Tan Zhiyong

$Q = CV$

$E = \frac{1}{2}CV^2$ or $\frac{1}{2}QV$

CAPACITANCE

Capacitor in series

Conservation of charge $Q_1 = Q_2$ $V_1 + V_2 = V$

conservation of energy

$V_1 = V_2$

$Q_T = Q_1 + Q_2$

Content

1. Capacitors and capacitance
2. Energy stored in a capacitor

Learning Outcomes

At the end of the lesson, you should be able to:

(a) show an understanding of the function of capacitors in simple circuits.

(b) define capacitance and the farad.

(c) recall and solve problems using C = Q/V

(d) derive, using the formula C = Q/V, conservation of charge and the addition of p.d.'s formulae for capacitors in series and in parallel.

(e) solve problems using formulae for capacitors in series and in parallel.

(f) deduce from the area under a potential-charge graph, the equation $W = \frac{1}{2}QV$ and hence $W = \frac{1}{2}CV^2$.

1. WHAT IS A CAPACITOR?

A capacitor is *any* ***conducting device*** *that can store* charge Q.

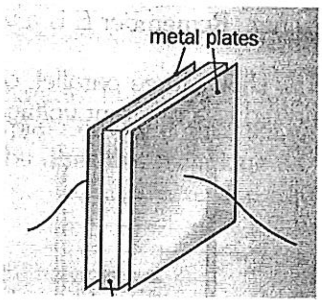
It usually consists of two conducting surfaces separated by an insulator.

The action of storing up this electrical energy is somewhat similar to that of a gas tank used for the storage of gas.

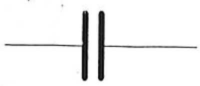
The amount of charges which a capacitor will hold depends on the voltage. V applied to the capacitor just as the amount of gas a tank will hold depends upon the pressure.

If the pressure is doubled on the gas tank, twice the amount of gas is forced into the tank, and if the voltage applied to a capacitor is doubled, twice the amount of charges will be forced into the capacitor. $Q \propto V$

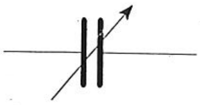
Any electrical capacitor has three essential parts two of which are usually metal conductors separated and insulated by the third part called the dielectric.



Electrical symbol **:**



Fixed capacitor



Variable capacitor

The simplest form of capacitor consists of two electrodes or conducting plates separated by air. This represents a capacitor having a gaseous dielectric.

The dielectric of a capacitor is one of three essential parts. It may be found in solid, liquid or gaseous form or even in combinations of these forms in a given capacitor.

Other dielectrics in common use are mica, paper, glass, sulphur, mineral and vegetable oils, waxes and synthetic insulating compounds such as the chlorinated groups.

It is a common practice to divide or identify capacitors in accordance with the type of dielectric employed in their structures. For example, there are mica capacitors, air capacitors, oil capacitors and paper capacitors.

When analyzing a single conducting surface, we can consider the second conducting surface to be placed at infinity. For example, an isolated spherical conductor is a capacitor that can store charges on its outer surface.

2. WHAT IS CAPACITANCE?

Capacitors have different capacities to store charges. This ability to store charges is called capacitance. C.

Definitions: The capacitance ***C*** of a system is the ***ratio*** of the **magnitude of charge Q** on *either conductor* to the **potential difference V** between the conductors.

i.e. $\boxed{C = \frac{Q}{V}}$ or $Q = CV$

Unit : The SI unit of capacitance is the Farad. F.

$\boxed{1F = \frac{1C}{1V}}$

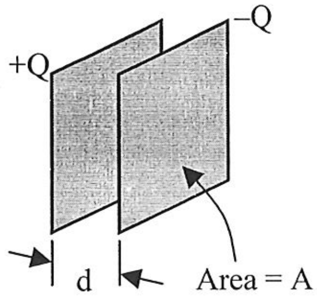
1 F is the capacitance of a capacitor which stores 1 C of charges with a pd of 1 V.

The farad, unfortunately, represents a capacitance so enormous that such a capacitor is rarely if ever produced or used. For practical purposes therefore, a small multiple of the farad is used. 1 microfarad, μF (× 10^{-6}); 1 nanofarad, nF (× 10^{-9}); or 1 picofarad, pF(× 10^{-12}).

3. WHAT DETERMINES THE CAPACITANCE OF A CAPACITOR?

It is not the amount of charges but the inherent characteristics of a capacitor that determines its capacitance. The capacitance of a capacitor is dependent upon the size and spacing of the conducting plates and the type of insulating or ***dielectric*** medium between the plates.

3.1 Simple parallel plate capacitor



It can be shown the capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon A}{d}$$

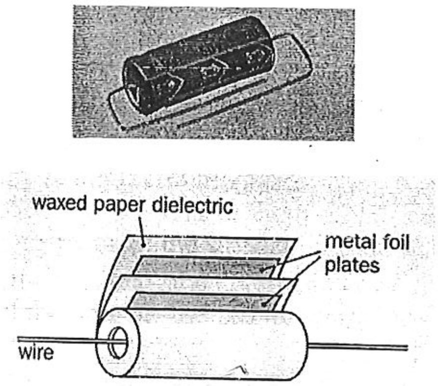
A: area of plates
d: separation distance
ε: permittivity of dielectric medium

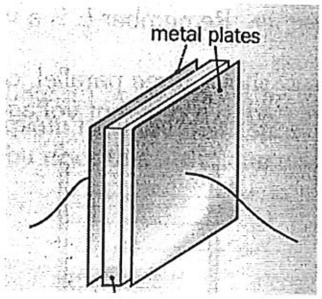
(Do not need to memorise this equation. It will be given if required.)

From the equation, the capacitance of the capacitor can be increased by
- increasing the surface area, ***A*** of the conducting plates;
- decreasing the separation distance ***d*** between both plates;
- introducing a dielectric of higher permittivity *ε* between the two plates.

