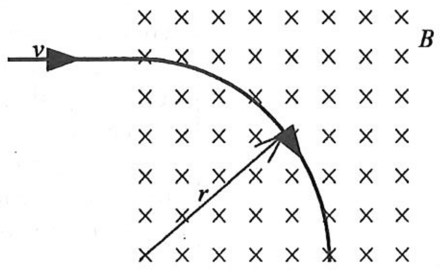
Note:

♦ Because the magnetic force is of constant magnitude and always at right angles to the velocity, the charged particle moves in a **circular motion** with constant tangential speed v.

♦ The magnetic force, which the charged particle experiences in a magnetic field, is a **deflecting force.** In contrast to the electric and gravitational forces, this magnetic force is not in the direction of the field. The direction of the magnetic force is not constant as shown below.



♦ The magnetic force causes no change in speed, it merely causes a change in direction of motion. This is because the direction of the magnetic force is always **perpendicular** to the velocity of the charged particle. As a result, **no work is done** by the magnetic force in increasing the kinetic energy of the charged particle i.e. speed of particle is **unchanged.** Instead, the force serves to **deflect** the charged particle to follow a circular path i.e. it is a **centripetal force**.

♦ For a body in uniform circular motion, the centripetal acceleration a of the body is

$$a = \frac{v^2}{r}$$

From Newton's 2nd Law, the equation of motion of the charged particle is given by

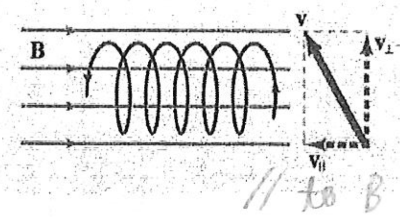
$$Bqv = \frac{mv^2}{r}$$

Hence, charged particle when projected perpendicularly into an uniform magnetic field follows a circular path of radius

momentum of charge

$$\boxed{r = \frac{mv}{Bq}}$$

iii) If a charged particle moves in a uniform magnetic field with its velocity v at some angle θ ($0^\circ < \theta < 90^\circ$) to the magnetic field, ***the path is a helix***.



Why helical?

The velocity vector v can be broken down into components parallel v_{11} and perpendicular $v_\perp$ to the field. The velocity component parallel to the field lines experiences no force, and thus this component remains constant. The velocity component perpendicular to the field results in circular motion about the field lines. Putting these two motions together produces a helical (spiral) motion around the field lines.

Example 11

When protons enter a uniform magnetic field of 1.0 T, they are rotated in a circular orbit of radius *r*. Show that the number of revolutions per second, *f*, is independent of *r* and calculate *f*. What is the radius of the protons' path if they travel with speed $v = 1.05 \times 10^6$ ms^{-1}? Assume that the specific charge for the proton is 1.0×10^8 C kg^{-1}.

Magnetic force Bqv provides the centripetal force that causes the protons to move in a circle.

$$Bqv = ma_c$$
$$= mv\omega$$
$$Bq = m(2\pi f)$$
$$f = \frac{Bq}{2\pi m}$$

f is independent of r. $f = 1.6 \times 10^{-7}$ Hz

$$Bqv = \frac{mv^2}{r}$$
$$r = (1.05 \times 10^6) \div (1.0)(1.0 \times 10^8)$$
$$\approx 0.011 \text{ m}$$

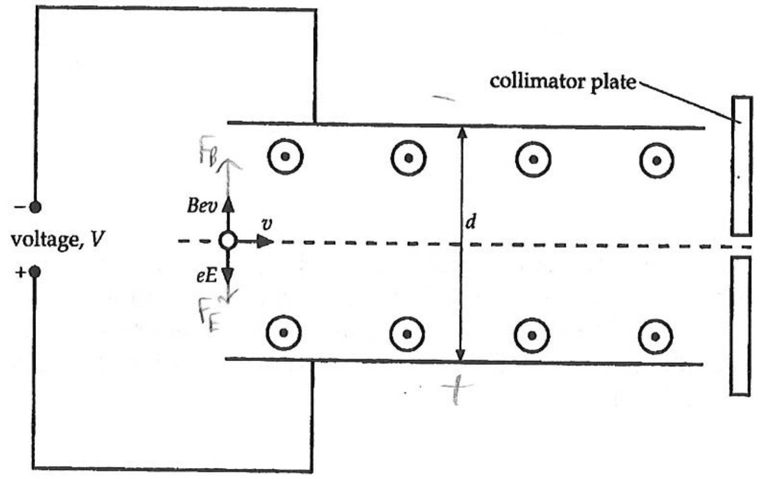
Refer to Appendix : For Mass Spectrometer

Concept test 4

4. Charged particles in E and B Fields (Cross Field)

4.1 Effect of mutually perpendicular E and B fields (Cross field) on electrons

- Consider an electron of velocity v projected into a crossed E- and B- fields as shown in the diagram.



- Upon entering the crossed fields, the electrons experienced two forces, F_E and F_B due to the E- and B-fields respectively.
- Now, $F_E = eE$ (directed downwards)
- Also, $F_B = BeV$ (directed upwards)
- Thus, by adjusting B and E such that $F_E = F_B$, the electron remains undeflected when passing through the crossed fields.
- This means that, $eE = BeV$
- That is, $\boxed{V = \frac{E}{B}}$
- Thus v can be determined from the values of B and E.

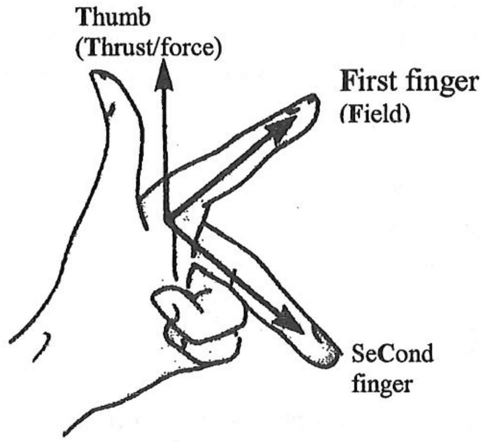
- **Velocity Selection**

Velocity	Rel. Mag of F_E & F_B	Deflection in Cross Field
$v = E/B$	$F_E = F_B$	undeflected
$v > E/B$	$F_B > F_E$	deflected upwards
$v < E/B$	$F_B < F_E$	deflected downwards

- For a beam of charged particles having a range of velocities, only those moving with velocity equal to E/B remain undeflected. Hence, only particles of this particular velocity will emerge at the slit while the rest are blocked. This is the working principle of the ***Velocity Selector.***

b) Direction of the force

The direction of the force on a moving charge in the B field is given by **Fleming's left-hand rule.**



Note:
1. FLHR works for direction in which a **positive** charged particle is moving. i.e. it is defined for conventional current.
2. The direction of the magnetic force **is always perpendicular** to the plane in which magnetic field and charge's velocity lie.

c) Effect of Uniform B-field on Motion of Charged Particles

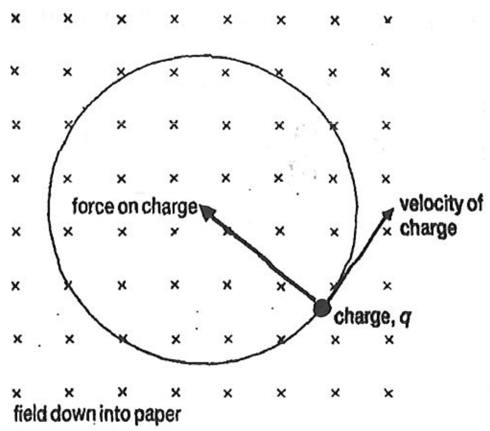
i) If the velocity of the charged particles, v is in the direction of magnetic field B (i.e. Charge, q projected parallel or anti-parallel to field).

$\theta = 0$, $F = B\,q\,v\,\sin\theta = 0$
ie no magnetic force experienced by charged particle.
Therefore, no effect on motion.

Path is a straight line.

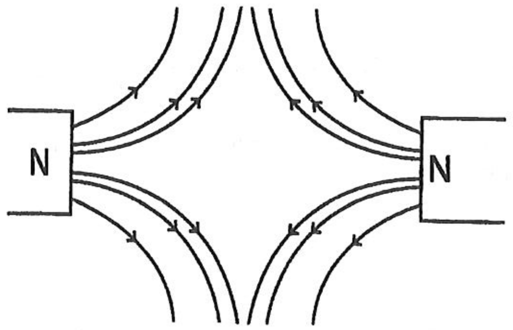
ii) If the velocity of the charged particle v is perpendicular to B (i.e. Charge, q projected perpendicular to field),

$\theta = \pi/2$, $F = B\,q\,v\,\sin\theta = B\,q\,v$

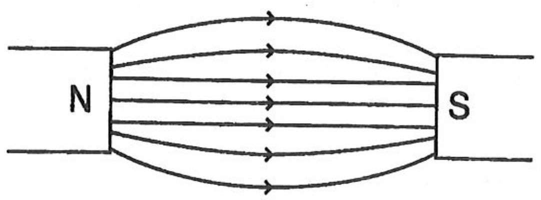


Applying FLHR, F is always directed towards the center of a circle of radius r. F is perpendicular to both B and v and also v is perpendicular to B.

Path is a circle.



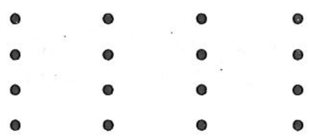
Resultant lines of force between two North poles



Resultant lines of force between a North pole and South pole

- Convention for representing Field Directions
  x represents a field directed into the plane of paper.
  • represents a field directed out of the plane of paper.

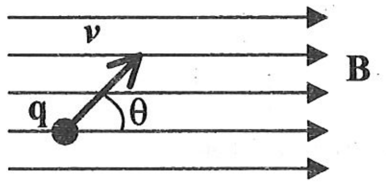
magnetic field into plane of paper



magnetic field out of plane of paper

3.2 Force on a moving charge in a magnetic field

a) Magnitude of the magnetic force on moving charged particles



Equation

$$\boxed{F_B = Bqv\sin\theta}$$

where θ = angle between B and v.
B = magnetic flux density (tesla, T)
v = velocity of the charge (ms^{-1})

If the charge moves at right angles to the field B, $\theta = 90^o$

$$\boxed{F = Bqv}$$

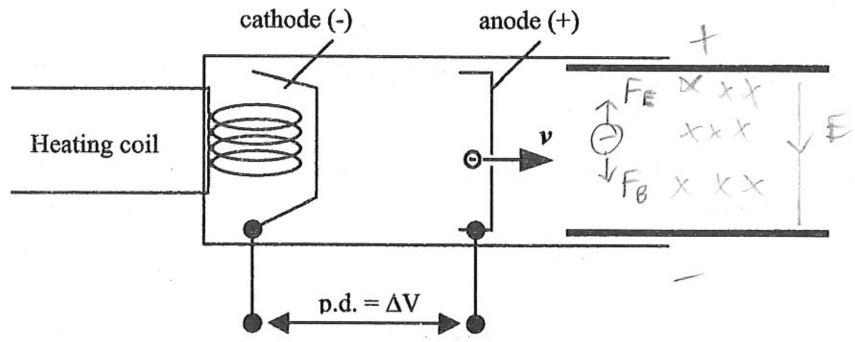
Example 14 (*Motion of charged particles in cross-field*)

A beam of electrons in a cathode-ray tube is accelerated by a p.d. of 1000 V between the cathode and the anode **as in Example 7**. It passes through a small hole in the anode into a pair of electrostatic deflecting plates which produce a field of 5.0×10^4 Vm^{-1} perpendicular to the initial direction of the beam.

(a) How should the magnetic field be oriented to allow the beam to pass undeviated? Draw a sketch to illustrate your answer.
(b) What is the magnetic flux density?

① F_E
② F_B
③ FLHR
↳ B

(a)



(b) When the electron beam is undeviated in the crossed field,

$$v = \frac{E}{B}$$

$$1.88 \times 10^7 = \frac{5.0 \times 10^4}{B}$$

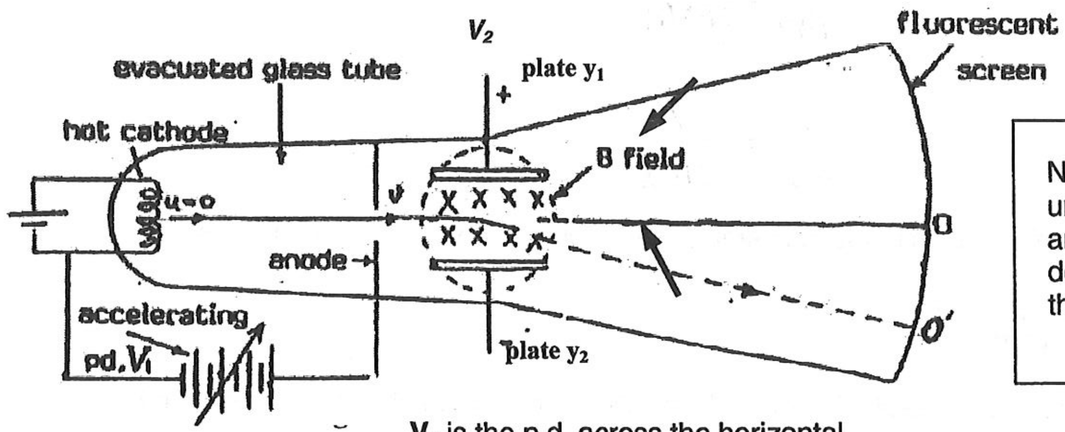
from eg. 7

$$B = 2.66 \times 10^{-3}\ T$$

$$q\Delta V = \frac{1}{2} m_e v^2$$

4.2 JJ Thomson's Cathode Ray Tube (Hutchings, p. 471)
- Principles of Determining Velocity, v and Specific Charge, e/m_e

- This section describes the principles behind measurement of v and e/m_e of electrons based on JJ Thomson landmark experiment that discovered the fundamental particle of electricity, the electron. The setup is as follows.

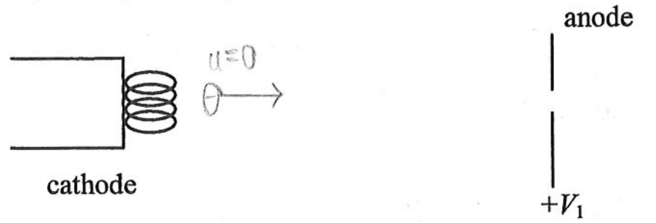


Need to understand and know the details for this method.

V_2 is the p.d. across the horizontal deflection plates y_1y_2 of separation **d**

Determining of v by Work-energy Theorem

- A common source of electrons is the cathode ray tube. The main components of this tube are a heated cathode and an accelerating anode. Electrons are obtained from the cathode through the process known as thermionic emission. They are then accelerated by the positive anode, kept at a positive p.d. V_1 w.r.t. to the cathode, to a certain speed v.



- Assuming that the electrons are at rest when emitted from the cathode, then it is accelerated through the Cathode-anode p.d., V_1.

