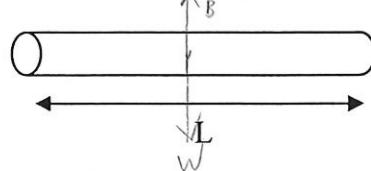


EXAMPLE 3

A wire of superconducting niobium of radius 0.100 cm can carry a current of 1500 A. The density of niobium is 8600 kg m^{-3} . What field must the wire be placed in so that it levitates, that is, it floats with no visible means of support? (The length of the wire is not needed to solve the problem.)



$$\begin{aligned} I &= 1500 \text{ A} \\ \text{Wire floats when } F_B &= BIL = mg \\ BIL &= \rho L (\pi r^2) g \\ B &= \frac{8600 \times \pi \times \left(\frac{0.100}{100}\right)^2 \times 9.81}{1500} \\ &= 1.77 \times 10^{-4} \text{ T} \end{aligned}$$

EXAMPLE 4

In an electric motor a rectangular coil of wire has 150 turns and is 0.20 m long and 0.12 m wide. The coil has a current of 0.26 A through it and is parallel to a field of magnetic flux density 0.36 T, as shown in the diagram. Find the torque which is exerted on the coil.

$$F = N B I L \sin \theta$$

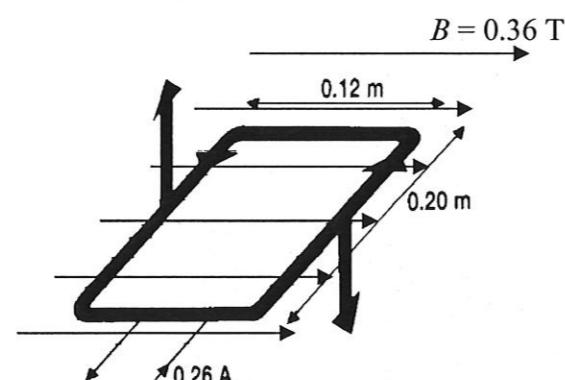
$$\begin{aligned} &= 150 \times 0.36 \times 0.26 \times 0.20 \\ &= 2.81 \end{aligned}$$

$$\tau = Fd$$

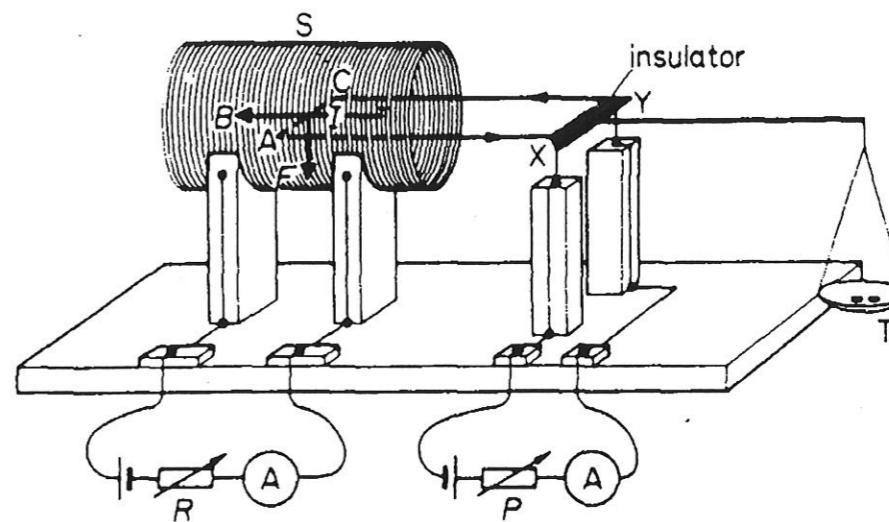
$$= 2.81 \times 0.12$$

$$= 0.337$$

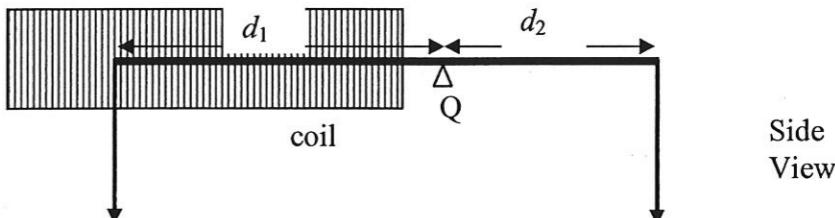
$$= 0.34 \text{ Nm}$$

**3.4 Current Balance**

The figure shows a simple form of current balance. The solenoid, S generates a magnetic field B directed towards the left. A current I is made to flow in the light rectangular coil by an e.m.f. source in an external circuit. The side AC is placed perpendicular to the magnetic field to be measured with the latter directed towards the right.



If the length AC is L , then neglecting Earth's field, the force acting on AC is BIL downwards. The anticlockwise torque about the axis, XY, is $BILd_1$. A rider, T of mass m is moved a distance d_2 from the axis until the plane of the coil is horizontal. The clockwise torque is mgd_2 .



For equilibrium:

Taking moments about XY,

Sum of anti-clockwise moments = Sum of Clockwise moments

$$BIL \cdot d_1 = mg \cdot d_2$$

$$B = \frac{mg \cdot d_2}{IL \cdot d_1}$$

Pause & Think

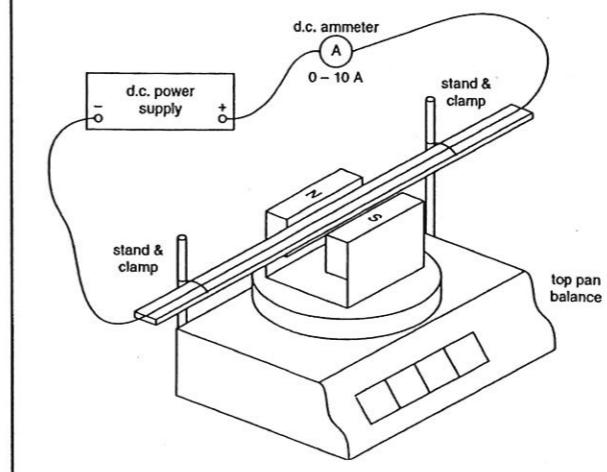
Discuss how the following setup can be used to determine the magnetic flux density of the permanent magnet.

Record:

1. length of magnet
2. current, I
3. change in balance reading, m

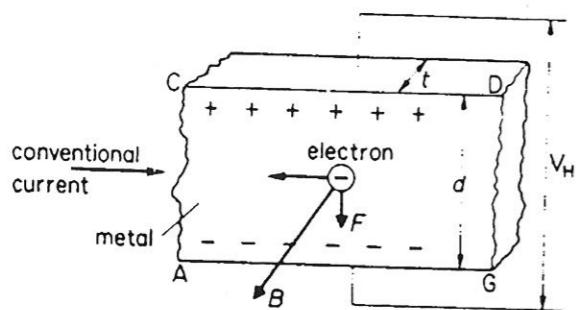
Equate

$$BIL = mg$$



3.5 Hall Effect & Hall Probe

The Hall effect is an effect that arises because of the force, which a magnetic field exerts, on a moving charge.