It is important that this fundamental fact be remembered, that doubling the area of the plates of a capacitor doubles the capacitance and reducing the thickness of the dielectric by one-half also doubles the capacitance of a capacitor.

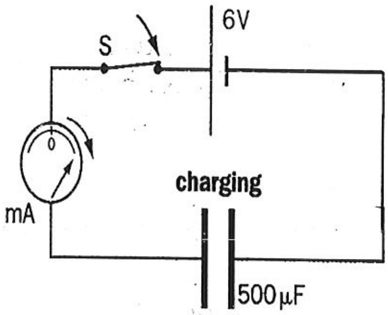
In order to increase the capacitance, how is it possible to increase the cross-sectional area of the parallel plates without making the capacitor too bulky?





4. CHARGE MOVEMENT IN A PARALLEL PLATE CAPACITOR

4.1 Charging a Capacitor



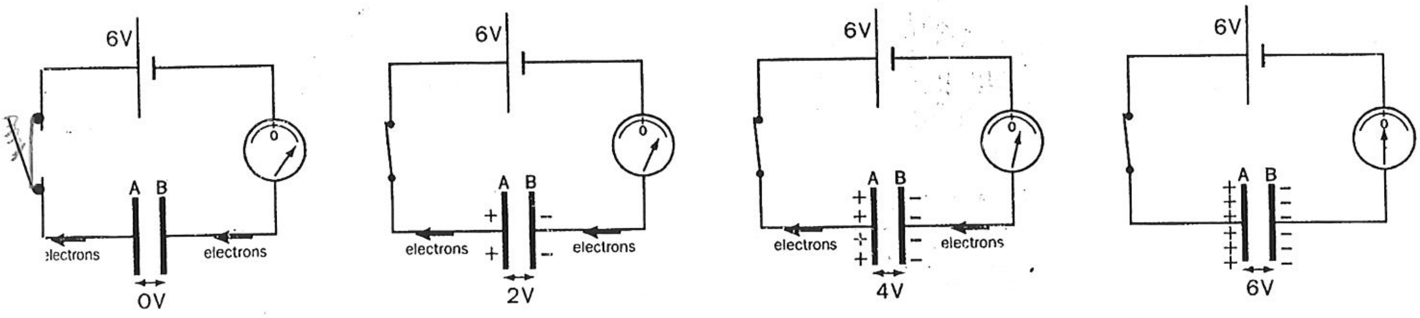
In order to charge up a capacitor, we can connect it to a battery:

What happens when switch S is closed?
The ammeter flicks and then go back to zero.
There is a surge of current (a flow of electrons) in the circuit; but only while the capacitor is charged up.

	Capacitance C	Charge Q	p.d. V	Current
Charging	Unchanged	↑	↑	High & slowly decreases
Discharging	Unchanged	↓	↓	High & slowly decreases

4.1.1 How doe a capacitor charge up

The capacitor plates are separated by an insulator. So how can charge flow?



(a) An initially uncharged capacitor has no net charges and hence no p.d. across plates A and B.

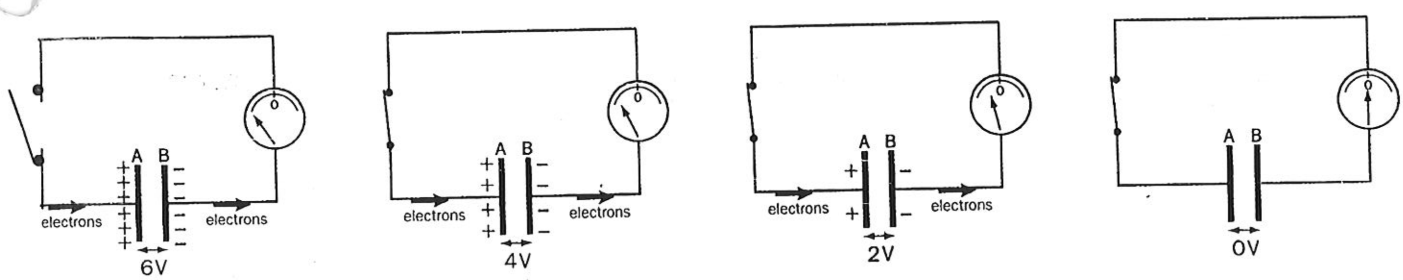
(b) When the switch is closed,
- Electrons flow from plate A (lower potential) to the positive terminal of the battery (higher potential). This leaves a net positive charge of +Q on plate A.
- Electrons flow from the negative terminal of the battery (lower potential) to plate B (higher potential). This leaves a net negative charge of –Q on plate B.

(c) As charges accumulate, the p.d. between the plates increases until it is ***equal*** to the emf of the battery, V. At this point, there is no potential difference between each of the plates and the terminal of the battery to which it is connected. Thus, no more electrons flow.

(d) The battery can be viewed as an electric pump pushing electrons from A round to B. Current is thus detected to be flowing from B to A during the charging process.

Note : Even though the capacitor acquires equal and opposite charge on both plates (ie. +Q on plate A and –Q on plate B), the capacitor is said to **store a charge Q**.

4.1.2 Discharging a Capacitor



(a) Since the capacitor is initially charged, *p.d.* exists between the terminals of the capacitor.

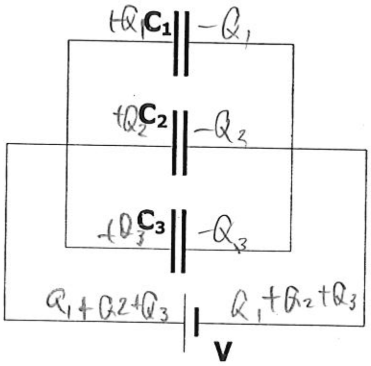
(b) When the switch is closed,
- Electrons flow from plate B (lower potential) towards A (higher potential).
- Hence the amount of net charge on the capacitor becomes lesser.
- Discharging stops when the net charge on the capacitor and the p.d. across it becomes ***zero***.

