- The regular arrangement of the atoms act as scattering points for the electrons (compare with diffraction grating).
- If electrons behave like wave, constructive interference will be observed at fixed angles.
- Since diffraction pattern is obtained, electrons must have exhibited wave behaviour.

[Similar diffraction patterns are obtained if X-ray were to be used instead of electrons]

Internet Resource:

http://www.colorado.edu/physics/2000/schroedinger/two-slit3.html

4.3 Wave Nature of Particles : De Broglie Wavelength

- It is found that all other particles (protons, neutrons etc.) also exhibit similar wave behaviour. De Broglie suggested that a particle of momentum p behaves in some ways like wave of wavelength λ given by

$$\boxed{\lambda = \frac{h}{p}} \qquad p = mv$$

- λ is known as the de Broglie wavelength of the particle. This formula can be applied to both particle and photon.

Does this relationship apply to all particles? Consider a pitched baseball:

$\longrightarrow$ v = 40 m/s = 90 mi/hr

m=0.15 kg $\quad \lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J s}}{(0.15\ \text{kg})(40\ \text{m/s})} = 1.1 \times 10^{-34}\ \text{m}$

10^{-10}m Atomic diameter

10^{-14}m Nuclear Diameter

For an electron accelerated through 100 Volts: $v = 5.9 \times 10^6$ m/s

$$\lambda = \frac{6.626 \times 10^{-34}\text{Js}}{(9.11 \times 10^{-31}\text{kg})(5.9 \times 10^6\ \text{m/s})} = 1.2 \times 10^{-10} = 0.12\ \text{nm}$$

This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

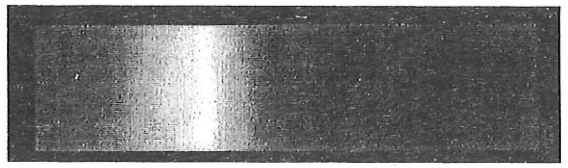
- According to the equation, a particle which has low momentum has long wavelength and thus exhibit obvious wave behaviour.
- For massive particles, p is large, λ is small and hence wave property is almost non-observable.
- Whether a particle exhibits wave or particle behaviour depends on its environment.
  - If the de Broglie wavelength is much smaller than the physical dimension of the surroundings, the particle will exhibit particle behaviour (e.g. electrons in vacuum tube).
  - If the particle is constrained to move in space comparable with its wavelength, wave behaviour becomes pronounced (e.g. electron in atom, electrons striking layers of atom in electron diffraction experiment). **Why?**

- If water wavelength ≈ slit width ⇒ appreciable diffraction
- If water wavelength is much < slit width ⇒ little diffraction

5. Energy Levels In Atoms

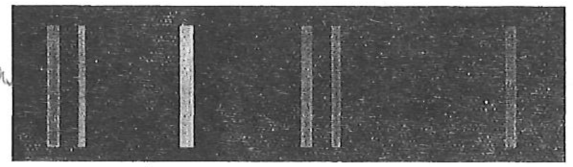
5.1 Historical Background – What are Atomic Spectra?

- When white light passes through a prism, a continuous spectrum of rainbow colour is observed.
- Similarly, when you analyse the white light from a filament lamp, through a diffraction grating, you are able to observe the continuous spectrum of colour at the various maxima position other than the central maximum. One example is shown below.



- But there are discontinuous spectra called line spectra – the radiation emitted by pure elements when they are heated or ' electrically disturbed'. [we shall discuss in later sections how such spectra can be obtained]
- Each element is found to have its own unique set of spectra. One example is shown below.

Discontinuous/discrete spectra due to electron transition between energy levels in an atom



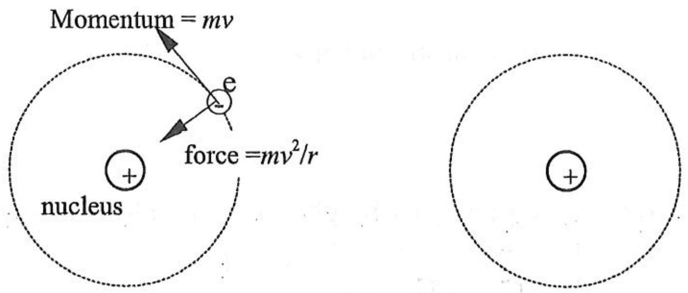
- This phenomenon could not be explained until 1913, when Bohr proposed his theory of atomic structure.

5.2 Model of the Hydrogen Atom

5.2.1 Early Model of the Hydrogen Atom

According to Rutherford:

- The electron revolves around the nucleus with a uniform motion due to Coulombic force, in agreement with Newton's law.

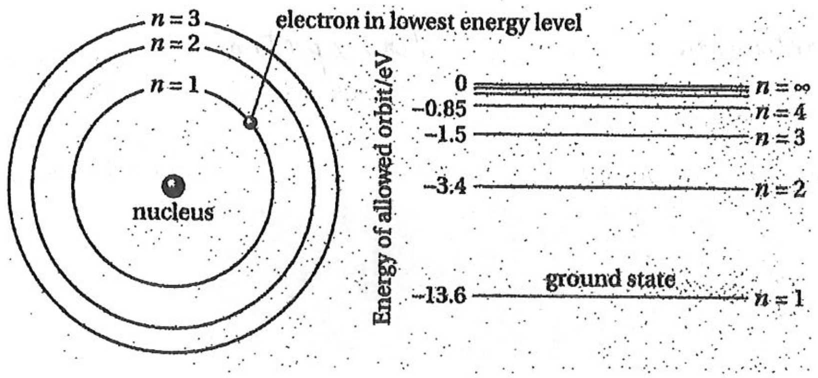


- There is a problem to this model. That is all charged particles emit radiation when they accelerate.
- The orbiting electron is accelerating towards the centre of its orbit as it is constantly changing direction.
- According to the laws of classical physics, the electron would be radiating energy all the time.
- As it radiates, it should loose energy, eventually spiralling down towards the nucleus.

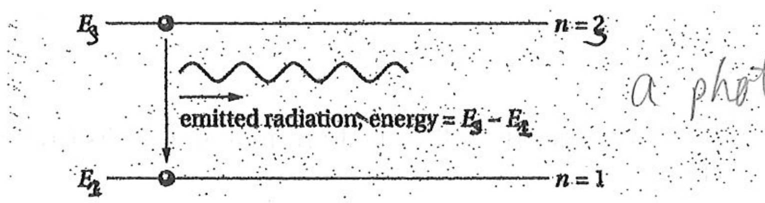
5.2.2 Bohr's Model of the Hydrogen Atom

According to Bohr:

- He suggested that the electron could travel in certain allowed orbits without losing energy. He called these allowed orbits Energy levels.
- The single electron is normally in the 'lowest orbit' or 'lowest energy level'. When you supply energy to a hydrogen atom, the electron can move to an orbit further from the nucleus.
- The hydrogen is said to be excited. Such excited atom is unstable, and the electron quickly falls back to its normal orbit. (the one nearest the nucleus)
- As it does so, the electron must give up the energy given to it during the 'excitation'. The energy given up is emitted in the form of EM radiation. That accounts for spectrum observed.



Allowed orbits and energy levels in a Bohr's hydrogen atom



a photon given out

An electron transition

- Using conservation of energy, the frequency of the photon can be found from

$$\boxed{E_1 - E_2 = hf}$$

$$f = \frac{E_3 - E_1}{h}$$

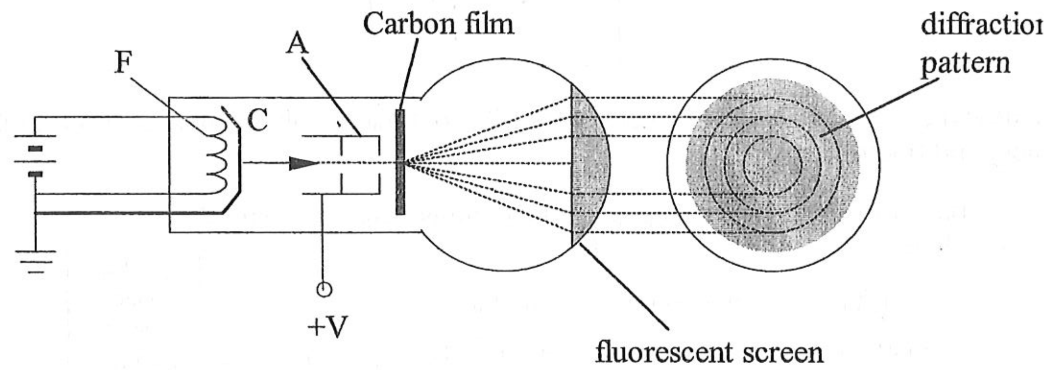
4. Wave-Particle Duality

4.1 Historical Background

- As a young student at the University of Paris, Louis DeBroglie had been impacted by relativity and the photoelectric effect, both of which had been introduced in his lifetime. The photoelectric effect pointed to the particle properties of light, which had been considered to be a wave phenomenon. He wondered if electons and other "particles" might exhibit wave properties.
- His proposal was not taken seriously until 1928 when Davisson and Germer, two American scientists, showed that electrons could be diffracted in the same way as light and water waves.
- Confirmation of the DeBroglie hypothesis came in the Davisson- Germer experiment. The modified experiment is described as below:

4.2 Electron Diffraction Experiment

The diagram below shows an apparatus used to demonstrate electron diffraction.