Consider a wire of rectangular cross-section placed in a magnetic field,

- An individual electron, travelling with velocity v through the strip, experiences a force F_B where $F_B = Bqv$
- This force causes the electron to accelerate towards the side AG of the strip.
- This would have the effect of giving AG a negative potential and CD a positive potential, setting up an electric field which will act a force on the moving electrons.
- The buildup of charge continues until the potential difference becomes so large that it prevents any further increase. This maximum potential difference is called the Hall voltage.
- Equilibrium is set up and this electric force is equal and opposite to that exerted by the magnetic field.
- The other electrons will then drift through the strip without being deflected to one side.

At equilibrium,

$$\mathbf{F}_B = \mathbf{F}_E$$

$$B q v = q E = (V_H/d) q$$

$$V_H = Bvd$$

where E = strength of the uniform electric field between CD and AG

V_H is called the Hall voltage and is a potential difference set up across the width of a current-carrying conductor in a magnetic field.

Now, given $I = nevA$

where n = number of electrons per unit volume;
 v = drift velocity; e = charge of electrons;
 A = cross sectional area of the material.

Rearranging, $v = I / neA$

$$\text{Substituting, } V_H = B(I/neA)d = \frac{Bid}{ne(td)} = \frac{BI}{net} \quad \text{since } A = td$$

$$\therefore B = \frac{V_H net}{I}$$

Hence, the flux density of the magnetic field can be determined. If we can keep n , e , t and I constant, the flux density will be directly proportional to the Hall voltage, V_H . ie.

$$B \propto V_H$$

3.3 Magnetic flux density and Tesla

Definition: Magnetic flux density is defined as the force acting on a conductor of unit length carrying unit current placed perpendicular to the magnetic field.

$$B = \frac{F}{IL}, \theta = 90^\circ$$

Direction of the field is at right angles to both the force and the current and is determined by Fleming's left-hand rule.

(The quantitative value of magnetic field strength is called the magnetic flux density. It is a measure of the strength of the magnetic field in a particular region of space.)

The SI unit of magnetic flux density, B is Tesla (T).

Definition: A tesla is the magnetic flux density if a force of 1N acts on a wire of length 1m, carrying a current of 1A placed perpendicular to the magnetic field.

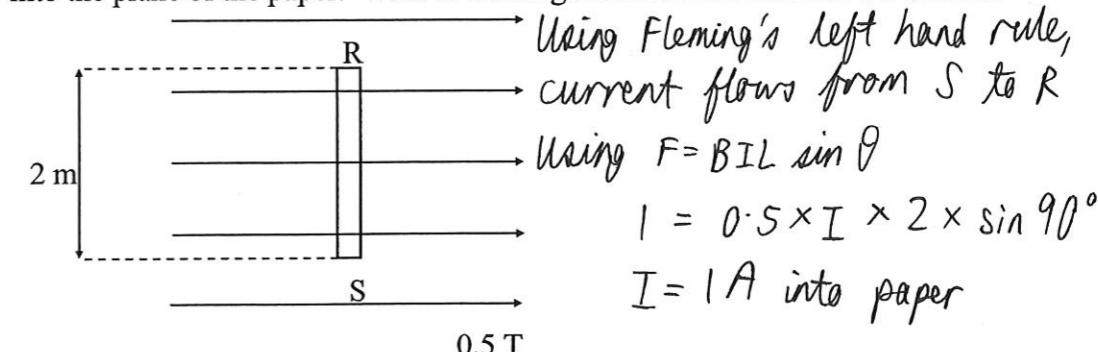
$$1T = \frac{1N}{1A \times 1m}, \theta = 90^\circ$$

Note:

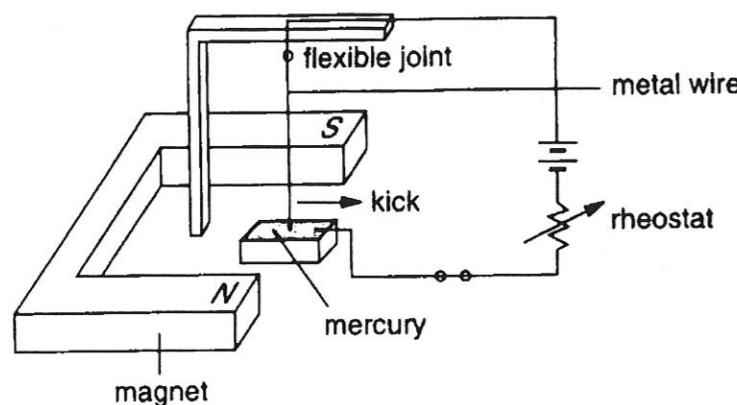
- A field of about 0.1 T can be achieved with big magnets.
- With electromagnets, fields of about 1 T can be attained.
- Earth's magnetic field is about 50 μ T.
- Gigantic field values of up to 20 T can be found in many stars and in the interiors of many kinds of atoms.

EXAMPLE 2

The diagram shows a current-carrying conductor RS of length 2 m placed perpendicularly to a magnetic field of flux density 0.5 T. The resulting force on the conductor is 1 N acting into the plane of the paper. What is the magnitude and direction of the current?



3.1 The Kicking Wire Experiment



The lower part of the wire is immersed in a pool of mercury which completes the circuit. Current flows down the wire. The magnetic field is provided by the U-shaped magnet. The wire experiences a kick in the direction indicated and breaks the circuit when its lower end comes out of the pool of mercury. It falls back due to gravity, re-establishes contact with the mercury and experiences a force again. The kicking action is thus repeated.

3.2 Force on Current-Carrying Conductor

Consider a metallic conductor carrying a current I and inclined at an angle θ to a magnetic field of flux density B . The conductor will experience a force into the plane of the paper.

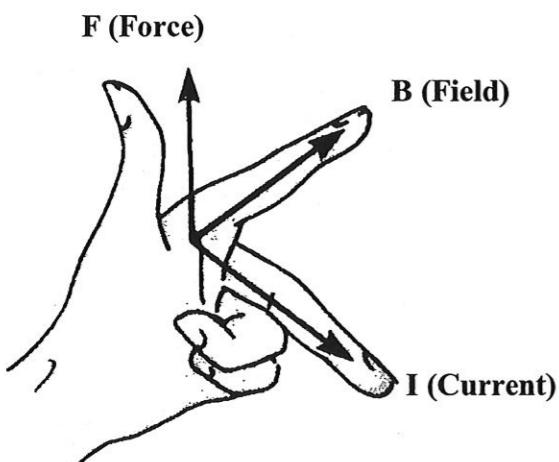
Experiments show that:

- (a) $F \propto B$;
- (b) $F \propto I$;
- (c) $F \propto L$, where L is the length of the conductor in field;
- (d) $F \propto \sin\theta$.

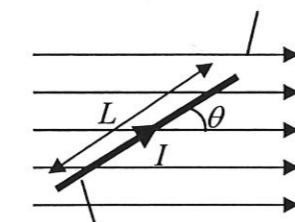
Hence,

$$F = BIL \sin\theta$$

When $\theta = 90^\circ$, $F = BIL$. The direction of the force is given by Fleming's Left Hand Rule.