(c) Current is detected to be flowing from the positively charged plate A to the negatively charged plate B.

5. CAPACITOR NETWORKS

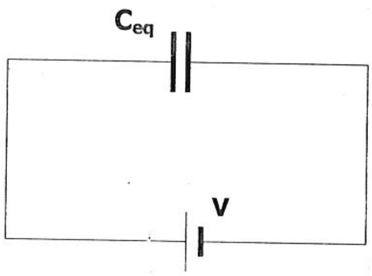
Like resistors, capacitors are also connected up in different ways to form different capacitance that may not be obtainable commercially.

5.1 Capacitors In Parallel



1. Since the capacitors are connected in parallel to the voltage source, the _p.d_ across all capacitors connected in parallel must be the same. Hence, the potential across each capacitor is _V_, the emf of the cell.
2. The charge on each capacitor is given by

$$\boxed{Q_1 = C_1V\ ;\ Q_2 = C_2V\ ;\ Q_3 = C_3V}$$



3. An equivalent capacitor (fully charged with the same cell) must store the same amount of charge that the individual capacitors store in total.

Hence, by Conservation of Charges, the total charge on the equivalent capacitor is

$$Q_T = Q_1 + Q_2 + Q_3$$

4. An equivalent capacitor would also have a potential difference of V, the emf of the cell. Hence,

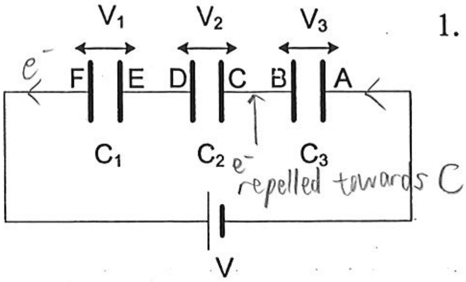
$$\boxed{C_{eq}V = C_1V + C_2V + C_3V}$$

Generalising the result to n capacitors, we have

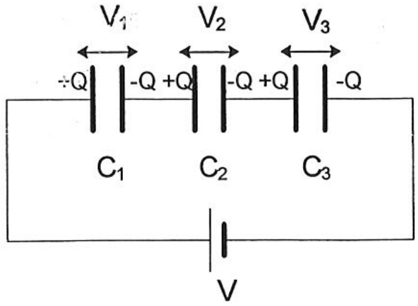
$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

- The total capacitance of a *parallel* capacitor circuit is equal to the *sum of the individual capacitances.*
- The equivalent capacitance of a *parallel* combination of capacitors is *larger than* any of the individual capacitances.

5.2 Capacitors In Series



1. When the cell is connected across the ends of the system, plate A will be charged –Q while plate F will be charged +Q as explained in **Section 4.1**. At the same time, plate A repels electrons away from plate B towards plate C until plate B is charged +Q while plate C is charged –Q. Similarly, plates D will be charged +Q while plate E will be charged –Q. *The cell caused a charge of Q to flow through the whole circuit.*

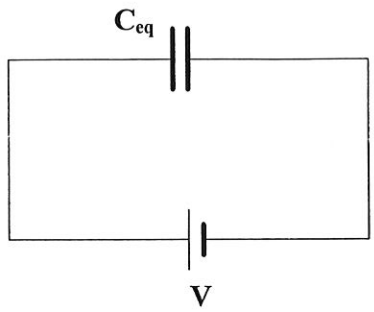


2. Each capacitor connected in series *carries the same amount of charge*, regardless of their capacitance.

3. The p.d. across each capacitor is given by

$$\boxed{V_1 = Q/C_1\ ;\ V_2 = Q/C_2\ ;\ V_3 = Q/C_3}$$

4. By conservation of energy, the three voltages *must add up to the total voltage, V.*



5. If we replace the 3 capacitors with an equivalent capacitor, the equivalent capacitor would acquire the charge Q from the cell voltage V, hence

$$V = \frac{Q}{C_{eq}}$$

The charge on the equivalent capacitor is Q and not $3Q$. This can be seen from the fact that *the cell caused a charge of Q to flow through the whole circuit.*

(3) & (4)

6. Equating the two expressions for V gives

$$\boxed{Q/C_{eq} = Q/C_1 + Q/C_2 + Q/C_3}$$

Hence, $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$

We can generalise the result to n capacitors.

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}}$$

- The *reciprocal* of the equivalent capacitance of a *series* capacitor circuit is equal to the *sum of the individual reciprocal capacitances.*
- The equivalent capacitance of a series combination is always *less than* any individual capacitance in combination.

6. OBSERVING PATTERNS

6.1 Comparison Between Series and Parallel Capacitors

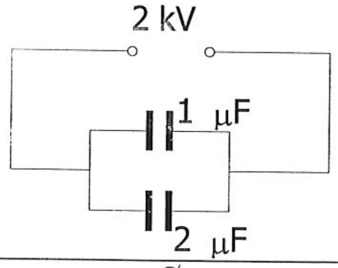
	Series	Parallel
Charges across each capacitor	Q (determined by current flowing through circuit.)	$Q_1, Q_2, \ldots Q_n$.
P.d. across each capacitor	$V_1, V_2, \ldots V_n$.	V (determined by voltage souce.)
Charge on equivalent capacitor total charge	$Q = Q_T$	$Q_T = Q_1 + Q_2 + \ldots + Q_3$
P.d. of equivalent capacitor	$V = V_1 + V_2 + \ldots + V_n$	$V = \frac{1}{V_1} + \frac{1}{V_2} + \ldots \frac{1}{V_n}$
Capacitance of equivalent capacitor	$1/C_{eq} = 1/C_1 + 1/C_2 + \ldots + 1/C_n$	$C_{eq} = C_1 + C_2 + \ldots + C_n$

