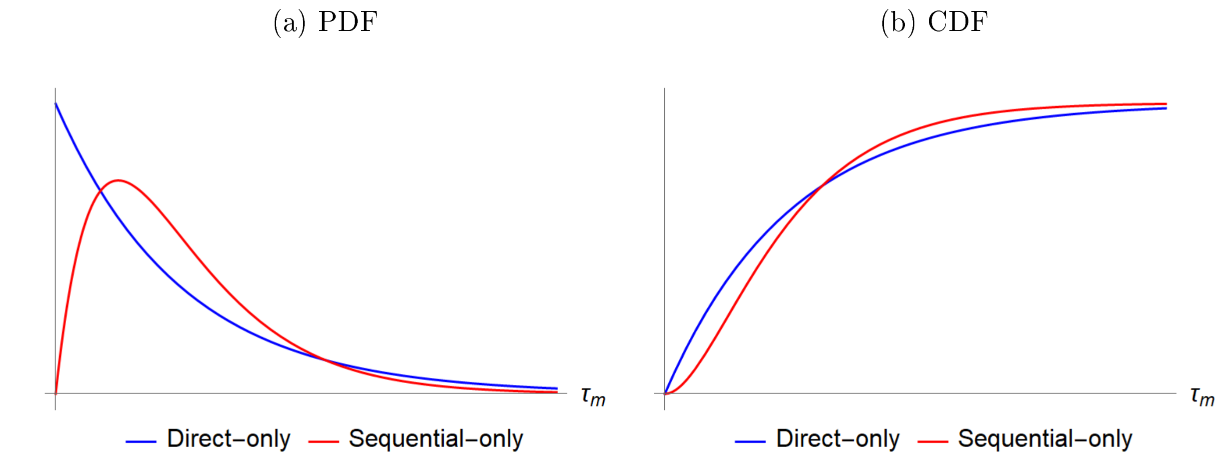

Figure 1: The probability and cumulative distribution functions of τ_m for direct-only and sequential-only contracts without deadlines

without a deadline is given by $\lambda_S e^{-\lambda_S \tau_s}$. Then, we can derive the probability distribution of τ_m for the sequential-only contract as follows:

$$\int_0^{\tau_m} \lambda_S e^{-\lambda_S(\tau_m-\tau_s)} \cdot \lambda_S e^{-\lambda_S \tau_s} d\tau_s = \lambda_S^2 \tau_m e^{-\lambda_S \tau_m}.$$

Figure 1 illustrates the probability and cumulative distribution functions of a date of a main breakthrough τ_m for both contracts. We can observe that the probability distribution of the direct-only contract is decreasing whereas that of the sequential-only contract is hump-shaped. In addition, the probability distribution of the direct-only contract has a fatter tail than that of the sequential-only contract. Moreover, up to a certain date, the cumulative probability of making a breakthrough by the date for the direct-only contract is higher than that for the sequential-only contract, after which the reverse holds. These observations will be useful in the analysis of the following sections.

By using the probability distributions, the expected profits for both contracts can be derived as follows:

$$P_0(\Gamma_G) = \int_0^{\infty} \left(\Pi - \int_0^{\tau_m} c\ dt\right) \lambda_G e^{-\lambda_G \tau_m} d\tau_m = \Pi - \frac{c}{\lambda_G},$$

$$P_0(\Gamma_S) = \int_0^{\infty} \left(\Pi - \int_0^{\tau_m} c\ dt\right) \lambda_S^2 \tau_m e^{-\lambda_S \tau_m} = \Pi - \frac{2c}{\lambda_S}.$$

Since neither contract has a deadline and the agent never shirks, the project is surely completed and the principal receives Π (recall that both parties do not discount). The second

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terms of the expected payoffs come from the cumulative costs. In the direct-only contract without a deadline, the expected duration of the project is $1/\lambda_G$, thus the expected cost is c/λ_G. In the sequential-only project without a deadline, the expected duration for each subproject is $1/\lambda_S$, thus the expected cost is $2c/\lambda_S$. Therefore, by comparing $2\lambda_G$ and λ_S, the principal can determine which contract is more profitable. The following proposition shows that one of these two benchmark contracts is indeed the first-best contract (or both of them are if $2\lambda_G = \lambda_S$).

Proposition 3.1. *Suppose the agent's allocation of effort is observable to the principal, $\lambda_G \Pi \geq c$, and $\lambda_S \Pi \geq 2c$. If $2\lambda_G > \lambda_S$, the direct-only contract is the first-best contract, i.e., it gives the highest expected profit to the principal. If $\lambda_S > 2\lambda_G$, the sequential-only contract is the first-best contract. If $\lambda_S = 2\lambda_G$, both contracts are the first-best contracts.*

In this paper, I focus on the case where splitting the project potentially harms the efficiency of the project, i.e., $2\lambda_G$ is greater than or equal to λ_S. Let $\eta \equiv \lambda_S/\lambda_G - 1$ denote the efficiency of the sequential approach. If η is equal to one, the sequential approach is equally efficient to the direct approach. As η decreases, the efficiency loss from monitoring increases. In the following two sections, I derive optimal contracts respectively for the case of no efficiency loss ($\eta = 1$) and the case of efficiency loss ($\eta < 1$).

4 Optimal Contracts for No Efficiency Loss

In this section, I assume that the principal cannot observe the agent's effort allocation and there is no efficiency loss from monitoring. I characterize the optimal contracts under these assumptions. The presence of moral hazard distorts the first-best contract in two ways: (i) the principal always sets a finite deadline; (ii) the recommended action for the agent may change at some point.

4.1 An Immediate Payment vs. A Deadline Extension

In this subsection, I illustrate that the principal's problem comes down to choosing one of two incentive schemes: an immediate payment to induce the agent to go for the project and a deadline extension to induce the agent to split the project. Then, I explain how the principal decides between these incentive schemes.

I begin by solving the principal's problem given that the subproject is already completed. Since it only requires one more breakthrough, it is identical to the single-stage benchmark of Green and Taylor (2016a), thus I can directly use their results. Let u_S be the agent's promised

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utility for this case.[15] Define the value function $V_S(u_S)$ as the function that maximizes the principal's expected utility $\hat{P}(\hat{\Gamma})$ subject to the promise keeping constraint $\hat{U}(\hat{\Gamma}) = u_S$ and the incentive compatibility condition. Then, they derive $V_S(u_S)$ as follows:

$$V_S(u_S) = \left(\Pi - \frac{c}{\lambda_S} - u_S\right) - \left(\Pi - \frac{c}{\lambda_S}\right) e^{-\frac{\lambda_S}{\phi} u_S}, \tag{4.1}$$

where the first term is the first-best payoff minus the promised utility u_S and the second term is the agency cost. This payoff can be realized by the contract with a finite deadline, $\hat{T} = u_S/\phi$, and a diminishing payoff, $R_{\tau_m} = \phi(1/\lambda_S + \hat{T} - \tau_m)$.[16]

I now consider the case where the agent has not yet had the success of the first subproject. Given a contract Γ, let $U_t(\Gamma)$ denote the agent's maximized continuation utility at time t, that is,

$$U_t(\Gamma) \equiv \sup_{(a,b)\in\mathcal{A}_t} \mathbb{E}^{a,b}\left[\begin{array}{c} R_{\tau_m}\cdot \mathbf{1}_{\{\tau_m\le\tau_s\wedge T\}} + \hat{U}_{\tau_s}(\hat{\Gamma}^{\tau_s})\cdot\mathbf{1}_{\{\tau_s<\tau_m\wedge T\}} \\ +\int_t^{T\wedge\tau_m\wedge\tau_s}\phi(1-a_s-b_s)\,ds \end{array}\ \middle|\ t\le\tau_m\wedge\tau_s\right], \tag{4.2}$$

where $\mathcal{A}_t \equiv \{\{a_s, b_s\}_{t\le s\le T} \ :\ (a_s, b_s, a_s+b_s)\in[0,1]^3\}$. Then, the following lemma characterizes the evolution of $U_t(\Gamma)$.

Lemma 4.1. *Given a contract $\Gamma = \{T, a, b, R, \hat{\Gamma}\}$, suppose that a continuous and differentiable process $\{u_t\}_{0\le t\le T}$ satisfies $u_T = 0$ and the following HJB equation:*

$$0 = \sup_{\substack{a_t,b_t\ge 0,\\ a_t+b_t\le 1}} \dot{u}_t + \phi(1-a_t-b_t) + (R_t - u_t)\lambda_G a_t + (u_S^t - u_t)\lambda_S b_t, \tag{PK}$$

where $\dot{u}_t \equiv du/dt$ and u_S^t is the agent's promised utility for the success of the subproject at time t, i.e., $u_S^t = \hat{U}^t(\hat{\Gamma}^t)$. Then, $u_t = U_t(\Gamma)$.

The HJB equation (PK) provides a clear interpretation of the agent's behavior. The first term is the drift term of the agent's continuation utility from no success and the second term is the benefit from shirking. When the main breakthrough arrives at rate $\lambda_G a_t$, the agent receives the immediate payment R_t but he loses the continuation utility since the contract is terminated. When the intermediate breakthrough arrives at rate $\lambda_S b_t$, the new phase of the contract with the promised utility u_S^t begins and he loses the continuation utility since the current phase of the contract is done.

[15] To derive optimal contracts, I consider the agent's promised utility as a state variable and write a contract recursively. This is a typical approach in the dynamic contract literature (see, e.g., Spear and Srivastava, 1987).

[16] See Proposition 1 in Green and Taylor (2016a).

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Observe that (PK) is linear in a_t and b_t. Hence, we can easily derive necessary conditions for inducing $a_t > 0$ or $b_t > 0$.

Corollary 4.2. *Given a promised utility u_t, to induce the agent to choose the direct approach ($a_t > 0$), R_t should be greater than or equal to $u_t + \phi/\lambda_G$. In order to induce the agent to choose the sequential approach ($b_t > 0$), u_S^t should be greater than or equal to $u_t + \phi/\lambda_S$.*

In the optimal contract, the principal sets $R_t = u_t + \phi/\lambda_G$ or $u_S^t = u_t + \phi/\lambda_S$. To see why, note that the principal will never recommend shirking because the principal incurs costs but cannot receive any benefit. Thus, at least one of a_t and b_t should be positive, then by the above corollary, either $R_t \geq u_t + \phi/\lambda_G$ or $u_S^t \geq u_t + \phi/\lambda_S$ holds. Moreover, if the constraint is not binding, the principal can improve the contract by lowering either the payment upon success (R_t) or the new promised utility (u_S^t).[17]

Given the above result, the principal chooses the recommendation by comparing her expected payoff from recommending the direct approach ($a_t = 1$) with the minimum incentive ($R_t = u_t + \phi/\lambda_G$) and that from recommending the sequential approach ($b_t = 1$) with the minimum incentive ($u_S^t = u_t + \phi/\lambda_S$).[18] The first option can be simply interpreted as an *immediate payment* since the principal immediately pays the agent when the breakthrough is made. I argue that the second option can be considered as a *deadline extension*. To see why, note that (PK) is equivalent to $\dot{u}_t = -\phi$ for both cases and the contract will be terminated at time T with $u_T = 0$. Therefore, when the agent is promised the continuation utility u unless any breakthrough has been made, the principal would terminate the contract at time u/ϕ. In other words, the contract has the deadline u/ϕ. Recall that the principal imposes the deadline u_S/ϕ when the subproject is completed and the agent's promised utility is u_S. By plugging in $u_S = u + \phi/\lambda_S$, we can see that the updated deadline is $u/\phi + 1/\lambda_S$. Thus, when the intermediate breakthrough is made, the deadline is extended by $1/\lambda_S$. The agent is paid if he achieves the final breakthrough by the extended deadline. Hence, the principal's problem becomes a choice between an immediate payment to induce the agent to go for the project and a deadline extension to induce the agent to split the project.

When the principal chooses between these two incentive schemes in no efficiency loss case, two economic forces come into play: *monitoring* and *time pressure*. For the deadline extension scheme, the former force is beneficial but the latter is disadvantageous. First, if the principal uses a deadline extension to induce splitting the project, she can monitor the agent's intermediate progress. On the other hand, if the principal chooses an immedi-

[17] Since I have not yet introduced the principal's problem, this argument is not straightforward. However, I guess the value function by assuming that the constraint binds, then verify that it is optimal to make the equality hold. See Appendix B for details.

[18] In Lemma B.2, I show that we can focus on the case where the agent's recommended action is pure.

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ate payment scheme to induce going for the project, the principal cannot observe progress until the main project is done. Therefore, the deadline extension incentive scheme has a comparative advantage in supervising the agent and can potentially lessen the moral hazard problem. Second, however, when the deadline is close by, the principal may prefer a path that requires one breakthrough (inducing going for the project) to a path that requires two breakthroughs (inducing splitting the project). This is because it is relatively difficult to make two breakthroughs under shorter time horizons as illustrated in Figure 1. Thus, with the presence of time pressure, the principal may choose an immediate payment to induce the agent to go for the project.

4.2 Contracts with at most One Switch

I now construct contracts that deliver the principal's maximal expected payoff given the agent's promised utility levels. I begin by introducing some definitions. Let $V(u)$ denote the principal's value function, which represents her maximized expected utility $P_0(\Gamma)$ subject to $U_0(\Gamma) = u$ and the incentive compatibility condition. If a contract Γ satisfies $P_0(\Gamma) = V(u)$ and $U_0(\Gamma) = u$, Γ is said to *implement* a pair of expected payoffs $(V(u), u)$. The derivation of the value function V is relegated to Appendix B. In the main text, I focus on introducing the contract that implements the value function.

According to the intuition from the previous subsection, we might guess that the optimal incentive scheme is to recommend the direct approach when the deadline is close by and to recommend the sequential approach when it is distant. I construct a contract that provides a minimum incentive not to shirk and potentially has a switch of recommendations from the sequential approach to the direct approach.

Definition 4.1. Define a contract Γ as *a minimum incentive contract with at most one switch* if it satisfies the following.

(i) There is a deadline $T < \infty$ and a potential switching point $0 \le S \le T$.

(ii) For all $0 \le t < S$, $(a_t, b_t, R_t) = (0, 1, 0)$, i.e., the sequential approach is recommended and the agent is not paid even if he makes the success on the main project. Moreover, $T^t = T + 1/\lambda_S$, and $(a^t_s, R^t_s) = (1, \phi(T^t - s + 1/\lambda_S))$ for all $s \in [t, T^t]$. It means that the deadline is extended by $1/\lambda_S$ and the agent's payment upon success diminishes linearly over time.

(iii) For all $S < t < T$, $(a_t, b_t, R_t) = (1, 0, \phi(T - t + 1/\lambda_G))$, i.e., the direct approach is recommended and the agent's payment upon success diminishes linearly over time.

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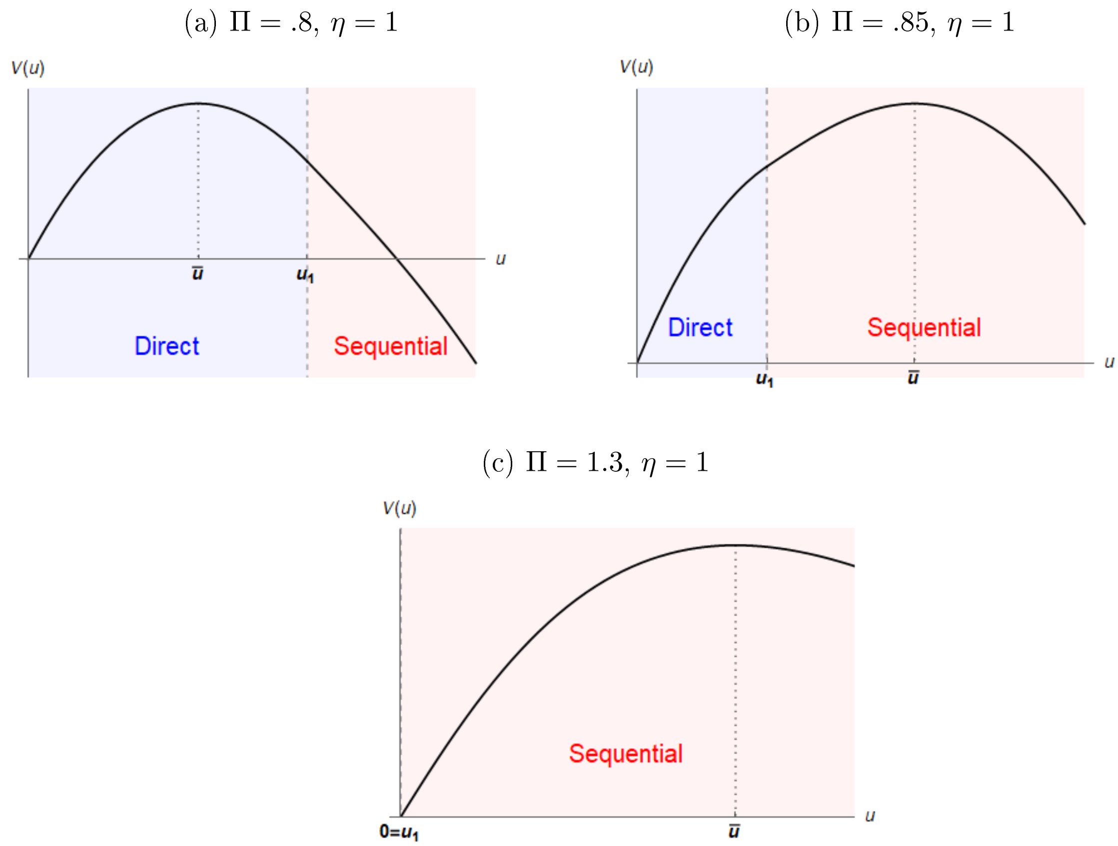

Figure 2: Value functions when there is no efficiency loss

Moreover, $T^t = t$, i.e., when the subproject is completed, the principal terminates the contract without paying the agent.

The following proposition confirms the conjecture that when there is no efficiency loss, the principal's maximized expected payoff can be implemented by a minimum incentive contract with at most one switch.

Proposition 4.1. *Suppose that there is no efficiency loss from monitoring ($\eta = 1$). A pair of the agent's promised utility and the corresponding maximized expected payoff of the principal $(u, V(u))$ is implemented by a minimum incentive contract with at most one switch. If the switch happens, it is from the sequential approach to the direct approach.*

This result is illustrated in the graphs in Figure 2. The horizontal axis represents the agent's promised utility u and the vertical axis represents the principal's value function $V(u)$. The level u_1 represents the promised utility level at which the recommendation switches. If u is in the blue region, i.e., below u_1, it is optimal for the principal to recommend the direct

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approach ($a = 1$). If u is in the red region, i.e., above u_1, it is optimal for the principal to recommend the sequential approach ($b = 1$). Recall that $\dot{u} = -\phi$ when the principal employs the minimum incentive contract. First, consider a case where u is less than u_1 in Figure 2a or Figure 2b. Note that the principal always recommends the direct approach until the deadline since $[0, u]$ is in the blue region. In other words, the principal would employ the minimum incentive contract with $T = u/\phi$ and $S = 0$ and this contract would implement $(u, V(u))$. Denote a contract with $S = 0$ and $T > 0$ by *a direct-only contract with a deadline T*. Second, consider the case where u is greater than u_1 in Figure 2a or Figure 2b. In this case, the principal's recommendation switches at the point that the promised utility is equal to u_1. Then, $(u, V(u))$ can be implemented by a minimum incentive contract with $T = u/\phi$ and $S = (u - u_1)/\phi$. Denote a contract with $T > S > 0$ by *a contract with a switch from sequential to direct at S and a deadline T*. Last, consider the case in Figure 2c. In this case, the principal would never recommend going for the project, that is, $S = T$. Thus, the principal would employ a minimum incentive contract with $S = T = u/\phi$ and the contract implements $(u, V(u))$. Denote a contract with $S = T$ by a *a sequential-only contract with a deadline T*.

4.3 The Optimal Contract

Now that I have characterized the contract that maximizes the principal's expected payoff under the constraint that the agent's promised utility is equal to u, the next step is to pin down the optimal promised utility level. To derive the optimal contract, the principal solves

$$\max_{u \geq 0} V(u). \tag{MP}$$

Let $\bar{u}$ be the solution to (MP). Then, the optimal contract is the contract that implements $(V(\bar{u}), \bar{u})$. To characterize the optimal contract, we need to answer (i) whether the switching point $\bar{u}$ is greater than 0 or not; (iii) whether $\bar{u}$ is greater than u_1 or not; (iii) whether u_1 is greater than 0 or not.

When $\bar{u}$ is equal to zero, the principal's expected payoff is maximized at $u = 0$. In this case, it is optimal for the principal not to initiate the contract in the first place, i.e., the project is infeasible. Second, if u_1 is greater than $\bar{u}$, the principal can implement the maximum expected payoff by a direct-only contract with a deadline $\bar{u}/\phi$. This case is illustrated in Figure 2a. Third, if u_1 is greater than 0 but less than $\bar{u}$, the optimal contract involves a switch of recommendations. Therefore, the optimal contract takes the form of a contract with a switch from sequential to direct at $(\bar{u} - u_1)/\phi$ and a deadline $\bar{u}/\phi$. This case is illustrated in Figure 2b. Last, if u_1 is equal to zero, the principal always recommends to split the

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project. Therefore, to implement $(\bar{u}, V(\bar{u}))$, the principal employs a sequential-only contract with a deadline $\bar{u}/\phi$. This case is illustrated in Figure 2c. The following theorem shows that the payoff from the success Π determines which case will be applied.

Theorem 1. *Suppose that there is no efficiency loss from monitoring ($\eta = 1$). There exist thresholds $\Pi_G(1)$ and $\Pi_S(1)$ with $\Pi_S(1) > \Pi_G(1) > \Pi_F \equiv (c + \phi)/\lambda_G$ such that the optimal contract is determined as follows:*

- *(a) when $\Pi \leq \Pi_F$, the project is infeasible;*
- *(b) when $\Pi_G(1) \geq \Pi > \Pi_F$, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with a deadline $\bar{u}/\phi$;*
- *(c) when $\Pi_S(1) > \Pi > \Pi_G(1)$, there exists $u_1 \in (0, \bar{u})$ such that $(\bar{u}, V(\bar{u}))$ is implemented by a contract with a switch from splitting the project to going for the project at $(\bar{u} - u_1)/\phi$ and a deadline $\bar{u}/\phi$;*
- *(d) when $\Pi \geq \Pi_S(1)$, $(\bar{u}, V(\bar{u}))$ is implemented by a sequential-only contract with a deadline $\bar{u}/\phi$.*

Why does Π determine the form of the optimal contract?—When Π is very small, the project is not appealing to the principal. Thus, the project would be infeasible in this case. Next, consider the case where Π is not very small, thus the project is feasible. The principal determines which approach to recommend by comparing the expected payoffs from recommending each approach. The expected payoff is the probability of success times the payoff from the project net of the expected running cost. When we compare an immediate deadline incentive to a deadline extension incentive, the deadline extension provides a higher probability of success, but the expected running cost is also higher. Therefore, if the payoff from a successful project is high, it is more advantageous for the principal to recommend the sequential approach.

Why can a sequential-only or direct-only contract be optimal?—As Π grows, the value of monitoring increases. If Π is above a certain threshold, monitoring outweighs time pressure and the principal prefers to recommend the sequential approach even at the deadline. On the other hand, a direct-only contract can be an optimal contract when Π is small enough. This is because a smaller Π not only makes the direct approach more appealing but also makes the length of the contract shorter. In this case, time pressure is a more important factor than monitoring even at the beginning of the contract.

What is the real-world implication of this result?—In the software development application mentioned in the introduction, this result explains why the agile methodology is

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preferred for a small-sized project and the waterfall methodology is preferred for a large-sized project. In addition, it suggests that the mixture of two methodologies—starting with the waterfall then switching to the agile—is the optimal strategy for a mid-sized project.[19] The theorem also has implications for scientific research. The scale of applied research is typically large due to its practical nature (e.g., clinical trials). Thus, this result implies that the sequential approach is popular for applied research. In contrast, the scale of basic research is usually small because its goals are less product oriented (e.g., in-vitro experiments). Therefore, this finding suggests that the direct approach is common for basic research.

5 Optimal Contracts under Efficiency Loss

I now introduce an efficiency loss from monitoring, that is, η is less than 1. In this case, we need to consider *efficiency* as another economic force determining the optimal contract in addition to monitoring and time pressure. For longer time horizons, the sum of expected payoffs for both players from the contract converges to the first-best contract, that is, efficiency determines which approach should be recommended. Since we focus on the case where the sequential approach is less efficient than the direct approach, the principal would recommend the agent to go for the project when the deadline is far off. Based on this intuition, I conjecture that the principal prefers to recommend a more efficient approach under longer time horizons. By adding this feature to the contract with one switch, I construct a contract that provides a minimum incentive not to shirk and potentially has two switches from the direct approach to the sequential approach, then to the direct approach again. The proposition following the definition shows that when there is an efficiency loss, the principal's maximized expected payoff can be implemented by this contract.

Definition 5.1. Define a contract Γ as *a minimum incentive contract with at most two switches* if it satisfies the following.[20]

(i) There is a deadline $T < \infty$ and two potential switching points $0 \leq S_1 \leq S_2 \leq T$.

(ii) For all $0 \leq t < S_1$, $(a_t, b_t, R_t) = (1, 0, \phi(T - t + 1/\lambda_G))$ and $T^t = t$, i.e., the direct approach is recommended with a minimum incentive.

[19] This conjecture is based on the assumption that there is no 'switching cost.' However, in practice, there may exist some learning costs when programmers switch from one methodology to another. These switching costs may eliminate the mid-sized project switching region as a matter of practicality.

[20] When I say that there are two switches, I implicitly assume that the first switch is from the direct approach to the sequential approach and the second switch is from the sequential approach back to the direct approach.