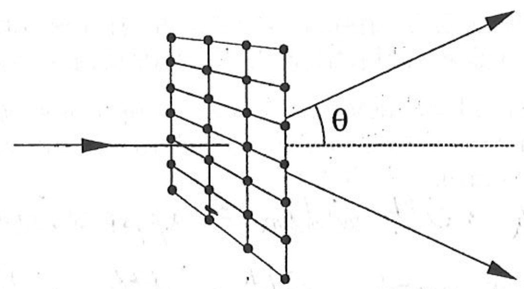
Set-up

- The cathode C is heated by filament F, so that electrons are emitted by thermionic emission.
- These electrons are then accelerated through a high p.d. of V, by making the anode (A) positive in potential. The thin slit on the anode causes a fine beam of electrons to emerge.
- These electrons will then pass through the thin film of crystal made of graphite. The dispersion pattern of the emergent electron is displayed on the coated fluorescent screen.

Observation

- The pattern produced on the screen is observed to be similar to the interference pattern produced by electromagnetic radiation. This is known as electron diffraction phenomenon.

Explanation



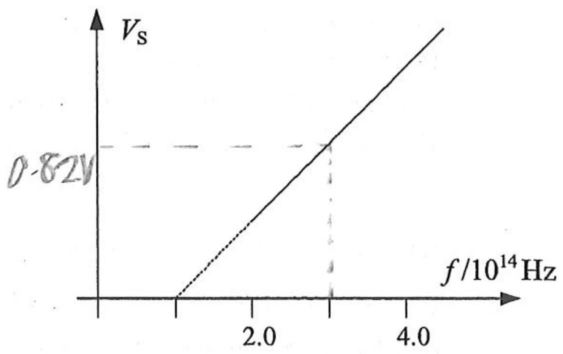
3.5 The figure shows how the stopping potential of photoelectrons vary with frequency of radiation illuminated on a metal.

(i) What is the gradient of the graph?

(ii) What is the threshold frequency for the metal?

(iii) What is the work function of the metal?

(iv) If radiation of 9×10^{14} Hz falls on the plate, what is maximum kinetic energy of electrons emitted?



(i) gradient $= \frac{h}{e}$

$= 4.1 \times 10^{-15}$

(ii) From the graph, $f_0 = 1.0 \times 10^{14}$ Hz

(iii) $\phi = hf_0$

$= 6.6 \times 10^{-20}$ J

(iv) $KE_{max} = hf - \phi$

$= (6.63 \times 10^{-34})(9 \times 10^{14}) - 6.6 \times 10^{-20}$

$hf - \phi$

$= hf - hf_0$

$= h(f - f_0)$

$= h(9.0 \times 10^{14} - 1.0 \times 10^{14})$

$= 1.3 \times 10^{-19}$ J

What is the significance of line spectra?

The fact that the spectra produced by pure elements are discrete indicates that the electrons in the atom can occupy certain allowed orbits, hence have fixed energy levels.

But how?

The electron can only move from one allowed energy level to another by gaining or losing specific amount of energy. That is why only certain frequencies of light appear in the line spectrum.

Pair exercise: Discuss the following review questions to check your understanding!

What are atomic spectra?

Spectra of radiations due to transitions of electrons between energy levels in an atom.

How do you describe line spectra?

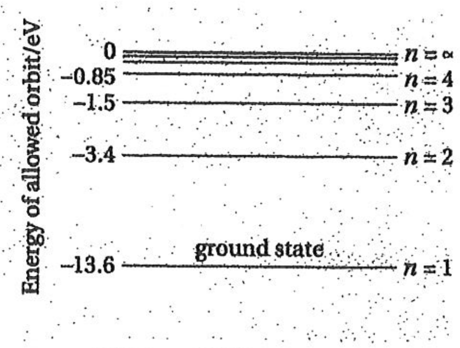
A line spectra consist of a pattern of closely spaced lines

A single photon is emitted when an electron moves from a higher energy level to a lower one. What happens when the electron moves from a lower to a higher energy level?

Energy is absorbed by the electron when it moves from lower to higher energy level.

5.3 Energy Levels in Atoms

- The energy levels within an atom which can be occupied by the electrons are often represented by horizontal lines arranged one above the other like a ladder with unequal spacing. The type of diagram shown in the figure on the right is called an **energy level diagram**.



Why are the energy levels negative?

Because the electron is in a bound state – it is tied to the atom.

- Just as the satellite is attracted to the earth and the satellite has a negative potential energy, the electron is also attracted to the nucleus of the atom.
- The energy level of a free electron that corresponds to n = ∞ has $E_\infty = 0$.
- The most stable state of the electron is when n = 1 when its energy is most negative, indicating it is most tightly bound to the nucleus.
- In atoms with more electrons, it is more complicated because the electrons also interact with each other.
- In such cases, we do not talk about energy state of each electron, but energy state of the entire atom.

Terminology

Some terms you should acquaint with:

Ground State -	This is the most stable, and lowest energy level available in the atom.
Excited State -	All energy levels above the ground state are called excited states.
Excitation Energy -	This is the energy required to promote the atom from the ground state to the excited state.
Ionisation Potential -	The p.d. required to remove an electron from an atom.
Ionisation Energy -	The energy that must be absorbed to remove an electron from an isolated atom. This is the found by difference in energy between that the present energy level of the atom and the energy level at $n = \infty$ (usually $E = 0$).

3.3 A monochromatic light source of constant intensity (wavelength 300 nm) is used to illuminate the surface of a clean piece of metal plate in a photoelectric emission experiment. It is found that a maximum current of 5 nA is obtained as the potential of the metal plate is varied and the current drops to zero when the potential difference between the emitter and collector exceeds 1.0 V.

(i) Sketch the graph of current vs p.d. between the emitter and collector.

(ii) Deduce the maximum kinetic energy of the photoelectrons.

(iii) Deduce the work function of the emitter.

Max $KE = eV_s$

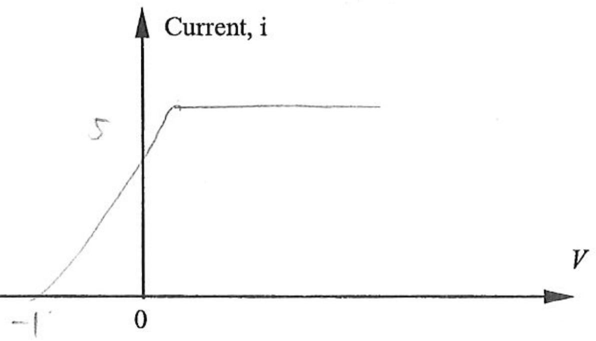
$= (1.6 \times 10^{-19})(1.0)$

$= 1.6 \times 10^{-19}\ J$

$\phi = hf - KE_{max}$

$= \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{300 \times 10^{-9}} - 1.6 \times 10^{-19}$

$= 5.03 \times 10^{-19}\ J$



3.4 The diagram shows the *i*-*V* graph when ultraviolet is incident on a piece of sodium strip. Sketch changes, if any, in the *i*-*V* graph if each of the following is carried out separately.

(a) Intensity of radiation is increased.

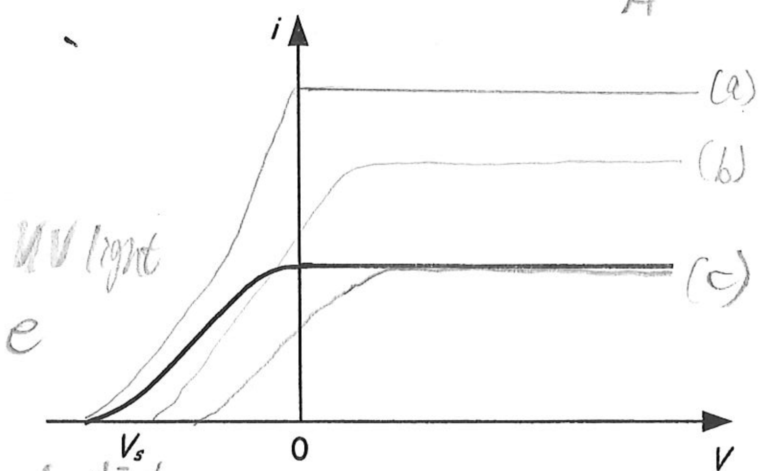
(b) Green light is used instead.

(c) Sodium is replaced with zinc.

$I_{green} = \frac{n h f_{green}}{A}$

$I_{uv} = \frac{n' h f_{uv}}{A}$

a)



b) f of green light < f of UV light
Hence max KE of emitted e will be lesser
Hence smaller stopping potential
Why is I ↑ at +ve voltage? > e collected at anode
∴ more photons incident on metal surface

c) ϕ(zinc) > ϕ(sodium)
Hence $hf - \phi$ is smaller
Hence KE_{max} is smaller. A smaller V_s.

III The stopping potential is determined as the frequency is varied. As mentioned earlier,

$$\frac{1}{2}mv_{max}^2 = hf - \phi = hf - hf_0 \qquad (1)$$

and

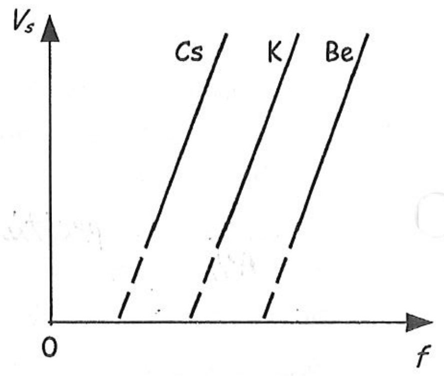
$$\frac{1}{2}mv_{max}^2 = eV_s$$

Since the work function, ϕ, is the minimum energy an electron in a given metal to escape, the minimum energy, E_0, a photon must have to liberate it is given by

$$E_0 = \phi$$

As $E = hf$, there exist a threshold frequency, f_0, below which there is no photoelectric emission. Hence,

$$hf_0 = \phi$$



Thus, equation (1) becomes

$$eV_s = hf - hf_0$$

$$V_s = \frac{h}{e}f - \frac{h}{e}f_0$$

Thus the graphs of V_s vs f for various metals are straight lines of the same gradient h/e but of different intercepts as the metals have their characteristic work functions hence threshold frequencies.

