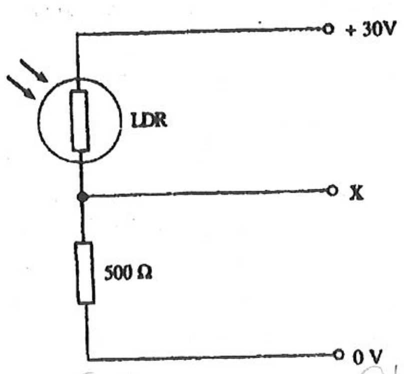
Worked Example 10

The light dependent resistor (LDR) and a 500 Ω resistor form a potential divider between voltage lines held at +30V and 0 V as shown in the diagram.
Given that the resistance of the LDR is 1000 Ω in the dark but drops to 100 Ω in bright light, find the corresponding change in the potential at X.



$V_R = \left(\frac{R}{R_1+R_2}\right)V$

$V_x = \left(\frac{1000}{1500}\right)(30)$ $= 10$

$V_x = \left(\frac{100}{600}\right)30$ $= 5$

15

$V_{BC} = V_x - 0$
$= \left(\frac{500}{1500}\right)30$
$= 10V$ (in dark)
In bright light,
$V_x = 25V$

Change in potential of x
= increase of 15 V

Worked Example 11

A student decided to build a temperature probe and set up the circuit as shown in the figure. The battery has e.m.f. 9.0 V and negligible internal resistance.

The voltmeter has infinite resistance. The calibration curve for the thermistor is shown in the figure below.

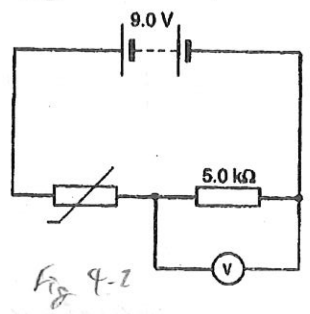


Fig 4-2

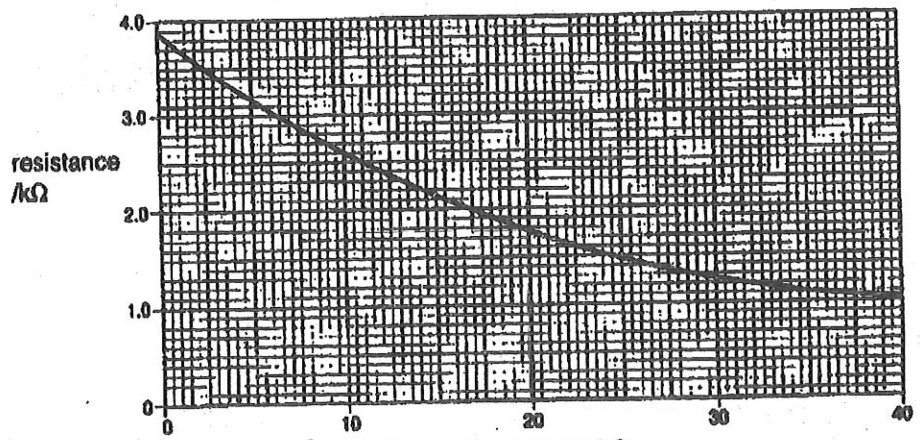


Fig 4-3

(i) Suggest why it is necessary to include a fixed resistor in the circuit of Fig. 4.2.
(ii) Use Fig. 4.3 to find the resistance of the thermistor when the probe is at 30 °C.
(iii) Hence calculate the reading on the voltmeter for the temperature of 30 °C.

(i) The p.d. across the voltmeter, , $V_R = [5/(5+R_t)](9)$, where R_T is the resistance of the thermistor.

As R_T changes with temperature, a variable p.d. can be obtained across the voltmeter. If the 5 kW resistor is left out, and the voltmeter is placed across the thermistor, the p.d. measured is the constant p.d. across the 9 V cell.

(ii) From the graph, R_T = 1.8 kW

(iii) V_R = 6.62 V $V_R = \left(\frac{5}{5+1.8}\right)9 = 6.62V$

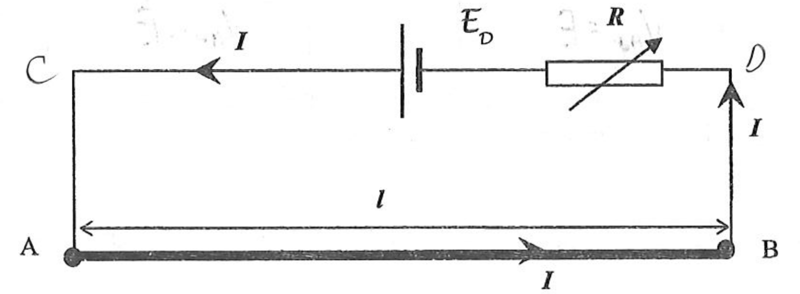
Potentiometer

recall and understand the principle of the potentiometer as a means of comparing potential differences.

Potentiometer Circuit

A potentiometer is an arrangement which measures p.d. accurately. Basically, it consists of a series circuit with **driver cell, variable resistor** and a length of **resistance** wire with uniform cross-sectional area.

In a potentiometer:

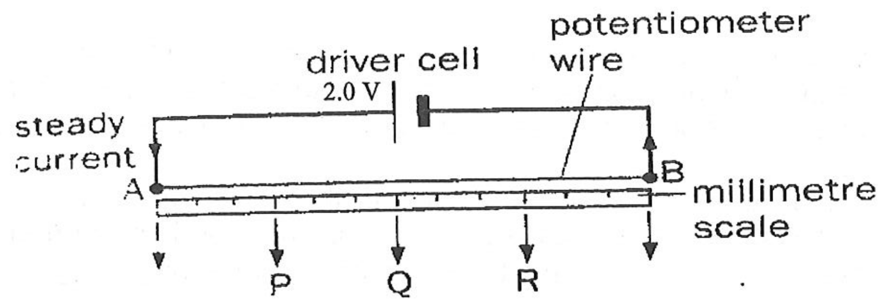


In a potentiometer, a p.d. is set up along the wire AB by the driver cell, E_D. which drives a **steady current, *I* that always flows in this main loop**.

- **About wire AB:**

  AB is assumed to have **uniform cross-section and resistivity**. Hence its resistance per unit length, r is a constant;

  There is a p.d. between any two points on the wire which is proportional to their distance apart. We say that the potential difference, V per unit length is a constant. This potential difference per unit length is called the potential gradient.

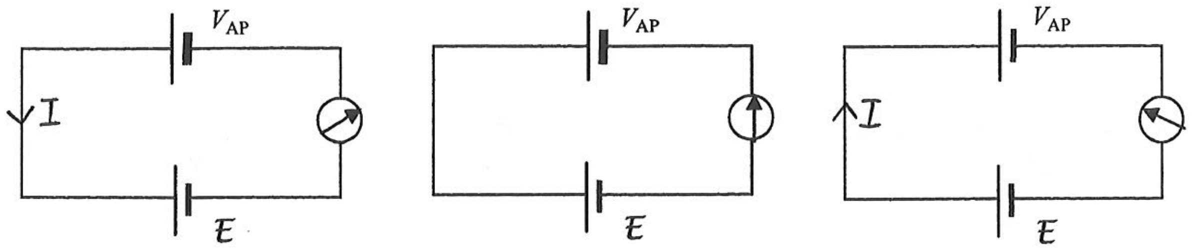


$V_{AP} = 0.50V$ ($\frac{1}{4}$ length of AB)
$V_{AQ} = 1.00V$ ($\frac{1}{2}$ length of AB)
$V_{AR} = 1.50V$ ($\frac{3}{4}$ length of AB)
$V_{QB} = 1.00V$ ($\frac{1}{2}$ length of AB)

p.d. V is proportional to the length of wire L

The principle of potentiometer:

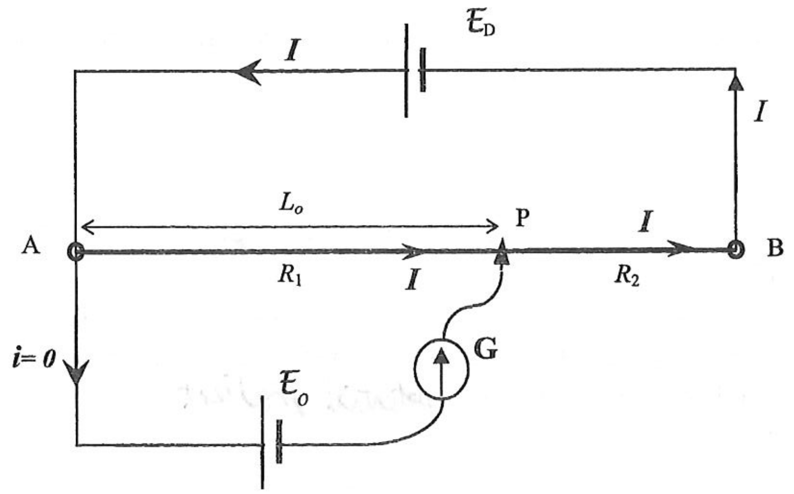
Consider two cells connected back to back with one another,



$\boxed{V_{AP} > E}$ $\boxed{V_{AP} = E}$ $\boxed{V_{AP} < E}$

When two identical cells (or p.d.) are placed back to back with one another in a circuit, the galvanometer registers null deflection (. i.e. no current)

If the cell with unknown emf, E_o, together with a galvanometer is now attached to the potentiometer as follows:



As you move the jockey along the wire (which is the sliding contact), V_{AP} changes.
If $V_{AP} < E_o$, current flows through the resistance wire in the potentiometer circuit as well as the galvanometer, G, in the lower portion of the circuit. Hence the galvanometer deflects.

There exists a point on the wire where a null deflection of the galvanometer is obtained. This means **no current** flows in the **lower circuit**. This position of the jockey on the wire is called the balance point, P and the length AP is called the balance length.

At balance point, e.m.f E_o is equal to V_{AP}. But V_{AP} is proportional to L_o, i.e.

$$\boxed{V_{AP} = kL_o}$$

Hence, the unknown emf $E_o = kL_o$ where k is a constant called the potential gradient of the resistance wire.

Light Dependent Resistor

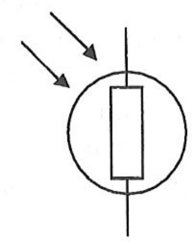


Fig P

The LDR is made by linking two metal electrodes with a film of cadmium sulphide.
In complete darkness, it has a resistance of about 10 MΩ but, in bright sunlight, its resistance falls to about 100 Ω. Light intensity ↑, R_{LDR} ↓,
The symbol for an LDR is shown in Fig. P. ∴ p.d across LDR ↓
Figure Q shows a circuit where the variation in light intensity incident on the LDR over a period of time can be represented as a voltage. The variation with time of this voltage follows that of the light intensity.

∴ Light intensity ↑
∴ R_{LDR} ↓
∴ V_{LDR} ↓
∴ V_R ↑

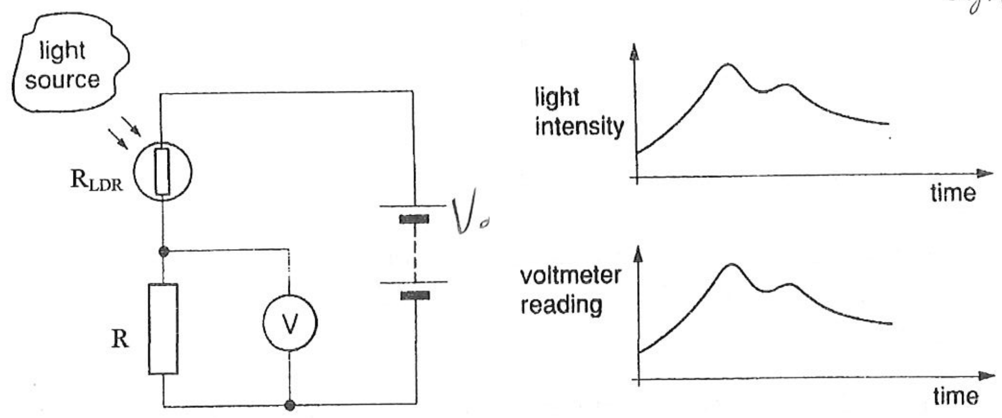


Fig. Q

- The resistance of the light dependent resistor (LDR) decreases with increasing **intensity of light.**
- A potential divider controlled by illumination can be set up as shown.
  The voltage across the R, V_R, is given by

$$\boxed{V_R = \left[\frac{R}{R+R_{LDR}}\right] V_o}$$

As the light intensity increases, R_{LDR} decreases and V_R increases.

[As light intensity decreases, resistance of LDR increases, potential across the LDR increases and potential difference across fixed resistor decreases .]

Applications of LDR

1) The circuit above (Fig Q) can be used as a light meter.
2) LDR is used in doors to prevent it from closing on a passenger entering the lift; another similar usage is a burglar alarm.
3) Light sensitive switches: lights can be turned on automatically when the surrounding is dark.

Thermistors & Light Dependent Resistors

describe and explain the use of thermistors and light dependent resistors in potential dividers to provide a potential difference which is dependent on temperature and illumination respectively

Thermistor

The thermistor is a temperature-dependent resistor which is manufactured in a number of different shapes and sizes using the oxides of various metals.

Negative temperature coefficient types have a resistance which becomes smaller as the temperature increases. The symbol for a thermistor is shown in Fig X.

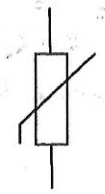


Fig X

- They are heated either externally from the surroundings or internally by the current flowing through them.
- Figure Y shows a circuit where the variation in temperature of a thermistor over a period of time can be represented by a voltage reading.
  The variation with time of this voltage follows that of the temperature.

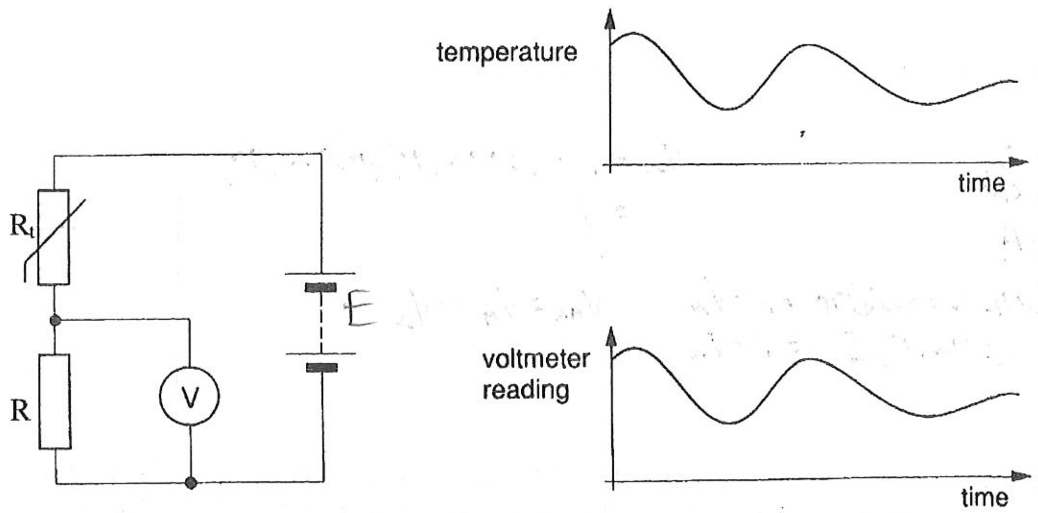


Fig Y

The voltage across R, V_R is $\boxed{V_R = \left(\frac{R}{R+R_t}\right) \cdot E}$

When temperature increases, R_t decreases and V_R increases.

[When temperature decreases, resistance of thermistor increases, voltage across fixed resistor decreases.]

Applications of Thermistor

The circuit above enables the thermistor to be used for temperature measurement. The galvanometer scale can be calibrated to read temperature.
This is used in
- measurement of respiration frequency of aircraft pilots.
- cardiac pacer to sense temperature of blood.

Uses Of The Potentiometer

Potentiometer can be used to:
(A) To compare or measure the e.m.f.s of cells.
(B) To compare or measure resistances.
(C) To measure internal resistance of a cell.

(A) To compare or measure the e.m.f.s of cells.

When switch, S is at position 1, the balance length found is l_1, where C is the balance point.

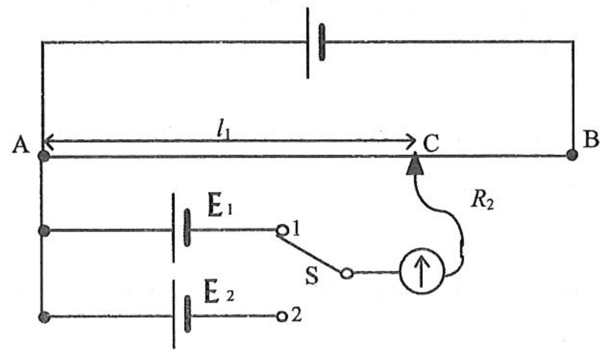
$E = kl_B \quad E_1 = kl_1$

Then $E_1 = l_1E$ $\quad k = \frac{E}{l_B} \quad = \left(\frac{E}{l_B}\right)l_1 = \left(\frac{l_1}{l_B}\right)E$

When switch is at position 2, the balance length found is l_2 where C′ is the new balance point.

$E_2 = kl_2$

Then $E_2 = l_2E$ $\quad = \left(\frac{E}{l_{AB}}\right)l_2 = \left(\frac{l_2}{l_{AB}}\right)E$



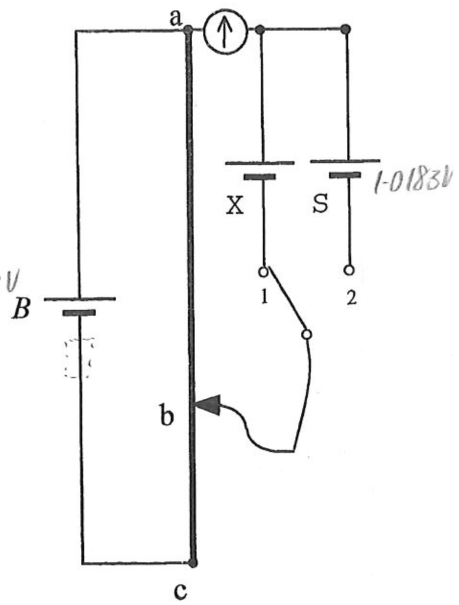
Taking ratio, $\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$

If E_1 is a known emf source, E_2 can be found.