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(a) $\Pi = .8$, $\eta = .9$

(b) $\Pi = .95$, $\eta = .9$

(c) $\Pi = 2$, $\eta = .9$

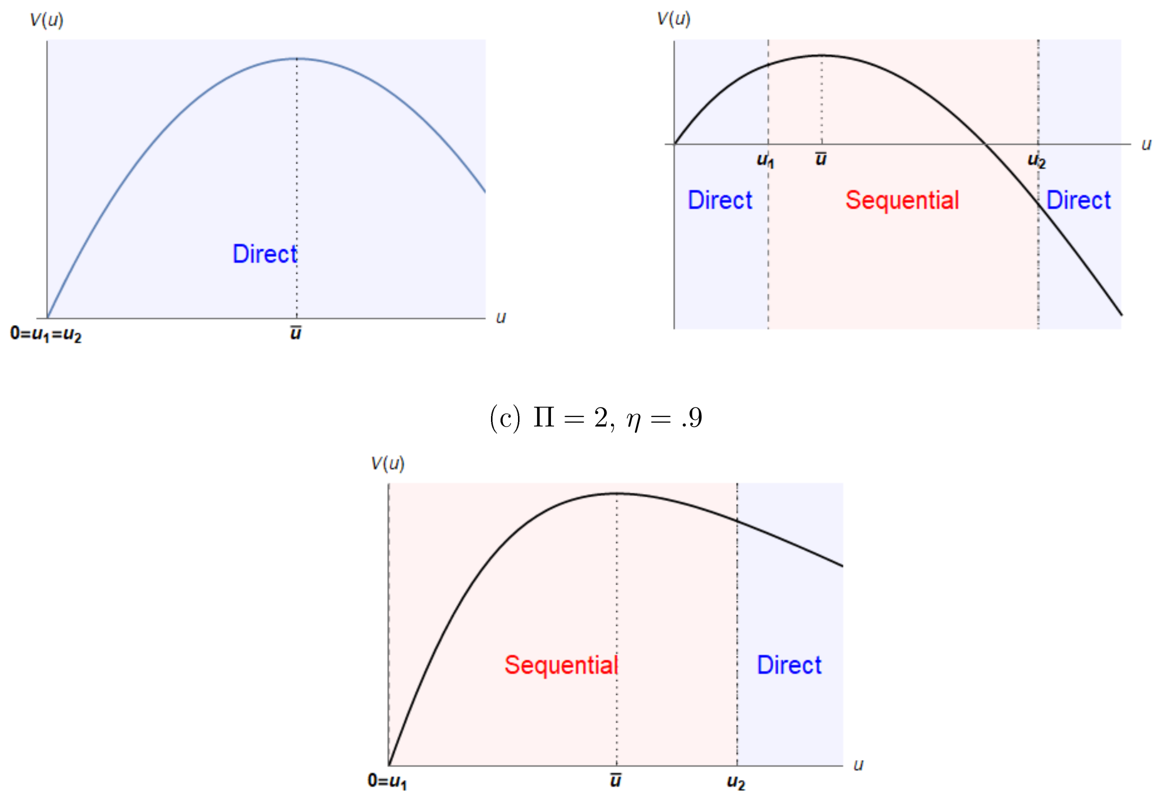

Figure 3: Value functions when the efficiency loss is small

(iii) For all $S_1 < t < S_2$, $(a_t, b_t, R_t) = (0, 1, 0)$, $T^t = T + 1/\lambda_S$, and $(a^t_s, R^t_s) = (1, \phi(T^t - s + 1/\lambda_S))$ for all $s \in [t, T^t]$, i.e., the sequential approach is recommended with a minimum incentive.

(iv) For all $S_2 < t \leq T$, $(a_t, b_t, R_t) = (1, 0, \phi(T - t + 1/\lambda_G))$ and $T^t = t$, i.e., the direct approach is recommended with a minimum incentive.

Proposition 5.1. *Suppose that there is an efficiency loss from monitoring* $(0 < \eta < 1)$*. A pair of the agent's promised utility and the corresponding maximized expected payoff of the principal* $(u, V(u))$ *is implemented by a minimum incentive contract with at most two switches.*

The main difference from the case of no efficiency loss is that the principal would recommend the direct approach when the agent's promised utility is large, i.e., the deadline is far away. We can see this feature in the graphs in Figure 3 and 4. In these plots, u_1 represents the promised utility level at which the recommendation switches from the sequential approach to the direct approach as time goes by, i.e., u decreases. Similarly, u_2 represents

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(a) $\Pi = .8$, $\eta = .55$ (b) $\Pi = 4$, $\eta = .55$

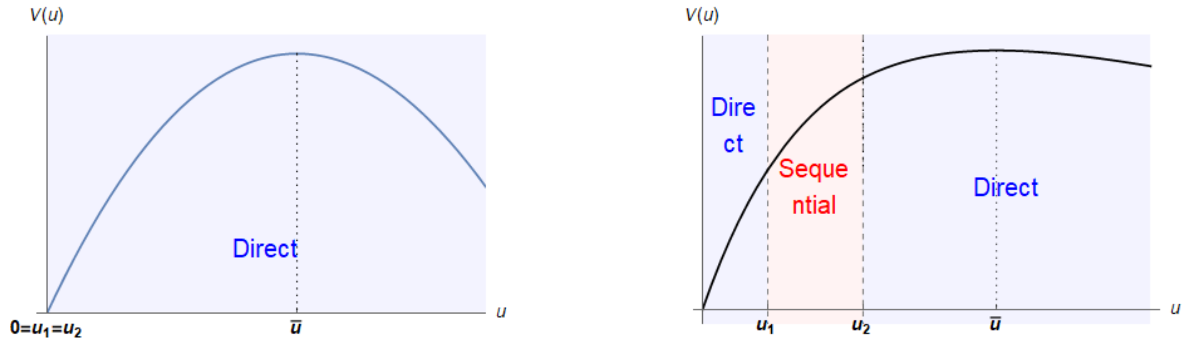

Figure 4: Value functions when the efficiency loss is large

the promised utility level at which the recommendation switches from the direct approach to the sequential approach. In Figure 4b, the pair $(\bar{u}, V(\bar{u}))$ can be implemented by a minimum incentive contract with $T = \bar{u}/\phi$, $S_1 = (\bar{u} - u_2)/\phi$ and $S_2 = (\bar{u} - u_1)/\phi$, thus the optimal contract in this case involves exactly two switches. Denote a contract with $T > S_2 > S_1 > 0$ by *a contract with two switches at* S_1 *and* S_2 *and a deadline* T. Another difference is that the principal may recommend the direct approach whatever the promised utility level is. This possibility is illustrated in Figure 3a and 4a. In the no efficiency loss case, since the principal has a monitoring advantage from splitting the project without loss, she always splits the project when the agent's promised utility is large enough (i.e., the horizon is long enough). However, this is not necessarily the case when there is an efficiency loss.

By Proposition 5.1, we know that the optimal contract involves at most two switches of recommendations. To obtain more refined results, we need to compare two switch points u_1 and u_2 (if they exist) with the profit maximizing promised utility level $\bar{u}$ as we did in the previous subsection. In this case, the type of the optimal contract depends not only on Π but also on η. For the rest of this section, I characterize optimal contracts for two cases: (i) when η is above $\max\{\sqrt{c/(c+\phi)},\ 1/(e-1)\}$, i.e., the efficiency loss is small; (ii) when η is below $\min\{1/(e-1),\ c/(c+\phi)\}$, i.e., the efficiency loss is large.[21]

Theorem 2. *Suppose that* η *is greater than* $\sqrt{c/(c+\phi)}$ *and* $1/(e-1)$*, i.e., the efficiency loss from monitoring is small. There exist thresholds* $\Pi_G(\eta)$ *and* $\Pi_S(\eta)$ *with* $\Pi_S(\eta) > \Pi_G(\eta) > \Pi_F = (c+\phi)/\lambda_G$ *such that the optimal contract is determined as follows:*

(a) when $\Pi \leq \Pi_F$*, the project is infeasible;*

[21] These cases do not cover the case where the efficiency loss is intermediate. In that case, the form of the optimal contract depends highly on parameter values η and Π and there are many subcases to consider.

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(b) when $\Pi_G(\eta) \geq \Pi > \Pi_F$, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with a deadline $\bar{u}/\phi$;

(c) when $\Pi_S(\eta) \geq \Pi > \Pi_G(\eta)$, there exists $u_1 \in [0, \bar{u}]$ such that $(\bar{u}, V(\bar{u}))$ is implemented by a contract with a switch from splitting the project to going for the project at $(\bar{u} - u_1)/\phi$ and a deadline $\bar{u}/\phi$;

(d) when $\Pi > \Pi_S(\eta)$, $(\bar{u}, V(\bar{u}))$ is implemented by a sequential-only contract with a deadline $\bar{u}/\phi$.

This result is similar to Theorem 1. Intuitively, when the efficiency loss is small, it only drives the principal to recommend the direct approach when the deadline is fairly far away and it may go further than the optimal length of the contract. Thus, the switch from the direct approach to the sequential approach may not occur in the optimal contract. The condition that η is greater than $\sqrt{c/(c+\phi)}$ ensures that the switching point u_2 would be always above the optimal promised utility level $\bar{u}$ (Figure 3b and 3c), or the switching point does not exist at all (Figure 3a).[22] Also, the condition that η is greater than $1/(e-1)$ makes the principal prefer the split project even at the deadline when Π is large enough (Figure 3c).[23]

These results imply that the findings from the no efficiency loss case are robust to the introduction of small efficiency costs. However, the results break down when the efficiency loss is quite large.

Theorem 3. *Suppose that η is less than $c/(c+\phi)$ and $1/(e-1)$, i.e., the efficiency loss from monitoring is large. There exists a threshold $\Pi_M(\eta)$ with $\Pi_M(\eta) > \Pi_F = (c+\phi)/\lambda_G$ such that the optimal contract is determined as follows:*

(a) when $\Pi \leq \Pi_F$, the project is infeasible;

(b) when $\Pi_M(\eta) \geq \Pi > \Pi_F$, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with a deadline $\bar{u}/\phi$;

(c) when $\Pi > \Pi_M(\eta)$, there exist u_1 and u_2 such that $0 < u_1 < u_2 < \bar{u}$ and $(\bar{u}, V(\bar{u}))$ is implemented by a contract with two switches at $(\bar{u} - u_2)/\phi$ and $(\bar{u} - u_1)/\phi$ and a deadline $\bar{u}/\phi$.

This result is quite different from the previous ones in that the optimal contract involves either zero or two switches. Intuitively, when there is a large efficiency loss from monitoring,

[22]Refer to Lemma B.16 for details.
[23]Refer to Proposition B.2 for details.

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the principal would recommend the direct approach for the majority of the time and recommend the sequential approach for only short periods. If Π is not that large, the principal would not employ the sequential approach at all (Figure 4a). However, if Π is large enough, the principal would recommend the sequential approach in the middle of the optimal contract (Figure 4b). To get this result, we need to show that $\bar{u} > u_2 > u_1 > 0$. The condition that η is less than $c/(c+\phi)$ ensures that u_2 is always below $\bar{u}$.[24] Also, the condition that η is less than $1/(e-1)$ makes the principal recommend going for the project at the deadline no matter what Π is.[25]

Intuitively, the principal generally prefers to recommend the direct approach since there is a large efficiency loss from recommending the sequential approach. Nevertheless, when Π is large enough, the principal may take advantage of the monitoring benefit by recommending the sequential approach. If the principal decides to monitor at some point, it is optimal to monitor in the middle of the contract. This is because efficiency outweighs monitoring at the beginning of the contract and time pressure outweighs monitoring at the end of the contract. Hence, the optimal contract involves two switches when Π is large.

One application of this result pertains to a situation with a certification process for skill competency. The agent can try to complete the project with a basic skill: the direct approach. Alternatively, he can try to obtain a certificate–which is observable and contractually verifiable–then try to complete the project with an advanced skill: the sequential approach. Since a certification process generally requires substantially more than project-specific skills, the efficiency loss from monitoring would be large. Theorem 3 implies that the principal would not recommend the agent to acquire a certification unless the project size is large enough.

For a very large-scale project, the theorem suggests that another type of contract involving all three economic forces is optimal. At the beginning of the contract, the principal recommends the agent use the direct approach because it is more efficient (i.e., efficiency is initially the dominant concern). When the success is not delivered by a specified time, the principal begins to monitor the agent more closely by recommending a switch to the sequential approach (i.e., monitoring becomes the primary concern). She extends the deadline if the agent makes intermediate progress, but if he does not make progress before the deadline is near, the principal then recommends a switch back to the direct approach in a "last-ditch" attempt at getting the job done (i.e., time pressure becomes the preeminent motivation).

[24] Refer to Lemma B.15 for details.

[25] Refer to Proposition B.2 for details.

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6 Extensions

I explore three variations of the model that reflect relevant features in some economic applications. First, I analyze the case of asymmetric arrival rates of subprojects and ask whether this can change the form of the optimal contract. Next, I consider outside options for both players that emerge after the completion of the subproject under the sequential approach and investigate how they affect the optimal contract. Last, I introduce discounting and discuss how it affects the desirability of each approach.

6.1 Asymmetric Arrival Rates of Subprojects

Here I investigate a setting where the arrival rates for the subprojects in the sequential approach are no longer the same. Let $\lambda_{S,1}$ and $\lambda_{S,2}$ denote the arrival rates for the first and second subprojects. The ratio between these two arrival rates, $\lambda_{S,2}/\lambda_{S,1}$, is denoted by κ. In Theorem 4, I show that the form of the optimal contract depends crucially on κ.

To simplify discussion, I restrict attention to the case where there is no efficiency loss from monitoring. I begin by considering the first-best scenario to derive the condition that fulfills this restriction. Under the assumption that the agent's allocation of effort is observable to the principal, the principal's expected profit from the sequential-only contract without a deadline is derived as follows:

$$\int_0^\infty \int_{\tau_s}^\infty \left(\Pi - \int_0^{\tau_m} c\ dt\right) \lambda_{S,2} e^{-\lambda_{S,2}(\tau_m - \tau_s)} d\tau_m\ \lambda_{S,1} e^{-\lambda_{S,1}\tau_s} d\tau_s = \Pi - \frac{c}{\lambda_{S,1}} - \frac{c}{\lambda_{S,2}}.$$

Note that the principal's expected profit from the direct-only contract without a deadline is $\Pi - c/\lambda_G$. By using similar steps to those in Section 3.3, we can show that there is no efficiency loss from monitoring if and only if $1/\lambda_G = 1/\lambda_{S,1} + 1/\lambda_{S,2}$.

Under this condition, by using the definition of κ, we can derive that $\lambda_{S,1} = (1 + 1/\kappa)\,\lambda_G$ and $\lambda_{S,2} = (1 + \kappa)\lambda_G$. From these equations, we observe that higher κ implies that it is harder to achieve the first subproject (lower $\lambda_{S,1}$) but it is easier to complete the second subproject (higher $\lambda_{S,2}$). Note that the completion of the first subproject is monitored by the principal. If the first subproject becomes more difficult and the second subproject becomes easier, we can think of monitoring as relatively more effective. Thus, κ can be interpreted as a parameter that measures the *effectiveness* of monitoring. The following theorem characterizes the optimal contract under an arbitrary κ.

Theorem 4. *Suppose that there is no efficiency loss from monitoring and the arrival rates of the subprojects are not necessarily symmetric. There is a threshold $\kappa^* > 0$ such that the optimal contract is implemented as follows.*

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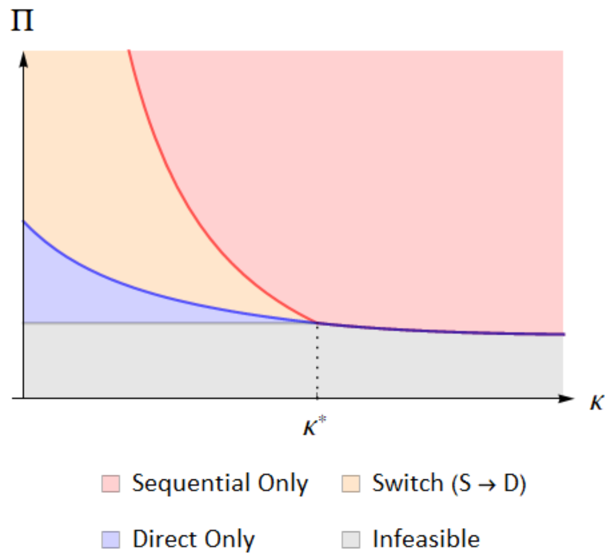

Figure 5: Optimal contracts under asymmetric arrival rates and no efficiency loss

(a) If $\kappa < \kappa^*$, there exist $\Pi_S^A(\kappa) > \Pi_G^A(\kappa) > \Pi_F^A(\kappa) = \Pi_F = (c+\phi)/\lambda_G$ such that the optimal contract is determined as follows:

(i) when $\Pi \leq \Pi_F^A(\kappa) = \Pi_F$, the project is infeasible;

(ii) when $\Pi_G^A(\kappa) \geq \Pi > \Pi_F^A = \Pi_F$, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with a deadline $\bar{u}/\phi$;

(iii) when $\Pi_S^A(\kappa) > \Pi > \Pi_G^A(\kappa)$, there exists $u_1 \in (0, \bar{u})$ such that $(\bar{u}, V(\bar{u}))$ is implemented by a contract with a switch from splitting the project to going for the project at $(\bar{u} - u_1)/\phi$ and a deadline $\bar{u}/\phi$;

(iv) when $\Pi \geq \Pi_S^A(\kappa)$, $(\bar{u}, V(\bar{u}))$ is implemented by a sequential-only contract with a deadline $\bar{u}/\phi$.

(b) If $\kappa \geq \kappa^*$, there exists $\Pi_F^A(\kappa) \leq \Pi_F$ such that the optimal contract is determined as follows:

(i) when $\Pi \leq \Pi_F^A(\kappa)$, the project is infeasible;

(ii) when $\Pi > \Pi_F^A(\kappa)$, $(\bar{u}, V(\bar{u}))$ is implemented by a sequential-only contract with a deadline $\bar{u}/\phi$.

This theorem is illustrated in Figure 5. When monitoring is relatively ineffective, i.e., κ is below κ^*, optimal contracts are characterized in the same way as in Theorem 1: (i) a

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sequential-only contract when Π is high; (ii) a contract with one switch when Π is intermediate; (iii) a direct-only contract when Π is relatively small; (iv) infeasible when Π is very small. However, when monitoring is relatively effective, i.e., κ is above κ^*, optimal contracts are characterized in a different manner. In this case, the optimal contract is in the form of a sequential-only contract when Π is not small, and the project is infeasible when Π is small.

From this exercise, we can conclude that the form of the optimal contract is determined not only by the size of the project (Π) but also the effectiveness of monitoring (κ). In particular, when monitoring is effective enough, monitoring is a more important factor than time pressure. Thus, even when the size of the project is small, the principal always recommends the sequential approach so long as the project is feasible.

6.2 Independent Values from a Subproject

In many relevant applications, both players may benefit from the completion of the subproject. In particular, their outside options will differ before and after the intermediate breakthrough. For example, by adding the experience to his resume, the agent can get a better offer from other firms. For the principal, even if the agent does not finish the main project, she may be able to license out the current progress to another company.

The goal of this section is to investigate how these outside options affect the optimal contract. I address this question by exploring a numerical example. I fix parameter values as follows: $\Pi = 5$, $c = 1$, $\phi = .5$, $\lambda_G = 1$, $\lambda_S = 2$. Note that λ_S is equal to $2\lambda_G$ which means that two approaches are equally efficient. Moreover, Π is greater than $\Pi_S(1)$ implying that the optimal contract takes a form of a sequential-only contract with a deadline when there are no intermediate values from the subproject.

Let $B_P \geq 0$ and $B_A \geq 0$ denote the outside options for the principal and the agent when the subproject is completed. We can interpret a high B_P involving an active buyout market for projects and a high B_A involving an active labor market for experienced workers. The goal is to provide comparative statics on the deadlines and the principal's expected payoffs from the optimal contracts with respect to B_A and B_P. To derive the optimal contracts, we need to reformulate the principal's problem by adding B_A and B_P.

I begin by rewriting the promise keeping constraint (PK). Once I introduce the outside option B_A for the agent, the effective promised utility for the agent at time t would be $u_S^{B,t} \equiv u_S^t - B_A$ where u_S^t is the agent's promised utility for the success of the subproject at t. Then, (PK) can be rewritten as follows:

$$0 = \sup_{\substack{a_t, b_t \geq 0, \\ a_t + b_t \leq 1}} \dot{u}_t + \phi(1 - a_t - b_t) + (R_t - u_t)\lambda_G a_t + (u_S^{B,t} - u_t + B_A)\lambda_S b_t. \qquad (\text{PK}_B)$$

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From this equation, we can infer that to induce the sequential approach, $u_S^{B,t}$ has to be greater than or equal to $u_t + \phi/\lambda_S - B_A$. To simplify discussion, I focus on the case where the principal "extends" the deadline after the completion of the subproject, i.e., $B_A < \phi/\lambda_S = .25$.

Next, we need to reconsider the principal's expected payoff after the completion of the intermediate breakthrough. Note that the additional value for the second subproject would be $\Pi - B_P$ since the principal has the outside option of B_P. In addition, the principal needs to work with the agent's effective promised utility rather than the original promised utility. Then, given the intermediate breakthrough, the principal's value function can be written as in (4.1):

$$V_S^B(u_S^B) = \left(\Pi - B_P - \frac{c}{\lambda_S}\right)\left(1 - e^{-\frac{\lambda_S}{\phi}u_S^B}\right) - u_S^B. \tag{6.1}$$

This value stands for the principal's expected payoff additional to the outside option B_P when the agent's effective promised utility is u_S^B. Thus, $V_S^B(u_S^B) + B_P$ is the principal's expected payoff after the completion of the subproject. By replacing (PK) with (PK_B) and $V_S(u)$ with $V_S^B(u_S^B) + B_P$, we can rewrite the principal's problem and derive the optimal contract as in the previous sections. The details are relegated to Supplementary Appendix SA.3.

In Figure 6, I illustrate the comparative statics on the optimal deadlines and the principal's expected payoffs under four different scenarios. In Figure 6a and 6b, I demonstrate the comparative statics with respect to B_A when B_P is 0 or 3. In Figure 6c and 6d, I display the comparative statics with respect to B_P when B_A is 0 or .2. In the left panels, the green curves demonstrate the optimal deadlines and the blue curves display the switching time from the sequential approach to the direct approach. If the blue curves are not present, it means that the sequential approach is recommended until the deadline. The principal's expected payoffs (before the intermediate breakthrough) are shown in the right panels.

First, we can observe that the principal's expected payoffs are increasing in B_A or B_P in every panel in Figure 6. This means that the principal benefits from both the active buyout market for projects and the labor market for experienced workers. Since the active buyout market allows the principal to liquidate the project easier, it directly helps the principal. When the labor market for experienced workers becomes more active, it is easier for the principal to incentivize the agent to split the project. Given the intermediate breakthrough, while the principal needs to extend the deadline by $1/\lambda_S$ without the labor market, she only needs to extend the deadline by $1/\lambda_S - B_A/\phi$ with the labor market. This numerical example shows that the principal benefits from this decreased incentive under the restriction that B_A is less than ϕ/λ_S.[26]

[26]The case where B_A is greater than ϕ/λ_S is problematic for the following reason. In this case, if the

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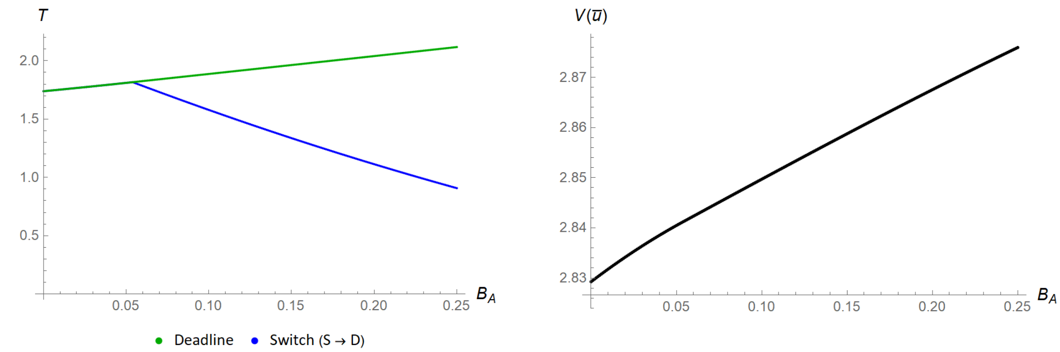
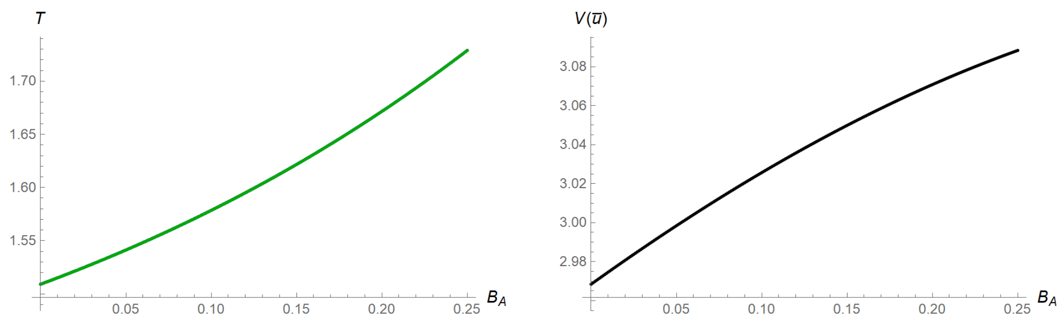
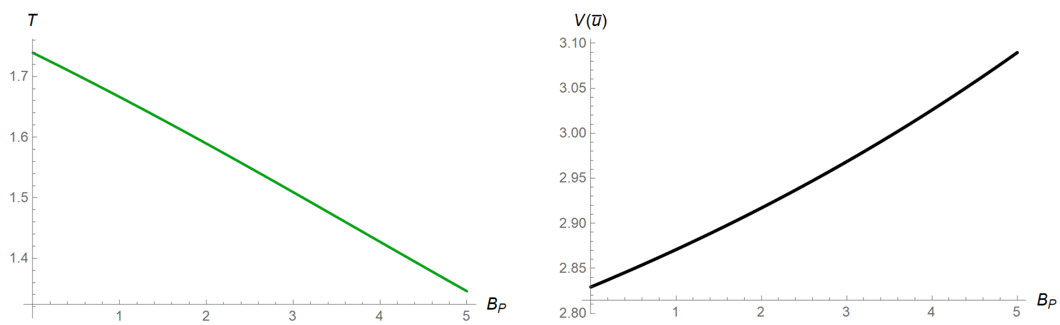
(a) Deadlines and the principal expected payoffs when $B_P = 0$



(b) Deadlines and the principal expected payoffs when $B_P = 3$



(c) Deadlines and the principal expected payoffs when $B_A = 0$



(d) Deadlines and the principal expected payoffs when $B_A = .2$

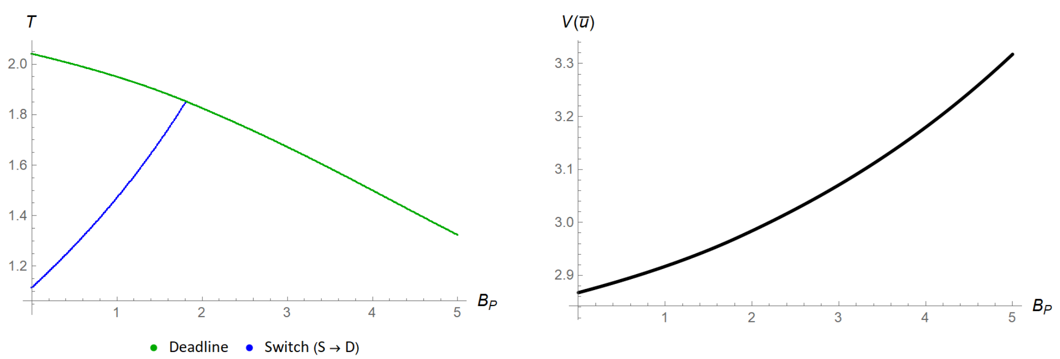


Figure 6: Comparative Statics with respect to B_A and B_P

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Next, in the left panels of Figure 6a and 6b, we can see that the optimal deadline increases as B_A increases. The principal expects that the deadline extension would be shortened as B_A increases, thus she preemptively prolongs the deadline. Another interesting feature is that switching to the direct approach occurs when B_A is large and B_P is small as in the left panel of Figure 6a. As B_A rises, the sequential approach becomes less appealing to the principal since the probability of success is lowered by the shortened deadline extension. Therefore, it may be more profitable to switch to the direct approach near the deadline. When B_P is large, even though the sequential approach becomes less desirable, it may still be better than the direct approach. Thus, there may not exist a switch at all as in 6b.