Exercises

3.2 Monochromatic light of wavelength 550 nm is incident on a metal surface in a vacuum photocell to produce photoelectric emission. The emission can be stopped by applying a positive potential of 1.40 V to the metal plate. Calculate

a) the work function of the metal,

b) the maximum K.E. of emitted photoelectrons.

$eV_s = hf - \phi$

$\phi = \frac{hc}{\lambda} - eV_s$

$= 1.38 \times 10^{-19} J$

Max KE = Work done

$= eV_s$

$= 1.6 \times 10^{-19} \times 1.40$

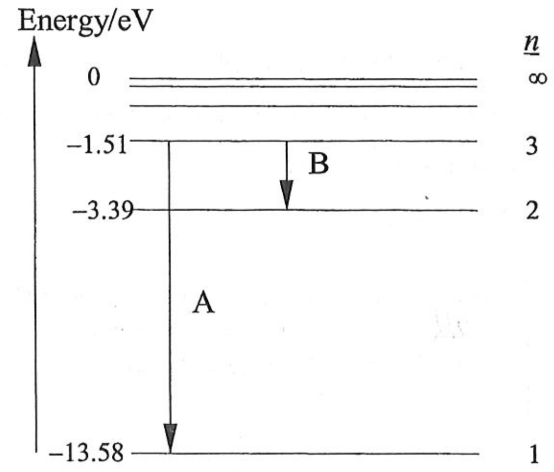
$= 2.24 \times 10^{-19} J$

Exercise

5.1. The figure shows the energy level associated with each principal quantum level of the hydrogen atom.

(i) Calculate the wavelength of the photon emitted in transitions A and B. In what part of the electromagnetic spectrum does this wavelength lies?

(ii) How much energy is required ionise the hydrogen atom in its ground state?



For transition A, Difference in energy levels,

$E_3 - E_1 = -1.51 - (-13.58)$

$= 12.07 eV$

Energy of a photon $= hf$

$= \frac{hc}{\lambda}$

$\lambda_A = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{12.07 \times 1.6 \times 10^{-19}}$

$= 1.03 \times 10^{-7} m$

UV radiation

For transition B, difference in energy level

$= E_2 - E_1$

$= -1.51 - (-3.39)$

$= 1.88 eV$

$\lambda_B = \frac{6.63 \times 10^{-34}(3.0 \times 10^8)}{1.88(1.6 \times 10^{-19})}$

$= 6.6 \times 10^{-7} m$

Red/orange light

5.3.1 How to excite an electron to a higher energy level?

A single photon is emitted whenever an electron moves from a higher energy level to a lower one. How can we excite the electron to a higher energy level in the first place?

- Energy is absorbed by the electron when it is excited to a higher energy level.
- To excite an electron, one can

transfer photon energy by

- Shining light on the substance

transfer thermal energy by

- Heating the gaseous element

transfer kinetic energy by

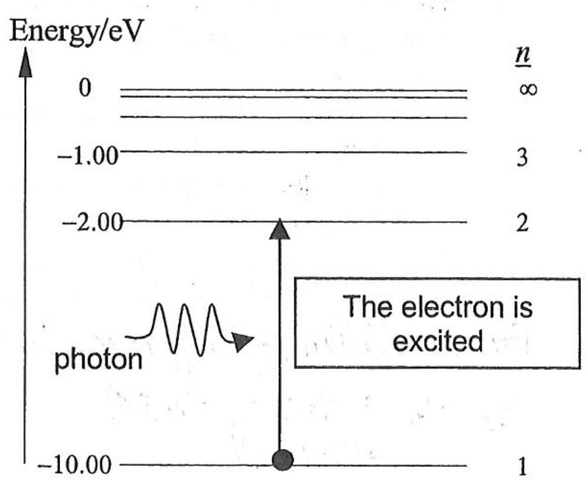
- Bombarding the gaseous atoms with stream of electrons/neutrons

5.3.2 More on Electron Excitation

Supposedly we shine blue light on the substance whose atomic energy level is shown on the right, the photon energy = hf_{blue}.

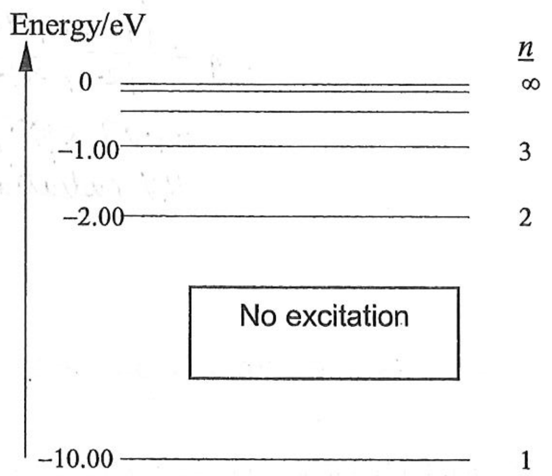
(i) If hf_{blue} = 8.00 eV,

The electron at the ground state will absorb all the photon energy. It is then excited to the energy level, n=2.



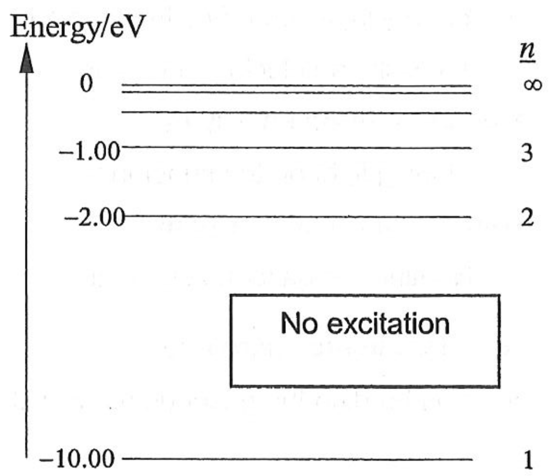
(ii) If hf_{blue} < 8.00 eV,

The photon energy is less than the minimum required. So **no excitation** takes place.



(iii) If hf_{blue} = 8.50 eV,

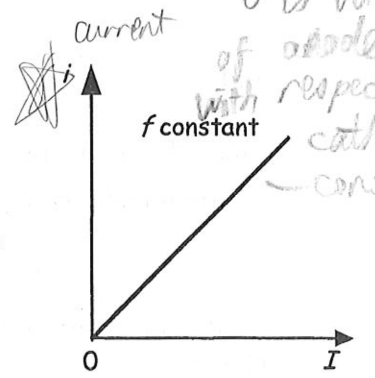
Remember that the photon either gives up all its energy or retains all the energy; The photon cannot keep part of the energy. If the photon gives up all its energy to the electron at the ground state, there is no allowable energy level which the electron can 'jump' to. So, no excitation takes place.



<u>Exercise: Try to describe the results as seen from the graph and then explain using Einstein's explanantion of the phenomenon of photoelectric effect.</u>

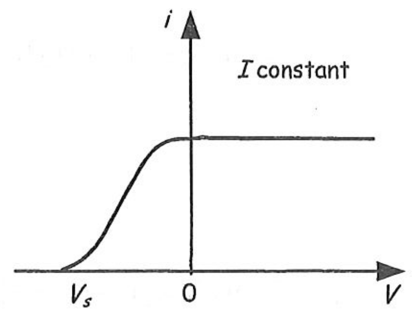
I The intensity of the light, I, is increased, it is found that the current, i, detected increases proportionately.
This is because as I increases, the number of photons incident on the metal per unit time increases. This results in a corresponding increase in the number of photoelectrons emitted and hence in the current detected. Note: provided $f > f_0$ or $hf > \phi$

current $\rightarrow \frac{Ne}{t}$



II For a given frequency and intensity of light, the current, i, detected is measured as the potential difference, V, is varied using the potentiometer.

When the V is positive, the potential at C (photo cathode) is lower than that at A. All photoelectrons emitted are collected at the anode A and hence, current detected is a maximum (saturation).



Note that V only affects the number of photoelectrons **detected**. The number of photoelectrons **emitted** is only affected by the intensity which is kept constant. Hence the current is unchanged as V becomes more positive.

When V is negative, only some of the electrons emitted are able to reach A. This is because only the more energetic electrons with kinetic energy, $\frac{1}{2}mv^2$ can overcome the potential barrier. In fact, the condition for the electrons to reach A is

$$\tfrac{1}{2}mv^2 > eV$$

As V becomes more negative, less electrons are collected and hence current drops. When V reaches V_s such that i becomes zero, even the most energetic electron just failed to reach A. V_s is called the stopping potential. In fact, eV_s measures KE_{max} of emitted electrons

$$\tfrac{1}{2}mv^2_{max} = eV_s$$

How will a change in intensity affect the graph? Sketch the graph of i vs. I above for incident light of intensity higher than in the case above.

Larger I_{max} due to ↑ photons per unit time

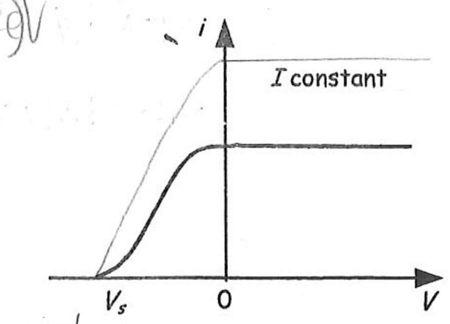
$i=0$ remains unchanged as photon energy remains unchanged

$K_i + (-e)(0) = K_f + (-e)V$

$K_i = K_f - eV$

$eV = K_f - K_i$

$= \frac{1}{2}mV^2_{max}$



$\frac{1}{2}mu^2 + 0 = 0 + (e)(V_s)$

$eV_s = \frac{1}{2}mV_{max}^2$

→ V_s is such that most energetic e⁻ is stopped

anode is negative & repels e⁻

Saturation current
⇒ all ejected e⁻ reaches anode
∴ anode attracts all e⁻

Exercises 1eV is energy required to accelerate 1e through potential of 1V. eV is a unit for energy $1eV = 1.6\times10^{-19}J$

3.1 The work function of potassium is 2.86 eV.

(a) Calculate the threshold frequency for the emission of electrons to occur in potassium.