Worked Example 12

A potentiometer is set up to measure the e.m.f. of cell X. B is a battery whose e.m.f. is approximately 3.0 V and whose internal resistance is unknown. S is a cell of 1.0183 V. The switch is set at position 2, placing the standard cell in the galvanometer circuit. When the tap b is 0.36 of the distance from a to c the galvanometer G reads zero.

a) What is the p.d. across the entire length of resistor ac? 3.0V
b) The switch is then set at position 1 and a new zero reading of the galvanometer obtained when b is 0.47 of the distance from a to c. What is the e.m.f. of cell X ?



a) $V_{ac} = kL_{ac}$ —— (1) $\quad \frac{V_{ac}}{V_{ab}} = \frac{kL_{ac}}{kL_{ab}}$

$V_{ab} = kL_{ab}$ —— (2) $\quad V_{ac} = \left(\frac{L_{ac}}{0.36L_{ac}}\right)1.0183$

$V_{ac} = \left(\frac{L_{ac}}{L_{ab}}\right)V_{ab} \quad = 2.83V$

$= 2.83 V$

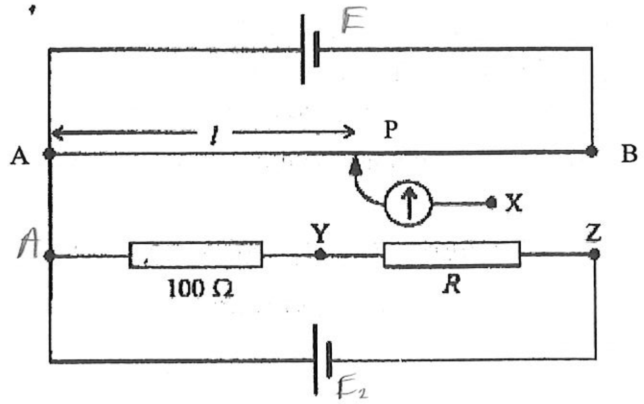
b) $\frac{V_{ab}}{V_{ac}} = \frac{L_{ab}}{L_{ac}} = 0.47$

$V_{ab} = 1.33V \quad \left(\frac{0.47L_{ac}}{L_{ac}}\right)2.83$

(B) To compare or measure resistances.

Worked Example 13

The diagram shows a circuit which may be used to compare the resistance R of an unknown resistor with a 100 Ω standard. The distance l from one end of the potentiometer slide-wire to the balance points are 400 mm and 588 mm when X is connected to Y and to Z respectively. The length of the slide wire is 1.00 m. What is the value of R?



When X is connected to Y and balanced point is obtained,,

$V_{AP} = l_{AP} V_{AB} = \frac{100}{100+R} V_{AZ}$ —— (1)

When X is connected to Z and with balanced point,

$V_{AP'} = k l_{AP'} = V_{AZ}$ —— (2)

Taking ratio of the two equations,

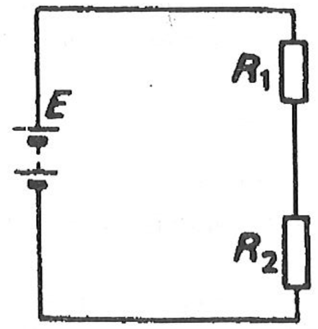
$$\frac{k l_{AP}}{k l_{AP'}} = \frac{\frac{100}{100+R} V_{AZ}}{V_{AZ}}$$

$R = 47\,\Omega$

Summary

Worked Example 8

A battery of e.m.f. E and negligible internal resistance is connected to two resistors of resistances R_1 and R_2 as shown in the circuit diagram. What is the p.d. across resistor R_1?



$I = \frac{E}{R_1 + R_2}$

P.d across R_1, $V_1 = IR_1$

$= \frac{R_1}{(R_1+R_2)} \cdot E$

Worked Example 9

Two resistors, of resistance 200 kΩ and 1 MΩ respectively, form a potential divider with outer junctions maintained at potentials of +3 V and – 15 V.

What is the potential at the junction X between the resistors?

$I = \frac{3-(-15)}{1200 \times 10^3}$

$= 0.15\,mA$

P.d across 200kΩ resistor $= 3 - V_x$

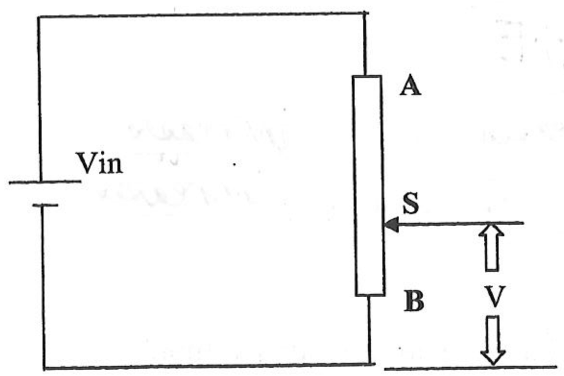
$(200 \times 10^3) I = 3 - V_x$

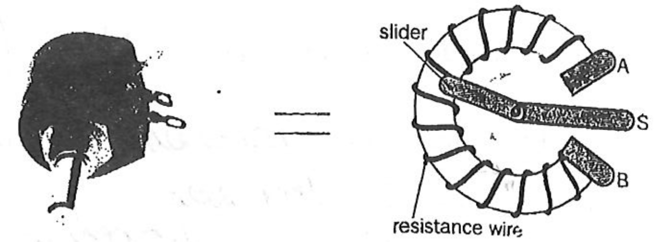
$V_x = 3 - (200 \times 10^3)(0.15 \times 10^{-3})$

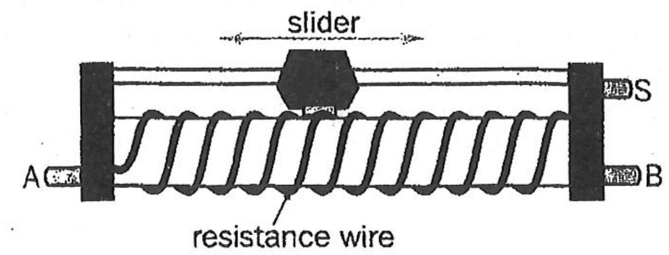
$= 0$

$V_{AX} = V_A - V_x$

The simplest form of a practical potential divider makes use of a variable resistor as shown below. The desired voltage is a fraction of the p.d. from the cell and is tapped off by varying the sliding contact along the resistor.







3 Balanced Potentials

Potential Divider

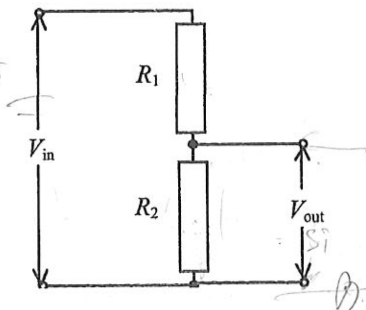
understand the use of a potential divider as a source of variable p.d..

$V = IR^{-1}$

You have a 1 V small light bulb but a 6 V cell. What should you do to light up the bulb without causing damage to the bulb?

To obtain only part of the voltage provided by a battery, we use a potential divider.

- Basically, a potential divider consists of two resistors connected in series, across a fixed voltage source, so that
- the same current flows through the two resistors and
- the p.d. across the resistors is divided into two portions.



- Simplest form of a potential divider:

V_{in} : input p.d. applied across R_1 and R_2.

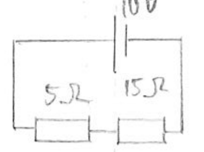
V_{out} : output p.d. taken across R_2.

If output current is zero then the current I flowing through R_1 also flows through R_2.

$$V_{in} = I(R_1 + R_2)$$
$$I = \frac{V_{in}}{R_1 + R_2}$$
$$V_{out} = IR_2 = \frac{R_2}{(R_1 + R_2)} \times V_{in}$$

Note:

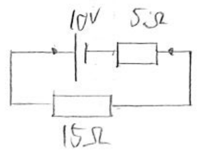
By adjusting the values of R_1 *and* R_2, the desired values of V_{out} can be obtained. This way the potential divider can be used to control the p.d. V applied across any device.

1) 

Find p.d across 15 Ω resistor

$V_{15} = \frac{15}{15+5} \times 10$

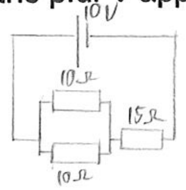
$= 7.5\ V$

2) 

Find p.d across 5 Ω internal resistor

$V_5 = \left(\frac{5}{5+15}\right)10$

$= 2.5\ V$

3) 

i) Find p.d across 15 Ω resistor

ii) Find current through 10 Ω resistor

$R_{\parallel} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1}$

$= 5\ \Omega$

$V_{15} = \left(\frac{15}{15+5}\right)10$

$= 7.5\ V$

$E = IR_T$

$10 = I(15+5)$

$I = 0.5\ A$

$V_{15} = I(15)$

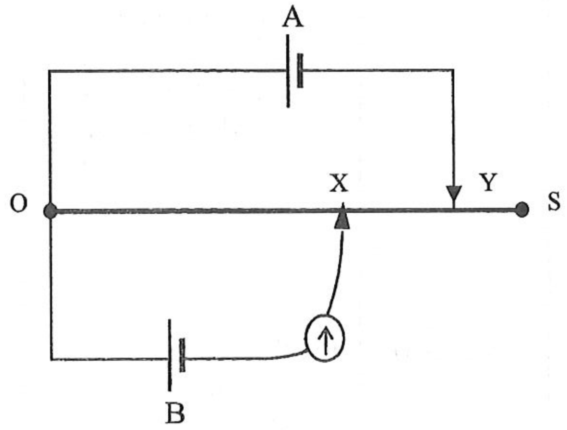
$7.5 = I(15)$

$I = 0.5\ A$

(C) To measure internal resistance

Worked Example 14

Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts X and Y as shown. The slide wire is 1.0 m long and has a resistance of 12 Ω. With OY 75 cm, the galvanometer shows no deflection when OX is 50 cm. If y is moved to touch the end of wire at S, the value of OX which gives no deflection is 62.5 cm. The e.m.f of cell B is 1.0 V.



Calculate

(a) the p.d. across OY when Y is 75 cm from O (with galvanometer balanced),

$\frac{V_{OY}}{V_{OX}} = \frac{V_{OY}}{E_B} = \frac{75}{50} = 1.5$

$V_{OY} = 1.5(1.0)$

$= 1.5\ V$

(b) the p.d. across OS when Y touches S (with galvanometer balanced),

$\frac{V_{OS}}{V_{OX}} = \frac{kL_{OS}}{kL_{OX}} = \frac{1.0}{0.625}$

$V_{OS} = \frac{1.0}{0.625} V_{OX}$

$= 1.6\ V$

(c) the internal resistance and the e.m.f of cell A.

$E_A = V_{OS} + Ir$

$= V_{OS} + \left(\frac{V_{OS}}{R_{OS}}\right) r$

$= 1.6 + \left(\frac{1.6}{12}\right) r$ — (1)

$E_A = V_{OY} + Ir$

$= V_{OY} + \left(\frac{V_{OY}}{0.75 \times 12}\right) r$

$= 1.5 + 0.167 r$ — (2)

solving (1) & (2)

$E_A = 2.0\ V$

$r = 3\ \Omega$

$\frac{\Delta l}{l}$ small, smaller % error

longer length to balance unknown cell is good

driver cell close to normal cell

raise R

Appendix

Electrical measuring instruments

	Ideal	Practical	Connections
Voltmeter	• of **infinite resistance** • doesn't draw current from circuit • reads the theoretical value of p.d. across the points connected.	• of **finite resistance** • draws a small current from circuit • reads a p.d. that is smaller than its theoretical value. The closer its resistance is to the circuit's other resistors, the larger the discrepancy.	• Always to be connected in **parallel** to the two points whose p.d. is to be measured. • Polarities needed to be observed for correct connections.
Ammeter	• of **zero resistance** • with zero p.d. across its ends • reads the theoretical value of current in the path posited.	• of **small finite resistance** • with finite p.d. across its ends • reads a current that is smaller than its theoretical value.	• Always to be connected in **series** along the circuit path where the current is to be measured. • Polarities needed to be observed for correct connections.

Moving coil ammeter

For the given circuit, theoretically, $I = V/R$

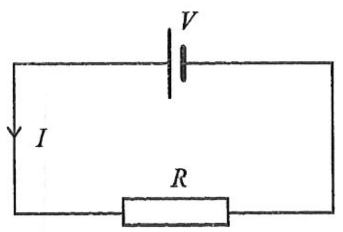


Fig. 1

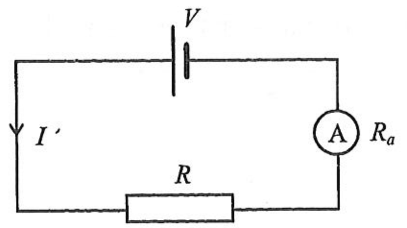


Fig. 2

With the real ammeter with resistance R_a, the current I' is given by

$$I' = \frac{V}{R+R_a}$$

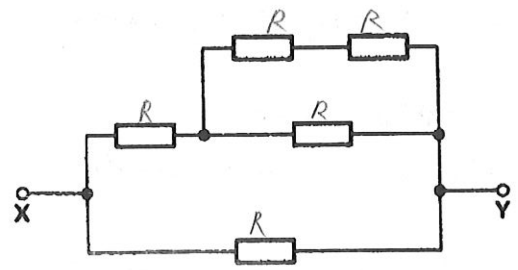
which is smaller than I.

Note:

For I' to be very close to I, R_a must be very small compared to R.

Worked Example 6

The circuit diagram shows a network of resistors each of resistance R. What is the effective resistance between the points X and Y?



$R_{XY} = 0.625R$

$\frac{1}{2R} + \frac{1}{R} = \frac{3}{2R}$

$\frac{2}{3}R$

$R + \frac{2}{3}R = \frac{5}{3}R$

$\frac{1}{R} + \frac{3}{5R} = \frac{5+3}{5R}$

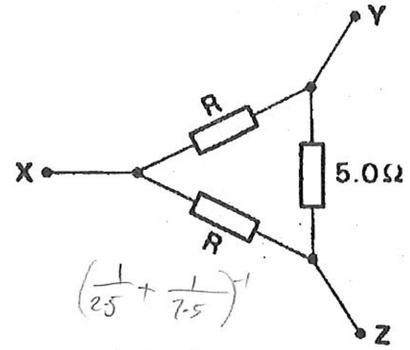
$= \frac{8}{5R}$

Tot $R = \frac{5}{8}R$

$= 0.625R$

Worked Example 7

The diagram shows a network of three resistors. Two of these, marked R, are identical. The other one has a resistance of 5.0 Ω. The resistance between Y and Z is found to be 2.5 Ω What is the resistance between X and Y?

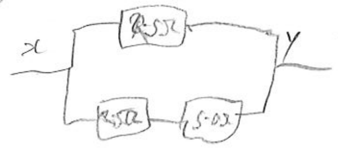


$\frac{1}{5} + \frac{1}{2R} = 2.5$

$\frac{1}{2} + \frac{2.5}{2R} = 1$

$\frac{2.5}{2R} = \frac{1}{2}$

$R = 2.5$



$\left(\frac{1}{2.5} + \frac{1}{7.5}\right)^{-1}$

$R_{XY} = 1.875\ \Omega$

$= 1.88\ \Omega$ (3s.f)

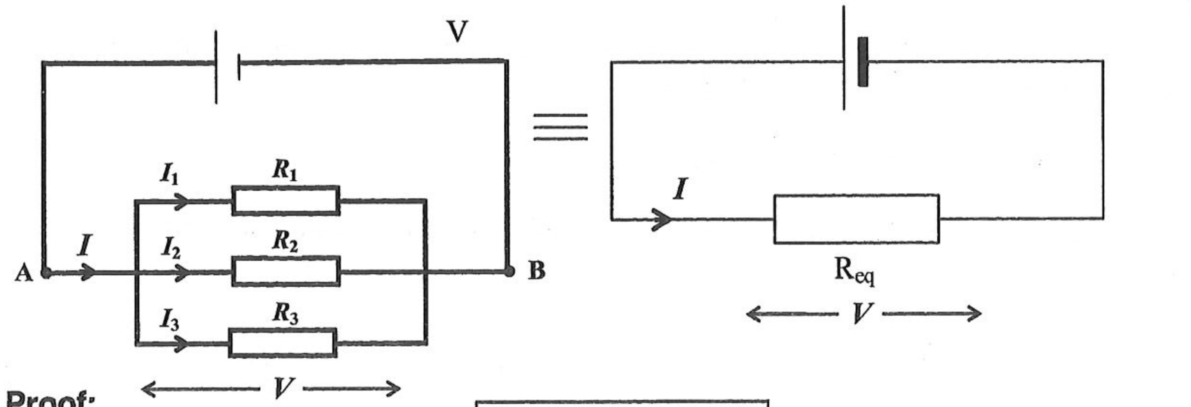
Summary

B. Resistances in Parallel

If the load consists of two or more resistors in parallel, by applying Kirchhoff's laws, the following can be proven to be true:

1. the current passing through the load is the sum of the individual currents through the resistors.

2. all the resistors share the same p.d. which is the overall p.d. across the load.



Proof:

Applying Kirchhoff's First Law, $\boxed{I = I_1 + I_2 + I_3}$

Using Kirchhoff's Second Law, the p.d. *V* across all three resistances are the same. (all 3 share the same 2 end points A and B)

V = Potential at A – Potential at B, so that

$$\boxed{V = I_1 R_1 = I_2 R_2 = I_3 R_3}$$

But *V/I* is the total resistance *R* between A and B, therefore $I = \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

giving $I = \frac{V}{R} = I_1 + I_2 + I_3$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

Note: R_{tot} is less than the smallest R.

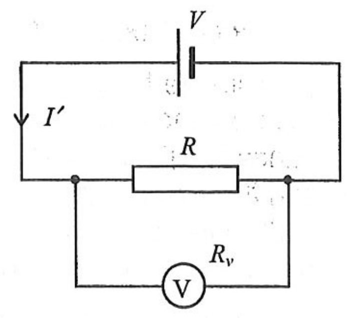
In general, for a parallel circuit with N number of resistors,

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}$$

For n identical resistors of resistance R connected in parallel, $R_{total} = R/n$

Moving coil voltmeter

For a similar circuit, theoretically $I = V/R$



- voltmeter of resistance R_v, connected in parallel to *R*
- effective external resistance is **lowered.**
- voltmeter will draw current from the circuit
- the measured p.d. across across R will be **lower**.
- choose a voltmeter with R_v that is **very much larger** than *R*

Note:

Its reading will be lesser than theoretically expected **only if** there is internal resistance or other external resistors present in the circuit.

As you have seen, in order to measure the current, an ammeter is placed in series, in the circuit.

What effect might this have on the size of the current?

An ideal ammeter has zero resistance, so that placing it in the circuit does not make the current smaller. Real ammeters do have very small resistances – around 0.01 Ω. So by placing real ammeters in series with the circuit, the total resistance increases and lower the total current flowing in the circuit.

A voltmeter is placed in parallel with a component, in order to measure the p.d. across it.

Why can this increase the current in the circuit?

Since the voltmeter is placed parallel with the component, their combined resistance is less than the component's resistance and hence increasing the total current flowing drawn from the cell. The ideal voltmeter has infinite resistance and takes no current. Digital voltmeters have very high resistances, around 10 MΩ, and so they have little effect on the circuit they are placed in.

Some practical points to note on use of potentiometer:

1. It is desirable to obtain a long balance length as far as possible. This is to ensure that the percentage uncertainty in the reading of balance length is small. To obtain a long balance length the series variable resistor will have to be adjusted such that the p.d along the metre wire is only slightly larger than the p.d to be measured ------ remember that the function of the series resistor is to vary the potential gradient of the metre wire such that it is appropriate for the determination of p.d.
2. To maintain a constant potential gradient along the wire, the e.m.f. of the driver cell must be constant. Hence it is important that the circuit should not be switched on unless taking measurements.
3. One should not slide the jockey along the wire as that will cause non-uniformity in the wire.
4. A balanced point cannot be achieved if both cells (driver and test cells) are driving currents in the same direction. They must, therefore, have both positive (or both negative) terminals connected together.