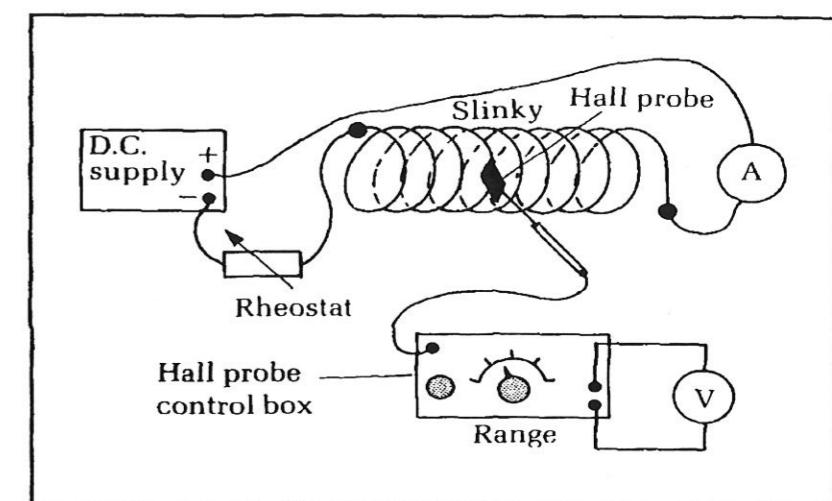
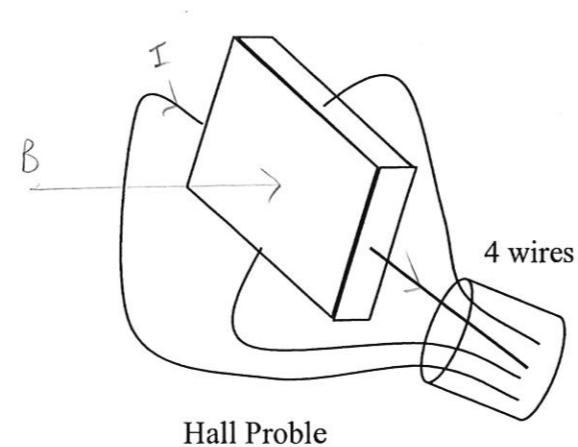
Uniform magnetic field in the plane of the paper and of flux density B .



Conductor carrying a current I in the plane of the paper

B & I are independent of each other
 B not due to I

3.5.1 Use of a Hall Probe



The Hall probe is a slice of doped semiconductor with 4 wires attached to its sides. Two connecting wires on both ends of the slice supply a constant current. The slice is positioned so that the magnetic field lines are perpendicular to its plane. The Hall voltage set up is measured via the other two connecting wires.

The probe is calibrated using a magnetic field of known flux density. It is inserted into the known field and the voltmeter reading is noted.

When the probe is inserted into an unknown field, the voltmeter reading is recorded and the field strength can then be calculated using proportionality.

Applications of Electromagnetism

1) DC Motor

2) Relay

3) Loudspeaker

4. FORCE BETWEEN CURRENT CARRYING CONDUCTORS

If two current-carrying conductors are placed near to one another, then each of the conductors is in the magnetic field which the current in the other creates. When the currents in the two long straight conductors X and Y are in the same direction, there is a force of attraction between them which tends to pull the conductors towards each other [see figure (i)]. When the currents flow in opposite directions, there is a repulsive force between them which tends to pull the conductors away from one another [see figure (ii)]. Fleming's left hand rule confirms the direction of the forces.

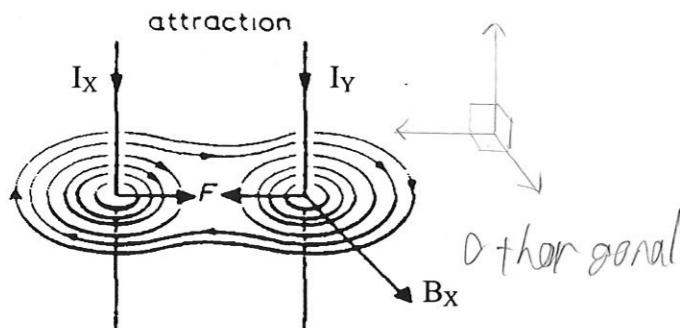


Figure (i): Forces between currents in same directions

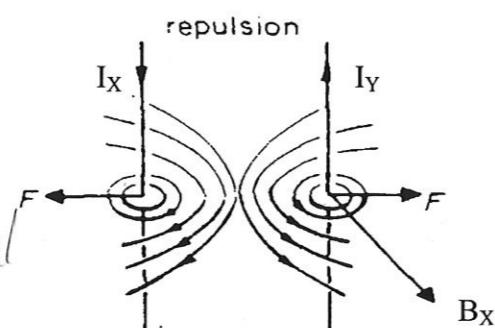
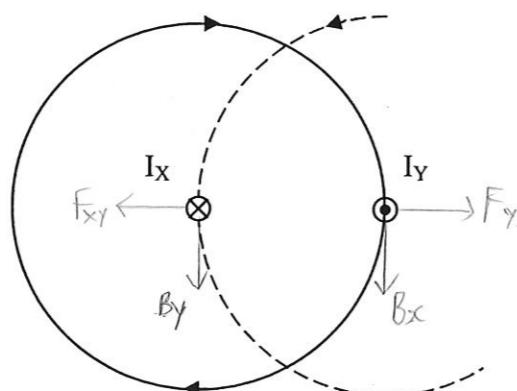
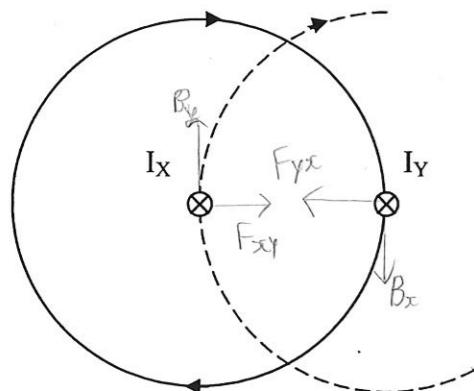


Figure (ii): Forces between currents in opposite directions



In **figure (i)**, at Y, the flux density B_x due to the conductor X is perpendicular to Y. Thus, from Fleming's rule, the direction of the force F on Y is towards X.

Similarly, at X, the flux density B_y due to the conductor Y is perpendicular to X. Thus, from Fleming's rule, the direction of the force F on X is towards Y.

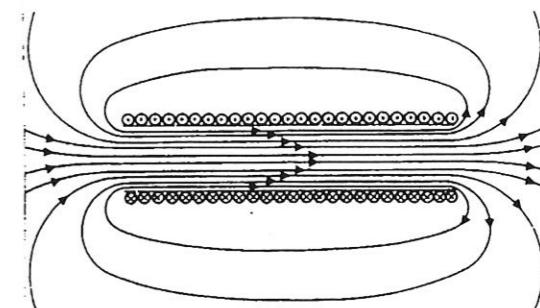
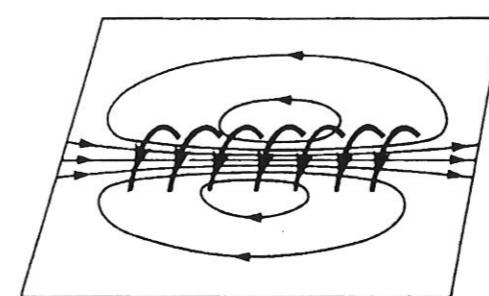
Hence the conductors **attract** each other.

