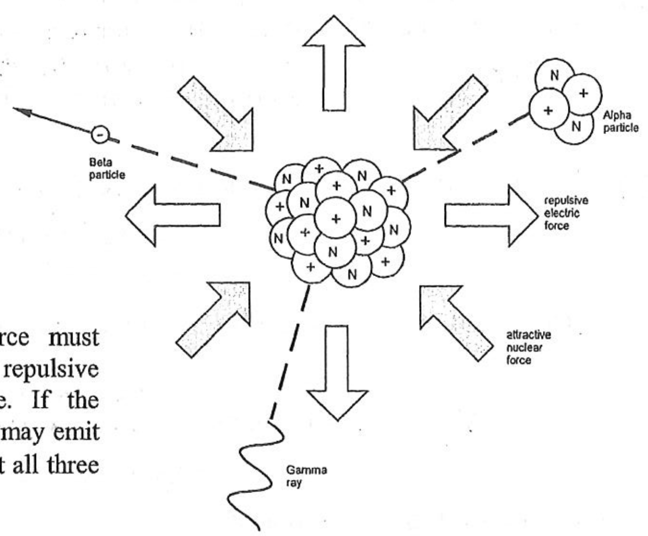
5. Radioactivity

Radioactivity is a spontaneous nuclear transformation with the emission of energy.

5.1 Nuclear Decay

- A process whereby an unstable 'parent' nucleus undergoes **spontaneous** disintegration (to form a stable 'daughter' nucleus or stable 'daughter' nuclei) is called **nuclear or radioactive decay**.
- Nuclear decay is a nuclear process. Nucleon number, proton number, mass-energy and linear momentum are all Conserved.
- The binding energy per nucleon in a nucleus gives an indication of the stability of the nucleus. Nuclear decay is a process by which unstable nuclides form more stable ones.
- A nucleus can decay if there are possible products with **lower** total mass that can be reached by any of the 3 types of decay process. Since mass-energy is conserved, energy must be emitted and the products of the decay are in a more stable state than the unstable parent nucleus.
- Such a decay process may result in an emission of any one of the following:
  (a) α-particles
  (b) β-particles
  (c) γ-rays



The attractive nuclear strong force must overcome the enormous electric repulsive force to keep the nucleus stable. If the nucleus is unstable (radioactive), it may emit α- or β-particles, or γ-rays. (But not all three together.)

5.2 Nature of Nuclear Decay: Spontaneous and Random

- It is ***spontaneous*** *means that it is not triggered by* external factors. In fact, the process is unaffected by extremes of temperature, pressure, electric fields, magnetic fields, luminous intensity, and so on. Nor is it affected by chemical combination. This is because it is a nuclear change not an atomic one.
- The decay process of unstable nuclei is entirely **random**. *It is impossible to say exactly* which *nucleus or exactly* when *a particular nucleus will disintegrate*, although statistically, it is possible to predict the fraction of nuclei of a given pure radioactive element that will decay after a certain length of time provided its decay characteristics are known.
- Demonstrations of the random nature of radioactive decay may be carried out with a Geiger-Muller tube connected to a *ratemeter* and a *loudspeaker*. The ratemeter measures the rate of arrival of particles on a micro-ammeter calibrated in counts per second, whereas the loudspeaker indicates the particle arrivals with a series of clicks. A radioactive source placed in front of the G-M tube gives a **randomly varying reading** on the ratemeter and a *series of clicks at random time intervals* on the loudspeaker.

5.3 Types of Radiation

The distinguishing factors between the three types of radiation will now be considered. However, the terms 'ionising' and 'range' must first be understood.

Ionising: Radiation passing through matter can cause its atoms to give up an electron resulting in positively charged (or ionised) particles. Since the alpha and beta particles and gamma rays can do this, they are also refered to as ionising radiation.

Range: The distance the ionising emission travels through a material before it has lost nearly all its energy. The energy is lost in the process of ionisation.

Notice the **more** ionising the radiation, the more energy will be lost and thus the **shorter** the range and vice versa.

Summary of properties of nuclear radiations :-

Properties	α	β	γ
Nature	Helium Nucleus	Fast Electron	Electromagnetic Radiation
Charge	$+3.2 \times 10^{-19}C$	$-1.6\times10^{-19}C$	0
Rest mass	$6.7 \times 10^{-27}kg$	$9.1 \times 10^{-31}kg$	0
Speed	0.1 c	0.9 c	c
Effect of E field	Deflected*	Deflected*	Undeflected
Effect of B field	Deflected*	Deflected*	Undeflected
Penetrating properties	Stopped by $\approx$ 0.1 mm Al $\approx$ 6 cm of air	stopped by $\approx$ a few mm of Al	stopped by $\approx$ several cm of Pb
Ionising effect	A great deal	some	very little

* Provided the field is not parallel to the initial velocity of the particle

Notes :

→ α-particles are emitted with definite kinetic energies hence have certain definite speeds. The greater the speed, the greater the energy.

→ β-particles, on the other hand, are emitted with variable kinetic energies hence have variable speeds reaching up to 0.995c.

5.4 Natural Transmutation

When an α- or a β-particle is emitted from a nucleus, a ___different___ element is formed. The changing of one chemical element to another is called **transmutation.**

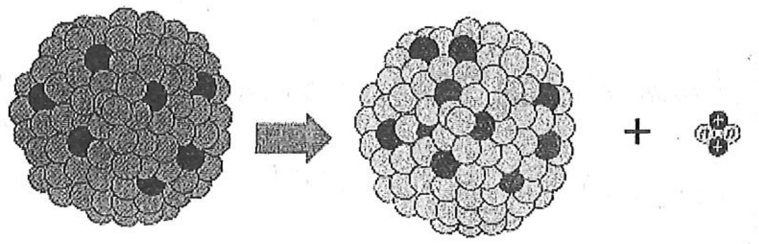
α-decay

$$^{A}_{Z}X \longrightarrow \ ^{A-4}_{Z-2}Y + \alpha$$

(Since the number of positive charges on the nucleus determines the number of electrons circling the nucleus which in turn determines the chemical nature of the atom, it follows that a new chemical element is formed after the emission of an α-particle.)

E.g. 5.4a When an α-particle is ejected from uranium, thorium is formed.

$$^{238}_{92}U \longrightarrow \ ^{234}_{90}Th + \ ^{4}_{2}He$$



^{238}U ^{234}Th ^{4}He

When this happens, energy is released, partly in the form of γ-radiation, partly in the kinetic energy of the α-particle and partly in the kinetic energy of the thorium nucleus.

β-decay*

A neutron can undergo transmutation in a nucleus by changing to a proton, which remains in the nucleus, and into an electron, which is ejected as a β-particle. Observe that charge and mass are conserved!

$$^{1}_{0}n \longrightarrow \ ^{1}_{1}p + \ ^{0}_{-1}e$$

$$X \longrightarrow \ ^{A}_{Z+1}Y + \beta\,(^{0}_{-1}e)$$

E.g. 5.4b Thorium, which is radioactive, decays by emitting a β-particle to form a new element, protactinium.

$$^{234}_{90}Th \longrightarrow \ ^{234}_{91}Pa + \beta\,(^{0}_{-1}e)$$

From E.g. 5.4a and 5.4b, the process will continue until radioactive $^{238}_{92}U$ decays to $^{206}_{82}Pb$.

When a radioactive atom disintegrates, the newly formed atom may still be radioactive and disintegrate by ejection of an α or a β particle to become another different element. This transformation continues until a stable element is formed.

E.g. 5.4c $^{226}_{88}Ra \xrightarrow{\alpha} \ ^{222}_{86}Rn \xrightarrow{\alpha} \ ^{218}_{84}Po \longrightarrow \ .... \longrightarrow \ ^{206}_{82}Pb$

- In practice, it is not possible to make atoms by taking protons, electrons and neutrons and forming them into atoms. The fact that there is a very strong ___repulsion___ between protons makes union between them and the required number of neutrons impossible at ___ordinary___ temperatures. What is possible is a ___rearrangement___ of the protons, electrons and neutrons in a particular nuclide to form different nuclides.

Worked example 8

(a) Explain why very high temperatures are required for nuclear fusion.

