

## Force and Field

Coulomb's Law:  $F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$

$$E = \frac{Q_1}{4\pi\epsilon_0 r^2}$$

This equation is used to find the force between two point charges or between 2 charged spheres.

Electric Field Strength,  $E$  at a point is the electrostatic force acting on unit positive charged placed at that point.

Relation between Force and Electric Field Strength:  $F = qE$

Resultant electric field strength at a point or resultant force acting on a charge due to multiple source charges can be found by vector addition

## Potential and Potential Energy

Electric Potential,  $V$  at a point in an electric field is the work done by an external agent in bringing a unit positive charge from infinity to that point.

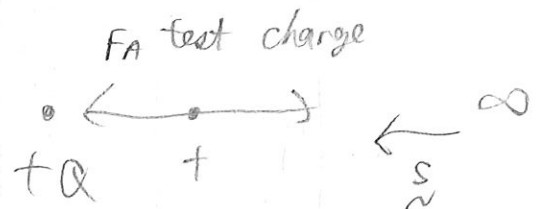
If a charge  $q$  is placed at that point, it will have an electric potential energy of  $qV$

For positive source charge (+Q):  $V = \frac{+Q}{4\pi\epsilon_0 r}$  (1)

For negative source charge (-Q):  $V = \frac{-Q}{4\pi\epsilon_0 r}$  (2)

Equations (1) & (2) are valid only for:

- (i) Point charges or
- (ii) Charged spheres where  $r > R$



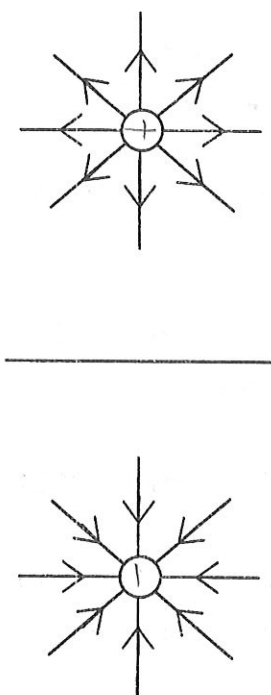
$$E.P.E = W = F_A \cdot s = +ve$$

$\therefore F_A$  and  $s$  are pts in same direction

## Electric Field

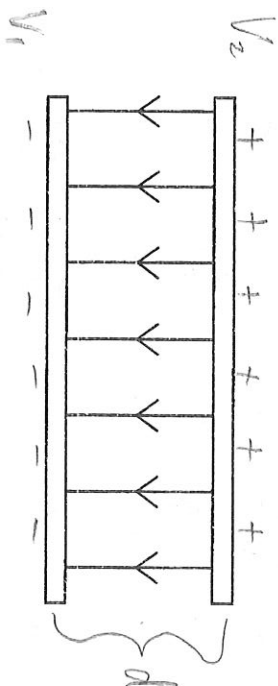
### Radial Field

- (1) Point Charge
- (2) Charged Sphere



### Uniform Field

- (1) Between 2 parallel metal plates that carry opposite charges.



Direction of the arrow show the direction that a unit positive test charge would move if place at a point in the field.

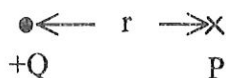
$$\text{Field Strength} = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\text{Field Strength} = \frac{V_2 - V_1}{d}$$

## Charge and Field Summary

### A. Radial Field – (due to a single point charge or a sphere)

1. Drawing the field lines associated with a point charge (pg 2).  
What is the significance of the arrows along the field lines?  
(a) Direction of the force experienced by a test charge (positive).  
(b) Points from higher potential to lower potential.
2. Drawing the equipotential lines due to a point charge (pg 13).
3. **Electric field strength** due to charge +Q at a point P.



$$|E| = \frac{Q}{4\pi\epsilon_0 r^2}$$

r is the distance measured from the centre of the charge.

4. **Electric potential** due to charge Q at point P:  $V = \frac{+Q}{4\pi\epsilon_0 r}$

Why does the electric potential carries a positive sign when the source charge is positive (i.e. +Q)?

Because to bring a unit (+ve) charge towards +Q, work must be done by the external agent on the unit charge in order to overcome the mutual repulsion between the two charges. When work is done on the charge, it gains potential energy. At infinity,  $V = 0$  (by definition). As the unit (+ve) charge gets closer to +Q, its potential energy increases. Hence V carries a positive sign (i.e.  $V \geq 0$ ).