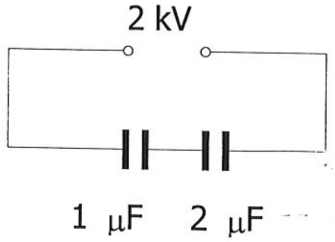
6.2 Comparison Between Capacitor and Resistor

	Capacitor	Resistor
Series	$1/C_{eq} = 1/C_1 + 1/C_2 + \ldots + 1/C_n$	$R_{eq} = R_1 + R_2 + \ldots + R_n$
Parallel	$C_{eq} = C_1 + C_2 + \ldots + C_n$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$

Example 4 :

Find the amount of charge and the p.d. across each capacitor in the circuits shown below:





$C_1 = \frac{Q}{V}$

$1 \times 10^{-6} = \frac{Q}{2 \times 1000}$

$\therefore Q = 2 \times 10^{-3}\,C$

$C_2 = \frac{Q}{V}$

$Q = (2 \times 10^{-6})(2000)$

$= 4 \times 10^{-3}\,C$

$C_1 V_1 = C_2 V_2 \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1}$

Also, $= V_1 + V_2 = 2000$

$V_1 = 2000 - V_2$

$\frac{2000 - V_2}{V_2} = \frac{2 \times 10^{-6}}{1 \times 10^{-6}}$

$2 \times 10^{-6} V_2 = 0.002 - V_2(1 \times 10^{-6})$

$V_2 = 667V$

$\Rightarrow V_1 = 1333V$

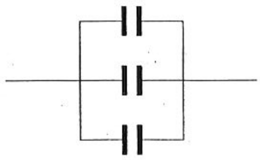
$Q = C_1 V_1$

$= 1.33 \times 10^{-3}\,C$

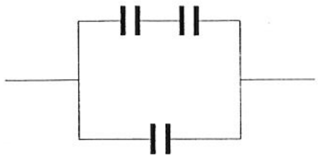
Example 5 :

Find the combined capacitance of the following circuits. (All capacitors are 1 μF).

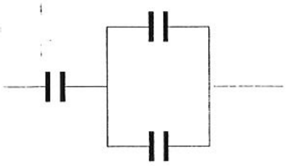
$C_{eq} = \frac{1}{3}\,\mu F$



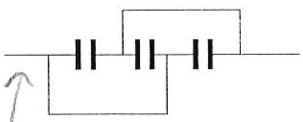
$C_{eq} = 3\,\mu F$



$C_{eq} = 1.5\,\mu F$



$C_{eq} = \frac{2}{3}\,\mu F$



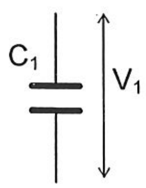
$C_{eq} = 3\,\mu F$

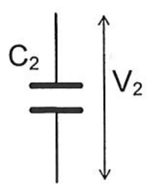
rearrange

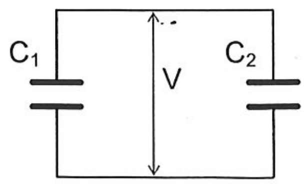
Example 6 :

Two capacitors of capacitances C_1 and C_2 are charged to potentials V_1 and V_2 respectively. They are then connected so as to provide a larger capacitance C. Derive an expression for the potential V of this larger capacitor in terms of C_1, C_2, V_1 and V_2.

N80/III/4 (part)







$Q_1 = C_1V_1$; $Q_2 = C_2V_2$

Larger Capacitance ⇒ // (charges redistribute until both capacitors have same p.d.)

By conservation of charges,

$$Q_1' + Q_2' = Q_1 + Q_2$$

$$C_1V + C_2V = C_1V_1 + C_2V_2$$

$$\therefore V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

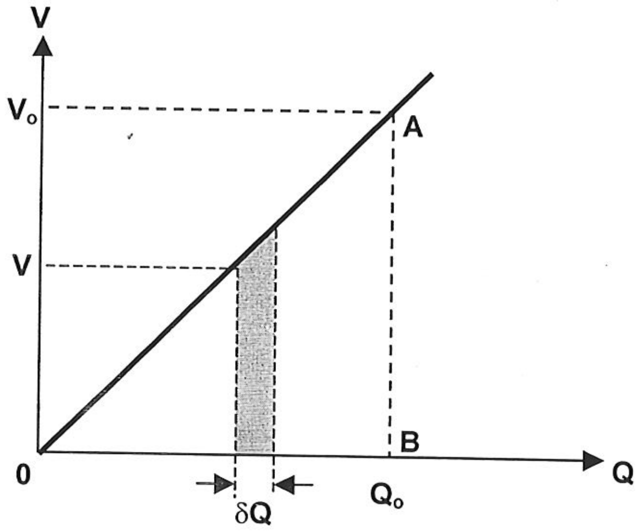
7. ENERGY STORAGE IN A CAPACITOR

7.1 Potential – Charge Graph (V vs Q)

If the capacitance of a capacitor remains constant as it is being charged,

$V \propto Q$, since $Q = CV$

Hence, the V vs Q graph will be a straight line passing through the origin.



7.2 Energy Stored in a Capacitor

Consider a capacitor already charged to a p.d. of V with a charge of Q.