In **figure (ii)**, the current in Y is opposite to that in **figure (i)**. From Fleming's left hand rule, the forces F on Y is now away from X and therefore the force is **repulsive**.

$$\frac{\mu_0(2I)}{2\pi r}(I)L = \frac{\mu_0 I}{2\pi r}(2I)L$$

2.3 Solenoid

A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonable uniform magnetic field can be produced in the space surrounded by the turns of wire.



The magnetic flux density at the centre of the solenoid is given by

$$B = \mu_0 n I \quad (\text{do not memorise this formula})$$

where $n = \text{no. of turns per unit length of the solenoid}$

Note:

1. The B -field is constant throughout the whole inner region, and over the whole cross-section.
2. The B -field is almost zero outside the solenoid, except near the ends.
3. The B -field is independent of the cross-sectional area or the shape of the solenoid.
4. The B -field can be greatly increased by the insertion of a ferromagnetic material such as iron, cobalt or nickel into its core.

Ferrous iron core is a soft magnetic material and can be easily magnetised. The field generated by the current in the solenoid partially aligns the domains in the iron, creating a much stronger field in the same direction. The magnetic flux density in iron B_{iron} is many hundreds of times larger than B_{air} .

3. FORCE ON CURRENT-CARRYING CONDUCTOR

Let's examine these arguments:

What will happen when you place 2 magnets near to one another?	They attract or repel ie. exert a force on each other
What will be generated around a wire carrying current?	Magnetic field
What will happen when a wire carrying current is placed near a magnet?	They exert a force on each other.

The interaction of 2 or more magnetic fields from different sources will result in a **magnetic force**. The following experiment demonstrates the force experienced by a wire carrying current placed in a magnetic field.

EXAMPLE 1

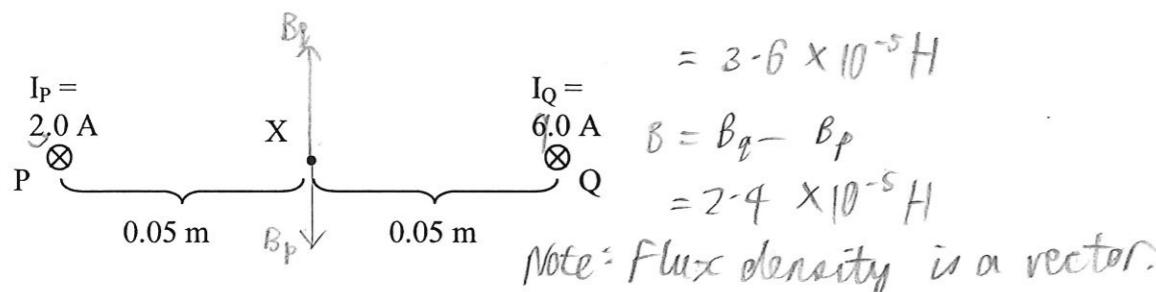
P and Q are straight wires 0.10 m apart each carrying 2.0 A and 6.0 A respectively, flowing into the plane of the paper as shown below. X is a point midway between P and Q.

Given that $B = \frac{\mu_0 I}{2\pi r}$, determine the magnitude and indicate on the diagram the direction of magnetic flux density at point X

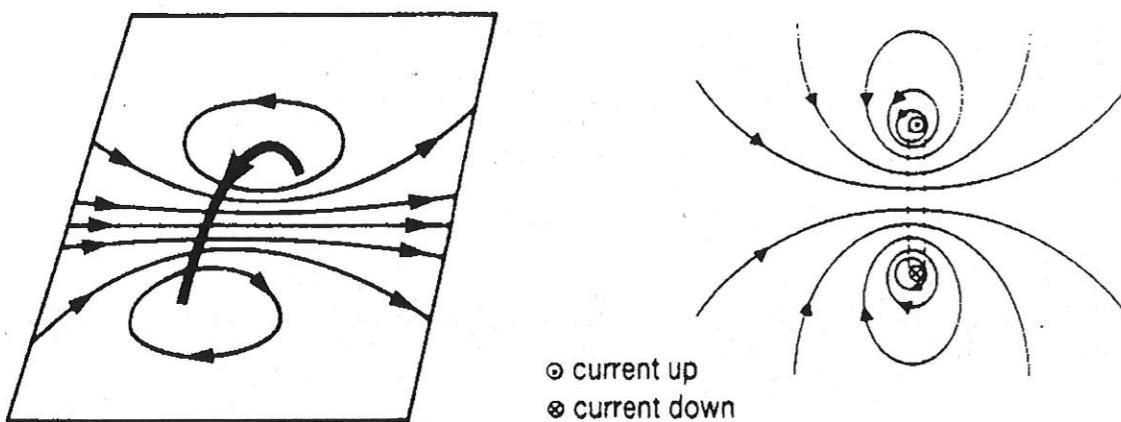
$$B_p = \frac{(4\pi \times 10^{-7})(3)}{2\pi(0.05)} \\ = 1.2 \times 10^{-5} \text{ H}$$

- (a) B_p , due to current in P only;
- (b) B_Q , due to current in Q only;
- (c) B , due to both currents in P and Q, ie. resultant field at X.

$$B_Q = \frac{(4\pi \times 10^{-7})(6)}{2\pi(0.05)} \\ = 3.6 \times 10^{-5} \text{ H}$$

**2.2 Circular coil**

The diagram below shows a wire bent into a single circular coil. This coil can be considered as made up of many small, straight segments each adding its individual magnetic field together at the centre of the loop where the field is strongest and will be directed through the coil as shown. The direction of the field lines are determined by the **right-hand grip rule**. In this case, the fingers point the direction of the current while the thumb points the direction of the field.



Suppose the coil has a radius r and carries a current I , the magnetic flux density at the centre of the coil is given by

$$B = \frac{\mu_0 I}{2r} \quad (\text{do not memorise this formula})$$

Note:

This is an example of Newton's Third Law. The force that each wire experiences are equal in magnitude and opposite in direction, regardless of the current flowing in each wire.

In figure (i), suppose $I_X = 2 I_Y$. Since magnetic flux density due to a current is directly proportional to the current, hence $B_X = 2 B_Y$.