5. Relation between field strength, E and potential, V:  $|E| = \left| \frac{dV}{dr} \right|$

6. If a negative charge  $-q$  is placed at point P:

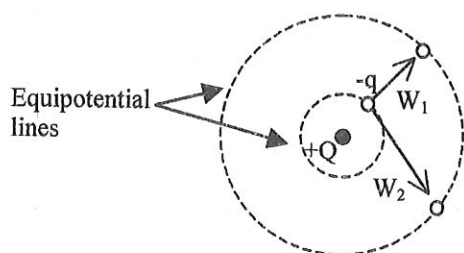
(a) Force exerted by +Q on -q:  $|F| = |qE| = \frac{qQ}{4\pi\epsilon_0 r^2}$

(b) Electric potential energy of  $-q$ :  $E_p = -qV$

Why is  $E_p$  negative? When  $-q$  is infinitely far away from +Q, the potential energy of  $-q$  is zero (by definition). The mutual attraction will bring  $-q$  towards +Q. Work is done by the electric force. Hence the  $-ve$  charge loses potential energy (i.e.  $E_p \leq 0$ ). Hence  $E_p$  carries a negative sign.

7. Sketching graphs of  $|E|$  versus r and V versus r. pg 14.

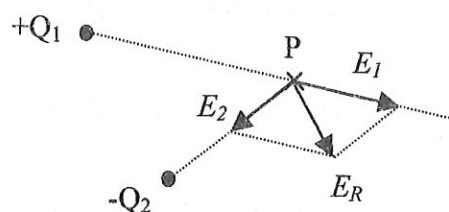
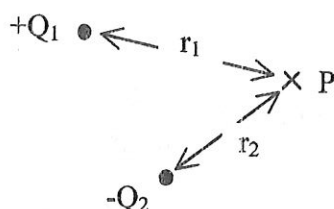
8. Work done in bringing a charge (e.g.  $-q$ ) from one point to another = change in electric potential energy = Final  $E_p$  – Initial  $E_p$ .



$$W_1 = W_2 = -qV_f - (-q)V_i \\ = (-q)\Delta V$$

## B. Two or more point charges / spheres

1. Finding resultant electric field strength due to 2 or more source charges.



- Field strength due to  $+Q_1$  at point P:  $|E_1| = \frac{Q_1}{4\pi\epsilon_0 r_1^2}$
- Field strength due to  $-Q_2$  at point P:  $|E_2| = \frac{Q_2}{4\pi\epsilon_0 r_2^2}$
- Draw the directions of  $E_1$  and  $E_2$  on a vector diagram.
- Perform **vector addition** (pay particular attention to the direction of the arrows for  $E_1$  and  $E_2$ ).
- Practice 1** of tutorial.

2. Finding the total electric potential due to 2 or more source charges.

- Potential due to  $+Q_1$  at point P:  $V_1 = \frac{+Q_1}{4\pi\epsilon_0 r_1}$
- Potential due to  $-Q_2$  at point P:  $V_2 = \frac{-Q_2}{4\pi\epsilon_0 r_2}$
- Total potential =  $V_1 + V_2$  (**scalar addition**)
- Practice 2** of tutorial.

3. Sketching field lines due to 2 point charges (pg 2).
  - (a) Like charges of equal magnitude.
  - (b) Like charges of different magnitude.
  - (c) Unlike charges of equal magnitude.
  - (d) Unlike charges of different magnitude.
  - (e) Remember to draw more field lines for the bigger charge.
4. Sketching E-r and V-r graphs for two point charges. (Practice 3 of tutorial)

**C. Uniform Field (applicable to E-fields between parallel plates).**

1. Drawing field lines (Practice 3 of tutorial)
  - (a) Arrow points from positive plate to negative plate.
  - (b) Lines parallel and evenly spaced.
  - (c) Strong field  $\Rightarrow$  lines are more closely spaced together.
2. Sketching equipotential lines (pg 13).
  - (a) Equipotential lines must be perpendicular to field lines.
  - (b) To find the values of potential at different points within a uniform field, use  $|E| = \frac{\Delta V}{d}$  (refer to **topical quiz 3 Question 1**)
  - (c) Note that in a uniform E-field,  $|E|$  is the same everywhere but V depends on the location within the E-field.
3. For uniform field,  $E$  is constant in magnitude and direction everywhere.
4. Force experienced by a charged particle ( $-q$ ) in a uniform field:
  - (a) Magnitude of  $F$ :  $|F| = |qE|$
  - (b) Direction of  $F$ : Particle will be pulled towards the plate carrying the opposite charge.
5. Work done (by external agent) in moving a charge particle ( $-q$ ) between two points in a uniform field that has a potential difference or p.d. of  $\Delta V$  volts.
 