As a small amount of charge δQ is added to the capacitor,

Work done = $\delta W = V\,\delta Q$ = area under the graph. $= \frac{1}{2}QV$

Hence total work done in charging a capacitor to Q_o, $W = \int_0^{Q_o} V\,dQ$

$E = \frac{\Delta V}{d}$

Thus in general, energy stored, E in the capacitor,

$\boxed{E = \frac{1}{2}QV}$ or $\boxed{E = \frac{1}{2}CV^2}$ or $\boxed{E = \frac{1}{2}\frac{Q^2}{C}}$

$Q = CV$ $\quad$ $V = Q / C$

Example 7 :

A photographic flash unit consists of a xenon-filled tube energised by the discharge of a capacitor previously charged by a 1000 V source. The average power delivered to the flash tube is 2000 W and the flash lasts 0.040 s. Estimate the capacitance of the capacitor. *J86/III/5*

Total energy delivered, $E = Pt$

$= 2000 \times 0.040$

$= 80 J$

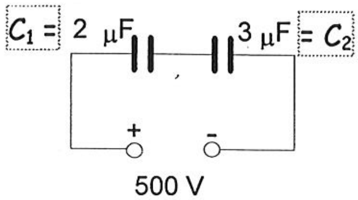
Assume all energy from the capacitor is delivered to flash tube,

$E = \frac{1}{2}CV^2$

$80 = \frac{1}{2}C(1000)^2$ $C = 1.6 \times 10^{-4}$ F

Example 8 :

Find the energy stored in each of the capacitor shown below:



Example 9 :

An uncharged capacitor is connected in series with a battery of e.m.f. V and a switch. When the switch is closed, a charge Q flows through the battery. Write down expressions in terms of V and Q for

(i) the energy supplied by the battery,

(ii) the energy stored in the capacitor.

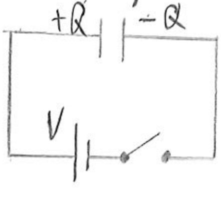
Comment on your answers with reference to the law of conservation of energy.

J89/III/11 (part)

(i) energy supplied by battery = QV

(ii) energy stored in capacitor = $\frac{1}{2}QV$

Exactly $\frac{1}{2}$ of the energy supplied by the battery is stored in the capacitor while the other $\frac{1}{2}$ is dissipated as heat in the resistance of the circuit



8. USES OF THE CAPACITOR

The charging and discharging properties of capacitors through resistors have many applications:

1. **Medical Application**
   Used in device for shocking heart attack victims by providing a current of 20A. In emergency situations, without a large enough power supply, only a capacitor can provide the required power of 100 kW.

2. **Computers**
   Used in RAM of computers to store information temporarily. Once computer is switched off, data will disappear.

3. **Charge Storage in Devices**
   Store charges *temporarily* so that it can be used to create a large current.
   eg. camera flash (takes time to recharge each time after taking a photo)

4. **Kinetic Watches**
   Ultracapacitors is used to store energy by virtue of its small size and high capacitance.

5. **Smoothing Circuits**
   Large capacitors are used at the rectified output stage for smoothing of the d.c. voltage. The large capacitance gives rise to a slower discharge and hence a steadier output current. (See **Appendix A&B**. Review this concept with your tutor after you have learnt the topic on **Alternative Current**.)

6. **Amplifier Circuits**
   In audio amplifiers, capacitors are used in combination with inductors to allow certain frequencies to pass through while other frequencies are "filtered-out".

7. **Tune Circuits**
   Variable capacitors are used in radio and television sets to tune to different stations.

REFERENCES

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Electrolytic capacitors- http://www.faradnet.com/deeley/book_toc.htm

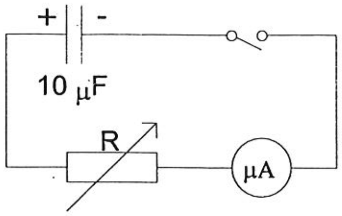
Books

Advanced Physics for you by Keith Johnson and Simmone Hewett
Physics by Robert Hutchings
Advanced Lĕvel Physics by Nelkon and Parker
Physics in Focus by Michael Brimicombe
Physics by Patrick Fullock
Essential Principles of Physics by Whelan and Hodgson
Advanced Physics by Keith Gibbs
A-level Physics by Roger Muncaster

TUTORIAL QUESTIONS

Self Attempt:

1. The circuit shown below was used to discharge a charged capacitor of 10 μF capacitance.

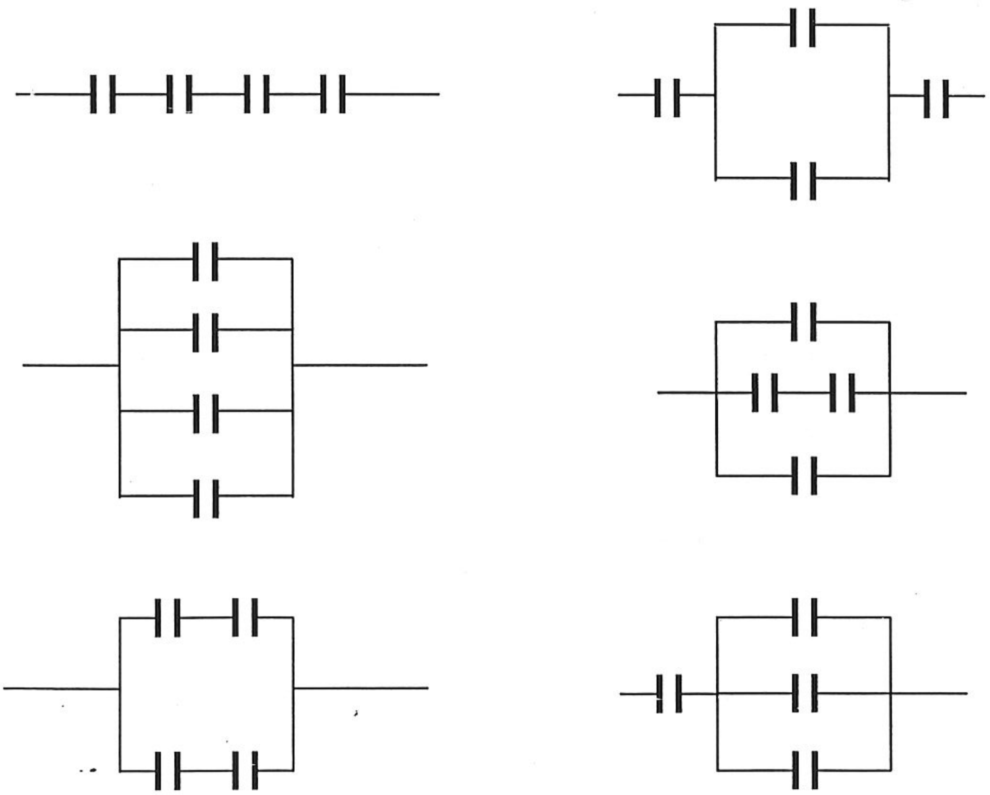


For a period of 40s, the current was kept constant at 20 μA by the continuous adjustment of *R*. By how much did the potential difference across the capacitor change during this period?
Answer: 80 V