(b) What is the range of kinetic energies and velocities of the electrons emitted when it is illuminated by radiation of wavelength 80 nm?

$\phi_K = 2.86\times1.6\times10^{-19} = 2.09\times10^{-19}J$

$hf_0 = 2.09\times10^{-19}$

$f_0 = 4.4\times10^{14}$ Hz

In general, $K.E = hf - W$

For $W=\phi$, $K.E_{max} = hf - \phi = \frac{hc}{\lambda} - \phi$

$KE_{max} = 2.03\times10^{-18}J$

$KE < 2.03\times10^{-18}J$

$0 < KE \le 2.03\times10^{-18}J$

$0 < \frac{1}{2}mv^2 \le 2.03\times10^{-18}J$

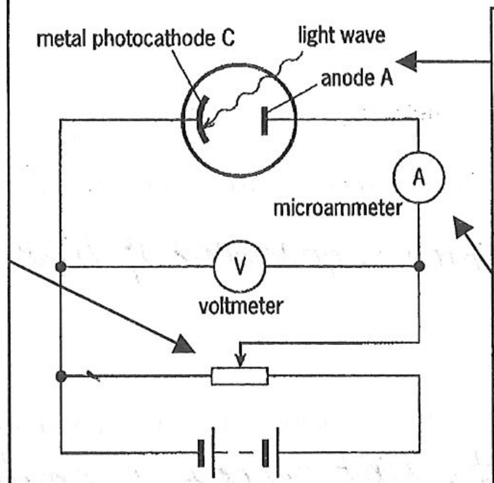
$0 < v \le 2.11\times10^{-6} ms^{-1}$

Mass of $e = 9.11\times10^{-31}$ kg

3.4 Experimental Set-up Investigating Photoelectric Effect – Results and Explanation

Due to photoelectric effect, light incident on the metal photocathode causes electrons to be emitted with a certain kinetic energy. The electrons are collected at the anode resulting in a current through the circuit. The important variables are indicated on the diagram below.

A **positive potential** may be applied to the anode A with the batteries thus accelerating the electrons to the anode.
A **negative potential** can also be applied to anode A which can repel the electrons. Energy required to move electron through the potential difference is eV.
Stopping potential, V_S is the negative potential which just stops all electrons (i.e. no current is registered) reaching the anode.
Thus, eV_S is the *maximum kinetic energy of electrons emitted.*



Intensity, I (brightness)
Frequency, f (colour)
Of the incident light

Current, i flowing through the ammeter indicates *number of electrons which reach anode A.*

How to collect more electrons? -- C +++ increase potential

How to impede e collected at A? +++ C --

Electron will not reach anode unless it has a ↑ K.E

Assume f = constant $I\uparrow \to N\uparrow$

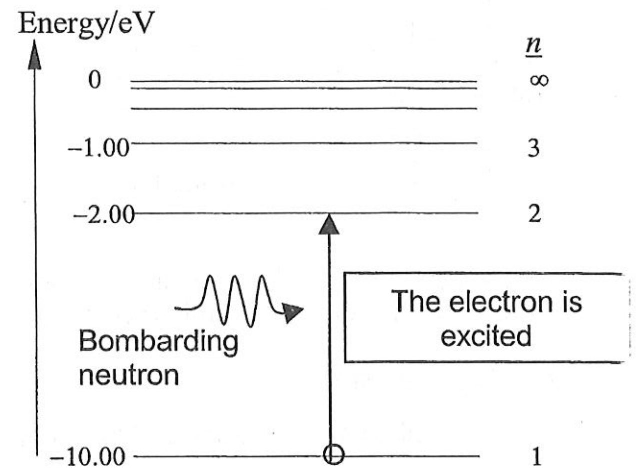
$f > f_0$ $I = \frac{N(hf)}{t \cdot A}$

V = constant

Supposedly we direct a stream of neutrons at the gaseous element, the bombarding neutrons would have KE = ½ mv^2.

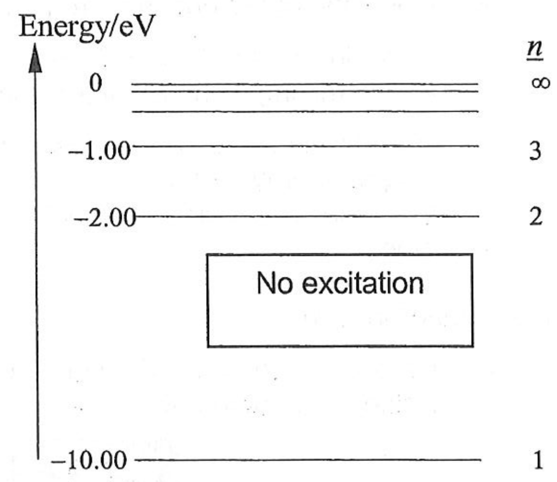
(i) If KE = 8.00 eV,

The electron at the ground state absorbs the KE of the incoming bombarding neutron. Excitation takes place from energy level n=1 to n=2.



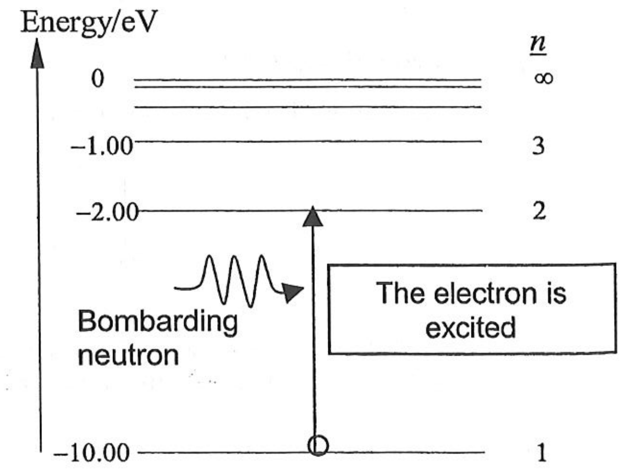
(ii) If KE < 8.00 eV,

The KE of the bombarding neutron is less than the minimum required. So **no excitation** takes place.



(iii) If KE = 8.50 eV,

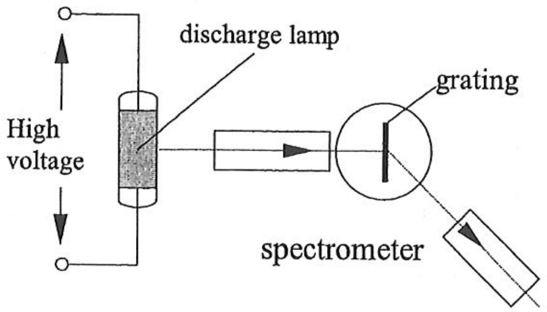
The KE of the bombarding neutrons is more than the minimum required. The bombarding electrons **transfer part** of its KE (8.00 eV) to excite the electrons at the ground state to n=2 while they are reflected with the remaining KE (0.50 eV).



Bombarding neutrons are deflected

6. Line Spectra

6.1 Emission Spectra



How does this work?
The gaseous atoms in the discharge lamp are constantly bombarded by free electrons which are accelerated between the electrodes of the lamp.
The atoms are excited to higher states. Being unstable, the electrons fall back to ground state, emitting photons of specific energy, hf. This corresponds to certain frequency.

These are patterns observed in a spectrometer using light emitted from matter.
There are three types of emission spectra:

(a) Line Spectra

- This consists of separate bright lines of definite wavelengths on a dark background.

- Line spectra are obtained from atoms in gases such as hydrogen or neon.
- Each line is due to transition of electrons from a higher energy level to a lower energy level, emitting a photon of fixed wavelength.
- No two elements give the same line spectrum because each element have their own unique energy levels.
- Line spectra provide convincing evidence for the existence of discrete energy levels in atom.

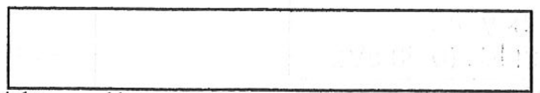
(b) Band Spectra

- Each band consists of a series of lines very close together at the head of the band and farther apart at the other end.

- Band spectra are essentially due to molecules.

(c) Continuous Spectra

- The entire background is bright.



- These are obtained from matter where energy levels are not quantised e.g. solids, liquids, plasma. Hence light from heated filament is continuous spectra.

3.3 More on Work Function, ϕ

- There are forces which normally hold the electron inside the metal, and it needs some **minimum energy** to do work against these forces before it escapes from the metal. This minimum escape energy ϕ is called the work function and is **unique** to each metal. The table shows the work functions for various metals.

Metal	W/eV	W/10^{-19} joules
sodium	2.28	3.65
aluminium	4.08	6.53
copper	4.7	7.52
zinc	4.31	6.9
silver	4.73	7.57
platinum	6.35	10.2
lead	4.14	6.62
iron	4.5	7.2
potassium	1.81	2.9

- If an electron absorbs energy E from the electromagnetic radiation to be ejected from the metal, by conservation of energy,

K.E. of electron = Energy absorbed from single photon – Work done in removing electron

- Hence the maximum kinetic energy, $\frac{1}{2}mv^2_{max}$, an escaped electron can possess is given by

$$\boxed{\frac{1}{2}mv^2_{max} = E - \phi}$$

- Since $E = hf$, so we have:

$$\frac{1}{2}mv^2_{max} = hf - \phi \quad \text{or} \quad hf = \frac{1}{2}mv^2_{max} + \phi$$

If electron has been emitted from deeper within the metal surface it may have a lower value of kinetic energy.