$$\begin{aligned}
 \text{Work done} &= \text{Change in electric potential energy.} \\
 &= (-q)V_f - (-q)V_i \\
 &= (-q)(V_f - V_i) \\
 &= (-q) \Delta V
 \end{aligned}$$
6. Sketching E-x and V-x graphs for parallel plates. **Practice 3** of tutorial
7. Motion of charged particles in uniform electric field: If particle is moving perpendicular to the direction of the E-field: the path will be parabolic i.e. similar to projectile motion. Use equations of motion under constant acceleration to solve problem. **Practice 4 & 5. Tutorial questions D6 and D7.**
8. Millikan's Oil drop experiment. **Pg 22 to 24, Practice 6, 7 and D8.**

#### D. Magnetic Field

1. Drawing magnetic lines of force. **Pg 25.**
2. Magnetic force on a moving charge:  $|F| = Bqv \sin \theta$  **(Pg 26).**
3. Direction of force: Left Hand Rule.
  - a. Note that Second finger represents the direction of the current (= flow of positive charges).
4. Path of Charged particles in uniform B-field. **Pg 27 to 29, pg 33.**
  - a.  $\theta = 0^\circ$ : Path is a straight line because  $F_B = 0$ .
  - b.  $\theta = 90^\circ$ : Path is a circle.  $F_B = Bqv$
  - c.  $0^\circ < \theta < 90^\circ$ : Path is helical.

#### E. Cross Field

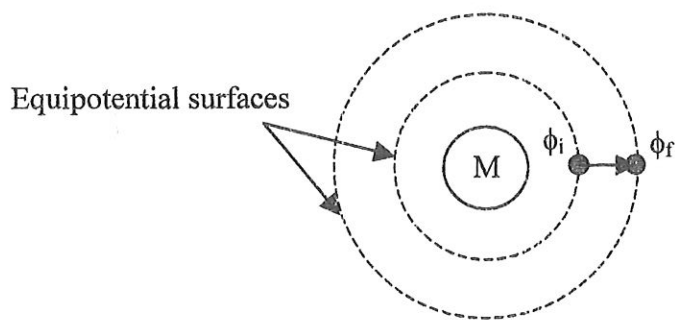
1. Refer to tutorial question **Practice 9** and **D11**.

#### F. Definitions – Need to remember the following definitions:

1. Electric field strength.
2. Electric potential.

#### G. Gravitational Field

1. Newton's Law of universal gravitation:  $F = \frac{GMm}{r^2}$ 
  - (a)  $r$  represents the centre to centre separation.
2. Gravitational field strength:  $g = \frac{F}{m} = \frac{GM}{r^2}$ 
  - (a)  $M$  represents the mass of the source.
  - (b) At the surface of Earth,  $g = 9.81 \text{ Nkg}^{-1}$ .
3. Gravitational potential at a point  $r$  away from the source  $M$ :  $\phi = -\frac{GM}{r}$
4. If a mass  $m$  is placed at that point, it will have G.P.E. =  $m\phi$



5. Work done in bringing a mass,  $m$  from one point to the other:

$$\begin{aligned} W &= \Delta E_p = m(\phi_f - \phi_i) \\ &= m(\Delta\phi) \end{aligned}$$

6. Escape velocity. Use conservation of energy.

$$(ke + gpe) \text{ at surface of planet} = (ke + gpe) \text{ at infinity}$$

$$\frac{1}{2}mu^2 - \frac{GMm}{r} = 0 + 0$$

7. **Satellite Orbits (Circular)**

- (a) Force exerted by Earth ( $M$ ) on satellite ( $m$ ):

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \text{ (circular motion)} \quad \text{-----} \quad (1)$$

OR  $\frac{GMm}{r^2} = mr\omega^2 \quad \text{-----} \quad (2)$

- (b) Equation (2) is used to find the **period T** of the satellite and to prove **Kepler's 3<sup>rd</sup> law** (i.e.  $T^2 \propto r^3$ )

8. Definitions:

- (a) Gravitational field strength.
- (b) Gravitational potential.
- (c) Geosynchronous orbit.