Last, the left panels of Figure 6c and 6d show that the optimal deadline decreases as B_P rises. This is simply because an "early buyout" becomes more attractive as the buyout market grows. Moreover, as B_P increases, the sequential approach becomes more desirable. The left panel of Figure 6d shows that the direct approach is optimal when B_P is small, but the sequential approach eventually dominates as B_P grows.

6.3 The Role of the Discount Rate

Finally, I introduce discounting to the model and investigate how it distorts the efficiency of each approach. Let $r \geq 0$ denote the common discount rate for the principal and the agent. The expected payoffs for a direct-only and a sequential-only contracts without deadlines defined in Section 3.3 can be rewritten respectively as follows:

$$\begin{aligned}
F_G(r) \equiv P_0(\Gamma_G) &= \int_0^\infty \left(e^{-r\tau_m}\Pi - \int_0^{\tau_m} e^{-rt} c dt \right) \lambda_G e^{-\lambda_G \tau_m} d\tau_m \\
&= \int_0^\infty \left(\frac{\lambda_G}{\lambda_G + r} \cdot \left(\Pi + \frac{c}{r} \right) \cdot (\lambda_G + r) e^{-(\lambda_G + r)\tau_m} - \frac{c}{r} \cdot \lambda_G e^{-\lambda_G \tau_m} \right) d\tau_m \\
&= \frac{\lambda_G}{\lambda_G + r} \left(\Pi - \frac{c}{\lambda_G} \right) = \frac{\lambda_G}{\lambda_G + r}\Pi - \frac{1}{\lambda_G + r} c,
\end{aligned}$$

$$\begin{aligned}
F_S(r) \equiv P_0(\Gamma_S) &= \int_0^\infty \int_{\tau_s}^\infty \left(e^{-r\tau_m}\Pi - \int_0^{\tau_m} e^{-rt} c dt \right) \lambda_S e^{-\lambda_S(\tau_m - \tau_s)} d\tau_m \; \lambda_S e^{-\lambda_S \tau_s} d\tau_s \\
&= \int_0^\infty \left(\frac{\lambda_S}{\lambda_S + r} \cdot \left(\Pi + \frac{c}{r} \right) \cdot e^{-r\tau_s} - \frac{c}{r} \right) \lambda_S e^{-\lambda_S \tau_s} d\tau_s \\
&= \frac{\lambda_S^2}{(\lambda_S + r)^2} \left(\Pi + \frac{c}{r} \right) - \frac{c}{r} = \frac{\lambda_S^2}{(\lambda_S + r)^2}\Pi - \frac{2\lambda_S + r}{(\lambda_S + r)^2} c.
\end{aligned}$$

principal provides the minimum incentive not to shirk, the deadline would be shortened after the completion of the subproject. This lowers the probability for the success of the project implying that it may not be optimal to employ "the minimum incentive contract" in the first place. This possibility significantly complicates analysis of the model even numerically.

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From these expressions, when $r > 0$, we observe that the efficiency relationship depends not only on the arrival rates but also on the project size (Π) and the flow cost (c). This is significantly different from the no-discounting case. When $r = 0$, the efficiency relationship is simply determined by comparing the expected durations under the two approaches ($1/\lambda_G$ and $2/\lambda_S$). Thus, the presence of the discount factor complicates the analysis.

To simplify the argument, I focus on the case where there is no efficiency loss from monitoring; i.e., $2\lambda_G = \lambda_S$. The following proposition shows how the discount rate distorts the efficiency of each approach.

Proposition 6.1. *Suppose that $2\lambda_G = \lambda_S$ and $\Pi > c/\lambda_G$. Then, for all $r > 0$, the following inequality holds: $F_G(0) = F_S(0) > F_G(r) > F_S(r)$.*

This proposition says that the introduction of the discount rate harms the efficiency of the sequential approach more than that of the direct approach. In other words, if players begin to discount the future, the sequential approach becomes less appealing in terms of efficiency. As illustrated in Figure 1, the sequential approach is less advantageous in the short run. Thus, a high discount rate distorts the efficiency of the sequential approach more than that of the direct approach.

7 Conclusion

In this paper, I study the economic tradeoffs between a direct approach and a sequential approach for achieving a discrete goal in the context of a principal-agent setting. The optimal contract is determined by the interplay of monitoring, efficiency, and time pressure. I show that the form of the optimal contract depends on the size of the project. When the efficiency loss from monitoring does not exist or is small enough, the direct approach will only be chosen if the project size is small, whereas the sequential approach will only be chosen if the project size is large. If the project size is intermediate, it is optimal to begin with the sequential approach then switch to the direct approach. When the efficiency loss is large, the principal generally recommends the direct approach. However, if the project size is above a certain cutoff, she may recommend the sequential approach for a short period of time in the middle of the contract (i.e., there may be two switches). In addition, I explore three variations of the model. I show that the sequential approach will only be recommended if monitoring is effective enough. I also present a numerical example illustrating how the optimal deadlines and the principal's expected payoffs vary as the outside options to both parties for completion of the subproject change. Last, I demonstrate that the discount factor makes the sequential approach less desirable than the direct approach.

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There are many other possible avenues for further research. For example, the principal may be able to design the approaches directly. In this paper, I assume that the two approaches are exogenously given and the principal chooses between them. However, in the real world, a project manager often designs how many subprojects to partition the main project into and how difficult each subproject is. We could also add the 'learning' feature of the agile methodology to the model. In this paper, I fix the arrival rate for the direct approach. If we assume that the agent learns from early errors, the arrival rate of the main breakthrough would increase over time.[27] Finally, we might consider competition between firms. Many technology companies are often exposed to competition and this may significantly influence which approaches project managers will take. For instance, competitive pressure may manifest as increased time sensitivity tipping the choice of approach toward the more efficient direct (e.g., agile) methodology. I leave these intriguing questions–and others–for future work.

[27] This possibility contrasts with the setting considered by Schneider and Wolf (2020), where learning causes the expected arrival rate to fall.

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Appendix

A Proof for Section 3.3

Proof of Proposition 3.1. Let $\hat{W}$ be the principal's maximized payoff when the agent's action is observable and the subproject is already completed. In this case, the principal's maximization problem can be written as follows:

$$\hat{W} = \max_{T,\ (a_t,R_t)_{t\in[0,T]}} \int_0^T \left(\lambda_S a_t(\Pi - R_t) - c\right) e^{-\lambda_S \int_0^t a_s ds} dt$$

subject to $0 \le a_t \le 1$ and $R_t \ge 0$.

We can easily see that the principal would set $R_t = 0$. If $T < \infty$, since $\lambda_S \Pi > c$, the principal can have a higher expected payoff by setting $a_t = 1$ for $t \in [T, T+\Delta]$. Thus, the finite T cannot maximize the objective function, that is, $T = \infty$. Therefore, we can rewrite $\hat{W}$ as follows:

$$\hat{W} = \max_{(a_t)_{t\ge 0}} \int_0^\infty \left(\lambda_S a_t \Pi - c\right) \cdot e^{-\lambda_S \int_0^t a_s ds} dt.$$

Then, the HJB equation of $\hat{W}$ is derived as follows:

$$\hat{W} = \max_{a\in[0,1]} \left(-c + \lambda_S a \Pi\right) dt + (1 - \lambda_S a dt)\hat{W}$$

$$\Longrightarrow \qquad 0 = \max_{a\in[0,1]} -c + \lambda_S(\Pi - \hat{W})a$$

The right hand side is maximized at $a = 0$ or $a = 1$. If $a = 0$, the RHS is equal to $-c < 0$. Hence, a should be equal to 1 and $\hat{W} = \Pi - c/\lambda_S$. Also note that $\lambda_S(\Pi - \hat{W}) = c > 0$, thus, $a = 1$ is induced in the maximization problem.

Now consider the case where the subproject has not completed. Let W be the principal's maximized payoff for this case. The principal's maximization problem can be written as follows:

$$W = \max_{T,\ (a_t,b_t,R_t)_{t\in[0,T]}} \int_0^T \left[\lambda_G a_t(\Pi - R_t) + \lambda_S b_t \hat{W} - c\right] \cdot e^{-\lambda_G \int_0^t a_s ds - \lambda_S \int_0^t b_s ds} dt$$

subject to $0 \le a_t \le 1$, $0 \le b_t \le 1 - a_t$ and $R_t \ge 0$.

As in the previous case, we can set $R_t = 0$ and $T = \infty$ from $\lambda_G \Pi \ge c$. Then, the HJB

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equation of W is derived as follows:

$$W = \max_{a,b\in[0,1],a+b\leq 1} \left(-c + \lambda_G a\Pi + \lambda_S b\hat{W}\right) dt + (1 - \lambda_G a dt - \lambda_S b dt)W$$

$$\Rightarrow \qquad 0 = \max_{a,b\in[0,1],a+b\leq 1} -c + \lambda_G(\Pi - W)a + \lambda_S(\hat{W} - W)b \tag{A.1}$$

Since the maximization problem is linear in a and b, the optimal solution pair (a, b) would be one of $(1, 0)$, $(0, 1)$, $(0, 0)$. Note that $(0, 0)$ cannot be optimal because $-c < 0$.

When $a = 1$, solving (A.1) gives $W = \Pi - c/\lambda_G$ and $\lambda_G(\Pi - W) \geq \lambda_S(\hat{W} - W)$ is required to induce $a = 1$. The inequality is equivalent to $2\lambda_G \geq \lambda_S$.

When $b = 1$, solving (A.1) gives $W = \hat{W} - c/\lambda_S = \Pi - 2c/\lambda_S$ and $\lambda_G(\Pi - W) \leq \lambda_S(\hat{W} - W)$ is required to induce $b = 1$. The inequality is equivalent to $2\lambda_G \leq \lambda_S$.

Therefore, if $2\lambda_G > \lambda_S$, the principal's maximized payoff W is equal to $P_0(\Gamma_G)$, i.e., the go-for-it only contract is the first-best contract. If $\lambda_S > 2\lambda_G$, W is equal to $P_0(\Gamma_S)$, i.e., the split-it only contract is the first-best contract. If $\lambda_S = 2\lambda_G$, W is equal to $P_0(\Gamma_G)$ and $P_0(\Gamma_S)$, thus, both contracts are first-best contracts. $\square$

B Analysis

B.1 The Agent's Problem

In this subsection, I formally derive the agent's continuation utility and prove Proposition 4.1.

I begin by specifying the probability distribution functions for possible events given an admissible action profile $(a, b) \in \mathcal{A}_t$ conditional on no breakthrough has made by time t. The probability distribution function that the main breakthrough arrives at time s and the intermediate breakthrough has not arrived by that time ($s = \tau_m < \tau_s$) is $\lambda_G a_s e^{-\lambda_G \int_t^s a_l dl} \cdot e^{-\lambda_S \int_t^s b_l dl}$. Similarly, the probability distribution function that the intermediate breakthrough arrives at time s and the intermediate breakthrough has not arrived by that time ($s = \tau_s < \tau_m$) is $\lambda_S b_s e^{-\lambda_S \int_t^s b_l dl} \cdot e^{-\lambda_G \int_t^s a_l dl}$. The probability distribution function that one of breakthroughs arrives at time s and the other breakthrough has not arrived by then, i.e., $\tau_s \wedge \tau_m = s$, is $(\lambda_G a_s + \lambda_S b_s)e^{-\lambda_G \int_t^s a_l dl - \lambda_S \int_t^s b_l dl}$. Last, the probability that any of breakthroughs arrives by time T is $e^{-\lambda_G \int_t^T a_s ds - \lambda_S \int_t^T b_s ds}$

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Based on the above results, we can derive that

$$\mathbb{E}^{a,b}\left[R_{\tau_m}\cdot\mathbf{1}_{\{\tau_m\leq\tau_s\wedge T\}}\mid t\leq\tau_m\wedge\tau_s\right]=\int_t^T R_s\cdot\lambda_G a_s\cdot e^{-\lambda_G\int_t^s a_l dl-\lambda_S\int_t^s b_l dl}ds,$$

$$\mathbb{E}^{a,b}\left[\hat{U}_{\tau_s}(\hat{\Gamma}^{\tau_s})\cdot\mathbf{1}_{\{\tau_s<\tau_m\wedge T\}}\mid t\leq\tau_m\wedge\tau_s\right]=\int_t^T\hat{U}_s(\hat{\Gamma}^s)\cdot\lambda_S b_s\cdot e^{-\lambda_G\int_t^s a_l dl-\lambda_S\int_t^s b_l dl}ds,$$

and

$$\begin{aligned}
&\mathbb{E}^{a,b}\left[\int_t^{T\wedge\tau_m\wedge\tau_s}\phi(1-a_s-b_s)\mid t\leq\tau_m\wedge\tau_s\right]\\
&=\int_t^T\left[\int_t^s\phi(1-a_l-b_l)dl\right]\cdot(\lambda_G a_s+\lambda_S b_s)e^{-\lambda_G\int_t^s a_l dl-\lambda_S\int_t^s b_l dl}ds\\
&\quad+\left[\int_t^T\phi(1-a_s-b_s)ds\right]\cdot e^{-\lambda_G\int_t^T a_s ds-\lambda_S\int_t^T b_s ds}\\
&=\int_t^T\phi(1-a_s-b_s)\cdot e^{-\lambda_G\int_t^s a_l dl-\lambda_S\int_t^s b_l dl}ds.
\end{aligned}$$

Then, by plugging the above expressions into (4.2), $U_t(\Gamma)$ can be rewritten as follows:

$$U_t(\Gamma)=\sup_{(a,b)\in\mathcal{A}_t}\int_t^T\left[R_s\cdot\lambda_G a_s+\hat{U}_s(\hat{\Gamma}^s)\cdot\lambda_S b_s+\phi(1-a_s-b_s)\right]e^{-\lambda_G\int_t^s a_l dl-\lambda_S\int_t^s b_l dl}ds. \tag{B.1}$$

Now we can prove Proposition 4.1 by using the above equation. The proof is inspired by Proposition 3.2.1 in Bertsekas (1995).

Proof of Proposition 4.1. Consider an arbitrary admissible action $(\tilde{a},\tilde{b})\in\mathcal{A}_t$. By rearranging (PK), we can derive that

$$-\dot{u}_s+(\lambda_G\tilde{a}_s+\lambda_S\tilde{b}_s)u_t\geq R_s\lambda_G\tilde{a}_s+u_S^s\lambda_S\tilde{b}_s+\phi(1-\tilde{a}_s-\tilde{b}_s)$$

and it is equivalent to

$$\begin{aligned}
&\frac{d}{ds}\left[-u_s\cdot e^{-\lambda_G\int_t^s\tilde{a}_l dl-\lambda_S\int_t^s\tilde{b}_l dl}\right]\\
&\geq\left[R_s\lambda_G\tilde{a}_s+u_S^s\lambda_S\tilde{b}_s+\phi(1-\tilde{a}_s-\tilde{b}_s)\right]\cdot e^{-\lambda_G\int_t^s\tilde{a}_l dl-\lambda_S\int_t^s\tilde{b}_l dl}.
\end{aligned}$$

By integrating the above inequality from t to T and using $u_T=0$, we can derive that

$$u_t\geq\int_t^T\left[R_s\lambda_G\tilde{a}_s+u_S^s\lambda_S\tilde{b}_s+\phi(1-\tilde{a}_s-\tilde{b}_s)\right]\cdot e^{-\lambda_G\int_t^s\tilde{a}_l dl-\lambda_S\int_t^s\tilde{b}_l dl}ds$$

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for all $(\tilde{a}, \tilde{b}) \in \mathcal{A}_t$.

Suppose that $(a^*, b^*) \in \mathcal{A}_t$ attains the maximum in the equation (PK) for all $0 \le t \le T$. Then, we have

$$\begin{aligned} u_t &= \int_t^T \left[R_s \lambda_G a_s^* + u_S^s \lambda_S b_s^* + \phi(1 - a_s^* - b_s^*)\right] \cdot e^{-\lambda_G \int_t^s a_l^* dl - \lambda_S \int_t^s b_l^* dl} ds \\ &\ge \int_t^T \left[R_s \lambda_G \tilde{a}_s + u_S^s \lambda_S \tilde{b}_s + \phi(1 - \tilde{a}_s - \tilde{b}_s)\right] \cdot e^{-\lambda_G \int_t^s \tilde{a}_l dl - \lambda_S \int_t^s \tilde{b}_l dl} ds \end{aligned}$$

for all $(\tilde{a}, \tilde{b}) \in \mathcal{A}_t$. Therefore, by (B.1), we have $u_t = U_t(\Gamma)$. □

B.2 The Principal's Problem

The value function $V(u_t)$ can be heuristically written as follows:[28]

$$V(u_t) = \max_{\substack{R_t \ge 0,\ u_S^t \ge 0, \\ a_t, b_t \ge 0,\ 1 \ge a_t + b_t}} (\Pi - R_t)\, \lambda_G a_t dt + V_S(u_S^t) \lambda_S b_t dt + (1 - \lambda_G a_t dt - \lambda_S b_t dt) V(u_{t+dt}).$$

The HJB equation for the value function $V(u)$ can be derived as follows:

$$0 = \max_{\substack{R \ge 0,\ u_S \ge 0, \\ a, b \ge 0,\ 1 \ge a + b}} -c + (\Pi - R - V(u)) \lambda_G a + (V_S(u_S) - V(u)) \lambda_S b + V'(u)\, \dot{u}. \tag{HJB}$$

Then, the principal's problem is to solve (HJB) subject to (PK) with the boundary condition $V(0) = 0$. The following lemma shows that the solution of the problem maximizes the principal's expected payoff subject to a promise keeping constraint $U_0(\Gamma) = u$.

Lemma B.1 (Verification Lemma)**.** *Suppose that a concave $\mathcal{C}^1$ function $\tilde{V}$ solves* (HJB) *subject to* (PK) *with the boundary condition $\tilde{V}(0) = 0$. Then, for any feasible contract Γ with $U_0(\Gamma) = u$,*

$$\tilde{V}(u) \ge P_0(\Gamma).$$

Moreover, the following lemma allows us to focus on pure actions.

Lemma B.2. *For given u and $V(u)$, if a mixed effort level (i.e., $a, b > 0$) solves* (HJB) *subject to* (PK)*, then the pure effort levels (i.e., $a = 1$ or $b = 1$) also solve* (HJB) *subject to* (PK)*.*

[28] The principal's problem can also be rigorously derived as in the agent's problem in the previous section. For simplicity, I use a heuristic derivation to obtain the HJB equation, but Lemma B.1 ensures that this HJB equation is the valid one.

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B.3 Benchmark Value Functions

In this subsection, I introduce three benchmark value functions. The guess of the value functions are based on the following intuitions: (i) the principal recommends the direct approach when the deadline is close by; (ii) the principal recommends the sequential approach when the deadline is moderately far way; (iii) the principal recommends the direct approach when the deadline is far away.

1. Let $V^g : \mathbb{R}_+ \to \mathbb{R}$ be the value function that induces the agent to go for the project ($a = 1$) with $R = u + \phi/\lambda_G$ for all $u \geq 0$. Then, (HJB) becomes

$$0 = -c + \lambda_G \left(\Pi - \frac{\phi}{\lambda_G} - u - V^g(u) \right) - \phi V^{g\prime}(u) \tag{B.2}$$

with the boundary condition $V^g(0) = 0$.

V^g takes the same form as V_S in (4.1) except that the arrival rate goes from λ_S to λ_G. Therefore, we obtain

$$V^g(u) = \left(\Pi - \frac{c}{\lambda_G} \right) \left(1 - e^{-\frac{\lambda_G}{\phi} u} \right) - u. \tag{B.3}$$

2. Let $V^{gs}(\cdot|u_1) : [u_1, \infty) \to \mathbb{R}$ be the value function that induces the agent to go for the project ($a = 1$) with $R = u + \phi/\lambda_G$ for $0 \leq u < u_1$ and to split the project ($b = 1$) with $u_S = u + \phi/\lambda_S$ for $u \geq u_1$. Then, (HJB) for $u \geq u_1$ becomes

$$0 = -c + \lambda_S \left(V_S \left(u + \phi/\lambda_S \right) - V^{gs}(u|u_1) \right) - \phi V^{gs\prime}(u|u_1) \tag{B.4}$$

with the boundary condition $V^{gs}(u_1|u_1) = V^g(u_1)$.

By multiplying $e^{\lambda_S u/\phi}/\phi$ to the HJB equation (B.4), the equation can be rewritten as follows:

$$\frac{\lambda_S}{\phi} e^{\frac{\lambda_S}{\phi} u} V^{gs}(u|u_1) + e^{\frac{\lambda_S}{\phi} u} V^{gs\prime}(u|u_1) = \frac{\lambda_S}{\phi} e^{\frac{\lambda_S}{\phi} u} \left(V_S \left(u + \phi/\lambda_S \right) - \frac{c}{\lambda_S} \right).$$

The left hand side is equal to $\frac{d}{du} \left(e^{\frac{\lambda_S}{\phi} u} V^{gs}(u|u_1) \right)$. By plugging the closed form solution

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of V_S into the equation, the right hand side can be rewritten as follows:

$$\begin{aligned}&\left(\Pi-\frac{2c}{\lambda_S}-u\right)\frac{\lambda_S}{\phi}e^{\frac{\lambda_S}{\phi}u}-e^{\frac{\lambda_S}{\phi}u}-\left(\Pi-\frac{c}{\lambda_S}\right)\frac{\lambda_S e^{-1}}{\phi}\\=&\frac{d}{du}\left[\left(\Pi-\frac{2c}{\lambda_S}-u\right)e^{\frac{\lambda_S}{\phi}u}-\left(\Pi-\frac{c}{\lambda_S}\right)\frac{\lambda_S e^{-1}}{\phi}\cdot u\right]\end{aligned}$$

Then, integrating the HJB equation from u_1 to u gives

$$\begin{aligned}&e^{\frac{\lambda_S}{\phi}u}V^{gs}(u|u_1)-e^{\frac{\lambda_S}{\phi}u_1}V^{gs}(u_1|u_1)\\=&\left(\Pi-\frac{2c}{\lambda_S}-u\right)e^{\frac{\lambda_S}{\phi}u}-\left(\Pi-\frac{c}{\lambda_S}\right)\frac{\lambda_S e^{-1}}{\phi}\cdot u\\&-\left(\Pi-\frac{2c}{\lambda_S}-u_1\right)e^{\frac{\lambda_S}{\phi}u_1}+\left(\Pi-\frac{c}{\lambda_S}\right)\frac{\lambda_S e^{-1}}{\phi}\cdot u_1.\end{aligned}$$

By rearranging the above equation and using the boundary condition ($V^{gs}(u_1|u_1)=V^g(u_1)$), we obtain

$$\begin{aligned}V^{gs}(u|u_1)=&\left(\Pi-\frac{2c}{\lambda_S}\right)\left(1-e^{\frac{\lambda_S}{\phi}(u_1-u)}\right)+(V^g(u_1)+u_1)e^{\frac{\lambda_S}{\phi}(u_1-u)}\\&-\left(\Pi-\frac{c}{\lambda_S}\right)\frac{\lambda_S}{\phi}(u-u_1)e^{-\frac{\lambda_S}{\phi}u-1}-u.\end{aligned}\tag{B.5}$$

3. Let $V^{gsg}(\cdot|u_1,u_2):[u_2,\infty)\to\mathbb{R}$ be the value function that induces the agent to go for the project ($a=1$) with $R=u+\phi/\lambda_G$ for $0\le u<u_1$, to split the project ($b=1$) with $u_S=u+\phi/\lambda_S$ for $u_1\le u<u_2$, and to go for it ($a=1$) with $R=u+\phi/\lambda_G$ for $u\ge u_2$. Then, (HJB) for $u\ge u_2$ becomes same as (B.2) with the boundary condition $V^{gsg}(u_2|u_1,u_2)=V^{gs}(u_2|u_1)$. Then, we obtain

$$\begin{aligned}V^{gsg}(u|u_1,u_2)=&\left(\Pi-\frac{c}{\lambda_G}\right)\left(1-e^{\frac{\lambda_G}{\phi}(u_2-u)}\right)\\&+(V^{gs}(u_2|u_1)+u_2)e^{\frac{\lambda_G}{\phi}(u_2-u)}-u.\end{aligned}\tag{B.6}$$

The following lemma is useful for the derivation of the value funcion.

Lemma B.3. *Suppose that $2\lambda_G\ge\lambda_S>\lambda_G$ and $\Pi>c/\lambda_G$. Then, the following statements hold:*

(a) $V^g(u)<\Pi-c/\lambda_G-u$, ${V^g}'(u)>-1$ *and* ${V^g}''(u)<0$;

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(b) *Suppose that* u_1 *satisfies* $V^{g\prime}(u_1) \leq V^{gs\prime}(u_1|u_1)$. *Then, for all* $u \geq u_1$, $V^{gs}(u|u_1) < \Pi - c/\lambda_G - u$, $V^{gs\prime}(u|u_1) > -1$ *and* $V^{gs\prime\prime}(u|u_1) < 0$.

(c) *Suppose that* u_1 *and* u_2 *satisfy* $V^{g\prime}(u_1) \leq V^{gs\prime}(u_1|u_1)$ *and* $u_2 > u_1$. *Then, for all* $u \geq u_2$, $V^{gsg}(u|u_1, u_2) < \Pi - c/\lambda_G - u$, $V^{gsg\prime}(u|u_1, u_2) > -1$ *and* $V^{gsg\prime\prime}(u|u_1, u_2) < 0$.