Pair Exercise: Discuss the following questions

1. From the equation, how does kinetic energy of the emitted electron vary with the frequency of incident photon?
K.E of emitted electrons ∝ frequency of light $KE_{max} = hf - \phi$

2. From the equation, deduce the conditions under which will no electrons be emitted.
When $hf < \phi$ i.e. the light energy is smaller than the work function of the metal.

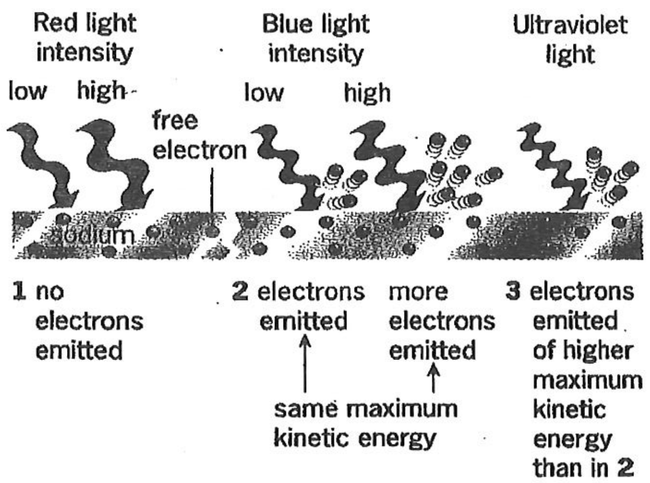
Are the emitted electrons more energetic when a brighter light is used?

What the wave-model of light would predict?	Einstein's prediction
According to the wave theory, the greater the intensity of the incident light, the greater the maximum k.e. of the photoelectrons as more energy would be imparted to the electrons per unit time. Electrons should then also be emitted for any frequency as long as intensity is high enough.	For a fixed frequency, the energy of each photon is the same regardless of the light intensity. Thus, the ejected electrons always have the same KE_{max} at a particular f.

What it really is in the Photoelectric Experiment..........

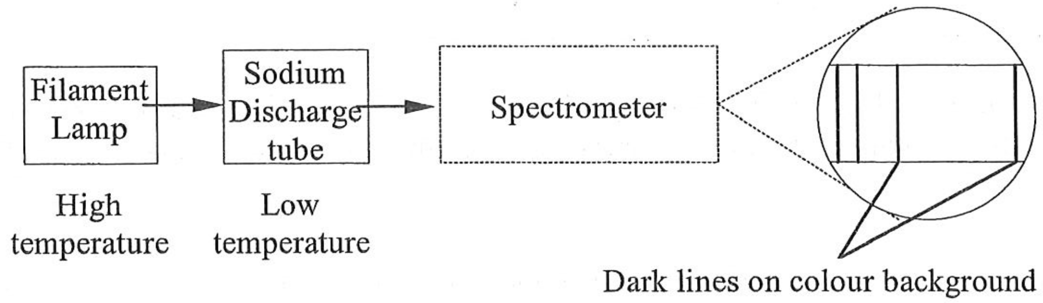
-More electrons should come out (because there are more photons to hit them), but they won't come out any faster, because each individual photon still has the same energy
-For incident light of frequency greater than f_0, the maximum kinetic energy of the photoelectrons is independent of intensity of the light and only varies with its frequency.

In Summary........



6.2 Absorption Spectra

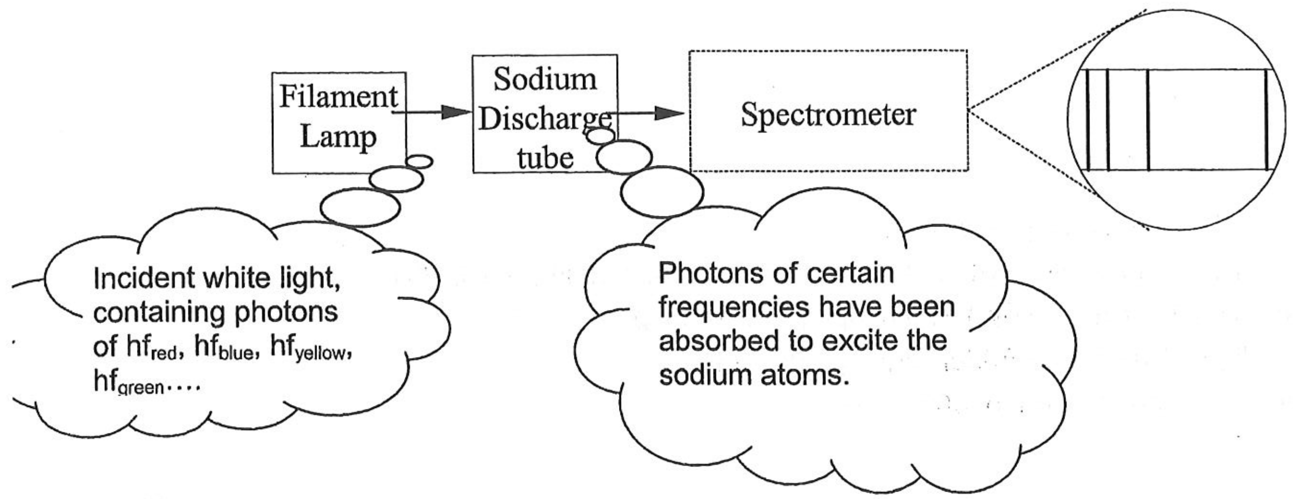
With the apparatus shown in the figure, the sodium to be analysed is kept at very low pressure in a discharge tube. However this time no high voltage is connected across the tube. Instead a source of white light (from the filament lamp) is shone through the tube.



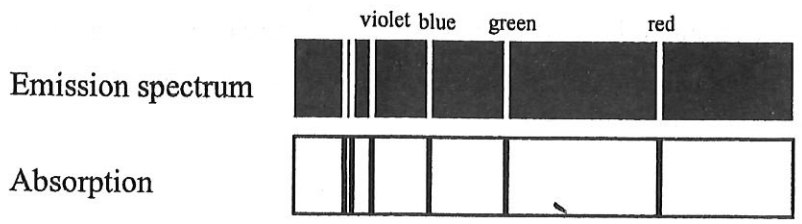
What would you observe instead if you remove the sodium discharge tube?
A continuous spectrum consisting of rainbow colours.
The light of the filament lamp is white light.
After passing through the grating in the spectroscope, it is diffracted into its components

When the photons of white light (made up from every possible frequency of radiation within the visible part of the electromagnetic spectrum) pass through the tube, the vast majority of the incident photons emerge. **However, a small number of incident photons will have energies which match *exactly* the energies needed to move the electrons in ground states into higher energy levels. The atoms in the low pressure gas absorb these photons and use their energy to promote electrons into higher levels. Consequently, photons of these precise energies are missing from the radiation emerging from the tube.**

When the radiation is analysed with a grating (in the spectrometer), the coloured spectrum of the white light is *almost* but not *quite* complete. Black lines can be seen at certain points in the spectrum. These are caused by the absorption of photons which have the precise energy required to cause upward transitions within the energy levels of the sodium atoms.



- The absorption spectra is the inverse of the emission spectra for the same element.



Photons are absorbed when the atoms get in the excited state but the photons will be re-emitted when the atoms return to the ground state. Why then should there be dark lines?

The atoms excited by absorbing photons of energy from the incident white light can return to their ground state by emitting photons. The energy of the emitted photons will be exactly the same as the energy of those photons absorbed in the first place. However only part of this radiation is emitted in the same direction as the incident white light so the parts of the spectrum corresponding to these frequencies are seen as dark by comparison with the intensities of the other frequencies which were not absorbed.

Internet resources:
www.sciencejoywagon.com/physicszone/lessonch/09waves/spectrum/emisndop.htm

Reference:

1. Physics by Hutchings, p.519-534
2. The Physics of Everyday Phenomena by W. Thomas Griffith, p.355-361
3. Understanding Physics by J.Breithaupt, p. 428-433, p.454-459
4. New Higher Physics by Adrian Watt, p.208-221
5. Physics by Ken Robson, p. 360 – 369

Are more electrons knocked out from the metal surface if I use a brighter light?

What the wave-model of light would predict?	Einstein's prediction
According to the wave theory, the greater the intensity of the incident light, the more energy would be imparted to the electrons per unit time.	For f < threshold frequency, no electrons are emitted irregardless of the light intensity. For f > threshold frequency, as light intensity increases, more photons per second strike the metal, and consequently more electrons per second are ejected. *More* electrons should come out (because there are more photons to hit them), but they won't come out any *faster*, because each individual photon still has the same energy.

What it really is in the Photoelectric Experiment.........

- Brighter light means more photons per second, not more energetic photons.
- The number of photoelectrons emitted increases with increasing intensity of the incident light.

Will there be a slight delay before electrons are emitted from a metal surface if I use a faint light?

What the wave-model of light would predict?	Einstein's Prediction
A dim light has less energy. So if a dim red light is shone a metal surface, it will take some time for the electrons at the metals surface to build up its absorption of energy before they break free.	**All** the energy carried by **one** photon is given to **one** electron. Just one photon is enough to cause an electron to be emitted – there is no need to wait for the energy to 'build up' as long as the frequency of the light is high enough since energy of a photon is hf. The energy of a photon cannot be shared by more than one electron nor can a single electron absorb the energy of more than one photon.

What it really is in the Photoelectric Experiment..........

For frequencies of radiation greater than f_0, photoelectrons are emitted as soon as the radiation is shone on the metal surface.