(b) A future fusion reactor might use the reaction

$$^{2}_{1}H + \ ^{2}_{1}H \rightarrow \ ^{4}_{2}He + \text{energy}$$

to produce useful energy. Calculate the number of reactions required to produce 1 J of energy.

(c) Calculate the mass of $^{2}_{1}H$ required to provide 1 J of energy.

(Mass of $^{2}_{1}H$ = 2.0136 *u*; mass of $^{4}_{2}He$ = 4.0015 *u*.)

(a) For nuclear fusion to take place, the hydrogen atoms must be brought close enough in order for them to fuse. This means that energy is required to overcome the ___very large___ Coulombic repulsion. At very high temperatures, the energy acquired is ___high___ enough to accomplish this.

(b) Energy produced by 1 reaction = $(2 \times 2.0136 - 4.0015)uc^2$

$= 0.0257 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2$

$= 3.84 \times 10^{-12}$ J

∴No. of reactions required to produce 1 J of energy = $\frac{1}{3.84} \times 10^{-12}$ = $\underline{2.60 \times 10^{11}}$

(c) Mass of $^{2}_{1}H$ required = $\frac{2 \times 2.60 \times 10^{-11}}{6.02 \times 10^{23}} \times 0.002$

= ~~8.65 × 10^-16 kg~~ 1.73×10^{-15} kg

Summary (Nuclear Processes)

- All nuclear reactions obey the Conservation of ___Proton Number___, Nucleon Number, ___Mass-Energy___ and ___Linear Momentum___.
- Number of ___protons___ and number of ___neutrons___ need not be conserved, as a neutron can undergo transmutation to form a proton and an electron (details under "Radioactivity"). Mass or energy ___alone___ needs not be conserved, because any change in the total mass can be compensated by a total change in energy.
- For a nuclear reaction to be spontaneous, the total rest mass energy (using $E = mc^2$) and KE of the *reactants* must be ___greater___ than the total rest mass energy of the *products*. The excess energy in a spontaneous reaction will either be in the form the ___KE___ of the products or energy of any ___radiations___ emitted.

Worked example 7

An α-particle of energy 1.23×10^{-12} J is incident on a stationary nitrogen nucleus. The following reaction is expected:

$$\alpha \quad + \quad {}^{14}_{7}\mathrm{N} \quad \rightarrow \quad {}^{17}_{8}\mathrm{O} \quad + \quad {}^{1}_{1}\mathrm{p}$$

$$4.002603\,u \qquad 14.003074\,u \qquad 16.999131\,u \qquad 1.007825\,u$$

Can this reaction take place? What is the total energy of the product?

LHS : $m_\alpha c^2 + m_N c^2 + E_\alpha$ $= (4.002603 + 14.003074)uc^2 + E_\alpha$
$= 18.005677uc^2 + 1.23 \times 10^{-12}$ J

RHS : $m_o c^2 + m_p c^2$ $= (16.999131 + 1.007825)uc^2$
$= 18.006956uc^2$ J

For the above reaction to take place,

$$m_\alpha c^2 + m_N c^2 + E_\alpha > m_o c^2 + m_p c^2$$

$\Delta mc^2 = (18.006956 - 18.005677)uc^2$
$= 1.91 \times 10^{-13}$ J

∴ Total energy of the product $= 1.23 \times 10^{-12} - 1.91 \times 10^{-13} = 1.04 \times 10^{-12}$ J

released

4.3* Fission And Fusion

Fission	Fusion
Fission refers to the disintegration of heavier unstable nuclides to lighter, more stable nuclides, releasing energy in the process.	Fusion refers to the combination of lighter nuclides to form heavier, more stable nuclides, releasing energy in the process.
For example, $^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{88}_{38}\mathrm{Sr} + {}^{136}_{54}\mathrm{Xe} + 12{}^{1}_{0}\mathrm{n} + \text{energy}$ or $^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3{}^{1}_{0}\mathrm{n} + \text{energy}$	For example, ${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + \text{energy}$

- The energy released in fusion and fission is usually in the form of kinetic energies of products which ultimately manifests as thermal energy.
- When we compare the total mass of the reactants and that of the products, we find that the mass of the products will be less than that of the reactants
- The mass used in calculation is often called the rest mass, i.e., the mass of the nucleus when it is at rest. This is to distinguish from the actual **inertial mass** which increases as a particle gains kinetic energy (consequence of Einstein's special relativity).
- This energy, as we have discussed in the previous section, can alternatively be seen to be due to the change in binding energy of the nuclides, which can be found from the mass defect.

Practical Considerations

- In nuclear fusion, light nuclei collide at very high speed and fuse together to release binding energy; the initial kinetic energy is necessary to overcome the electrostatic repulsion of the nuclei.

Worked Example 9

A $^{238}_{92}\mathrm{U}$ nucleus, originally at rest, spontaneously decays to form a thorium (Th) nucleus and an α-particle. No γ-ray is emitted.

(a) Write down the equation for this disintegration.

(b) The α-particle produced in this disintegration travelled 25 mm in a cloud chamber. Given that, on average, an α-particle creates 5.0×10^3 ion pairs per mm of track in the cloud chamber, and that the energy required to produce an ion pair is 5.2×10^{-18} J, find the kinetic energy with which the α-particle was emitted. Assume mass of thorium to be 234.1165u kg and mass of α-particle to be 4.0026u kg.

(c) Hence deduce the initial velocities of the α-particle and the thorium nucleus.

a) $^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$

b) KE of an α particle $= (5.2 \times 10^{-18})(5.0 \times 10^3)(25)$
$= 6.50 \times 10^{-13}$ J

c) $KE_\alpha = \frac{1}{2} m_\alpha v_\alpha^2$, $m_\alpha = 4.0026u$ kg
$v_\alpha = 1.399 \times 10^7\ \mathrm{m^{-1}}$

By conservation of linear momentum,

$$m_\alpha v_\alpha = m_{Th} v_{Th}$$

$$v_{Th} = \frac{m_\alpha v_\alpha}{m_{Th}}$$

$$= \frac{(4.0026)(1.399 \times 10^7)}{234.1165}$$

$$= 2.39 \times 10^5\ \mathrm{ms^{-1}}$$

5.5 Radioactive Decay Law

The **activity** of a radioactive material is the number of disintegrations of its atoms per unit time, i.e.

$$A = -\frac{dN}{dt}$$

where A is the activity, and N is the number of atoms present. The negative sign indicates that the number of atoms of the radioactive material is decreasing.

Activity can be measured in terms of the **becquerel** (Bq) and the **curie** (C).

1 Bq = 1 disintegration per second

1 C = 3.70×10^{10} disintegrations per second = 3.70×10^{10} Bq

The rate of decay follows the laws of chance. It is proportional to the number of atoms of the original kind present at a certain instant, i.e.,

$$-\frac{dN}{dt} \propto N$$

$$\boxed{-\frac{dN}{dt} = \lambda N}$$

where λ is called the **decay constant** or **disintegration constant**. probability per unit time for particle to decay

Thus

$$\boxed{A = \lambda N}$$

The final number of atoms N after time t is related to the original number of atoms N_0 (at time t= 0) as follows:

$$\boxed{N = N_0 e^{-\lambda t}}$$

The exponential equation given above only tells us probably how many unchanged atoms are left after a certain time span. At the end of the time span the number of unchanged atoms left may actually be slightly more or less than the value predicted by the equation. This is because of the random nature of radioactivity.

The activity A of the sample is proportional to N, since $A = \lambda N$.

The count rate c, received by a radioactive detector, is proportional to A, hence, c is also proportional to N.
(When time $t = 0$, $N = N_0$, $A = A_0$, and $c = c_0$.)

What is the difference between activity, A and count rate, c?

Count rate refers to the amount of radiations detected per unit time. It is normally less than the activity since most detectors will not be able to detect all the emitted radiations.

Thus, we have

$$\boxed{\begin{aligned} &N = N_0 e^{-\lambda t} \text{ and } A = \lambda N \\ &\therefore A = A_0 e^{-\lambda t} \\ &\quad C = C_0 e^{-\lambda t} \end{aligned}}$$

Worked example 6
A stationary radium nucleus emits an α-particle spontaneously.

$$\begin{array}{ccccc} {}^{226}_{88}\text{Ra} & \rightarrow & {}^{222}_{86}\text{Rn} & + & {}^{4}_{2}\text{He} \\ 226.0254\ u & & 222.0176\ u & & 4.0026\ u \end{array}$$

(i) Determine the mass difference. How can we account for the difference in the total mass of the reactants with that of the products?