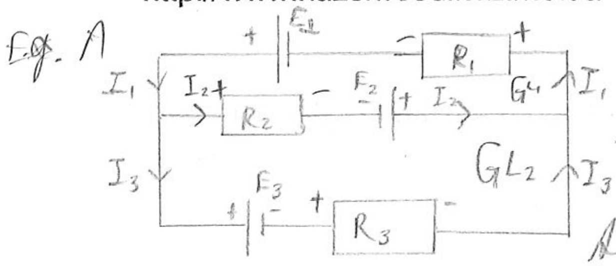
Internet Resources

http://www.walter-fendt.de/ph14e/combres.htm

http://lectureonline.cl.msu.edu/~mmp/kap20/RR506a.htm

http://www.hazelwood.k12.mo.us/~grichert/sciweb/electric.htm

Eg. 1



Step 1: Assign current direction

Step 2: Put down polarity for batteries

Step 3: Put down polarity for resistors

Step 4: Assign loop direction.

Step 5: Apply Kirchoff's Laws (1st, 2nd)

$\Sigma E = \Sigma V$

$\Sigma E - \Sigma V = 0$

- Follow loop direction
- If "+" is encountered first, then there is a potential drop, so the term is "−"

Loop L_1: $E_1 - I_2R_2 + E_2 - I_1R_1 = 0$

Loop L_2: $-E_2 + I_2R_2 - E_3 - I_3R_3 = 0$

Loop L_3: $E_1 - E_3 - I_3R_3 - I_1R_1 = 0$

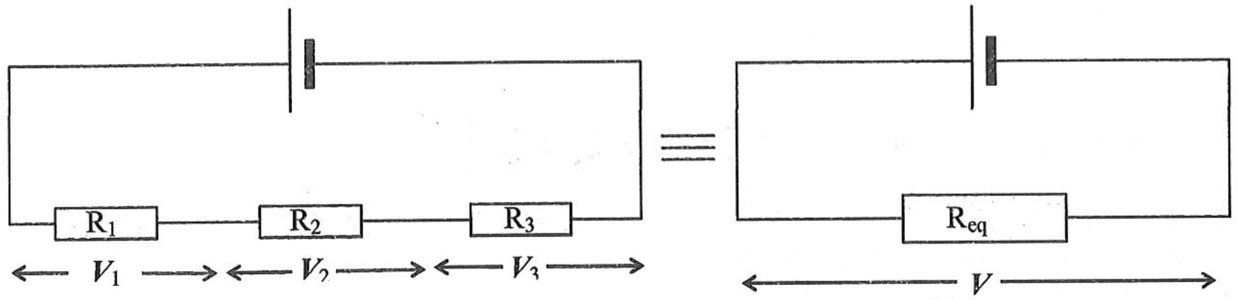
Resistance in Series and Parallel

derive , using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in series.
solve problems using the formula for the combined resistance of two or more resistors in series.
derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in parallel.
solve problems using the formula for the combined resistance of two or more resistors in parallel.

A. Resistances in Series

If a load in the circuit consists of two or more resistors in series, by applying Kirchhoff's Laws, the following can be proven to be true:

1. the same current passes through them.
2. the p.d. across the load is the sum of the individual p.d. across the resistors.



Proof:

By Kirchhoff's 1st Law, the current through each resistor is the same.
Replace the resistors with an effective resistor.

$V = V_1 + V_2 + V_3$

$IR = I(R_1 + R_2 + R_3)$

$R = R_1 + R_2 + R_3$

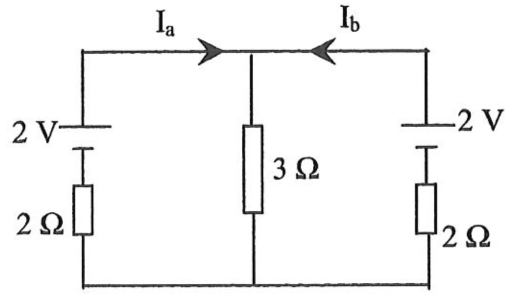
Note: **Total resistance R_{tot} must be greater than any of the individual resistances.**

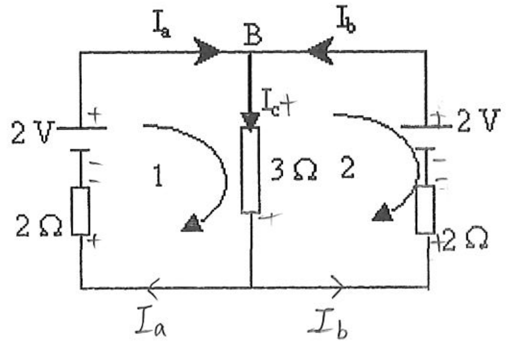
In general, for a series circuit with N number of resistors,

$$\boxed{R_{tot} = R_1 + R_2 + R_3 + \ldots + R_N}$$

Worked Example 5

Use Kirchhoff's Laws to deduce values of the currents as shown in the circuit below.





Using Kirchhof's 1st Law for junction B,

$I_c = I_a + I_b$ —— (1)

Using Kirchhof's 2nd Law for loop 2,

$-2 - 2I_b + 3I_c = 0$ —— (2)

Using Kirchhof's 2nd Law for loop 1,

$+2 - 3I_c - 2I_a = 0$ —— (3)

From (2) & (3), $I_a = I_b$

Hence from (1), $I_c = 2I_b$

Solving (3), $I_b = 0.25A$

$= I_A$

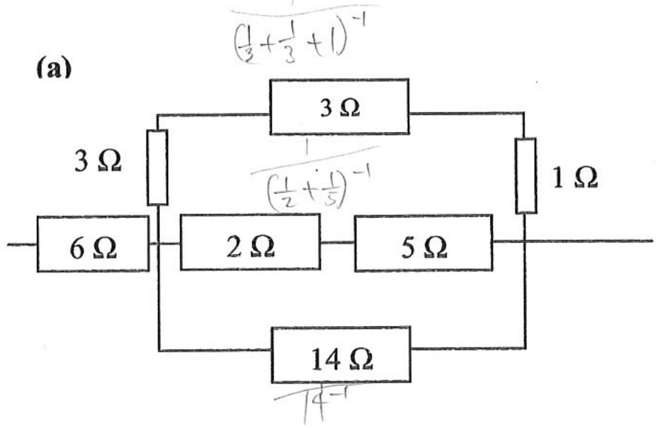
$I_c = 0.50A$

Direct Current Circuits Tutorial

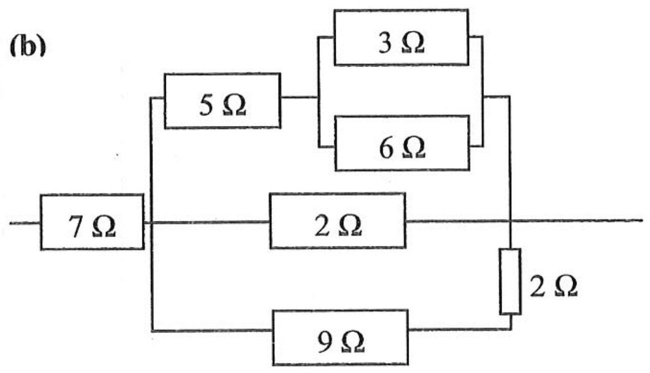
SELF-ATTEMPT::

S1. What is the equivalent resistance in the following?

(a)

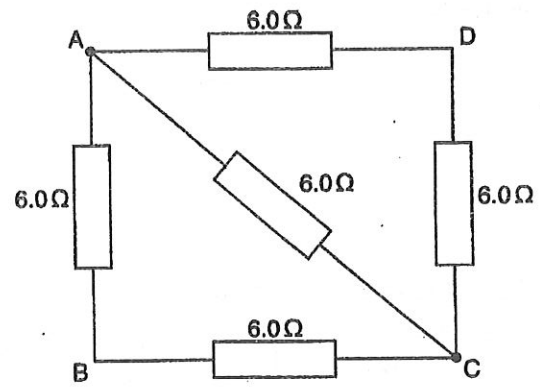


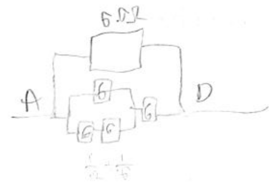
(b)



[8.8 Ω, 8.36 Ω]

S2. A network of resistors, each of resistance 6.0 Ω is constructed as shown below.

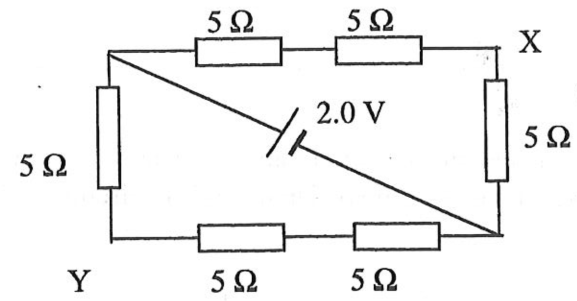


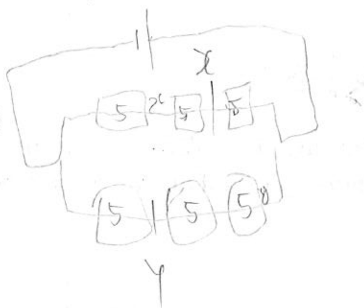


Determine the total resistance of the network between
(a) terminals A and C,
(b) terminals A and D.

[3.0 Ω, 3.75 Ω]

S3. What is the current supplied by the cell? Hence or otherwise determine the p.d. across XY.

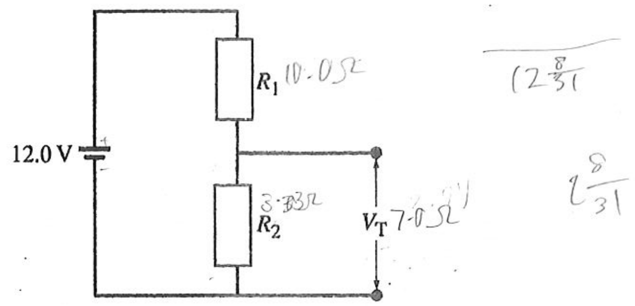




[0.267A,- 2/3V]

VD

S4. A power supply has a fixed output voltage of 12.0 V, but you need V_T = 3.0 V for an experiment.
(a) Using the voltage divider shown , what should R_2 be if R_1 is 10.0 Ω?
(b) What will the terminal voltage V_T be if you connect a load to the 3.0 V terminal, which has a resistance of 7.0 Ω?

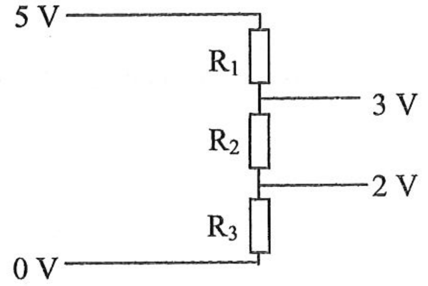


[3.33 Ω, 2.2 V]

VD

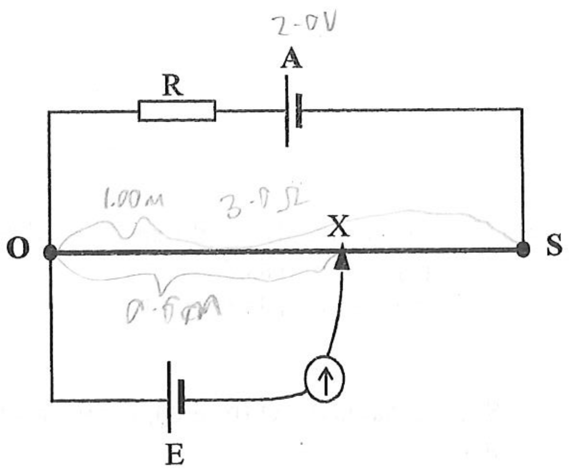
S5. A potential divider is used to give outputs of 2 V and 3 V from a 5 V source.
Give the ratio of resistances R_1, R_2 and R_3 that will give the correct voltages.

[2:1:2]



S6. The driver cell **A** of a potentiometer has an e.m.f of 2.0 V and negligible internal resistance. The potentiometer wire **OS** has a resistance of 3.0 Ω. Calculate the resistance R needed in series with the wire if a p.d. of 5.0 mV is required across the whole wire.

The wire is 100 cm long and a balance length of 60 cm is obtained for an e.m.f of E. What is the value of E?

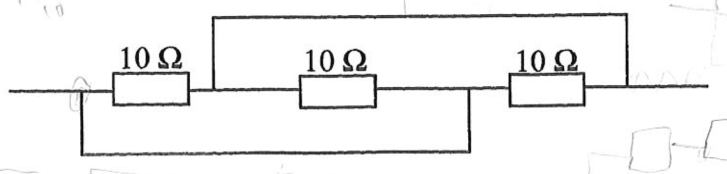


[1197 Ω, 3.0 mV]

DISCUSSION::

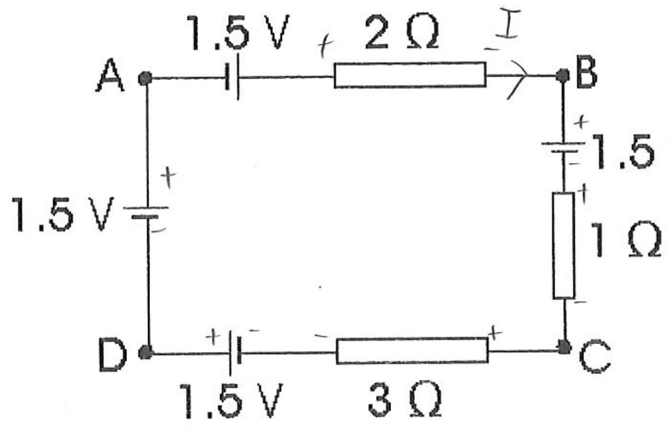
D1. Redraw the following circuit so that the resistors can be clearly seen to be in series, and/or parallel arrangements and determine the resistance of the circuit.

[3.33 Ω]



Worked Example 3

Using Kirchhoff's Laws, determine the current flowing in the circuit.

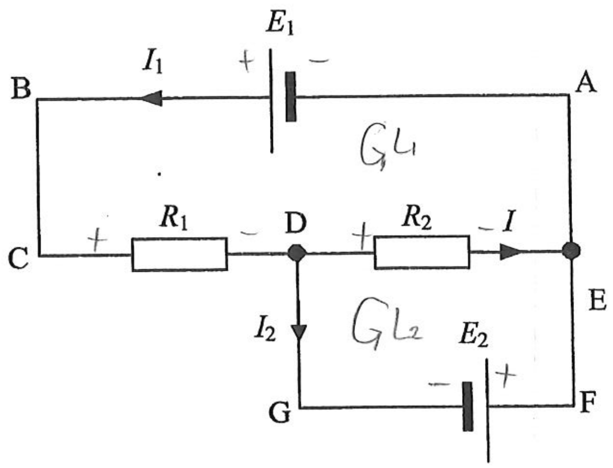


A → B, +1.5 – 2I
B → C, -1.5 – I
C → D, +1.5 – 3I
D → A, +1.5

Net: 3 – 6I = 0
I = 0.50 A

Worked Example 4

For the given circuit, use Kirchhoff's laws to write down expressions in terms of E_1, E_2, I, I_1, I_2, R_1 and R_2 (where appropriate) for the following:
(a) the currents at junction D,
(b) the circuit loop ABCDEA
(c) the circuit loop GFEDG.



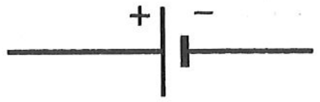
$I_1 = I + I_2$
Loop L_1: $E_1 - IR_2 - I_1R_1 = 0$
Loop L_2: $E_2 + IR_2 = 0$

Here is a good set of guidelines when applying Kirchhoff's Second Law:

1. Assign arbitrary directions of the currents in all branches of the circuit in accordance with Kirchhoff's First Law if they are not already specified.

2. Label on all electrical components (i.e., resistors, e.m.f.s, etc.) with "+" and "–" to indicate the polarity of the potential difference across each of them.

3. For any e.m.f., the 'sense of e.m.f.' is usually given. This does not depend on the direction of the current which is flowing through it. For example,



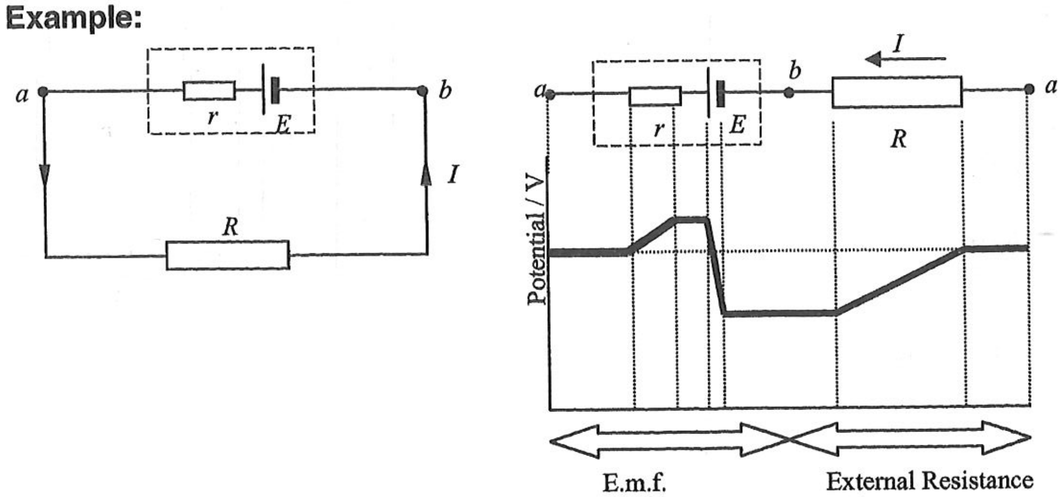
4. For any resistor, the polarity of the potential difference across it depends on the direction of the current which is flowing through it. For example,

5. Next choose the direction of traversing (either clockwise or anticlockwise) for each loop. Usually traversing a loop yields one equation. Traverse the required number of loops to get as many equations as there are unknowns.

6. For each loop, start at any point and traverse round the loop.
For any electrical component, if "+" is encountered before "–", then it means that there is a decrease in potential. The term for this 'drop' should have a minus sign.
Conversely, if "–" is encountered before "+", then it means that there is an increase in potential. The term for this 'rise' should have a positive sign.