Quantum Physics Tutorial

Use $h = 6.63 \times 10^{-34}$ Js; $e = 1.60 \times 10^{-19}$ C; $c = 3.00 \times 10^{8}$ ms^{-1}

Self Attempt

1 **(a)** Calculate the energy of a photon of electromagnetic radiation with a wavelength of:
(i) 200 nm, (ii) 10 mm.
(b) In which part of the electromagnetic spectrum is each of the above photons?

2 **(a)** What is the wavelength of a 5.0 MeV γ-ray?
(b) Show that, for a photon, the product of the energy of a photon (E) and its wavelength (λ) is a constant. What is the value of this constant?

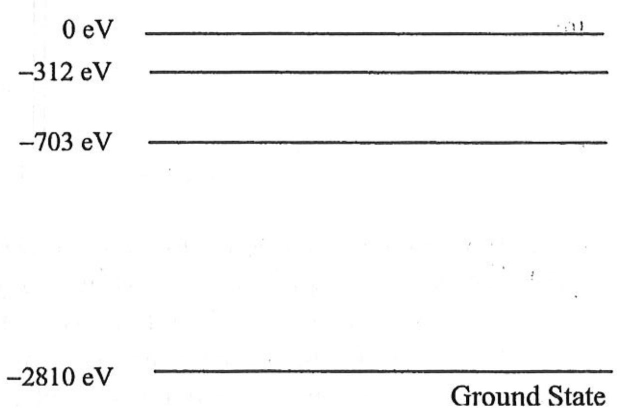
3 Energy from the Sun strikes the Earth surface at a rate of about 1000 Wm^{-2}. What is the rate of arrival of photons per unit area assuming that the average wavelength of light is 500 nm?

4 **(a)** A 60 W light bulb converts electrical energy to visible light with an efficiency of 8%. Calculate the visible light intensity 2 m away from the light bulb.
(b) The average energy of the photons emitted by the light bulb in the visible region is 2 eV. Calculate the number of these photons received per square metre per second at this distance from the light bulb.

5 Light quanta of frequency 5.0×10^{14} Hz fall on the cathode of a photocell. The current through the cell is just reduced to zero by applying a stopping potential of 0.25 V.
(a) What is the work function of the metal?
If light quanta of frequency 6.5×10^{14} Hz were used,
(b) what is the maximum kinetic energy of electrons emitted?
(c) what is the stopping potential required?

6 Electron diffraction experiments show that the wavelength associated with a certain electron beam is 0.15 nm.
(a) Find the momentum of an electron in the beam.
(b) Through what potential difference should the electrons be accelerated from rest to acquire this momentum?

7 A muon is a particle which has the same charge as an electron but its mass is 207 times the mass of an electron.
(a) An unusual atom similar to hydrogen has been created, consisting of a muon orbiting a single proton. An energy level diagram for this atom is shown.
(i) State the ionisation energy of this atom.
(ii) Calculate the maximum possible wavelength of a photon which, when absorbed, would be able to ionise this atom.
(iii) To which part of the electromagnetic spectrum does this photon belong?



(b) Calculate the de Broglie wavelength of a muon travelling at 11% of the speed of light.

Discussion

8 Ultraviolet radiation within intensity of 1.0×10^3 Wm^{-2} shines on an area of 4.0×10^{-4} m^2 of a copper surface.

(a) How much energy reaches the copper surface each second?

There are 1.0×10^{20} copper atoms per square metre of surface. Assume that only electrons in these atoms absorb energy. The minimum energy required for electrons to leave the surface is 4.50 eV. Assume that two electrons per atom of copper have this ionisation energy. The remainder are too strongly bound for light to eject them. (For the chemists among you, the two accessible electrons are valence electrons and the remainder of the electrons are in the core of the atom.)

First we will test what the *wave* model would predict in this situation.

(b) Assuming a wave model, how much energy per second is gained by each surface electron?

(c) How long would it take for an electron to gain sufficient energy to escape?

(d) Experimentally it is found that there is only about a 10^{-9} s delay between the time radiation first falls on the surface and the time electrons are emitted. Compare this with your answer to (b) and comment.

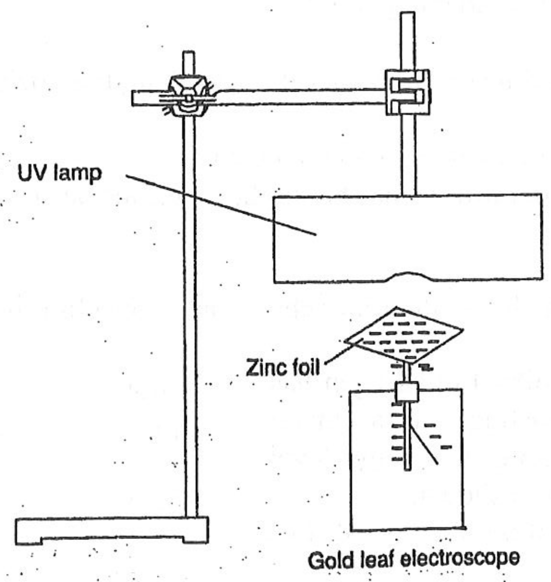
Now we explore the *particle* model predictions.

(e) What is the minimum photon energy required to eject electrons from copper? Calculate the frequency of this radiation. What other frequencies can eject copper electrons?

(f) If all of the photons arriving at the copper surface have this minimum energy, how many photons arrive each second when the light intensity is 1.0×10^3 Wm^{-2}?

(g) What will be the value of the maximum photocurrent of electrons accelerated to the anode?

9 A student studying the photoelectric effect directs electromagnetic radiation from different sources onto a charged zinc plate placed on top of an electroscope.



(a) Explain the meaning of the terms threshold frequency and work function.

(b) (i) When the plate is negatively charged, ultraviolet radiation will discharge the electroscope. Explain this observation.

(ii) Describe and explain the effect of increasing the intensity of the uv radiation.

(c) Explain why, when the plate is negatively charged, light from a sodium lamp will not discharge the electroscope.

3.2 Features of Photoelectric Effect

Does it make a difference whether I use a blue or red light to shine at a metal surface? As long as they are bright enough, will electrons be emitted?

What the wave-model of light would predict?	**Einstein's prediction**
Be it blue light or red light, as long as they are bright enough, when they are shone on a piece of metal, photoelectrons should be emitted. Light energy $\propto$ Intensity of light $\propto$ amplitude square of vibration	Light arrives in packets with energy, E = hf. If hf > ϕ, photoelectrons will be emitted. So a dim blue light may emit photoelectrons from the metal surface but a bright red light may not. Energy of light doe not depend on intensity!

What it really is in the Photoelectric Experiment..........

For a given metal, no photoelectrons were emitted when it was illuminated by light beneath a certain minimum frequency (threshold frequency f_0)

3. Photoelectric Emission Of Electrons

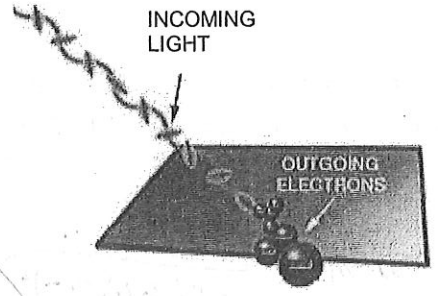
3.1 Photoelectric Effect – Historical Background and What is it?

- When light is shone on a clean metal surface, the surface may become positively charged. This happens because the energy carried by light waves is absorbed by free electrons in the metal, giving them enough kinetic energy to 'jump' out of the metal. This is the **Photoelectric Effect**.
- http://www.sciencejoywagon.com/physicszone/lessonch/10modern/photoelc/index.htm

Photoelectric effect is the emission of electrons from surface of metals when they absorb energy from electromagnetic radiation.

Because the electrons are ejected with the aid of light, they are called **photoelectrons.**

minimum amt of energy to emit e^-



Energy from the incident light is used for: 1. removing the electron from the metal as they have negative potential and are thus bound. This energy is known as the work function, ϕ and is unique to each metal. Its value is fixed for a particular metal. 2. The rest of the energy is imparted to the electron's kinetic energy as it is ejected from the metal.

K.E. of electron = Energy absorbed from single photon – Work done in removing electron $\frac{1}{2}mv^2_{max} = E - \phi$

$\frac{1}{2}mv^2 = E - W$

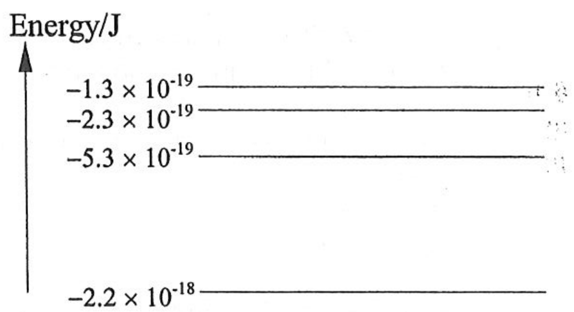
10 Red light, however bright it is, cannot produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Explain why this is.

11 Ultraviolet light of wavelength 12.2 nm is shone on to a metal surface. The work function of the metal is 6.20 eV.
(a) Calculate the maximum kinetic energy of the emitted photoelectrons.
(b) Show that the maximum speed of these photoelectrons is approximately 6 x 10^6 ms^{-1}.
(c) Calculate the de Broglie wavelength of photoelectrons with this speed. Explain why these photoelectrons would be suitable for studying the crystal structure of a molecular compound.

12 Monochromatic light of wavelength 450 nm falls on the cathode of a photocell at a rate of 25 μW. Only 10% of the photons produce an electron and all these electrons produce a current, *I*, in an external circuit.
(a) Calculate
(i) the energy of one photon in J,
(ii) the number of photons falling on the cathode per second,
(iii)the current, *I*.
(b) The same amount of energy per second falls on the cathode in the form of light of wavelength 600 nm. The work function of the material of the cathode is 3.0×10^{-19} J. Show that a photoelectric current flow.