(ii) Find the velocities of the products.

(i) Mass Difference = $(226.0254 - 222.0176 - 4.0026)u = 5.2 \times 10^{-3} u$ kg
The **decrease** in mass can be accounted for by the energy that is released in the reaction in the form of the kinetic energy of the products.

$$m_{Ra}c^2 = m_{Rn}c^2 + m_\alpha c^2 + E$$

$$\Rightarrow E = (226.0254 - 222.0176 - 4.0026)uc^2$$
$$= 7.77 \times 10^{-13}\ \text{J}$$

(ii) By conservation of linear momentum,

$$0 = P_{Rn} + P_\alpha \Rightarrow \frac{V_\alpha}{V_{Rn}} = \frac{m_{Rn}}{m_\alpha} = \frac{222}{4} \qquad V_{Rn} = V_\alpha \frac{4}{222}$$

Now, by conservation of energy,

$$\frac{1}{2} m_\alpha V_\alpha^2 + \frac{1}{2} m_{Rn} V_{Rn}^2 = 7.77 \times 10^{-13}\ J$$

$$\Rightarrow \frac{1}{2}u\left(4v_\alpha^{\ 2} + 222 \cdot \frac{4^2}{222^2} v_\alpha^{\ 2}\right) = 7.77 \times 10^{-13}\ \text{J}$$

$\Rightarrow$ $v_\alpha = 1.52 \times 10^7\ \text{ms}^{-1}$
and $v_{Rn} = 2.73 \times 10^5\ \text{ms}^{-1}$ *in opposite directions to each other*

4.2 Determining whether a nuclear reaction is spontaneous

When reactants have NO initial KE, how can we determine if a nuclear reaction can occur spontaneously and calculate the energy released?

- Compare the total rest mass of the reactants and products. If that of the products is lower, the reaction can occur spontaneously
- To calculate energy released in a single reaction, we use E = mc^2 where m is the mass difference. The rest mass of the system can be converted to E_k of the products and/or energy of the emitted radiation.

Is it then possible for reactions where the total rest mass of the reactants is less than the products to occur? Yes, if the reactant particles have **initial KE**. In such a reaction, we have to take this KE into account in order to determine if a reaction can occur.

When reactants have initial KE, how can we determine if a nuclear reaction can occur and calculate the KE of the products?

- Compare the total rest mass energy (using E = mc^2) and KE of the reactants with the total rest mass energy of the *products*. If that of the products is lower, the reaction can occur spontaneously.
- To calculate the energy released in a single reaction, we simply take the difference in the values we compared above.

4 Nuclear Processes

Nuclei may disintegrate or collide with each other to form new nuclei. Such reactions can be represented by nuclear equations. For example,

$$^{14}_{7}\mathrm{N} + ^{4}_{2}\mathrm{He} \rightarrow ^{17}_{8}\mathrm{O} + ^{1}_{1}\mathrm{H}$$

When writing down a nuclear equation, bear in mind that a few physical quantities must be conserved:

1. **Proton Number Conservation** : In the absence of other positive particles, proton number must be conserved due to the conservation of charge.
2. **Nucleon Number Conservation** : Total number of nucleons before the reaction should be the same as the total number of nucleons after the reaction.
   Note :- Nucleon number and proton number conservation **do not imply** that number of protons or number of neutrons is also necessarily conserved (Refer to the process of β-decay discussed in Worked Example 4).
   - Electrons are assigned atomic mass of 0 and atomic number of -1.
3. **Mass-Energy Conservation** : Although this is not reflected in the equation, it is used for calculation.
4. **Linear Momentum Conservation** : This applies to the parent, daughter nuclides and other particles liberated during nuclear reactions, notably β-particles or electrons, α-particles, photons, neutrinos and antineutrinos. (See Example 6)

Worked Example 4

When ^{14}N is bombarded with neutrons, one of the product is ^{1}H. What is the other product?

$$^{14}_{7}N + ^{1}_{0}n \rightarrow ^{1}_{1}H + ^{14}_{6}C$$

This product undergoes β-decay to release an electron from the nucleus. Write down the equation for this reaction.

$$^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e$$

Worked Example 5

^{4}He bombards an unknown nucleus to produce $^{30}_{15}$P and a neutron. What is the unknown element?

$$^{4}_{2}He + ^{A}_{Z}X \rightarrow ^{30}_{15}P + ^{1}_{0}n$$

$4 + A = 30 + 1$

$A = 27$

$Z + 2 = 15$

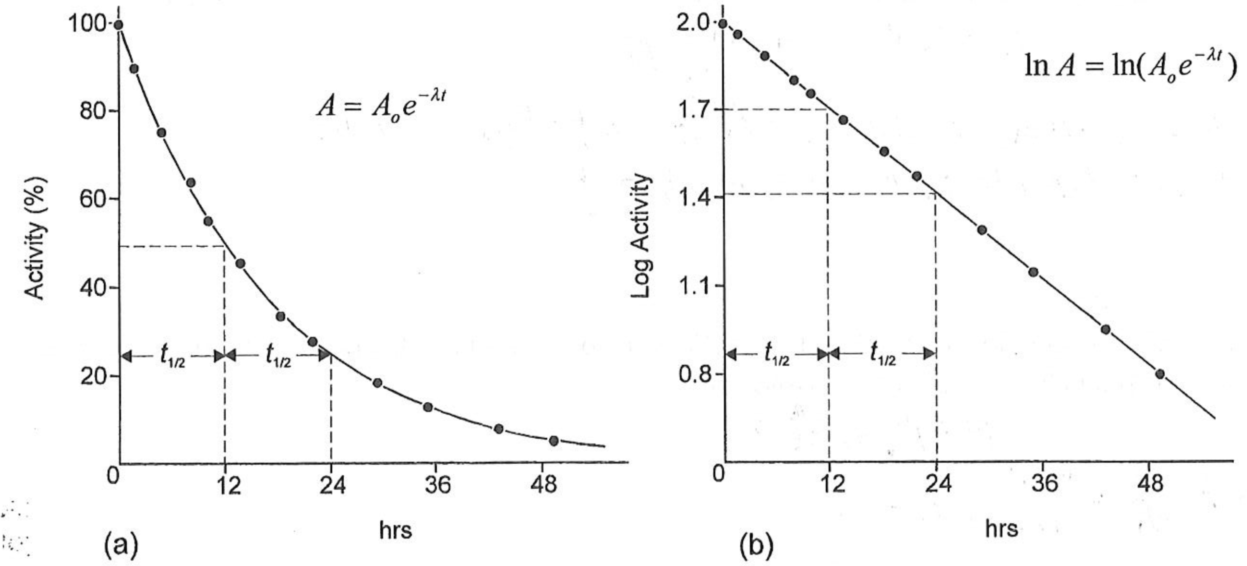
$Z = 13$

$\therefore\ ^{A}_{Z}X = ^{27}_{13}Al$

4.1 Mass Difference and Energy considerations

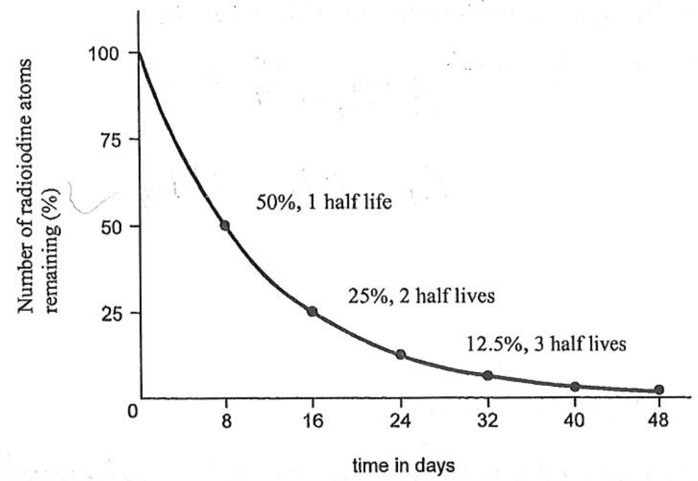
In **Newtonian Mechanics**, mass is strictly conserved and energy is also strictly conserved. However, in **nuclear reactions**, it is mass-energy that is conserved. As a result, the mass of the system in a nuclear reaction can increase or decrease as can the KE. The meaning of this is best illustrated with an example.