The velocity it thus acquires v can be obtained from increase in kinetic energy of the electrons = work done by the accelerating p.d.

$$\tfrac{1}{2}m_e v^2 - 0 = eV_1 \qquad (1)$$

S paper!
Know how to

- Hence, $$v = \sqrt{\frac{2eV_1}{m_e}}$$

  which can be determined. prove

Determining of e/m$_e$ by Zero Deflection in Cross Field

- The accelerated electron next enters the region of crossed fields at speed, v.
  Adjustments for zero net deflection yields
  $$eE = Bev \qquad (2)$$
- Combining (1) and (2), $$\frac{e}{m_e} = \frac{E^2}{2B^2V_1}$$
- But $E = V_2/d$, where V_2 is the p.d. across the horizontal deflection plates y_1y_2 of separation d.

  therefore, $$\frac{e}{m_e} = \frac{V_2^2}{2B^2V_1d}$$

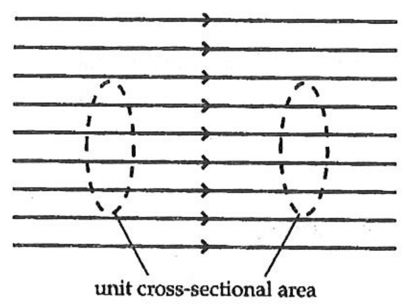
- Making $V_1 = V_2 = V$, $$\frac{e}{m_e} = \frac{V}{2B^2d^2}$$
- P.d. across the plates V_2 is made the same as the accelerating p.d. V_1 by connecting together
  a) cathode and plate y_2
  b) anode and plate y_1
- Since V, B and d can be easily determined, specific charge of the electron can be found.
  $$e/m_e = 1.76\times10^{11}\ \text{C kg}^{-1}$$

3. Magnetic field (B field)

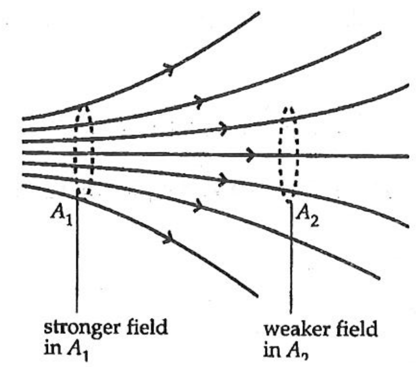
- A magnetic field is a region of space in which **a moving charge** located in it will experience a **force**.
- The magnetic field may exist at a point as a result of the presence of either a permanent magnet or a conductor carrying an electric current, in the vicinity of the point.

3.1 Magnetic Lines of Force

- **Magnetic field lines** are used to represent a magnetic field just as electric field lines are used to represent an electric field.
- By convention, magnetic filed lines leave the north pole and enters the south pole of a magnet.
- If the lines of force are parallel, the associated field is uniform. This means that the number of lines passing perpendicularly through unit area at all cross-section in a magnetic field are the same.
- A non-uniform field is represented by non-parallel lines. The number of the magnetic field lines varies at different unit cross sections.

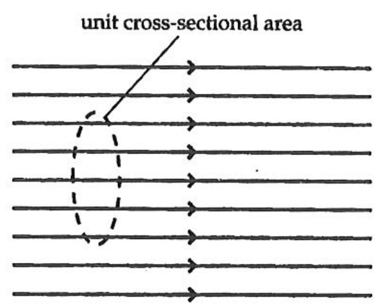


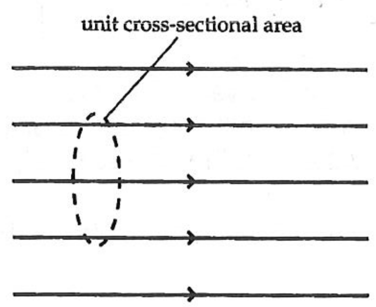
Uniform magnetic field



Non-uniform magnetic field

- The field is said to be **strong** if the lines of force are crowded very closely together and **weak** when they are widely separated from one another.





- Magnetic field lines **do not intersect** with one another.
- When magnetic filed lines are superposed, a result line of force is obtained, The field lines are not rigid but can be deformed and give way to each other.

Example 9 (Charge quantisation)

The results of 14 experiments to determine the charge on oil drops were as follows.

negative charge on oil drop /10^{-19} C

1.605	1.608	3.199	1.598	1.606	1.602	4.806
3.214	4.803	1.610	1.599	3.217	8.001	1.607

Use these results to deduce a value for the charge on the electron.

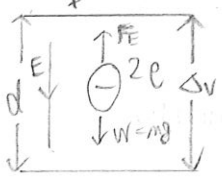
	$Q = Ne / 10^{-19}$C	Q / smallest charge	N (nearest integer)
	1.605	1.00	1
	1.608	1.01	1
	3.199	2.00	2
	1.598	1.00	1
	1.606	1.01	1
	1.602	1.00	1
	4.806	3.01	3
	3.214	2.01	2
	4.803	3.01	3
	1.610	1.01	1
	1.599	1.00	1
	3.217	2.01	2
	8.001	5.01	5
	1.607	1.01	1
Total	40.075		25

Hence, $e = \frac{\text{Total } Q}{\text{Total } N}$

$= \frac{40.075\times10^{-19}}{25} = 1.603\times10^{-19}\ C$

Example 10 (Stationary oil drop in E-field) Always draw diagrams

Taking this electronic charge to be -1.60 x 10^{-19} C, calculate the potential difference in volts necessary to be maintained between two horizontal conducting plates, one 5 mm above the other, so that a small oil drop, of mass 1.31 x 10^{-14} kg with two electrons attached to it, remains in equilibrium between them. Which plate would be at the positive potential?



Oil drop remains stationary

$\Rightarrow \quad W = F_E$

$mg = q\left(\frac{\Delta V}{d}\right)$

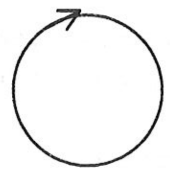
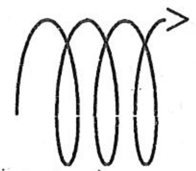
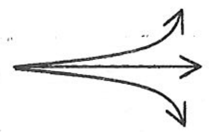
$\Delta V = \frac{mgd}{q}$

$= \frac{(1.31\times10^{-14})(9.81)(5\times10^{-3})}{(2\times1.6\times10^{-19})}$

$= 20000V$

Upper plate would be at the positive potential

4.3 Brief Review of Possible Types of Motion in Different Fields

	Projection of charged Particles with respect to direction of field			
Fields	Stationary	Parallel	Perpendicular	At an angle
E	**Accelerate linearly with direction depending on type of charge.**	**Same as for stationary case.**	**Projectile motion.** 	**Projectile motion (symmetrical).** 
B	**No effect.**	**No effect on motion of particle.**	**Circular motion.** 	**Helical or spiral motion.** 
Cross	**Cannot be easily deduced.**	**Cannot be easily deduced.**	**Either deflected upward, downward or undeflected, depending on its speed.** 	**Cannot be easily deduced.**

5. Appendix

SOME APPLICATIONS OF ELECTRIC FIELD

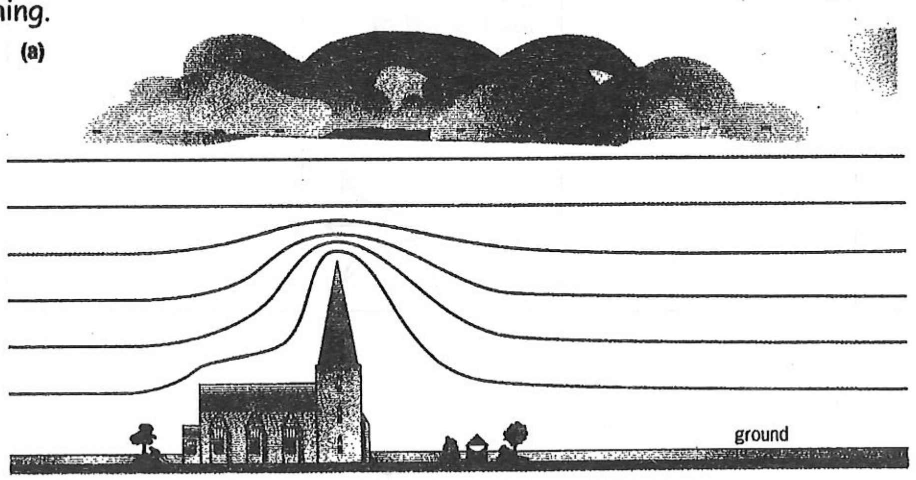
NON-UNIFORM FIELDS

In non-uniform fields the potential gradient varies. Fig (a) shows the electric field below a thundercloud. Notice how the potential gradient increases around the pointed church spire. The pointed spire distorts the field pattern, making the field stronger around the point.

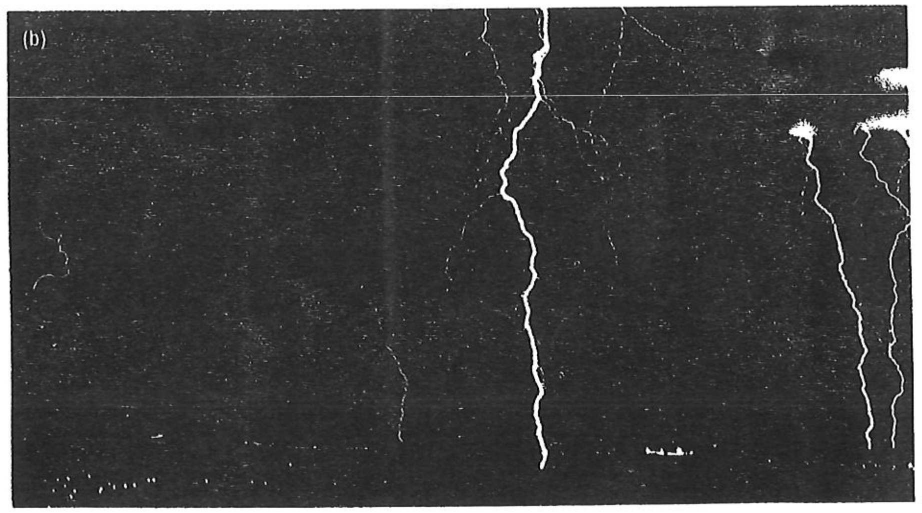
This property is used in a lightning conductor, which is basically a pointed metal rod. One end is earthed and the pointed end extends above the top of the building to which it is attached. A thundercloud often has a concentration of negative charge at its base. This generates an electric field between the base and the ground. Below the thundercloud, the field at the point of the lightning conductor is strong enough to ionize air molecules around the point.

The resulting charged molecules - ions - move in the field: positive ions go towards the cloud base and electrons go to earth through the lightning conductor. This mechanism allows the thundercloud to discharge slowly before enough charge builds up to cause lightning.

(a)

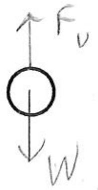


(b)



- **Step 1**: E = 0

For an oil droplet falling through air at terminal velocity



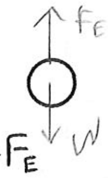
$$W = F_v$$
$$\frac{4}{3}\pi a^3 \rho . g = 6\pi\eta a V_t$$
$$a = \left(\frac{9\eta V_t}{2 \cdot 2\, g}\right)^{\frac{1}{2}}$$

By measuring the terminal velocity v_t, a *(radius of oil drop)* can be worked out.

- **Step 2:** E applied

ΔV across the plates are adjusted until the drop is stationary.

For the stationary oil droplet between the two Millikan metal plates,



> Understand the basic principles and apply the result.

Therefore,

$$W = F_E$$
$$\frac{4}{3}\pi a^3 \rho g = q\left(\frac{\Delta V}{d}\right)$$
$$q = \frac{4}{3}\pi a^3 \rho g \left(\frac{d}{\Delta V}\right)$$

Thus, q can be calculated from the equation, since a was worked out in Step 1.

Step 1 and 2 are repeated using different oil drop.

Summary

Field	Oil Drop	Equation	Steps to take
E = 0	Falling at terminal velocity	$W = F_V$	Measure v_t to find a .
E applied	Stationary	$W = F_E$	Sub in a to find q.

Conclusion

- Working with thousands of oil drops, q was found to be an integral (whole number) multiple of a basic unit e (i.e. 1.6 x 10^{-19} C)

  $q = ne$ where $n = 0, \pm1, \pm2, \pm3$ ...

  Millikan concluded that all charges are built up of units of e.

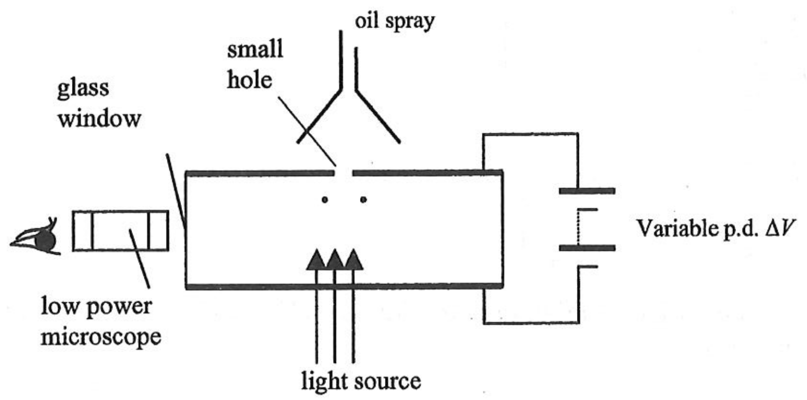
- The Millikan experiment is said to provide an experimental evidence for the QUANTISATION OF CHARGE.

- **Quantisation of Charge**: It refers to the discrete or quantum nature of charge. In another words, any charge can be expressed as an integral multiple of a basic unit of charge.

2.6 The charge on the electron

***Milikan's Oil Drop Experiment* (Hutchings, p. 473 - 475)**

During the period 1909 to 1913, Robert A. Millikan (1868 – 1953) performed a brilliant set of experiments in which he measured the elementary charge on an electron, e, and demonstrated the quantised nature of the electronic charge.



- The experimental set up consists of two parallel charged plates with potential difference ΔV applied across them.
- Oil droplets from an atomiser are allowed to pass through a small hole in the upper plate.
- The oil droplets are charged by friction when emerging from the nozzle of the atomiser.
- A light beam is used to illuminate the falling oil droplets and they are observed as tiny bright specks of light against a dark background.
- A low power microscope with a scale can then determine the velocity of an oil droplet.
- The potential difference ΔV applied between the plates can be adjusted until a drop is stationary.

Key Quantities

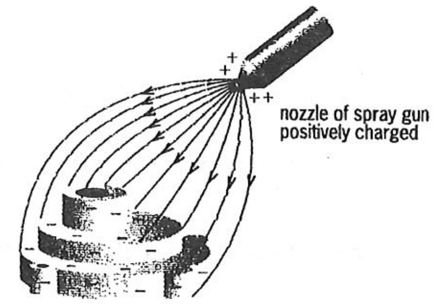
Fixed Variables	**Control Variables**	**Response Variables**
Density of oil drop, ρ Viscosity of medium, η Acceleration due to gravity, g Distance betw. 2 plates, d	Oil drop – charge polarity Charge magnitude, q, Radius of oil drop, a	Terminal velocity, v_t p.d across the 2 plates, ΔV

Relevant Forces

- **Weight** of oil drop, W = mass of oil drop x g
  = volume of oil drop x ρ x g
  = $\left(\frac{4}{3}\pi a^3\right)$ x ρ x g
- **Viscous** force, F_V = $6\pi\eta\, a\, v_t$ (motion -air resistance)
- **Electric** force, F_E = $qE = q\Delta V/d$
- Please note that **upthrust** on oil droplets is **ignored** because the density of oil is very much greater than that of air.

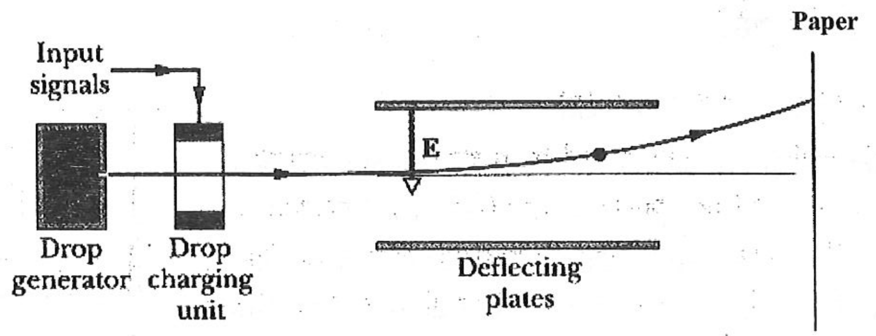
ELECTROSTATIC PAINT SPRAYING

Particles of paint are given a positive charge as they leave the nozzle of a spray gun as shown below. The object to be painted is earthed so that there is an electric field between the nozzle and the object. The charged paint droplets follow the field lines and are deposited evenly over the surface of the object.