13 An experiment is set up to measure the maximum energy of photoelectrons emitted from a metal. In such an experiment, the photoelectric current i depends on the frequency f of the incident light, and potential difference V between the collector and emitter, and the incident power P.
Draw sketch graphs to show how
(a) i varies with P; f and V remaining constant;
(b) i varies with V; P and f remaining constant,
(c) stopping potential varies with f; (NB: V is not stopping potential)
Give an explanation, in terms of the photon theory of light, of the main features of these graphs.

14 The figure represents the energy levels of the four lowest states of the hydrogen atom.
(a) Calculate the longest wavelength which might be emitted by a spectral transition between any pair of these four levels.
(b) Determine the total number of difference spectral lines, which might be detected in the emission spectrum of atomic hydrogen, due to transitions between these four states.



A photon of wavelength 82 nm strikes a hydrogen atom in its ground state. Using the information in the above figure, calculate the maximum velocity of the ejected electron.

15 The diagram is a simplified representation of the 5 lowest energy levels of the outermost electron in the sodium atom.

(a) Considering transitions between only these levels

(i) which spectral transition has the shortest wavelength (give your answer in terms of level numbers),

(ii) how many spectral emission lines might be produced by transitions among these levels?

(b) If the sodium atoms are initially in the ground state, how many absorption lines might be detected?

(c) Cool sodium vapour at low pressure is bombarded with electrons of kinetic energy E.

ground state

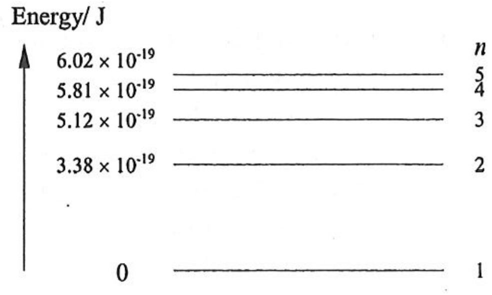
Which transitions would you expect to observe if E has the value

(i) 3.0×10^{-19} J,

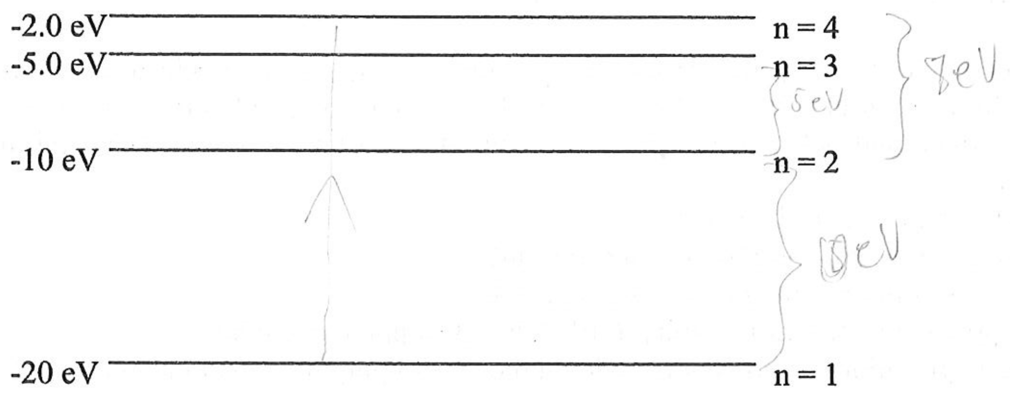
won't collide & excite each other's electrons

(ii) 4.0×10^{-19} J,

(iii) 5.5×10^{-19} J



16 The energy level scheme for the mythical one-electron element Searsium is shown in the diagram below. The potential energy of an electron is taken to be zero at a infinite distance from the nucleus.



(a) How much energy (in electronvolts) does it take to ionise an electron from the ground state?

(b) A 15 eV photon is absorbed by a cool Searsium atom. When the atom returns to its ground state, what possible energies can the emitted photon have?

(c) State and explain what will happen if a photon strikes a cool Searsium atom with an energy of

(i) 17 eV

(ii) 25 eV

(d) If photons emitted from Searsium transitions n = 4 to n = 2 and from n = 2 to n = 1 will eject photoelectrons from an unknown metal, but photons emitted from the transactions n = 3 to n = 2 will not, what is the range of values within which the work function of the unknown metal lie?

3eV 7eV 10eV

5eV

Exercises

2.3 In 1 second 10^{20} photons each having energy of 3.0×10^{-19} J fall on a solar cell of area 10 cm^2. What is the intensity of the incident light?

$$I = \frac{Power}{A}$$

$$= \frac{nE}{A}$$

$$= \frac{10^{20} \times (3.0 \times 10^{-19})}{10 \times 10^{-4}}$$

$$= 3.0 \times 10^{4}\ Wm^{-2}$$

2.4 The Andromeda Nebula, at a distance of 2.0×10^{22} m from the Earth, radiates 8.0×10^{27} W in the spectral line of frequency 1420 MHz. Estimate the number of photons received per second when the nebula is observed by a radio telescope of collecting area 100 cm^2.

I of radiation reaching Earth $= \frac{8.0 \times 10^{27}}{4\pi(2.0\times 10^{22})^2}$

$= 1.59 \times 10^{-18}\ Wm^{-2}$

P of radiation reaching telescope $= 1.59 \times 10^{-8} \times 100 \times 10^{-4}$

$= 1.59 \times 10^{-20}\ W$

$P = nhf$ $\qquad n = \frac{1.59 \times 10^{-20}}{hf}$

$= 1.7 \times 10^{4}$

2.2 Intensity, Inverse Square Law and Quantum Theory

- According to the quantum theory, the intensity of radiation is proportional to the rate of number of photons per unit area.
- Hence an area experiencing low intensity of light has less photons falling on it per second than an area experiencing high intensity.
- In fact,

$$\text{Intensity} = \frac{\text{Power of light falling on area}}{\text{Area}}$$

$$\frac{E}{t \cdot A} \xrightarrow{Q.P} \frac{n(hf)}{t \cdot A}$$

$$= \frac{\text{No. of incident photons per unit time} \times \text{Energy of one photon}}{\text{Area}}$$

$$= \frac{n \times hf}{A}$$

- For a radial source of energy,

$$\text{Intensity} = \frac{\text{No. of incident photons per unit time} \times \text{Energy of one photon}}{4\pi r^2}$$

i.e. $I \propto 1/r^2$, which is still in agreement with the classical wave theory.

Pair Exercise: Compare between the two views of the Nature of light

Classical Wave Theory of Light	Quantum Theory of Light
Energy: -Light energy is spread out in space as it is emitted.	Energy: -Light energy is emitted in discrete amounts called photons which is proportional to light frequency.
Intensity: $I \propto A^2$ $I \propto \frac{1}{r^2}$	Intensity: $I \propto \frac{n}{t}$ $I \propto \frac{1}{r^2}$

17 Electron diffraction is observed when a beam of electrons is fired at a graphite crystal in which rows of carbon atoms act as the grating. The carbon atoms are in rows with a spacing of 1.23×10^{-10} m.

(a) In an experiment, a beam of electrons was diffracted by graphite so that the second order maximum was at an angle of 0.167 radian. Calculate the wavelength of the electrons.

(b) The electrons were accelerated by a potential difference of 5000 V. Calculate the kinetic energy and hence the momentum of the electrons.

Answers :

1 (i) 9.9×10^{-19} J, (ii) 2.0×10^{-23} J

2 2.5×10^{-13} m

3 2.5×10^{21} $m^{-2}s^{-1}$

4 (a) 0.095 Wm^{-2} (b) 3.0×10^{17} $m^{-2}s^{-1}$

5 2.9×10^{-19} J; 1.4×10^{-19} J; 0.87 V

6 4.4×10^{-24} kg ms^{-1}; 67 V

7 (a)(i) 2810 eV (ii) 4.42×10^{-10} m (iii) X-rays (b) 1.07×10^{-13} m

8 0.40 J (b) 5.0×10^{-18}J (c) 0.14s (e) 4.50 eV, 7.2×10^{-19} J; 1.09×10^{15} Hz (f) 5.56×10^{17} photons per second (g) 5.56×10^{17} electrons per second is a current of 0.089 A.

11 95 eV; 1.25×10^{-10} m

12 a(i) 4.42×10^{-19} J (ii) 5.66×10^{13} s^{-1} (iii) 9.05×10^{-7} A

14 2.0×10^{-6} m; 6; 7.0×10^{5} ms^{-1}

15 (d) 2.11 V < V < 5.13 V, 3.44×10^{-22} J

16 20 eV; 5 eV, 10 eV and 15 eV; 5 eV < ϕ < 8 eV

17 1.02×10^{-11} m, 8.0×10^{-16} J, 3.8×10^{-23} $kgms^{-1}$

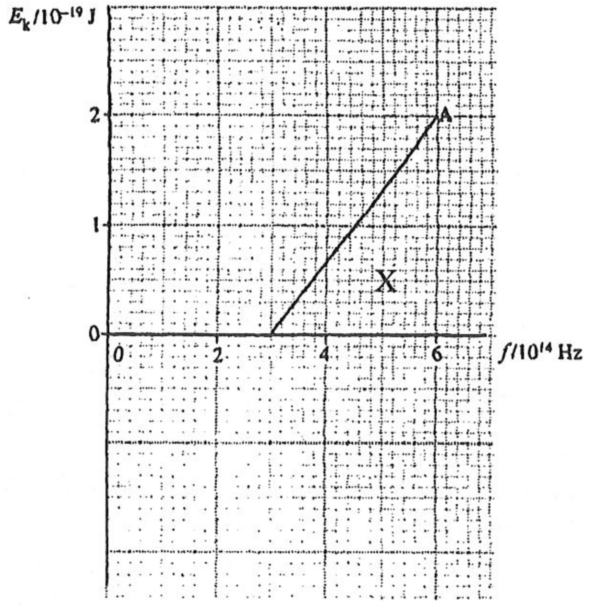
(17b) gain in K.E = loss in e.p.e

= $q(5000)$

$K_f - K_i$ =

Assignment

18(a) A metal surface A is illuminated with light of varying frequencies and electrons are emitted. The maximum kinetic energy of the emitted electrons is measured. The figure shows how the maximum kinetic energy of the emitted electron varies with the frequency of the incident light.