B.4 Conditions for Smooth Pasting

In this subsection, I identify the conditions under which we can smoothly paste the benchmark value functions introduced in the previous subsection. Since the value function implicitly depends on Π, λ_S and λ_G, the condition would depend on these parameters. I define $\eta \equiv \lambda_S/\lambda_G - 1$ and note that $1 \geq \eta > 0$ from the assumption $2\lambda_G \geq \lambda_S > \lambda_G$. In the following lemmas, I introduce threshold of Π as functions of η and characterize the conditions for smooth pasting by using those thresholds.

Lemma B.4. *If* η *is less than or equal to* $1/(e-1)$, *the inequality* $V^{g\prime}(0) > V^{gs\prime}(0|0)$ *always holds. If* η *is greater than* $1/(e-1)$, $V^{g\prime}(0) > V^{gs\prime}(0|0)$ *holds if and only if*

$$\Pi < \Pi_S(\eta) \equiv \frac{e-1}{(e-1)\eta - 1} \cdot \frac{c}{\lambda_G}. \tag{B.7}$$

Moreover, if $\Pi = \Pi_S(\eta)$, *then* $V^{g\prime}(0) = V^{gs\prime}(0|0)$ *and* $V^{g\prime\prime}(0) < V^{gs\prime\prime}(0|0)$.

Lemma B.5. *There exists* $\Pi_M(\eta) \geq c/\lambda_G$ *such that* $V^{g\prime}(u) > V^{gs\prime}(u|u)$ *for all* u *if and only if* $c/\lambda_G < \Pi < \Pi_M(\eta)$. *In addition,* $\Pi_M(1) = c/\lambda_G$. *Moreover, if* $\Pi_S(\eta) > \Pi_M(\eta)$ *and* $\Pi_S(\eta) \geq \Pi > \Pi_M(\eta)$, *there exists* $u \geq 0$ *such that* $V^{g\prime}(u) = V^{gs\prime}(u|u)$ *and* $V^{g\prime\prime}(u) < V^{gs\prime\prime}(u|u)$.

Lemma B.6. *Suppose that* $\Pi > c/\lambda_G$ *and either* $V^{g\prime}(u_1) < V^{gs\prime}(u_1|u_1)$ *or* $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ *&* $V^{g\prime\prime}(u_1|u_1) < V^{gs\prime\prime}(u_1)$.

(a) *If* $\eta = 1$, $V^{gs\prime}(u|u_1) > V^{gsg\prime}(u|u_1, u)$ *for all* $u > u_1$.

(b) *If* $\eta < 1$, *there exists* $u_2 > u_1$ *such that* $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ *&* $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$ *and such* u_2 *is unique. Moreover,* $V^{gs\prime}(u|u_1) > V^{gsg\prime}(u|u_1, u)$ *for all* $u \in (u_1, u_2)$.

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Lemma B.7. *Suppose that* $\Pi > c/\lambda_G$*,* $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ *and* $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$*. Then,* $\lambda_S\left(V_S(u + \phi/\lambda_S) - V^{gsg}(u|u_1, u_2)\right) - \phi V^{gsg\prime}(u|u_1, u_2) - c < 0$ *for all* $u > u_2$*.*

Corollary B.8. *Suppose that* η *is equal to* 1*.*

(a) If $\Pi \in (c/\lambda_G, \Pi_S(1)]$*, there exists* $u_1 \geq 0$ *such that* $V^{g\prime}(u) > V^{gs\prime}(u|u)$ *for all* $0 \leq u < u_1$*,* $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ *&* $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$*, and* $V^{gs\prime}(u|u_1) > V^{gsg\prime}(u|u, u_1)$ *for all* $u > u_1$*.*

(b) If $\Pi > \Pi_S(1)$*,* $V^{gs\prime}(0|0) > V^{g\prime}(0)$ *and* $V^{gs\prime}(u|0) > V^{gsg\prime}(u|0, u)$ *for all* $u \geq 0$*.*

Corollary B.9. *Suppose that* η *is less than* 1*.*

(a) If $c/\lambda_G < \Pi < \Pi_M(\eta)$*,* $V^{g\prime}(u) > V^{gs\prime}(u|u)$ *for all* $u \geq 0$*.*

(b) If $\eta \leq 1/(e-1)$ *&* $\Pi \geq \Pi_M(\eta)$ *or* $1/(e-1) < \eta < 1$ *&* $\Pi \in [\Pi_M(\eta), \Pi_S(\eta)]$*, there exist a pair* (u_1, u_2) *with* $u_2 \geq u_1 \geq 0$ *such that:*

(i) $V^{g\prime}(u) > V^{gs\prime}(u|u)$ *for all* $0 \leq u < u_1$*,*

(ii) $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ *&* $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$*,*

(iii) $V^{gs\prime}(u|u_1) > V^{gsg\prime}(u|u, u_1)$ *for all* $u_2 > u > u_1$*,*

(iv) $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ *&* $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$*.*

(v) $V^{gsg\prime}(u|u_1, u_2) > \frac{1}{\phi}\left[\lambda_S(V_S(u + \phi/\lambda_S) - V^{gsg}(u|u_1, u_2)) - c\right]$ *for all* $u > u_2$*.*

(c) If $1/(e-1) < \eta < 1$ *&* $\Pi > \Pi_S(\eta)$*,* $V^{gs\prime}(0|0) > V^{g\prime}(0)$ *and there exists* $u_2 > 0$ *such that* $V^{gs\prime}(u_2|0) = V^{gsg\prime}(u_2|0, u_2)$*,* $V^{gs\prime}(u|0) > V^{gsg\prime}(u|0, u)$ *for all* $u_2 > u \geq 0$*, and* $V^{gsg\prime}(u|0, u_2) > \frac{1}{\phi}\left[\lambda_S(V_S(u + \phi/\lambda_S) - V^{gsg}(u|0, u_2)) - c\right]$ *for all* $u > u_2$*.*

B.5 Value Function Derivation

B.5.1 Functions for Deviation

In this subsection, I introduce functions that specify deviations from the given value functions. Then I present some properties of these functions.[29]

1. Functions for deviation given V^g

[29] This approach is inspired by the tangible first breakthrough case of Green and Taylor (2016b). In their paper, they only need to consider the deviation from working to shirking. In this paper, we also need to consider the deviation from an approach to another approach. Thus, we need to define two functions for each case.

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(a) Define

$$L_1^a(u,R) \equiv \lambda_G(\Pi - R - V^g(u)) - c - \lambda_G(R-u)V^{g\prime}(u).$$

Given u, maximizing this function with respect to $R \geq u + \phi/\lambda_G$ is equivalent to maximizing the right hand side of (HJB) under the condition that $a = 1$ solves (PK).

(b) Define

$$\begin{aligned} L_1^b(u,w) \equiv& \lambda_S(V_S(w) - V^g(u)) - c - \lambda_S(w-u)V^{g\prime}(u) && \text{(B.8)} \\ =& \lambda_S \left[\left(\Pi - \frac{c}{\lambda_S}\right)\left(1 - e^{-\frac{\lambda_S}{\phi}u - \frac{\lambda_S}{\phi}(w-u)}\right) \right. && \text{(B.9)} \\ & \left. - \left(\Pi - \frac{c}{\lambda_G}\right)\left(1 - e^{-\frac{\lambda_G}{\phi}u}\right) - (w-u)\left(\frac{\lambda_G \Pi - c}{\phi}\right)e^{-\frac{\lambda_G}{\phi}u}\right] - c. \end{aligned}$$

Given u, maximizing this function with respect to $w \geq u + \phi/\lambda_S$ is equivalent to maximize the right hand side of (HJB) under the condition that $b = 1$ solves (PK).

Lemma B.10. *Suppose that $\Pi > c/\lambda_G$ and $2\lambda_G \geq \lambda_S > \lambda_G$ are satisfied. Then, L_1^a and L_1^b satisfy the following properties:*

(a) $L_1^a(u,R) \leq 0$ for all $u \geq 0$ and $R \geq u + \phi/\lambda_G$.

(b) If $\Pi \leq \Pi_M(\eta)$, $L_1^b(u,w) \leq 0$ for all $u \geq 0$ and $w \geq u + \phi/\lambda_S$.

(c) If $\Pi_M(\eta) < \Pi < \Pi_S(\eta)$, there exists $u_1 > 0$ such that $L_1^b(u,w) \leq 0$ for all $u \in [0,u_1]$ and $w \geq u + \phi/\lambda_S$.

2. Functions for deviation given V^{gs}

(a) Define

$$L_2^a(u,R|u_1) \equiv \lambda_G\left(\Pi - R - V^{gs}(u|u_1)\right) - c - \lambda_G(R-u)V^{gs\prime}(u|u_1).$$

(b) Define

$$L_2^b(u,w|u_1) \equiv \lambda_S\left(V_S(w) - V^{gs}(u|u_1)\right) - c - \lambda_S(w-u)V^{gs\prime}(u|u_1).$$

Lemma B.11. *Suppose that $2\lambda_G \geq \lambda_S > \lambda_G$, $\Pi > \Pi_M(\eta)$ and $V^{g\prime}(u_1) < V^{gs\prime}(u_1|u_1)$ or $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ & $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$ are satisfied. Then, L_2^a and L_2^b satisfy the following properties:*

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(a) $L_2^b(u, w|u_1) \leq 0$ *for all* $u \geq u_1$ *and* $w \geq u + \phi/\lambda_S$.

(b) When $\eta = 1$, *for all* $u > u_1$ *and* $R \geq u + \phi/\lambda_G$, $L_2^a(u, R|u_1) < 0$ *is satisfied.*

(c) When $\eta < 1$, *there exists* $u_2 > u_1$ *such that* $L_2^a(u_2, u_2 + \phi/\lambda_G|u_1) = 0$, $L_2^a(u, R|u_1) < 0$ *for all* $u_1 < u < u_2$ *and* $R \geq u + \phi/\lambda_G$, *and* $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ *&* $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$.

3. Functions for deviation given V^{gsg}

(a) Define

$$L_3^a(u, R|u_1, u_2) \equiv \lambda_G \left(\Pi - R - V^{gsg}(u|u_1, u_2)\right) - c - \lambda_G(R - u)V^{gsg\prime}(u|u_1, u_2).$$

(b) Define

$$L_3^b(u, w|u_1, u_2) \equiv \lambda_S \left(V_S(w) - V^{gsg}(u|u_1, u_2)\right) - c - \lambda_S(w - u)V^{gsg\prime}(u|u_1, u_2).$$

Lemma B.12. *Suppose that* $2\lambda_G \geq \lambda_S > \lambda_G$, $\Pi > \Pi_M(\eta)$, $V^{g\prime}(u_1) < V^{gs\prime}(u_1|u_1)$ *or* $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ *&* $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$, $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ *and* $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$ *are satisfied. Then,* L_3^a *and* L_3^b *satisfy the following properties:*

(a) $L_3^a(u, R|u_1, u_2) \leq 0$ *for all* $u \geq u_2$ *and* $R \geq u + \phi/\lambda_G$.

(b) $L_3^b(u, w|u_1, u_2) \leq 0$ *for all* $u \geq u_2$ *and* $w \geq u + \phi/\lambda_S$.

B.5.2 Verifying the Value Function

By using the lemmas in the previous subsection, I verify that each segment of the principal's value function is the same as one of the benchmark value functions introduced in Appendix B.3. I begin by characterizing the value function for no efficiency loss case.

Proposition B.1. *Suppose that* η *is equal to* 1. *Then, the following statements hold:*

(a) When $\Pi \in (c/\lambda_G, \Pi_S(1))$, *for some* $u_1 > 0$, $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ *and*

$$V(u) = \begin{cases} V^g(u) & \text{if } 0 \leq u \leq u_1, \\ V^{gs}(u|u_1) & \text{if } u_1 < u \end{cases} \tag{B.10}$$

solves (HJB) *subject to* (PK).

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(b) *When* $\Pi \geq \Pi_S(1)$*,* $V(u) = V^{gs}(u|0)$ *solves* (HJB) *subject to* (PK)*.*

Proof of Proposition B.1. (a) By (a) and (c) of Lemma B.10, there exists $u_1 > 0$ such that for all $a, b \in [0,1]$ with $a + b \leq 1$, $u \in [0, u_1]$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_1^a(u, R) + b \cdot L_1^b(u, w),$$

and $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ & $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$. Since $L_1^a(u, u+\phi/\lambda_G) = 0$, for all $u \in [0, u_1]$, $V^g(u)$ solves (HJB) subject to (PK).

By (a) and (b) of Lemma B.11, for all $a, b \in [0,1]$ with $a+b \leq 1$, $u > u_1$, $R \geq u+\phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_2^a(u, R|u_1) + b \cdot L_2^b(u, w|u_1),$$

whereas $L_2^b(u, u + \phi/\lambda_S|u_1)$. Thus, for all $u > u_1$, $V^{gs}(u|u_1)$ solves (HJB) subject to (PK).

Therefore, the value function specified in (B.10) solves (HJB) subject to (PK).

(b) By Lemma B.4, $V^{gs\prime}(0|0) > V^{g\prime}(0)$ $(\Pi > \Pi_S(1))$ or $V^{gs\prime}(0|0) = V^{g\prime}(0)$ & $V^{gs\prime\prime}(0|0) > V^{g\prime\prime}(0)$ $(\Pi = \Pi_S(1))$. By (a) and (b) of Lemma B.11, for all $a, b \in [0,1]$ with $a+b \leq 1$, $u \geq 0$ $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_2^a(u, R|0) + b \cdot L_2^b(u, w|0),$$

whereas $L_2^b(u, u + \phi/\lambda_S|0) = 0$. Therefore, $V^{gs}(u|0)$ solves (HJB) subject to (PK).

□

The following proposition characterizes the value function for the case where there is an efficiency loss.

Proposition B.2. *Suppose that* η *is less than* 1*. Then, the following statements hold:*

(a) *When* $\Pi_M(\eta) \geq \Pi$*,* $V(u) = V^g(u)$ *for all* $u \geq 0$ *solves* (HJB) *subject to* (PK)*.*

(b) *When* $\eta \leq 1/(e-1)$ *&* $\Pi > \Pi_M(\eta)$ *or* $1/(e-1) < \eta < 1$ *&* $\Pi \in (\Pi_M(\eta), \Pi_S(\eta))$*, for some* $u_2 > u_1 > 0$*,* $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ *&* $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$*, and*

$$V(u) = \begin{cases} V^g(u) & \text{if } 0 \leq u \leq u_1, \\ V^{gs}(u|u_1) & \text{if } u_1 < u \leq u_2, \\ V^{gsg}(u|u_1, u_2) & \text{if } u_2 < u, \end{cases} \tag{B.11}$$

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solves (HJB) *subject to* (PK).

(c) *When* $1/(e-1) < \eta < 1$ *&* $\Pi \geq \Pi_S(\eta)$*, for some* $u_2 > 0$*,* $V^{gs\prime}(u_2|0) = V^{gsg\prime}(u_2|0,u_2)$*, and*

$$V(u) = \begin{cases} V^{gs}(u|0) & \text{if } 0 \leq u \leq u_2, \\ V^{gsg}(u|0,u_2) & \text{if } u_2 < u, \end{cases} \tag{B.12}$$

solves (HJB) *subject to* (PK).

Proof of Proposition B.2. (a) By (a) and (b) of Lemma B.10, for all $a, b \in [0,1]$ with $a + b \leq 1$, $u \geq 0$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$, $0 \geq a \cdot L_1^a(u,R) + b \cdot L_1^b(u,w)$ whereas $L_1^a(u, u + \phi/\lambda_G) = 0$. Therefore, $V^g(u)$ solves (HJB) subject to (PK).

(b) By (a) and (c) of Lemma B.10, there exists $u_1 > 0$ such that for all $a, b \in [0,1]$ with $a + b \leq 1$, $u \in [0, u_1]$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_1^a(u,R) + b \cdot L_1^b(u,w),$$

and $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ & $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$. Since $L_1^a(u, u + \phi/\lambda_G) = 0$, for all $u \in [0, u_1]$, $V^g(u)$ solves (HJB) subject to (PK).

By (a) and (c) of Lemma B.11, there exists $u_2 > u_1$ such that $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1,u_2)$, $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1,u_2)$, and for all $a, b \in [0,1]$ with $a + b \leq 1$, $u \in (u_1, u_2]$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_2^a(u, R|u_1) + b \cdot L_2^b(u, w|u_1),$$

whereas $L_2^b(u, u + \phi/\lambda_S|u_1) = 0$. Thus, for all $u \in (u_1, u_2]$, $V^{gs}(u|u_1)$ solves (HJB) subject to (PK).

By Lemma B.12, for all $a, b \in [0,1]$ with $a + b \leq 1$, $u > u_2$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_3^a(u, R|u_1,u_2) + b \cdot L_3^b(u, w|u_1,u_2),$$

whereas $L_3^a(u, u + \phi/\lambda_S|u_1,u_2) = 0$. Thus, for all $u > u_2$, $V^{gsg}(u|u_1,u_2)$ solves (HJB) subject to (PK).

Therefore, the value function specified in (B.12) solves (HJB) subject to (PK).

(c) By Lemma B.4, $V^{gs\prime}(0|0) > V^{g\prime}(0)$ $(\Pi > \Pi_S(1))$ or $V^{gs\prime}(0|0) = V^{g\prime}(0)$ & $V^{gs\prime\prime}(0|0) > V^{g\prime\prime}(0)$ $(\Pi = \Pi_S(1))$. By (a) and (c) of Lemma B.11, there exists $u_2 > 0$ such that

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$V^{gs\prime}(u_2|0) = V^{gsg\prime}(u_2|0, u_2)$, $V^{gs\prime\prime}(u_2|0) < V^{gsg\prime\prime}(u_2|0, u_2)$, and for all $a, b \in [0, 1]$ with $a + b \leq 1$, $u \in [0, u_2]$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_2^a(u, R|0) + b \cdot L_2^b(u, w|0),$$

whereas $L_2^b(u, u + \phi/\lambda_S|0) = 0$. Therefore, for all $u \in [0, u_2]$, $V^{gs}(u|0)$ solves (HJB) subject to (PK).

By Lemma B.12, for all $a, b \in [0, 1]$ with $a + b \leq 1$, $u > u_2$, $R \geq u + \phi/\lambda_G$ and $w \geq u + \phi/\lambda_S$,

$$0 \geq a \cdot L_3^a(u, R|0, u_2) + b \cdot L_3^b(u, w|0, u_2),$$

whereas $L_3^a(u, u + \phi/\lambda_S|u_1, u_2) = 0$. Thus, for all $u > u_2$, $V^{gsg}(u|0, u_2)$ solves (HJB) subject to (PK).

Therefore, the value function specified in (B.11) solves (HJB) subject to (PK).

□

Note that each segment of the value function is strictly concave by Lemma B.3. Moreover, the above propositions imply that they are smoothly pasted. Therefore, the value function is strictly concave.

Corollary B.13. *The principal's value function that solves* (HJB) *subject to* (PK) *is concave.*

Then, by Lemma B.1, we can verify that $V(u)$ in Proposition B.1 or B.2 is the maximum of the principal's expected payoff subject to $U_0(\Gamma) = u$.

B.5.3 Implementation

Next, I show that benchmark value functions can be implemented by the contracts introduced in the main text.

Proposition B.3. *The following statements hold:*

(a) A direct-only contract with the deadline u/ϕ implements a pair of expected payoffs of the principal and the agent $(V^g(u), u)$.

(b) When $0 \leq u_1 \leq u$, a contract with a switch from splitting the project to going for the project at $(u - u_1)/\phi$ and the deadline u/ϕ implements $(V^{gs}(u|u_1), u)$.

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(c) A sequential-only contract with the deadline u/ϕ implements $(V^{gs}(u|0), u)$.

(d) When $0 \le u_1 < u_2 \le u$, a contract with two switches at $(u-u_2)/\phi$ and $(u-u_1)/\phi$ and the deadline u/ϕ implements $(V^{gsg}(u|u_1, u_2), u)$.

Proof. (a) Let $\Gamma_g(T)$ denote a direct-only contract with the deadline T. The agent's expected payoff is

$$
\begin{aligned}
U_0(\Gamma_g(T)) &= \int_0^T R_{\tau_m} \lambda_G e^{-\lambda_G \tau_m} d\tau_m \\
&= \int_0^T \phi \left[T - \tau_m + 1/\lambda_G\right] \lambda_G e^{-\lambda_G \tau_m} d\tau_m \\
&= - \phi \left(T - \tau_m\right) e^{-\lambda_G \tau_m} \Big|_0^T \\
&= \phi T.
\end{aligned}
$$

Therefore, $U_0(\Gamma_g(u/\phi)) = u$.

Also note that

$$
\begin{aligned}
P_0(\Gamma_g(T)) + U_0(\Gamma_g(T)) &= \int_0^T \left(\Pi - c\tau_m\right) \lambda_G e^{-\lambda_G \tau_m} d\tau_m - cTe^{-\lambda_G T} \\
&= - \left(\Pi - c\tau_m - c/\lambda_G\right) e^{-\lambda_G \tau_m} \Big|_0^T - cTe^{-\lambda_G T} \\
&= - \left(\Pi - cT - c/\lambda_G\right) e^{-\lambda_G T} + \left(\Pi - c/\lambda_G\right) - cTe^{-\lambda_G T} \\
&= \left(\Pi - c/\lambda_G\right) \left(1 - e^{-\lambda_G T}\right).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
P_0(\Gamma_g(u/\phi)) &= \left(\Pi - \frac{c}{\lambda_G}\right) (1 - e^{-\frac{\lambda_G}{\phi} u}) - U_0(\Gamma_g(u/\phi)) \\
&= \left(\Pi - \frac{c}{\lambda_G}\right) (1 - e^{-\frac{\lambda_G}{\phi} u}) - u = V^g(u).
\end{aligned}
$$

(b) Let $\Gamma_{sg}(T_1, T)$ denote a contract with a switch from splitting the project to going for the project at T_1 and the deadline T. The subcontract at time $t \le T_1$ is denoted by $\hat{\Gamma}_{sg}(t|T_1, T)$. Then, the agent's expected payoff for the subcontract $\hat{\Gamma}_{sg}(t|T_1, T)$ at time

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t is

$$\begin{aligned}U_t(\hat{\Gamma}_{sg}(t|T_1,T)) =& \int_t^{T+1/\lambda_S} \phi\left(T+1/\lambda_S-\tau_m+1/\lambda_S\right)\lambda_S e^{-\lambda_S(\tau_m-t)}d\tau_m \\ =& -\phi\left(T+1/\lambda_S-\tau_m\right)e^{-\lambda_S(\tau_m-t)}\Big|_t^{T+1/\lambda_S} \\ =&\phi\left(T+1/\lambda_S-t\right).\end{aligned}$$

Also note that

$$\begin{aligned}\int_0^{T_1} U_{\tau_s}(\hat{\Gamma}_s(\tau_s|T_1,T))\lambda_S e^{-\lambda_S\tau_s}d\tau_s =& \int_0^{T_1}\phi(T+1/\lambda_S-\tau_s)\lambda_S e^{-\lambda_S\tau_s}d\tau_s \\ =& -\phi(T-\tau_s)e^{-\lambda_S\tau_s}\Big|_0^{T_1} \\ =&\phi T-\phi(T-T_1)e^{-\lambda_S T_1}\end{aligned}$$

Then, the agent's expected payoff at time 0 is

$$\begin{aligned}U_0(\Gamma_{sg}(T_1,T)) =& \int_0^{T_1} U_{\tau_s}(\hat{\Gamma}_s(\tau_s|T_1,T))\lambda_S e^{-\lambda_S\tau_s}d\tau_s \\ &+ e^{-\lambda_S T_1}\int_{T_1}^{T}\phi(T+1/\lambda_G-\tau_m)\lambda_G e^{-\lambda_G(\tau_m-T_1)}d\tau_m \\ =&\phi T-\phi(T-T_1)e^{-\lambda_S T_1}-e^{-\lambda_S T_1}\left[\phi\left(T-\tau_m\right)e^{-\lambda_G(\tau_m-T_1)}\Big|_{T_1}^{T}\right] \\ =&\phi T.\end{aligned}$$

Thus, $U_0(\Gamma_{sg}(T_1,u/\phi))=u$.

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The sum of expected payoffs for the subcontract is

$$
\begin{aligned}
&P_t(\hat{\Gamma}_{sg}(t|T_1,T)) + U_t(\hat{\Gamma}_{sg}(t|T_1,T)) \\
=&\int_t^{T+1/\lambda_S} (\Pi - c(\tau_m - t))\lambda_S e^{-\lambda_S(\tau_m - t)} d\tau_m \\
&- c\left(T + \frac{1}{\lambda_S} - t\right) e^{-\lambda_S\left(T+\frac{1}{\lambda_S}-t\right)} \\
=&-\left(\Pi - \frac{c}{\lambda_S} - c(\tau_m - t)\right) e^{-\lambda_S(\tau_m-t)}\Big|_t^{T+1/\lambda_S} \\
&- c\left(T + \frac{1}{\lambda_S} - t\right) e^{-\lambda_S\left(T+\frac{1}{\lambda_S}-t\right)} \\
=&-\left(\Pi - \frac{c}{\lambda_S} - c\left(T + \frac{1}{\lambda_S} - t\right)\right) e^{-\lambda_S\left(T+\frac{1}{\lambda_S}-t\right)} + \Pi - \frac{c}{\lambda_S} \\
&- c\left(T + \frac{1}{\lambda_S} - t\right) e^{-\lambda_S\left(T+\frac{1}{\lambda_S}-t\right)} \\
=&\left(\Pi - \frac{c}{\lambda_S}\right)\left(1 - e^{-\lambda_S\left(T+\frac{1}{\lambda_S}-t\right)}\right)
\end{aligned}
$$

Also note that

$$
\begin{aligned}
&\int_0^{T_1} \left[P_{\tau_s}(\hat{\Gamma}_{sg}(\tau_s|T_1,T)) + U_{\tau_s}(\hat{\Gamma}_{sg}(\tau_s|T_1,T)) - c\tau_s\right] \lambda_S e^{-\lambda_S \tau_s} d\tau_s \\
=&-\left(\Pi - 2c/\lambda_S - c\tau_s\right)\Big|_0^{T_1} - \left(\Pi - c/\lambda_S\right) e^{-\lambda_S(T+1/\lambda_S)}\tau_s\Big|_0^{T_1} \\
=&\left(\Pi - \frac{2c}{\lambda_S}\right)\left(1 - e^{-\lambda_S T_1}\right) + cT_1 e^{-\lambda_S T_1} - \left(\Pi - \frac{c}{\lambda_S}\right) T_1 e^{-\lambda_S\left(T+\frac{1}{\lambda_S}\right)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_{T_1}^{T} \left(\Pi - c(\tau_m - T_1)\right) \lambda_G e^{-\lambda_G(\tau_m - T_1)} d\tau_m - c(T-T_1)e^{-\lambda_G(T-T_1)} \\
=&V^g((T-T_1)/\phi) + (T-T_1)/\phi = V^g(u_1) + u_1.
\end{aligned}
$$

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Then, we can derive that

$$
\begin{aligned}
&P_0(\Gamma_{sg}(T_1,T))+U_0(\Gamma_{sg}(T_1,T))\\
=&\int_0^{T_1}\left[P_{\tau_s}(\hat{\Gamma}_s(\tau_s|T))+U_{\tau_s}(\hat{\Gamma}_s(\tau_s|T))-c\tau_s\right]\lambda_S e^{-\lambda_S\tau_s}d\tau_s-cT_1e^{-\lambda_S T_1}\\
&+e^{-\lambda_S T_1}\left[\int_{T_1}^{T}\left(\Pi-c(\tau_m-T_1)\right)\lambda_G e^{-\lambda_G(\tau_m-T_1)}d\tau_m-c(T-T_1)e^{-\lambda_G(T-T_1)}\right]\\
=&\left(\Pi-\frac{2c}{\lambda_S}\right)(1-e^{-\lambda_S T_1})-\left(\Pi-\frac{c}{\lambda_S}\right)T_1e^{-\lambda_S\left(T+\frac{1}{\lambda_S}\right)}+e^{-\lambda_S T_1}\left(V^g(u_1)+u_1\right)\\
=&\left(\Pi-\frac{2c}{\lambda_S}\right)\left(1-e^{\frac{\lambda_S}{\phi}(u_1-u)}\right)+(V^g(u_1)+u_1)e^{\frac{\lambda_S}{\phi}(u_1-u)}\\
&-\left(\Pi-\frac{c}{\lambda_S}\right)\frac{\lambda_S}{\phi}(u-u_1)e^{-\frac{\lambda_S}{\phi}u-1},
\end{aligned}
$$

thus $P_0(\Gamma_{sg}(T_1,T))=V^{gs}(u|u_1)$.