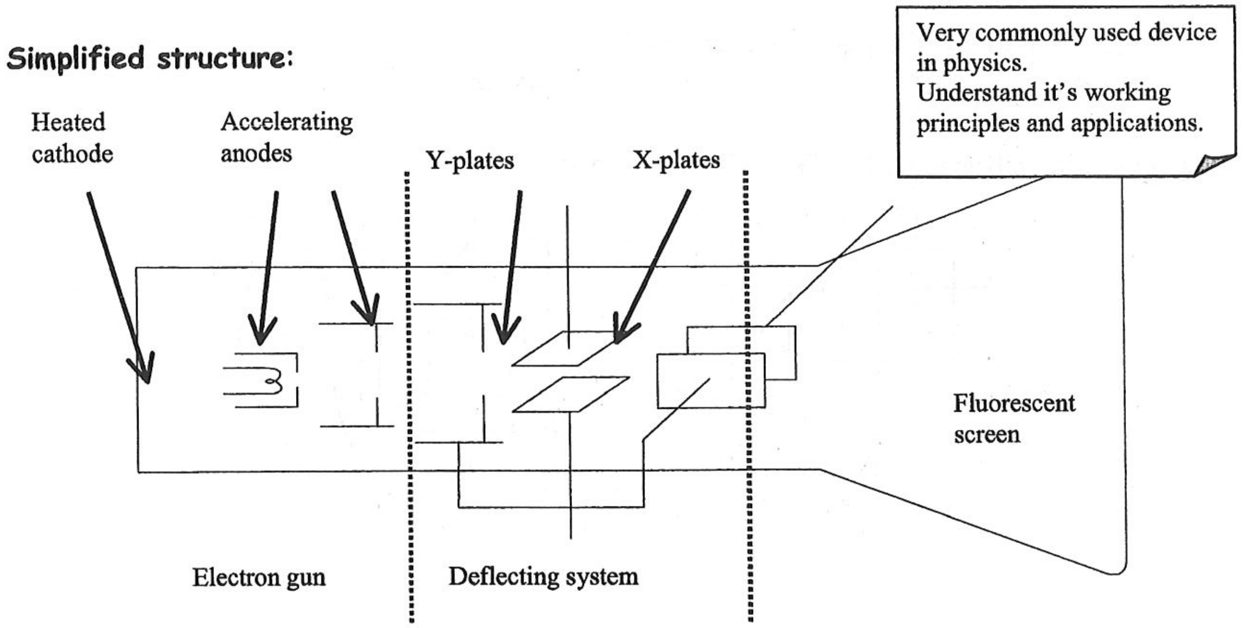
INKJET PRINTER (RESNICK, P.567)

The essential features of an ink-jet printer: An input signal from a computer controls the charge given to each drop and thus the position on the paper at which the drop lands. About 100 drops are needed to form a single character.



CATHODE-RAY OSCILLOSCOPE (C.R.O.) (HUTCHINGS, P. 477 - 480)

Simplified structure:



a) Electron gun

- It produces a narrow beam of electrons.
- An indirectly heated cathode C is the source of the electrons.
- Electrons are then focused and accelerated by electrodes.

b) Deflecting System

There are 2 sets of deflecting plates: X and Y plates

- The Y-plates (hor. plates) causes vertical deflection when a p.d. is applied to them.
- The X-plates (vert. plates) causes horizontal deflection when a p.d. is applied to them.

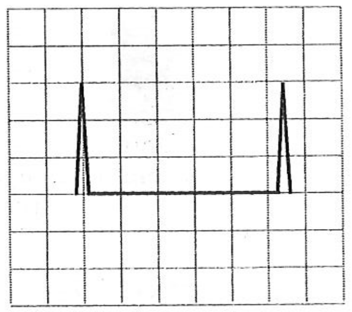
c) Fluorescent Screen

- The inside of the wide end of the tube is usually coated with zinc sulphide which emits light when struck by fast-moving particles.

Simple Uses of the CRO

1 Measurement of Small Time Interval

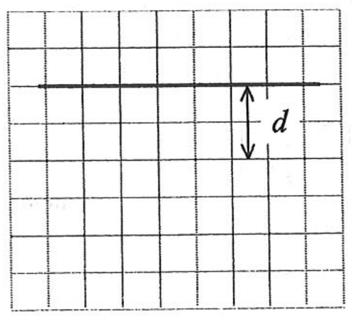
- If two events which are separated by a small time interval are each to cause a voltage pulse on the CRO, the horizontal separation of the pulse can be used together with the time base setting to estimate the time interval between the events.

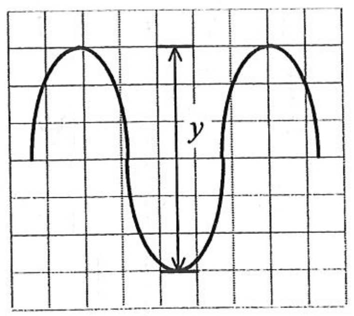


2 Measurement of Voltages (AC or DC)

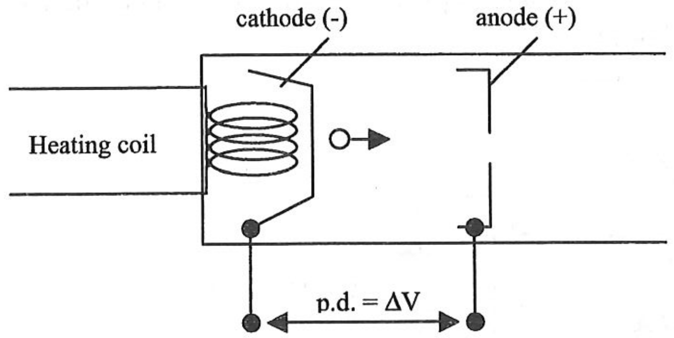
- The voltage to be measured is applied to the Y-plates.

Display of the voltages on the CRO with time base on :





Example 7 (*Motion of charged particles parallel to E-field*)
A potential difference of 1000 V is maintained between two electrodes in a vacuum tube as shown below. An electron is emitted with negligible velocity from the negative electrode (cathode). Calculate its velocity when it reaches the positive electrode (anode).
($e = 1.60\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ kg)



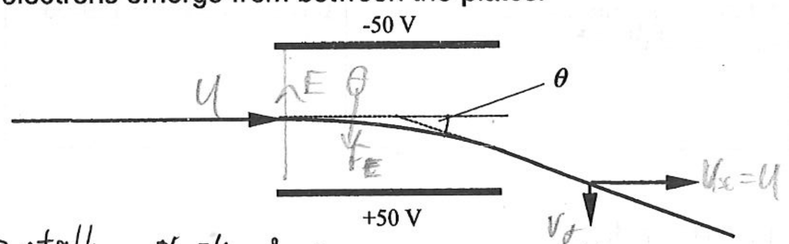
Work done **by** the field **on** the electron (i.e. electron loses its EPE) will go into increasing the kinetic energy of the electron.

By Work-energy theorem,
W.D. by electric field = Kinetic energy gained by q

$$q\Delta V = \frac{1}{2} m_e v^2 - 0$$
$$eV = \frac{1}{2} m_e v^2$$
$$v = \sqrt{\frac{2eV}{m_e}}$$
$$= 1.88\times10^{7}\ ms^{-1}$$

Example 8 (*Motion of charged particles perpendicular to E-field*)

A beam of electrons travelling at 1.35×10^7 ms^{-1} enters a uniform electric field between two plates of length 0.060 m, separated by a distance 0.020 m. One plate is at a potential of +50 V and the other at -50 V as shown below. Find θ, the angular deflection of the beam as the electrons emerge from between the plates.



Remember the equations of motion from kinematics.

Horizontally, $v_x = u$ & $x = ut$

Vertically, $\Delta V = 5-(-50)$
$= 100V$

$$E = \frac{\Delta V}{d}$$
$$a_y = \cancel{\frac{E}{m_e}} = \frac{eE}{m_e}$$

$$v_y = u_y + a_y t$$
$$= \left(\frac{eE}{m_e}\right)\left(\frac{x}{u}\right) \text{ cause } u_y = 0$$

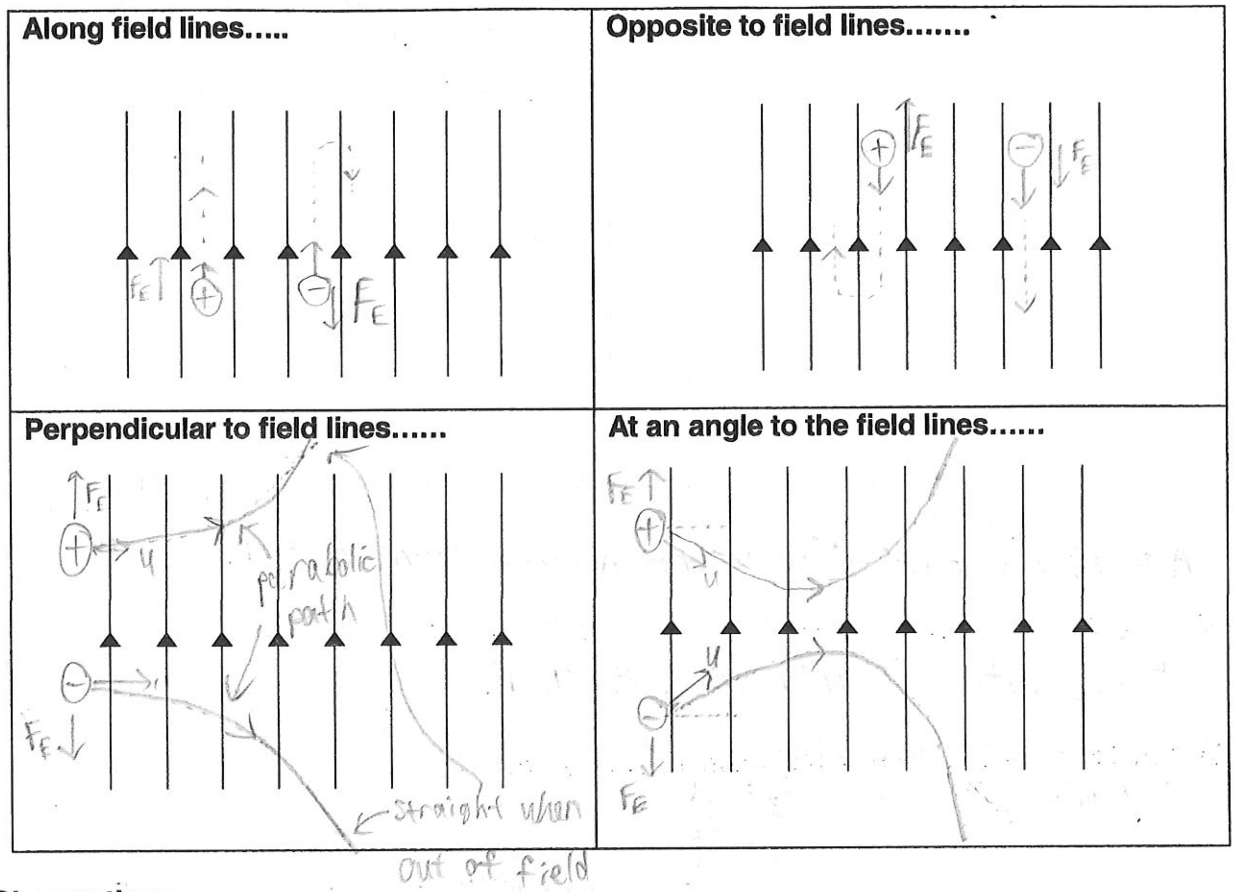
Hence $\tan\theta = \frac{v_y}{v_x}$

$$\theta = 16.1^\circ$$

Refer to Appendix : For applications of E field and CRO

ii) Effect of Uniform E-field on Motion of Charged Particles

Different ways of projecting a charged particle and their subsequent motion:



Observations

- when positive charge is projected along the direction of the field, charge undergoes linear acceleration.
- when negative charge is projected along the direction of the field, charge undergoes deceleration.
- when positive charge is projected against the direction of the field, charge undergoes deceleration.
- when negative charge is projected against the direction of the field, charge undergoes linear acceleration.
- When charge is projected at an angle of the direction of the field, charge undergoes linear acceleration along the direction of the field, i.e. **parabolic path.**

Questions to Ponder

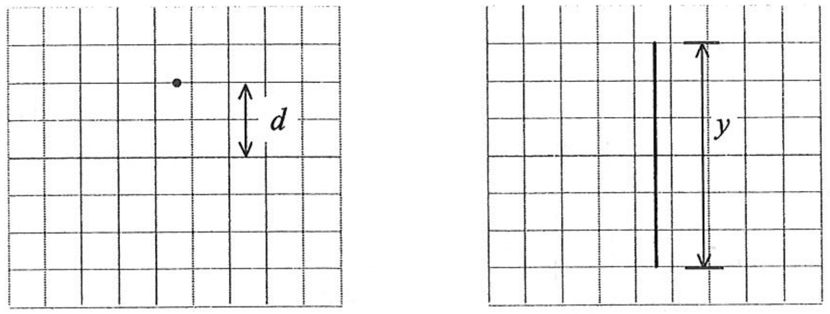
- Do charged particles always move in the same direction as the electric force that is exerted on them?

No, for example, in projectile motion, the forces only deflects the direction of the moving charged particles but the result motion is due to the vector addition of the horizontal and vertical velocity vectors of the particle.

- What will happen to the particles after they leave the field?

They will move in linear motion in the direction at which they leave the field.

Display of the voltages on the CRO with time base off :



- For DC, $V = d \times k$

  where k = Y-sensitivity in volts per cm or per division.

- For AC, $2V_o = y \times k$

  where V_o = peak voltage of a.c.
  k = Y-sensitivity in volts per cm or per division.

- The CRO is particularly useful for measuring voltages because :

i) it has nearly infinite resistance to DC and very high impedance to AC i.e. it draws very little current.

ii) it has instantaneous response.

3 Measurement of Frequencies

Apply the signal whose frequency is to be measured to the Y-plates, with the time base (calibrated) on.