2. A capacitor of capacitance 64 μF, which is initially uncharged, is connected to a supply which gives a constant current of 0.16 mA. Calculate
(i) the time taken for the potential difference across the capacitor to become 30 V,
(ii) the energy stored by the capacitor when the potential difference across is 30 V.
Answer: 12 s, 0.0288 J

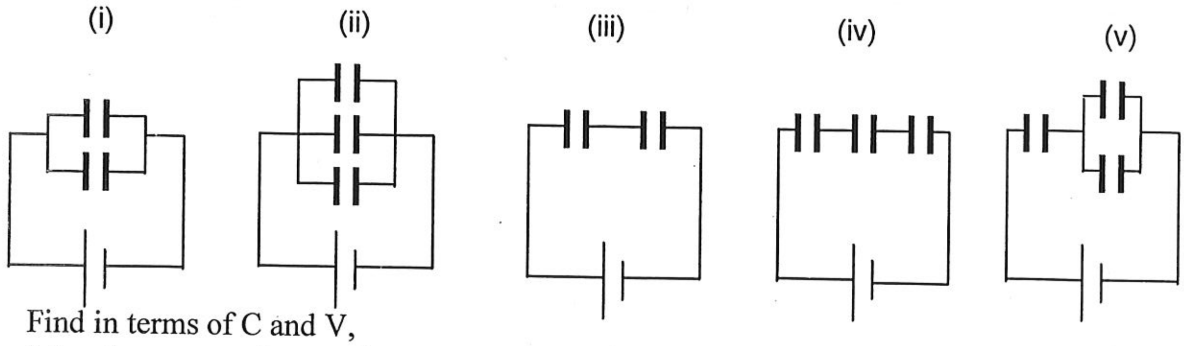
3. Each capacitor in the systems shown below has capacitance C. Calculate their combined capacitance.

$\frac{1}{C_{eq}} = \frac{1}{C}$ $\frac{C}{2} + C = 2\frac{1}{2}C$



Answer: C/4, 4C, C, 2C/5, 3C/4

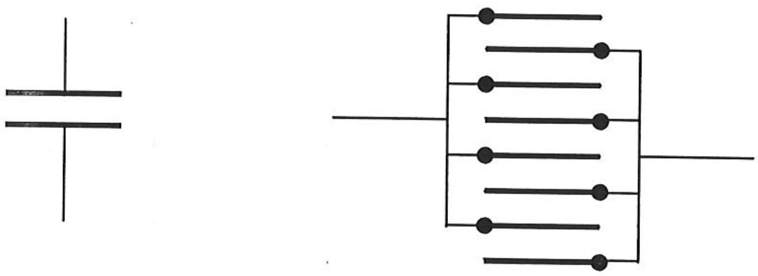
4. For each of the diagrams below, each capacitor has capacitance C. The cells have e.m.f. of V.

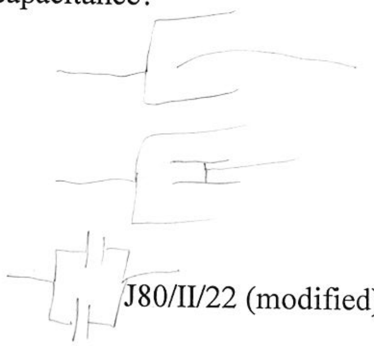


Find in terms of C and V,
(a) p.d. across each capacitor;
(b) the charge on each capacitor;
(c) the total energy stored in the capacitors.

Answer: (i) V, CV, CV^2; (ii) V, CV, $3CV^2/2$; (iii) $V/2$, $CV/2$, $CV^2/4$; (iv) $V/3$, $CV/3$, $CV^2/6$; (v) $2V/3$, $V/3$, $CV/3$, $CV^2/3$

5. Two metal plates form a parallel plate capacitor of capacitance C. Four such capacitors are connected as shown in the diagram below. What is the combined capacitance?





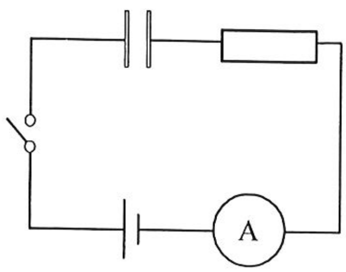
J80/II/22 (modified)

Answer: 7C

Discussion:

6. Discuss the following questions:
A capacitor is connected across a battery.
(a) What is the difference between a battery and a capacitor?
(b) Why does each plate receive a charge of exactly the same magnitude?
(c) Is it true if the plates are of different size?
(d) What is the total charge on a fully charged capacitor?