(i) Use the graph to obtain a value for the Planck constant.

(ii) Determine the minimum photon energy required to emit an electron. [5]

(b) Copy the graph in the figure and using the same axes draw a second graph showing the result of using a metal B with a lower work function and a light of higher intensity. Label this line B. Explain your answers. [5]

(c) The metal B is replaced with metal C. One more measurement of stopping potential V_o for incident light photons of frequency 5.0 x 10^{14} Hz was made, and plotted on the original graph as point X. Plot the rest of the graph for metal C on the figure you have reproduced for 17(b). Label this line C. [2]

(d) (i) Explain what is meant by the duality of electrons.

(ii) State the relationship between the electron mass, the electron velocity and the wavelength for a monoenergetic beam of electrons. [3]

(e) The spacing of atoms in a crystal is 1.0 x 10^{-10}m. The mass of the electron is 9.1 x 10^{-31}kg.

Estimate the speed of electrons which would give detectable diffraction with such crystals. [4]

(f) Give one piece of evidence to demonstrate that electrons have particle properties. [1]

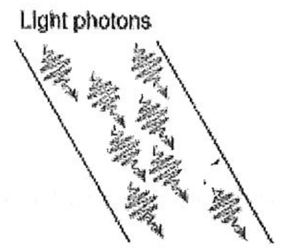
2.1 Quantum Theory of Radiation – Planck's astonishing hypothesis

On December 14, 1900, Planck unveiled his hypothesis which was based on one assumption: light energy was not emitted in the continuous flow as common sense suggested but as discrete bursts of energy.

- Electromagnetic radiation consists of packets of electromagnetic energy.
- Each packet is called a quantum (latin for "how much") of energy or a **photon.**
- Planck postulated that the energy contained in a photon for radiation of frequency *f* is given by

$$\boxed{E = hf}$$

where *h* is a constant known as Planck's constant (approximately 6.63 × 10^{-34} Js).



- His theory, he stressed was concerned with the interaction between radiation and matter, not with the nature of radiation on its journey between emission and reception. He was not sure how light could 'travel as a wave' but be emitted and absorbed as 'particles' (packets of energy). His theory was by his own admission an "act of desperation" to solve the problem of the ultraviolet catastrophe. (You can read more about this in any first year university physics textbook.)

Exercises

2.1 How much energy is contained in a photon of light of wavelength 500 nm?

$E = hf$

$= \frac{hc}{\lambda}$

$= 3.98 \times 10^{-19} J$

2.2 How many photons are emitted per second for a 60 W lamp giving light of this wavelength?

Power of lamp $= nE$ $(P = \frac{NE}{t})$

where n = No. of photons emitted per unit time

$n = \frac{\text{Power}}{\text{Energy of a photon}}$

$= \frac{60}{3.98 \times 10^{-19}}$

$= 1.51 \times 10^{20} s^{-1}$

1. Overview

Around the end of the 19th century, several seemingly unrelated developments in physics converged to provide the basis for a surprising new model of atomic structure.

- One was the observations from the study of atomic spectra.
- Another was Planck's IDEA of light emitted from an incandescent object.
- Planck's IDEA was in turn used by Einstein to explain an observation known as the photoelectric effect
- Then Bohr applied the idea with some of his own ideas to account for the phenomenon of atomic spectra. His ideas led directly to the development of modern quantum and its applications, including lasers and semiconductors.

This topic focuses on the experimental data in two phenomena

- Atomic spectra and
- Photoelectric effect

and how they were explained using ideas previously unknown to physicists. These new ideas had to do with the nature of light and its interaction with matter.

2. The Nature of Light

<u>Individual Exercise: Consider each of the following questions. What do you know about the nature of light at this moment?</u>

What have you learned about the nature of light from the topic of waves (i.e. classical theory)?

Is light considered a wave? What phenomena support your belief?

Reflection Polarization
Refraction
Interference
Diffraction

What does the energy of light depend on?

- $E \propto A^2$, given f of oscillation is fixed
- The light is emitted as a continuous flow of energy
- $I \propto \frac{1}{r^2}$, given light source is a point source

19 A photographic exposure meter may have a photocell with the cathode coated with a material which emits electrons when visible light shines on it.

(a) When light of wavelength 550 nm shines on it, electrons are emitted with negligible kinetic energy.
(i) What is the threshold frequency of this material? [1]
(ii) What is the work function in joules? [1]

(b) The wavelength of the light is changed to 400 nm, at the same intensity.
(i) What is the maximum possible kinetic energy of an ejected electron? [2]
(ii) Determine the stopping potential for this maximum kinetic energy and explain why the photoelectrons are emitted with different speeds though the energy of each photon is fixed. [2]
(iii) The intensity is then doubled. What is the effect on the kinetic energy of the ejected electrons and the number of electrons emitted per second? [2]

(c) (i) Distinguish between absorption and emission line spectra. [2]
(ii) Excited electrons will move to lower level almost immediately after excitation, each releasing a photon in the process.

Figure 6.1 shows the 4 lowest energy levels in a hydrogen atom.

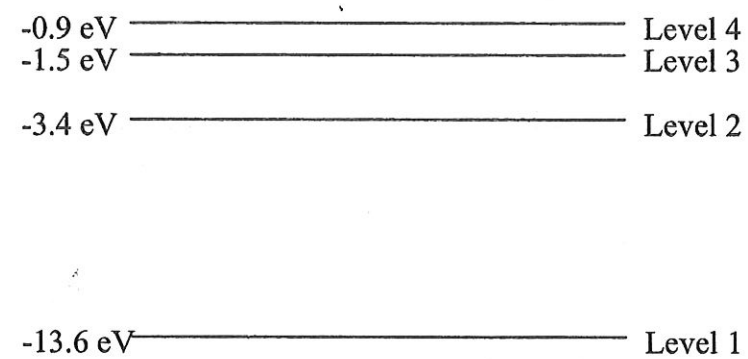


Figure 6.1

Figure 6.2 shows the theoretical emission line spectrum that results from transitions of electrons initially excited to levels 3 and 4.

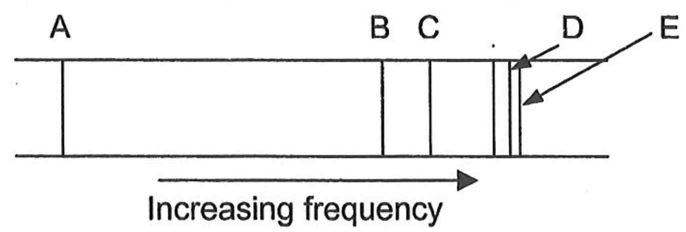


Figure 6.2

1. Copy Figure 6.1 and sketch the 5 transitions which give rise to the 5 emission lines **A to E**. Label these accordingly. [3]

2. State which of the lines A to E will be observed, noting that the wavelength of visible light is between 400 nm and 700 nm. Explain your reasoning clearly. [3]

$\mathcal{E} = hf$
$= \frac{hc}{\lambda}$ } photons

Electrons $\rightarrow \lambda_B$ $p = \frac{h}{\lambda}$

$K.E = \frac{p^2}{2m}$ (particles with mass)

$\mathcal{E} \propto f$

$\mathcal{E} \propto f^2 x_0^2$

$\frac{1}{2} m \omega^2 x_0^2$

$I = \frac{N(hf)}{t.A}$
$= \frac{n\,dq}{dt}$

$hf = \phi + \frac{1}{2} m V_{max}^2$ $f > f_0$ $\frac{1}{2} m V_{max}^2 = eV_s$ when no e^- detected

$1eV = 1.6 \times 10^{-19} J$

1) Energy of e-m waves of freq f, $E = hf = \frac{hc}{\lambda}$

2) $I = \frac{N(hf)}{t.A}$ $\frac{E \text{ (arriving)}}{At}$

3) Photoelectric effect $\phi = 1st$ ionization energy

ionisation energy $\rightarrow$ energy needed to remove an e^- from an atom

work function $\rightarrow$ energy needed to remove an electron from the metallic lattice

Energy of an e^- is quantised only when the electron is in a bound (confined) state

Excitation — 1) By photons (energy of incident photon must be exactly equal to energy diff between levels)
2) By electrons (energy of incident electron $\geq$ energy diff between levels)

Quantum Physics

Content :

1. *Introduction*
2. *The Nature of Light*
3. *Photoelectric Emission of Electrons*
4. *Wave-Particle Duality*
5. *Energy Levels in Atoms*
6. *Line Spectra*

Objectives: Students should be able to

(a) show an appreciation of the particulate nature of electromagnetic radiation.

(b) recall and use $E = hf$

(c) describe the phenomena of the photoelectric effect.

(d) recall the significance of the threshold frequency.

(e) explain why the maximum photoelectric energy is independent of intensity, and why the photoelectric current is proportional to intensity.

(f) explain photoelectric phenomena in terms of photon energy and work function energy.

(g) recall, use and explain the significance of $hf = \phi + (1/2)mv_{max}^2$

(h) appreciate that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence of a wave nature.

(i) describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles.

(j) recall and use the relation for the de Broglie wavelength $\lambda = h/p$.

(k) understand the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and explain how this leads to spectral lines.

(l) distinguish between emission and absorption line spectra.

(m) recall and use the relation $hf = E_1 - E_2$.

Max Planck

Albert Einstein

Niels Bohr