Say the time base was set at 10 ms cm^{-1}

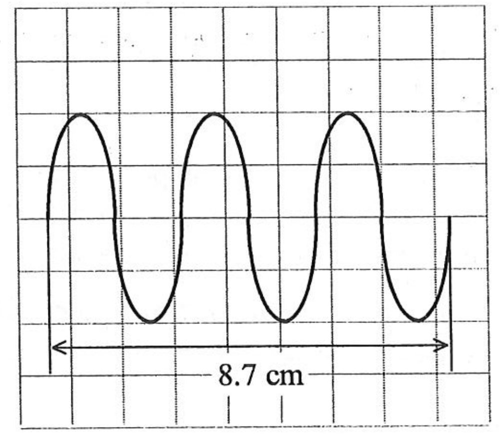
3 complete cycles occupy 8.7 cm

1 complete cycle occupies = 2.9 cm

Period T = 2.9 x 10 ms

= 2.9 x 10^{-2} s

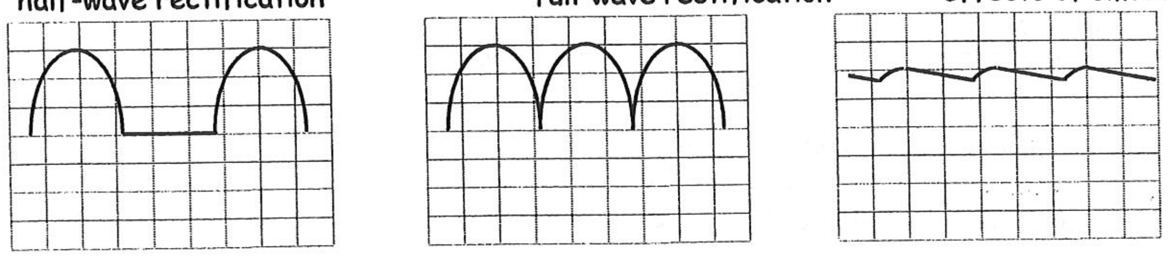
$\therefore$frequency = $1/T$ = 34.5 Hz



4 Display Waveforms

- Connect the signal to be examined to the Y-plates and with the time base on. (i.e. time base to the X-plates).
- Some examples of the waveforms that it can display :

half-wave rectification | full-wave rectification | effects of smoothing

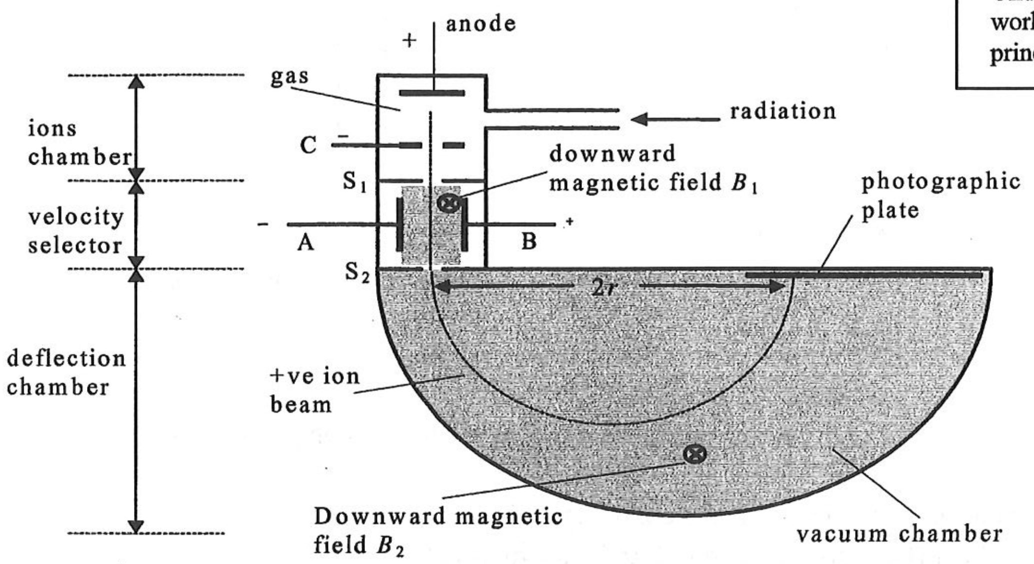


MASS SPECTROMETER (HUTCHINGS, P. 483)

This is an instrument used to separate isotopes of an element, which cannot be separated by chemical means.

Setup

Understand the working principles.



Ions Chamber

- The substance to be measured is first vaporised, then bombarded by high energy radiations such as X-rays, γ-rays or electrons to ionise the atoms or molecules. The positive ions are then accelerated between the positive anode and negative plate C.
- They emerge through the slits S_1 at various speeds and enter the velocity selector.

Velocity Selector

- Particles with this particular speed will travel straight through the slit S_2 to enter the deflection chamber. Note that the particles that pass through does not depend on their masses or their charge, but solely on their velocity.
- We can select ions travelling at a certain speed to pass through the slit S_2 by adjusting either E or B_1. Ions of all other speeds will be blocked.

Deflection Chamber

Magnetic field B_2 applied perpendicular to the line of motion of the ions provides the centripetal force for them to move in semicircular path. Photographic plate or an electrometer can be used to detect the ions.

$$\boxed{\frac{mv^2}{r}} = qvB_2$$

$$\frac{m}{q} = \frac{B_2 r}{v} = \frac{B_2 B_1 r}{E} \qquad \text{(using } v = E/B_1\text{)}$$

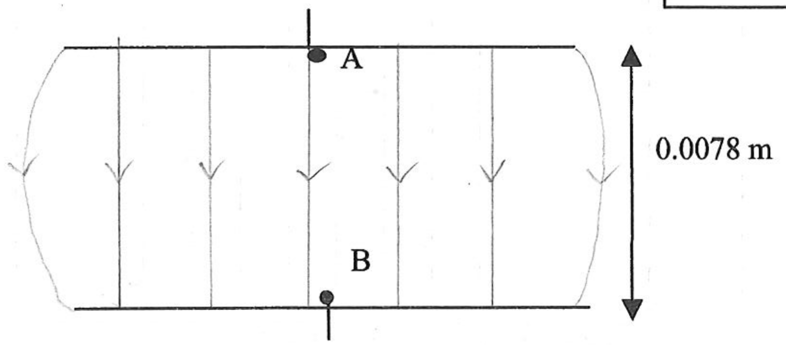
For constant values of B_1, B_2 and E, $r \propto \dfrac{1}{q/m}$.

Hence ions of certain q/m ratio will have a certain radius.

Example 6 (*Motion of charged particles parallel to E-field*)

A proton is moved in a vacuum by a uniform electric field of 2.7 x 10^5 NC^{-1} from A to B, a distance of 0.0078 m.

Electric field is doing work on charge here. (Contrast with Eg. 4)



a) Sketch the lines representing the electric field between 2 charged plates. State whether A or B is at a higher potential.

A is at higher potential as proton moves from A to B.

b) Describe the motion of the proton as it moves from A to B.

It accelerates linearly from A to B.

c) What is the acceleration of the proton as it moves from A to B.

Newton's 2^{nd} law, $F_E = ma$

$$a = \frac{F_E}{m} = \frac{qE}{m} = \frac{(1.6\times10^{-19})(2.7\times10^5)}{(1.67\times10^{-27})} = 2.6\times10^{13}\ \text{ms}^{-2}$$

d) Calculate the difference in potential between A and B.

$$E = \frac{V}{d}$$

$$\therefore V = Ed = (2.7\times10^5)(0.0078) = 2.1\,kV$$

e) What is the change in electric potential energy of the proton?

$$\Delta U_E = q\Delta V = (1.6\times10^{-19})(2.1\times10^3) = 3.36\times10^{-16}\,J$$

f) Is there a gain or loss of electric potential energy? Explain.

Proton loses its electric potential energy as there is work done by field on proton.

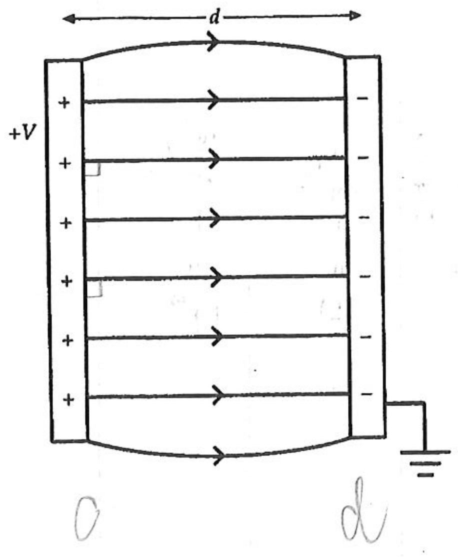
g) What is the change in the kinetic energy of the proton? Is there a gain or a loss?

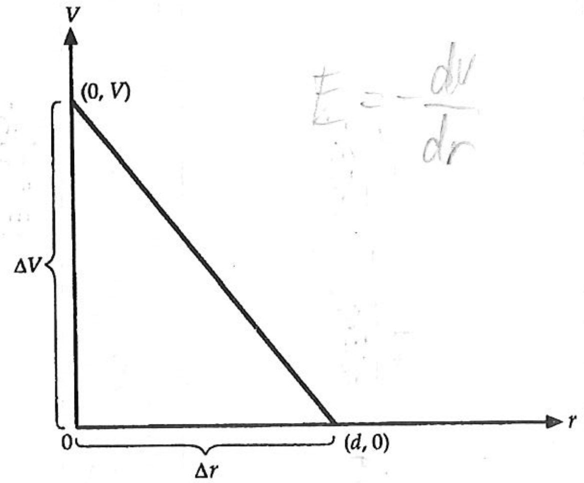
By conservation of energy,

$$K.E \text{ gained} = E.P.E \text{ lost} = 3.36\times10^{-16}\,J$$

2.5 A study of Uniform Field (Charged Parallel Plates)

i) Characteristics

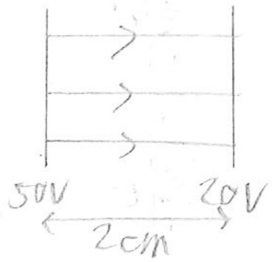




$E = -\frac{dV}{dr}$

- A **potential difference** exists between the two charged plates.
- Electric field strength is **a constant**. It is given by the **potential gradient** of the field.
- Numerically,

$$\boxed{E = \frac{V}{d}}$$



$E = \frac{50-20}{0.02}\ Vm^{-1}$

where *V* is the *difference in potential between two plates.*
d is the *distance between two plates*

- Electric force acting on a charge q placed inside the field is also **constant** in the region of the E-field.

$$\boxed{\begin{aligned} F &= qE \\ &= q\left(\frac{V}{d}\right) \end{aligned}}$$

- Electric field lines are **equally spaced.**
- Equipotential lines are **perpendicular** to the field lines and **equally spaced out**.
  (Refer to page 13)

Internet resources:

1) Useful Links: www.phschool.com/science/cpsurf/elec-mag/5simu.html
2) *Demo of field lines and **potential**: www.cco.caltech.edu/~phys1/java/phys1/EField/EField.html
3) Demo on force:www.colorado.edu/physics/2000/waves_particles/wavpart2.html
4) *Dipole **field** and potential: surendranath.tripod.com
5) Dipole: www.bekkoame.ne.jp/~kamikawa/electricfield/elefi_e.htm
6) Trace field lines: www.gel.ulaval.ca/~mbusque/elec/main_e.html
7) Dramatic demo on field and potential :www.dcc.uchile.cl/~sebrodri/JAVA/Proyecto/ProyectI.html

➤ Note: Use google search engine to find other related websites.

Simulations

- CDRom : MEPI (available from the library)

References:

1. Physics (2nd Edition) by Robert Hutchings, Nelson 2000.
2. Fundamentals of Physics Extended (5th Editioon) by Resnick, Halliday and Walker.
3. The Handy Physics Answer Book by P. Erik Gunderson.
4. Physics (2nd Edition) by Ken Dobson, David Grace & David Lovett, Collins Advanced Science.

TUTORIAL

SELF-ATTEMPT

Some Standard Data

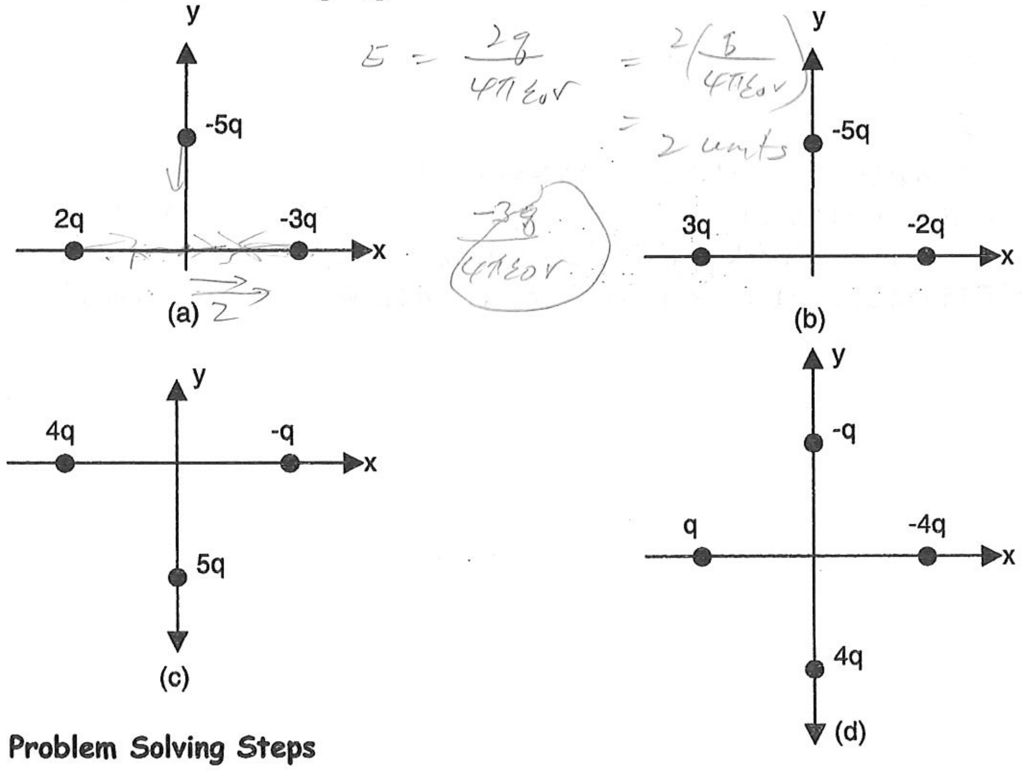
- Mass of an electron : 9.1×10^{-31} kg
- Charge on an electron: -1.6×10^{-19} C
- Gravitational constant, $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-1}
- Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}$ C^{2}m^{-2}

Mass of a proton : 1.6×10^{-27} kg
Charge on a proton: $+1.6 \times 10^{-19}$ C

Radial Electric Field (point charge)

Practice 1

The figure below shows four situations in which charged particles are at equal distances from the origin. Rank the situations according to the magnitude of the net electric field at the origin, greatest first.



Problem Solving Steps

Vector Addition of Electric Field Strength at a Point

When performing calculations to find resultant E-field at a point due to a number of charges, it is easy to be confused by the signs of the charges. Adopt the following steps to calculate the resultant E-field at a point:

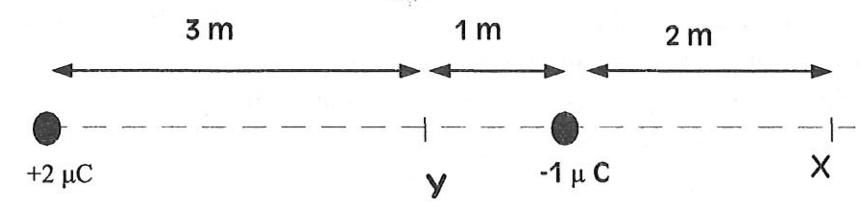
1. Determine the **directions** and draw out the field strength due to each of the charges at the point. Ignore the negative signs due to negative charges once the directions are determined.
2. Calculate the **magnitudes** of the field strength at the point due to each of the charges.
3. Add the field strengths as in vectors.
4. To find resultant force acting on a charge q, simply multiply the resultant E field by charge q.

		Gravitational	Electrostatic
9	Potential due to 2 point masses M / charges Q system	m_1, m_2, r_1, r_2, A; $\phi_A = \left(\frac{-Gm_1}{r_1}\right) + \left(\frac{-Gm_2}{r_2}\right)$	$+Q_1$, $-Q_2$, r_1, r_2, A; $V_A = \left(\frac{+Q_1}{4\pi\varepsilon_o r_1}\right) + \left(\frac{-Q_2}{4\pi\varepsilon_o r_2}\right)$
10	Potential energy (definition) **scalar**	Work done by an external agent in bringing **a mass** from infinity to that point in the gravitational field (J)	**Work done** by an external agent in **bringing a positive charge** from infinity to that point in the electric field (J)
11	Potential energy of 2 masses / 2 charges system	M, m, r; $U_G = -\frac{GMm}{r}$ (always negative)	Q, q, r; $U_E = \frac{Qq}{4\pi\varepsilon_o r}$ U_E : +ve if Q & q are like charges U_E : -ve if Q & q are opposite charges
12	Relationship between field and potential	$g = -\frac{d\phi}{dr}$	$E = -\frac{dV}{dr}$
13	Relationship between force and potential energy	$F_G = -\frac{dU_G}{dr}$	$F_E = -\frac{dU_E}{dr}$

		Gravitational	Electrostatic
5	Field strength due to an isolated point mass M / charge Q	$g_A = \frac{GM}{r^2}$	$E_A = \frac{Q}{4\pi\varepsilon_0 r^2}$
6	Field strength due to 2 point masses M / charges Q system	At A, $g_1 = \frac{GM_1}{r_1^2}$, $g_2 = \frac{GM_2}{r_2^2}$ resultant g = vector sum of g_1 & g_2	At A, $E_Q = \frac{Q}{4\pi\varepsilon_0 r_1^2}$ & $E_q = \frac{q}{4\pi\varepsilon_0 r_2^2}$ resultant E = vector sum of E_Q and E_q
7	Potential (definition) **scalar**	**Work done** by an external agent in bring **unit mass** from infinity to that point in the gravitational field (J kg^{-1}) $\phi = \frac{W}{m}$	**Work done** by an external agent in bring **unit positive charge** from infinity to that point in the electric field (J C^{-1}) $V = \frac{W}{q}$
8	Potential due to an isolated point mass M / charge Q	$\phi_A = -\frac{GM}{r}$	$V_A = \frac{+Q}{4\pi\varepsilon_0 r}$ if Q is +ve charge $V_A = \frac{-Q}{4\pi\varepsilon_0 r}$ if Q is -ve charge

Electric potential

Practice 2



a) Find the potential at points X and Y.

b) Is there a point along the axis joining the two charges where the electric potential is zero? If so, calculate the position where it occurs. Is the electric field also zero at this point?