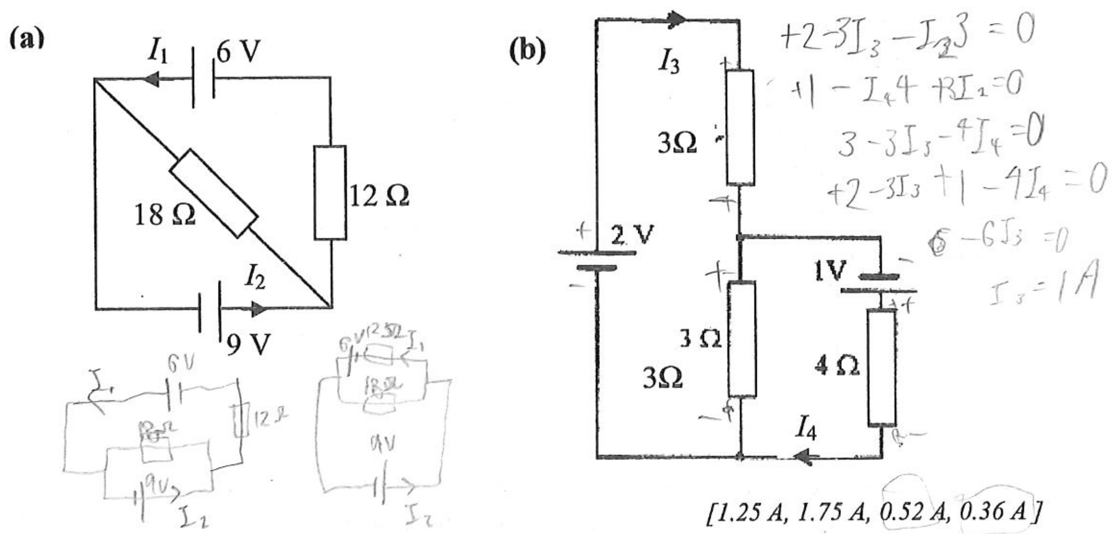
> ***Important to note:***
> Be careful with the signs assigned to the terms. They determine the correctness of your equations.

Example:



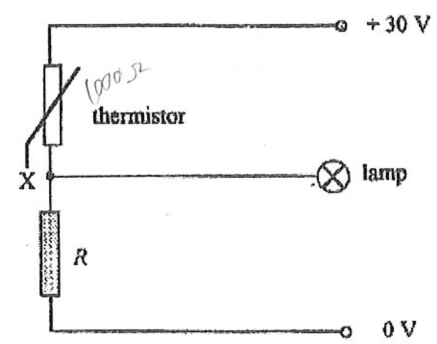
7. After traversing the whole loop, the sum of all increases and decreases in potential must be zero since the potential of the start cum end point should not change.

D2. Using Kirchhoff's laws, find the currents through each cell from I_1 to I_4.



[1.25 A, 1.75 A, 0.52 A, 0.36 A]

D3. A thermistor and an unknown resistor R form a potential divider between voltage lines held at +30 V and 0 V as shown.

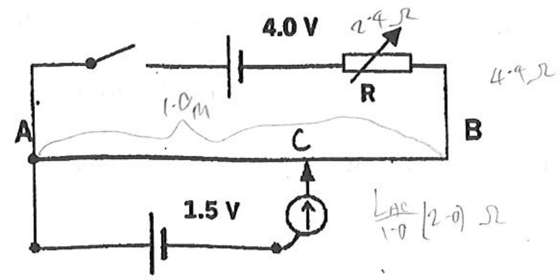


The resistance of the thermistor is 1000 Ω at a temperature of 300 K.

If the indicator lamp lights up when the potential at X is more than 10 V, suggest a suitable value for R so that the lamp will automatically light up when the temperature exceeds 300K.

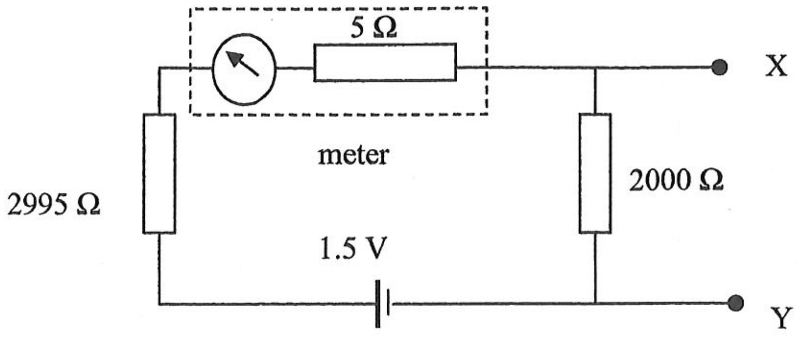
[500 Ω]

D4. A simple potentiometer circuit is set up as follow, using a uniform wire AB, 1.0 m long, which has a resistance of 2.0 Ω. The resistance of the 4 V battery is negligible. If the variable resistor R were given a value of 2.4 Ω, what would be the length AC for zero galvanometer deflection?
If R were made 1.0 Ω and the 1.5 V cell and galvanometer were replaced by a voltmeter of resistance 20 Ω, what would be the reading of the voltmeter if the contact C were placed at the midpoint of AB?



[82.5 cm, 1.29V]

D5. The meter in the circuit shown below has an uncalibrated linear scale. With the circuit as shown, the scale reading is 20. Find the scale reading when another 2000 Ω resistor is connected across XY.



[25]

D6. The current that passes through a certain diode varies with the potential difference across it as shown in fig.a. PQ is a straight line. Two such diodes are connected in parallel with a milliammeter of resistance 100 Ω as shown in fig. b. What is the value of the direct current I when the I_g through the milliammeter is 8 mA?

$V = 100(0.008)$
$= 0.8V$

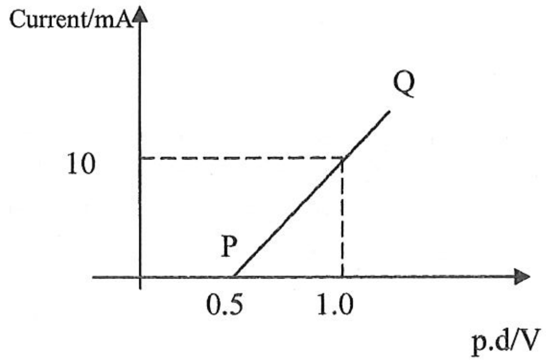


Fig. a.

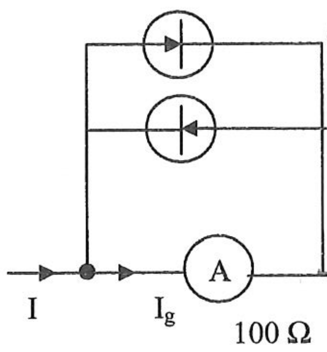


Fig. b.

Grad $PQ = \frac{10}{0.5}$
$= 20$
$y - 0 = 20(x - 0.5)$
$y = 20x - 10$
$y = 20(0.8) - 10$
$= 6mA$
$6 + 8 = 14$

[14 mA]

Assignment

$P = IV$
$V = 12$
$I = \frac{P}{V}$

1(a) A car headlamp is marked 12 V, 72 W. It is switched on for a 20 minute journey. Calculate

(i) the current in the lamp,

(ii) the charge which passes through the lamp during the journey,

(iii) the energy supplied to the lamp during the journey,

(iv) the working resistance of the lamp.

Kirchhoff's Second Law (The Voltage Law)

Round any closed circuit, the algebraic sum of the emfs is equal to the algebraic sum of the potential difference of all the individual components.

Or mathematically,

$$\Sigma E = \Sigma V$$
$$\Sigma E = \Sigma IR$$

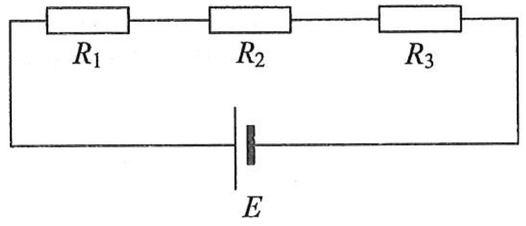
if the components are purely resistive.

- This is a consequence of the ***conservation of energy***. The electrical energy produced by the source is equal to the sum of the electrical energy consumed by all the components.

Proof:

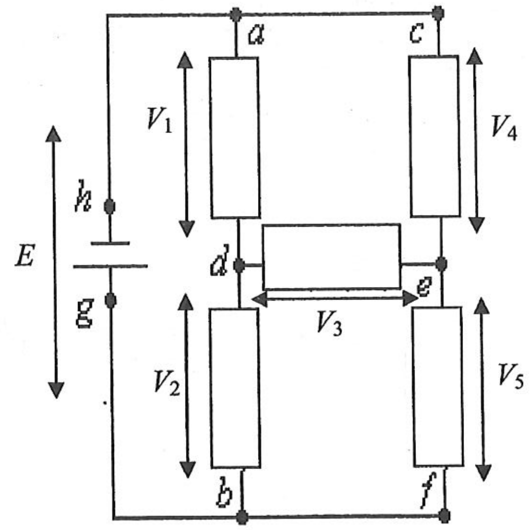
Rate of electrical energy supplied by battery = Rate of energy dissipated in resistors

$IE = I^2(R_1 + R_2 + R_3)$
$E = IR_1 + IR_2 + IR_3$
$= V_1 + V_2 + V_3$



Worked Example 2

A **loop** is any closed conducting path. ~~*aceda*~~ hadfbgh, *defbd*, and *hadefbgh* are examples of loops.



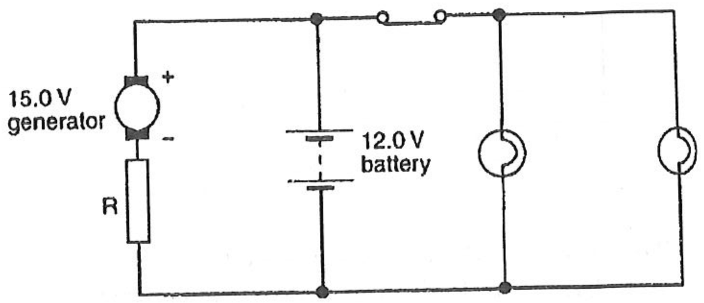
Loop ~~aceda~~, hadfbgh
$E_1 = V_1 + V_2$

Loop defbd,
$\Sigma E = \Sigma V$
$0 = V_3 + V_5 + V_2$

Loop hadefbgh
$E_1 = V_1 + V_3 + V_5$

$E_1 - V_1 + V_3 + V_5 = 0$

(b) Two of the headlamps referred to in part (a) are connected into the circuit shown in the figure, in which one source of e.m.f. (the generator of the car) is placed in parallel with the car battery and the two lamps. Both lamps are on and are working normally.



The battery has an e.m.f. of 12.0 V and negligible internal resistance: the generator has an e.m.f. of 15.0 V and negligible internal resistance. The generator is in series with a variable resistor R.

(i) The value of R is adjusted so that there is no current in the battery when the lamps are on.

Calculate

1. the current in the generator,
2. the value of the resistance of R.

(ii) Calculate the current in the battery when both lamps are switched off, the value of R remaining the same as in (i).

(c) Suggest two advantages which the circuit, as shown in the figure, has over a single power source.

2 The variation with current of the potential difference (p.d) across a component X is shown in the figure below.

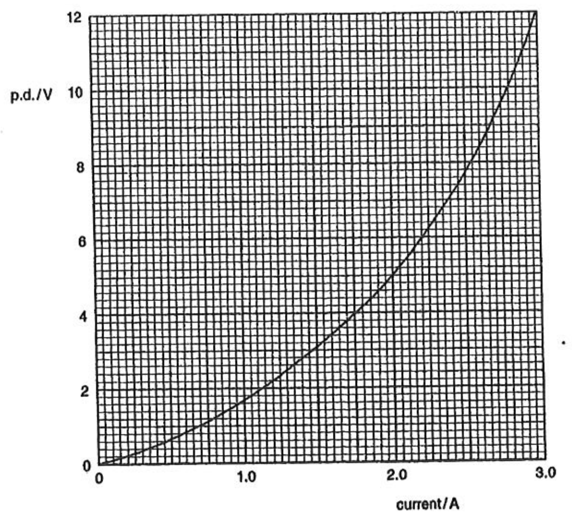


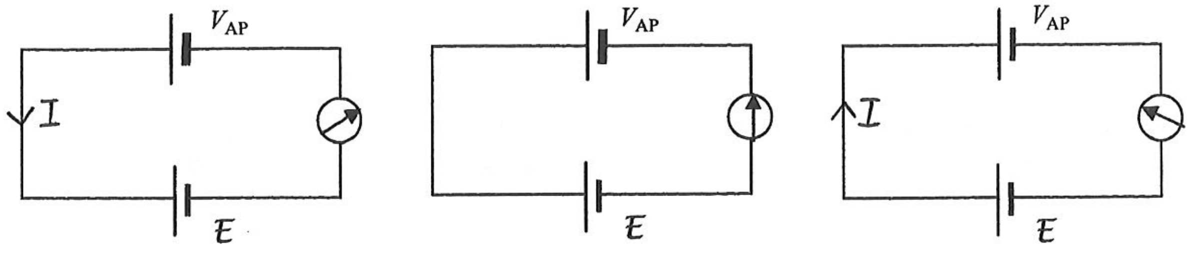
Figure a

(a)(i) State how the resistance of component X varies, if at all, with increase of current.

(ii) Copy the figure and sketch a line to show the variation with the current of the p.d. across a resistor R of constant resistance 3.0 Ω.

The principle of potentiometer:

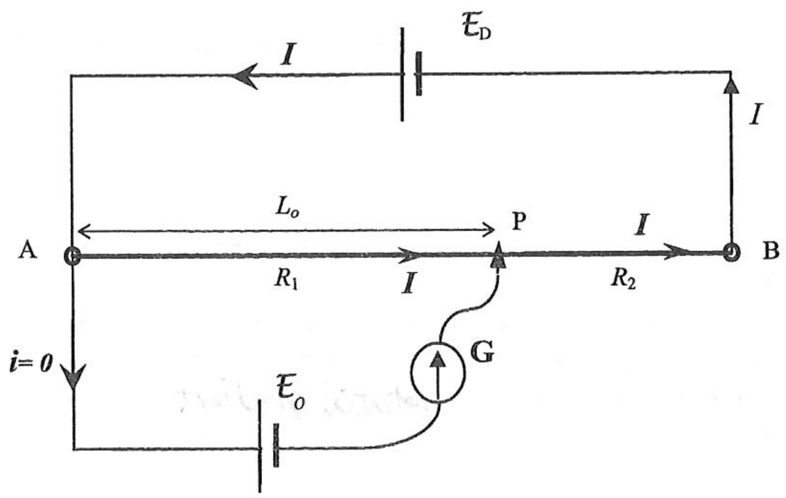
Consider two cells connected back to back with one another,



$\boxed{V_{AP} > E}$ $\boxed{V_{AP} = E}$ $\boxed{V_{AP} < E}$

When two identical cells (or p.d.) are placed back to back with one another in a circuit, the galvanometer registers null deflection (. i.e. no current)

If the cell with unknown emf, E_o, together with a galvanometer is now attached to the potentiometer as follows:



As you move the jockey along the wire (which is the sliding contact), V_{AP} changes.
If $V_{AP} < E_o$, current flows through the resistance wire in the potentiometer circuit as well as the galvanometer, G, in the lower portion of the circuit. Hence the galvanometer deflects.

There exists a point on the wire where a null deflection of the galvanometer is obtained. This means **no current** flows in the **lower circuit**. This position of the jockey on the wire is called the balance point, P and the length AP is called the balance length.

At balance point, e.m.f E_o is equal to V_{AP}. But V_{AP} is proportional to L_o, i.e.

$$\boxed{V_{AP} = kL_o}$$

Hence, the unknown emf $E_o = kL_o$ where k is a constant called the potential gradient of the resistance wire.

Light Dependent Resistor

The LDR is made by linking two metal electrodes with a film of cadmium sulphide.
In complete darkness, it has a resistance of about 10 MΩ but, in bright sunlight, its resistance falls to about 100 Ω. Light intensity ↑, R_{LDR} ↓, ∴ p.d across LDR ↓
The symbol for an LDR is shown in Fig. P.
Figure Q shows a circuit where the variation in light intensity incident on the LDR over a period of time can be represented as a voltage. The variation with time of this voltage follows that of the light intensity.

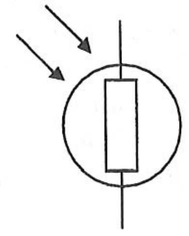
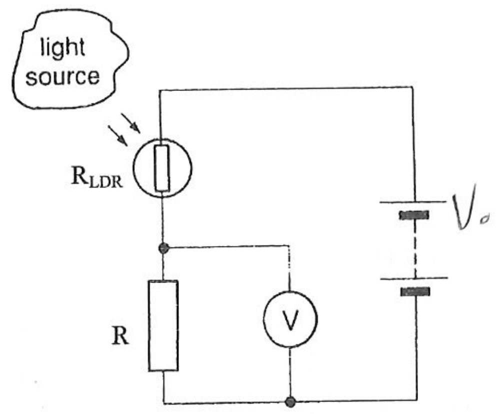


Fig P

∵ Light intensity ↑
∴ R_{LDR} ↓
∴ V_{LDR} ↓
∴ V_R ↑



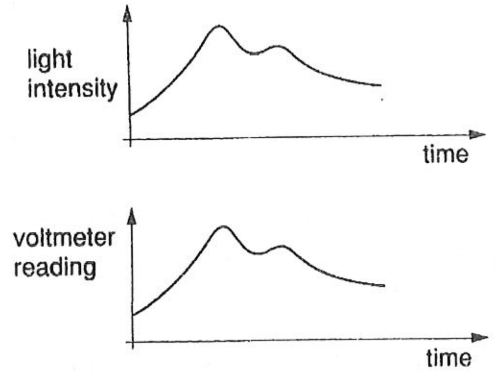


Fig. Q

- The resistance of the light dependent resistor (LDR) decreases with increasing **intensity of light.**
- A potential divider controlled by illumination can be set up as shown.
  The voltage across the R, V_R, is given by

$$\boxed{V_R = \left[\frac{R}{R+R_{LDR}}\right] V_o}$$

As the light intensity increases, R_{LDR} decreases and V_R increases.

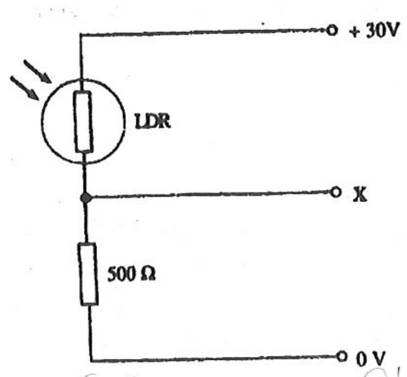
[As light intensity decreases, resistance of LDR increases, potential across the LDR increases and potential difference across fixed resistor decreases.]

Applications of LDR

1) The circuit above (Fig Q) can be used as a light meter.
2) LDR is used in doors to prevent it from closing on a passenger entering the lift; another similar usage is a burglar alarm.
3) Light sensitive switches: lights can be turned on automatically when the surrounding is dark.

Worked Example 10

The light dependent resistor (LDR) and a 500 Ω resistor form a potential divider between voltage lines held at +30V and 0 V as shown in the diagram.
Given that the resistance of the LDR is 1000 Ω in the dark but drops to 100 Ω in bright light, find the corresponding change in the potential at X.