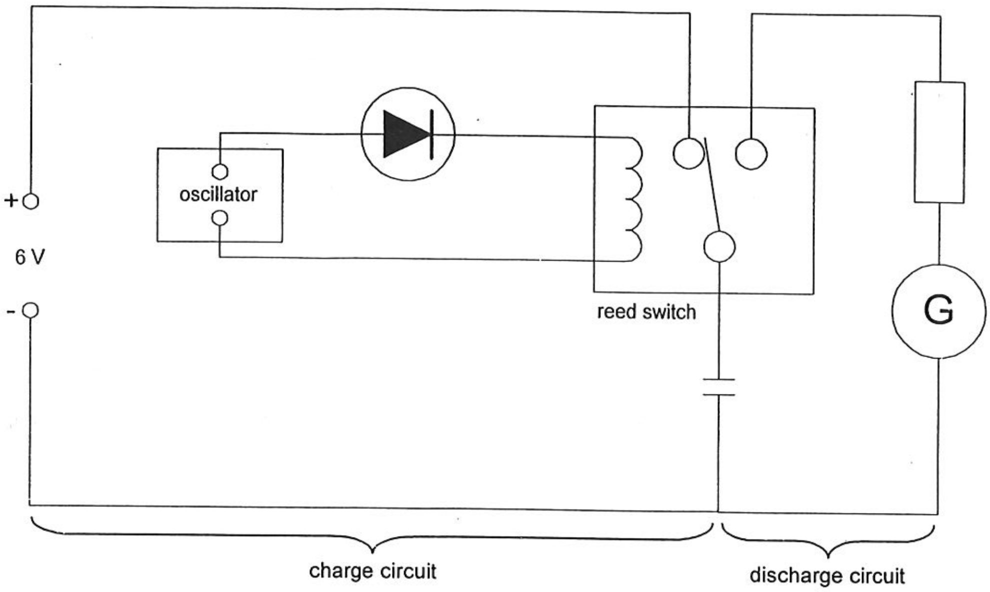
7. The circuit shows a typical charging circuit for a capacitor. Discuss whether you expect the current flowing in the circuit to be constant after the switched is closed.



8. Give a number of capacitors each of 2 μF and a maximum safe working potential difference of 10 V, how would you construct capacitors of
(a) 1 μF suitable for use up to 20 V;
(b) 2 μF suitable for use up to 20 V?

J87/III/3

9. A method of measuring the capacitance of a parallel plate capacitor is illustrated in the figure below. The **reed switch** is controlled by an oscillator, in such a way that the capacitor is charged from the supply and discharged through the galvanometer f times per second. The resistor in the discharge circuit is included to protect the galvanometer. Assume that the capacitor completely discharge itself before being recharged via the reed switch.



(a) The galvanometer records the average current, I, in the discharge circuit. Show that the charge stored on the capacitor when it is charged is given by $Q=I/f$.

(b) The information below shows some data for a particular parallel plate capacitor inserted in the circuit. Use it to calculate the value of its capacitance.

Frequency of oscillator, $f = 50$ Hz
Supply emf, V = 6.0 V
Galvanometer reading, I = 7.3 μA

Answer: 2.5×10^{-8} F

(c) It can be shown the capacitance of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon A}{d}$$

A: area of plates
d: separation distance of plates
ε: permittivity of dielectric

Show that the permittivity of a dielectric can be calculated by

$$\varepsilon = \frac{Id}{fVA}$$

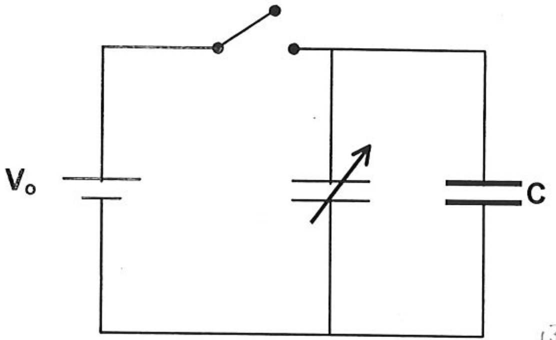
(d) The above experiment can be used to determine the relative permittivity of glass, $\varepsilon_r = \varepsilon / \varepsilon_0$. Describe the additional steps required to calculate for ε_r.

10. A 2 μF capacitor is charged to a potential of 200 V and then isolated. When it is connected in parallel with a second capacitor of capacitance 1 μF which is initially uncharged, find the final charge on each capacitor after the connection.
Answer: 2.67 x 10^{-4} C, 1.33 x 10^{-4} C *N96/III/4(b)*

11. A 1 μF capacitor and a 2 μF capacitor are connected in series across a 1200 V supply line.
   (a) Find the charge and potential difference across each capacitor.
   (b) The charged capacitors are disconnected from the line and each other, and reconnected with terminals of like signs together. Find the final charge and potential difference across each capacitor.
Answer: (a)800 V, 400 V, 8 x 10^{-4} C; (b) 533 V, 5.33 x 10^{-4} C, 10.7 x 10^{-4} C

Assignment:

1. In the figure, X is a fixed capacitor of capacitance C, and Y is a variable capacitor. The e.m.f. of the battery is V_o.



$Q_I = 2CV_0$

$Q_F = \frac{5}{4}C$

$2CV_0 = \frac{5}{4}C \times \frac{8}{5}V_0$

$\frac{1}{2}CV_0^2 = \frac{1}{2}(\frac{5}{4}C)(\frac{8}{5}V_0)^2$

$1\frac{3}{5}CV_0^2 - CV_0^2 = \frac{3}{5}CV_0^2$

   (a) Initially, the switch is closed and Y is adjusted so that its capacitance is also C. Find, in terms of C and V_o, the total energy stored in the two capacitors.
   (b) The switch is then opened and Y is adjusted so that its capacitance becomes C/4. $Q = CV$
      (i) What is the resulting p.d. across the capacitors? $\frac{8}{5}V_0$
      (ii) How much work is done against electrical forces in reducing the capacitance of Y?