Therefore,

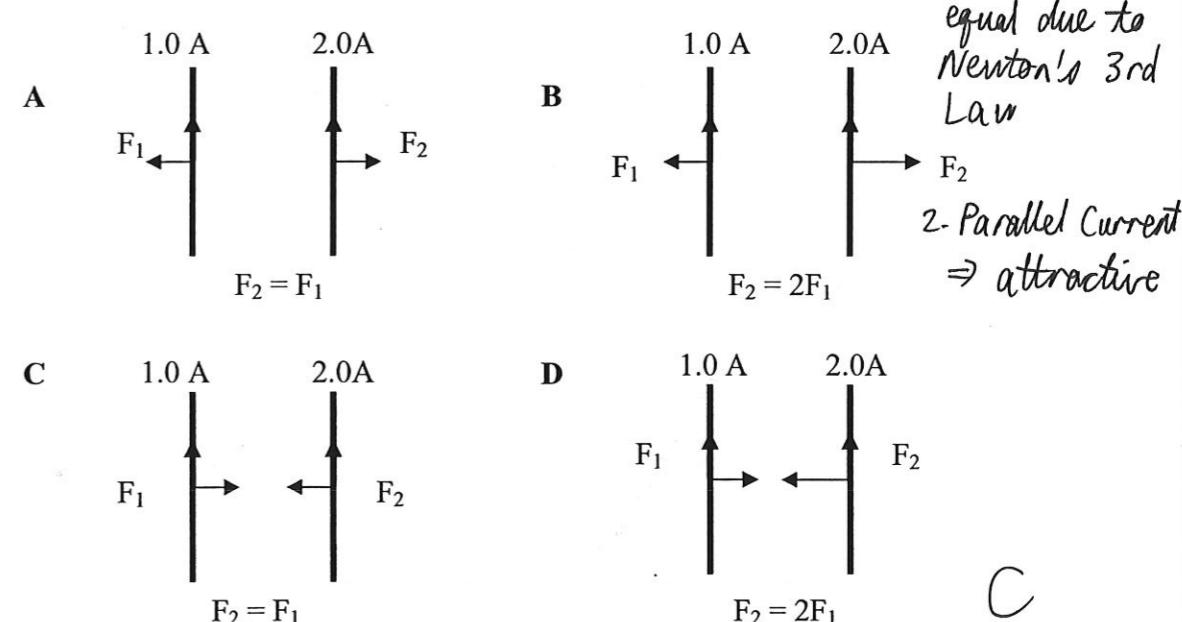
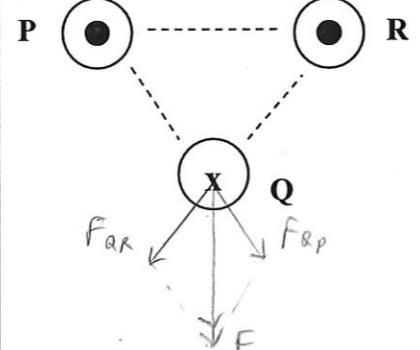
Force experienced by X, $F_{XY} = B_y I_x L$ where L = length of the wire

Force experienced by Y, $F_{YX} = B_x I_y L$

Notice that even though the current in X is greater, the force X experiences also depends on the magnetic flux density due to Y which is lesser. As a result, the force experienced by X is exactly equal in magnitude to the force experienced by Y.

EXAMPLE 6

Two long, straight, parallel wires carry currents of 1.0A and 2.0A. Which diagram shows the directions and relative magnitudes F_1 and F_2 of the forces per unit length on each of the wires? [N96/I/18]

**EXAMPLE 7**

Three long wires pass through the corners of an equilateral triangle PQR. They carry equal currents into or out of the paper in the directions shown in the diagram. By drawing a vector diagram, show the direction of the resultant force F on the wire at Q.

Magnetic Field & Electromagnetism Tutorial**Self Attempt**

- S1. A wire carrying a current 10 A and 2.0 m in length is placed in a field of flux density 0.15 T. What is the force on the wire if it is placed

- (a) at right angles to the field,
- (b) at 45° to the field.

Determine the angle of the wire to the field when there is no force acting on the wire.

(3.0 N, 2.1 N, 0°)

- S2. A straight horizontal rod X, of mass 50 g and length 0.5 m, is placed in a uniform horizontal magnetic field of 0.2 T perpendicular to X. Calculate the current in X if the force acting on it just balances its weight. Draw a sketch showing the directions of the current, field and force. (4.9 A)

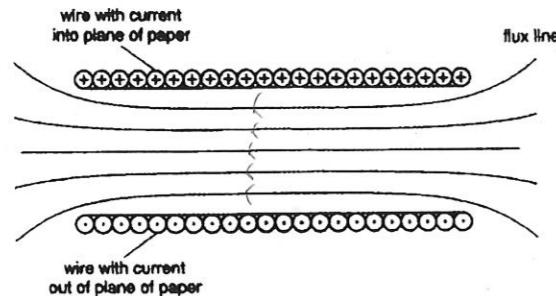
- S3. (a) A current of 1.5 A flows in a solenoid of 5 turns cm⁻¹. Calculate B, the flux density of the magnetic field at the centre of the solenoid is given by the expression $B = \mu_0 n I$ where n = no. of turns per unit length. (0.94 mT)

- (b) When a Hall probe is calibrated by placing it in the middle of this solenoid, a Hall voltage of 9.8 V is noted. The calibrated Hall probe is then placed 1.0 cm away from a straight current-carrying conductor and a Hall voltage of 1.1 V is recorded. Given that the magnetic flux density due to current in a straight conductor is $B = \frac{\mu_0 I}{2\pi r}$, determine the current flowing in the conductor. (5.3 A)

Bx Venet

Discussion Questions

- D1. The figure illustrates the patterns of the magnetic flux due to a current in a solenoid.



- (a) On the figure,
 (i) draw arrows to show the direction of the magnetic field in the solenoid,
 (ii) draw a line to represent a current-carrying conductor in the magnetic field which does not experience a force due to the magnetic field. Label the conductor C.
 (b) The coils of wire on an electromagnet are usually wound on a ferrous core. State two properties of the core which are important in its use in an electromagnet.

2. MAGNETIC FIELDS DUE TO CURRENT

A current flowing in a wire will generate a magnetic field. A greater current will result in a stronger magnetic field, ie. greater flux density. On the other hand, the flux density at a point further away from the wire will be smaller. These relationships can be generalized by the following formula:

$$B \propto \frac{I}{r}$$

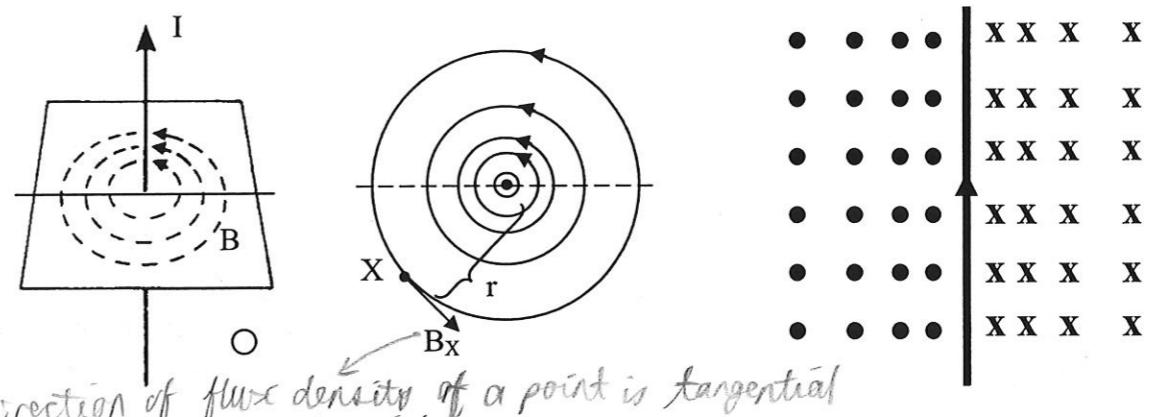
where

B is the magnetic flux density (a measure of the strength of the field)
 I is the current in the wire
 r is the distance of the point from the wire

2.1 Straight wire

For a straight wire, the field lines are a series of concentric circles centered on the wire. Further away from the wire, the magnetic field is weaker and hence the concentration of the field lines are lesser.