E-r and V-r Graphs

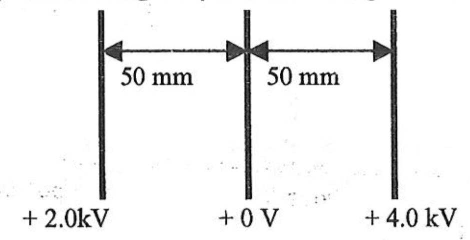
Practice 3

Sketch the E-field lines, V-r and E-r graphs for the follow cases:

a) Two positive charges

b) One positive charge and one negative charge

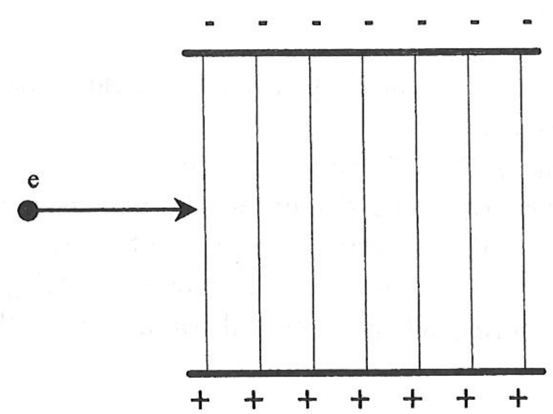
c) The setup consisting of parallel charged conducting plates shown below



Uniform E field

Practice 4

An electron is projected into the evacuated space between two charged plates of length 5.0 cm, as shown below. The electric field strength E is 3.5×10^4 NC^{-1}. The velocity of the electron, v_1 is 3.2×10^7 ms^{-1}. Neglect any gravitational effect.

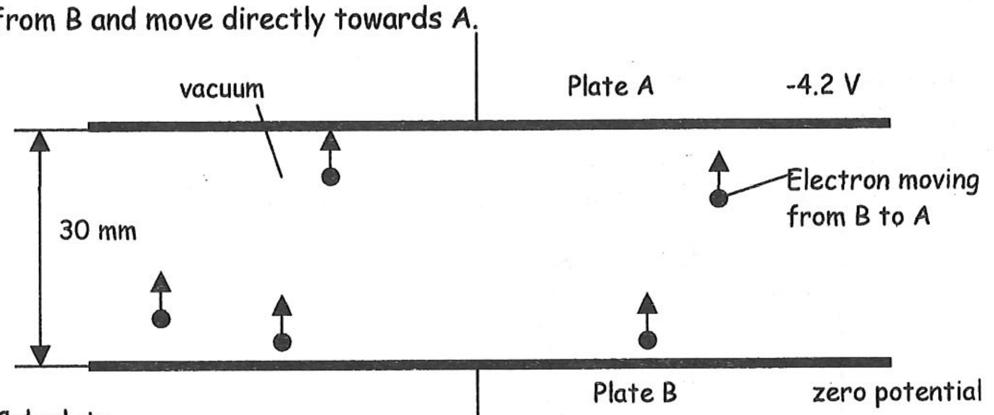


Apply equations of motion like in G field

a) Sketch the path taken by the electron (within and after the two charged plates) as it enters perpendicular to the field. Where have you encountered motion of this nature before? What are some similarities and differences in the two situations?
b) What is the size and direction of the electric force on the electron?
c) What is the horizontal velocity of the electron as it leaves the plate?
d) How long does it take to pass between the plates?
e) What is the vertical acceleration of the electron?
f) What is the vertical velocity on leaving the plates?
g) What is the size and direction of the final velocity?

Practice 5

The diagram below shows two plates A and B, a distance 30 mm apart in a vacuum, with A at a potential of -4.2 V and B at zero potential. Electrons are emitted at high speed from B and move directly towards A.



Calculate
a) the electric field, assumed uniform, between the plates;
b) the change in electric potential energy of an electron as it moves from plate B to plate A;
c) the velocity with which the electrons need to be emitted in order to reach plate A.
State, with a reason, what your answer to c) would have been if the distance between the plates had been doubled to 60 mm, while keeping the potentials the same.

Millikan's Oil Drop Experiment

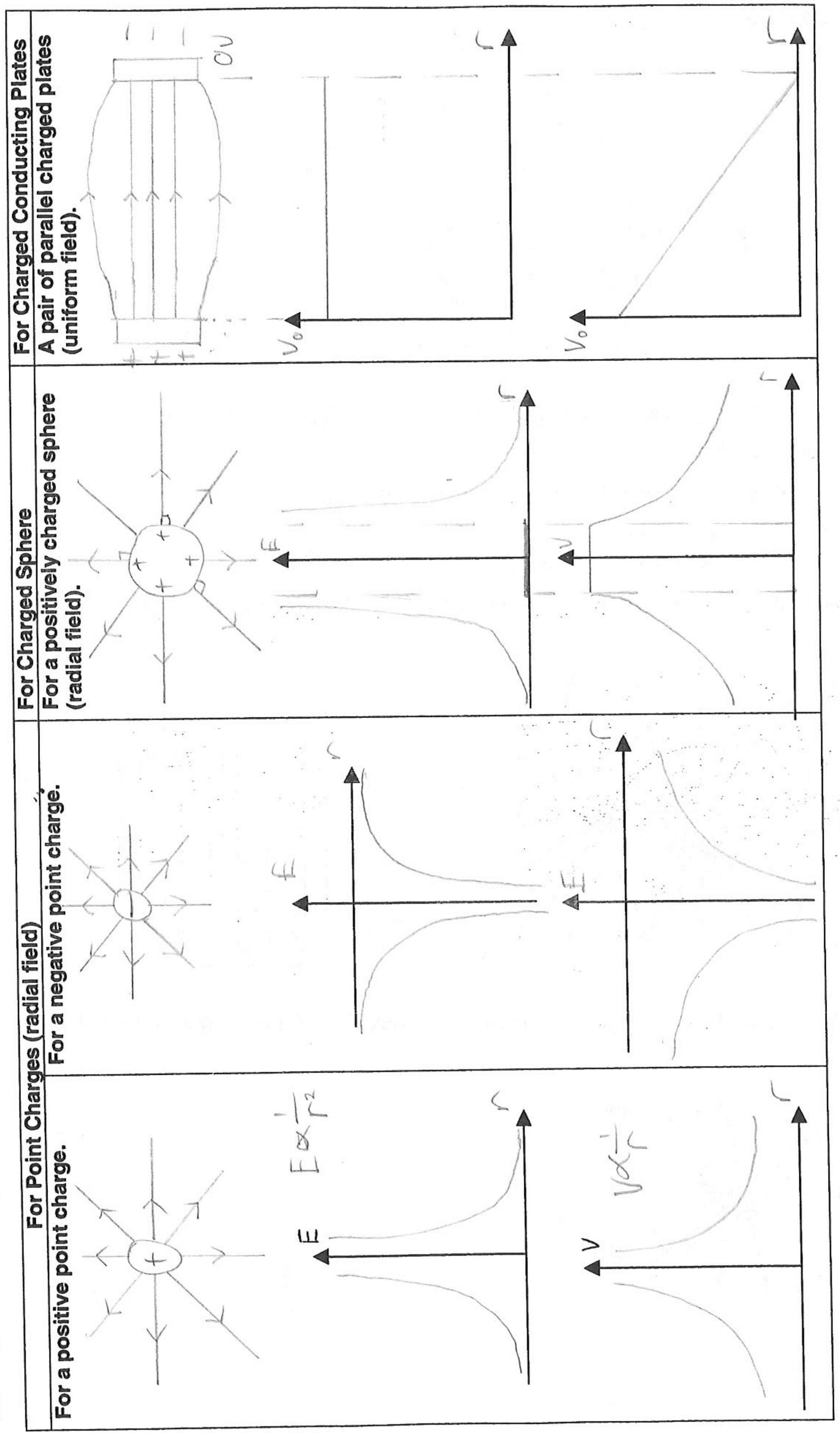
Practice 6

Millikan performed an experiment using oil drops in air to measure the elementary charge e. Discuss if the following is correct.
(a) When an oil drop is stationary, it must carry a charge.
(b) When an oil drop becomes charged, the size of the charge must equal e.
(c) If an oil drop moves upwards, the electric force must act on it.
(d) If an oil drop moves downwards, only the force of gravity is acting on it.
(e) Without the electric field acting, all drops move downwards with the same constant velocity.

2.4 Comparison between gravitational and electric fields

		Gravitational	Electrostatic
1	Origin of force	Due to mass interaction	Due to charge interaction
2	Nature of force	Only attraction	Either attractive (between opposite charges) or repulsive between similar charges) repulsive attractive
3	Inverse square Law	Newton's law of gravitation: $F_G = \frac{GMm}{r^2}$	Coulomb's law: $F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$
4	Field strength (definition) **Vector**	Force per unit mass $g = \frac{F_G}{m}$ $N\,kg^{-1}$ or $m\,s^{-2}$	Force per unit positive charge $E = \frac{F_E}{q}$ $N\,C^{-1}$ or $V\,m^{-1}$

2.3 E-r and V-r Graphs



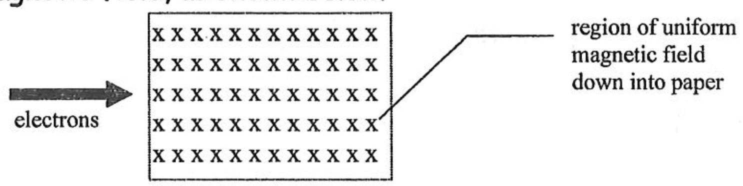
Practice 7

In an experiment to measure the electron charge, an oil drop with an excess of two electron charges is held stationary between two parallel horizontal plates when the potential difference between them is 150 V. A second oil drop of mass twice that of the first, is held stationary with 200 V between the plates. Neglecting the upthrust of the air, how many excess electron charges does the second drop carry?

Magnetic field

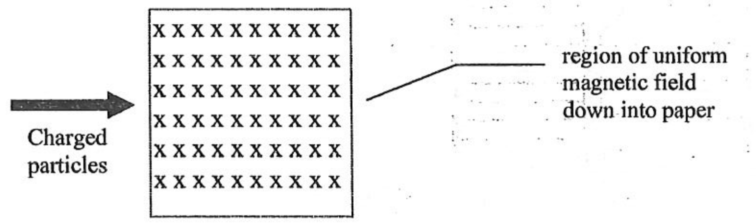
Practice 8

(a) A stream of electrons with a variety of different speeds enters a region of uniform magnetic field, as shown below.



Draw the path of three electrons when in the field and after they leave the field. Label the path of the slowest and of the fastest electron.

(b) Another stream of negatively charged particles with different specific charge (Q/m ratio) are then projected at the same initial velocity into the field.



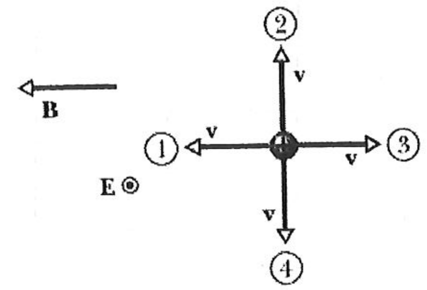
Draw the path of three charged particles when in the field and after they leave the field. Label the path of the particle with the lowest and of the highest specific charge.

(c) What difference would it make to your answers in a) and b) if
   (i) particles considered become positively charged, with field direction unchanged.
   (ii) field direction is reversed but the sign of the particles' charges is unchanged.
   (iii) both the sign of the particles' charge and the field's direction are reversed.

Cross field

Practice 9

The figure shows four directions for the velocity vector v of a positively charged particle moving through a uniform *E* field (directed out of page) and *B* field.



(a) Rank directions 1, 2, and 3 according to the magnitude of the net force on the particle, greatest first.

(b) Of all four directions, which might result in a net force of zero?

(c) Would your answers to (a) and (b) be different if the particle is an electron? Explain.

Solution Outline to Practice Questions

Practice 1

The magnitude and direction of the resultant field is the same in all four situations.

E_{total} = 7.1 q directed at 45° anti-clockwise from the x-axis.

Practice 2

a) $V_x = [1 / 4\pi (8.85 \times 10^{-12})] [2 \times 10^{-6} / 6 + (-1 \times 10^{-6}) / 2] = -1.5 \times 10^{9}$ V

$V_y = [1 / 4\pi (8.85 \times 10^{-12})] [2 \times 10^{-6} / 3 + (-1 \times 10^{-6}) / 1] = -3.0 \times 10^{9}$ V

b) Yes.

$V_x + V_y = 0$

$V_x = -V_y$

Let the distance from that point to the +2C charge be r.

$$\frac{2x10^{-6}}{4\pi\varepsilon_o r} = \frac{1x10^{-6}}{4\pi\varepsilon_o (4-r)}$$

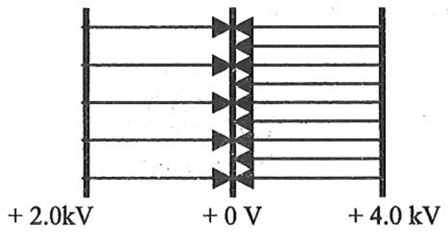
r = 2.7 m

No, E is not equal to zero even though V is equal to zero.