Activity in a sample containing ^{42}K measured as a function of time (a) linear, (b) semi-logarithmic scale.



Half life, $t_{\frac{1}{2}}$, is the time taken for half the number of radioactive nuclei in any given sample of a given isotope to decay.

Radioactive decay curve for iodine-131 (radioiodine).



The decay constant is related to the half life by

$$\boxed{\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}}$$

Can you prove the above relation?

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

$$-\ln\left(\frac{1}{2}\right) = \lambda t_{\frac{1}{2}}$$

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$$

Worked Example 10

A student stated that 'radioactive materials with a short half-life always have a high activity. Discuss whether the student's statement is valid. [N94/II/6]

$A = \lambda N \qquad A = \frac{\ln 2}{t_{\frac{1}{2}}} N$

- From the equation, A will be high if half-life is small
- But A also depends on N, the number of

Worked Example 11

Six hours after a sample of the β-emitter $^{24}_{11}Na$ has been prepared, only 6.25% of it remains undecayed. What is the half-life of this isotope?

$N = N_0 e^{-\frac{\ln 2}{t_{\frac{1}{2}}} t}$

Given that $\frac{N}{N_0} = \frac{6.125}{100}$, $t = 6h$

$t_{\frac{1}{2}} = 1.5h$

Worked Example 12

A radioactive source has a half-life of 1.80×10^5 s. At time $t = 0$, its activity is A_0. After a time of 6.05×10^5 s, its activity drops to 3.90×10^3 Bq.

What is the value of A_0?

$A = A_0 e^{-\frac{\ln 2}{t_{\frac{1}{2}}} t}$

$A_0 = 4.01 \times 10^4$ Bq

Summary (Radioactivity)

Spontaneous nature of nuclear decay: Not <u>triggered</u> by external factors and not <u>affected</u> by physical conditions or chemical combination.

Random nature of nuclear decay: Impossible to say exactly <u>which</u> nucleus or exactly <u>when</u> a particular nucleus will disintegrate.

Nature of α,β and γ radiations: Refer to table on page 14

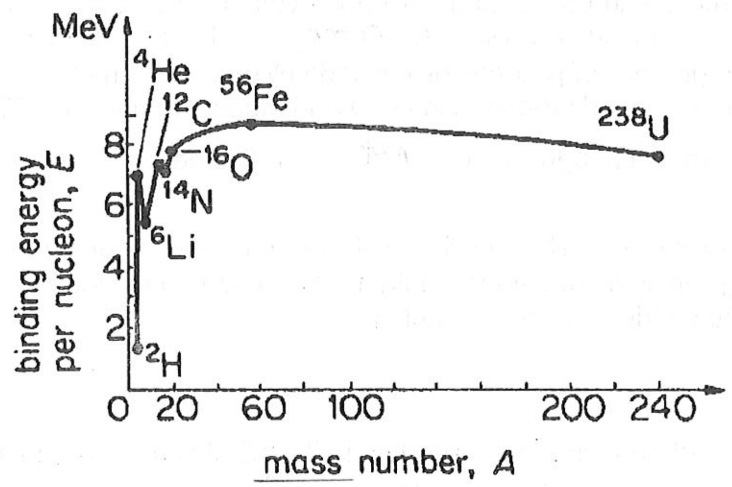
Equations: $A = \lambda N$; $A = A_0 e^{-\lambda t}$; $N = N_0 e^{-\lambda t}$; $c = C_0 e^{-\lambda t}$; $\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$

Activity is the rate of <u>disintegration</u> of radioactive nuclei, while count rate is the rate at which radiations are <u>detected</u>.

3.5 Binding Energy Per Nucleon and the Stability of a Nucleus

- The binding energy provides us with an idea of how stable the nucleus is. The higher the binding energy, the <u>more</u> energy is required to separate all nucleons from the nucleus. However this is not an accurate means of comparison between different nuclei as they have different number of nucleons. A better measure is the **binding energy per nucleons**, which is the ratio of the binding energy of a nucleus to the number of nucleons in that nucleus.
- We can think of binding energy per nucleon as the average energy needed to separate a nucleus into its individual nucleons. Thus, a nucleus which has high binding energy **per nucleon** is very <u>stable</u>.

The figure shows the variation of the binding energy per nucleon among the elements:



Know how to draw

Notable features:

- The graph shows the **relative stability** of the nuclides. A nuclide is stable if it is not <u>energetically</u> feasible for it to be changed into another nuclide.
- ^{56}Fe has the highest binding energy per nucleon compared to all the other nuclides and is one of the <u>most</u> stable nuclide. One of the reasons why there is so much iron in the universe is that the iron atom is the atom with the greatest mass defect.
- The rising part of the curve shows that elements with low mass number can combine to produce more stable, heavier nuclide, releasing energy in the process. This process is known as <u>fusion</u>. This process is the source of energy of the Sun and other stars. (Refer to page 11)
- Similarly the other part of the graph shows that heavier nuclides tend to disintegrate into lighter, more stable nuclides. This process is known as <u>fission</u>. It is the source of energy of all nuclear power stations. (Refer to page 11)
- Nuclides with big number (>200) are unstable, they may decay <u>spontaneously</u>. These nuclides are said to be <u>radioactive</u>.

Summary (Mass Defect and Nuclear Binding Energy)

Mass-Energy relationship	Binding Energy	Binding Energy per Nucleon
$E = \Delta mc^2$	Amount of work needed to take all the constituent nucleons apart so that they are separated an infinite distance from one another.	Total binding energy of nucleus divided by the total number of nucleons. A nucleus with a higher binding energy per nucleon is <u>more stable</u>.

Binding energy of a nucleus:

Amount of work needed to take all its constituent nucleons apart so that they are separated an infinite distance from one another.

That is, Energy required to hold nucleons together

Mass of nucleus × c^2 + Binding Energy = Mass of uncombined nucleons × c^2

Symbolically,

$m_{nucleus}\, c^2$ + Binding Energy $= [\, Z \times m_p + (A - Z) \times m_n]\, c^2$

Rearranging gives,

Binding Energy $= [\, Z \times m_p + (A - Z) \times m_n]\, c^2 - m_{nucleus}\, c^2$

= Mass Defect × c^2

Mass defect = Binding Energy/c^2

$\Delta m = \frac{E}{c^2}$

Hence mass defect is a measure of the binding energy.

Note:

1. Because typical nuclear binding energies are usually of the order ~ 10^{-11} – 10^{-12} J, it is more convenient to express them in units of MeV as its order of magnitude is much larger in MeV.

   Energy equivalent of a mass **1 *u*** = uc^2 $= 1.66 \times 10^{-27} \times (3 \times 10^8)^2$ J ÷ $(10^6 \times 1.6 \times 10^{-19})$

   = 934 MeV

2. **Binding energies is not an energy that resides in the nucleus. Rather, it is a difference in mass energy between a nucleus and its individual nucleons.**

Worked Example 3

Find the binding energy of **(i)** the helium nucleus; **(ii)** the carbon-12 nucleus.

[Mass of helium = 4.0015 *u*, mass of proton = 1.0073 *u*, mass of neutron = 1.0087 *u*]

$2(1.0087u)c^2 + 2(1.0073u)c^2 = (4.0015u)c^2 + B.E$

$B.E = (-4.0015 + 2.0174 + 2.0146) \times 934\,MeV$

$= 28.5\ MeV$ or $4.56 \times 10^{-12}\ J$

$B.E = [(6)(1.0073) + (6)(1.0087)] - 12.000 \times 934\ MeV$

$= 99.7\ MeV$ or

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Self Attempt

S1 (a) If the mass of a deuterium atom, ${}^2_1H$, is 2.008032u, calculate the mass defect.

(b) Hence calculate the binding energy per nucleon for deuterium in J and MeV. (Use the constants provided in the notes)
[1.34×10^{-29}kg, 0.605×10^{-12} J]

S2 What is the binding energy per nucleon for ${}^{120}Sn$?