(c) Note that a sequential-only contract with a deadline T is equivalent to a contract with a switch from splitting the project to going for the project at $T_1=T$ and a deadline T. Therefore, by the previous result, a sequential-only contract with the deadline u/ϕ implements $(V^{gs}(u|0),u)$.

(d) Let $\Gamma_{gsg}(T_1,T_2,T)$ denote a contract with two switches at T_1 and T_2 and a deadline T. Note that at time T_1 (if the project has not been successful), the remaining contract is equivalent to $\Gamma_{sg}(T_2-T_1,T-T_1)$. Then, the agent's expected payoff at time 0 is

$$
\begin{aligned}
U_0(\Gamma_{gsg}(T_1,T_2,T))&=\int_0^{T_1}\phi(T+1/\lambda_G-\tau_m)\lambda_G e^{-\lambda_G\tau_m}d\tau_m+e^{-\lambda_S T_1}U_0(\Gamma_{sg}(T_2-T_1,T-T_1))\\
&=\phi T-\phi(T-T_1)e^{-\lambda_G T_1}+e^{-\lambda_G T_1}\phi(T-T_1)\\
&=\phi T.
\end{aligned}
$$

Thus, $U_0(\Gamma_{gsg}(T_1,T_2,u/\phi))=u$.

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Also note that

$$
\begin{aligned}
&P_0(\Gamma_{gsg}(T_1,T_2,T)) + U_0(\Gamma_{gsg}(T_1,T_2,T)) \\
&= \int_0^{T_1} (\Pi - c\tau_m)\lambda_G e^{-\lambda_G \tau_m} d\tau_m - cT_1 e^{-\lambda_G T_1} \\
&\quad + e^{-\lambda_G T_1}(P_0(\Gamma_{sg}(T_2-T_1, T-T_1)) + U_0(\Gamma_{sg}(T_2-T_1, T-T_1))) \\
&= \left(\Pi - \frac{c}{\lambda_G}\right)\left(1 - e^{\lambda_G T_1}\right) + \left(V^{gs}(\phi(T-T_1)|\phi(T-T_2)) + \phi(T-T_1)\right) e^{-\lambda_G T_1} \\
&= \left(\Pi - \frac{c}{\lambda_G}\right)\left(1 - e^{\frac{\lambda_G}{\phi}(u_2-u)}\right) + \left(V^{gs}(u_2|u_1) + u_2\right) e^{\frac{\lambda_G}{\phi}(u_2-u)},
\end{aligned}
$$

thus $P_0(\Gamma_{gsg}(T_1,T_2,T)) = V^{gsg}(u|u_1,u_2) - u$.

□

Now we are ready to prove Proposition 4.1 and 5.1.

Proof of Proposition 4.1. From Proposition B.1, we can observe that the value function takes a form of V^g or V^{gs}. By (a)–(c) in Proposition B.3, we can see that $(V(u), u)$ can be implemented by a minimum incentive contract with at most one switch. □

Proof of Proposition 5.1. From Proposition B.2, we can observe that a point on the value function can be represented by one of the benchmark value functions. Note that every contract specified in Proposition B.3 is a minimum incentive contract with at most two switches. Therefore, $(V(u), u)$ can be implemented by a minimum incentive contract with at most two switches. □

B.6 The Optimal Contract

B.6.1 Feasibility (Π_F)

Now that we characterized the value function that solves (HJB) subject to (PK), the next step is to solve (MP) subject to $u \geq 0$. I begin by checking the feasibility of the project. If the maximum of the value function V is greater than 0, the principal earns positive expected payoff from the contract, thus the project is feasible. If $V'(0) > 0$, there exists $u > 0$ such that $V(u) > 0$. Thus, the project is feasible. On the other hand, if $V'(0) \leq 0$, the maximum of the value function is 0 at $u = 0$ since V is strictly concave (Corollary B.13). Thus, the

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project is infeasible. Note that from (HJB), $V'(0) > 0$ is equivalent to

$$\max\left[\lambda_G\Pi - \phi, \lambda_S V_S\left(\phi/\lambda_S\right)\right] > c, \tag{B.13}$$

i.e., the project is feasible if at least one of the instantaneous payoff at the deadline covers the operating cost c. Note that

$$\begin{aligned}\lambda_S V_S(\phi/\lambda_S) =& \lambda_S(\Pi - c/\lambda_S)\left(1 - e^{-1}\right) - \phi \\ =& (1+\eta)(1-e^{-1})\lambda_G\Pi - (1-e^{-1})c - \phi.\end{aligned}$$

Then, we can derive that $\lambda_S V_S(\phi/\lambda_S) > c$ is equivalent to

$$\Pi > \frac{(2-e^{-1})c + \phi}{(1+\eta)(1-e^{-1})\lambda_G},$$

whereas $\lambda_G\Pi - \phi > c$ is equivalent to $\Pi > \dfrac{c+\phi}{\lambda_G}$. By simple algebra, we can show that

$$\frac{(2-e^{-1})c + \phi}{(1+\eta)(1-e^{-1})\lambda_G} > \frac{c+\phi}{\lambda_G}.$$

Therefore, $\Pi > \Pi_F \equiv \dfrac{c+\phi}{\lambda_G}$ is equivalent to (B.13) and the project is feasible if this condition is satisfied.

B.6.2 The Length of the Contract (Π_G, Π_{SG})

Given that the project is feasible, there exists $\bar{u} > 0$ that maximizes $V(u)$. Since V is concave and differentiable, $\bar{u}$ is the solution of $V'(\bar{u}) = 0$. To check what type of contract would be utilized to implement $(\bar{u}, V(\bar{u}))$, we need to compare $\bar{u}$ with switch points u_1 and u_2 defined in (B.5) and (B.6).

If Π is less than or equal to $\Pi_M(\eta)$, by Proposition B.1 and B.2, the value function is equal to $V^g(u)$, i.e., it does not have any switch point. Therefore, it is enough to restrict attention to the case where Π is greater than $\Pi_M(\eta)$.

If Π is greater than $\Pi_M(\eta)$ and less than $\Pi_S(\eta)$, there exists a switch point $u_1 > 0$ such that $V^g(u_1) = V^{gs\prime}(u_1|u_1)$. Note that if Π is greater than or equal to $\Pi_S(\eta)$ and η is greater than $1/(e-1)$, the value function near $u = 0$ corresponds to $V^{gs}(u|0)$ by Corollary B.8 and B.9. Thus, in this case, we can consider u_1 as 0. The following lemma characterizes the threshold of Π for comparing $\bar{u}$ and u_1.

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Lemma B.14. *Suppose that* $\Pi > \Pi_M(\eta)$ *and* $2\lambda_G \geq \lambda_S > \lambda_G$ *are satisfied. Then, there exists* $\Pi_G(\eta) \geq \Pi_M(\eta)$ *such that* $u_1 < \bar{u}$ *if and only if* $\Pi > \Pi_G(\eta)$*. Moreover, if* $\eta \leq \sqrt{c/(c+\phi)}$*,* $\Pi_G(\eta)$ *is equal to* $\Pi_M(\eta)$*.*

Now I compare $\bar{u}$ and the second switching point u_2. The following lemma shows that u_2 is less than $\bar{u}$ if η is sufficiently small. Thus, in this case, the optimal contract involves two switches of recommended approaches.

Lemma B.15. *Suppose that* $\Pi > \Pi_M(\eta)$ *and* $\eta \leq c/(c+\phi)$ *are satisfied. Then,* u_2 *is less than* $\bar{u}$*.*

On the other hand, the following lemma shows that u_2 is greater than $\bar{u}$ if η is close to 1. Thus, in this case, the optimal contract involves at most one switch of recommended approaches.

Lemma B.16. *Suppose that* $\Pi \geq \Pi_G(\eta)$ *and* $\eta \geq \sqrt{c/(c+\phi)}$ *are satisfied. Then,* u_2 *is greater than or equal to* $\bar{u}$*.*

B.6.3 Proofs of Theorems

Proof of Theorem 1. I will use functions Π_S and Π_G defined in Lemma B.4 and B.14.

(a) By the argument in Appendix B.6.1, the project is infeasible when Π is less than Π_F.

(b) Note that $\Pi_M(1) = c/\lambda_G$ from Lemma B.5. When $\Pi_G(1) \geq \Pi > c/\lambda_G$, by Lemma B.14, the switching point u_1 is greater than or equal to $\bar{u}$. Then, $V(\bar{u}) = V^g(\bar{u})$ by (a) of Proposition B.1. In both cases, by (a) of Proposition B.3, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with the deadline $\bar{u}/\phi$.

(c) When $\Pi \in (\Pi_G(1), \Pi_S(1))$, u_1 is greater than 0 and less than $\bar{u}$ by Lemma B.4 and Corollary B.8 (b). Then, $V(\bar{u}) = V^{gs}(\bar{u}|u_1)$ by Proposition B.1. By (b) of Proposition B.3, $(\bar{u}, V(\bar{u}))$ is implemented by a contract with a switch from splitting the project to going for the project at $(\bar{u} - u_1)/\phi$ and the deadline $\bar{u}/\phi$.

(d) When $\Pi \geq \Pi_S(1)$, $V(u) = V^{gs}(u|0)$ by (b) of Proposition B.1. By (c) of Proposition B.3, $(\bar{u}, V(\bar{u}))$ is implemented by a sequential-only contract with the deadline $\bar{u}/\phi$.

$\square$

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Proof of Theorem 2. Note that u_2 is greater than $\bar{u}$ by Lemma B.16 since $\eta > \sqrt{c/(c+\phi)}$. Also note that Π_S is properly defined as in (B.7) since η is greater than $1/(e-1)$. When $\Pi \in [\Pi_F, \Pi_M(\eta)]$, $V(\bar{u}) = V^g(\bar{u})$ by (a) of Proposition B.2. Then, by (a) of Proposition B.3, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with the deadline $\bar{u}/\phi$. For other cases, the statements can be similarly proved as in Theorem 1, except that we need to use Proposition B.2 instead of Proposition B.1. □

Proof of Theorem 3. Proof for part (a) is same as Theorem 1, thus, it is enough to show (b) and (c).

(b) I will use the function Π_M defined in Lemma B.5. When $\Pi \in [\Pi_F, \Pi_M(\eta)]$, $V(\bar{u}) = V^g(\bar{u})$ by (a) of Proposition B.2. By (a) of Proposition B.3, $(\bar{u}, V(\bar{u}))$ is implemented by a direct-only contract with the deadline $\bar{u}/\phi$.

(c) When $\Pi > \Pi_M(\eta)$, since $\eta < c/(c+\phi)$ is assumed, u_2 is less than $\bar{u}$ by Lemma B.15. Also note that (b) of Proposition B.2 applies since $\eta < 1/(e-1)$. Therefore, $V(\bar{u})$ is equal to $V^{gsg}(u|u_1, u_2)$. By (a) of Proposition B.3, $(\bar{u}, V(\bar{u}))$ is implemented by a contract with two switches at $(\bar{u}-u_2)/\phi$ and $(\bar{u}-u_1)/\phi$ and the deadline $\bar{u}/\phi$.

□

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Supplementary Appendix for "the Direct vs. the Sequential Approach in Project Management"

SA.1 Proofs of Lemmas in Appendix

SA.1.1 Proof of Lemma in Section B.2

Proof of Lemma B.1. Consider an arbitrary (deterministic) contract Γ where the agent's expected payoff is given by . Let $A_t \equiv \int_0^t a_s ds$ and $B_t = \int_0^t b_s ds$. The payoff to the principal under Γ is

$$\begin{aligned} P_0(\Gamma) = &\int_0^T \lambda_G a_t e^{-\lambda_G A_t - \lambda_S B_t} \left(\Pi - R_t - c\ t\right) dt \\ &+ \int_0^T \lambda_S b_t e^{-\lambda_G A_t - \lambda_S B_t} \left(V_S(u_{S,t}) - c\ t\right) dt + e^{-\lambda_G A_T - \lambda_S B_T}(-c\ T) \\ = &\int_0^T e^{-\lambda_G A_t - \lambda_S B_t} \left((\Pi - R_t)\lambda_G a_t + V_S(u_{S,t})\lambda_S b_t - c\right) dt \end{aligned}$$

where $u_{S,t} = \hat{U}_t(\hat{\Gamma}^S)$.

Since $\tilde{V}$ solves the HJB equation, we have

$$0 \geq -c + (\Pi - R_t - \tilde{V}(u_t))\lambda_G a_t + (V_S(u_{S,t}) - \tilde{V}(u_t))\lambda_S b_t + \tilde{V}'(u_t)\dot{u}_t.$$

By rearranging and multiplying by $e^{-\lambda_G A_t - \lambda_S B_t}$,

$$\begin{aligned} &(\lambda_G a_t + \lambda_S b_t) e^{-\lambda_G A_t - \lambda_S B_t} \tilde{V}(u_t) - e^{-\lambda_G A_t - \lambda_S B_t} \tilde{V}'(u_t)\dot{u}_t \\ \geq &e^{-\lambda_G A_t - \lambda_S B_t} \left((\Pi - R_t)\lambda_G a_t + V_S(u_{H,t})\lambda_S b_t - c\right) \end{aligned} \tag{SA.1.1}$$

Note that

$$\frac{d}{dt}\left(-e^{-\lambda_G A_t - \lambda_S B_t}\tilde{V}(u_t)\right) = (\lambda_G a_t + \lambda_S b_t) e^{-\lambda_G A_t - \lambda_S B_t}\tilde{V}(u_t) - e^{-\lambda_G A_t - \lambda_S B_t}\tilde{V}'(u_t)\dot{u}_t.$$

Then, by integrating (SA.1.1) over $[0, T]$ and noting that $u_T = 0$, we have

$$\begin{aligned} &\tilde{V}(u_0) = \tilde{V}(u_0) - e^{-\lambda_G A_T - \lambda_S B_T}\tilde{V}(u_T) \\ \geq &\int_0^T e^{-\lambda_G A_t - \lambda_S B_t} \left((\Pi - R_t)\lambda_G a_t + V_S(u_{S,t})\lambda_S b_t - c\right) dt = P_0^L(\Gamma^L). \end{aligned}$$

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Therefore, $\tilde{V}(u_0)$ is greater than or equal to any deterministic contract where the agent's expected payoff is equal to u_0. Since $\tilde{V}$ is assumed to be concave, it is greater than or equal to any randomized contract. □

Proof of Lemma B.2. Suppose that (a^*, b^*, R^*, u_H^*) with $a^*, b^* > 0$ solves (HJB) subject to (PK). Note that $(R^* - u)\lambda_G - \phi = (u_H^* - u)\lambda_S - \phi \geq 0$ from the maximization of (PK). Also note that $(\Pi - R^* - V(u))\lambda_G = (V_S(u_H^*) - V(u))\lambda_S \geq 0$ from the maximization of (HJB).

Then, $(a, b, R, u_H) = (1, 0, R^*, 0)$, $(a, b, R, u_H) = (0, 1, 0, u_H^*)$ and $(a, b, R, u_H) = (a^*, b^*, R^*, u_H^*)$ have the same values for the RHS of (HJB) and the RHS of (PK). Since (a^*, b^*, R^*, u_H^*) solves (HJB) subject to (PK), $(1, 0, R^*, 0)$ and $(0, 1, 0, u_H^*)$ also solve (HJB) subject to (PK). □

SA.1.2 Proof of Lemma in Section B.3

Proof of Lemma B.3. (a) By (B.3) and $e^{-\lambda_G u/\phi} > 0$, $V^g(u) < \Pi - c/\lambda_G - u$. By differentiating (B.3), we have

$$V^{g\prime}(u) = \left(\Pi - \frac{c}{\lambda_G}\right) \frac{\lambda_G}{\phi} e^{-\frac{\lambda_G}{\phi} u} - 1 > -1.$$

By differentiating once again, we have

$$V^{g\prime\prime}(u) = -\left(\Pi - \frac{c}{\lambda_G}\right) \frac{\lambda_G^2}{\phi^2} e^{-\frac{\lambda_G}{\phi} u} < 0.$$

(b) By (B.5), $\Pi - 2c/\lambda_S \leq \Pi - c/\lambda_G$, $V^g(u_1) + u_1 < \Pi - c/\lambda_G$ and $u \geq u_1$, we can derive that

$$V^{gs}(u|u_1) < \Pi - \frac{c}{\lambda_G} - u.$$

From (4.1), (B.2) and (B.4), we can derive that

$$\begin{aligned} V^{g\prime}(u_1) &= -\frac{c}{\phi} - 1 + \frac{\lambda_G}{\phi} \left(\Pi - u_1 - V^g(u_1)\right), \\ V^{gs\prime}(u_1|u_1) &= -\frac{c}{\phi} - 1 + \frac{\lambda_S}{\phi} \left(\left(\Pi - \frac{c}{\lambda_S}\right) \left(1 - e^{-\frac{\lambda_S}{\phi} u_1 - 1}\right) - V^g(u_1) - u_1 \right). \end{aligned}$$

Then, $V^{g\prime}(u_1) \leq V^{gs\prime}(u_1|u_1)$ is equivalent to

$$u_1 + V^g(u_1) \leq \Pi - \frac{c}{\lambda_S - \lambda_G} - \frac{\lambda_S}{\lambda_S - \lambda_G} \left(\Pi - \frac{c}{\lambda_S}\right) e^{-\frac{\lambda_S}{\phi} u_1 - 1}. \tag{SA.1.2}$$

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By differentiating (B.5) twice, we have

$$
\begin{aligned}
V^{gs''}(u|u_1) = &- \left(\frac{\lambda_S}{\phi}\right)^2 e^{\frac{\lambda_S}{\phi}(u_1-u)} \left[\left(\Pi - \frac{2c}{\lambda_S}\right) - (V^g(u_1)+u_1)\right] \\
&- \left(\Pi - \frac{c}{\lambda_S}\right)\left(\frac{\lambda_S}{\phi}\right)^2 \left[-2 + \frac{\lambda_S}{\phi}(u-u_1)\right] e^{-\frac{\lambda_S}{\phi}u-1} \\
= &\left(\frac{\lambda_S}{\phi}\right)^2 e^{\frac{\lambda_S}{\phi}(u_1-u)} \left[(V^g(u_1)+u_1) - \left(\Pi - \frac{2c}{\lambda_S}\right) + 2\left(\Pi - \frac{c}{\lambda_S}\right) e^{-\frac{\lambda_S}{\phi}u_1-1}\right] \\
&- \left(\Pi - \frac{c}{\lambda_S}\right)\left(\frac{\lambda_S}{\phi}\right)^3 (u-u_1) e^{-\frac{\lambda_S}{\phi}u-1}.
\end{aligned}
$$

By using (SA.1.2), we can show that

$$
\begin{aligned}
V^{gs''}(u|u_1) \leq &\left(\frac{\lambda_S}{\phi}\right)^2 e^{\frac{\lambda_S}{\phi}(u_1-u)} \left[\frac{\lambda_S - 2\lambda_G}{\lambda_S(\lambda_S-\lambda_G)} c + \frac{\lambda_S - 2\lambda_G}{\lambda_S-\lambda_G}\left(\Pi - \frac{c}{\lambda_S}\right) e^{-\frac{\lambda_S}{\phi}u_1-1}\right] \\
&- \left(\Pi - \frac{c}{\lambda_S}\right)\left(\frac{\lambda_S}{\phi}\right)^3 (u-u_1) e^{-\frac{\lambda_S}{\phi}u-1}.
\end{aligned}
$$

Then, from $2\lambda_G \geq \lambda_S > \lambda_G$ and $\Pi > c/\lambda_S$, we can derive that $V^{gs''}(u|u_1) \leq 0$.

Note that

$$
\begin{aligned}
V^{gs'}(u|u_1) = &\frac{\lambda_S}{\phi} e^{\frac{\lambda_S}{\phi}(u_1-u)} \left[\left(\Pi - \frac{2c}{\lambda_S}\right) - (V^g(u_1)+u_1)\right] \\
&- \left(\Pi - \frac{c}{\lambda_S}\right)\left(\frac{\lambda_S}{\phi}\right) \left[1 - \frac{\lambda_S}{\phi}(u-u_1)\right] e^{-\frac{\lambda_S}{\phi}u-1} - 1, \\
\lim_{u\to\infty} V^{gs'}(u|u_1) = &-1.
\end{aligned}
$$

Then, by the concavitiy of $V^{gs}(u|u_1)$, $V^{gs'}(u|u_1) > -1$.

(c) By (B.6), $V^{gs}(u_2|u_1) + u_2 < \Pi - c/\lambda_G$ and $u \geq u_2$, we can derive that

$$
V^{gsg}(u|u_1,u_2) < \Pi - \frac{c}{\lambda_G} - u.
$$

By differentiating (B.6) once, we have

$$
V^{gsg'}(u|u_1,u_2) = \frac{\lambda_G}{\phi}\left(\Pi - \frac{c}{\lambda_G} - (V^{gs}(u_2|u_1,u_2)+u_2)\right) e^{\frac{\lambda_G}{\phi}(u_2-u)} - 1.
$$

By the previous result ($V^{gs}(u_2|u_1,u_2)+u_2 < \Pi - c/\lambda_G$), we can derive that $V^{gsg'}(u|u_1,u_2) >$

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-1.

By differentiating (B.6) twice, we can derive that

$$V^{gsg\prime\prime}(u|u_1,u_2) = -\left(\frac{\lambda_G}{\phi}\right)^2 \left(\Pi - \frac{c}{\lambda_G} - (V^{gs}(u_2|u_1,u_2) + u_2)\right) e^{\frac{\lambda_G}{\phi}(u_2-u)} < 0.$$

□

SA.1.3 Proofs of Section B.4

Proof of Lemma B.4. By (B.2), (B.4) and $V^g(0) = V^{gs}(0|0) = 0$, we have

$$\phi V^{g\prime}(0) = \lambda_G \Pi - \phi - c,$$

$$\phi V^{gs\prime}(0|0) = \lambda_S V_S(\phi/\lambda_S) - c = \lambda_S (\Pi - c/\lambda_S)(1 - e^{-1}) - \phi - c.$$

Then, we can simply show that $V^{g\prime}(0) > V^{gs\prime}(0|0)$ is equivalent to

$$\begin{aligned} & \lambda_S (\Pi - c/\lambda_S)(1 - e^{-1}) < \lambda_G \Pi \\ \iff & \big(\lambda_S(1 - e^{-1}) - \lambda_G\big)\Pi < (1 - e^{-1})c \\ \iff & ((e-1)\eta - 1)\Pi < (e-1)\frac{c}{\lambda_G} \end{aligned}$$

Therefore, if $\eta \le 1/(e-1)$, $V^{g\prime}(0) > V^{gs}(0|0)$ holds always. If $\eta > 1/(e-1)$, $V^{g\prime}(0) < V^{gs\prime}(0|0)$ is equivalent to $\Pi < \Pi_S(\eta)$. Moreover, $V^{g\prime}(0) = V^{gs\prime}(0|0)$ when $\Pi = \Pi_S(\eta)$.