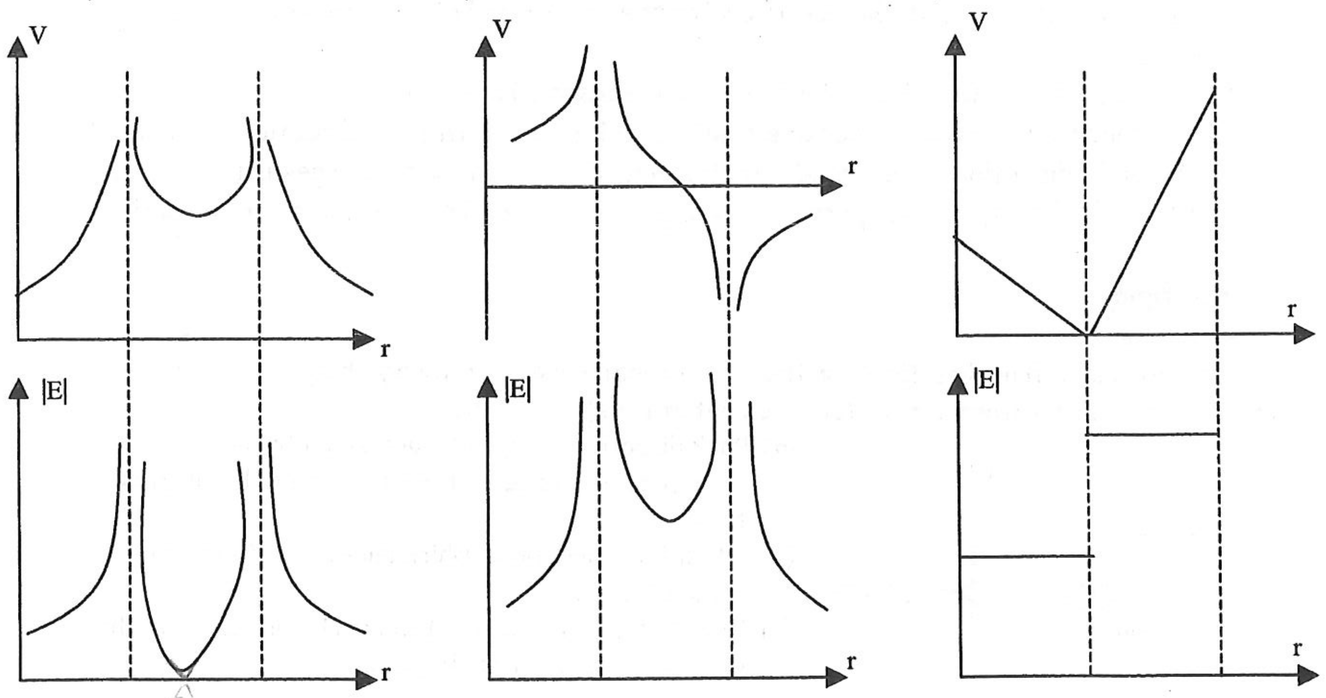
Practice 3

Refer to notes for E-field lines of a) and b).

c)

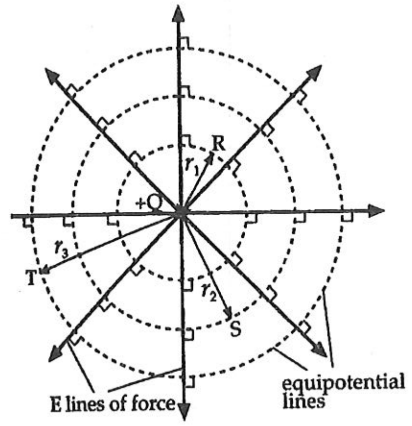


a) b) c)

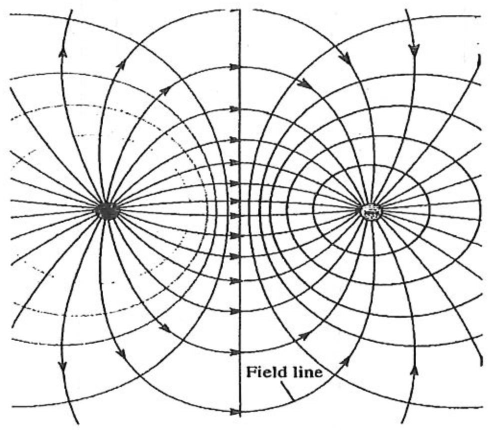


touches 0

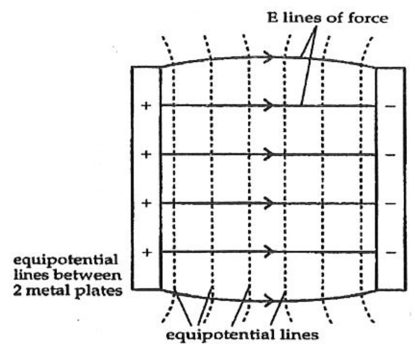
vi) Equipotential lines (surfaces)



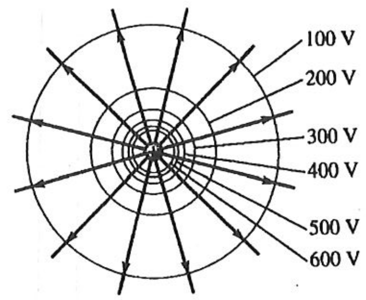
- Equipotential lines (or surfaces) are the locus of points that have the same potential.
- Line cuts E-field lines **perpendicularly**.
- Lines nearer to the positive charge have higher potential ($V_R > V_S > V_T$).
- Lines never **cross** each other.
- Scalar quantity, hence no direction or arrows.
- Moving a charge along an equipotential line does not require any work, as $\Delta V = 0$.



Equipotential lines between 2 point charges



Equipotential lines between 2 metal plates



iv) **Electric Potential and Electric Field Strength (Radial Field)**

From previous section,

$W.D = q(\text{final } V - \text{initial } V)$

$= q\Delta V$

Also $W.D = -\int_{r_i}^{r_f} F\,dr' = -F\Delta r$

And $F = qE$

Hence, we see that $-(qE)\Delta r = q\Delta V$

$$E = -\frac{\Delta V}{\Delta r}$$

Time to put a few things together...

In the limit $\Delta r \to 0$, $\boxed{E = -\frac{dV}{dr}}$ (potential gradient)

Observations

- The electric field strength at a point is **numerically** equal to the **potential gradient** at that point.
- The negative sign indicates that the **potential decreases** along the direction of the E field.
- E = 0 when V is constant, but V needs not necessarily be zero.
- Based on this relation, another unit for *E* is *Vm* $^{-1}$.

v) **Relation between F, E, U and V**

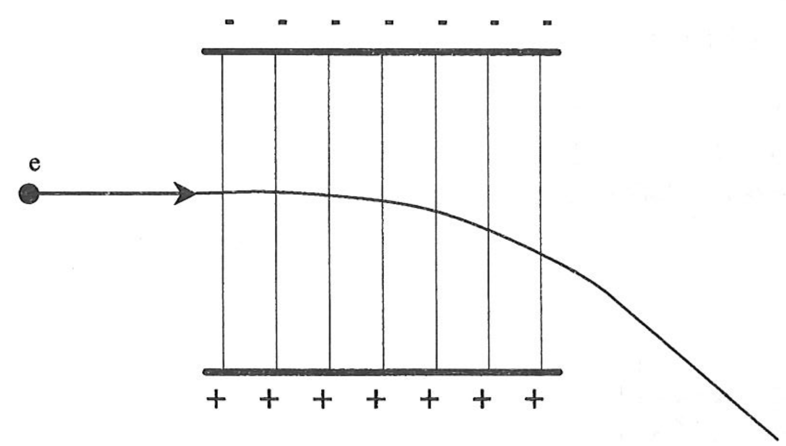
$$\begin{tikzcd} \mathbf{F} \arrow[r, "\div q"] & \mathbf{E} \\ \mathbf{U} \arrow[u, "-\frac{dU}{dr}"] \arrow[r, "\div q"] & \mathbf{V} \arrow[u, "-\frac{dV}{dr}"'] \end{tikzcd}$$

Same relationships in G-field, except that mass is replaced by charge.

Concept test 3

Practice 4

a)



<u>Similarities</u>:

1. The trajectories of both are parabolic, involving horizontal and vertical components of motion.
2. A constant force is experienced by both in the direction perpendicular to the projection.

<u>Differences</u>:

1. Magnitude of electric force is much smaller, but acceleration is much larger. Magnitude of velocity is also much larger.
2. Acceleration for E-field may be directed towards positive or negative plates depending on the charge. Gravitational acceleration is always directed downwards towards earth, i.e. direction is constant.

b) $F = qE = 1.6 \times 10^{-19} \times 3.5 \times 10^{4} = 5.6 \times 10^{-15}$ N downwards

c) $v_x = 3.2 \times 10^{7}$ ms^{-1}.

d) $t = 5.0 \times 10^{-3} / 3.2 \times 10^{7} = 1.6 \times 10^{-9}$ s

e) $a = F/m = 5.6 \times 10^{-15} / 9.1 \times 10^{-31} = 6.1 \times 10^{15}$ ms^{-2}

f) $v_y = u_y + at = 0 + 6.1 \times 10^{15} \times 1.6 \times 10^{-9} = 9.8 \times 10^{6}$ ms-1 downwards

g) $v_f = \sqrt{(9.8\text{x}10^{6})^2 + (3.2\text{x}10^{7})^2} = 3.3 \times 10^{7}$ ms^{-1}

$\tan\theta = 9.8 \times 10^{6} / 3.2 \times 10^{7}$; $\theta = 17°$ wrt horizontal component.

Practice 5

a) $E = \Delta V/x = 4.2 / 30 \times 10^{-3} = 140$ V

b) $\Delta U = q\Delta V = 1.6 \times 10^{-19} \times 4.2 = 6.7 \times 10^{-19}$ J

c) By conservation of energy : $\frac{1}{2}mu^2 = 6.7 \times 10^{-19}$; $u = 1.2 \times 10^{6}$ ms^{-1}
No change as the p.d. between the two plates remains unchanged.

Practice 6

(a) True. Electrostatic force is needed to balance the weight.
(b) False. Charge may be an integral multiple of e.
(c) True. Electrostatic force causes it to move upward.
(d) False. Electrostatic force may act downwards, depending on field direction and charge polarity.
(e) False. Terminal velocity depends on the radius of the drop.

Practice 7

At equilibrium,

mg = qV/d where q = ne

mg = 2e (150)/d (1)

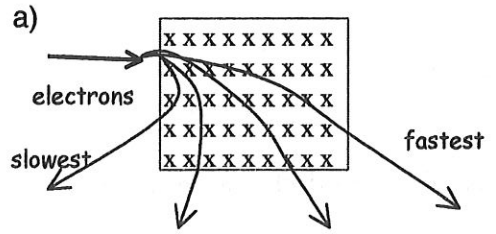
2mg = Ne (200)/d (2)

(2) / (1), we have 2 = 200N / 300

Hence, <u>N = 3</u>.

Practice 8

a)



b) Same as for a) except that path with largest radius means smallest q/m ratio and vice versa.

Note : when charged particles exit a field, it will resume linear motion at constant speed.

c) i) Circular motion will be anticlockwise.

ii) same as i).

iii) No effect on motion.

Practice 9

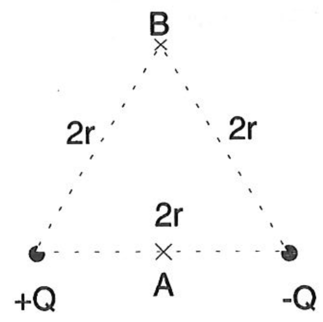
(a) 2, then tie of 1 and 3. In direction 2, both the E and B fields will cause a resultant deflection in the same direction (outward), while in 1 and 3, only E field will act on the particle as motion in parallel or antiparallel to B field.

(b) 4. Forces due to E and B fields are in opposite directions, hence cancelling each other.

(c) If particle were an electron, answers to (a) and (b) will be unchanged since the direction of forces due to E and B fields will both switch. The resultant force on electron in direction 2 will be inward instead of outward.

Example 5

Find the electric potential at the points A and B.



Problem Solving Technique

The potential at a point is equal to the **scalar** sum of the potential due to each of the individual point charges at that point.

$$V_A = \frac{+Q}{4\pi\varepsilon_0 r} + \frac{-Q}{4\pi\varepsilon_0 r} = 0$$

$$V_B = \frac{+Q}{4\pi\varepsilon_0 (2r)} + \frac{-Q}{4\pi\varepsilon_0 (2r)} = 0$$

Compare your answers here to those in Example 3.

- What do you notice?

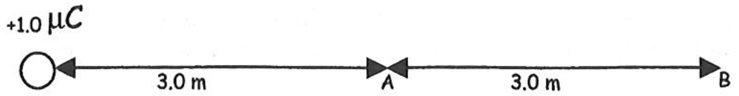
$V=0$ at equidistant from the two equal and opposite charges

- What general conclusion(s) can you draw?

1. $E \neq 0$ at equidistant from the two equal and opposite charges but the electric potential can be 0.
2. When $V=0$, it does not imply that $E=0$.

- So what is the relationship between E and V ?

Example 4



a) Calculate the electric potential at A, 3.0 m away from a point charge of +1.0 μC.

$$V_A = \frac{Q}{4\pi\varepsilon_0 r}$$
$$= \frac{(+1.0\times10^{-6})}{4\pi(8.85\times10^{-12})(3)}$$
$$= 3.0\times10^{3}\ V$$

b) Calculate the electric potential at B, 6.0 m away from the same charge of + 1.0 μC.

$$V_B = \frac{(+1.0\times10^{-6})}{4\pi(8.85\times10^{-12})(6)}$$
$$= 1.5\times10^{3}\ V$$

c) How much work would be done in moving a +0.2 μC charge from A to B?

Work done by external agent $= q(V_B - V_A)$
$$= (+0.2\times10^{-6})\left[(1.5\times10^{3}) - (3.0\times10^{3})\right]$$
$$= -3.0\times10^{-4}\ J$$

i.e. charge loses its electric potential energy when it moves from A to B.

d) How much work would be done by an external agent in moving a unit positive charge from infinity to point A ?

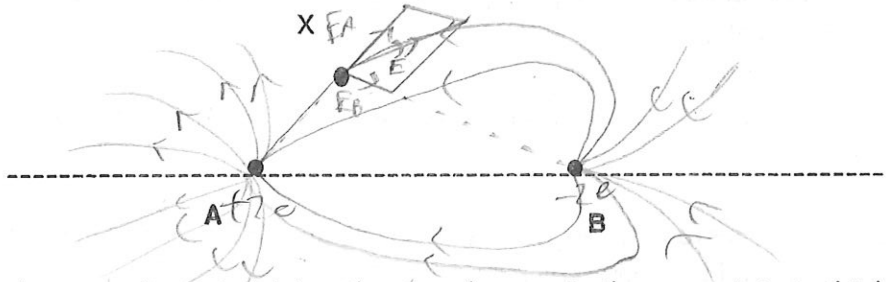
By definition, the work done by external agent in moving unit positive charge from infinity to point A is the potential due to charge (+1.0 μC) at point A

$\therefore$ work done $= V_A$
$$= 3.0\times10^{3}\ J$$

DISCUSSION

Charged Particles in E-field

D1 Two ions **A** and **B** are separated by a distance in a vacuum as shown below. **A** has a charge of + 3.2 x 10^{-19} C and **B** has a charge of -1.6 x 10^{-19} C.

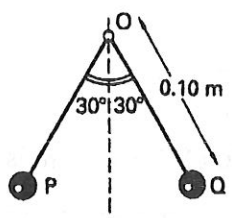


(a) Deduce where on the axis joining the two charges is there a point at which the net electric field is zero: between the charges, to their left or to their right?

(b) Sketch labelled arrows on the figure to represent
- (i) the field E_A at the point X due to the charge at **A** only,
- (ii) the field E_B at the point X due to the charge at **B** only,
- (iii) the resultant field E at X due to both charges.

(c) Sketch, on the same diagram, lines representing the electric field caused by the two ions in the region. Include the field line passing through X.

D2 Two small conducting spheres P and Q, each of mass 1.5 X 10^{-5} kg, are suspended from the same point O by insulating threads 0.10 m long. When the spheres are charged (with equal charges) they come to rest with both threads inclined at 30° to the vertical, as shown in the diagram.



(a) Show that the force of repulsion between the spheres is 85 μN.
(b) Calculate the distance between the spheres.
(c) What is the charge on each sphere?
(d) When half the charge has leaked off one sphere, but none has leaked off the other, it is found that **both** threads are then inclined at 16° to the vertical. Explain why the angles must be equal.

D3 Consolidation questions:

(a) Are we free to call the potential of the earth +100 V instead of zero? What effect would such an assumption have on the measured values of (i) potential and (ii) potential difference?

(b) If the potential is zero at a given point, must the electric field also be zero at this point? Give an example.

(c) If the field is zero at a given point, must the potential also be equal to zero for that point? Give an example.

(d) If the potential is constant throughout a given region of space, what can you say about the electric field in that region?

(e) If the electric field is constant throughout a given region of space, what can you say about the potential in this region?

(f) If the electric potential at some point is zero, can you conclude that there is no charge in the vicinity of that point?