Atomic mass of Hydrogen	=	1.007825u
Mass of neutron	=	1.008665u
Mass of Sn	=	119.902199u

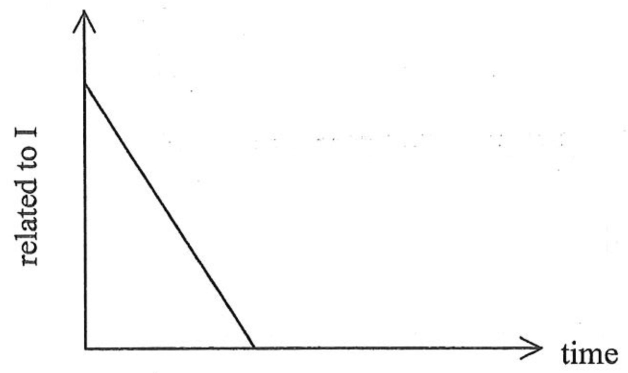
S3 One reaction which might be used for controlled nuclear fission is shown.

$${}^7_3Li + {}^2_1H \rightarrow 2({}^4_2He) + X$$

What is the particle X?

S4 The half-life of ${}^{238}_{92}U$ is 1.42×10^9 years. What mass of this nuclide would give an emission of one α-particle per second?
[2.56×10^{-5} g]

S5 An ionisation chamber is a sealed box where all charge produced is detected. It contains a short-lived radioactive gas. The current created is monitored and recorded as the gas decays. A straight-line graph is drawn relating the current and the time, as shown in figure below.



a. From what you know about the way the activity of a radioactive material varies with time, write an equation describing the behaviour of the current with time.
b. What must have been plotted on the vertical axis to produce this graph?
c. What can you find out from the intercept on this axis?
d. What can you find from the gradient?
Justify each of your answers.

S6 Radioactive decay is a **spontaneous** and **random** process. Explain the meaning of the terms in bold.
At a certain instant, a piece of radioactive material contains 10^{12} atoms. The half life of the material is 30 days. What is meant by **half-life** of the material?
(i) What is the decay constant?
(ii) Calculate the number of disintegrations per second.
(iii) How long will elapse before 10^4 atoms remain?
(iv) What is the activity at this time?
[2.67×10^{-7} s^{-1}, 2.67×10^5 Bq, 6.89×10^7 s, 2.67×10^{-3} Bq]

The Nuclear Atom

D1 (a) An alpha-particle travels from a great distance directly towards a gold ($^{197}_{79}Au$) nucleus, which can be assumed to remain stationary throughout the interaction. The alpha-particle returns along the same path without penetrating the nucleus. If x is the separation of the alpha-particle and the nucleus, draw on the same axis labelled graphs showing for the alpha-particle

(i) the electrostatic potential energy V,

(ii) the kinetic energy T.

Mark on the x-axis the distance x_0 of closest approach of the particles. State the relation between the two graphs. *N86/II/12*

(b) The figure below shows the path of an alpha particle approaching the gold nucleus head on.

At a great distance from the gold $^{197}_{79}Au$ nucleus, the alpha particle has kinetic energy 4.8 MeV.

The distance of closest approach of the alpha particle to the gold nucleus is d.

(i) Name the form of energy into which the kinetic energy of the alpha particle has been converted when it is at a distance of closest approach.

(ii) State the value in joules, of 1.0 MeV of energy

(iii) Deduce a value for the distance d of closest approach. [6]

(c) With reference to your answer to (b) (iii), comment on the radius of the gold nucleus. [1]

[1.6×10^{-13} J, 4.74×10^{-14}m] *N01/II/8*

Mass Defect and Nuclear Binding Energy

D2 Explain what is meant by the *nuclear binding energy* of a nucleus.

The conventional notation for a certain nuclide is $^{81}_{35}Br$.

(a) The mass of the nucleus is 80.8971 u. Taking the proton and neutron masses to be 1.0073 u and 1.0087 u respectively, deduce the binding energy per nucleon. [1.40×10^{-12}J]

(b) Sketch a labelled graph showing how the binding energy per nucleon varies with mass number, indicating the approximate position of $^{81}_{35}Br$ on the curve.

(c) Explain the implications for the stability of a given nuclide of a low value of the binding energy per nucleon.

(d) Account also for why fusion of nuclei having high mass numbers is not associated with a release of energy. *N85/II/11 (modified)*

- From the graph, fusion of 2 nuclius of ↑ mass result in nuclei with ↑ BE/neuclean.
- B.E reactant > BE product
- energy required to break reactant > energy released in form of product nuclei
- ∴ this does not lead to release of energy

3.3 Mass and Energy Relationship

- In his theory of special relativity, Einstein postulated the equivalence of mass and energy:

$$\boxed{E = mc^2}$$

Any change in the energy E of a body implies a corresponding change Δm in its inertial mass. The relation between E and Δm is given by

$$\boxed{E = \Delta mc^2}$$

- This equation shows that if a body emits energy in the form of radiation, its mass must decrease by an amount proportional to the energy of the emitted photon.
- Alternatively, if energy is released from nuclear reactions, the mass of the system must have decreased as well.
- Due to the factor of c^2, a small change in mass can result in a large amount of energy liberated. For most changes in energy experienced, the change in mass is usually too small to be measurable.

3.4 Making the Links! What is Binding Energy?

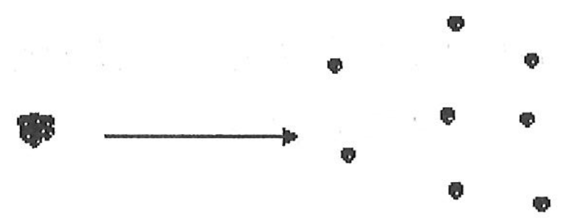
Energy is needed to break the nucleus into its constituent nucleons since there is a net attractive force holding them together.

The mass of the nucleus is less than the sum of the mass of its constituent nucleons.

Energy supplied to break down a nucleus into its constituent nucleons lead to an increase in mass. Energy and mass are equivalent. $E = \Delta mc^2$ In this case: E is binding energy Δm is mass defect

⇓

- Using Einstein's mass-energy relation, we can conclude that the mass of the nucleus must be **less** than the sum of masses of the uncombined nucleons (protons + neutrons) since **energy must be provided** to separate the nucleons. (Can take it as: The energy released when the nucleus is formed from its constituent nucleons, 'makes up' for the missing mass that accompanies the formation of the nucleus. We do not explain it this way because this does not occur actually.)



3. Mass Defect and Nuclear Binding Energy

3.1 What keeps the nucleons in the nucleus together?

What holds the protons and the neutrons in the nucleus together?

- Protons in the nucleus repel each other due to repulsive forces. These forces are long-range so that every proton in the nucleus experiences these forces from the others. Also known as Coulombic (electrostatic) forces, they cause instability in the nucleus.
- In order to keep the nucleus intact, there must be attractive forces capable of providing a **resultant attractive force** larger than the repulsive electrostatic force. This is provided by the short range nuclear *force*, where one nucleon can interact with its immediate neighbour only, i.e., between a pair of neutrons, a pair of protons, or a pair of proton and neutron.
- As a result of the net attractive forces within the nucleus,

energy **is needed to take apart a nucleus into its constituent nucleons.**

3.2 The Mystery of the Missing Mass of the Nucleus

We would expect that the mass of a nucleus would be the same as the mass of all its constituent nucleons if found separately. However, ...

Worked Example 2
Compare the mass of the helium nucleus with that of its constituent nucleons.
[Mass of helium = 4.0015 *u*, mass of proton = 1.0073 *u*, mass of neutron = 1.0087 *u*]

Soln:
Mass of Constituents of He = $2(1.0073u) + 2(1.0087u)$
$= 4.0320u$

This is greater than the mass of helium, 4.0015 u.
The missing mass = $0.0305u$

This missing mass is called the **mass defect. Where has the mass gone?**

Mass Defect

The difference between the sum of the **rest masses of the constituent nucleons** of a particular nucleus and the **mass of the nucleus** itself. It is a positive value.

Mass defect is the difference in mass that account for the energy released when a nucleus is formed from initially separated protons & neutrons

Nuclear Processes

D3 Given the following atomic masses:

^{238}U = 238.05079u
^{234}Th = 234.04363u
^{237}Pa = 237.05121u
^{4}He = 4.00260u
^{1}H = 1.00783

(a) Calculate the energy released during the alpha decay of ^{238}U.
(b) Show that ^{238}U cannot spontaneously emit a proton.