The direction of the field can be found by using the **right-hand grip rule**. Grip the wire using the right hand with the thumb pointing in the direction of the current – the fingers then point in the direction of the field.



Direction of flux density of a point is tangential to field line at that point PLAN

Suppose X is a point at a perpendicular distance r from the wire carrying a current I. The magnetic flux density at X is then given by

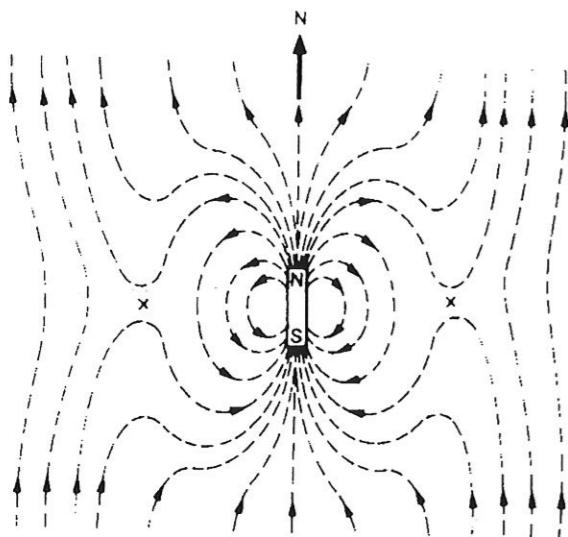
$$B_x = \frac{\mu_0 I}{2\pi r} \quad (\text{do not memorise this formula})$$

Permeability (μ) of a medium indicates the ability of a medium to concentrate the magnetic field lines. Magnetic materials are able to concentrate field lines significantly (by aligning their domains) and therefore have greater permeability than non-magnetic materials.

μ_0 is known as the permeability of free space. It can be found in the table of constants:

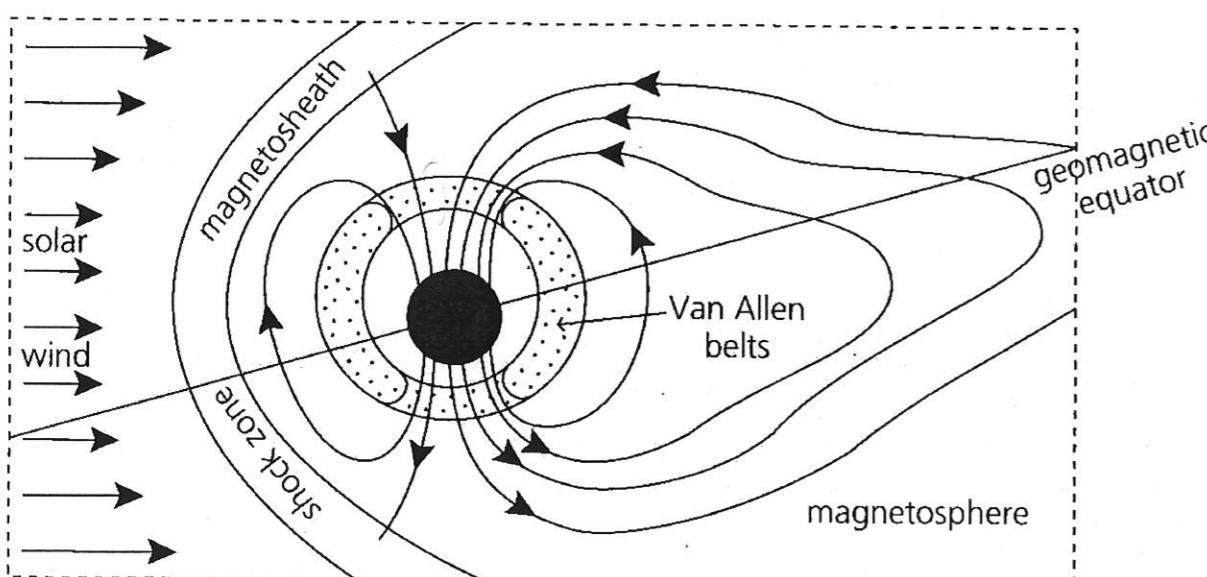
$$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$$

Note that the “Earth’s magnetic North Pole” acts like the “South Pole of a magnet”. Hence, if a magnet is suspended freely, it will orientate itself such that the “North pole of a magnet” will point towards the “Earth’s magnetic North Pole”. This is the working principle of a compass.



Field pattern showing a magnet in the Earth's magnetic field.

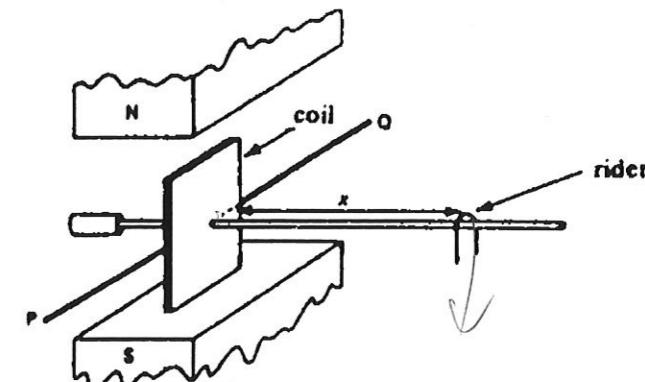
The presence of the Earth's magnetic field protects the inhabitants of the earth from solar winds which comprises of radioactive particles that come from the sun. These particles are trapped in regions called Van Allen radiation belts.