D4

a) Two small positively charged spheres are a large distance apart. They are then brought closer together so that they nearly touch.
   i) Explain why work has to be done to bring the spheres closer together.
   ii) Suggest why this work done may be referred to as electric potential energy.

b) The electric potential energy E_p of two point charges Q and q separated by a distance r is given by the expression

$$E_p = \frac{8.9 \times 10^9 \times Qq}{r}$$

A proton of mass 1.67×10^{-27} kg and radius 1.2×10^{-15} m is brought towards a lithium nucleus ${}^{7}_{3}\mathrm{Li}$ of radius 2.3×10^{-15} until they just touch. The elementary charge (charge on one electron or proton) is 1.6×10^{-19} C and the charges on the proton and the lithium nucleus may be assumed to be point charges.

i) Calculate the electric potential energy of the proton when it is just in contact with the lithium nucleus.

ii) Determine the initial speed of the proton so that it just reaches the lithium nucleus.
Hint: Li nucleus has 3 protons.

iii) Electric Potential

- So what is the difference between electric potential and electric potential energy?
  It is the same as EPE, except that it is WD on **unit charge** rather than just any charge q.

Definition

The electric potential at a point in an electric field is the **work done** by an external agent in bringing **a unit positive charge** from infinity to that point.

Hence, electric potential at a point wrt infinity is given by

$V = \frac{U_E}{q}$ Unit: JC^{-1}

or $V = \frac{Q}{4\pi\varepsilon_0 r}$ (scalar)

For **positive** point charge: $V = \frac{+Q}{4\pi\varepsilon_0 r}$

For **negative** point charge: $V = \frac{-Q}{4\pi\varepsilon_0 r}$

In the limit $r \rightarrow \infty$, $V \rightarrow 0$. Hence the potential at infinity is taken to be zero since it is not under the influence of the field. This is the **Theoretical zero.**

➤ Note that practically, the potential of the earth is also taken to have zero potential. Hence, a conductor which is earthed also has zero potential. This is the **Practical zero.**

- Generally, for two finite points A and B,

  WD by external agent in moving charge q from A to B
  $W = U_B - U_A$
  $= q\,V_B - q\,V_A$
  $= q\,(V_B - V_A)$

  Thus, potential difference, $V = V_B - V_A = W / q$

  The potential difference between two points in an electric field is equal to the work done by an external force in moving a unit positive charge from a point at a lower potential to a point at higher potential.

Concept test 2

ii) Electric Potential Energy (EPE)

- What is EPE of a charge q at a point in space?
  It is the **work done** by an external agent in bringing **a charge** q from infinity to that point.

- What is the link between *EPE* and *work done* on a charge?

  By Conservation of Energy,

  WD in moving charge q between 2 points
  = Change in EPE
  = Final EPE – Initial EPE
  = $U_f - U_i$

Work done and EPE

- +ve work done **by** external agent **on** charge : charge **gains** electric potential energy
- -ve work done **by** external agent **on** charge : charge **loses** electric potential energy

- Why is kinetic energy not involved here?
  KE will not be involved as a static condition was assumed.

- How can we show the change in EPE mathematically?
  To show this mathematically, we need to find out how U varies with r.

  Suppose we set point X to be at infinity, then WD by ext. agent in bringing charge q from infinity to Y (which is r from the charge Q) is as follows: $x = \infty$

F_A F_E

$$U_E = -\int_{\infty}^{r} F_A \, dr'$$
$$= -\int_{\infty}^{r} \left(\frac{Qq}{4\pi\varepsilon_0 r^2}\right) dr'$$
$$= -\left[-\frac{Qq}{4\pi\varepsilon_0 r}\right]_{\infty}^{r}$$
$$= \frac{Qq}{4\pi\varepsilon_0 r}$$

$$F_A = F_E = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

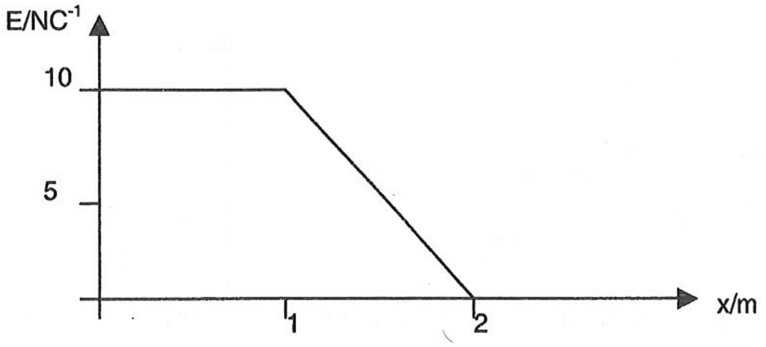
> Gravitational P.E is
> $U_g = -GMm / r$
> Minus sign indicates the gravity is always an attractive force. If two particles move closer together, gravity does positive work, GPE decreases.

Generally, EPE at any point **w.r.t to infinity** is given by

$$U_E = \frac{Qq}{4\pi\varepsilon_0 r} \qquad \text{Unit: } J$$

Observations
- U_E is inversely proportional to r. Hence, in the limit $r \to \infty$, $U_E \to 0$.
- For like charges, the force is repulsive. U_E is positive, i.e. U_E increases as charges move closer together.
- For opposite charges, the force is an attractive one. U_E is negative, i.e. U_E decreases as the charges move closer together.

D5 Shaped electrodes are used to produce an electric field with a field intensity that varies numerically with distance, as shown below.

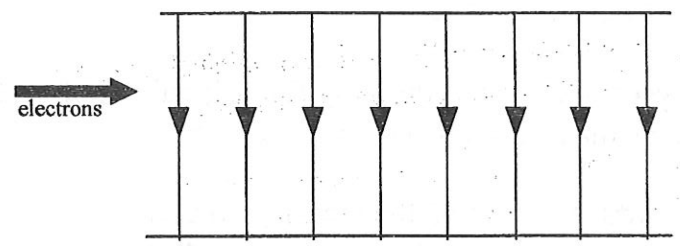


> **Remember:**
> E = V/d
> numerically

a) Sketch the V-x graph with appropriate labels on the x and y axes (You may ignore the negative sign).

b) i) What is the potential difference between the origin and the point 2 m away?
   ii) What is the change in electrical potential energy of a charge of +1.6 x 10^{-19} C moving from the origin to a point 2 m away?

D6 A stream of electrons with a variety of different speeds enters a region of uniform electric field, as shown below.

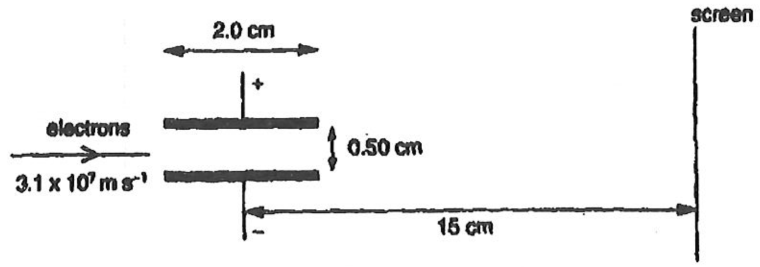


(a) Draw the path of three electrons when in the field and after they leave the field. Label the path of the slowest and of the fastest electron.

(b) What difference would it make to your answers in a) if in each of the cases below
  (i) the distance between the plates become larger.
  (ii) the potential difference applied across the plates become smaller.
  (iii) positively charged particles are used instead of electrons, with field direction unchanged.
  (iv) field direction is reversed.
  (v) both the sign of the particles' charge and the field's direction are reversed.

(c) Does the mass of the ~~electron~~ charged particles have any effect on its path taken? Explain why or why not.

D7 In one type of c.r.o., the electrostatic deflection system consists of two parallel metal plates, each of length 2.0 cm, with a separation of 0.50 cm, as shown below.



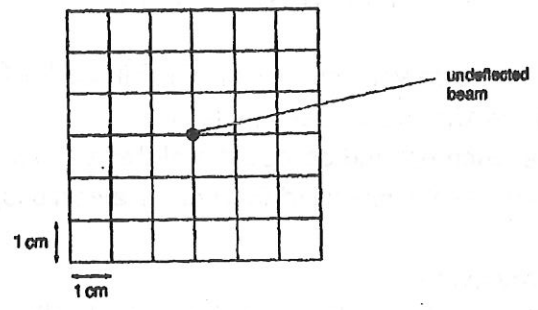
The centre of the plates is situated 15 cm from a screen. A potential difference of 80 V between the plates provides a uniform electric field in the region between the plates.

(a) Electrons of speed 3.1×10^7 ms^{-1} enter this region at right angles to the field. Calculate

(i) the time taken for an electron to pass between the plates,
(ii) the electric field strength between the plates,
(iii) the force on an electron due to the electric field,
(iv) the acceleration of the electron along the direction of the electric field,
(v) the speed of the electron at right angles to its original direction of motion as it leaves the region between the plates.

(b) Hence, by considering your answer to (a)(v) and the original speed of the electron, estimate the deflection of the electron beam on the screen.

(c) The figure below represents the front of the screen of the c.r.o.
Copy the figure on to your paper and mark on your diagram the position of the deflected beam of electrons.



c) Work Done, Electric Potential Energy and Electric Potential (Radial Field)

i) Work done

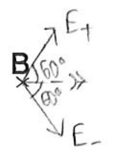
Let F_E be the force on test charge due to E-field & F_A be the force on test charge due to *external agent*. External force, F_A is applied to prevent the charge from accelerating, i.e. to keep the motion *static*. W_A refers to the work done by external agent on charge.

Positive test charge brought closer to positive source charge +Q (+) (+) F_A ← +q → F_E F_E : repulsive F_A : pushing Hence, W_A is positive.	Positive test charge brought closer to negative source charge (–) Q + ← → F_A F_E F_E :attractive F_A : restraining Hence, W_A is negative.
Positive test charge brought further from positive source charge Q (+) F_A ← (+) → F_E (+) q F_E :repulsive F_A : restraining Hence, W_A is negative.	Positive test charge brought further from negative source charge (–) Q F_E ← (+) → F_A (+) q F_E : attractive F_A : pushing Hence, W_A is positive.

Conclusion

- If external force is in **same** direction as motion, it is **pushing against** the natural force of attraction or repulsion.
  WD by external agent on charge, W_A is **positive.**
- If external force is in **opposite** direction to motion, it is **restraining** the charge from being accelerated by the field.
  WD by external agent on charge, W_A is **negative.**

Electric field strength at point B



$$E_+ = E_-$$
$$= \frac{Q}{4\pi\varepsilon_0 (2r)^2}$$
$$= \frac{Q}{16\pi\varepsilon_0 r^2}$$

+Q -Q

$(E_-)_x = E_+ \cos 60°$, $(E_-)_y = E_- \sin 60°$

$$E_x = (E_+)_x + (E_-)_x$$
$$= 2\left(\frac{Q}{16\pi\varepsilon_0 r^2}\right)\cos 60°$$
$$= \frac{Q}{16\pi\varepsilon_0 r^2}$$

$$E_y = (E_+)_y - (E_-)_y$$
$$= 0$$

At B, $E_{net} = E_x = \frac{Q}{16\pi\varepsilon_0 r^2}$ directed to the right

Concept test 1

D8 In a certain Millikan experiment, the necessary adjustments are made to keep a charged droplet of mass m stationary between the plates.

(a) Show and label on a free-body diagram the forces acting on the stationary charged oil droplet and give the relationship between the forces. Indicate clearly the charge, q and the direction of the electric field, E. Write down an equation to show how the forces balances up.

(b) Describe and explain what would happen to the droplet if the separation of the plates were then slowly reduced, the potential difference between the plates remaining constant.

(c) In one experiment Millikan found that the charge Q on a particular drop had the following values at various times.

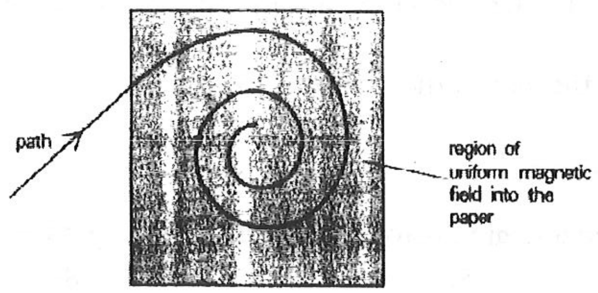
Q / 10^{-9} e.s.u. (electrostatic unit)					
6.87	4.44	8.37	5.39	1.97	2.96

Use these results to find a value for the electron charge in e.s.u. Deduce the conversion factor between the SI unit of charge (Coulomb) and the e.s.u. of charge.

(d) What is meant by quantisation of charge?
Explain why the Millikan experiment is said to provide experimental evidence for the quantisation of charge.

Charged Particles in B-field

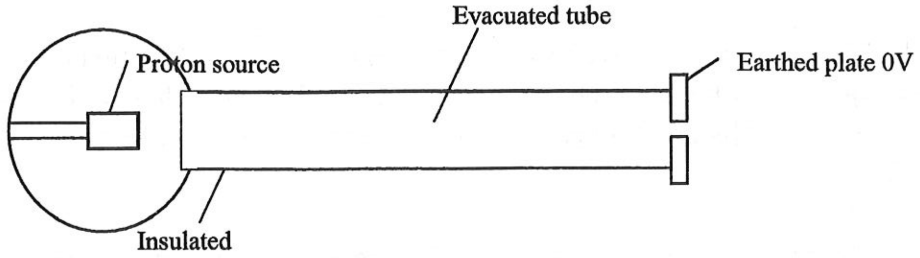
D9 A common way of investigating charged particles is to observe how they move in a plane at right angles to a uniform magnetic field. The diagram shows the path of a certain particle.



Which of the following gives a satisfactory explanation for the path ?
(a) The momentum of the particle is increasing steadily. ↑r
(b) The charge on the particle is decreasing steadily. ↑r
(c) The magnetic flux density is decreasing steadily. ↑r
(d) The mass of the particle is increasing steadily. ↑r
(e) The speed of the particle is decreasing steadily. ↓r

D10

(a) A source of protons is situated inside a sphere and accelerated to earth along an evacuated tube.



For a proton accelerated from rest through a potential difference of 1.9×10^5 V,

(i) calculate the change in potential energy,

(ii) show that its speed is 6.0×10^6 ms^{-1}.

(b) The protons emerge from the evacuated tube into a region of uniform magnetic field of flux density 0.18 T. The region is evacuated and the magnetic field is normal to the direction of the motion of the protons.

(i) Sketch the path of the proton in the magnetic field and show the direction of the field.

(ii) Calculate the radius of the path of the protons in the magnetic field.

(iii) Measurement of this radius can be used as a means to determine the kinetic energy of the protons. State and explain what happens to the radius if the kinetic energy of the protons were to be reduced.

(c) If the proton source is replaced by a source emitting positive ions that has twice the specific charge as proton, state what change occurs in

(i) the speed of the ions entering into the regions of the electric and magnetic fields in (a).

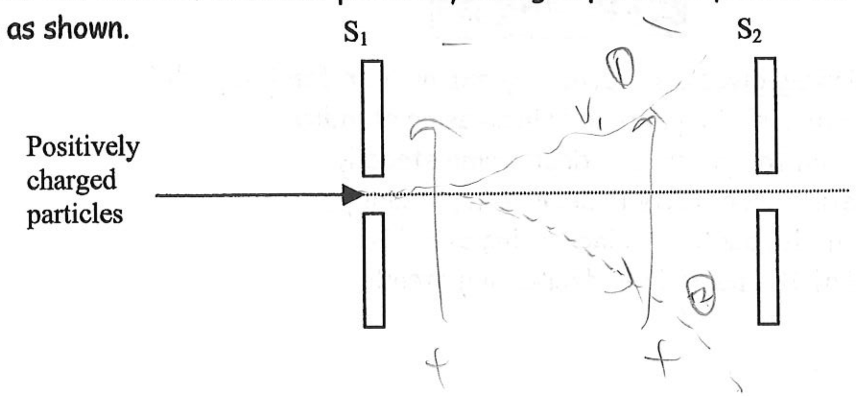
(ii) the path of the ions in the magnetic field in (b).