Also note that

$$\phi V^{g\prime\prime}(0) = -\lambda_G \left(1 + V^{g\prime}(0)\right) = -\frac{\lambda_G}{\phi}(\lambda_G \Pi - c),$$

$$\begin{aligned} \phi V^{gs\prime\prime}(0|0) &= \lambda_S V_S'(\phi/\lambda_S) - \lambda_S V^{gs\prime}(0|0) = \frac{\lambda_S}{\phi}\left(\phi V_S'(\phi/\lambda_S) - \phi V^{gs\prime}(0|0)\right) \\ &= \frac{\lambda_S}{\phi}\left(\phi V_S'(\phi/\lambda_S) - \lambda_S V_S(\phi/\lambda_S) + c\right) \\ &= \frac{\lambda_S}{\phi}\left(\lambda_S \Pi - 2\lambda_S (V_S(\phi/\lambda_S) + \phi/\lambda_S)\right) \end{aligned}$$

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When $\Pi = \Pi_S(\eta)$, $\lambda_G \Pi = \lambda_S \left(V_S(\phi/\lambda_S) + \phi/\lambda_S\right)$ from $V^{g\prime}(0) = V^{gs\prime}(0|0)$. Then,

$$\begin{aligned}
\phi^2 V^{gs\prime\prime}(0|0) - \phi^2 V^{g\prime\prime}(0) &= \lambda_S(\lambda_S \Pi_S(\eta) - 2\lambda_G \Pi_S(\eta)) + \lambda_G(\lambda_G \Pi_S(\eta) - c) \\
&= \lambda_G^2 \left[\left(\frac{\lambda_S}{\lambda_G} - 1 \right)^2 \Pi_S(\eta) - \frac{c}{\lambda_G} \right] \\
&= \lambda_G^2 \left[\eta^2 \Pi_S(\eta) - \frac{c}{\lambda_G} \right] \\
&= \lambda_G c \left[\frac{(e-1)\eta^2}{(e-1)\eta - 1} - 1 \right]
\end{aligned}$$

Since $(e-1)x^2 > (e-1)x - 1$ for all $x > 1/(e-1)$, we can see that $V^{gs\prime\prime}(0|0) > V^{g\prime\prime}(0)$. $\square$

Proof of Lemma B.5. Consider a function $H_1 : \mathbb{R}_+ \to \mathbb{R}$ defined as follows:

$$H_1(u) \equiv \phi V^{gs\prime}(u|u) - \phi V^{g\prime}(u). \tag{SA.1.3}$$

Then, $H_1(u)$ can be rewritten as follows:

$$\begin{aligned}
H_1(u) &= \lambda_S \left((\Pi - c/\lambda_S)(1 - e^{-\frac{\lambda_S}{\phi}u - 1}) - V^g(u) - u \right) - \lambda_G(\Pi - u - V^g(u)) \\
&= (\lambda_S \Pi - c)\left(1 - e^{-\frac{\lambda_S}{\phi}u-1}\right) - (\lambda_S - \lambda_G)(u + V^g(u)) - \lambda_G \Pi \\
&= (\lambda_S \Pi - c)\left(1 - e^{-\frac{\lambda_S}{\phi}u-1}\right) - (\lambda_S - \lambda_G)\left(\Pi - c/\lambda_G\right)(1 - e^{-\frac{\lambda_G}{\phi}u}) - \lambda_G \Pi \\
&= (\lambda_S/\lambda_G - 1)(\lambda_G \Pi - c)e^{-\frac{\lambda_G}{\phi}u} - (2 - \lambda_S/\lambda_G)c - (\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u-1} \\
&= \eta(\lambda_G \Pi - c)e^{-\frac{\lambda_G}{\phi}u} - (\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u-1} - (1-\eta)c.
\end{aligned}$$

Let u_* be the solution of $H_1'(u) = 0$. Then, we can derive that

$$u_* = \frac{\phi}{\lambda_S - \lambda_G}\left(\log\left[\frac{\eta+1}{\eta} \cdot \frac{\lambda_S \Pi - c}{\lambda_G \Pi - c} \right] - 1 \right)$$

Note that u_* be the global maximum of $H_1(u)$, thus $V^{g\prime}(u) > V^{gs\prime}(u)$ for all $u \geq 0$ is equivalent to $H_1(u_*) < 0$.[30] By plugging u_* into $H_1(u)$, we can derive that

$$H_1(u_*) = K \left(\frac{\lambda_G \Pi - c}{\lambda_S \Pi - c} \right)^{\frac{1}{\eta}} (\lambda_G \Pi - c) - (1-\eta)c$$

[30] To show that u_* be the global minimum, substitute x for $e^{-\frac{\lambda_G}{\phi}u}$ and show that the function satisfies the second order condition for x.

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where $K = \dfrac{\eta^2}{\eta+1}\left(\dfrac{\eta e}{\eta+1}\right)^{\frac{1}{\eta}}$. Consider the above equation as a function of Π and denote by $h(\Pi)$.

Note that $h(c/\lambda_G) = -(1-\eta)c \leq 0$, $\lim_{\Pi\to\infty} h(\Pi) = \infty$, and

$$h'(\Pi) = K(\lambda_G\Pi - c)^{1/\eta}(\lambda_S\Pi - c)^{-1/\eta-1}\lambda_G\lambda_S\Pi > 0.$$

Therefore, there exists a unique Π such that $h(\Pi) = 0$ and $\Pi \geq c/\lambda_G$. Let the solution of $h(\Pi) = 0$ be $\Pi_M(\eta)$. Also note that when $\eta = 1$, $h(c/\lambda_G) = 0$ thus $\Pi_M(1) = c/\lambda_G$. Then, if $c/\lambda_G \leq \Pi < \Pi_M(\eta)$, $h(\Pi) = H_1(u_*) < 0$, thus, $V^{g\prime}(u) > V^{gs\prime}(u|u)$ for all $u \geq 0$.

Note that by the definition of $\Pi_S(\eta)$, if $\Pi \geq \Pi_S(\eta)$, $H_1(0) \geq 0$. It implies that $h(\Pi) = H_1(u_*) \geq H_1(0) \geq 0$ and $\Pi \geq \Pi_M(\eta)$. Therefore, we can see that $\Pi_S(\eta) \geq \Pi_M(\eta)$.

If $\Pi_S(\eta) \geq \Pi > \Pi_M(\eta)$, $H_1(0) \leq 0$ and $H_1(u_*) > 0$. Then, there exists $u \geq 0$ such that $H_1(u) = 0$ and $H_1'(u) > 0$. Therefore, there exists $u \geq 0$ such that $V^{g\prime}(u) = V^{gs\prime}(u|u)$ and $V^{gs\prime\prime}(u|u) > V^{g\prime\prime}(u)$. □

Proof of Lemma B.6. Define a function $H_2 : [u_1, \infty) \to \mathbb{R}$ as $H_2(u) \equiv \phi V^{gsg\prime}(u|u_1, u) - \phi V^{gs\prime}(u|u_1)$. From $\phi V^{gsg\prime}(u|u_1, u) = -c + \lambda_G\left(\Pi - \phi/\lambda_G - u - V^{gsg}(u|u_1, u)\right)$ (by (B.2)), $V^{gsg}(u|u_1, u) = V^{gs}(u|u_1)$ and (B.5), $H_2(u)$ can be rewritten as follows:

$$\begin{aligned}
H_2(u) &= \lambda_G\left(\Pi - \frac{\phi}{\lambda_G} - u - V^{gs}(u|u_1)\right) - c - \phi V^{gs\prime}(u|u_1) \\
&= \frac{2\lambda_G - \lambda_S}{\lambda_S}c + (\lambda_S\Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1}\left[1 + \frac{\lambda_S - \lambda_G}{\lambda_S}\frac{\lambda_S}{\phi}(u_1 - u)\right]e^{\frac{\lambda_S}{\phi}(u_1-u)} \\
&\quad - (\lambda_S - \lambda_G)\left[\Pi - \frac{2c}{\lambda_S} - (V^g(u_1) + u_1)\right]e^{\frac{\lambda_S}{\phi}(u_1-u)} \\
&= \frac{1-\eta}{1+\eta}c + (\lambda_S\Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1}\left[1 + \frac{\eta}{1+\eta}\frac{\lambda_S}{\phi}(u_1 - u)\right]e^{\frac{\lambda_S}{\phi}(u_1-u)} \\
&\quad + \eta\left[\frac{1-\eta}{1+\eta}c - (\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u_1}\right]e^{\frac{\lambda_S}{\phi}(u_1-u)}
\end{aligned}$$

Define $x \equiv e^{\frac{\lambda_S}{\phi}(u_1-u)}$. Then, $H_2(u)$ can be rewritten as follows:

$$\begin{aligned}
\tilde{H}_2(x) \;\equiv\; &\frac{1-\eta}{1+\eta}c + (\lambda_S\Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1}\left[1 + \frac{\eta}{1+\eta}\log x\right]x \\
&+\eta\left[\frac{1-\eta}{1+\eta}c - (\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u_1}\right]x.
\end{aligned} \tag{SA.1.4}$$

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Note that $\tilde{H}_2(1) = H_2(u_1) = \phi V^{gsg\prime}(u_1|u_1, u_1) - \phi V^{gs\prime}(u_1|u_1) = \phi V^{g\prime}(u_1) - \phi V^{gs\prime}(u_1|u_1) \leq 0$. By differentiating $\tilde{H}$ twice, we have

$$\tilde{H}_2''(x) = \frac{\eta}{1+\eta}(\lambda_S \Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1}\frac{1}{x} > 0.$$

Since $\Pi > c/\lambda_G > c/\lambda_S$, $\tilde{H}_2$ is strictly convex in x. Also note that

$$\lim_{x\to 0} \tilde{H}_2(x) = \frac{1-\eta}{1+\eta}c$$

and we simply denote by $\tilde{H}_2(0)$.

(a) Suppose that $\eta = 1$. Then, by the strict convexity of $\tilde{H}_2$, for all $x \in (0,1)$,

$$\tilde{H}_2(x) < (1-x)\tilde{H}_2(0) + x\tilde{H}_2(1) \leq 0.$$

Therefore, for all $u > u_1$, $H_2(u) < 0$, i.e., $V^{gs\prime}(u|u_1) > V^{gsg\prime}(u|u_1, u)$.

(b) Suppose that $\eta < 1$. Then, $\tilde{H}_2(0) > 0$. If $V^{gs\prime}(u_1|u_1) < V^{g\prime}(u_1)$, $\tilde{H}_2(1) < 0$. In this case, there exists $x_2 \in (0,1)$ such that $\tilde{H}_2(x_2) = 0$. Let $u_2 = u_1 - \frac{\phi}{\lambda_S}\log x_2$. Then, $H(u_2) = 0$, i.e., $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$.

Next, consider the case with $V^{gs\prime}(u_1|u_1) = V^{g\prime}(u_1)$ and $V^{gs\prime\prime}(u_1|u_1) > V^{g\prime\prime}(u_1)$. By differentiating (B.2) and (B.4) once, we have

$$\begin{aligned}\phi V^{g\prime\prime}(u_1) &= -\lambda_G(1 + V^{g\prime}(u_1)),\\ \phi V^{gs\prime\prime}(u_1|u_1) &= \lambda_S\left(V_S'(u + \phi/\lambda_S) - V^{gs\prime}(u|u_1)\right).\end{aligned}$$

Then, from the above expressions and $V^{gs\prime}(u_1|u_1) = V^{g\prime}(u_1)$, $V^{gs\prime\prime}(u_1|u_1) > V^{g\prime\prime}(u_1)$ is equivalent to :

$$\begin{aligned} & \lambda_S\left(V_S'(u_1 + \phi/\lambda_S) + 1\right) > (\lambda_S - \lambda_G)(1 + V^{g\prime}(u_1)) \\ \iff \quad & \frac{\lambda_S}{\phi}\lambda_S\left(\Pi - \frac{c}{\lambda_S}\right)e^{-\frac{\lambda_S}{\phi}u_1 - 1} > (\lambda_S - \lambda_G)\frac{\lambda_G}{\phi}\left(\Pi - \frac{c}{\lambda_G}\right)e^{-\frac{\lambda_G}{\phi}u_1} \\ \iff \quad & (\eta+1)(\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_1 - 1} > \eta(\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u_1}. \end{aligned} \tag{SA.1.5}$$

Note that $\tilde{H}_2(1) = (1-\eta)c + (\lambda_S\Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1} - \eta(\lambda_G\Pi - c)e^{-\frac{\lambda_S}{\phi}u_1} = 0$ from

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$V^{gs\prime}(u_1|u_1) = V^{g\prime}(u_1)$. Then,

$$\begin{aligned}\tilde{H}_2'(1) &= (\lambda_S\Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1}\left[1+\frac{\eta}{1+\eta}\right] + \eta\left[\frac{1-\eta}{1+\eta}c - (\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u_1}\right] \\ &= (\lambda_S\Pi - c)e^{-1-\frac{\lambda_S}{\phi}u_1} - \frac{\eta}{1+\eta}(\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u_1} \\ &\geq 0.\end{aligned}$$

The last inequality is due to (SA.1.5).

$\tilde{H}_2(1) = 0$ and $\tilde{H}_2'(1) > 0$ imply $\tilde{H}_2(1-\epsilon) < 0$ for small enough $\epsilon > 0$. Then, since $\tilde{H}_2(0) > 0$ and $\tilde{H}_2(1-\epsilon) < 0$, there exists $x_2 \in (0, 1-\epsilon)$ such that $\tilde{H}_2(x_2) = 0$ & $\tilde{H}_2'(x_2) < 0$, thus there exists u_2 such that $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ & $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$.

Suppose that there exists another u_2' with $H_2(u_2') = 0$. Consider corresponding x_2', then $\tilde{H}_2(x_2') = 0$. If $x_2' > x_2$,

$$0 = \tilde{H}_2(x_2'|u_1) < \frac{1-x_2'}{1-x_2}\tilde{H}_2(x_2) + \frac{x_2'-x_2}{1-x_2}\tilde{H}_2(1) \leq 0$$

from $\tilde{H}_2(x_2) = 0$ and $\tilde{H}_2(1) \leq 0$. Similar logic holds for the case of $x_2' < x_2$. Therefore, there is unique u_2 satisfying $H_2(u_2) = 0$.

From $\tilde{H}_2(x_2) = 0$, $\tilde{H}_2(1) < 0$ and the strict convexity of $\tilde{H}_2$, for all $x \in (x_2, 1)$,

$$\tilde{H}_2(x) < \frac{1-x}{1-x_2}\tilde{H}_2(x_2) + \frac{x-x_2}{1-x_2}\tilde{H}_2(1) < 0.$$

Therefore, for all $u_2 > u > u_1$, $H_2(u) < 0$, i.e., $V^{gs\prime}(u|u_1) > V^{gsg\prime}(u|u_1, u)$.

□

Proof of Lemma B.7. By (B.2) and (B.4), we have

$$\phi V^{gs\prime}(u|u_1) = \lambda_S\left(V_S\left(u+\phi/\lambda_S\right) + u + \phi/\lambda_S\right) - \lambda_S\left(V^{gs}(u|u_1) + u\right) - c - \phi,$$

$$\phi V^{gsg\prime}(u|u_1, u_2) = \lambda_G\Pi - \lambda_G\left(V^{gsg}(u|u_1, u_2) + u\right) - c - \phi.$$

By differentiating above equations, we have

$$\phi V^{gs\prime\prime}(u|u_1) = \lambda_S\left(V_S'\left(u+\phi/\lambda_S\right) + 1\right) - \lambda_S\left(V^{gs\prime}(u|u_1) + 1\right),$$

$$\phi V^{gsg\prime\prime}(u|u_1, u_2) = -\lambda_G\left(V^{gsg\prime}(u|u_1, u_2) + 1\right).$$

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Then, $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1,u_2)$ and $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1,u_2)$ imply that

$$(\lambda_S - \lambda_G)(1 + V^{gs\prime}(u_2|u_1)) > \lambda_S(V_S'(u_2 + \phi/\lambda_S) + 1)$$

$$\iff \qquad \eta(1 + V^{gs\prime}(u_2|u_1)) > (\eta+1)\left(\frac{\lambda_S\Pi - c}{\phi}\right)e^{-\frac{\lambda_S}{\phi}u_2 - 1}. \qquad \text{(SA.1.6)}$$

Define a function $H_3 : [u_2, \infty) \to \mathbb{R}$ as

$$\begin{aligned}
H_3(u) &= \lambda_S\left[V_S(u + \phi/\lambda_S) - V^{gsg}(u|u_1,u_2)\right] - \phi V^{gsg\prime}(u|u_1,u_2) - c \\
&= \lambda_S\left[V_S(u + \phi/\lambda_S) + u + \phi/\lambda_S\right] - \lambda_S\left[u + V^{gsg}(u|u_1,u_2)\right] \\
&\quad - \phi(1 + V^{gsg\prime}(u|u_1,u_2)) - c \\
&= \left(\frac{\lambda_S}{\lambda_G} - 2\right)c - (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u - 1} \\
&\quad + \left(\frac{\lambda_S}{\lambda_G} - 1\right)\left[\lambda_G\Pi - c - \lambda_G(V^{gs}(u_2|u_1) + u_2)\right]e^{\frac{\lambda_G}{\phi}(u_2 - u)} \\
&= (\eta - 1)\,c - (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}\cdot e^{\frac{\lambda_S}{\phi}(u_2 - u)} \\
&\quad + \eta\left[\lambda_G\Pi - c - \lambda_G\left(V^{gs}(u_2|u_1) + u_2\right)\right]e^{\frac{\lambda_G}{\phi}(u_2 - u)} \\
&= (\eta - 1)\,c - (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}\cdot e^{\frac{\lambda_S}{\phi}(u_2 - u)} + \eta\phi\left(V^{gs\prime}(u_2|u_1) + 1\right)e^{\frac{\lambda_G}{\phi}(u_2 - u)}.
\end{aligned}$$

Also note that

$$\begin{aligned}
H_3(u_2) &= \lambda_S\left[V_S(u_2 + \phi/\lambda_S) - V^{gs}(u_2|u_1)\right] - c - \phi V^{gsg\prime}(u_2|u_1,u_2) \\
&= \phi V^{gs\prime}(u_2|u_1) - \phi V^{gsg\prime}(u_2|u_1,u_2) = 0.
\end{aligned}$$

Define $x \equiv e^{\frac{\lambda_G}{\phi}(u_2 - u)}$. Then, $H_3(u)$ can be rewritten as follows:

$$\tilde{H}_3(x) = (\eta - 1)c - (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}x^{\eta+1} + \eta\phi\left(V^{gs\prime}(u_2|u_1) + 1\right)x$$

and $\tilde{H}_3(1) = H_3(u_2) = 0$.

Note that

$$\tilde{H}_3'(x) = -(\eta + 1)(\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}x^{\eta} + \eta\phi\left(V^{gs\prime}(u_2|u_1) + 1\right).$$

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By (SA.1.6), we can derive that

$$\tilde{H}_3'(1) = -(\eta+1)(\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1} + \eta\phi\left(V^{gs\prime}(u_2|u_1) + 1\right) > 0.$$

Also note that

$$\tilde{H}_3''(x) = -(\eta+1)\eta(\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}x^{\eta-1} < 0.$$

Therefore, $\tilde{H}_3'(x) > 0$ for all $0 < x < 1$. Since $\tilde{H}_3(1) = 0$, $\tilde{H}_3(x) < 0$ for all $x \in (0,1)$. Thus, $\lambda_S\left(V_S(u + \phi/\lambda_S) - V^{gsg}(u|u_1,u_2)\right) - \phi V^{gsg\prime}(u|u_1,u_2) - c < 0$ for all $u \geq u_2$. □

SA.1.4 Proofs of Lemmas in Section B.5.1

Proof of Lemma B.10. (a) Note that $\frac{\partial}{\partial R}L_1^a = -\lambda_G(1 + V^{g\prime}(u)) < 0$ by Lemma B.3. Therefore, for a fixed u, L_1^a is maximized at $R = u + \phi/\lambda_G$. Note that by the definition of V^g, $L_1^a(u, u+\phi/\lambda_G) = 0$, thus, $L_1^a(u,R) \leq 0$ for all $R \geq u + \phi/\lambda_G$.

(b) Define $x \equiv e^{-\frac{\lambda_G}{\phi}u}$ and $y \equiv w - u$. Note that $u \geq 0$, $w \geq u + \phi/\lambda_S$ and $2\lambda_G \geq \lambda_S > \lambda_G$ imply that $1 \geq x > 0$, $y \geq \phi/\lambda_S$ and $1 \geq \eta > 0$. Then, L_1^b can be rewritten as follows:

$$\tilde{L}_1^b(x,y) \equiv -\lambda_S\left[\left(\Pi - \frac{c}{\lambda_S}\right)e^{-\frac{\lambda_S}{\phi}y}\cdot x^\eta - \left(1 - \frac{\lambda_G y}{\phi}\right)\left(\Pi - \frac{c}{\lambda_G}\right)\right]x + (\eta-1)c.$$

By $\Pi > c/\lambda_G > c/\lambda_S$, $\eta > 0$ and $x > 0$,

$$\frac{\partial^2 \tilde{L}_1^b}{\partial x^2} = -\lambda_S\left(\Pi - \frac{c}{\lambda_S}\right)e^{-\frac{\lambda_S}{\phi}y}(\eta+1)\eta x^{\eta-1} < 0,$$

thus, $\tilde{L}_1^b$ is strictly concave in x.

By differentiating $\tilde{L}_1^b$ once by x,

$$\frac{\partial \tilde{L}_1^b}{\partial x} = -\lambda_S\left[(\eta+1)\left(\Pi - \frac{c}{\lambda_S}\right)e^{-\frac{\lambda_S}{\phi}y}x^\eta - \left(1 - \frac{\lambda_G y}{\phi}\right)\left(\Pi - \frac{c}{\lambda_G}\right)\right].$$

If $y \geq \phi/\lambda_G$ and $x \geq 0$, $\tilde{L}_1^b$ is decreasing in x, thus, $\tilde{L}_1^b$ is maximized at $x = 0$. Then, for all $1 \geq x > 0$ and $y \geq \phi/\lambda_G$, the following inequalities hold:

$$\tilde{L}_1^b(x,y) \leq \tilde{L}_1^b(0,y) = (\eta-1)c \leq 0,$$

thus, $L_1^b(u,w) \leq 0$ for all $u \geq 0$ and $w \geq u + \phi/\lambda_G$.

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When $\phi/\lambda_G > y \geq \phi/\lambda_S$ and y is fixed, since $\tilde{L}_1^b(x,y)$ is concave in x, $\tilde{L}_1^b(\cdot,y)$ is maximized at

$$x^*(y) \equiv \left[\frac{(\lambda_G\Pi - c)\,(1-\lambda_G y/\phi)\,e^{\frac{\lambda_S}{\phi}y}}{(\lambda_S\Pi - c)}\right]^{\frac{1}{\eta}}.$$

Define $g(y) \equiv (1-\lambda_G y/\phi)\,e^{(\lambda_S/\phi)y}$. Then, differentiating $g(y)$ gives

$$\begin{aligned} g'(y) =& -\frac{\lambda_G}{\phi}e^{\frac{\lambda_S}{\phi}y} + \frac{\lambda_S}{\phi}\left(1-\frac{\lambda_G y}{\phi}\right)e^{\frac{\lambda_S}{\phi}y} \\ =& \frac{\lambda_G\lambda_S}{\phi}e^{\frac{\lambda_S}{\phi}y}\left(-\frac{1}{\lambda_S}+\frac{1}{\lambda_G}-\frac{y}{\phi}\right). \end{aligned}$$

Note that since $y \geq \phi/\lambda_S$ and $2\lambda_G \geq \lambda_S$, $g(y)$ is decreasing in y, hence, $x^*(y)$ is also decreasing in y.

Now, restrict attention to $1 \geq x > 0$. If $x^*(y) < 1$, the maximum value of $\tilde{L}_1^b(\cdot,y)$ is

$$\begin{aligned} \tilde{L}_1^b(x^*(y),y) =& \eta\lambda_S\left(\Pi - \frac{c}{\lambda_S}\right)e^{-\frac{\lambda_S}{\phi}y}x^*(y)^{\eta+1} + (\eta-1)c \\ =& \eta\,(\lambda_S\Pi - c)\left[\frac{\lambda_G\Pi - c}{\lambda_S\Pi - c}\left(1-\frac{\lambda_G y}{\phi}\right)e^{\frac{\lambda_G y}{\phi}}\right]^{\frac{1+\eta}{\eta}} + (\eta-1)c \end{aligned}$$

Note that $(1-\lambda_G y/\phi)e^{\lambda_G y/\phi}$ is decreasing in y,[31] thus, $\tilde{L}_1^b(x^*(y),y)$ is also decreasing in y.

If $x^*(y) \geq 1$, since $\dfrac{\partial \tilde{L}_1^b}{\partial x}$ is negative for all $0 \leq x \leq 1$, the maximum value of $\tilde{L}_1^b(\cdot,y)$ is

$$\tilde{L}_1^b(1,y) = -\lambda_S\left[\left(\Pi - \frac{c}{\lambda_S}\right)e^{-\frac{\lambda_S}{\phi}y} - \left(1-\frac{\lambda_G y}{\phi}\right)\left(\Pi - \frac{c}{\lambda_G}\right)\right] + (\eta-1)c.$$

Note that $x^*(y) \geq 1$ implies that $0 > -\lambda_G y/\phi \geq (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}y} - (\lambda_G\Pi - c)$. Also note that

$$\frac{\partial\,\tilde{L}_1^b(1,y)}{\partial y} = \frac{\lambda_S}{\phi}\left[(\lambda_S\Pi - c)\,e^{-\frac{\lambda_S}{\phi}y} - (\lambda_G\Pi - c)\right] < 0.$$

Therefore, $\tilde{L}_1^b(1,y)$ is decreasing in y.

When $x^*(\phi/\lambda_S) \leq 1$, $x^*(y) \leq 1$ holds for all $\phi/\lambda_G > y \geq \phi/\lambda_S$ since $x^*(y)$ is decreasing in y. Then,

$$\tilde{L}_1^b(x,y) \leq \tilde{L}_1^b(x^*(y),y) \leq \tilde{L}_1^b\left(x^*\left(\phi/\lambda_S\right),\phi/\lambda_S\right)$$

[31] Differentiating the term by y gives $-(\lambda_G^2 y/\phi^2)e^{\lambda_G y/\phi} < 0$.

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since $x^*(y)$ maximizes $\tilde{L}_1^b(x,y)$ and $\tilde{L}_1^b(x^*(y),y)$ is decreasing in y.

When $x^*(\phi/\lambda_S) > 1$, there exists $y^* \in (\phi/\lambda_S, \phi/\lambda_G)$ such that $x^*(y^*) = 1$ since $x^*(\phi/\lambda_G) = 0$. Then, $x^*(y) < 1$ for $y > y^*$ and $x^*(y) > 1$ for $y < y^*$. When $y < y^*$, by using the decreasingness of $\tilde{L}_1^b(1,y)$ for $x^*(y) > 1$,

$$\tilde{L}_1^b(x,y) \le \tilde{L}_1^b(1,y) \le \tilde{L}_1^b(1,\phi/\lambda_S).$$

When $y > y^*$,

$$\tilde{L}_1^b(x,y) \le \tilde{L}_1^b(x^*(y),y) \le \tilde{L}_1^b(x^*(y^*),y^*) = \tilde{L}_1^b(1,y^*) \le \tilde{L}_1^b(1,\phi/\lambda_S).$$

By combining the above results, we can show that

$$\max_{\substack{1\ge x>0,\\ \phi/\lambda_G > y \ge \phi/\lambda_S}} \tilde{L}_1^b(x,y) = \begin{cases} \tilde{L}_1^b\left(x^*\left(\phi/\lambda_S\right),\ \phi/\lambda_S\right) & \text{if } \ x^*\left(\phi/\lambda_S\right) \le 1, \\ \tilde{L}_1^b\left(1,\ \phi/\lambda_S\right) & \text{if } \ x^*\left(\phi/\lambda_S\right) > 1. \end{cases}$$

Note that

$$\tilde{L}_1^b(x,\phi/\lambda_S) = L_1^b(u, u+\phi/\lambda_S) = \phi V^{gs\prime}(u|u) - \phi V^{g\prime}(u)$$

where $u = -\frac{\phi}{\lambda_G}\log x$. Then, by Lemma B.5, if $c/\lambda_G < \Pi \le \Pi_M(\eta)$, $\tilde{L}_1^b(x,\phi/\lambda_S) \le 0$ for all $x \in (0,1]$. Therefore, if $c/\lambda_G < \Pi \le \Pi_M(\eta)$, for all $x \in (0,1]$, $y \ge \phi/\lambda_S$,

$$\tilde{L}_1^b(x,y) \le \tilde{L}_1^b(x^*(\phi/\lambda_S)\wedge 1, \phi/\lambda_S) \le 0,$$

thus, $L_1^b(u,w) \le 0$ for all $u \ge 0$ and $u + \phi/\lambda_G > w \ge u + \phi/\lambda_S$.