D4 The fusion of deuterium nuclei can be represented by the equation

$$^2_1H + ^2_1H \rightarrow ^4_2He$$

calculate the energy released by this reaction.

(a) 2 kg of deuterium are caused to fuse and it is proposed that the energy released by the fusion is used to generate electricity in power station. If the efficiency of the conversion process is 52 % and the output of the station is to be 5.0 MW, for how long would the fuel (deuterium) last?

(b) What are the difficulties associated with using fusion as a source of power?

[relative molecular mass of deuterium = 2, Nuclear Masses ^2_1H = 2.01419u, ^4_2He = 4.00277u, 1 u is equivalent to 1.49×10^{-10} J]
[3.82×10^{-12} J, 1.18×10^{8} s]

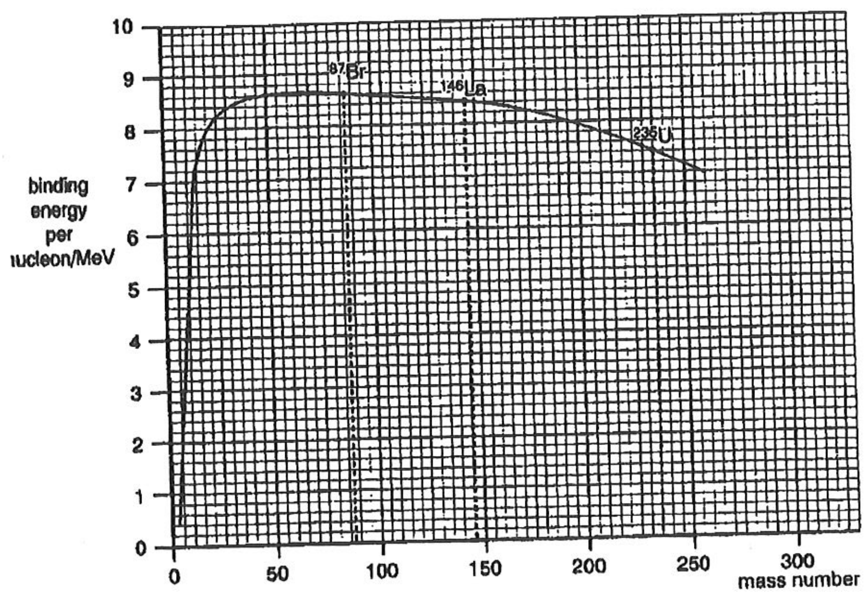
(c) Why does a release of energy occur when there is an *increase* in binding energy?

D5 The nuclear equation

$$^{235}_{92}U + ^1_0n \rightarrow ^{146}_{57}La + ^{87}_{35}Br + 3^1_0n + \text{energy}$$

represents a nuclear process which occurs in the core of a nuclear reactor.

(a) What is the name given to this type of nuclear process?
(b) The figure shows the variation with nucleon number (mass number) of binding energy per nucleon.



Use the curve to calculate a value for the energy, which would be available from the fission of a $^{235}_{92}U$ nucleus into a $^{146}_{57}La$ nucleus and a $^{87}_{35}Br$ nucleus.
The power output from a typical nuclear power station is 2000MW. If the efficiency of the power station is 25%, calculate a value for the number of fissions per second of uranium-235 nuclei in order to generate this power.

Radioactivity

D6 Explain how, when making measurements of radioactivity, practical steps can be taken to overcome problems caused by

(i) the random nature of radioactivity,
(ii) background radiation levels

3.36×10^{15} atoms
10^{-6} g
10^{3}
0.0001 kg

D7 The half-life of the polonium-210 is about 140 days. During this period, the average number of α-emissions per day from 1 x 10^{-6} g of polonium is about 12 x 10^{12}. Assuming that one emission takes place per atom and the approximate density of polonium is 10 gcm^{-3}, estimate the number of atoms in 1 cm^3 of polonium.

[3.36×10^{22} atoms]

D8 Carbon-14, with a half-life of 5730y, is produced in the upper atmosphere by cosmic ray bombardment of carbon atoms in carbon dioxide. As a result, every living thing, which is interacting with the carbon cycle in nature has within its carbon atoms a definite proportion of carbon-14, which produces an activity of 0.26Bq per gram of carbon in living materials. When the organism dies, this interaction with the atmosphere ceases and the proportion of carbon-14 decreases. If 1.0g of carbon from an ancient manuscript was found to have an activity of 0.19 Bq, approximately how old is the manuscript?

1.0^{-6} —— 3.36×10^{15}

10 —— $10 \times \frac{3.36\times10^{15}}{10^{-6}}$

2. Nuclides & Isotopes

2.1 Notations

The constituents of the nucleus are called ___nucleons___. These can either be protons or neutrons. A ___nuclide___ is an atom as characterized by its nucleon and proton number. The nuclide notation is shown below:

Nucleon (Mass) Number
No. of Protons + Neutrons → A

$^{A}_{Z}X$ ← **Nuclide Symbol**

Proton (Atomic) Number
No. of Protons → Z

2.2 Isotopes

- The difference between the terms ***element*** and ***nuclide*** is a subtle but important one. Substances which have the same nucleon **and** proton numbers can be said to be the same ___nuclide___. However, those with the same proton number are said to be the same ___element___. In other words, even if the nucleon number is ___different___, it will still be the same element if the proton number is the ___same___.
- **Isotopes** are ___elements___ with the ___same___ **proton number** but ___different___ **nucleon number.** In other words, isotopes have the same number of protons but different number of neutrons. They are chemically indistinguishable since they have the same electronic configuration.
- Examples of isotopes:
  (a) $^{35}_{17}Cl$ (17 protons and 18 neutrons) and $^{37}_{17}Cl$ (17 protons and 20 neutrons) are the common isotopes of chlorine.
  (b) Hydrogen ($^{1}_{1}H$), Deuterium ($^{2}_{1}H$)and tritium ($^{3}_{1}H$) are all isotopes of hydrogen.

Worked Example 1
Chlorine exists in two isotopes of ^{35}Cl and ^{37}Cl having percentage abundance of 75.4% and 24.6% respectively. What is the molar mass of naturally occurring chlorine?

Molar mass of naturally occuring chlorine = $(0.754)(35) + (0.246)(37)$
$= 35.5g$
$= 0.355kg$

This gives the average mass of a chlorine atom as 35.5u. The knowledge of isotope helps us to understand why, unlike many atoms, chlorine does not have an atomic number which is nearly a whole number.

Summary (Nuclides and Isotopes)

Nuclide	Element	Isotope
___same___ nucleon no. and proton no. $^{A}_{Z}X$ $^{A}_{Z}Y$	___same___ proton number only	- same proton no. but different nucleon no. - $^{35}_{17}Cl$ and $^{37}_{17}Cl$ are isotopes of the ___same element, Cl___. $^{A}_{Z}X$ $^{B}_{Z}Y$

1.3 More About The Atom

The mass of the atom is very small. Thus, it is convenient to define a new unit:

Unified Atomic Mass Unit, u
1/12 the mass of the carbon atom.
Thus, mass of a neutral carbon atom = 12 u exactly

And you have learned in Chemistry that:

Mole
Amount of substance which contains as many molecules or atoms (or any other elementary entities) as there are atoms in 0.012kg of Carbon-12.

Avogadro's No., N_A
The number of molecules or atoms in one mole of substance = 6.02×10^{23}

- Since 1 mole is defined as the number of atoms in 0.012 kg (12 g) of carbon-12,

$$1\,u = \frac{0.012}{12 \times 6.02 \times 10^{23}} = 1.66 \times 10^{-27}\ \text{kg}$$

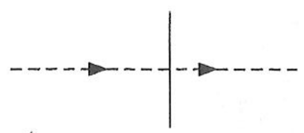
- According to this definition, the proton and neutron each have a mass of approximately 1 u, and the electron has a mass that is only a small fraction of this value:

Mass of proton = 1.007 276 u
Mass of neutron = 1.008 885 u
Mass of electron = 0.000 548 6 u

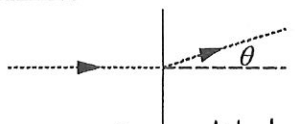
Summary (The Nuclear Atom)

Results of α-particle scattering experiment :

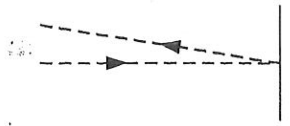
1. majority



2. least probable



3. very rare unlikely



Simple model of an atom

1. A lot of empty space.
2. A positive nucleus made up of proton and neutrons.
3. Electrons which are much less massive than protons and neutrons orbit around the atom.