$V_{BC} = V_x - 0$
$= \left(\frac{500}{1500}\right) 30$
$= 10V$ (in dark)
In bright light,
$V_x = 25V$

Change in potential of x
= increase of 15 V

$V_R = \left(\frac{R}{R_1+R_2}\right)V$

$V_x = \left(\frac{1000}{1500}\right)(30)$ = 20

$V_x = \left(\frac{100}{600}\right) 30$ = 5

15

Worked Example 11

A student decided to build a temperature probe and set up the circuit as shown in the figure. The battery has e.m.f. 9.0 V and negligible internal resistance.

The voltmeter has infinite resistance. The calibration curve for the thermistor is shown in the figure below.

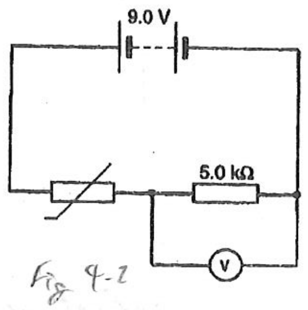


Fig 4.2

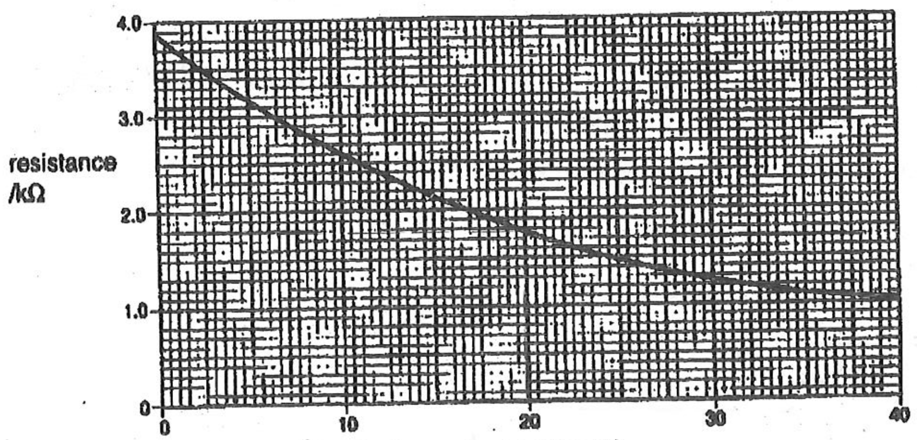


Fig 4.3

(i) Suggest why it is necessary to include a fixed resistor in the circuit of Fig. 4.2.
(ii) Use Fig. 4.3 to find the resistance of the thermistor when the probe is at 30 °C.
(iii) Hence calculate the reading on the voltmeter for the temperature of 30 °C.

(i) The p.d. across the voltmeter, , $V_R = [5/(5+R_t)](9)$, where R_T is the resistance of the thermistor.

As R_T changes with temperature, a variable p.d. can be obtained across the voltmeter. If the 5 kW resistor is left out, and the voltmeter is placed across the thermistor, the p.d. measured is the constant p.d. across the 9 V cell.

(ii) From the graph, R_T = 1.8 kW

(iii) V_R = 6.62 V $V_R = \left(\frac{5}{5+1.8}\right) 9 = 6.62V$

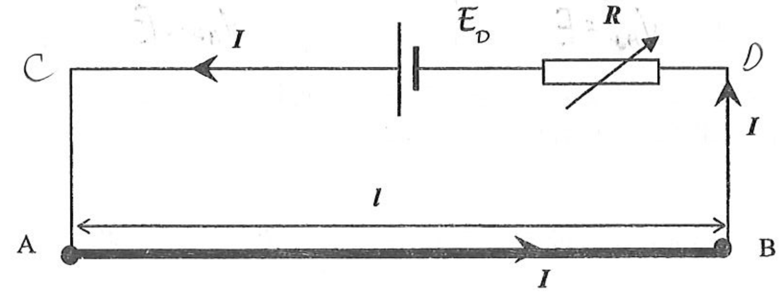
Potentiometer

recall and understand the principle of the potentiometer as a means of comparing potential differences.

Potentiometer Circuit

A potentiometer is an arrangement which measures p.d. accurately. Basically, it consists of a series circuit with **driver cell, variable resistor** and a length of **resistance** wire with uniform cross-sectional area.

In a potentiometer:

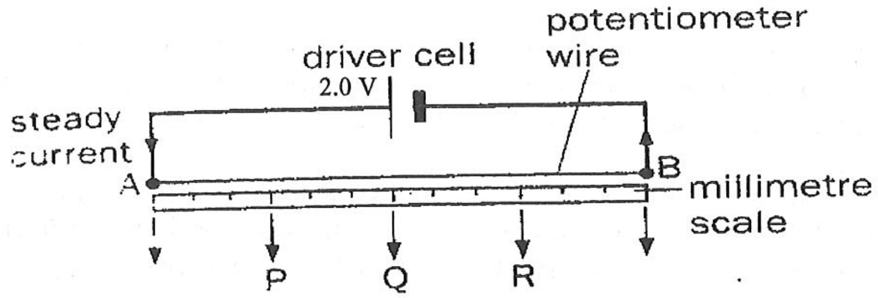


In a potentiometer, a p.d. is set up along the wire AB by the driver cell, E_D, which drives a **steady current, *I* that always flows in this main loop**.

- **About wire AB**:

  AB is assumed to have **uniform cross-section and resistivity**. Hence its resistance per unit length, r is a constant;

  There is a p.d. between any two points on the wire which is proportional to their distance apart. We say that the potential difference, V per unit length is a constant. This potential difference per unit length is called the potential gradient.

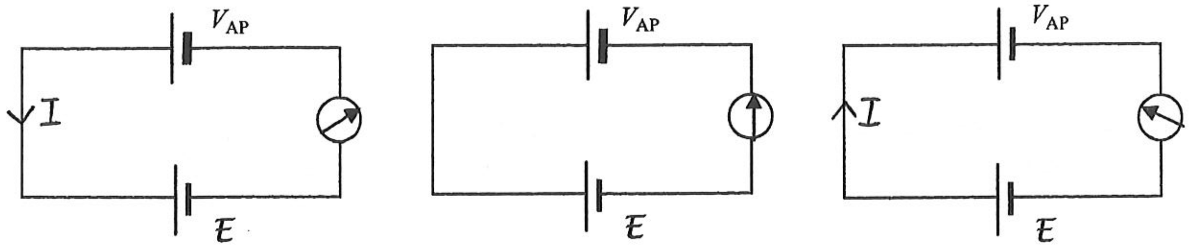


$V_{AP} = 0.50V$ ($\frac{1}{4}$ length of AB)
$V_{AQ} = 1.00V$ ($\frac{1}{2}$ length of AB)
$V_{AR} = 1.50V$ ($\frac{3}{4}$ length of AB)
$V_{QB} = 1.00V$ ($\frac{1}{2}$ length of AB)

p.d. V is proportional to the length of wire L

The principle of potentiometer:

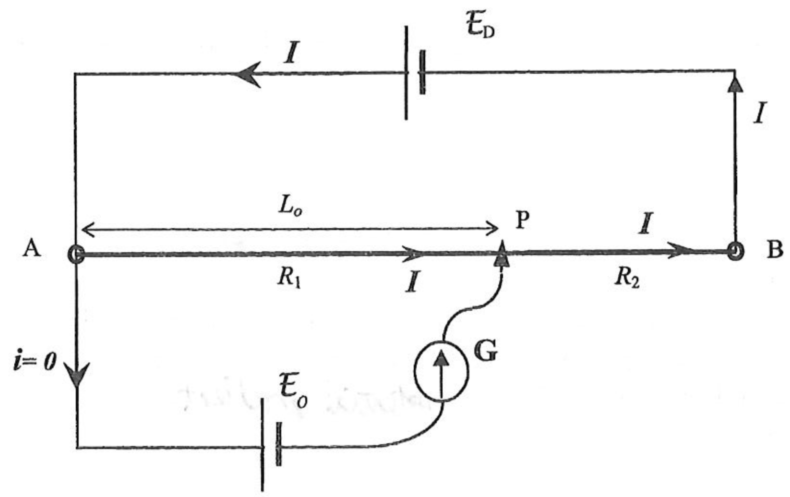
Consider two cells connected back to back with one another,



$\boxed{V_{AP} > E}$ $\boxed{V_{AP} = E}$ $\boxed{V_{AP} < E}$

When two identical cells (or p.d.) are placed back to back with one another in a circuit, the galvanometer registers null deflection (. i.e. no current)

If the cell with unknown emf, E_o, together with a galvanometer is now attached to the potentiometer as follows:



As you move the jockey along the wire (which is the sliding contact), V_{AP} changes.
If $V_{AP} < E_o$, current flows through the resistance wire in the potentiometer circuit as well as the galvanometer, G, in the lower portion of the circuit. Hence the galvanometer deflects.

There exists a point on the wire where a null deflection of the galvanometer is obtained. This means **no current** flows in the **lower circuit**. This position of the jockey on the wire is called the balance point, P and the length AP is called the balance length.

At balance point, e.m.f E_o is equal to V_{AP}. But V_{AP} is proportional to L_o, i.e.

$$\boxed{V_{AP} = kL_o}$$

Hence, the unknown emf $E_o = kL_o$ where k is a constant called the potential gradient of the resistance wire.

Light Dependent Resistor

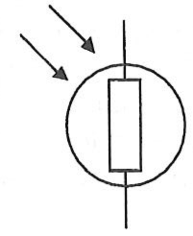


Fig P

The LDR is made by linking two metal electrodes with a film of cadmium sulphide.
In complete darkness, it has a resistance of about 10 MΩ but, in bright sunlight, its resistance falls to about 100 Ω. Light intensity ↑, R_{LDR} ↓,
The symbol for an LDR is shown in Fig. P. ∴ p.d across LDR ↓
Figure Q shows a circuit where the variation in light intensity incident on the LDR over a period of time can be represented as a voltage. The variation with time of this voltage follows that of the light intensity.

∴ Light intensity ↑
∴ R_{LDR} ↓
∴ V_{LDR} ↓
∴ V_R ↑

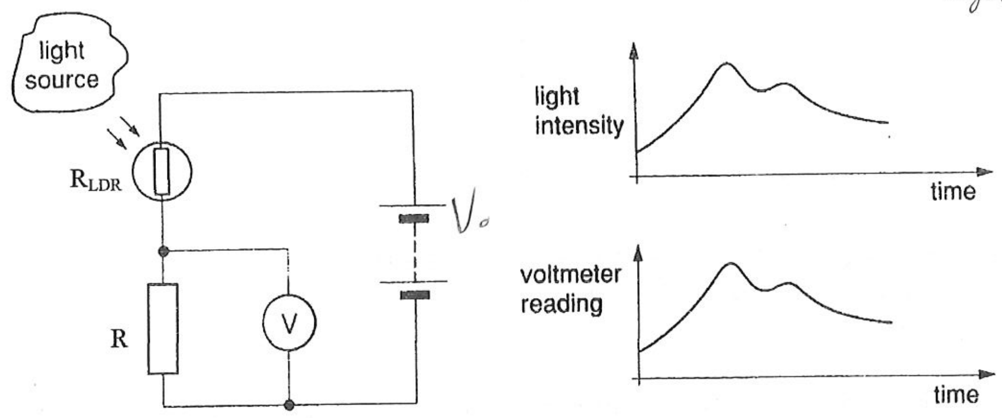


Fig. Q

- The resistance of the light dependent resistor (LDR) decreases with increasing **intensity of light.**
- A potential divider controlled by illumination can be set up as shown.
  The voltage across the R, V_R, is given by

$$\boxed{V_R = \left[\frac{R}{R + R_{LDR}}\right] V_o}$$

As the light intensity increases, R_{LDR} decreases and V_R increases.

[As light intensity decreases, resistance of LDR increases, potential across the LDR increases and potential difference across fixed resistor decreases.]

Applications of LDR

1) The circuit above (Fig Q) can be used as a light meter.
2) LDR is used in doors to prevent it from closing on a passenger entering the lift; another similar usage is a burglar alarm.
3) Light sensitive switches: lights can be turned on automatically when the surrounding is dark.

Thermistors & Light Dependent Resistors

describe and explain the use of thermistors and light dependent resistors in potential dividers to provide a potential difference which is dependent on temperature and illumination respectively

Thermistor

The thermistor is a temperature-dependent resistor which is manufactured in a number of different shapes and sizes using the oxides of various metals.

Negative temperature coefficient types have a resistance which becomes smaller as the temperature increases. The symbol for a thermistor is shown in Fig X.

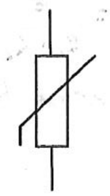


Fig X

- They are heated either externally from the surroundings or internally by the current flowing through them.
- Figure Y shows a circuit where the variation in temperature of a thermistor over a period of time can be represented by a voltage reading.
  The variation with time of this voltage follows that of the temperature.

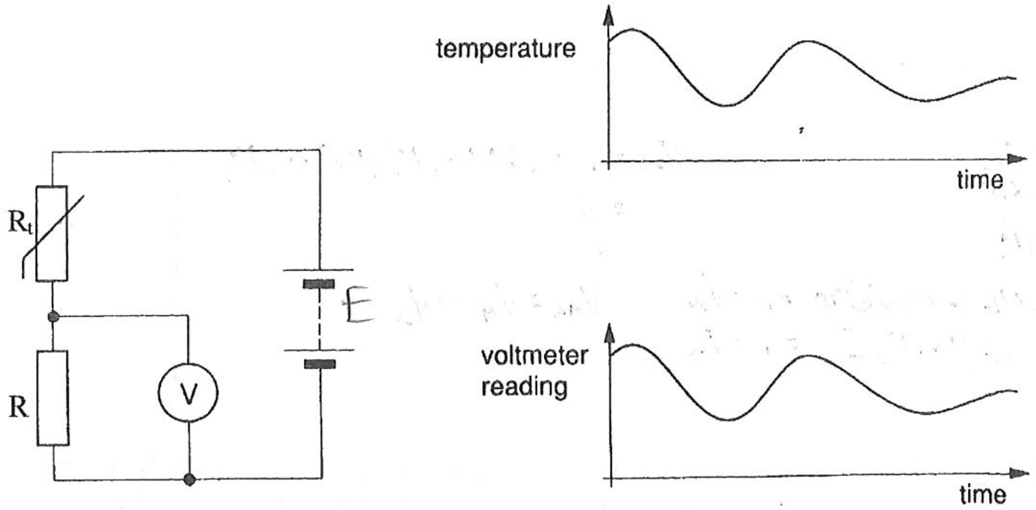


Fig Y

The voltage across R, V_R is $\boxed{V_R = \left(\frac{R}{R+R_t}\right)\cdot E}$

When temperature increases, R_t decreases and V_R increases.

[When temperature decreases, resistance of thermistor increases, voltage across fixed resistor decreases.]

Applications of Thermistor

The circuit above enables the thermistor to be used for temperature measurement. The galvanometer scale can be calibrated to read temperature.

This is used in
- measurement of respiration frequency of aircraft pilots.
- cardiac pacer to sense temperature of blood.

Uses Of The Potentiometer

Potentiometer can be used to:
(A) To compare or measure the e.m.f.s of cells.
(B) To compare or measure resistances.
(C) To measure internal resistance of a cell.

(A) To compare or measure the e.m.f.s of cells.

When switch, S is at position 1, the balance length found is l_1, where C is the balance point.

$E = kl_B$ $E_1 = kl_1$

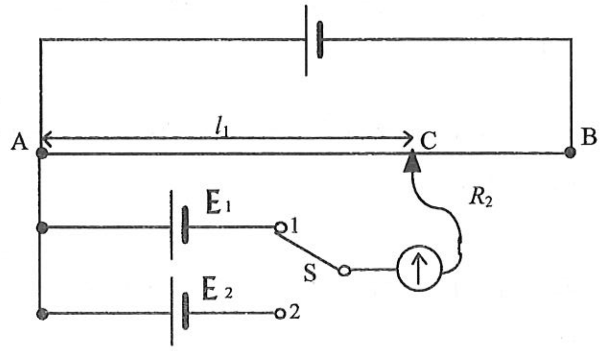
$k = \frac{E}{l_B}$ $= \left(\frac{E}{l_B}\right) l_1$

Then $E_1 = l_1E$

When switch is at position 2, the balance length found is l_2 where C′ is the new balance point. $= \left(\frac{l_1}{l_B}\right)E$

$E_2 = kl_2$

Then $E_2 = l_2E$ $= \left(\frac{E}{l_B}\right) l_2 = \left(\frac{l_2}{l_{AB}}\right)E$



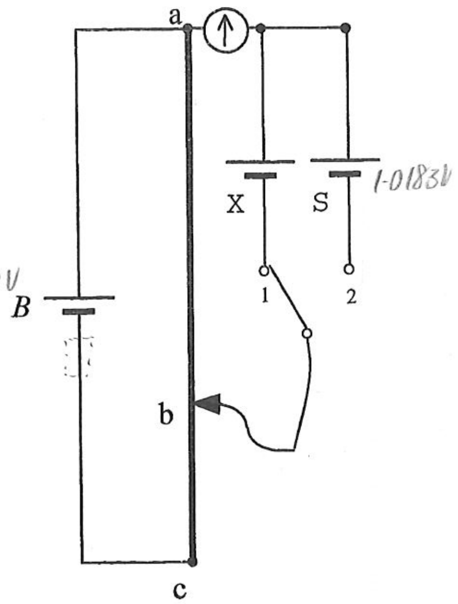
Taking ratio, $\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$

If E_1 is a known emf source, E_2 can be found.

Worked Example 12

A potentiometer is set up to measure the e.m.f. of cell X. B is a battery whose e.m.f. is approximately 3.0 V and whose internal resistance is unknown. S is a cell of 1.0183 V. The switch is set at position 2, placing the standard cell in the galvanometer circuit. When the tap b is 0.36 of the distance from a to c the galvanometer G reads zero.

a) What is the p.d. across the entire length of resistor ac?
b) The switch is then set at position 1 and a new zero reading of the galvanometer obtained when b is 0.47 of the distance from a to c. What is the e.m.f. of cell X ?



a) $V_{ac} = kL_{ac}$ — (1)

$V_{ab} = kL_{ab}$ — (2)

$V_{ac} = \left(\frac{L_{ac}}{L_{ab}}\right)V_{ab}$

$= 2.83\ V$

$\frac{V_{ac}}{V_{ab}} = \frac{kL_{ac}}{kL_{ab}}$

$V_{ac} = \left(\frac{L_{ac}}{0.36L_{ac}}\right) 1.0183$

$= 2.83V$

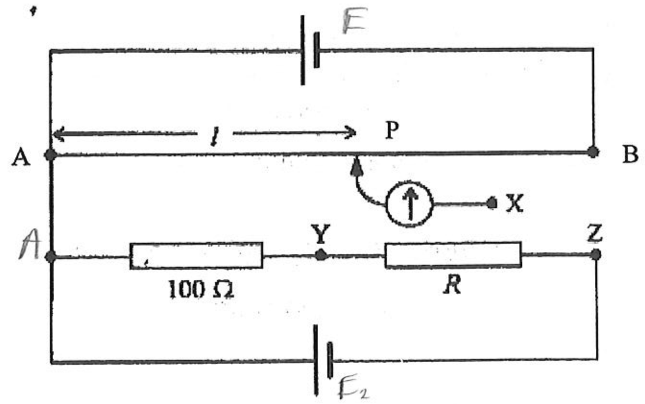
b) $\frac{V_{ab}}{V_{ac}} = \frac{L_{ab}}{L_{ac}} = 0.47$

$V_{ab} = 1.33V$ $\left(\frac{0.47L_{ac}}{L_{ac}}\right) 2.83$

(B) To compare or measure resistances.