- D2.** A narrow vertical rectangular coil is suspended from the middle of its upper side with its plane parallel to a uniform horizontal magnetic field of 0.020 T. The coil has 10 turn, and the lengths of its vertical and horizontal sides are 0.10 m and 0.050 m respectively. Calculate the torque on the coil when a current of 5.0 A is passed into it. Draw a sketch showing the directions of current, field and torque. (5.0×10^{-3} Nm)

- D3.(a)** A small coil of N turns has sides of length L and is mounted so that it can pivot freely about a horizontal axis PQ, parallel to one pair of sides of the coil, through its centre (see figure). The coil is situated between the poles of a magnet which produces a uniform magnetic field of flux density B . The coil is maintained in a vertical plane by moving a rider of mass M along a horizontal beam attached to the coil. When a current I flows through the coil, equilibrium is restored by placing the rider a distance x along the beam from the coil. Starting from the definition of magnetic flux density, show that B is given by the expression

$$B = \frac{Mgx}{IL^2N}$$

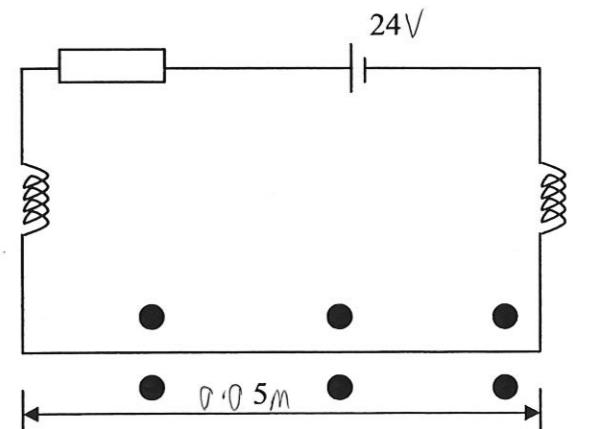


- (b) If the current is supplied by a battery of constant e.m.f. and negligible internal resistance, discuss the effect on x if the coil is replaced by one wound with similar wire but having
 (i) sides of length L with $2N$ turns,
 (ii) N turns with sides of length $L/2$.

- D4.** A straight wire of mass 10 g and length 5 cm is suspended from two identical springs which, in turn, form a closed circuit as shown. The total resistance of the circuit is 12Ω .

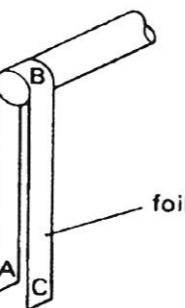
Each spring is stretched a distance of 0.5 m under the weight of the wire. When a magnetic field is turned on, directed out of the page (indicated by the dots), the springs are observed to stretch an additional 3 cm. Calculate the following

- (a) the force constant of each spring.
 (b) the strength of the magnetic field.
 $(0.0981 \text{ Nm}^{-1}, 0.0589 \text{ T})$



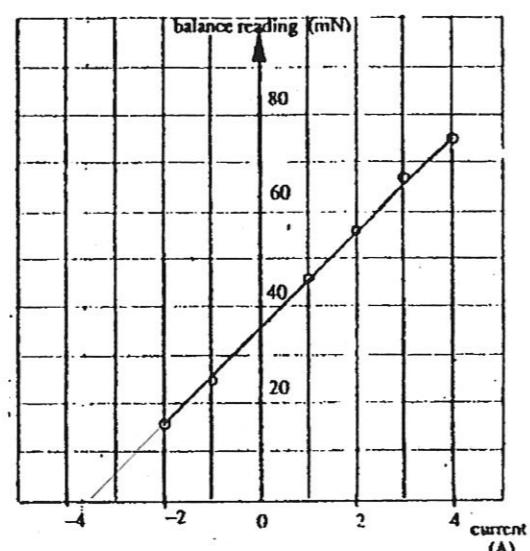
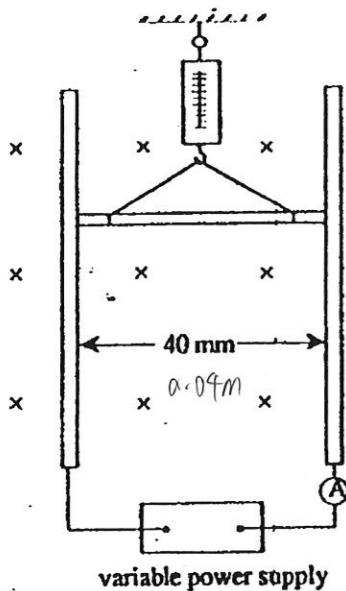
- D5.** A long strip ABC of aluminium foil is hung over a wooden peg as shown in the figure. A car battery is connected for a short time between A and C.

Describe and explain what will be seen to happen to the foil whilst there is a current.



Assignment Questions

- A1.** Two smooth electrically conducting vertical tracks are joined by a horizontal metal slide of length 40 mm. The slide is supported by a sensitive spring balance and is free to move up and down the tracks in a uniform horizontal magnetic field, B, at right angles to its length as shown.



The spring balance, which is calibrated in millinewton, reads zero when it is hanging vertically and supporting no load. The tracks are connected to a variable power supply which can provide a current in either direction through the circuit.

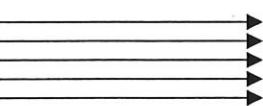
A student passes various currents through the circuit and notes corresponding readings on the spring. The results are shown in the graph above.

Use the graph to determine

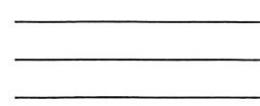
- the mass of the slide,
- the current flowing when the spring balance reads zero,
- Indicate the direction of the current on the diagram when the spring balance reads zero, and
- the value of the magnetic field B.

- A2.(a)** (i) Write down the equation defining magnetic flux density in terms of F the force it produces on a long, straight conductor of length L carrying a current I at an angle θ to the field.
(ii) Draw a clear diagram to illustrate the direction of the force relative to the current and magnetic field.
(iii) Hence, define the *tesla*.

Uniform fields:

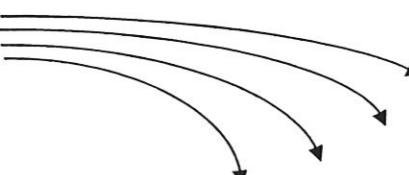


Strong field



Weak field

Non-uniform field:



1.5 Convention for representing Field Directions

• represents a field directed into the plane of paper.

• represents a field directed out of the plane of paper.



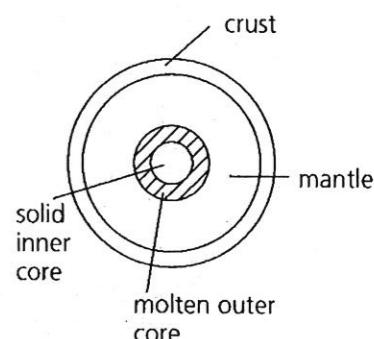
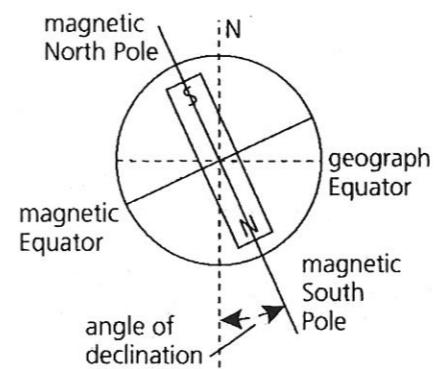
magnetic field into plane of paper



magnetic field out of plane of paper

1.6 Magnetic Field of the Earth

The earth has a magnetic field surrounding it which is called the *magnetosphere*. It is thought that the field is caused by the cyclic motion of the molten material, mainly iron and nickel, that makes the outer core of the earth. These motion, together with the spinning of the earth itself, produces electric currents that flow through the Earth and maintain the magnetic field.