J83/I/9

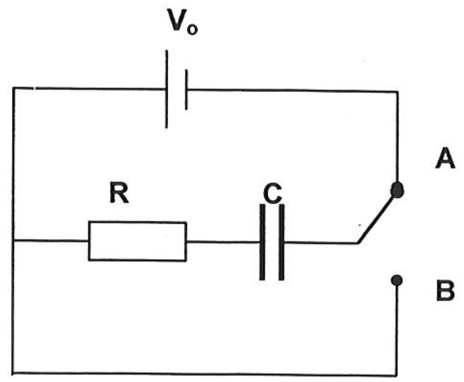
2. Two capacitors of capacitances 22 μF and 47 μF are connected in series with a constant 24 V supply. The capacitors are initially uncharged. Calculate

   (i) the total circuit capacitance,
   (ii) the charge delivered by the supply,
   (iii) the charge on each capacitor,
   (iv) the potential difference across the capacitor of capacitance 47 μF.

N96/III/4(c)

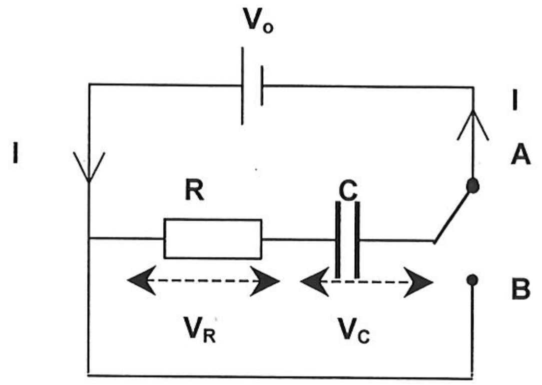
Appendix A

Graphs of Charging and Discharging

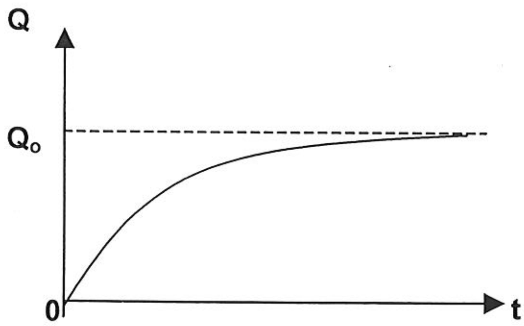


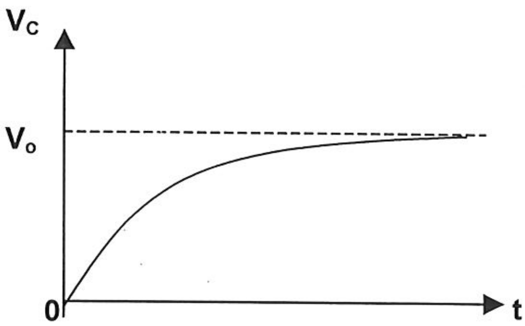
Charging

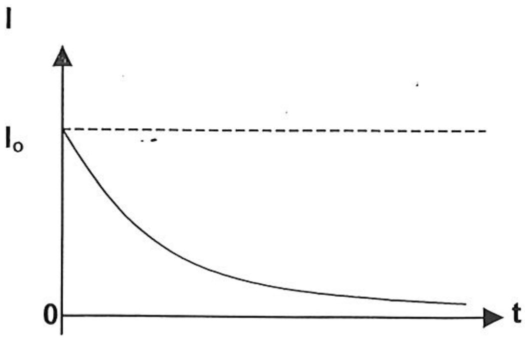
The switch will be thrown to A when the capacitor is uncharged.



Graphs of R-C Charging:



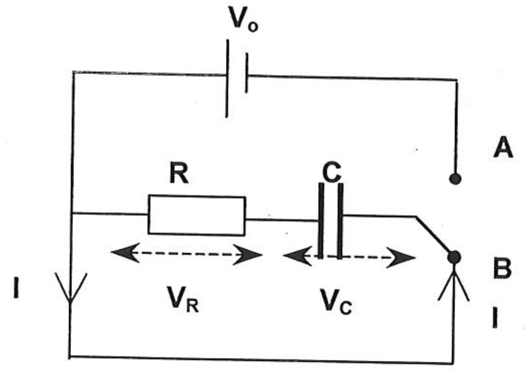




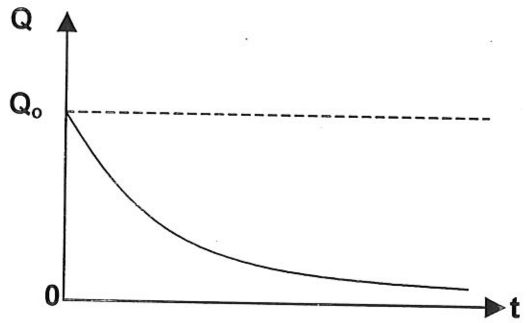
Appendix B

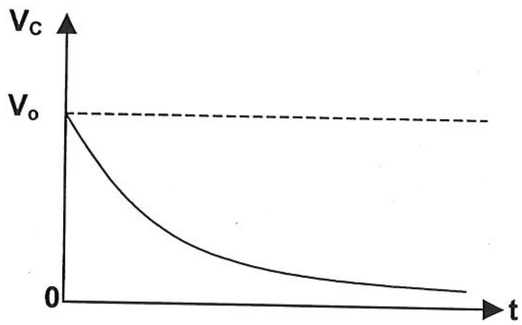
Discharging

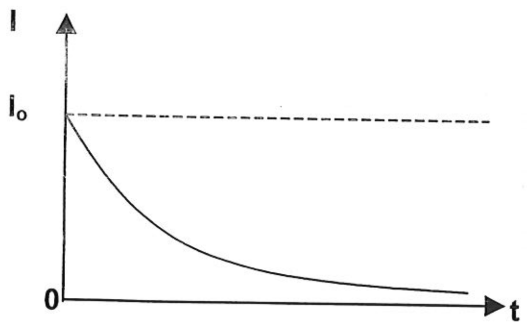
After a sufficient length of time, the switch will be swung to B.



Graphs of R-C Discharging:







Note that I vs t for discharging is the same of charging.

A capacitor can be made to discharge more quickly by having a

- lower capacitance;
- lower resistance connected to it.