Worked Example 13

The diagram shows a circuit which may be used to compare the resistance R of an unknown resistor with a 100 Ω standard. The distance l from one end of the potentiometer slide-wire to the balance points are 400 mm and 588 mm when X is connected to Y and to Z respectively. The length of the slide wire is 1.00 m. What is the value of R?



When X is connected to Y and balanced point is obtained,,

$V_{AP} = l_{AP}\, V_{AB} = \frac{100}{100+R} V_{AZ}$ —— (1)

When X is connected to Z and with balanced point,

$V_{AP'} = k\, l_{AP'} = V_{AZ}$ —— (2)

Taking ratio of the two equations,

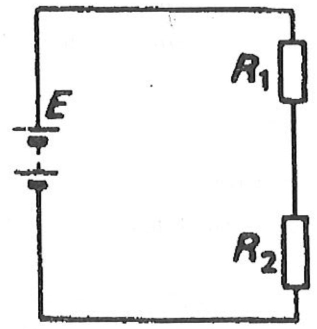
$$\frac{k\, l_{AP}}{k\, l_{AP'}} = \frac{\frac{100}{100+R} V_{AZ}}{V_{AZ}}$$

$R = 47\,\Omega$

Summary

Worked Example 8

A battery of e.m.f. E and negligible internal resistance is connected to two resistors of resistances R_1 and R_2 as shown in the circuit diagram. What is the p.d. across resistor R_1?



$I = \frac{E}{R_1 + R_2}$

P.d across R_1, $V_1 = IR_1$

$= \frac{R_1}{(R_1+R_2)} \cdot E$

Worked Example 9

Two resistors, of resistance 200 kΩ and 1 MΩ respectively, form a potential divider with outer junctions maintained at potentials of +3 V and – 15 V.

What is the potential at the junction X between the resistors?

$I = \frac{3-(-15)}{1200 \times 10^3}$

$= 0.15\,mA$

P.d across 200kΩ resistor $= 3 - V_x$

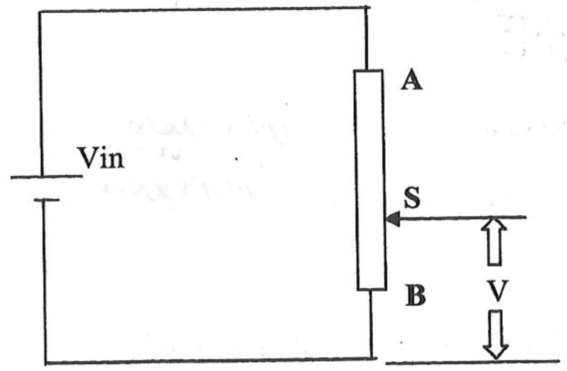
$(200 \times 10^3) I = 3 - V_x$

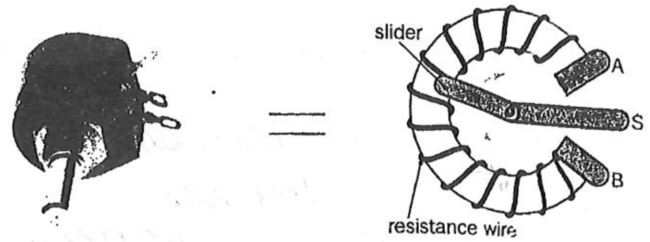
$V_x = 3 - (200 \times 10^3)(0.15 \times 10^{-3})$

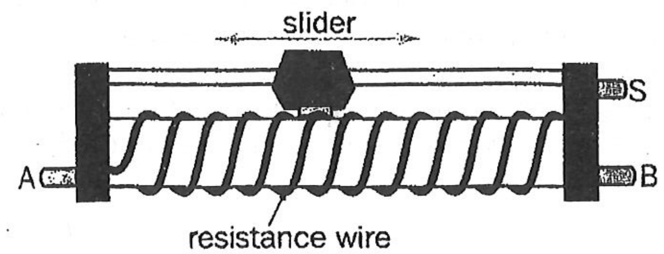
$= 0$

$V_{AX} = V_A - V_x$

The simplest form of a practical potential divider makes use of a variable resistor as shown below. The desired voltage is a fraction of the p.d. from the cell and is tapped off by varying the sliding contact along the resistor.







3 Balanced Potentials

Potential Divider

understand the use of a potential divider as a source of variable p.d..

$V = JC^{-1}$

You have a 1 V small light bulb but a 6 V cell. What should you do to light up the bulb without causing damage to the bulb?

To obtain only part of the voltage provided by a battery, we use a potential divider.

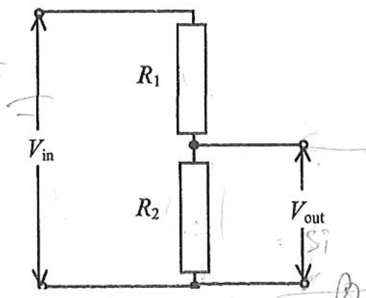
- Basically, a potential divider consists of two resistors connected in series, across a fixed voltage source, so that
- the same current flows through the two resistors and
- the p.d. across the resistors is divided into two portions.

- Simplest form of a potential divider:

V_{in} : input p.d. applied across R_1 and R_2.

V_{out} : output p.d. taken across R_2.

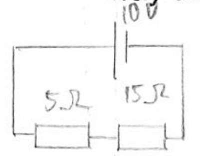
If output current is zero then the current I flowing through R_1 also flows through R_2.



$$V_{in} = I(R_1 + R_2)$$
$$I = \frac{V_{in}}{R_1 + R_2}$$
$$V_{out} = IR_2 = \frac{R_2}{(R_1 + R_2)} \times V_{in}$$

Note:

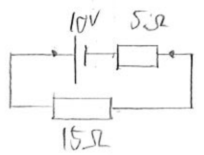
By adjusting the values of R_1 *and* R_2, the desired values of V_{out} can be obtained. This way the potential divider can be used to control the p.d. V applied across any device.

1)


Find p.d across 15 Ω resistor

$V_{15} = \frac{15}{15+5} \times 10$

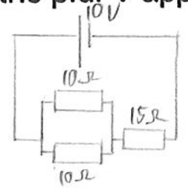
$= 7.5\ V$

2)


Find p.d across 5Ω internal resistor

$V_5 = \left(\frac{5}{5+15}\right)10$

$= 2.5\ V$

3)


i) Find p.d across 15Ω resistor

ii) Find current through 10Ω resistor

$R_{\parallel} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1}$

$= 5\ \Omega$

$V_{15} = \left(\frac{15}{15+5}\right)10$

$= 7.5\ V$

$E = IR_T$

$10 = I(15+5)$

$I = 0.5\ A$

$V_{15} = I(15)$

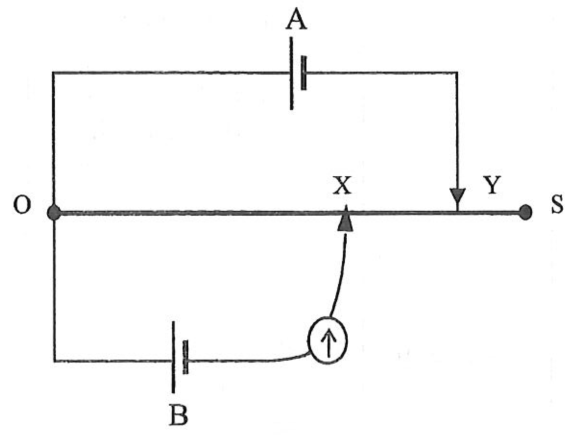
$7.5 = I(15)$

$I = 0.5\ A$

(C) To measure internal resistance

Worked Example 14

Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts X and Y as shown. The slide wire is 1.0 m long and has a resistance of 12 Ω. With OY 75 cm, the galvanometer shows no deflection when OX is 50 cm. If y is moved to touch the end of wire at S, the value of OX which gives no deflection is 62.5 cm. The e.m.f of cell B is 1.0 V.



Calculate

(a) the p.d. across OY when Y is 75 cm from O (with galvanometer balanced),

$$\frac{V_{OY}}{V_{OX}} = \frac{V_{OY}}{E_B} = \frac{75}{50} = 1.5$$
$$V_{OY} = 1.5(1.0) = 1.5\ V$$

(b) the p.d. across OS when Y touches S (with galvanometer balanced),

$$\frac{V_{OS}}{V_{OX}} = \frac{kl_{OS}}{kl_{OX}} = \frac{1.0}{0.625}$$
$$V_{OS} = \frac{1.0}{0.625} V_{OX} = 1.6\ V$$

(c) the internal resistance and the e.m.f of cell A.

$$E_A = V_{OS} + Ir = V_{OS} + \left(\frac{V_{OS}}{R_{OS}}\right) r = 1.6 + \left(\frac{1.6}{12}\right) r \quad \text{— (1)}$$
$$E_A = V_{OY} + Ir = V_{OY} + \left(\frac{V_{OY}}{0.75 \times 12}\right) r = 1.5 + 0.167r \quad \text{— (2)}$$

Solving (1) & (2)

$E_A = 2.0\ V$

$r = 3\ \Omega$

$\frac{\Delta l}{l}$ small, smaller % error

longer length to balance unknown cell is good

driver cell close to normal cell

raise R

Appendix

Electrical measuring instruments

	Ideal	Practical	Connections
Voltmeter	• of **infinite resistance** • doesn't draw current from circuit • reads the theoretical value of p.d. across the points connected.	• of **finite resistance** • draws a small current from circuit • reads a p.d. that is smaller than its theoretical value. The closer its resistance is to the circuit's other resistors, the larger the discrepancy.	• Always to be connected in **parallel** to the two points whose p.d. is to be measured. • Polarities needed to be observed for correct connections.
Ammeter	• of **zero resistance** • with zero p.d. across its ends • reads the theoretical value of current in the path posited.	• of **small finite resistance** • with finite p.d. across its ends • reads a current that is smaller than its theoretical value.	• Always to be connected in **series** along the circuit path where the current is to be measured. • Polarities needed to be observed for correct connections.

Moving coil ammeter

For the given circuit, theoretically, $I = V/R$

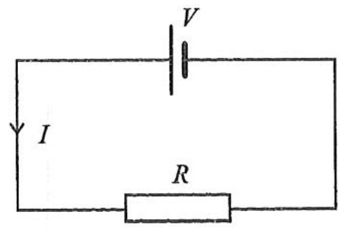


Fig. 1

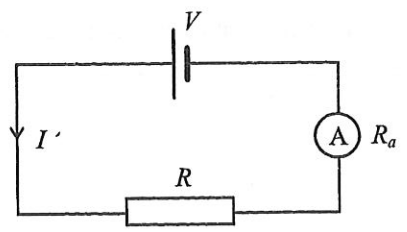


Fig. 2

With the real ammeter with resistance R_a, the current I' is given by

$$I' = \frac{V}{R + R_a}$$

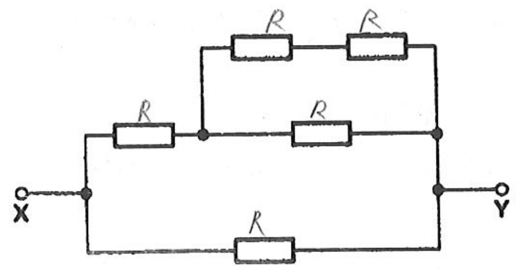
which is smaller than I.

Note:

For I' to be very close to I, R_a must be very small compared to R.

Worked Example 6

The circuit diagram shows a network of resistors each of resistance R. What is the effective resistance between the points X and Y?



$R_{XY} = 0.625R$

$\frac{1}{2R} + \frac{1}{R} = \frac{3}{2R}$

$\frac{2}{3}R$

$R + \frac{2}{3}R = \frac{5}{3}R$

$\frac{1}{R} + \frac{3}{5R} = \frac{5+3}{5R}$

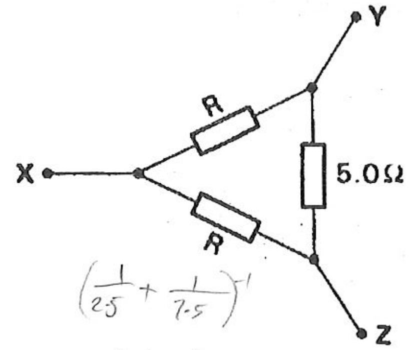
$= \frac{8}{5R}$

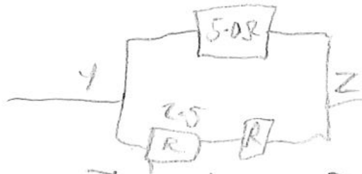
Tot $R = \frac{5}{8}R$

$= 0.625R$

Worked Example 7

The diagram shows a network of three resistors. Two of these, marked R, are identical. The other one has a resistance of 5.0 Ω. The resistance between Y and Z is found to be 2.5 Ω What is the resistance between X and Y?



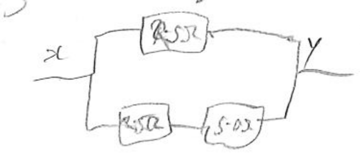


$\frac{1}{5} + \frac{1}{2R} = 2.5$

$\frac{1}{2} + \frac{2.5}{2R} = 1$

$\frac{2.5}{2R} = \frac{1}{2}$

$R = 2.5$



$\left(\frac{1}{2.5} + \frac{1}{7.5}\right)^{-1}$

$R_{XY} = 1.875\ \Omega$

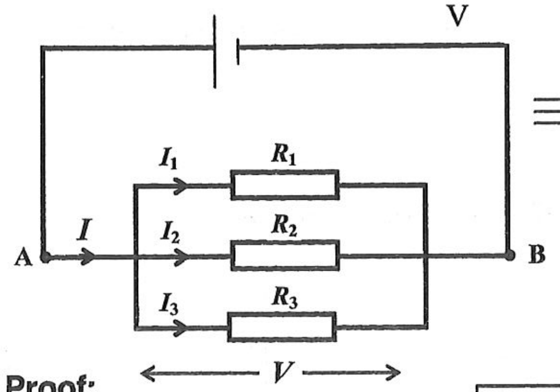
$= 1.88\ \Omega$ (3 s.f.)

Summary

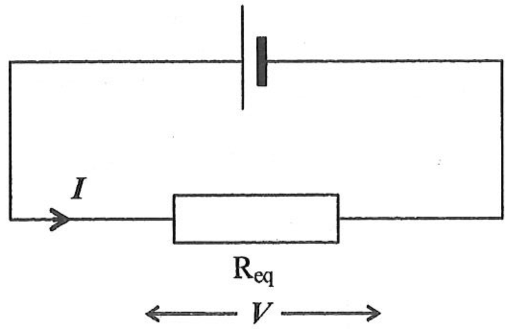
B. Resistances in Parallel

If the load consists of two or more resistors in parallel, by applying Kirchhoff's laws, the following can be proven to be true:

1. the current passing through the load is the sum of the individual currents through the resistors.
2. all the resistors share the same p.d. which is the overall p.d. across the load.



≡



Proof:
Applying Kirchhoff's First Law, $\boxed{I = I_1 + I_2 + I_3}$

Using Kirchhoff's Second Law, the p.d. V across all three resistances are the same. (all 3 share the same 2 end points A and B)
V = Potential at A – Potential at B, so that

$$\boxed{V = I_1 R_1 = I_2 R_2 = I_3 R_3}$$

But V/I is the total resistance R between A and B, therefore $I = \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

giving $I = \frac{V}{R} = I_1 + I_2 + I_3$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

Note: R_{tot} is less than the smallest R.

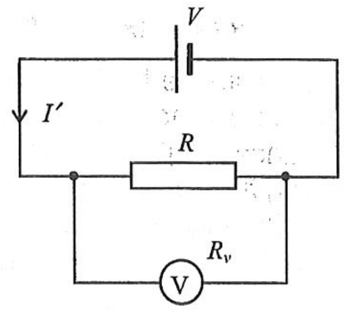
In general, for a parallel circuit with N number of resistors,

$$\boxed{\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}}$$

For n identical resistors of resistance R connected in parallel, $R_{total} = R/n$

Moving coil voltmeter

For a similar circuit, theoretically $I = V/R$



- voltmeter of resistance R_v, connected in parallel to R
- effective external resistance is **lowered.**
- voltmeter will draw current from the circuit
- the measured p.d. across across R will be **lower**.
- choose a voltmeter with R_v that is **very much larger** than R

Note:

Its reading will be lesser than theoretically expected **only if** there is internal resistance or other external resistors present in the circuit.

As you have seen, in order to measure the current, an ammeter is placed in series, in the circuit.

What effect might this have on the size of the current?

An ideal ammeter has zero resistance, so that placing it in the circuit does not make the current smaller. Real ammeters do have very small resistances – around 0.01 Ω. So by placing real ammeters in series with the circuit, the total resistance increases and lower the total current flowing in the circuit.

A voltmeter is placed in parallel with a component, in order to measure the p.d. across it.

Why can this increase the current in the circuit?

Since the voltmeter is placed parallel with the component, their combined resistance is less than the component's resistance and hence increasing the total current flowing drawn from the cell. The ideal voltmeter has infinite resistance and takes no current. Digital voltmeters have very high resistances, around 10 MΩ, and so they have little effect on the circuit they are placed in.

Some practical points to note on use of potentiometer:

1. It is desirable to obtain a long balance length as far as possible. This is to ensure that the percentage uncertainty in the reading of balance length is small. To obtain a long balance length the series variable resistor will have to be adjusted such that the p.d along the metre wire is only slightly larger than the p.d to be measured ------ remember that the function of the series resistor is to vary the potential gradient of the metre wire such that it is appropriate for the determination of p.d.
2. To maintain a constant potential gradient along the wire, the e.m.f. of the driver cell must be constant. Hence it is important that the circuit should not be switched on unless taking measurements.
3. One should not slide the jockey along the wire as that will cause non-uniformity in the wire.
4. A balanced point cannot be achieved if both cells (driver and test cells) are driving currents in the same direction. They must, therefore, have both positive (or both negative) terminals connected together.