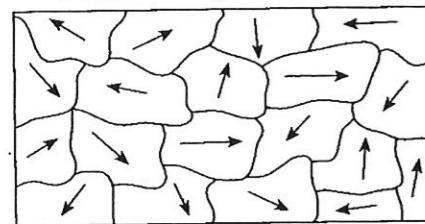
1. CONCEPT OF MAGNETIC FIELD

1.1 Theory of Magnetism

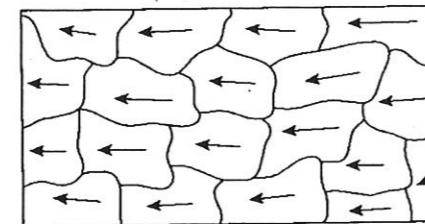
Magnetic substances are those that can be magnetised. The elements iron (Fe), cobalt (Co) and nickel (Ni), together with certain alloys, display the strongest magnetic properties.

Where does the magnetic properties of magnetic substances come from? Each electron in an atom act as a small magnet due to its motion of rotation and spin. In most materials the combined effect of many spinning electrons within the atom cancels out any net magnetism surrounding an individual atom or collective region of atoms within the material. In the case of ferromagnetic materials, the spin of the electrons does not cancel out but produces a magnetic effect associated with the atom. Adjacent atoms subsequently affect each other and become aligned over small zones or regions that are called magnetic domains of the materials.

(a) unmagnetised material—domains aligned randomly



(b) magnetised material—domains line up



1.2 Magnetic Field

A magnetic field may exist at a point as a result of the presence of either a permanent magnet or a conductor carrying an electric current, in the vicinity of the point.

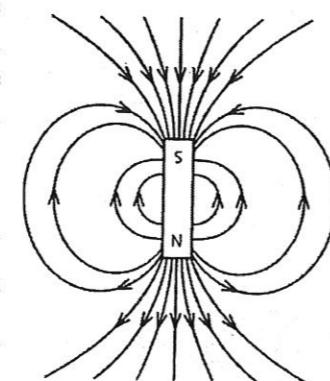
1.3 Definition of Magnetic field

A magnetic field is a region of space in which a magnetic material or moving charge located in it will experience a force.

1.4 Magnetic Lines of Force

A magnetic field can be represented by so-called lines of force. If the lines are parallel, the associated field is uniform. Otherwise, the field will vary in strength from one point to another. The field is said to be strong if the lines of force are crowded very closely together and weak when they are widely separated from one another.

Direction of the magnetic field, at a point, is taken as the direction in which a force is exerted on the North pole of a magnet placed at that point. The path that such a pole would follow is called a magnetic field line (or line of force).



A2.(b) Two long straight conducting wires are separated by a distance d . Each carries a current I in the same direction.

- Explain, with the aid of sketches, the forces which exist between the two wires. Predict the direction of the forces.

$$(ii) B, the magnetic flux density due to a long straight wire, is given by B = \frac{\mu_0 I}{2\pi r}.$$

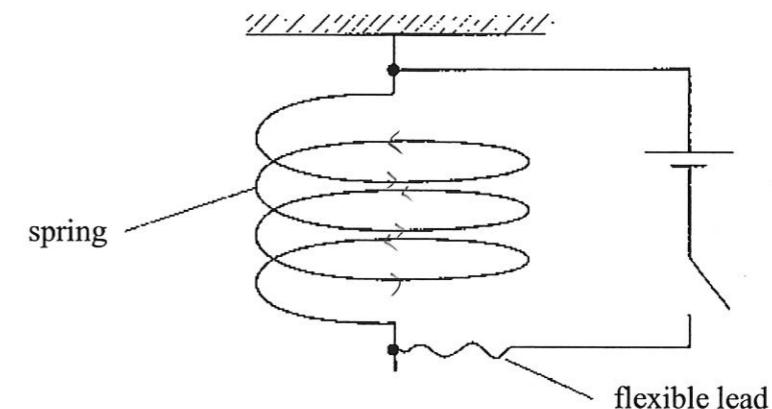
Derive an expression for the force per unit length between the two wires.

(c) One particular overhead powerline consists of two parallel cables with a separation of 6.0 m. The current in each cable is 200 A.

- Calculate the force per unit length on each cable.
- Hence, explain why it is not possible, by looking at the cables, to detect the instant at which the current is switched on.

A3.(a) There are two situations in which a charged particle in a magnetic field does not experience a magnetic force. State these two situations.

(b) A loosely-coiled spring is suspended from a fixed point as illustrated below.

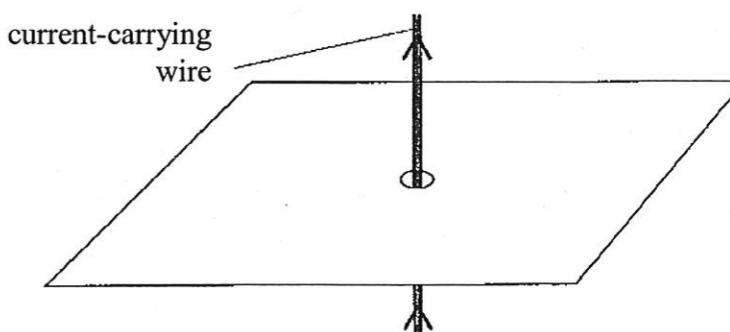


$$F = qvB \sin \theta$$



Electrical connections are made to the ends of the spring. When a current is switched on in the spring, there is a small change in the length of the spring.

- On the figure, mark the direction of the current in the individual turns of the spring.
- On the figure below, draw the pattern of the magnetic field due to a straight current-carrying wire.



- Using your answers to (i) and (ii), explain why the spring changes in length when the current is switched on.

- State whether the spring expands or is compressed.

Magnetic Field & Electromagnetism

Content

1. Concept of magnetic field
2. Magnetic fields due to currents
3. Force on a current-carrying conductor
4. Force between current-carrying conductors

Learning Outcomes

- (a) understand a magnetic field as an example of a field of force produced either by current-carrying conductors or by permanent magnets.
- (b) represent a magnetic field by field lines.
- (c) sketch flux patterns due to a long straight wire, a flat circular coil and a long solenoid.
- (d) understand how the field due to a solenoid may be influenced by the presence of a ferrous core.
- (e) appreciate that a force might act on a current carrying conductor placed in a magnetic field.
- (f) recall and use the equation $F = BIL\sin\theta$, with directions as interpreted by Fleming's left-hand rule.
- (g) define magnetic flux density and the tesla.
- (h) understand how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance.
- (i) describe the use a calibrated Hall probe to measure the flux density of a magnetic field.
- (j) explain the forces between current-carrying conductors and predict the direction of the forces.

References

1. Hutchings, Robert – Physics, Second Edition, Chapter 18
2. Walding, Richard – New Century Senior Physics, Chapter 25
3. Dobson, Ken – Physics, Second Edition, Chapter 10
4. Chan, KF – Comprehensive Physics for 'A' Level Vol 2, Third Edition, Ch 11

Summary

Magnetic effect of current: $B \propto \frac{I}{r}$ B due to I

Force: $F = BIL \sin\theta$ B not due to I