Assignment Exercise

A1(a) A deuterium reaction that occurs in experimental fusion reactions is

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow X + Y$$

where X is a charged particle and Y is an unknown element.

The process is followed by a reaction of the Y product with another deuterium atom. This second reaction is

$$Y + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$$

Given:

Nuclide	Rest Mass / u
${}^{1}_{1}p$	1.00814
${}^{1}_{0}n$	1.00898
${}^{2}_{1}H$	2.01474

(i) Name any two physical quantities that are conserved in such reactions. [2]
(ii) Deduce X and Y and write down their respective nucleon number and proton number. [3]
(iii) Suggest why such fusion reactions require very high temperatures. [1]
(iv) State what is meant by the binding energy of a nucleus. [1]
(v) Calculate the binding energy per nucleon of the ${}^{2}_{1}H$ nuclide. [2]
(vi) Hence, compare the nuclear stability of ${}^{2}_{1}H$ and ${}^{4}_{2}He$ nuclides in the light of their binding energies per nucleon. [Binding energy of ${}^{4}_{2}He = 4.564 \times 10^{-10}$ J] [4]

(b) ${}^{14}C$ emits β particles and has a half-life of 5600 years. The isotope is made by the interaction of cosmic rays with nitrogen in the upper atmosphere. It is then taken in by living organisms. When they die there is no further exchange of carbon so you can calculate by proportion of undecayed ${}^{14}C$ relative to the more stable ${}^{12}C$ present in the specimen. The ratio of ${}^{14}C$ to ${}^{12}C$ while a specimen is alive is $1:10^{10}$.

(i) Show that the number of ${}^{14}C$ atoms in 12g of carbon from a live specimen is 6.0×10^{13}. [1]
(ii) Hence calculate the activity of ${}^{14}C$ atoms in Bq in this live specimen. [3]
(iii) The activity measured from 12g of carbon from a bone is 30 Bq. Roughly how old is the bone? [3]

A2 Data Analysis (N2000/2/8) Pg 352, Q22

Appendix
Nuclear Fission

In 1932, James Chadwick (1891-1974) discovered a fragment of the nucleus, the neutrons, which has no electrical charge and can thus approach a nucleus without any electrostatic repulsion. It was soon discovered that neutron stick to the nuclei of many atoms.

Nuclear Fission was first observed in 1938, where Uranium (Z = 92) was bombarded with neutrons as follows:

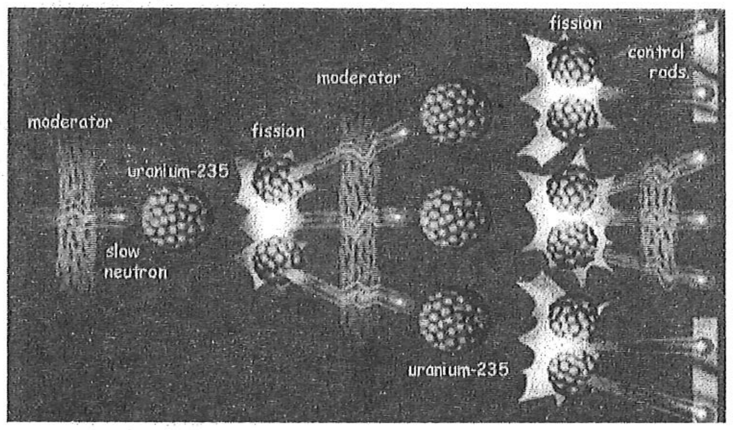
$${}_0^1n + {}_{92}^{235}U \rightarrow {}_{92}^{236}U^* \rightarrow x + y + neutrons$$

where x and y are called fission fragments. In any fission equation, there are many combination of x and y that satisfy the requirements of energy and charge. With Uranium for example, there are about 90 different daughter nuclei that can be formed. (refer to tutorial-Data analysis 2003)

A typical fission reaction is as follows:

$${}_0^1n + {}_{92}^{235}U \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\left({}_0^1n\right)$$

Measurement shows that about 200 MeV of energy was released in each fission event. most goes into the kinetic energy of the heavy fragment of Ba and Kr.
The neutrons created in this process could themselves caused other nuclei to fission, with the possibility of a chain reaction.



The result could be a catastrophic nuclear process in which many or event most of the nuclei in a piece of Uranium would shatter and release fantastic amount of energy.

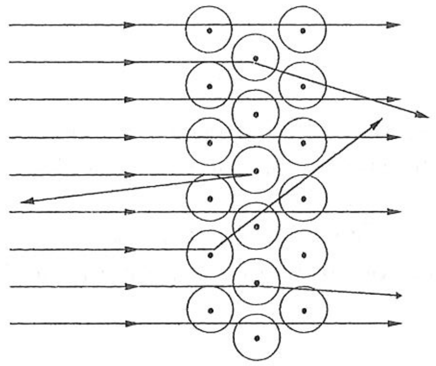
Energy Released by 1kg of Matter

Form of Matter	Process	Time*
Water	A 50 m waterfall	5s
Coal	Burning	8h
Enriched UO_2	Fission in a reactor	690y
^{235}U	Complete Fission	3×10^4 y
Hot Deuterium gas	Complete Fusion	3×10^4 y

* This column shows the time for which the generated energy could power a 100 W bulb.

Interpretations of Results

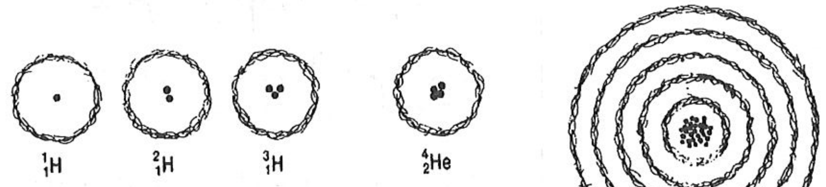
- Result **i** indicated that much of the atom is made up of empty space since the vast majority of the alpha particles managed to pass through without being deflected.
- Result **ii** showed that the α-particles were repelled by some positively charged object in the gold foil.
- Result **iii** revealed there is a very heavy centre core in the atom (nucleus) since the alpha particles must have collided with particles sufficiently massive to bring about their own about-turn negotiations.



1.2 Simple Model For Atom

From the experiment and subsequent discovery of the neucleus, scientists concluded that:

- The atom is largely made up of empty space. The diameter of the atom is about 10^{-10} m whereas the nucleus is about 10^{-15} m.
- The nucleus is made up of positively charged protons and neutral neutrons.
- The nucleus practically contributes to the entire mass of the atom. This is because the masses of protons and neutrons (which are almost the same) are about 1800 times heavier than the other component of the atom : electrons.
- Electrons orbit around the atom. In a neutral atom, the number of electrons is equal to the number of protons.



all these hydrogen nuclides have one orbiting electron

nucleus surrounded by a cloud containing 2 electrons

nucleus surrounded by cloud containing 83 electrons

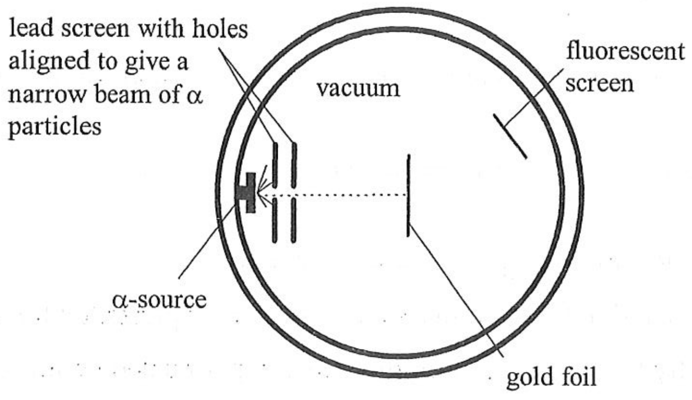
1. The Nuclear Atom

1.1 Alpha Particle Scattering Experiment

In 1911, Rutherford and his students Hans Geiger and Ernst Marsden performed a now classic experiment to demonstrate the existence of the nucleus in the atom.