(c) By Lemma B.4, if $\Pi_S(\eta) \ge \Pi$,

$$\tilde{L}_1^b(1,\phi/\lambda_S) = \phi V^{gs\prime}(u|u) - \phi V^{g\prime}(u) < 0.$$

In the previous case, we show that $\frac{\partial}{\partial y}\tilde{L}_1^b(1,y) < 0$, thus $\tilde{L}_1^b(1,y) < 0$ for all $y \ge \phi/\lambda_S$.

On the other hand, by Lemma B.5, if $\Pi > \Pi_M(\eta)$, there exists $u_1 > 0$ such that $V^{g\prime}(u_1) = V^{gs\prime}(u_1|u_1)$ and $V^{g\prime\prime}(u_1) < V^{gs\prime\prime}(u_1|u_1)$. Let $x_1 = e^{-\frac{\lambda_S}{\phi}u_1}$. Then,

$$\tilde{L}_1^b(x_1,\phi/\lambda_S) = L_1^b(u_1,\phi/\lambda_S) = \phi V^{gs\prime}(u_1|u_1) - \phi V^{g\prime}(u_1) = 0.$$

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Then, we can derive that

$$-(\lambda_S\Pi - c)e^{-1}x_1^{\eta+1} + \eta(\lambda_G\Pi - c)x_1 + (\eta-1)c = 0.$$

Note that

$$\begin{aligned}
\frac{\partial \tilde{L}_1^b}{\partial y}(x_1,y) =& \frac{\lambda_S}{\phi}\left[(\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}y}x_1^{\eta} - (\lambda_G\Pi - c)\right]x_1 \\
=& \frac{\lambda_S}{\phi}\left[(\lambda_S\Pi - c)x_1^{\eta}e^{-1}\cdot e^{-\frac{\lambda_S}{\phi}\left(y-\frac{\phi}{\lambda_S}\right)} - (\lambda_G\Pi - c)\right]x_1 \\
=& \frac{\lambda_S}{\phi}\left[(\eta(\lambda_G\Pi - c)x_1 + (\eta-1)c)\cdot e^{-\frac{\lambda_S}{\phi}\left(y-\frac{\phi}{\lambda_S}\right)} - (\lambda_G\Pi - c)x_1\right] \\
=& \frac{\lambda_S}{\phi}\left[(\lambda_G\Pi - c)\cdot\left(\eta e^{-\frac{\lambda_S}{\phi}\left(y-\frac{\phi}{\lambda_S}\right)} - 1\right)x_1 + (\eta-1)c\cdot e^{-\frac{\lambda_S}{\phi}\left(y-\frac{\phi}{\lambda_S}\right)}\right].
\end{aligned}$$

Since $\eta \le 1$, $y \ge \phi/\lambda_S$ and $\lambda_G\Pi > c$, $\dfrac{\partial \tilde{L}_1^b}{\partial y}(x_1,y) < 0$. Therefore, $\tilde{L}_1^b(x_1,y) < 0$ for all $y \ge \phi/\lambda_S$.

In the previous case, we show that $\tilde{L}_1^b$ is strictly concave, thus, for all $x_1 \le x \le 1$ and $y \ge \phi/\lambda_S$,

$$\tilde{L}_1^b(x,y) \le \frac{x-x_1}{1-x_1}\,\tilde{L}_1^b(1,y) + \frac{1-x}{1-x_1}\,\tilde{L}_1^b(x_1,y) \le 0.$$

Therefore, $L_1^b(u,w) \le 0$ for all $u \in [0,u_1]$ and $w \ge u + \phi/\lambda_S$.

□

Proof of Lemma B.11. (a) Note that $\frac{\partial}{\partial w}L_2^b = \lambda_S(V_S'(w) - V^{gs\prime}(u|u_1))$ and $\frac{\partial^2}{\partial w^2}L_2^b = \lambda_S V_S''(w) < 0$. By differentiating (B.4) once, we can derive that

$$\phi V^{gs\prime\prime}(u|u_1) = \lambda_S\left(V_S'\left(u + \frac{\phi}{\lambda_S}\right) - V^{gs\prime}(u|u_1)\right).$$

By (b) of Lemma B.3, $V^{gs\prime\prime}(u|u_1) < 0$. Thus, the following inequality holds:

$$0 > \frac{\partial}{\partial w}L_2^b(u, u+\phi/\lambda_S|u_1) = \lambda_S\left(V_S'\left(u + \frac{\phi}{\lambda_S}\right) - V^{gs\prime}(u|u_1)\right).$$

Then, $L_2^b(u,w|u_1)$ subject to $w \ge u + \phi/\lambda_S$ is maximized at $w = u + \phi/\lambda_S$ for a given u. Also note that $L_2^b(u, u+\phi/\lambda_S|u_1) = 0$ holds by (B.4). Therefore, $L_2^b(u,w|u_1) \le 0$ for all $u \ge u_1$ and $w \ge u + \phi/\lambda_S$.

(b) Note that if $V^{g\prime}(u_1) \le V^{gs\prime}(u_1|u_1)$, $\frac{\partial}{\partial R}L_2^a = -\lambda_G(1 + V^{gs\prime}(u|u_1)) < 0$ by Lemma B.3.

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Therefore, for all $u > u_1$ and $R \geq u + \phi/\lambda_G$,

$$L_2^a(u, R|u_1) \leq L_2^a(u, u + \phi/\lambda_G|u_1).$$

Note that

$$L_2^a(u, u + \phi/\lambda_G|u_1) = \phi V^{gsg\prime}(u|u_1, u) - \phi V^{gs\prime}(u|u_1)$$

By (a) of Lemma B.6, when $\eta = 1$, $V^{gsg\prime}(u|u_1, u) < V^{gs\prime}(u|u_1)$ for all $u > u_1$, thus, $L_2^a(u, R|u_1) < 0$ for all $u \geq u_1$ and $R \geq u + \phi/\lambda_G$.

(c) By (b) of Lemma B.6, when $\eta < 1$, there exists $u_2 > u_1$ such that $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$ and $V^{gs\prime\prime}(u_2|u_1) < V^{gsg\prime\prime}(u_2|u_1, u_2)$, thus $L_2^a(u_2, u_2+\phi/\lambda_G|u_1) = \phi V^{gsg\prime}(u_2|u_1, u) - \phi V^{gs\prime}(u_2|u_1) = 0$. Moreover, $V^{gsg\prime}(u|u_1, u) < V^{gs\prime}(u|u_1)$ for all $u \in (u_1, u_2)$, thus, $L_2^a(u, R|u_1) < 0$ for all $u \in (u_1, u_2)$ and $R \geq u + \phi/\lambda_G$.

□

Proof of Lemma B.12. (a) Note that $\frac{\partial}{\partial R} L_3^a(u, R) = -\lambda_G \left(1 + V^{gsg\prime}(u|u_1, u_2)\right)$. By (c) of Lemma B.3, $V^{gsg\prime}(u|u_1, u_2) > -1$, thus $\frac{\partial}{\partial R} L_3^a(u, R)$ and $L_3^a(u, R) \leq L_3^a(u, u + \phi/\lambda_G)$ for all $u \geq u_2$ and $R \geq u + \phi/\lambda_G$.

By (B.2),

$$L_3^a\left(u, u + \frac{\phi}{\lambda_G} \mid u_1, u_2\right) = \lambda_G(\Pi - u - \phi/\lambda_G - V^{gsg}(u|u_1, u_2)) - c - \phi V^{gsg\prime}(u|u_1, u_2) = 0.$$

Therefore, $L_3^a(u, R) \leq 0$ for all $u \geq u_2$ and $R \geq u + \phi/\lambda_G$.

(b) By differentiating L_3^b by w, we have

$$\begin{aligned}
\frac{\partial}{\partial w} L_3^b(u, w|u_1, u_2) &= \lambda_S \left[V_S'(w) - V^{gsg\prime}(u|u_1, u_2)\right] \\
&= \lambda_S \left[\frac{\lambda_S}{\phi}\left(\Pi - \frac{c}{\lambda_S}\right) e^{-\frac{\lambda_S}{\phi} w} - \frac{1}{\phi}\left(-c + \lambda_G(\Pi - u - V^{gsg}(u|u_1, u_2))\right)\right] \\
&= \frac{\lambda_S}{\phi}\left[(\lambda_S \Pi - c) e^{-\frac{\lambda_S}{\phi} w} - \lambda_G \left(\Pi - \frac{c}{\lambda_G} - (V^{gs}(u_2|u_1) + u_2)\right) e^{\frac{\lambda_G}{\phi}(u_2 - u)}\right].
\end{aligned}$$

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By $w \geq u + \phi/\lambda_S$,

$$\begin{aligned}\frac{\partial}{\partial w} L_3^b(u, w|u_1, u_2) <& \frac{\lambda_S}{\phi}\left[(\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u - 1} - \lambda_G\left(\Pi - \frac{c}{\lambda_G} - (V^{gs}(u_2|u_1) + u_2)\right)e^{\frac{\lambda_G}{\phi}(u_2 - u)}\right] \\ =& \frac{\lambda_S}{\phi}e^{\frac{\lambda_G}{\phi}(u_2-u)}\Bigg[(\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}e^{\frac{\lambda_S - \lambda_G}{\phi}(u_2 - u)} \\ & - \lambda_G\left(\Pi - \frac{c}{\lambda_G} - (V^{gs}(u_2|u_1) + u_2)\right)\Bigg].\end{aligned}$$

Since $u \geq u_2$ and $\lambda_S > \lambda_G$, we have

$$\frac{\partial}{\partial w} L_3^b(u, w|u_1, u_2) < \frac{\lambda_S}{\phi}e^{\frac{\lambda_G}{\phi}(u_2-u)}\left[(\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1} - \lambda_G\left(\Pi - \frac{c}{\lambda_G} - (V^{gs}(u_2|u_1) + u_2)\right)\right].$$

By (B.2), (B.4), $V^{gs}(u_2|u_1) = V^{gsg}(u_2|u_1, u_2)$ and $V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1, u_2)$, we can derive that

$$(\lambda_S \Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1} = (\lambda_S - \lambda_G)(\Pi - (V^{gs}(u_2|u_1) + u_2)) - c.$$

By plugging this into the above inequality, we have

$$\frac{\partial}{\partial w} L_3^b(u, w|u_1, u_2) < \frac{\lambda_S}{\phi}e^{\frac{\lambda_G}{\phi}(u_2-u)}(\lambda_S - 2\lambda_G)\left(\Pi - (V^{gs}(u_2|u_1) + u_2)\right),$$

thus, since $2\lambda_G \geq \lambda_S$ and $\Pi - c/\lambda_G > V^{gs}(u_2|u_1) + u_2$ ((b) of Lemma B.3), $L_3^b(u, w|u_1, u_2)$ is decreasing in w. Therefore, $L_3^b(u, w|u_1, u_2) \leq L_3^b(u, u + \phi/\lambda_S \mid u_1, u_2)$ for all $w \geq u + \phi/\lambda_S$.

Note that $L_3^b(u, u + \phi/\lambda_S|u_1, u_2) = H_3(u)$ and $H_3(u) \leq 0$ for all $u \geq u_2$ by Lemma B.7. Therefore, $L_3^b(u, w|u_1, u_2) \leq 0$ for all $u \geq u_2$ and $w \geq u + \phi/\lambda_S$.

□

SA.1.5 Proofs of Lemmas in Section B.6.2

Proof of Lemma B.14. If Π is greater than or equal to $\Pi_S(\eta)$ and η is greater than $1/(e-1)$, u_1 is equal to zero by Corollary B.8 and B.9. Note that

$$\Pi_S(\eta) = \frac{e-1}{(e-1)\eta - 1} \cdot \frac{c}{\lambda_G} \geq \frac{e-1}{e-2} \cdot \frac{c}{\lambda_G} > \frac{2c}{\lambda_G} \geq \frac{c+\phi}{\lambda_G} = \Pi_F.$$

Then, the project is feasible and $\bar{u}$ is greater than 0, thus, $\bar{u}$ is always greater than u_1.

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Now suppose that $\Pi_G(\eta) < \Pi < \Pi_S(\eta)$. Since V is strictly concave, $u_1 < \bar{u}$ is equivalent to $0 < V'(u_1) = V^{g\prime}(u_1)$. Note that $0 < V^{g\prime}(u_1)$ is equivalent to:

$$\frac{\phi}{\lambda_G \Pi - c} < e^{-\frac{\lambda_G}{\phi} u_1}.$$

Recall that u_1 is the solution of

$$-H_1(u) = ((\eta+1)\lambda_G \Pi - c)e^{-1} e^{-\frac{(\eta+1)\lambda_G}{\phi} u} - \eta(\lambda_G \Pi - c)e^{-\frac{\lambda_G}{\phi} u} + (1-\eta)c = 0.$$ [32]

Define $x_1 \equiv e^{-\frac{\lambda_G}{\phi} u_1}$. Then, x_1 is the solution of

$$\tilde{H}_1(x) = ((\eta+1)\lambda_G \Pi - c)e^{-1} x^{\eta+1} - \eta(\lambda_G \Pi - c)x + (1-\eta)c = 0,$$

and we need to identify a condition for $x_1 > \phi/(\lambda_G \Pi - c)$.

Note that

$$\frac{\partial^2 \tilde{H}_1}{\partial x^2}(x) = (\eta+1)\eta((\eta+1)\lambda_G \Pi - c)e^{-1} x^{\eta-1} > 0,$$

thus there exists $\underline{x} \in (0,1)$ that minimizes $\tilde{H}_1$. Also note that $\Pi_S(\eta) > \Pi > \Pi_M(\eta)$ imply that $\tilde{H}_1(1) > 0 > \tilde{H}_1(\underline{x})$ and $x_1 \in (\underline{x}, 1)$.

There are two possible cases that satisfy $x_1 > \phi/(\lambda_G \Pi - c)$: (i) $\underline{x} \geq \phi/(\lambda_G \Pi - c)$; (ii) $\phi/(\lambda_G \Pi - c) > \underline{x}$ and $\tilde{H}_1(\phi/(\lambda_G \Pi - c)) < 0$.

The first case is equivalent to $\tilde{H}_1'(\phi/(\lambda_G \Pi - c)) < 0$. By algebra, we can show that it is equivalent to

$$\frac{(\eta+1)\lambda_G \Pi - c}{(\lambda_G \Pi - c)^{\eta+1}} < \frac{\eta e}{\eta+1} \phi^{-\eta-1}. \tag{SA.1.7}$$

The second case is equivalent to $\tilde{H}_1'(\phi/(\lambda_G \Pi - c)) \geq 0$ and $\tilde{H}_1(\phi/(\lambda_G \Pi - c)) < 0$. By algebra, we can show that it is equivalent to

$$\frac{\eta e}{\eta+1} \phi^{-\eta} \leq \frac{(\eta+1)\lambda_G \Pi - c}{(\lambda_G \Pi - c)^{\eta+1}} < \left(\eta(c+\phi) - c\right) e \phi^{-\eta}. \tag{SA.1.8}$$

Lastly, by the proof of Lemma B.5, we can show that $\Pi > \Pi_M(\eta)$ is equivalent to

$$\frac{(\eta+1)\lambda_G \Pi - c}{(\lambda_G \Pi - c)^{\eta+1}} < \left(\frac{\eta^2}{1-\eta^2}\right)^{\eta} \frac{\eta e}{1+\eta} c^{-\eta}. \tag{SA.1.9}$$

Now I compare the above three conditions. When $\eta > \sqrt{c/(c+\phi)}$, by simple algebra,

[32] See the proof of Lemma B.5 for the definition of H_1.

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we can show that

$$\frac{\eta e}{\eta+1}\phi^{-\eta} < (\eta(c+\phi)-c)e\phi^{-\eta-1} \quad \& \quad (\eta(c+\phi)-c)\, e\phi^{-\eta} < (\eta(c+\phi)-c)e\phi^{-\eta-1}.$$

Therefore, the inequality

$$\frac{(\eta+1)\lambda_G\Pi - c}{(\lambda_G\Pi - c)^{\eta+1}} < (\eta(c+\phi)-c)\, e\phi^{-\eta}$$

imply that (SA.1.7), (SA.1.8) and (SA.1.9). Define $\Pi_G(\eta)$ be the value of Π that makes both sides of the above inequality equal. Then, $\Pi_G(\eta) > \Pi_M(\eta)$ since $\Pi < \Pi_M(\eta)$ implies $\Pi < \Pi_G(\eta)$. Therefore, there exists $\Pi_G(\eta) > \Pi_M(\eta)$ such that $u_1 < \bar{u}$ if and only if $\Pi > \Pi_G(\eta)$.

When $\eta \leq \sqrt{c/(c+\phi)}$, by simple algebra, we can show that

$$\frac{\eta e}{\eta+1}\phi^{-\eta} \geq (\eta(c+\phi)-c)e\phi^{-\eta-1} \quad \& \quad \frac{\eta e}{\eta+1}\phi^{-\eta} \geq \left(\frac{\eta^2}{1-\eta^2}\right)^{\eta}\frac{\eta e}{1+\eta}c^{-\eta}.$$

Therefore, (SA.1.8) cannot hold in this case and (SA.1.9) implies (SA.1.7). It means that $\Pi > \Pi_M(\eta)$ implies $\tilde{H}_1'(\phi/(\lambda_G\Pi - c)) < 0$. Moreover, $\Pi > \Pi_M(\eta)$ is necessary for the existence of u_1. Hence, $u_1 < \bar{u}$ holds if and only if $\Pi > \Pi_M(\eta)$. □

Proof of Lemma B.15. Since V is strictly concave, $u_2 < \bar{u}$ is equivalent to $0 < V'(u_2)$. By (b) of Proposition B.2, we have $V'(u_2) = V^{gs\prime}(u_2|u_1) = V^{gsg\prime}(u_2|u_1,u_2)$. By (B.2) and $V^{gsg}(u_2|u_1,u_2) = V^{gs}(u_2|u_1)$, $0 < V^{gsg\prime}(u_2|u_1,u_2)$ is equivalent to:

$$\lambda_G(u_2 + V^{gs}(u_2|u_1)) < \lambda_G\Pi - c - \phi. \tag{SA.1.10}$$

Also note that $V^{gsg\prime}(u_2|u_1,u_2) = \phi V^{gs\prime}(u_2|u_1)$ and $V^{gsg}(u_2|u_1,u_2) = V^{gs}(u_2|u_1)$ imply that

$$\lambda_G(\Pi - u_2 - V^{gs}(u_2|u_1)) = \lambda_S\left(V_S\left(u_2 + \frac{\phi}{\lambda_S}\right) + u_2 + \frac{\phi}{\lambda_S}\right) - \lambda_S\left(V^{gs}(u_2|u_1) + u_2\right)$$

by (B.2) and (B.4). By plugging (4.1) into the above equation, we can derive that

$$\begin{aligned}
(\lambda_S - \lambda_G)(V^{gs}(u_2|u_1) + u_2) =& \lambda_S\left(\Pi - \frac{c}{\lambda_S}\right)\left(1 - e^{-\frac{\lambda_S}{\phi}u_2 - 1}\right) - \lambda_G\Pi \\
\iff \qquad \eta\lambda_G(V^{gs}(u_2|u_1) + u_2) =& \eta\lambda_G\Pi - c - (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_2 - 1}.
\end{aligned}$$

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Then, by plugging this into (SA.1.10), $0 < V^{gsg\prime}(u_2|u_1,u_2)$ is equivalent to

$$(\eta-1)c+\eta\phi < (\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_2-1}.$$

Since $\Pi > c/\lambda_G > c/\lambda_S$, the right hand side of the above inequality is always greater than 0. Since it is assumed that $\eta > \dfrac{c}{c+\phi}$, the left hand side of the above inequality is always less than 0. Therefore, the above inequality always holds, i.e., u_2 is less than $\bar{u}$.

□

Proof of Lemma B.16. By following the proof of Lemma B.15, $u_2 \geq \bar{u}$ is equivalent to

$$\hat{y} \equiv \frac{(\eta-1)c+\eta\phi}{(\lambda_S\Pi - c)e^{-\frac{\lambda_S}{\phi}u_1-1}} \geq e^{\frac{\lambda_S}{\phi}(u_1-u_2)} \tag{SA.1.11}$$

By the proof of Lemma B.6, $x_2 \equiv e^{\frac{\lambda_S}{\phi}(u_1-u_2)}$ is the solution, which is not equal to 1, of $\tilde{H}_2(x) = 0$.[33]. Since $u_2 \geq u_1$, if $\hat{y} \geq 1$, the above inequality holds, thus, I restrict attention to the case of $\hat{y} < 1$. Observe that the inequality $\tilde{H}_2(\hat{y}) \leq 0$ implies (SA.1.11) because $\tilde{H}_2$ is strictly convex in x and $\tilde{H}_2(1) \leq 0$.

Note that $\tilde{H}_2(x)$ can be rewritten as follows:

$$\tilde{H}_2(x) = \frac{1-\eta}{1+\eta}c - H_1(u_1)x + \left[-\frac{1-\eta}{1+\eta}c + \frac{\eta}{1+\eta}(\lambda_S\Pi - c)e^{-\frac{\lambda_G}{\phi}u_1-1}\log x\right]x$$

where H_1 is a function defined in (SA.1.3). Also note that $H_1(u_1) = \phi V^{gs\prime\prime}(u_1|u_1) - \phi V^{g\prime}(u_1) \geq 0$.

By plugging the definition of $\hat{y}$ into the above equation, we can derive that

$$\tilde{H}_2(\hat{y}) = \frac{1-\eta}{1+\eta}c(1-\hat{y}) - H_1(u_1)\hat{y} + \frac{\eta}{1+\eta}\left((\eta-1)c+\eta\phi\right)\log\hat{y}.$$

Now define a new function G as follows:

$$G(y) \equiv \frac{1-\eta}{1+\eta}c(1-y) - H_1(u_1)y + \frac{\eta}{1+\eta}\left((\eta-1)c+\eta\phi\right)\log y,$$

and it is enough to show that $G(y) \leq 0$ for all $y < 1$.

Note that

$$G''(y) = -\frac{\eta}{1+\eta}\left(\frac{(\eta-1)c+\eta\phi}{y^2}\right) < 0$$

[33]The function $\tilde{H}_2$ is defined in (SA.1.4)

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from $\eta \geq \sqrt{c/(c+\phi)} > c/(c+\phi)$. Also note that

$$G'(1) = -H_1(u_1) + \frac{1}{1+\eta}\left((\eta^2-1)c + \eta^2\phi\right) < 0.$$

from $\eta \geq \sqrt{c/(c+\phi)}$ and $H_1(u_1) \geq 0$. Lastly, note that $G(1) = -H_1(u_1) \leq 0$. Therefore, for all $y < 1$, $G(y) \leq G(1) + G'(1)(1-y) \leq 0$. Therefore, $\tilde{H}_2(\hat{y}) \leq 0$ and $u_2 \geq \bar{u}$. □

SA.2 Asymmetric Arrival Rates for Subprojects

This section is largely based on the results from the previous version of this paper. In this material, I omit technical details such as the proof of verification lemma but focus on the derivation of Theorem 4.

SA.2.1 Benchmark Value Functions

Under the assumption that the arrival rates for the subprojects are no longer symmetric, the agent's HJB equation (PK) can be rewritten as follows:

$$0 = \sup_{\substack{a_t, b_t \geq 0, \\ a_t + b_t \leq 1}} \dot{u}_t + \phi(1 - a_t - b_t) + (R_t - u_t)\lambda_G a_t + (u_S^t - u_t)\lambda_{S,1} b_t. \tag{PK$_A$}$$

Then, we can observe that the sequential approach can be induced with a minimum incentive by setting $u_S^t = u_t + \phi/\lambda_{S,1}$.

Also note that the principal's value function given that the subproject, V_S^A, is already completed can be written as follows:

$$V_S^A(u_S) = \left(\Pi - \frac{c}{\lambda_{S,2}} - u_S\right) - \left(\Pi - \frac{c}{\lambda_{S,2}}\right) e^{-\frac{\lambda_{S,2}}{\phi} u_S}. \tag{SA.2.1}$$

Let the principal's value function prior to the completion of the subproject be V^A. The HJB equation for $V^A(u)$ is

$$0 = \max_{\substack{R \geq 0,\ u_S \geq 0, \\ a,b \geq 0,\ 1 \geq a+b}} -c + (\Pi - R - V^A(u))\lambda_G a + (V_S^A(u_S) - V^A(u))\lambda_{S,1} b + V^{A\prime}(u)\ \dot{u}. \tag{HJB$_A$}$$

Then, the principal's problem is to solve (HJB$_A$) subject to (PK$_A$). We can derive two benchmark value functions as follows:[34]

[34] Since I focus on the case of no efficiency loss from splitting the project, a benchmark value function

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1. When the agent's promised utility is lower than a switching point u_1, the principal would recommend the direct approach and the benchmark value function, V_A^g, would be the same as (B.3):

$$V_A^g(u) = \left(\Pi - \frac{c}{\lambda_G}\right)\left(1 - e^{-\frac{\lambda_G}{\phi}u}\right) - u.$$

2. When the agent's promised utility is higher than a switching point u_1, the principal would recommend the sequential approach. By following the similar steps in Section B.3, the benchmark value function, V_A^{gs}, is derived as follows:

$$\begin{aligned} V_A^{gs}(u|u_1) = &\left(\Pi - \frac{c}{\lambda_{S,1}} - \frac{c}{\lambda_{S,2}}\right)\left(1 - e^{\frac{\lambda_{S,1}}{\phi}(u_1-u)}\right) + (V_A^g(u_1) + u_1)e^{\frac{\lambda_{S,1}}{\phi}(u_1-u)} \\ &- \left(\Pi - \frac{c}{\lambda_{S,2}}\right)\frac{e^{\frac{\lambda_{S,2}}{\phi}(u_1-u)} - e^{\frac{\lambda_{S,1}}{\phi}(u_1-u)}}{\kappa - 1}e^{-\frac{\lambda_{S,1}}{\phi}u_1 - \kappa} - u.\text{[35]} \end{aligned} \tag{SA.2.2}$$

SA.2.2 Value Function Derivation

To derive the value function, we need to characterize a threshold of Π that determines the recommended approach at the deadline as in Lemma B.4.