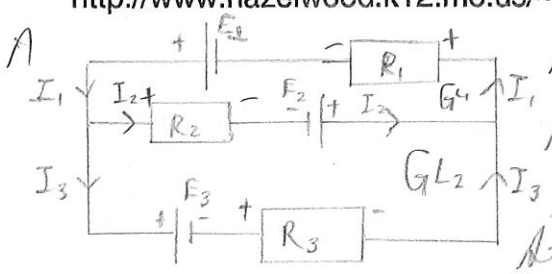
Internet Resources

http://www.walter-fendt.de/ph14e/combres.htm

http://lectureonline.cl.msu.edu/~mmp/kap20/RR506a.htm

http://www.hazelwood.k12.mo.us/~grichert/sciweb/electric.htm

Eg. 1



Step 1: Assign current direction

Step 2: Put down polarity for batteries

Step 3: Put down polarity for resistors

Step 4: Assign loop direction. L_1, L_3

Step 5: Apply Kirchoff's Laws (1st, 2nd)

$\Sigma E = \Sigma V$

$\Sigma E - \Sigma V = 0$

- Follow loop direction
- If "+" is encountered first, then there is a potential drop, so the term is "−"

Loop L_1: $E_1 - I_2R_2 + E_2 - I_1R_1 = 0$

Loop L_2: $-E_2 + I_2R_2 - E_3 - I_3R_3 = 0$

Loop L_3: $E_1 - E_3 - I_3R_3 - I_1R_1 = 0$

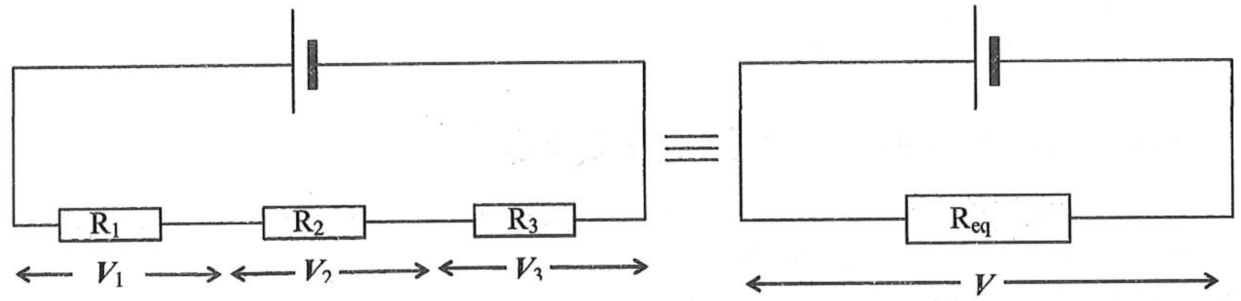
Resistance in Series and Parallel

derive , using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in series.
solve problems using the formula for the combined resistance of two or more resistors in series.
derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in parallel.
solve problems using the formula for the combined resistance of two or more resistors in parallel.

A. Resistances in Series

If a load in the circuit consists of two or more resistors in series, by applying Kirchhoff's Laws, the following can be proven to be true:

1. the same current passes through them.
2. the p.d. across the load is the sum of the individual p.d. across the resistors.



Proof:

By Kirchhoff's 1st Law, the current through each resistor is the same.
Replace the resistors with an effective resistor.

$V = V_1 + V_2 + V_3$

$IR = I(R_1 + R_2 + R_3)$

$R = R_1 + R_2 + R_3$

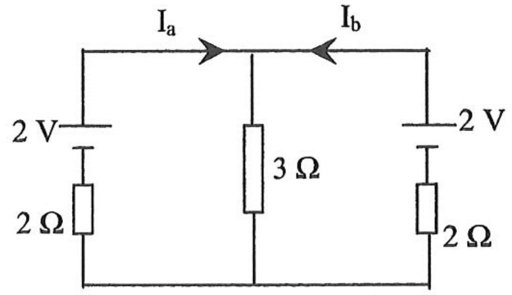
Note: **Total resistance R_{tot} must be greater than any of the individual resistances.**

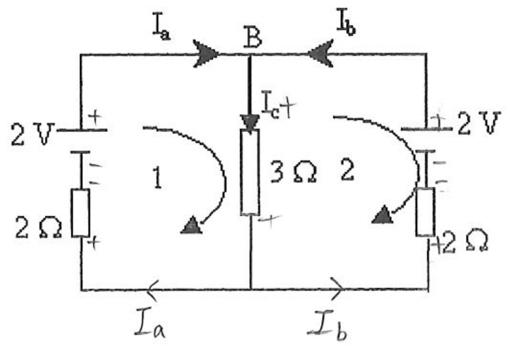
In general, for a series circuit with N number of resistors,

$$R_{tot} = R_1 + R_2 + R_3 + \ldots + R_N$$

Worked Example 5

Use Kirchhoff's Laws to deduce values of the currents as shown in the circuit below.





Using Kirchhof's 1st Law for junction B,

$I_c = I_a + I_b$ —— (1)

Using Kirchhof's 2nd Law for loop 2,

$-2 - 2I_b + 3I_c = 0$ —— (2)

Using Kirchhof's 2nd Law for loop 1,

$+2 - 3I_c - 2I_a = 0$ —— (3)

From (2) & (3), $I_a = I_b$

Hence from (1), $I_c = 2I_b$

Solving (3), $I_b = 0.25A$

$= I_A$

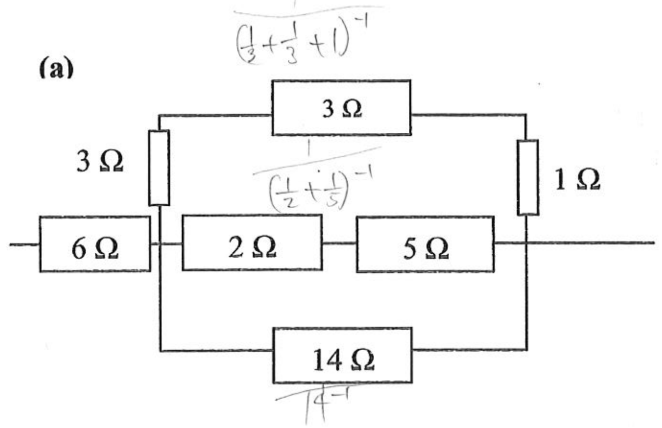
$I_c = 0.50A$

Direct Current Circuits Tutorial

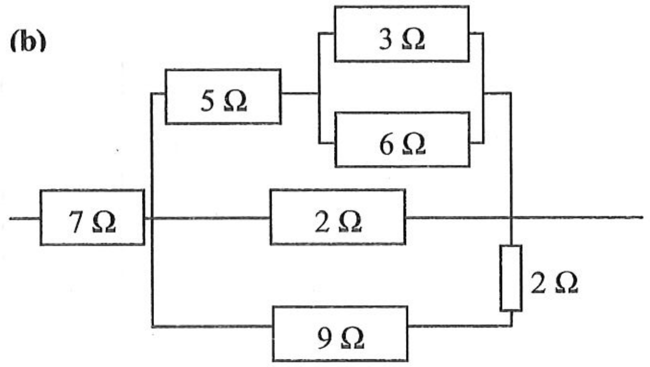
SELF-ATTEMPT::

S1. What is the equivalent resistance in the following?

(a)

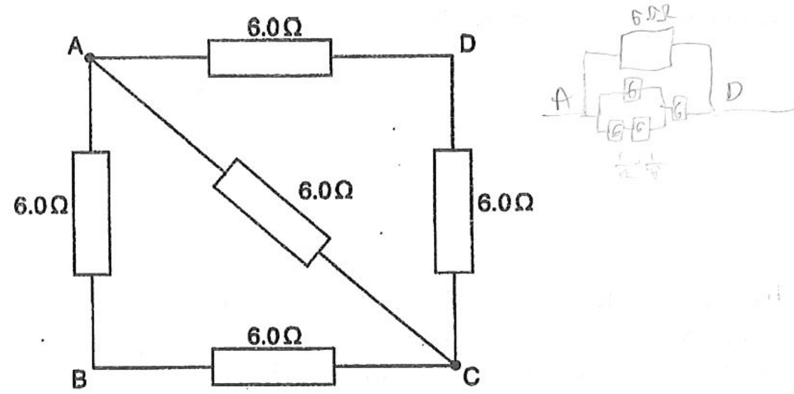


(b)



[8.8 Ω, 8.36 Ω]

S2. A network of resistors, each of resistance 6.0 Ω is constructed as shown below.



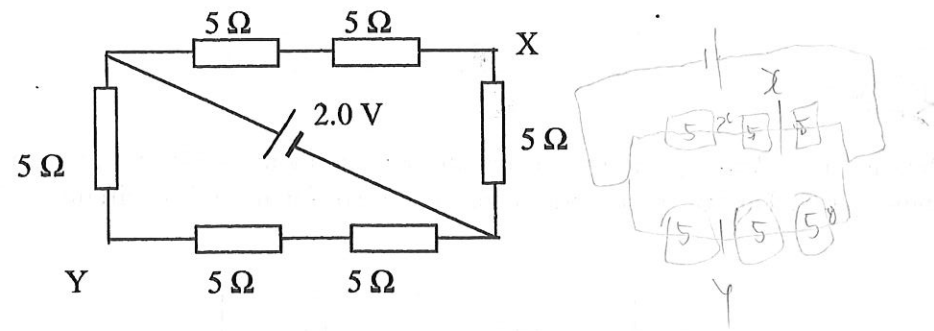
Determine the total resistance of the network between

(a) terminals A and C,

(b) terminals A and D.

[3.0 Ω, 3.75 Ω]

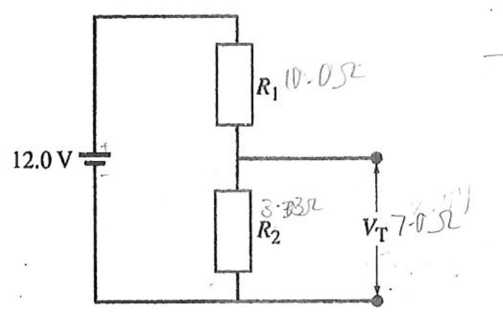
S3. What is the current supplied by the cell? Hence or otherwise determine the p.d. across XY.



[0.267A, - 2/3V]

VD

S4. A power supply has a fixed output voltage of 12.0 V, but you need $V_T = 3.0$ V for an experiment.
(a) Using the voltage divider shown, what should R_2 be if R_1 is 10.0 Ω?
(b) What will the terminal voltage V_T be if you connect a load to the 3.0 V terminal, which has a resistance of 7.0 Ω?

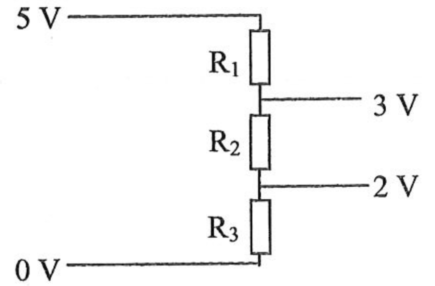


[3.33 Ω, 2.2 V]

VD

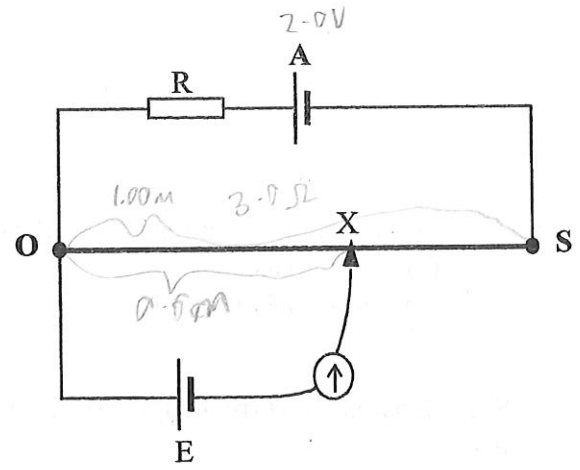
S5. A potential divider is used to give outputs of 2 V and 3 V from a 5 V source.
Give the ratio of resistances R_1, R_2 and R_3 that will give the correct voltages.

[2:1:2]



S6. The driver cell **A** of a potentiometer has an e.m.f of 2.0 V and negligible internal resistance. The potentiometer wire **OS** has a resistance of 3.0 Ω. Calculate the resistance R needed in series with the wire if a p.d. of 5.0 mV is required across the whole wire.

The wire is 100 cm long and a balance length of 60 cm is obtained for an e.m.f of E. What is the value of E?

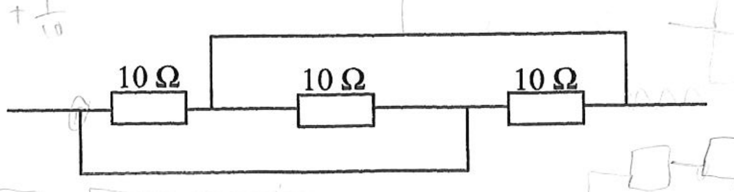


[1197 Ω, 3.0 mV]

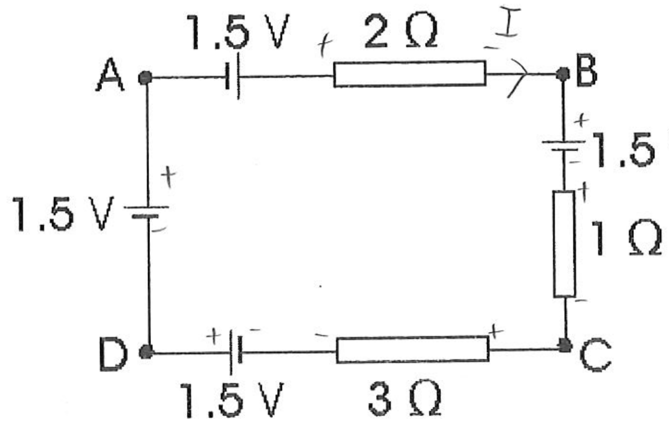
DISCUSSION::

D1. Redraw the following circuit so that the resistors can be clearly seen to be in series, and/or parallel arrangements and determine the resistance of the circuit.

[3.33 Ω]



Worked Example 3
Using Kirchhoff's Laws, determine the current flowing in the circuit.

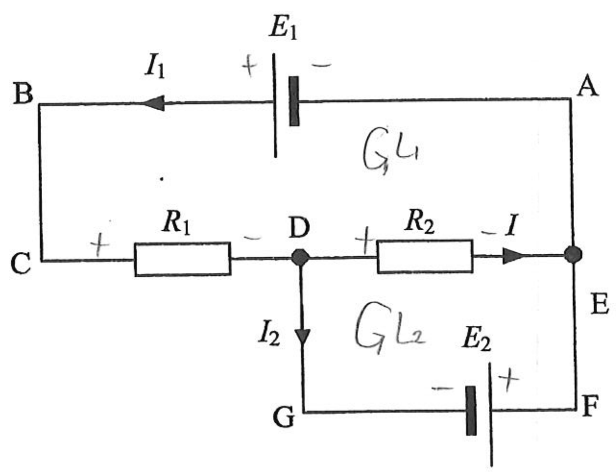


A → B, +1.5 – 2I
B → C, -1.5 – I
C → D, +1.5 – 3I
D → A, +1.5

Net: 3 – 6I = 0
I = 0.50 A

Worked Example 4

For the given circuit, use Kirchhoff's laws to write down expressions in terms of E_1, E_2, I, I_1, I_2, R_1 and R_2 (where appropriate) for the following:
(a) the currents at junction D,
(b) the circuit loop ABCDEA
(c) the circuit loop GFEDG.



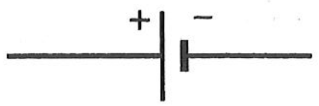
$I_1 = I + I_2$
Loop L_1: $E_1 - IR_2 - I_1R_1 = 0$
Loop L_2: $E_2 + IR_2 = 0$

Here is a good set of guidelines when applying Kirchhoff's Second Law:

1 Assign arbitrary directions of the currents in all branches of the circuit in accordance with Kirchhoff's First Law if they are not already specified.

2 Label on all electrical components (i.e., resistors, e.m.f.s, etc.) with "+" and "–" to indicate the polarity of the potential difference across each of them.

3 For any e.m.f., the 'sense of e.m.f.' is usually given. This does not depend on the direction of the current which is flowing through it. For example,



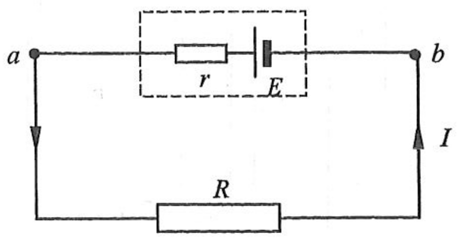
4 For any resistor, the polarity of the potential difference across it depends on the direction of the current which is flowing through it. For example,

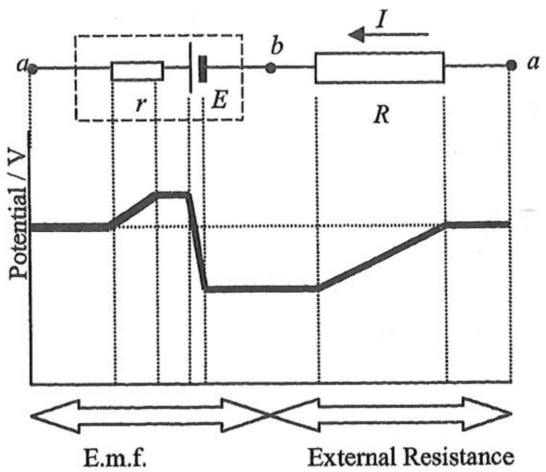
5 Next choose the direction of traversing (either clockwise or anticlockwise) for each loop. Usually traversing a loop yields one equation. Traverse the required number of loops to get as many equations as there are unknowns.

6 For each loop, start at any point and traverse round the loop.
For any electrical component, if "+" is encountered before "–", then it means that there is a decrease in potential. The term for this 'drop' should have a minus sign.
Conversely, if "–" is encountered before "+", then it means that there is an increase in potential. The term for this 'rise' should have a positive sign.

> ***Important to note:***
> Be careful with the signs assigned to the terms. They determine the correctness of your equations.

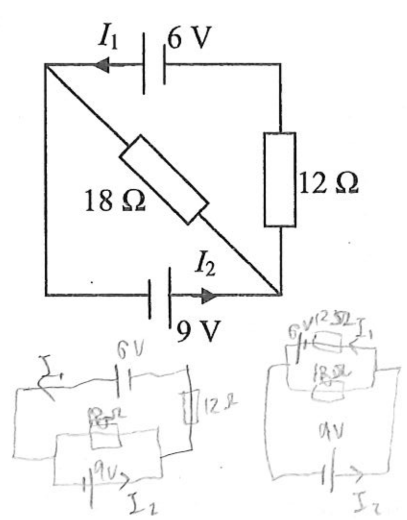
Example:

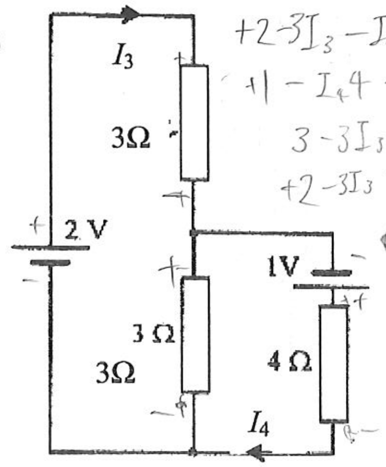




7 After traversing the whole loop, the sum of all increases and decreases in potential must be zero since the potential of the start cum end point should not change.

D2. Using Kirchhoff's laws, find the currents through each cell from I_1 to I_4.