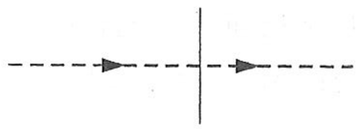
Experimental set-up

- A narrow beam of alpha particles was directed towards a thin layer of gold foil.
- A zinc sulphide screen (fluorescent material) was mounted so that it could be set at different angles to the gold foil.
- When a scintillation was observed in the zinc sulphide screen, it could be deduced that an alpha particle had been deflected from the gold foil to hit the zinc sulphide screen at that particular angle.
- The entire apparatus was set up in a vacuum chamber.



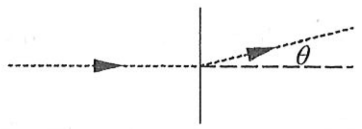
Results

i. Most scintillations were observed when the detector was aligned along the original path of the alpha particles. This indicated that a vast majority of the alpha particles managed to pass through the gold foil without being deflected.



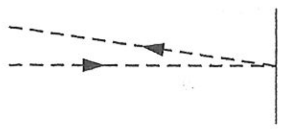
$\theta = 0°$

ii. Fewer scintillations were observed at an angle to the direct path on the same side of the gold foil. (The distribution of α-particles scattered at an angle θ was found to satisfy calculations performed by Ernest Rutherford based on Coulombic repulsion.)



$0° < \theta < 90°$

iii. Very rarely, but nevertheless present, were scintillations observed on the same side as the alpha beam. Some were even detected to reflect back along the path they came from. Scientists compared this result with that of shooting a bullet at a piece of paper and getting the bullet reflected from the paper!



$\theta > 90°$

Nuclear Fusion

The binding energy curve shows that energy can be released if two light nuclei combine to form a single larger nucleus, a process called nuclear fusion. This process is hindered by the coulomb repulsion that acts to prevent the two positively charged particles from getting close enough to be within range of their attractive nuclear forces and thus "fusing". To generate useful amount of energy, nuclear fusion must occur in bulk matter. The best hope for bringing this about is to raise the temperature of the material until the particles have enough energy to overcome the Coulomb barrier.

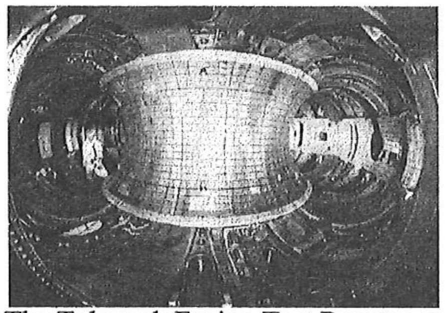
The Sun radiates energy at the rate of 3.9 x 10^{26} W and has been doing so for several billion years. The fusion reaction in the Sun is the process that is keeping the sun burning all these while. This fusion reaction is a multistep process in which hydrogen is burned into helium.

$$^1_1H+^1_1H\rightarrow^2_1H+\beta^++\upsilon$$
$$^1_1H+^2_1H\rightarrow^3_2H+\upsilon$$
$$^1_1H+^3_2H\rightarrow^4_2H+\beta^++\upsilon$$
$$^3_2H+^3_2H\rightarrow^4_2H+^1_1H+^1_1H$$

These are the basic reactions in what is called the proton-proton cycle, believed to be on of the basic cycles by which energy is generated in the Sun and other starts that have an abundance of hydrogen. Most of the energy production takes place in the Sun's interior, where the temperature is approximately 1.5 x 10^7 K. All the p-p cycle are exothermic. An overall view of the p-p cycle is that four proton combine to form an alpha particle and 2 positron (β^+), with the release of 25 MeV of energy.

Unfortunately, p-p interaction is not suitable for use in a fusion reactor on earth because the event requires very high pressures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior. The fusion reactor that appear most promising for a fusion power reactor involve deuterium (^2_1H) and tritium (^3_1H). This can be accomplished by heating the fuel to extremely high temperatures. At this high temperature, the atoms are ionised and the system consists of a collection of electrons and nuclei commonly referred to as a plasma.

A toroidal device called a **tokamak** (figure on the right) uses a combination of two magnetic fields to confine and stabilise the plasma. This prevents the plasma in coming into contact with the vacuum walls where its temperature would be reduced and contamination by impurities from the walls.



The Tokamak Fusion Test Reactor at Princeton University.

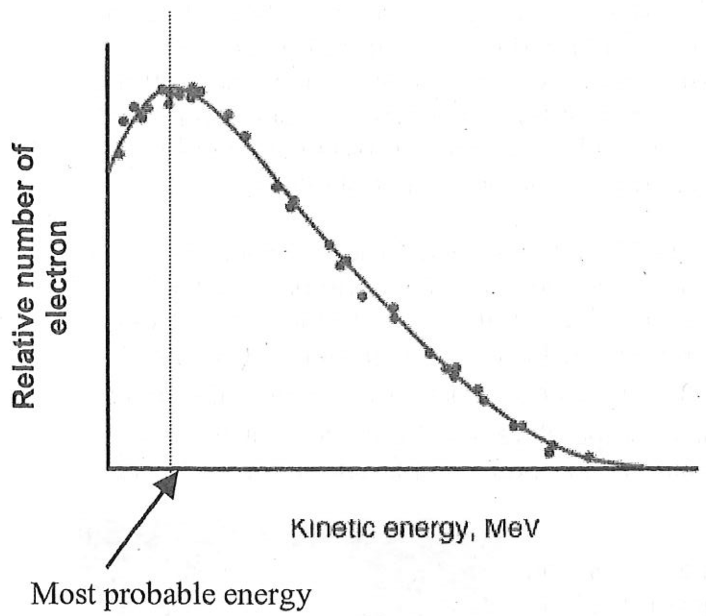
β - decay

The process described earlier in the lecture notes is actually not complete.

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \beta\,({}^{0}_{-1}e)$$

Considering the energy of the system before and after the decay, total energy must be conserved. Experimentally, it is found that the beta particles are emitted over a continuous range of energies. These results show that beta particles having different energies are emitted. The kinetic energy of the particles must be balanced by the decrease in the mass of the system. Since all decaying nuclei have the same initial mass, the law of conservation of energy seems to be violated!
After a great deal of experimental and theoretical study, Wolfgang Pauli suggested the existence of neutrino (little neutral ones) in 1930, which must be present to carry away the "missing" energy and momentum. Thus the above process can be re-written as:

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \beta\,({}^{0}_{-1}e) + \overline{\nu}$$



A typical beta decay curve is shown on the left. In the decay, energy is shared between the electron and the neutrino, in varying proportions. The most probable energy for the emitted electron corresponds to the peak.

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Nuclear Physics

Content

1. The Nuclear Atom
2. Nuclides and Isotopes
3. Mass Defect and Nuclear Binding Energy
4. Nuclear processes
5. Radioactivity

Objectives

(a) Infer from the results of the α-particle scattering experiment the existence and small size of the nucleus.

(b) Describe a simple model for the nuclear atom to include protons, neutrons and orbital electrons.

(c) Distinguish between nucleon number (mass number) and proton number (atomic number).

(d) Understand that an element can exist in various isotopic forms each with a different number of neutrons.

(e) Use the usual notation for the representation of nuclides.

(f) Appreciate the association between mass and energy as represented by $E = mc^2$.

(g) Illustrate graphically the variation of binding energy per nucleon with nucleon number.

(h) Explain the relevance of binding energy per nucleon to nuclear fusion and nuclear fission.

(i) Appreciate that nucleon number, proton number, energy and mass are all conserved in nuclear processes.

(j) Represent simple nuclear reactions by nuclear equations of the form

$$^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

(k) Show an appreciation of the spontaneous and random nature of nuclear decay

(l) Show an understanding of the nature of α, β and γ radiations

(m) Infer the random nature of radioactive decay from the fluctuations in count rate

(n) Define the terms activity and decay constant and recall and solve problems using $A = \lambda N$

(o) Infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_o \exp(-\lambda t)$ where x could represent activity, number of decayed particles and received count rate.

(p) Define half-life

(q) Solve problems using the relation $\lambda = 0.693 / t_{1/2}$