$qE = qBv$

$qE = qBv$

Charged Particles in Cross field

D11 A narrow beam of identical positively charged particles passes through two slits, S_1 and S_2 as shown.



$qE = qBv$

$\frac{E}{Bv} = \text{constant}$

$\frac{E}{Bv_1} = \frac{E}{Bv_2}$

Example 2

Find the force acting on an electron in an electric field strength of 3.1×10^6 Vm^{-1}.

$F_E = qE$

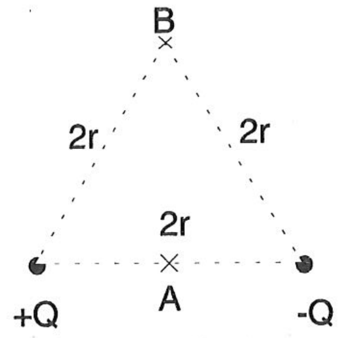
$= (1.6 \times 10^{-19})(3.1 \times 10^6)$

$= 5.0 \times 10^{-13}$ N

The magnitude of force is 5.0×10^{-13} N and is directly opposite to the direction of electric field.

Example 3

Find the electric field strength at the points A and B.



Problem Solving Technique

The electric field strength at a point is equal to the **vector** sum of the field due to each of the individual point charges at that point.

Steps to solve the problem:

1. Indicate the direction of Electric field strengths at A due to +Q and -Q
   i.e. +Q generates electric fields that radiate outward at A from the charge
   -Q generates electric fields that radiate inward at A from the charge
2. Calculate the magnitude of Electric field strengths at A due to +Q and -Q
   i.e. Calculate the E field strength at A due to +Q
   Calculate the E field strength at A due to -Q
3. Calculate the total Electric field strengths at A as the sum of the E fields generated by +Q and –Q

Electric field strength at point A

$E_+ = \frac{Q}{4\pi\varepsilon_0 r^2}$

$E_- = \frac{Q}{4\pi\varepsilon_0 r^2}$

A + A, $E_{net} = E_+ + E_-$ ($\because$ both in the same direction)

$= 2 \times \frac{Q}{4\pi\varepsilon_0 r^2}$

$= \frac{Q}{2\pi\varepsilon_0 r^2}$ directed to $-Q$

Nature of Electric Force, F
- when charges are of the same sign, the force is repulsive
- when charges are of opposite sign, the force is attractive
- whether the charges are of the same or different magnitude, the magnitude of the forces experienced by each charge is the same and acting in the opposite direction. This is consistent with Newton's 3rd Law

Example 1
A charge of +2 μC is placed 4 m away from a charge of -3 μC. Find the force acting on each of the charges.

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2} = \frac{(2\times10^{-6})(3\times10^{-6})}{4\pi(8.85\times10^{-12})(4)^2} = 3.4\times10^{-3}N$$

∴ the force has a magnitude of 3.4×10^{-3} N and is attractive.

b) Electric Field Strength of a point charge

Definition

Electric field strength, E at a point is the **electrostatic force** F acting on a **unit positive charge** placed at that point.

Electric Field Strength depends on
- charge Q
- $\frac{1}{r^2}$
- $\frac{1}{4\pi\varepsilon_0}$

> In topic Gravitation, the gravitational force acting on a unit mass of the object, we get the gravitational field strength.
> $g = F_G / m$

Equation

By definition: Unit: NC^{-1}

$$E = \frac{F_E}{q}$$

E at a point due to a point charge, $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

Representation
- vector
- E-field lines
- points in the same direction as the force experienced by a positive charge

A uniform magnetic field of flux density B is applied in the region between S_1 and S_2 in a direction out of the plane of the paper.

(a) A uniform electric field is applied in the space between the slits such that charged particles of only one speed v can pass through S_2. On the diagram, mark clearly with an arrow labelled E the direction of this electric field.

(b) Explain how this combination of magnetic and electric fields allows particles of only one speed to pass through S_2. Deduce an expression for v in terms of B and the electric field strength, E.

(c) Sketch on the diagram possible paths, in the region of the electric and magnetic fields, of particles having speed greater than v and of those having speed smaller than v.

Gravitational, Electric and Magnetic Fields

D12 Consolidation Questions

a) What similarities and differences do you observe between
   i) Newton's Law of Gravitation and Coulomb's Law for electrostatics
   ii) electric field strength, E and gravitational field strength, g
   iii) electric potential and gravitational potential
   iv) field lines and equipotential surfaces of E-field and G-field?

b) Is it necessary to consider the gravitational effects when calculating the electric force between two sub-atomic particles like a proton and an electron? Explain quantitatively.

c) Can you set a resting electron into motion with a magnetic field? With an electric field?

d) How can the motion of a moving charged particle be used to distinguish between a magnetic field, an electric field and a gravitational field?
circular parabolic min deflection

Numerical Answers to Selected Questions

Qn	Numerical Answers	Qn	Numerical Answers
D2	b) 0.10 m c) 9.7nC	D4	b) i) 1.95×10^{-13} J ii) 1.53×10^{7} ms^{-1}
D5	b) i) 15 V; ii) 2.4×10^{-18} J	D7	6.5×10^{-10}s, 1.6×10^{4} Vm^{-1}, 2.6×10^{-15} N, 2.8×10^{15} ms^{-2}, 1.8×10^{6} ms^{-1}, 0.88 cm
D8	4.9×10^{-10} e.s.u.; 1 C ≡ 3.1×10^{9} e.s.u.	D10	3.04×10^{-14} J; 0.35 m

Na

A

A

a)

b)

T

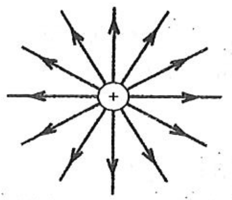
tl

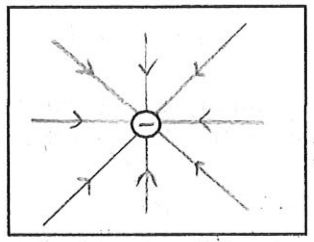
m

1. **Electric charge**
   - It is one of the fundamental properties of nature.
   - The forces between the charges within materials are responsible the strength of the materials and all their electrical properties.
   - There are two types of charge : positive and negative
   - It is observed that two charged objects of the **same charge** will push each other apart (**repel**) while charged objects of **opposite charge** will pull each other together (**attract**).
   - Surrounding every charged object is an electric field.

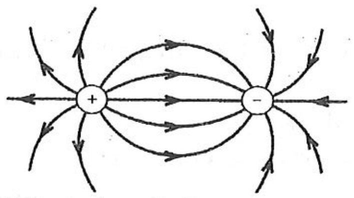
2. **Electric Field (E-field)**
   - An electric field is a region of space in which **a test charge** placed in it will experience a **force.** (A test charge by definition is so small that it will not affect the field that was originally present. By convention, test charge is chosen to be positive charge).

2.1 Electric Lines of Force
- Electric field is represented by **electric field lines.**
- Pattern depends on the shape and nature of the charged body.
- Examples of Electric Field Lines

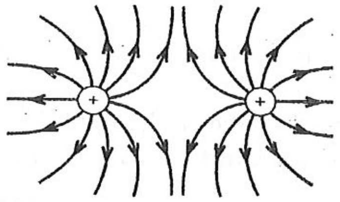




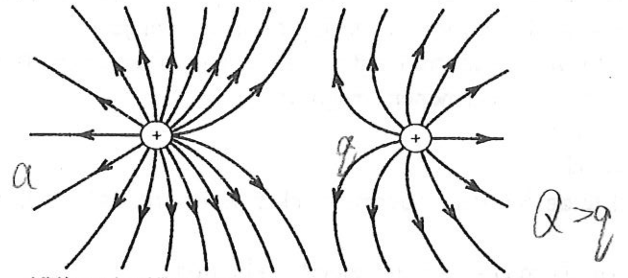
(a) Field due to single charge



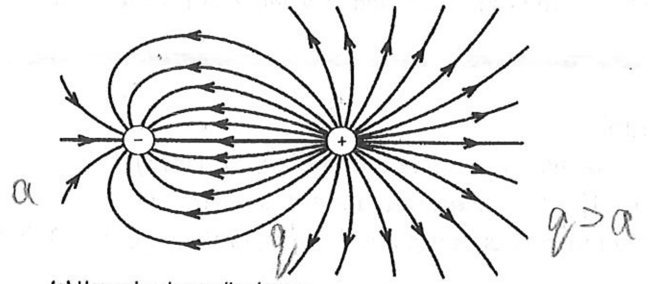
(b) Equal and opposite charges



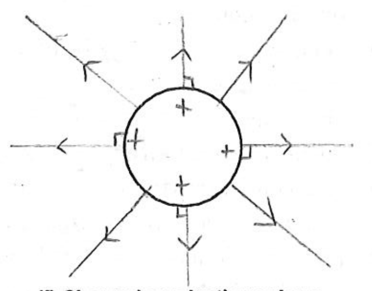
(c) Equal and like charges



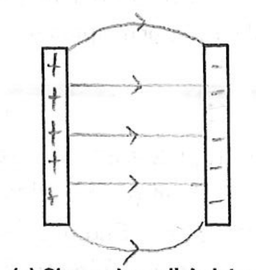
(d) Unequal and like charges



(e) Unequal and opposite charges



(f) Charged conducting sphere

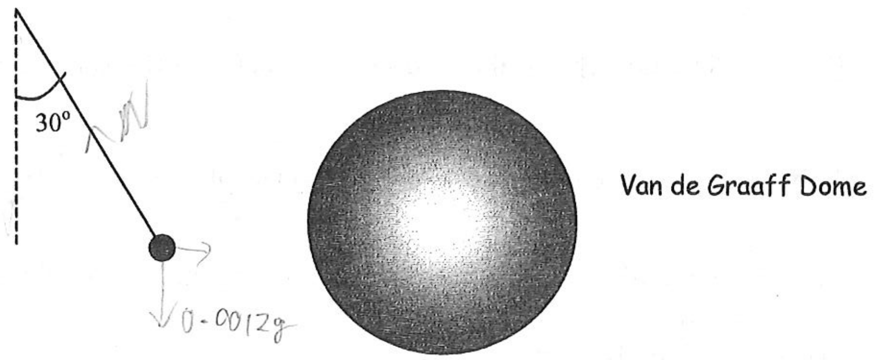


(g) Charged parallel plates

- What are some points to note when drawing electric field lines?

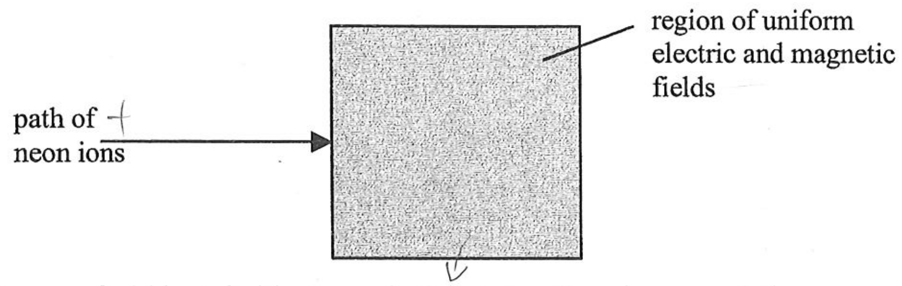
v) Calculate the force exerted on an electron when it is 0.30 m from the centre of the dome.

c) A small charged ball of mass 1.2 g hangs on a string close to the dome of a Van de Graaff generator as shown below. The string is pulled to 30o from the vertical by the electrical attraction on the ball.



i) Draw a free body diagram showing all the forces acting on the ball.
ii) Draw a vector triangle showing that the forces are in equilibrium.
iii) From your diagrams, work out the electric force on the ball.
iv) Supposing that the field strength in that position is 4.0×10^5 NC^{-1}. Calculate the charge on the ball.

A2. Singly charge neon ions are accelerated from rest in a vacuum through a potential difference of 1400 V. They are then injected into a region of space where there are uniform electric and magntic fields acting at right angles to the original direction of the motion of the ions.



(a) The electric field has field strength E and the flux density of the magnetic field is B.

(i) Copy the diagram and indicate clearly the directions of the electric and magnetic fields so that the ions pass through the region undeflected.

(ii) Calculate the speed of the accelerated ions on entry into the region of the electric and magnetic fields.

(iii) The electric field strength E is 6.2×10^3 Vm^{-1}. Calculate the magnitude of the magnetic flux density so that the ions are not deflected in the regions of the field.

(b) State and explain what would happen to the path of the ions if

(i) the magnetic field only is switched off.

(ii) the electric field only is switched off.

(iii) the charged particle is projected at an angle that is not perpendicular to the B-field. (no E field)

(iv) the direction of E field is adjusted to point in the same direction as the B field.

(c) If the neon ions become doubly rather than singly charged, state what change occurs in

(i) the speed of the ions entering into the regions of the electric and magnetic fields in (a).

(ii) the path of the ions in the cross field.

P.C Mole
Cell str
Atom MA
Transition
Homeostasis

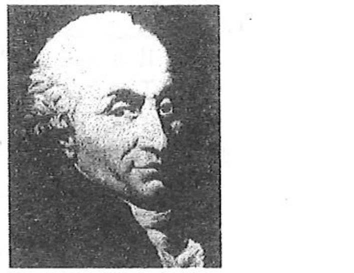
Easy
Electro
Organic

Analysis
Power
ODE
Integration
No. method
Func
Complex no
Trigo

Ideal gas
Osc
Wave
Super
Capac
Thermal pro
Quantum
C & F
Nuclear
Phy of fluid

W5 Ideal gas
thermal prop
Capacitance
W7 Fluids
Fields (all 3)
W9 Quantum physics

Charles Coulomb (1736 - 1806)
Coulomb's major contribution to science was in the field of electrostatics and magnetism.



Charge and Field

Assessment Objectives:

When you have completed work on this unit, you should be able to:

a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength as force per unit positive charge.
b) represent an electric field by means of field lines.
c) recall and use Coulomb's law in the form $F = Q_1Q_2/4\pi\varepsilon_o r^2$ for the force between two point charges in free space or air.
d) recall and use $E = Q/4\pi\varepsilon_o r^2$ for the field strength of a point charge in free space or air.
e) calculate the field strength of the uniform field between charged parallel plates in terms of potential difference and separation.
f) calculate the forces on charges in uniform electric fields.
g) describe the effect of a uniform electric field on the motion of charged particles.
h) define potential at a point in terms of the work done in bringing unit positive charge from infinity to the point.
i) state that the field strength of the field at a point is numerically equal to the potential gradient at that point.
j) use the equation $V = Q/4\pi\varepsilon_o r$ for the potential in the field of a point charge.
k) recognise the analogy between certain qualitative and quantitative aspects of electric and gravitational fields.
l) show an understanding of the main principles of determination of e by Millikan's experiment
m) summarise and interpret the experimental evidence for quantisation of charge.
n) show an understanding of the concept of a magnetic field as an example of a field of force produced either by current-carrying conductors or by permanent magnets.
o) represent a magnetic field by field lines.
p) calculate magnetic force on a moving charge
q) describe and analyse qualitatively the deflection of beams of charged particles by uniform electric and uniform magnetic fields.
r) explain how electric and magnetic fields can be used in velocity selection.
s) explain the principles of one method for the determination of v and e/m for electrons.

Previous topics essential for this topic:

1. Gravitation (Comparison between gravitational field and electric field)
2. Vectors and Scalars (Addition of field strengths and electric potentials)
3. Dynamics and Kinematics (Motion of a charged particle in an uniform E and B fields)

1) derive $v = u + at$.
By definition: $a = \frac{dv}{dt}$
Integrate both sides wrt time
$\int a\, dt = \int dv$
If a is constant, $at + u = v$

2) Eqn is valid only if a is constant (direction and magnitude must be a constant)