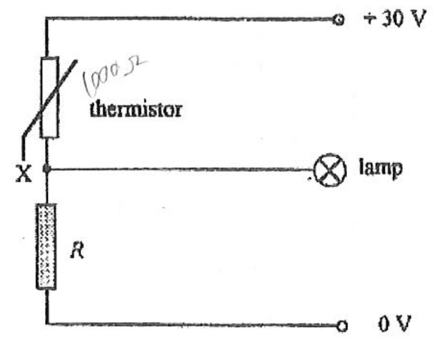




$+2-3I_3-I_2 3=0$
$+1-I_4 4+3I_2=0$
$3-3I_3-4I_4=0$
$+2-3I_3+1-4I_4=0$
$6-6I_3=0$
$I_3=1A$

[1.25 A, 1.75 A, 0.52 A, 0.36 A]

VD D3. A thermistor and an unknown resistor R form a potential divider between voltage lines held at +30 V and 0 V as shown.

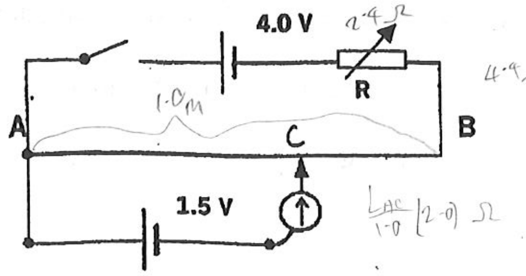


The resistance of the thermistor is 1000 Ω at a temperature of 300 K.

If the indicator lamp lights up when the potential at X is more than 10 V, suggest a suitable value for R so that the lamp will automatically light up when the temperature exceeds 300K.

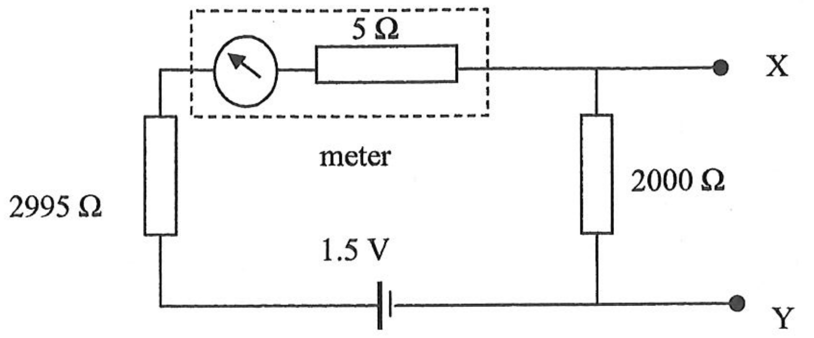
[500 Ω]

D4. A simple potentiometer circuit is set up as follow, using a uniform wire AB, 1.0 m long, which has a resistance of 2.0 Ω. The resistance of the 4 V battery is negligible. If the variable resistor R were given a value of 2.4 Ω, what would be the length AC for zero galvanometer deflection?
If R were made 1.0 Ω and the 1.5 V cell and galvanometer were replaced by a voltmeter of resistance 20 Ω, what would be the reading of the voltmeter if the contact C were placed at the midpoint of AB?



[82.5 cm, 1.29V]

V D5. The meter in the circuit shown below has an uncalibrated linear scale. With the circuit as shown, the scale reading is 20. Find the scale reading when another 2000 Ω resistor is connected across XY.



[25]

D6. The current that passes through a certain diode varies with the potential difference across it as shown in fig.a. PQ is a straight line. Two such diodes are connected in parallel with a milliammeter of resistance 100 Ω as shown in fig. b. What is the value of the direct current I when the I_g through the milliammeter is 8 mA?

$V = 100(0.008)$
$= 0.8V$
Grad PQ $= \frac{10}{0.5}$
$= 20$
$y - 0 = 20(x - 0.5)$
$y = 20x - 10$
$y = 20(0.8) - 10$
$6mA$
$6+8 = 14$

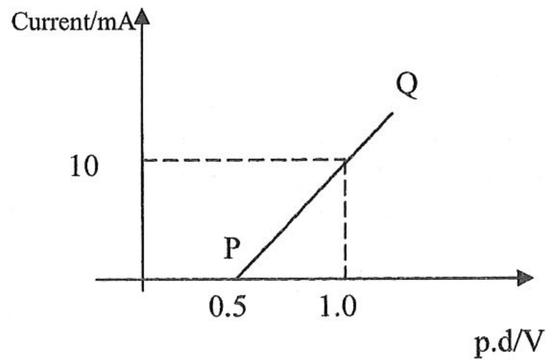


Fig. a.

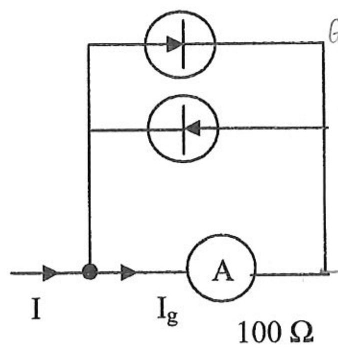


Fig. b.

[14 m A]

Assignment

$P = IV$
$V = 12$
$I = \frac{P}{V}$

1(a) A car headlamp is marked 12 V, 72 W. It is switched on for a 20 minute journey. Calculate
(i) the current in the lamp,

(ii) the charge which passes through the lamp during the journey,

(iii) the energy supplied to the lamp during the journey,

(iv) the working resistance of the lamp.

Kirchhoff's Second Law (The Voltage Law)

Round any closed circuit, the algebraic sum of the emfs is equal to the algebraic sum of the potential difference of all the individual components.

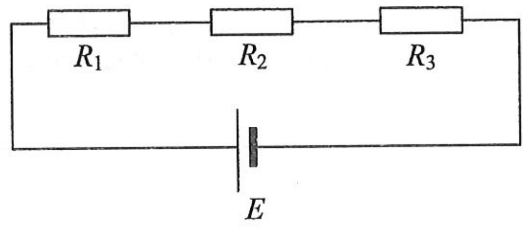
Or mathematically, $\Sigma E = \Sigma V$
$\Sigma E = \Sigma IR$
if the components are purely resistive.

- This is a consequence of the ***conservation of energy***. The electrical energy produced by the source is equal to the sum of the electrical energy consumed by all the components.

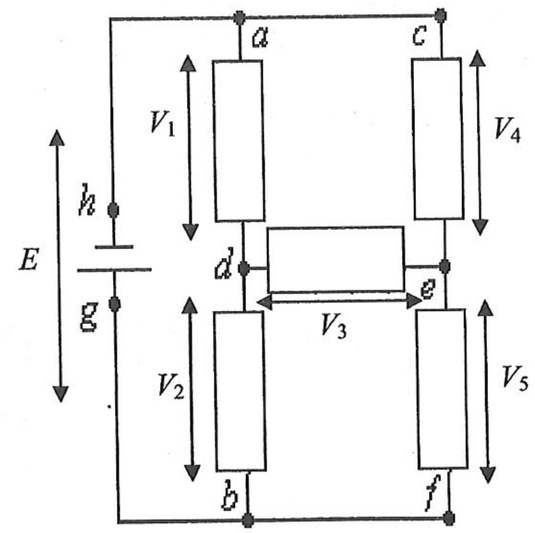
Proof:

Rate of electrical energy supplied by battery = Rate of energy dissipated in resistors

$IE = I^2(R_1 + R_2 + R_3)$
$E = IR_1 + IR_2 + IR_3$
$= V_1 + V_2 + V_3$



Worked Example 2
hadfbgh
A **loop** is any closed conducting path. ~~aceda,~~ *defbd*, and *hadefbgh* are examples of loops.



Loop ~~aceda~~, hadfbgh
$E_1 = V_1 + V_2$

Loop defbd,
$\Sigma E = \Sigma V$
$0 = V_3 + V_5 + V_2$

Loop hadefbgh
$E_1 = V_1 + V_3 + V_5$

$E_1 - V_1 + V_3 + V_5 = 0$

Kirchhoff's First Law (The Current Law)

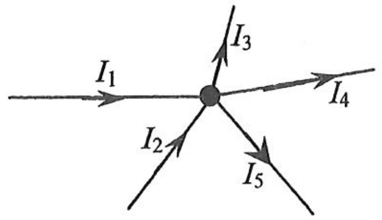
The algebraic sum of the currents at a junction is zero
i.e the sum of the currents going into the junction is equal to that coming out from it

Or mathematically, $\Sigma I = 0$

$I_1 + I_2 - I_3 - I_4 - I_5 = 0$

The diagram on the right illustrates this:

$I_1 + I_2 = I_3 + I_4 + I_5$

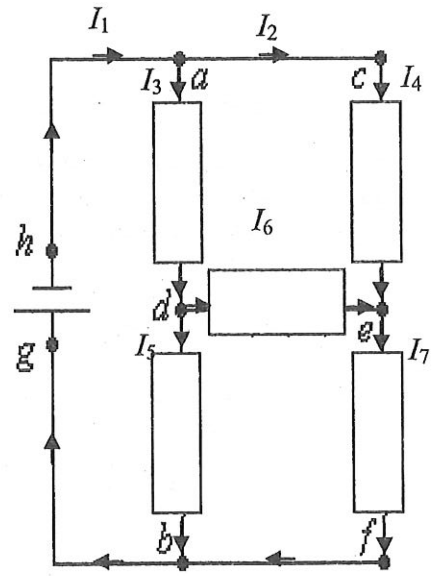


- This is a consequence of the ***conservation of charge***, which states that charges cannot be created nor destroyed. The total charge that leaves a junction per unit time must therefore be equal to the charge that arrives there per unit time. Charge cannot escape from a wire but has to flow around the circuit.

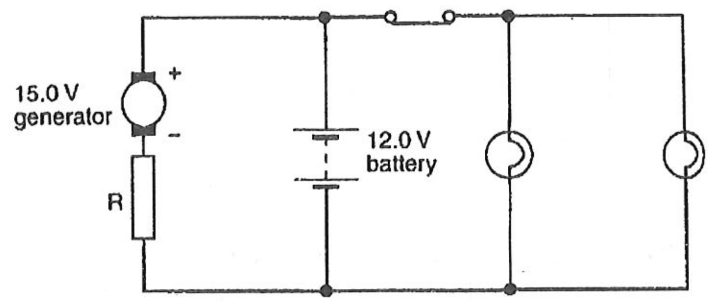
Worked Example 1

A **junction** is a point where three or more conductors are joined, e.g. *a, b, d, e.*

At junction a, $I_1 = I_2 + I_3$
At junction c, $I_2 = I_4$
At junction e, $I_4 + I_6 = I_7$
At junction b, $I_7 + I_5 = I_1$



(b) Two of the headlamps referred to in part (a) are connected into the circuit shown in the figure, in which one source of e.m.f. (the generator of the car) is placed in parallel with the car battery and the two lamps. Both lamps are on and are working normally.



The battery has an e.m.f. of 12.0 V and negligible internal resistance: the generator has an e.m.f. of 15.0 V and negligible internal resistance. The generator is in series with a variable resistor R.

(i) The value of R is adjusted so that there is no current in the battery when the lamps are on.
Calculate
1. the current in the generator,
2. the value of the resistance of R.

(ii) Calculate the current in the battery when both lamps are switched off, the value of R remaining the same as in (i).

(c) Suggest two advantages which the circuit, as shown in the figure, has over a single power source.

2 The variation with current of the potential difference (p.d) across a component X is shown in the figure below.

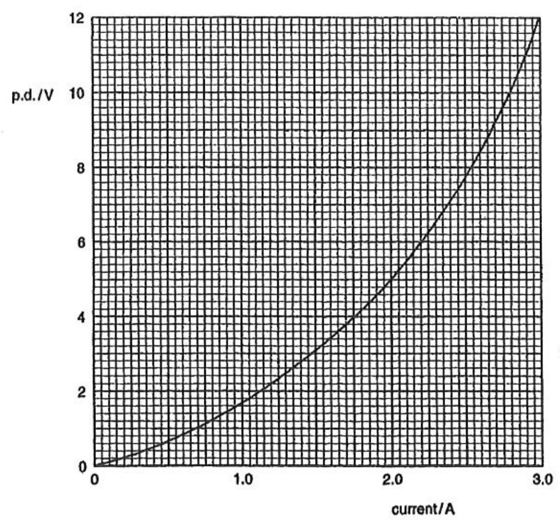
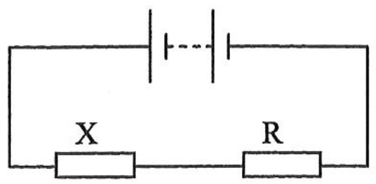


Figure a

(a)(i) State how the resistance of component X varies, if at all, with increase of current.

(ii) Copy the figure and sketch a line to show the variation with the current of the p.d. across a resistor R of constant resistance 3.0 Ω.

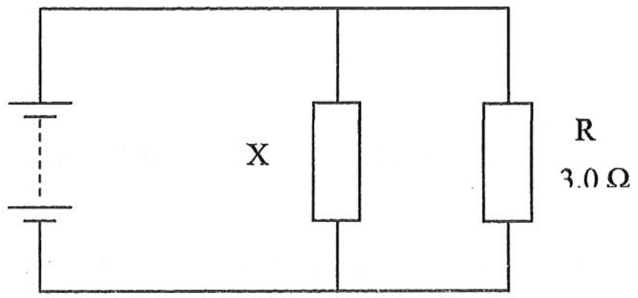
(b) The component X and the resistor R of resistance 3.0 Ω are connected in series with a battery of negligible internal resistance as shown below.



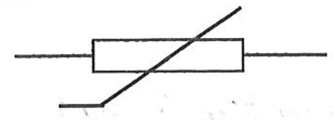
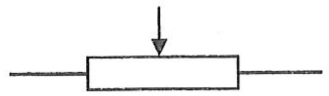
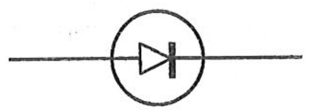
3.0 Ω

The current in the circuit is found to be 2.0 A.
(i) Use Fig.xx to determine the p.d. across component X.
(ii) Determine the p.d across R and the e.m.f of the battery.

(c) The resistor R and the component X are now connected in parallel with the battery as shown below.



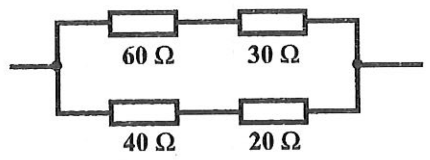
Using your answer to (b)(ii) and the graph of Fig.a, determine the current from the battery.
[N2003P2Q2]

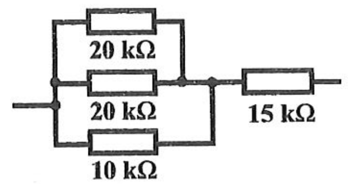
	Thermistor
	Switch
	Capacitor
	potential divider vary R
	diode

2 Conservation of Charge and Energy

recall Kirchhoff's first law and appreciate this as a consequence of conservation of charge.
recall Kirchhoff's second law and appreciate this as a consequence of conservation of energy.
apply Kirchhoff's laws to simple circuits.

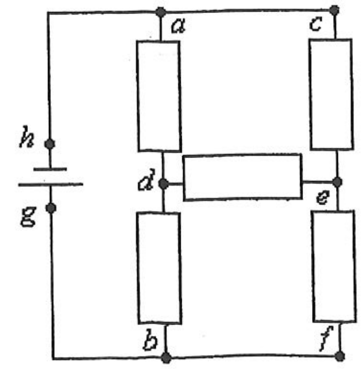
Kirchhoff's Laws





The illustrations above show two simple electric circuits with resistors arranged in parallel and series-parallel combinations.

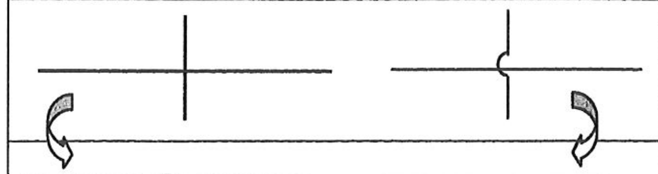
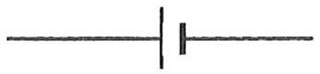
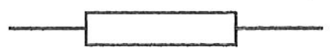
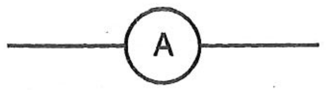
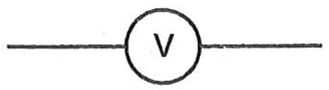
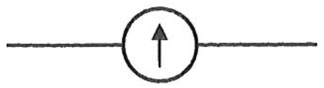
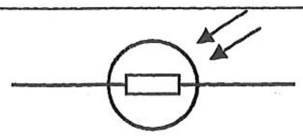
However, since not all circuits can be reduced to simple series-parallel combinations, we need new rules to work with when we have resistance networks of cross connections such as the illustration at right.

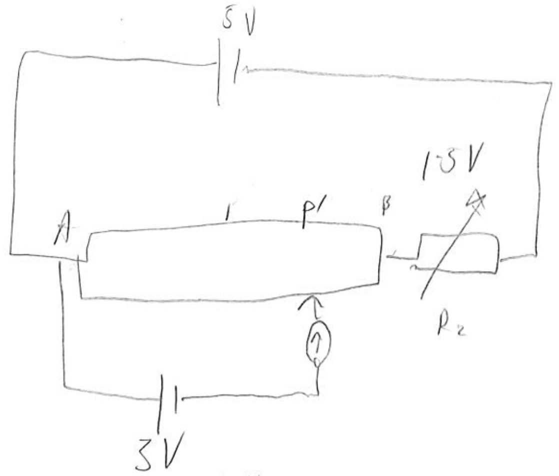


Now, concerning such networks, there are only two relatively simple rules known as Kirchhoff's rules (developed by Gustav Robert Kirchhoff (1824-1887)).

1 Practical Circuits

recall and use appropriate circuit symbols as set out in the current ASE Report — SI Units, Signs, Symbols and Abbreviations.
draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, any other type of component referred to the syllabus.

	conductors crossing with no connection
	junction of conductors
	earth connection (zero potential)
	primary or secondary cell
	battery of cells
	fixed resistor
	variable resistor
	filament lamp
	Ammeter
	Voltmeter
	Galvanometer (measures small current)
	light dependent resistor



$V_{AB} = 3.5\,V$

$k = 3.5\,Vm^{-1}$

$V_{AP'} = kL_{AP'}$

$3 = 3.5\,L_{AP'}$

$L_{AP'} = \frac{3}{3.5} = 0.857\,m$

D.C. CIRCUITS

CONTENT

1 Practical circuits.
2 Conservation of charge and energy.
3 Balanced potentials.

ASSESSMENT OBJECTIVES

Candidates should be able to:

(a) recall and use appropriate circuit symbols as set out in *SI Units, Signs, Symbols and Abbreviations* (ASE, 1981) and *Signs, Symbols and Systematics* (ASE, 1995).

(b) draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, any other type of component referred to the syllabus.

(c) recall Kirchhoff's first law and appreciate the link to conservation of charge.

(d) recall Kirchhoff's second law and appreciate the link to conservation of energy.

(e) derive , using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in series.

(f) solve problems using the formula for the combined resistance of two or more resistors in series.

(g) derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in parallel.

(h) solve problems using the formula for the combined resistance of two or more resistors in parallel.

(i) apply Kirchhoff's laws to solve simple circuit problems.

(j) show an understanding of the use of a potential divider circuit as a source of variable p.d..

(k) explain the use of thermistors and light dependent resistors in potential dividers to provide a potential difference which is dependent on temperature and illumination respectively

(l) recall and solve problems using the principle of the potentiometer as a means of comparing potential differences.