Lemma SA.2.1. *Suppose that there is no efficiency loss from splitting the project and $\kappa = \lambda_{S,2}/\lambda_{S,1}$. The inequality $V_A^{g\prime}(0) > V_A^{gs\prime}(0|0)$ holds if and only if*

$$\Pi < \Pi_S^A(\kappa) \equiv \frac{1 - e^{-\kappa}}{1 - (\kappa + 1)e^{-\kappa}} \cdot \frac{c}{\lambda_G}.\text{[36]} \tag{SA.2.3}$$

In addition, $\Pi_S^A(\kappa)$ is decreasing in κ.

involving two switches such as (B.6) is not needed.

[35] Note that

$$\lim_{\kappa \to 1} \frac{e^{\frac{\lambda_{S,2}}{\phi}(u_1-u)} - e^{\frac{\lambda_{S,1}}{\phi}(u_1-u)}}{\kappa - 1} = \frac{(u - u_1)e^{\frac{\lambda_{S,2}}{\phi}(u_1-u)}}{\phi}.$$

Hence, when $\kappa = 1$, i.e., $\lambda_{S,1} = \lambda_{S,2}$, (SA.2.2) is equivalent to (B.5).

[36] Note that $1 - e^{-\kappa} > 1 - (\kappa + 1)e^{-\kappa} > 0$ for all $\kappa > 0$, thus, $\Pi_S^A(\kappa) > c/\lambda_G$.

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Proof of Lemma SA.2.1. Note that

$$
\begin{aligned}
V_A^{g\prime}(0) &= \frac{1}{\phi}\left(\lambda_G \Pi - \phi - c\right), \\
V_A^{gs\prime}(0|0) &= \frac{1}{\phi}\left(\lambda_{S,1} V_S^A\left(\frac{\phi}{\lambda_{S,1}}\right) - c\right) \\
&= \frac{1}{\phi}\left(\left(1 + \frac{1}{\kappa}\right)(1 - e^{-\kappa})\lambda_G \Pi - \frac{1}{\kappa}(1 - e^{-\kappa})c - \phi - c\right).
\end{aligned}
$$

Therefore, $V_A^{g\prime}(0) > V_A^{gs\prime}(0|0)$ is equivalent to

$$
(1 - e^{-\kappa})c > (1 - (\kappa + 1)e^{-\kappa})\lambda_G \Pi.
$$

Since $1 - (\kappa + 1)e^{-\kappa} > 0$ for all $\kappa > 0$, the above inequality is equivalent to $\Pi_S^A(\kappa) > \Pi$.

By differentiating Π_S^A with respect ot κ, we have

$$
\Pi_S^{A\prime}(\kappa) = -\frac{e^{\kappa}\left(e^{-\kappa} + \kappa - 1\right)}{\left(1 - (\kappa + 1)e^{-\kappa}\right)^2} \cdot \frac{c}{\lambda_G}.
$$

Since $e^{-\kappa} > -\kappa + 1$, we have $\Pi_S^{A\prime}(\kappa) < 0$. Thus, Π_S^A is decreasing in κ. □

Proposition SA.2.1. *Assume that both technologies are equally efficient and $\lambda_{S,2}/\lambda_{S,1}$ is equal to κ. Then, the following statements hold:*

(a) When $\Pi \in (c/\lambda_G, \Pi_S^A(\kappa))$, for some $u_1 > 0$, $V_A^{g\prime}(u_1) = V_A^{gs\prime}(u_1|u_1)$ and

$$
V^A(u) = \begin{cases} V_A^g(u) & \text{if } 0 \le u \le u_1, \\ V_A^{gs}(u|u_1) & \text{if } u_1 < u \end{cases} \tag{SA.2.4}
$$

solves (HJB_A) subject to (PK_A).

(b) When $\Pi \ge \Pi_S^A(\kappa)$, $V^A(u) = V_A^{gs}(u|0)$ solves (HJB_A) subject to (PK_A).

SA.2.3 Thresholds

SA.2.3.1 Feasibility (Π_F^A)

Now I derive a threshold that determines the feasibility of the project. By using the same logic in Appendix B.6.1, the project is feasible if and only if $V'(0) > 0$, and it is equivalent to

$$
\max\left[\lambda_G \Pi - \phi,\ \lambda_{S,1} V_S^A(\phi/\lambda_{S,1})\right] > c.
$$

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Note that

$$\begin{aligned}\lambda_{S,1}V_S^A(\phi/\lambda_{S,1}) =& \lambda_{S,1}\left(\Pi - \frac{c}{\lambda_{S,2}}\right)(1-e^{-\kappa}) - \phi \\ =& \left(1+\frac{1}{\kappa}\right)\lambda_G(1-e^{-\kappa})\Pi - \frac{1-e^{-\kappa}}{\kappa}c - \phi.\end{aligned}$$

Therefore, $\lambda_{S,1}V_S^A(\phi/\lambda_{S,1}) > c$ is equivalent to

$$\Pi > \frac{1}{(\kappa+1)\lambda_G}\left[c + \frac{\kappa}{1-e^{-\kappa}}(c+\phi)\right].$$

Then, the project is feasible if and only if $\Pi > \min\left[\frac{c+\phi}{\lambda_G}, \frac{1}{(\kappa+1)\lambda_G}\left[c+\frac{\kappa}{1-e^{-\kappa}}(c+\phi)\right]\right] \equiv \Pi_F^A(\kappa)$.

SA.2.3.2 The Length of the Contract (Π_G^A)

Next, I derive another threshold that determines whether there is a switch in the optimal contract. Let $\bar{u}$ denote the value that maximizes $V^A(u)$. Then, we need to compare $\bar{u}$ with a switch point u_1. The following lemma characterizes the threshold Π_G^A.

Lemma SA.2.2. *Suppose that there is no efficiency loss from splitting the project and $\kappa = \lambda_{S,2}/\lambda_{S,1}$. Then, there exists $\Pi_G^A(\kappa)$ such that $u_1 < \bar{u}$ if and only if $\Pi > \Pi_G^A(\kappa)$.*

Proof of Lemma SA.2.2. I begin by deriving the closed form of u_1. Define a function H_1^A as in (SA.1.3):

$$H_1^A(u) \equiv \phi V_A^{gs\prime}(u|u) - \phi V_A^{g\prime}(u). \tag{SA.2.5}$$

Then, $H_1^A(u)$ can be rewritten as follows:

$$\begin{aligned}H_1^A(u) &= \lambda_{S,1}\left((\Pi - c/\lambda_{S,2})(1-e^{-\frac{\lambda_{S,2}}{\phi}u-\kappa}) - V_A^g(u) - u\right) - \lambda_G(\Pi - u - V_A^g(u)) \\ &= (\lambda_{S,1}\Pi - c/\kappa)\left(1-e^{-\frac{\lambda_{S,2}}{\phi}u-\kappa}\right) - (\lambda_{S,1}-\lambda_G)(u+V_A^g(u)) - \lambda_G\Pi \\ &= (\lambda_{S,1}\Pi - c/\kappa)\left(1-e^{-\frac{\lambda_{S,2}}{\phi}u-\kappa}\right) - (\lambda_{S,1}-\lambda_G)(\Pi - c/\lambda_G)(1-e^{-\frac{\lambda_G}{\phi}u}) - \lambda_G\Pi \\ &= (\lambda_{S,1}/\lambda_G - 1)(\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u} - (1/\kappa - \lambda_{S,1}/\lambda_G + 1)c - (\lambda_{S,1}\Pi - c/\kappa)e^{-\frac{\lambda_{S,1}}{\phi}u-\kappa} \\ &= (\lambda_G\Pi - c)e^{-\frac{\lambda_G}{\phi}u}/\kappa - (\lambda_{S,2}\Pi - c)e^{-\frac{\lambda_{S,1}}{\phi}u-\kappa}/\kappa.\end{aligned}$$

By the definition of u_1, it is the solution of $H_1^A(u) = 0$. Then, by using the above equation,

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we can derive that

$$u_1 = \frac{\phi}{\lambda_G}\left[\frac{1}{\kappa}\log\left(\frac{\lambda_{S,2}\Pi - c}{\lambda_G\Pi - c}\right) - 1\right].$$

The next step is to compare u_1 with $\bar{u}$. Since V^A is strictly concave, $u_1 < \bar{u}$ is equivalent to $0 < V^{A\prime}(u_1) = V_A^{g\prime}(u_1)$. By using the above equation, we can derive that $u_1 = \bar{u}$ is equivalent to

$$\frac{(\lambda_G\Pi - c)^{1+\kappa}}{(1+\kappa)\lambda_G\Pi - c}\left(\frac{e}{\phi}\right)^{\kappa} = 1. \tag{SA.2.6}$$

Note that the left hand side is increasing in Π, is equal to zero when Π is equal to c/λ_G, and diverges as Π goes to infinity. Therefore, there exists a unique solution that satisfies (SA.2.6) and I denote the solution as $\Pi_G^A(\kappa)$. Then, $\Pi > \Pi_G^A(\kappa)$ is equivalent to $\bar{u} > u_1$. $\square$

SA.2.3.3 Comparison of Thresholds

Lemma SA.2.3. *Let κ^* be the positive solution of the equation*

$$0 = \phi + (c+\phi)\kappa - \phi e^{\kappa}. \tag{SA.2.7}$$

Then,

1. *if $\kappa > \kappa^*$,*

$$\Pi_F^A(\kappa) = \frac{1}{(\kappa+1)\lambda_G}\left(c + \frac{\kappa}{1-e^{-\kappa}}(c+\phi)\right) > \max\left[\Pi_G^A(\kappa),\ \Pi_S^A(\kappa)\right],$$

2. *if $\kappa = \kappa^*$,*

$$\Pi_F^A(\kappa^*) = \frac{c+\phi}{\lambda_G} = \frac{1}{(\kappa^*+1)\lambda_G}\left(c + \frac{\kappa^*}{1-e^{-\kappa^*}}(c+\phi)\right) = \Pi_G^A(\kappa^*) = \Pi_S^A(\kappa^*);$$

3. *if $\kappa < \kappa^*$,*

$$\Pi_F^A(\kappa) = \frac{c+\phi}{\lambda_G} < \Pi_G^A(\kappa) < \Pi_S^A(\kappa);$$

4. *as $\kappa \to 0$,*

$$\lim_{\kappa\to 0}\Pi_G^A(\kappa) = \frac{c + \phi\cdot\psi\left(c/\phi\right)}{\lambda_G} \quad \textit{and} \quad \lim_{\kappa\to 0}\Pi_S^A(\kappa) = \infty,$$

   where $\psi : \mathbb{R}_+ \to [1,\infty)$ is the inverse function of $x\log(x)$ for $x \geq 1$.

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Proof of Lemma SA.2.3. Note that for all $\kappa > 0$ and $\lambda_G > 0$,

$$\begin{aligned}
& \frac{c+\phi}{\lambda_G} > \frac{1}{(\kappa+1)\lambda_G}\left(c + \frac{\kappa}{1-e^{-\kappa}}(c+\phi)\right) \\
\Leftrightarrow \quad & (c+\phi)(\kappa+1)(e^{\kappa}-1) > c(e^{\kappa}-1) + (c+\phi)(\kappa+1-e^{\kappa}) \\
\Leftrightarrow \quad & 0 > g(\kappa) \equiv \phi + (c+\phi)\kappa - \phi e^{\kappa}.
\end{aligned}$$

Also note that $g(\kappa)$ is concave in κ, $\lim_{\kappa\to 0} g(\kappa) = 0$, $\lim_{\kappa\to\infty} g(\kappa) = -\infty$ and $\lim_{\kappa\to 0} g'(\kappa) = c > 0$. Then, there exists a unique positive solution to $g(\kappa) = 0$, which is κ^*. Then, $g(\kappa) < 0$ is equivalent to $\kappa > \kappa^*$. Therefore,

$$\Pi_F^A(\kappa) = \begin{cases} \dfrac{c+\phi}{\lambda_G}, & \text{if } \kappa < \kappa^*, \\ \dfrac{c+\phi}{\lambda_G} = \dfrac{1}{(\kappa^*+1)\lambda_G}\left(c + \dfrac{\kappa^*}{1-e^{-\kappa^*}}(c+\phi)\right), & \text{if } \kappa = \kappa^*, \\ \dfrac{1}{(\kappa+1)\lambda_G}\left(c + \dfrac{\kappa}{1-e^{-\kappa}}(c+\phi)\right), & \text{if } \kappa > \kappa^*. \end{cases}$$

For $i \in \{F, W, S\}$, note that $\Pi_i(\kappa)$ can be considered as a unique solution (greater than c/λ) to the equation

$$L(\Pi) = R_i(\Pi|\kappa),$$

where

$$\begin{aligned}
L(\Pi) &= (\kappa+1)\lambda_G\Pi - c, \\
R_F(\Pi|\kappa) &= \begin{cases} \dfrac{\kappa}{1-e^{-\kappa}}(c+\phi) & \text{if } \kappa \geq \kappa^* \\ \kappa(c+\phi)+\phi & \text{if } \kappa \leq \kappa^* \end{cases}, \text{[37]} \\
R_W(\Pi|\kappa) &= \phi \cdot e^{\kappa} \cdot \left(\frac{\lambda_G\Pi - c}{\phi}\right)^{\kappa+1}, \\
R_S(\Pi|\kappa) &= \phi \cdot e^{\kappa} \cdot \left(\frac{\lambda_G\Pi - c}{\phi}\right).
\end{aligned}$$

Note that $L(c/\lambda_G) < R_i(c/\lambda_G|\kappa)$, $\lim_{\Pi\to\infty} L(\Pi) > \lim_{\Pi\to\infty} R_i(\Pi|\kappa)$ and L and $R_i(\cdot|\kappa)$ cross only once for all $i \in \{F, W, S\}$ and $\kappa > 0$.

[37] Note that $\kappa^*(c+\phi)/(1-e^{-\kappa^*}) = \kappa^*(c+\phi)+\phi$ by the definition of κ^*.

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If $R_i(\Pi_i(\kappa)|\kappa) > R_j(\Pi_i(\kappa)|\kappa)$,

$$L(\Pi_i(\kappa)) = R_i(\Pi_i(\kappa)|\kappa) > R_j(\Pi_i(\kappa)|\kappa),$$

and it implies that $\Pi_j(\kappa)$ is smaller than $\Pi_i(\kappa)$. Similarly, $R_i(\Pi_i(\kappa)|\kappa) = R_j(\Pi_i(\kappa)|\kappa)$ implies that $\Pi_j(\kappa)$ is equal to $\Pi_i(\kappa)$ and $R_i(\Pi_i(\kappa)|\kappa) < R_j(\Pi_i(\kappa)|\kappa)$ implies that $\Pi_j(\kappa)$ is greater than $\Pi_i(\kappa)$.

1. When $\kappa > \kappa^*$, to prove that $\Pi_F^A(\kappa) > \max\left[\Pi_G^A(\kappa), \Pi_S^A(\kappa)\right]$, it is enough to show that $R_F(\Pi_F^A(\kappa)|\kappa) < R_W(\Pi_F^A(\kappa)|\kappa)$ and $R_F(\Pi_F^A(\kappa)|\kappa) < R_S(\Pi_F^A(\kappa)|\kappa)$.

   Define $x(\kappa)$ as follows:
$$x(\kappa) = \frac{\kappa}{e^\kappa - 1}\left(\frac{c+\phi}{\phi}\right).$$
   Then, $x(\kappa) < 1$ is equivalent to $g(\kappa) < 0$, i.e., $\kappa > \kappa^*$. Also note that
$$\frac{\lambda_G \Pi_F^A(\kappa) - c}{\phi} = \frac{\kappa}{\kappa+1}\left(\frac{c + e^\kappa \phi}{e^\kappa - 1}\right) = \frac{x(\kappa)+\kappa}{\kappa+1}.$$

   By using the definition of $x(\kappa)$ and the above equation, we can see that
$$\begin{aligned} R_F(\Pi_F^A(\kappa)|\kappa) &= \phi \cdot e^\kappa \cdot x(\kappa), \\ R_W(\Pi_F^A(\kappa)|\kappa) &= \phi \cdot e^\kappa \cdot \left(\frac{x(\kappa)+\kappa}{\kappa+1}\right)^{\kappa+1}, \\ R_S(\Pi_F^A(\kappa)|\kappa) &= \phi \cdot e^\kappa \cdot \left(\frac{x(\kappa)+\kappa}{\kappa+1}\right). \end{aligned} \tag{SA.2.8}$$

   Consider a function $h(x) = \left(\frac{x+\kappa}{1+\kappa}\right)^{\kappa+1}$. Note that $h'(x) = \left(\frac{x+\kappa}{1+\kappa}\right)^{\kappa}$ and $h''(x) = \frac{\kappa}{1+\kappa}\left(\frac{x+\kappa}{1+\kappa}\right)^{\kappa-1} > 0$. Then, $h(x) > h(1) + h'(1)(x-1) = x$ for $x < 1$. Hence, $R_W(\Pi_F^A(\kappa)|\kappa) > R_F(\Pi_F^A(\kappa)|\kappa)$. Also, we can easily see that $\dfrac{x+\kappa}{\kappa+1} > x$ is equivalent to $x < 1$, i.e., $R_S(\Pi_F^A(\kappa)|\kappa) > R_F(\Pi_F^A(\kappa)|\kappa)$.

2. When $\kappa = \kappa^*$, to prove that $\Pi_F^A(\kappa) = \Pi_G^A(\kappa) = \Pi_S^A(\kappa)$, it is enough to show that $R_F(\Pi_F^A(\kappa)|\kappa) = R_W(\Pi_F^A(\kappa)|\kappa) = R_S(\Pi_F^A(\kappa)|\kappa)$.

   Note that $x(\kappa^*) = 1$. Hence, by (SA.2.8), $R_F(\Pi_F^A(\kappa)|\kappa) = R_W(\Pi_F^A(\kappa)|\kappa) = R_S(\Pi_F^A(\kappa)|\kappa)$.

3. When $\kappa < \kappa^*$, to prove that $\Pi_S^A(\kappa) > \Pi_G^A(\kappa) > \Pi_F^A(\kappa)$, it is enough to show that $R_F(\Pi_F^A(\kappa)|\kappa) > R_W(\Pi_F^A(\kappa)|\kappa)$ and $R_W(\Pi_S^A(\kappa)|\kappa) > R_S(\Pi_S^A(\kappa)|\kappa)$.

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In this case, $\Pi_F^A(\kappa) = (c+\phi)/\lambda_G$. Then, by the definition of R_F and R_W, $R_F(\Pi_F^A(\kappa)|\kappa) = \kappa(c+\phi)+\phi$ and $R_W(\Pi_F^A(\kappa)|\kappa) = \phi \cdot e^\kappa$. Since $\kappa < \kappa^*$ is equivalent to $\kappa(c+\phi)+\phi > \phi e^\kappa$, $R_F(\Pi_F^A(\kappa)|\kappa) > R_W(\Pi_F^A(\kappa)|\kappa)$.

Also note that

$$\frac{\lambda_G \Pi_S^A(\kappa) - c}{\phi} = \frac{\frac{1-e^{-\kappa}}{1-(\kappa+1)e^{-\kappa}}c - c}{\phi} = \frac{\kappa \cdot c}{(e^\kappa - (\kappa+1))\phi} > 1.$$

Then, since $R_W(\Pi_S^A(\kappa)|\kappa) = R_S(\Pi_S^A(\kappa)|\kappa) \cdot \left(\frac{\lambda_G \Pi - c}{\phi}\right)^\kappa$, $R_W(\Pi_S^A(\kappa)|\kappa) > R_S(\Pi_S^A(\kappa)|\kappa)$.

4. When $\kappa \to 0$, by L'Hôpital's Rule,

$$\lim_{\kappa \to 0} \Pi_S^A(\kappa) = \lim_{\kappa \to 0} \frac{1 - e^{-\kappa}}{1 - (\kappa+1)e^{-\kappa}} \cdot \frac{c}{\lambda_G} = \lim_{\kappa \to 0} \frac{e^{-\kappa}}{\kappa\ e^{-\kappa}} \cdot \frac{c}{\lambda_G} = \infty.$$

Define $y(\kappa) \equiv \left(\lambda_G \Pi_G^A(\kappa) - c\right)/\phi > 0$. Then, from (SA.2.6), $y(\kappa)$ satisfies the following equations for all $\kappa > 0$:

$$\begin{aligned} & y(\kappa)^{1+\kappa} \cdot e^\kappa = (1+\kappa) y(\kappa) + \frac{c}{\phi}\kappa \\ \Rightarrow \qquad & (1+\kappa)\log[y(\kappa)] + \kappa = \log\left[(1+\kappa)y(\kappa) + \frac{c}{\phi}\kappa\right]. \end{aligned}$$

By differentiating the above equation by κ, we have

$$\log[y(\kappa)] + 1 + \frac{1+\kappa}{y(\kappa)} y'(\kappa) = \frac{y(\kappa) + \frac{c}{\phi}}{(1+\kappa)y(\kappa) + \frac{c}{\phi}\kappa} + \frac{1+\kappa}{(1+\kappa)y(\kappa) + \frac{c}{\phi}\kappa} y'(\kappa).$$

By sending $\kappa \to 0$, we have

$$y(0) \cdot \log[y(0)] = \frac{c}{\phi},$$

i.e., $y(0) = \psi(c/\phi)$. Then, we have

$$\lim_{\kappa \to 0} \Pi_G^A(\kappa) = \frac{c + \phi \cdot y(0)}{\lambda_G} = \frac{c + \phi \cdot \psi\left(\frac{c}{\phi}\right)}{\lambda_G}.$$

□

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SA.2.4 Proof of Theorem 4

Proof of Theorem 4. (a) By Lemma SA.2.3, when $\kappa < \kappa^*$, $\Pi_S^A(\kappa) > \Pi_G^A(\kappa) > \Pi_F^A(\kappa) = (c+\phi)/\lambda_G$.

(i) By the argument in Section SA.2.3.1, the project is infeasible if $\Pi \leq \Pi_F^A(\kappa)$.

(ii) When $\Pi_G^A(\kappa) \geq \Pi > \Pi_F^A(\kappa)$, u_1 is greater than or equal to $\bar{u}$ by Lemma SA.2.2. By Proposition SA.2.1, the value function is $V^A(u) = V_A^g(u)$ for $u \leq \bar{u} \leq u_1$. Thus, the optimal contract is to execute the direct approach for all $u \leq \bar{u}$. Therefore, the direct-only contract with $T = \bar{u}/\phi$ implements $(\bar{u}, V(\bar{u}))$.

(iii) When $\Pi_S^A(\kappa) > \Pi > \Pi_G^A(\kappa)$, u_1 is less than $\bar{u}$ by Lemma SA.2.2 and greater than zero by Lemma SA.2.1. By Proposition SA.2.1, the value function is $V(u) = V_A^{gs}(u|u_1)$ for $u_1 < u < \bar{u}$ and $V(u) = V_A^g(u)$ for $0 \leq u \leq u_1$. Thus, the optimal contract is to execute the sequential approach for $u_1 < u \leq \bar{u}$ and the direct approach for $0 \leq u \leq u_1$. Therefore, the contract with a switch from the splitting the project to going for the project at $(\bar{u} - u_1)/\phi$ and a deadline $\bar{u}/\phi$.

(iv) When $\Pi \geq \Pi_S^A(\kappa)$, $V^A(u) = V_A^{gs}(u|0)$ by Proposition SA.2.1. Thus, the optimal contract is to execute the sequential approach for $0 \leq u \leq \bar{u}$. Therefore, the sequential-only contract with $T = \bar{u}/\phi$ implements $(\bar{u}, V(\bar{u}))$.

(b) By Lemma SA.2.3, when $\kappa \geq \kappa^*$, $\Pi_F \geq \Pi_F^A(\kappa) \geq \Pi_S^A(\kappa)$.

(i) By the argument in Section SA.2.3.1, the project is infeasible if $\Pi \leq \Pi_F^A(\kappa)$.

(ii) When $\Pi > \Pi_F^A(\kappa)$, Π is greater than $\Pi_S^A(\kappa)$. Thus, $V^A(u) = V_A^{gs}(u|0)$ by Proposition SA.2.1 and the sequential-only contract with $T = \bar{u}/\phi$ implements $(\bar{u}, V(\bar{u}))$.

□

SA.3 Additional Value on the Subproject

In this section, I rewrite the benchmark value function V^{gs} in Appendix B.3 by introducing B_A and B_P. Note that the introduction of B_A and B_P does not affect the direct approach, thus V^g remains the same. Since (PK) is replaced by (PK$_B$), when the sequential approach is recommended, we need to substitute $u_S^B = u + \phi/\lambda_S - B_A$ for u_S. The principal's value function after the intermediate breakthrough ($V_S(u_S)$) also needs to be replaced by $V_S^B(u_S^B) + B_P$. Then, the HJB equation (B.4) can be rewritten as follows:

$$0 = -c + \lambda_S \left(V_S^B\left(u + \phi/\lambda_S - B_A\right) + B_P - V_B^{gs}(u|u_1)\right) - \phi V_B^{gs\prime}(u|u_1) \tag{SA.3.1}$$

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where V_B^{gs} is the new value function.

By plugging (6.1) into (SA.3.1) and following the same steps in Appendix B.3, we can derive the closed form of V_B^{gs} as follows:

$$V_B^{gs}(u|u_1) = \left(\Pi - \frac{2c}{\lambda_S} + B_A\right)\left(1 - e^{\frac{\lambda_S}{\phi}(u_1-u)}\right) + (V^g(u_1) + u_1)e^{\frac{\lambda_S}{\phi}(u_1-u)}$$
$$- \left(\Pi - B_P - \frac{c}{\lambda_S}\right)\frac{\lambda_S}{\phi}(u-u_1)e^{\frac{\lambda_S}{\phi}(B_A-u)-1} - u.$$

If there is no switching point, $V_B^{gs}(u|0)$ would serve as the value function. If there exists a switching point, we can pin down the switching point by following the same steps in Appendix B.4. When the switching point is u_1, the value function for $u \leq u_1$ is $V^g(u)$ and that for $u > u_1$ is $V^{gs}(u|u_1)$.

SA.4 The Role of the Discount Rate

Proof of Proposition 6.1. Note that

$$F_G(0) = \Pi - \frac{c}{\lambda_G} = \Pi - \frac{2c}{\lambda_S} = F_S(0),$$

and

$$F_G(0) = \frac{\lambda_G\Pi - c}{\lambda_G} > \frac{\lambda_G\Pi - c}{\lambda_G + r} = F_G(r)$$

since $\Pi > c/\lambda_G$ and $r > 0$. Also note that

$$F_G(r) - F_S(r) = \frac{\lambda_G r}{(\lambda_G + r)(\lambda_S + r)^2}(r\Pi + c) > 0,$$

thus $F_G(r) > F_S(r)$. □

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